UNIVERSITÀ DEGLI STUDI DI PADOVA
FACOLTÀ DI INGEGNERIA
Master degree in Ingegneria dell'energia elettrica

Computer Assisted Electromagnetic Design

Exam report of:
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Contents

All codes file, both Matalb and Comsol, are attached to this report. They are organized in folders divided by test cases. Graphs are reported for give an idea of the fields behavior and other dimensions trends. They can be better visualize running the scripts provided and explore them with the Matlab plot designer.

Test Case 1 - 1D FEM

1.1 Assignements

The first test case is a simple application of FEM method in a one dimensional case. It was considered a capacitor with different potential plates. The distance between plates is equal to L and they are infinite extended along x and y axis. It's been studied both homogeneous and inhomogenous media in L.

1.2 Results

These plots are the result of post processing and represent the electric potential and the electric field E inside the capacitor. The computation is done with 8 discretization elements. A comparison between the computed solution with non linear media and the analytical one is proposed.

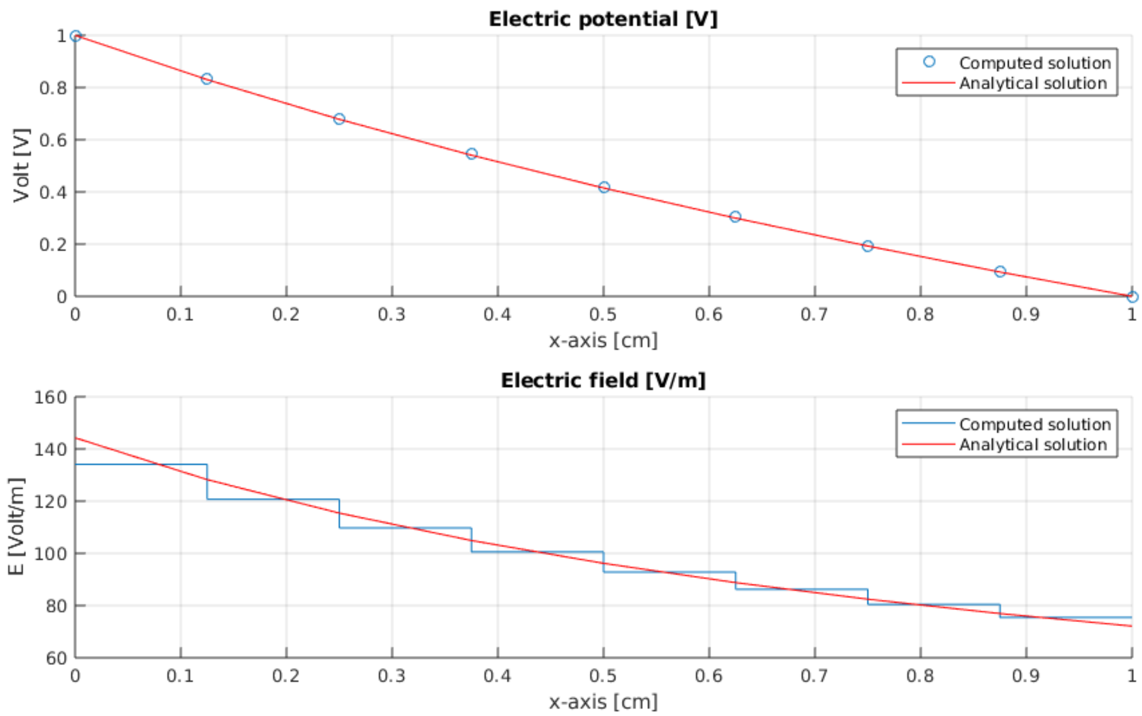

Figure 1.1: Computed and analytical solution comparison

The potential computation is very accurate and not depend on the number of discretization step, while the electric field is strongly dependent by discretization step as it can see by the second plot with increasing numbers of the mesh elements.

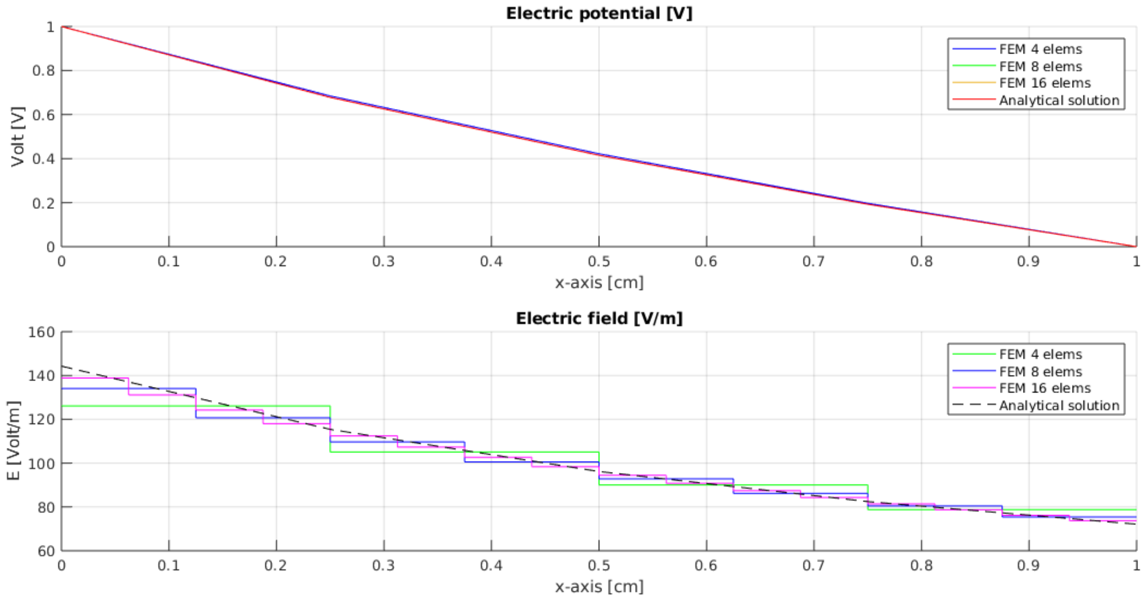

Figure 1.2: Refining mesh

Test Case 2 - 2D FEM

2.1 Assignements

The assignements of this test case consist in two classical benchmarks Poisson problems to solve with two dimensional FEM: a bump and an oscillating function. For each Poisson case is provided the function $f(x, y)$ and the potential $u(x, y)$ in a square domain. Dirichlet boundary condition is applied on all boundary.

2.2 Results

These graphs rapresent the results for a computation of both function with a mesh of 10000 nodes. The third subplot is a coarse mode to rapresent the computation error using the computed and the analytical solution, obtained through a simple difference.

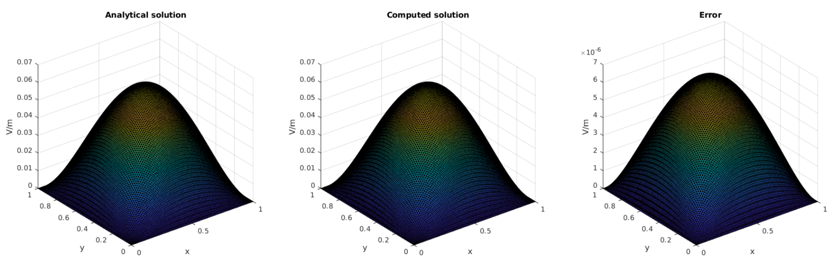

Figure 2.1: Bump function solution comparison

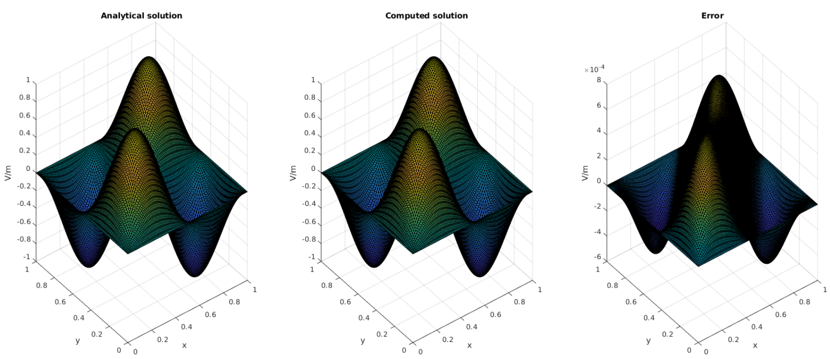

Figure 2.2: Oscillating function solution comparison

Computing the gradient of potential you can easly obtain the electric field distribution: $E = \nabla U$.

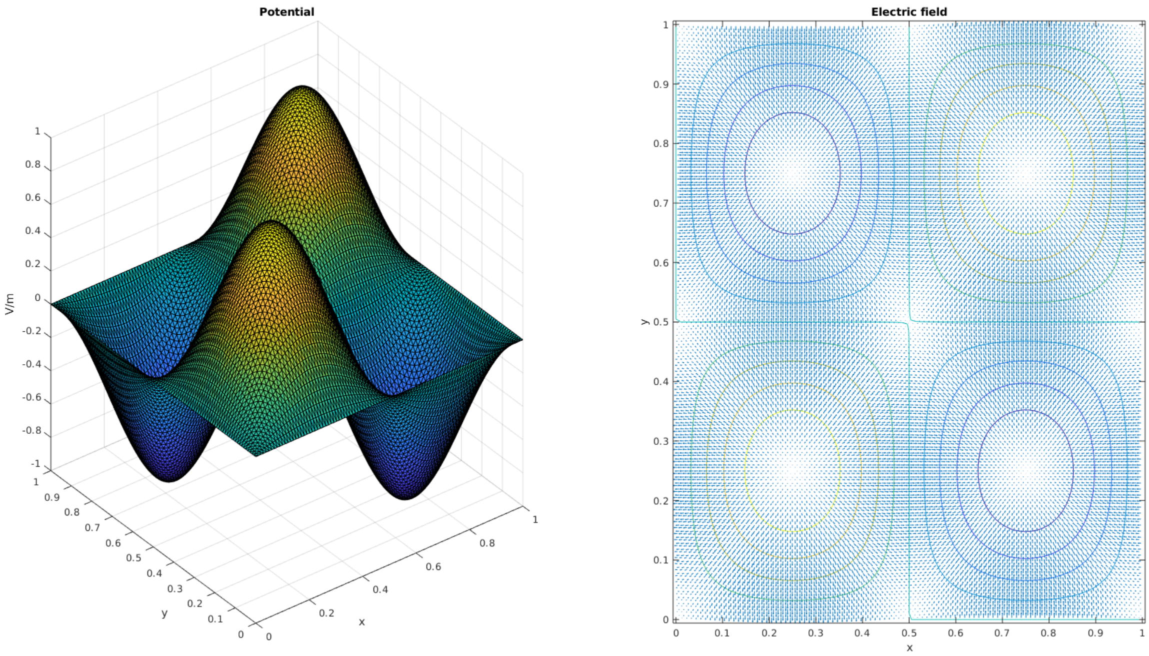

Figure 2.3: Potential and E field oscillating function

2.2.1 Computing Time

In order to reduce the computing time two solutions are adopted: fast assembly of matrices and free variables Dirichlet's boundary condition computing. The first approach use a vector combination of sparse matrix, that also allow to reduce the memory occurrence. Opposite you can use a for loop based method, this solution could be more simple to implement but is very time expensive (remember that Matalb is enemy of for loop). The free variables method for BC's condition allow to compute only in the not imposed potential nodes, thus reduce the computation time drastically.

An example to compare the two solution with 10000 nodes:

- Computed time of global matrices with for loop method: 2.697 sec
- Computed time of global matrices with fast method: 1.072 sec

The following graph shows the position of non zero element in the LHS matrix, as you expect they are around the main diagonal.

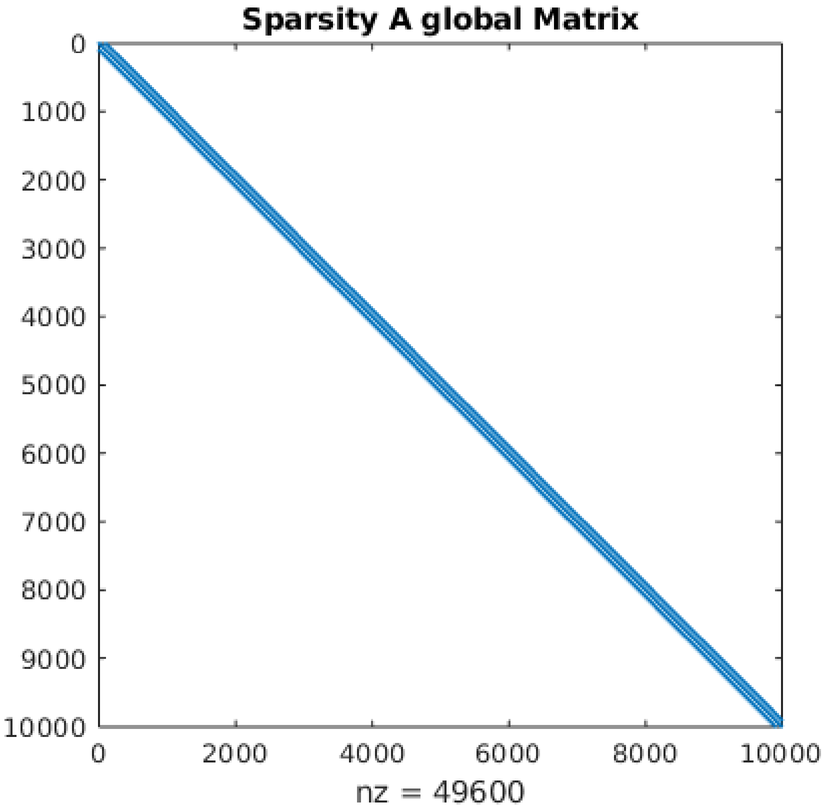

Figure 2.4: LHS sparsity

Test Case 3 - Electrostatic

3.1 Assignements

This test case simulate a parallel-plate capacitor with different permittivity media. It's composed by two conductive parallel electrodes infinite along z dimension, fed by two opposite potentials $+V$ and $-V$. Two cases are proposed: one with a relative permittivity of the internal medium $\varepsilon_r = 4\varepsilon_0$ and another with $\varepsilon_r = 16\varepsilon_0$.
Also in post processing phase computation of both electrostatic energy and capacitance is required.

3.2 Results

These two plots shown the typical behavoir of electric potential and field, caused by field displacement. Infact the equipotentials line are removed from the higher permittivity medium and the electric field is stronger near the edge of square shape. It is easy to notice from the two graphs below that the effect is much more intense with the higher permettivity value.

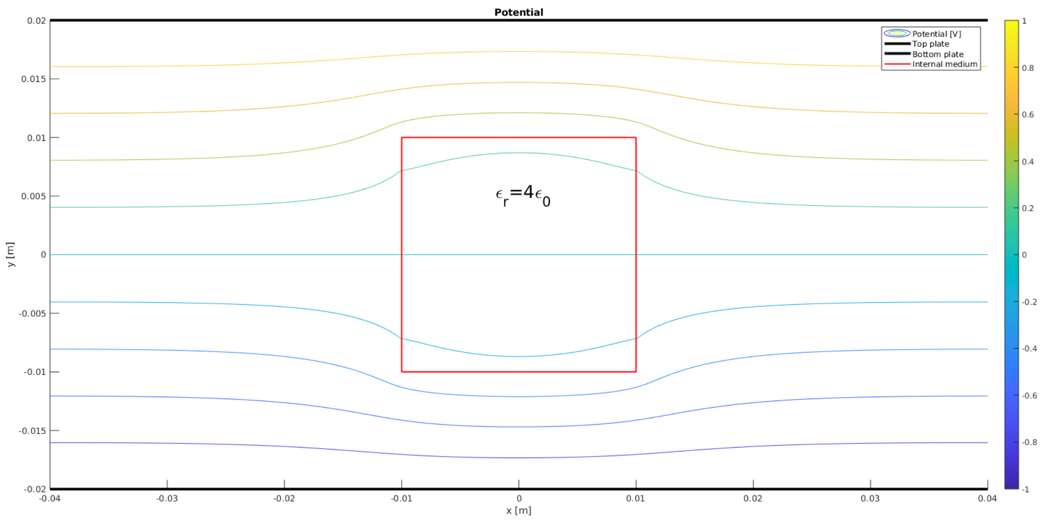

Figure 3.1: Electric potential distribution with $\varepsilon_r = 4\varepsilon_0$

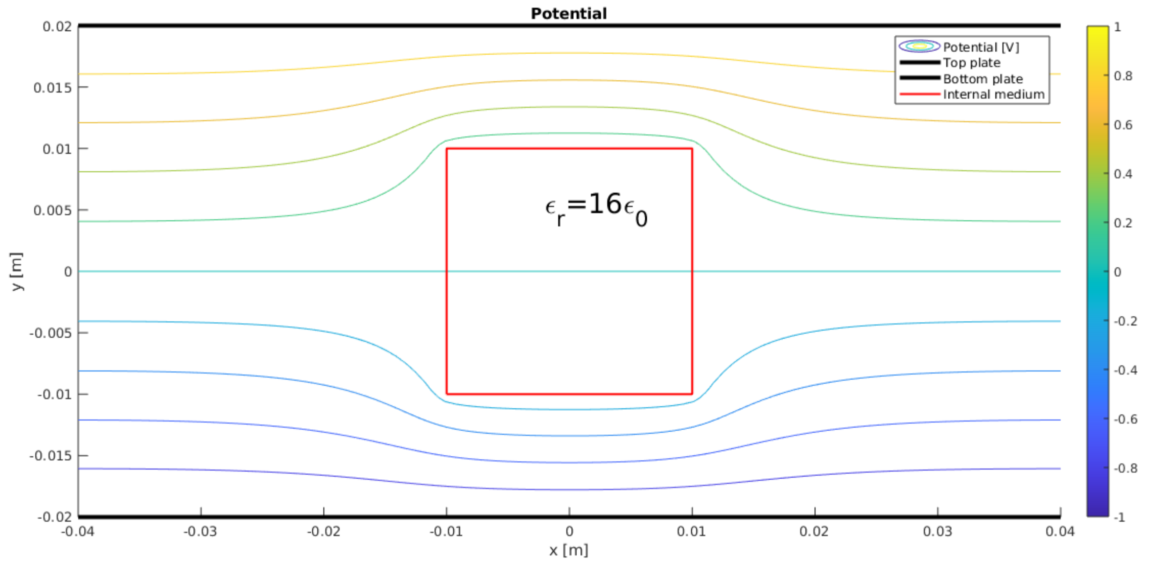

Figure 3.2: Electric potential distribution with $\varepsilon_r = 16\varepsilon_0$

With some simple operations of post processing it can be obtain the electric field E in the capacitor.

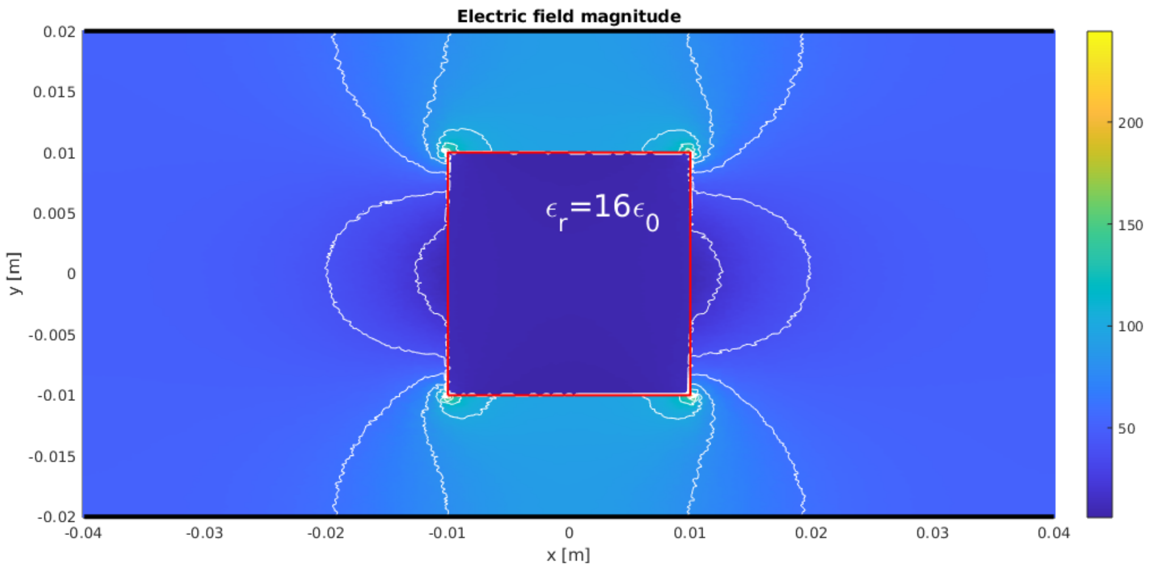

Figure 3.3: Electric field distribution

The value of electrostatic energy and capacitance for the case with $\varepsilon_r = 16\varepsilon_0$ computed with Matlab script are:

- Capacitance: 2.317381e-11 [F]
- Electrostatic energy: 4.634762e-11 [J]

The same values computed with Comosol software:

- Capacitance: 2.3420e-11 [F]
- Electrostatic energy: 4.6840-11 [J]

Refining the mesh size the values tends to converge to the exactly valor. I'm not convinced by his swinging behavior. It deserves a closer look. For example, one could expect an asymptotic trend towards the exact value, without damped oscillations in its surroundings.

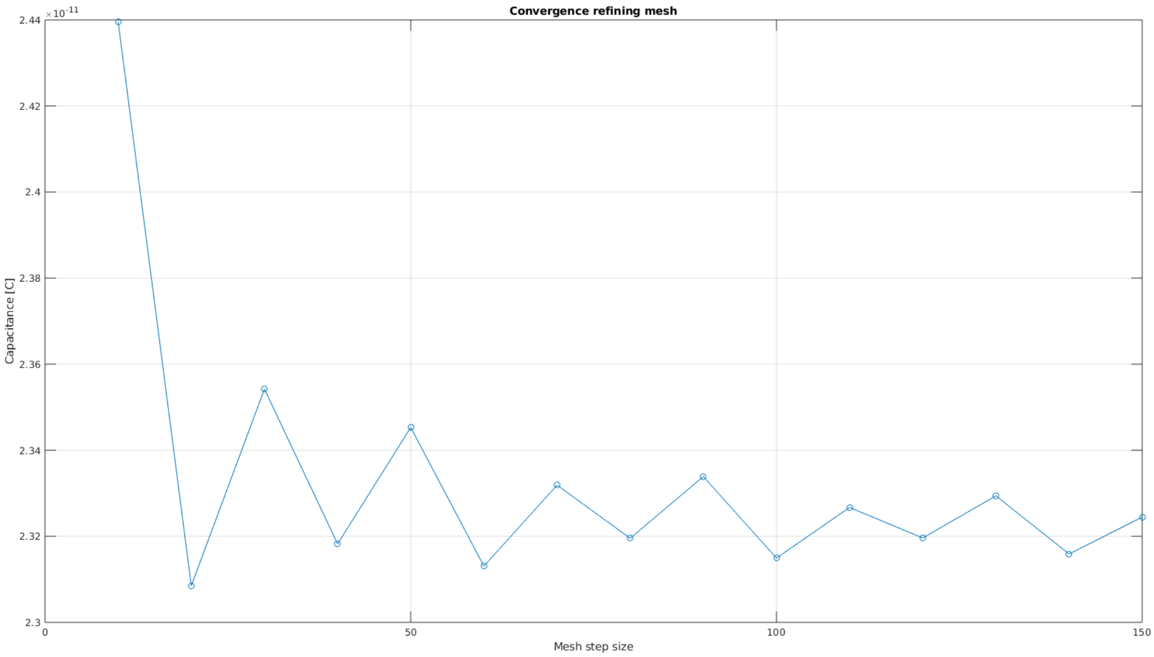

Figure 3.4: Convergence of capacity refining mesh

Test Case 4 - Magnetostatic

4.1 Assignements

A clapper solenoid actuator is studied in this homework. It's composed by three different parts an armature, a stator and a coil. It's studied exploiting symmetry in cylindrical coordinate and closing it in a air bounding box. Each part have specific parameter considering homogeneous and isotropic materials. The actuator is excited by a coil with an impressed current of 1 A and 2000 coils. The goal is studied the force in the air gap posed between armature and stator with different methods: analytically with the reluctance and Maxwell stress tensor with a FEM software.

4.2 Results

From these plots you can see that the magnetic flux density B is stronger near the coil edges (the source of the system). While the magnetic vector potential A', is also stronger near the coil and well distributed in all stator, reclosing in the armature.

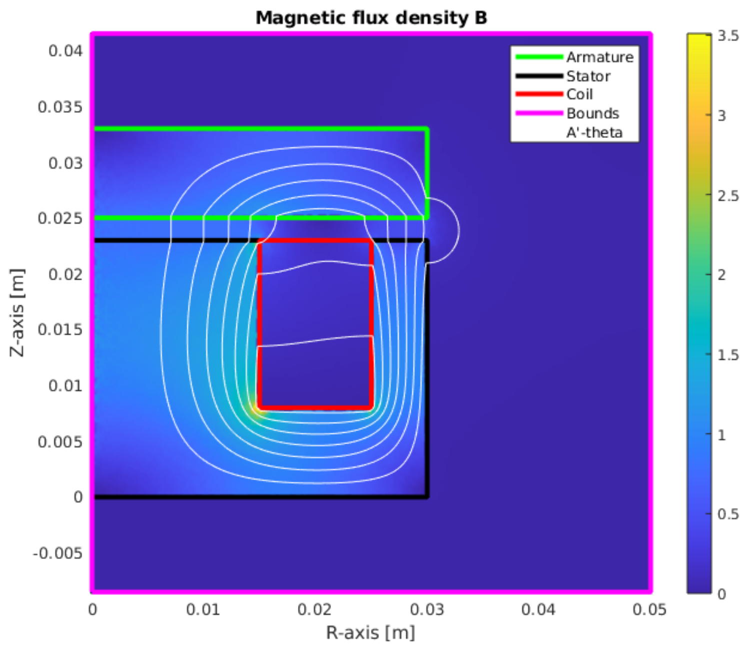

Figure 4.1: Magnetic flux density B distribution

Dividing the components of B in the air gap, it's easy to note that the r component is almost zero and the field in only along z component. All the Matlab solution are compared with Comsol results.

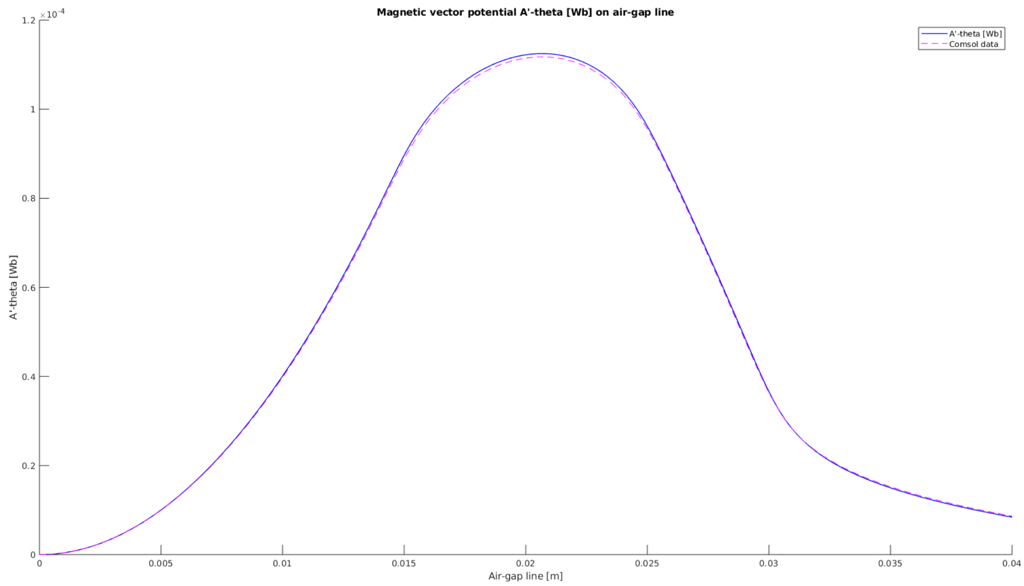

Figure 4.2: Magnetic vector potential in the air gap

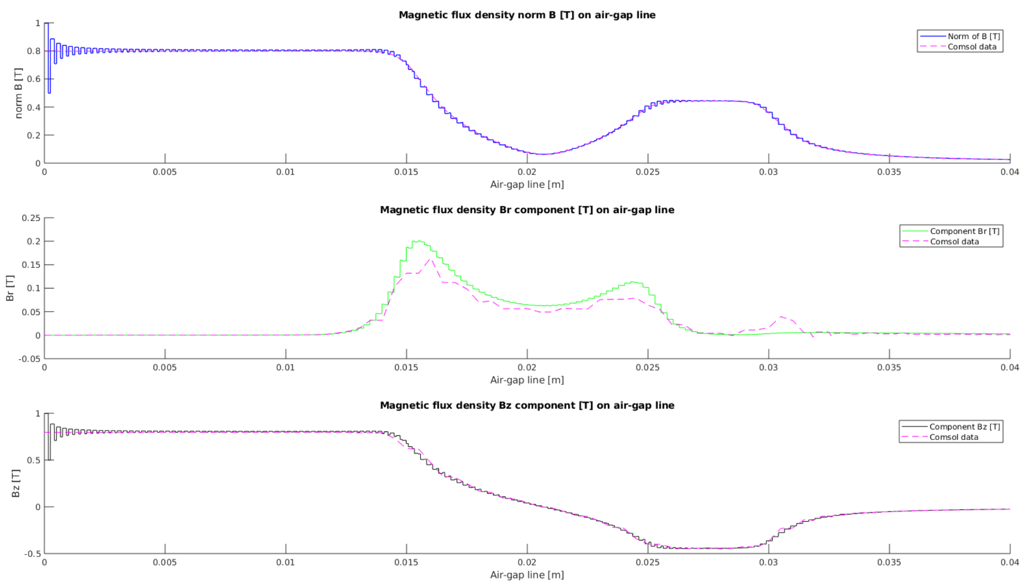

Figure 4.3: Magnetic flux B components in the air gap

The force values computed with the two different methods are:

- Reluctance method: inner z-axis=-131.992 N and outer z-axis=-107.994 N
- Maxwell stress tensor (Comsol FEM software): 277.35 N

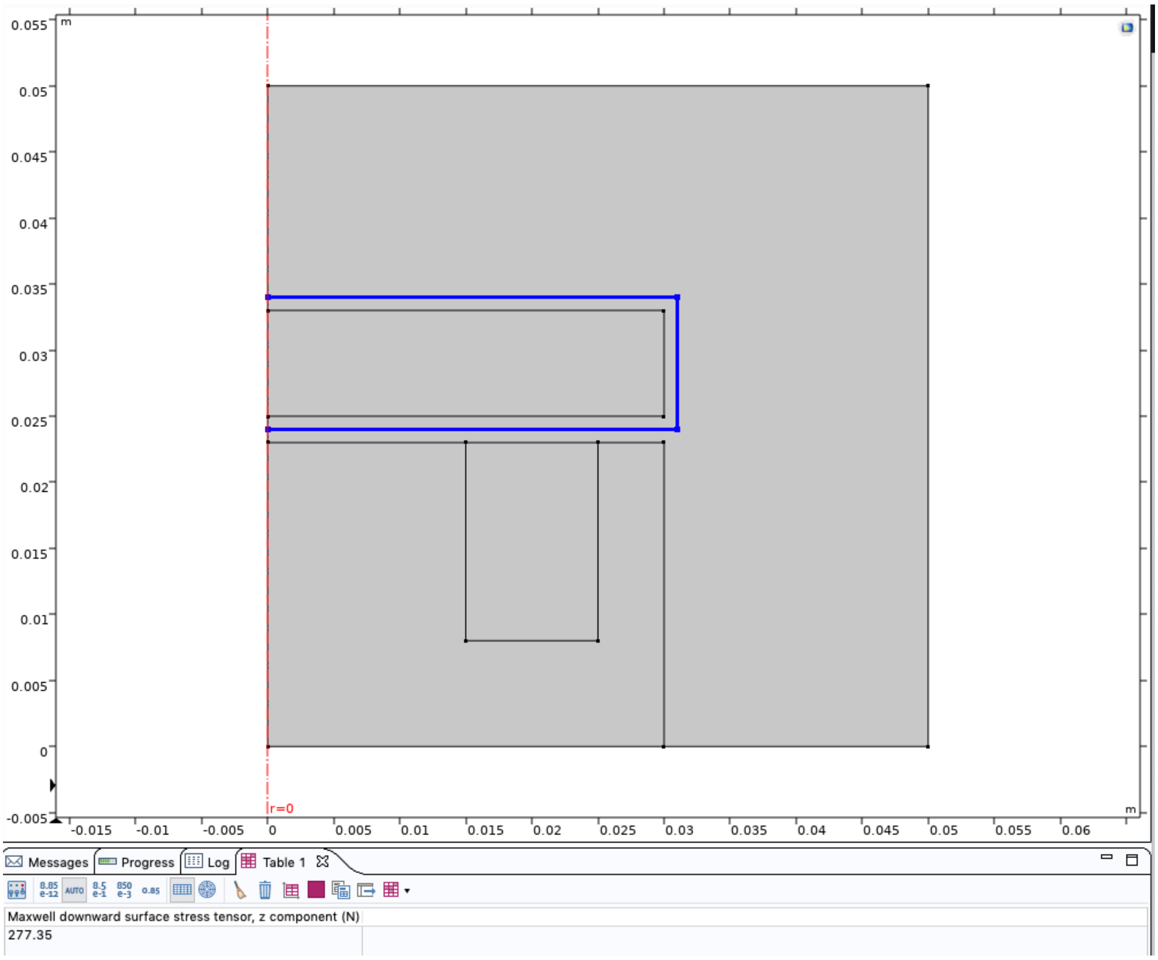

Figure 4.4: Comsol Maxwell stress tensor

Test Case 5 - Magnetodynamic

5.1 Assignements

For the magneto-dynamic problems has been analyzed a cylindrical inductor in two dimensional axial symmetry. Is provided a frequency of $100Hz$, in order to evaluate the time variation. The pre processing steps are the same for the other test cases, in addition it's only need compute a new LHS vector composed by the sum of stiffness matrix a new complex matrix, called mass matrix, that allow to consider the time derivation in the FEM computation.
The targets are analyze the trend of the magnetic vector potential and magnetic flux density components in a line located in the middle of the solenoid. Also is required to computed the Joule losses inside the core.

5.2 Results

You must take into account the imaginary part of the field, as showned in the following graphs (in order: the magnitude, the real part and the imaginary part).

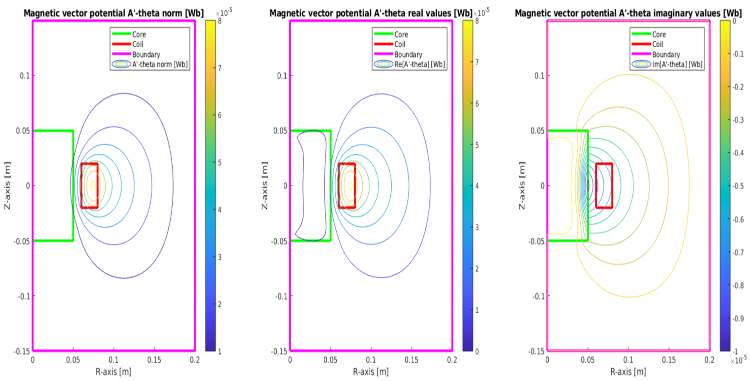

Figure 5.1: Magnetic vector potential A'_θ distribution

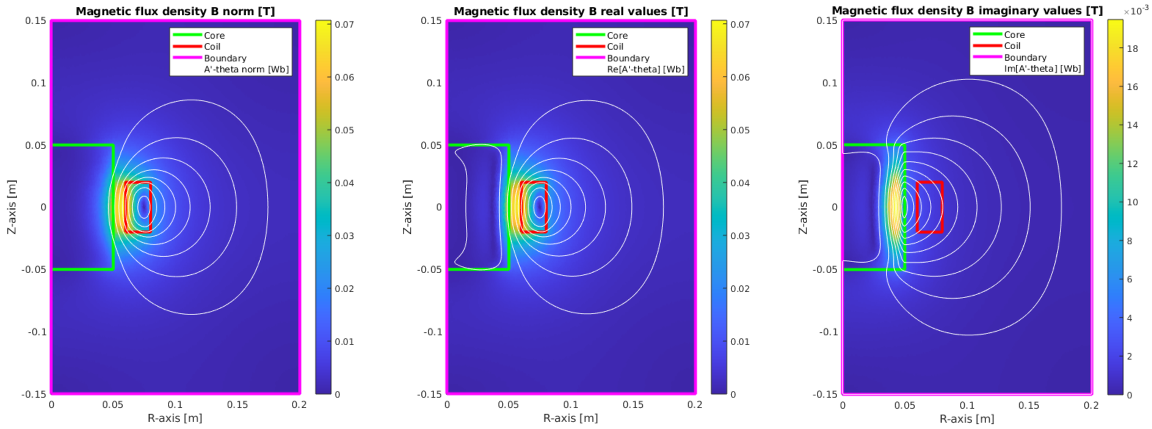

Figure 5.2: Magnetic flux density B distribution

Consider a line 5cm long in the middle of the core you can study the behavoir of the fields. In order to do a complete and accurate analisys of the field distribution in the inductor is necessary evaluate both real and imaginary part of each component. All results are confirmed with Comsol.

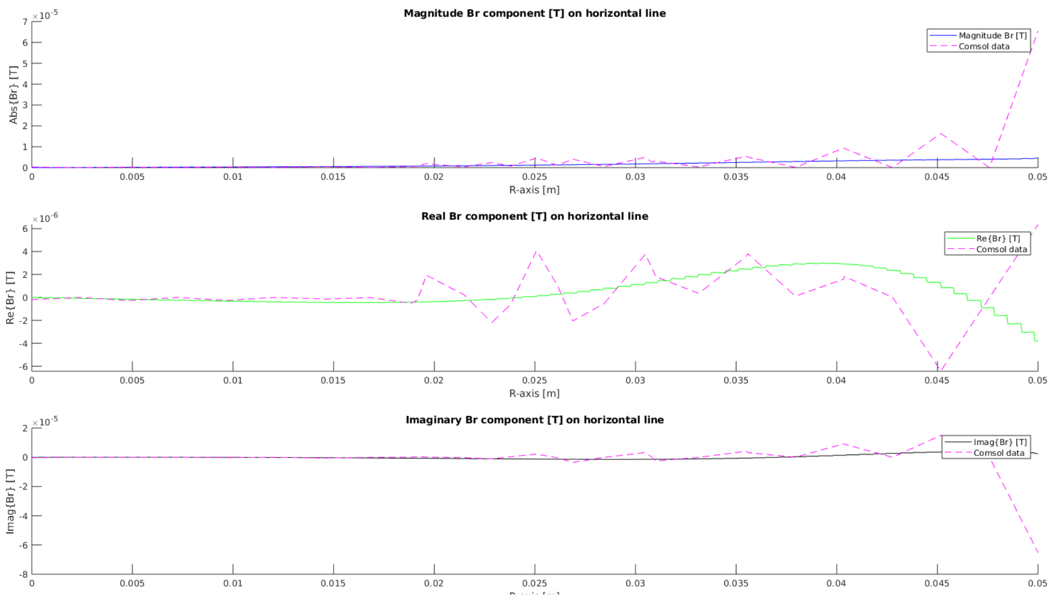

Figure 5.3: Magnetic flux density Br component in the air gap

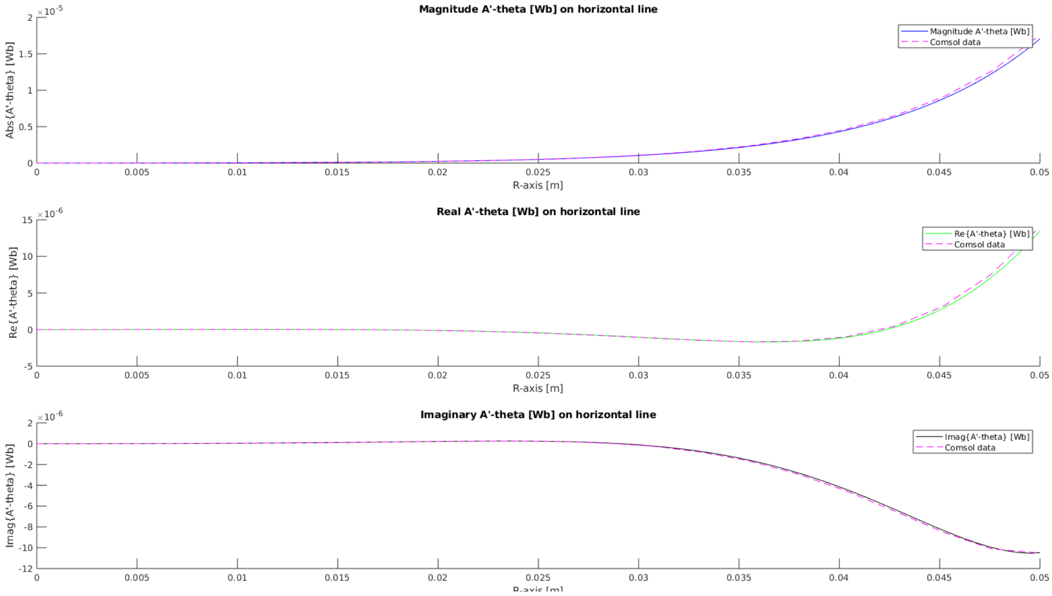

Figure 5.4: Magnetic flux density Bz component in the air gap

The Joule losses in the core computed with Matlab FEM code are 53.228 W. This value is confirmed by Comsol computation.

Test Case 6 - Heat transfer

6.1 Assignements

Another typical real problem solvable with FEM model is the heat transfer. In fact the physic can be modeled like a PDE equation in order to simulate it like a electromagnetic situation. To exercise has been studied an aluminum billet in two different regime: steady-state and transient. The first is useful to understand the behavior of this phenomena, while the second to consider the time variation evaluated with theta method. Convective BC's are applied on the boundary, axis-symmetry excluded.

6.2 Results

As the magnetic field it can plot the equipotential lines of T. All results (both steady state and transient) are compared with the Comsol ones.

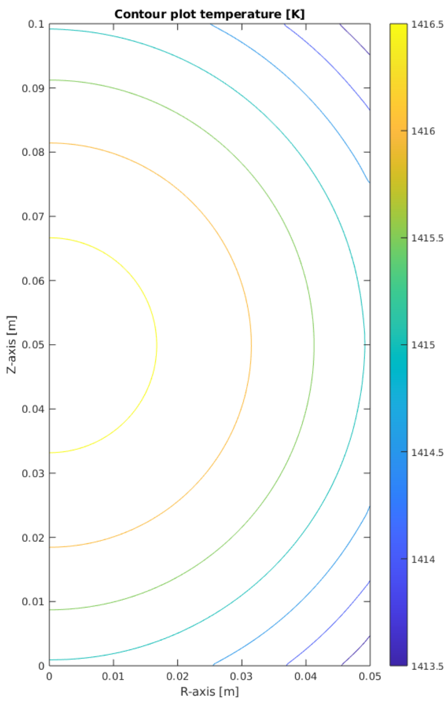

Figure 6.1: Temperature distribution

Consider a line 5 cm long in the middle of the billet you can study the behavoir of the temperature that increase approacching to the middle of the billet.

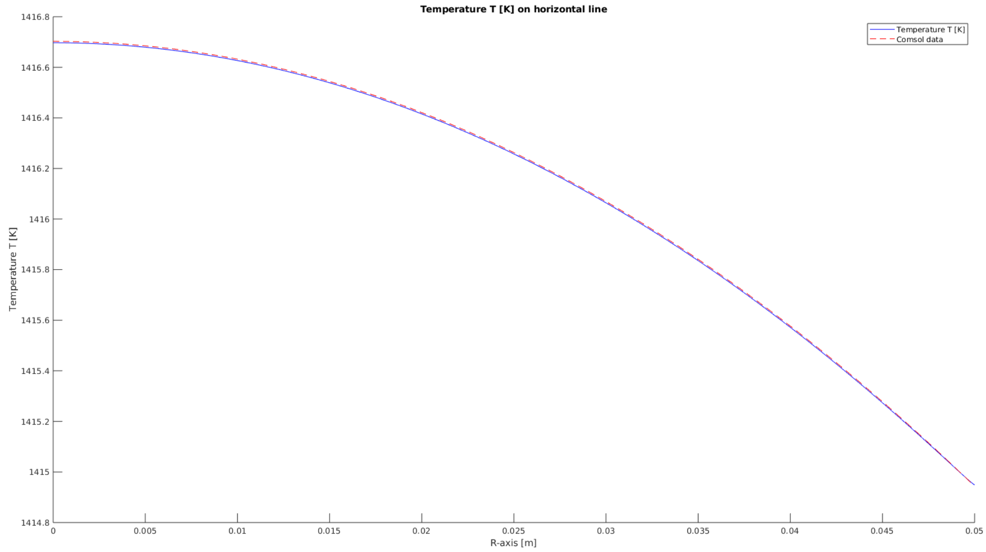

Figure 6.2: Temperature along core line

In the transient case the billet use about 8τ before arrive to regime.

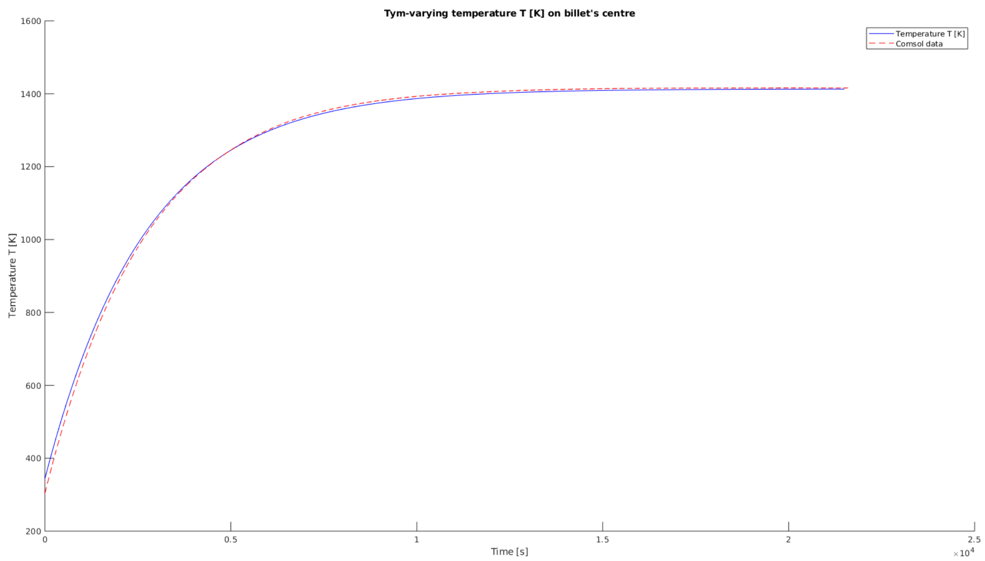

Figure 6.3: Temperature in time

Test Case 7 - Coupled problem

7.1 Assignements

Coupled problems are very frequent in reality. Their numerical simulation require a particular attention both to the study of their iteration each other and more computational cost required. In this test case a billet heated by AC current coil is proposed. The goal is to computed the temperature T at the center of the billet after 1000 seconds. Also the Joule net power losses in are required to compute.

7.2 Results

I tried to solve this optional test case joined the last two ones: the magnetodynamics and the transient thermal one. Infact they could be consider like two different problems in casacade. Convective BC for thermal problem and Dirichlet ones for electrical was applied on the relative domains. I think to have correctly develop the first steps: wrote the functions to compute the Joule power density and the updating of conductibility σ for solving the electrical problem. But I founded some difficulties with the mesh of the two problems that are refereed to different subdomain and during coding have a mismatching of nodes references. Theta method, for evaluating the transient, was implemented, but the the code don't reach a solution because of a different size matrices in solving procedure.

All the developing codes are attached.

Test Case 8 - Biot-Savart's integral

8.1 Assignements

Integral methods are another type of computational method to simulate real physic phenomena. They require more memory becouse can't use sparse matrix, but they allow to discretize only the sources. Avoid to consider large air domain like FEM. The real application is for example in a balanced three phases bus bar.

8.2 Results

The parabolic curve is the typical trend of B for this situation, but it could find some differences depending on the approach used. As usual all results are compared with Comsol software ones. Magnetic flux density B in a misure line (distance 50cm from cables) is computed for two situations: finite and infinite bus bar length.

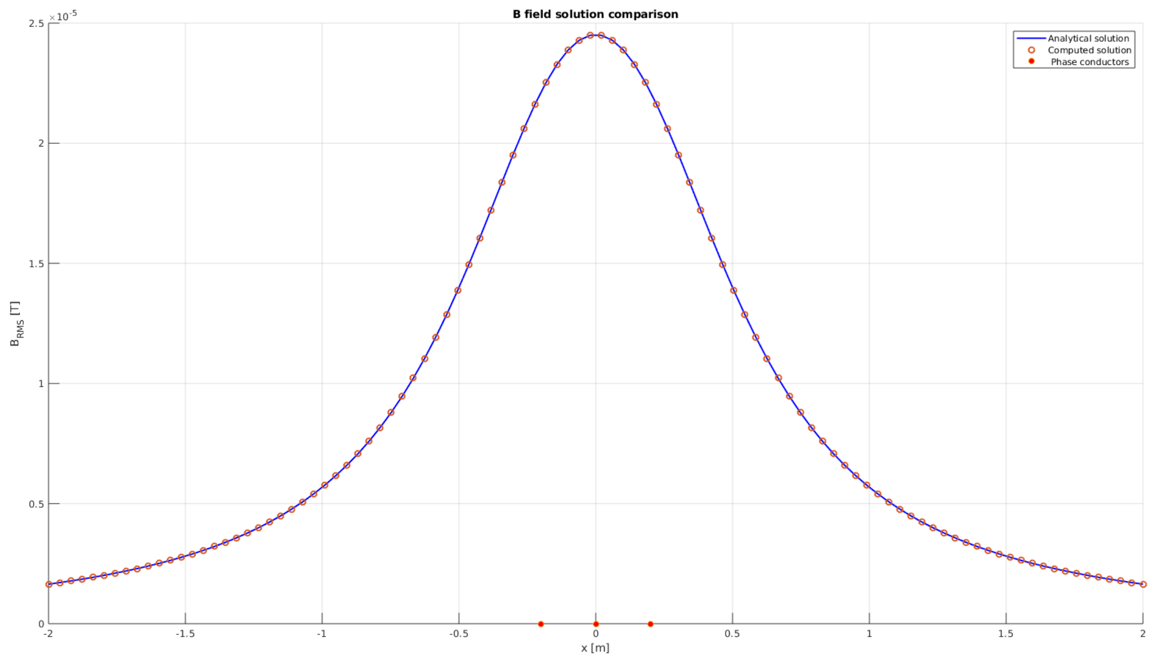

Figure 8.1: Magnetic flux density B for infinite bus bar

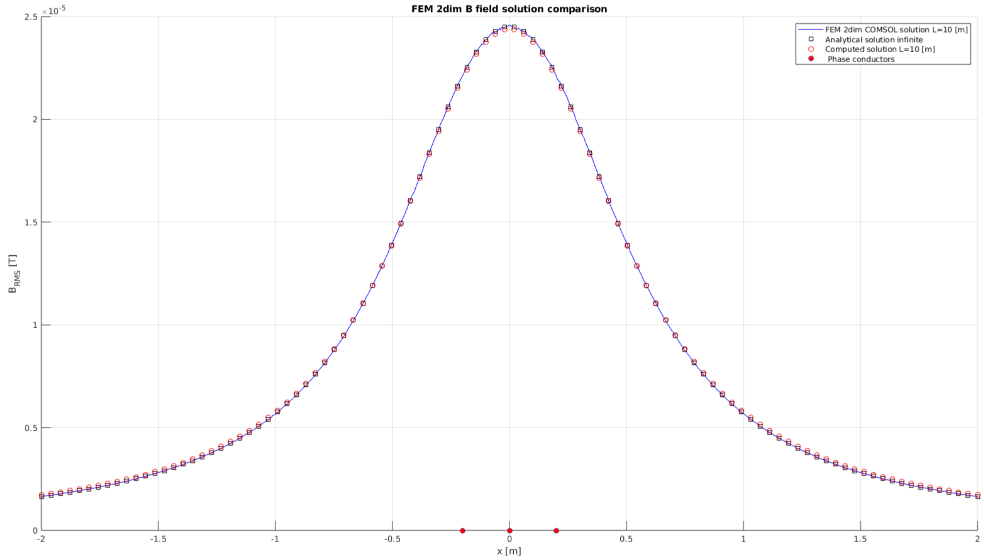

Figure 8.2: Magnetic flux density B for finite bus bar

Different lengths are consider to evaluate the accuracy of formulas. Lentgh about 20 meters allow to have a good approximation with infinite formulation, like it could notice in graph below.

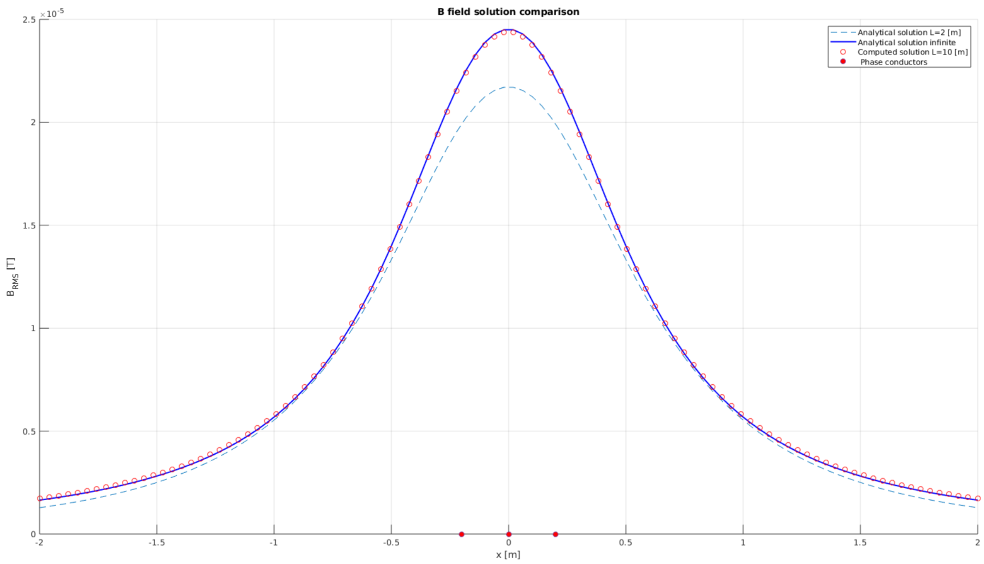

Figure 8.3: Results comparison for different lengths

Test Case 9 - PEEC

9.1 Assignements

PEEC is one of the integral method. It consinst in an aproximatation of the physic problem in a lumped parameters circuit rappresentation, solvable with well knowing circuit solving method like Modify Nodal Analysis. This test case study a MT-BT substation with a balanced three phase system.

9.2 Results

Initially the magnetic vector potential A is analyzed with numerical and analytical formula. Then through PEEC approach, it computed three-phase voltage from the three phase current provided. The mutual-inductance matrix is compared with analytical expression and as usual all results are compared with Comsol ones.

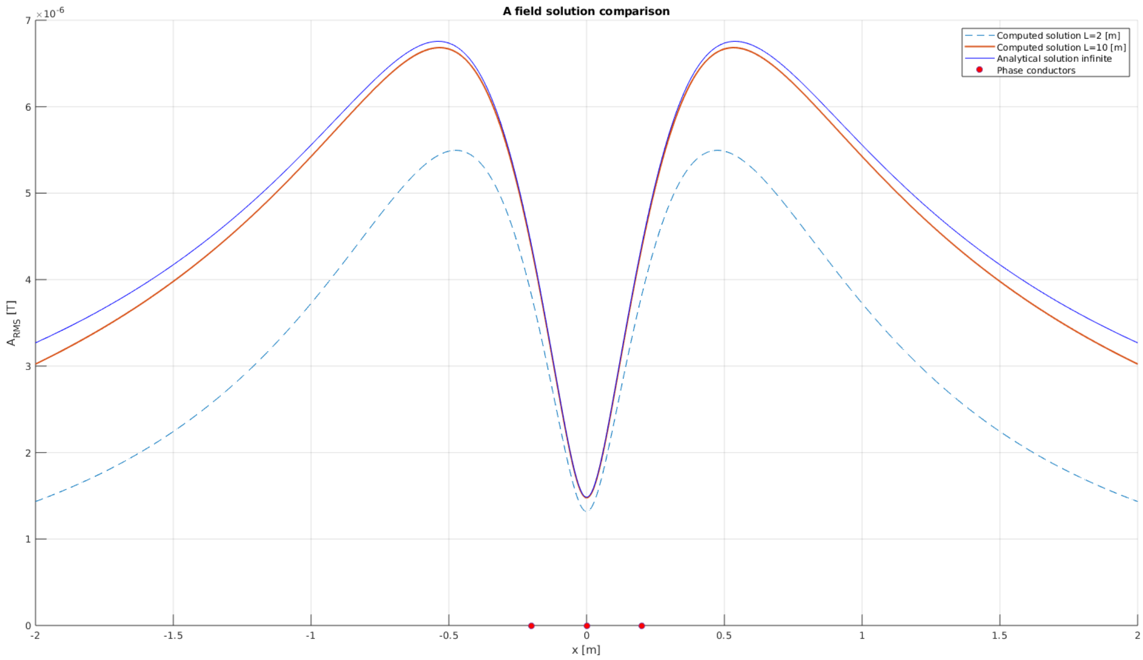

Figure 9.1: Magnetic vector potential along $50cm$ distance line

Discrepancy between analytical and numerical mutual-inductance matrices is 0.0016.

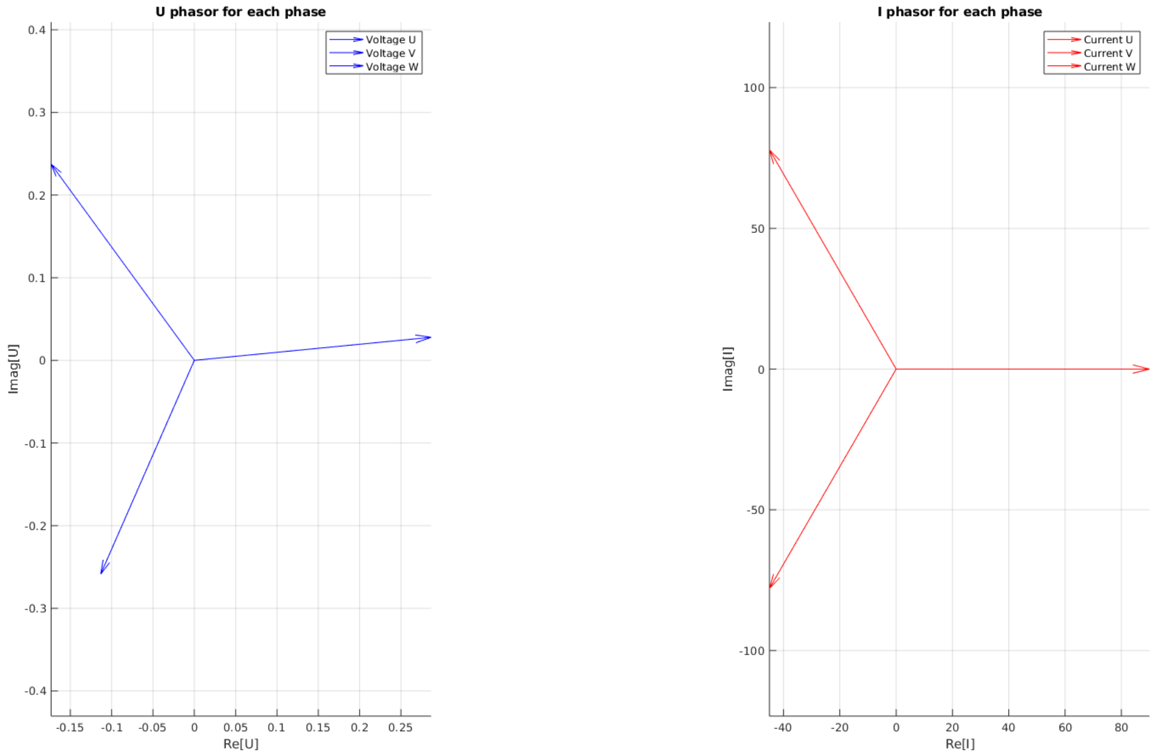

Figure 9.2: Voltage and current phasors in the bus bar

Test Case 10 - BEM

10.1 Assignements

BEEM is another integral method. This test case is very useful to understand the differences of this process respect FEM. Infact the same bump and oscillating functions analized in test case 2 is now computed with BEM approach.

10.2 Results

The results are more accurate to the detriment of computational time and memory resources occurence.

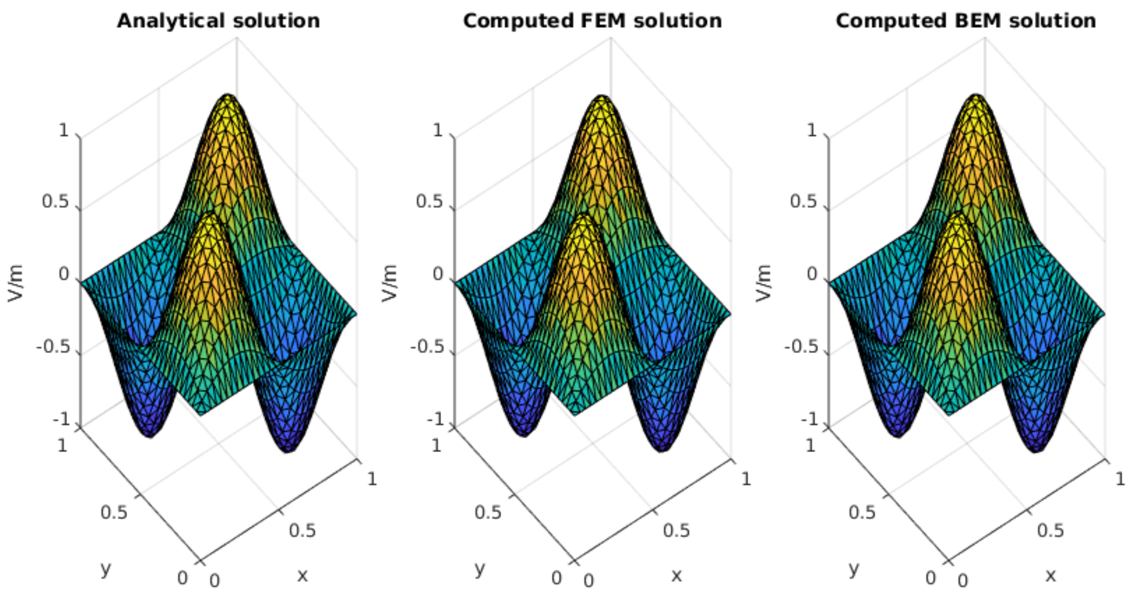

Figure 10.1: Comparison between BEM and FEM of oscillating function

A coarse method to evaluete the error of this approach is made with a simple difference between BEM and FEM solution. As you can see the shape respect the FEM error already seen in test case 2.

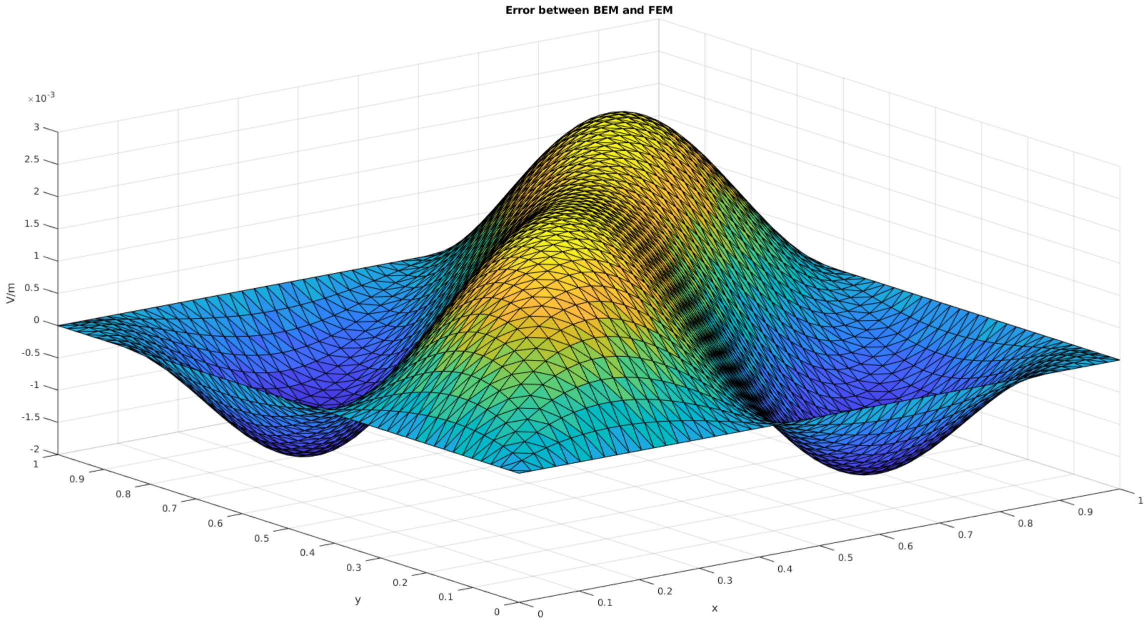

Figure 10.2: Coarse error rappresentation of oscillating function

The exactly numerical values are for a case with 2500 nodes:

- Error potential U with BEM: 1.356242e-02
- Error potential U with FEM: 2.816272e-01

By refining the mesh, i.e. reducing the size of the discretization step into which it divide the sides of the domain the error of both method are drastically reduced, but with different order of size.

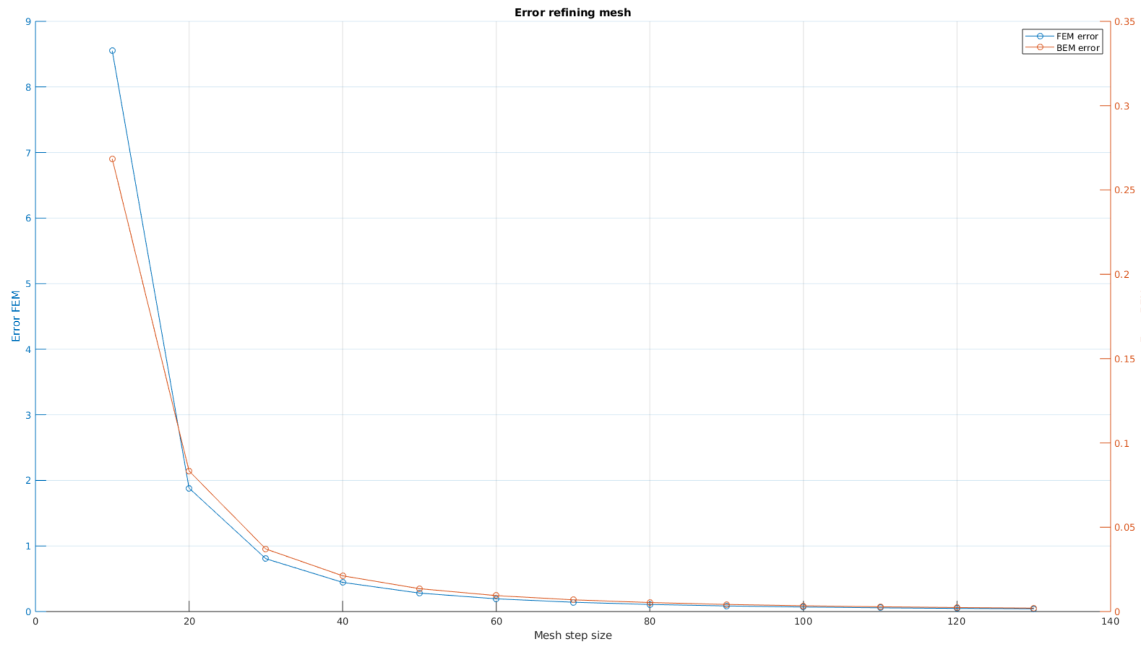

Figure 10.3: Error comparison between FEM and BEM

As you aspect the BEM LHS matrix is fully in the opposite of FEM that is very sparse. This have a confirm also in the computation time, an example with 2500 nodes:

- Computed time FEM 0.142 sec
- Computed time BEM 123.298 sec

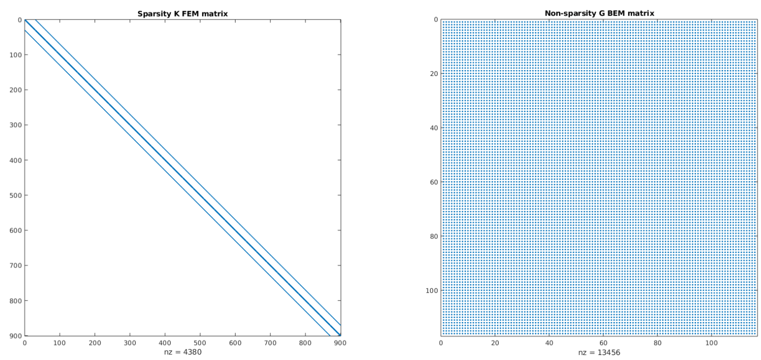

Figure 10.4: Comparison of LHS matrices between FEM and BEM

Test Case 11 - MoM

11.1 Assignements

Method of moments is most generic of integral methods, it's used for analize EM scattering problems becouse allow to operate at high frequency. Typical usage is to evaluate the antenna field, in particular this test case consider a straight wire half-wave dipole antenna. The assignemnts are to compute the current along the antenna with different approaches: first using the formulation proposed by Orfanidi book. Then make a comparison between approximate and accurate Green kernel evaluating. Finally analyze the behavoir in case of high number of nodes.

11.2 Results

These result are obtained with a mesh of 21 nodes. The solution provided by Orfanidi formulation is pretty the same computed with approximate Green kernel (infact they use the same analytical formulation), while there are some differences with the accurate one: in particular approaching the antenna center. Real and imaginary part of the current are evaluated.

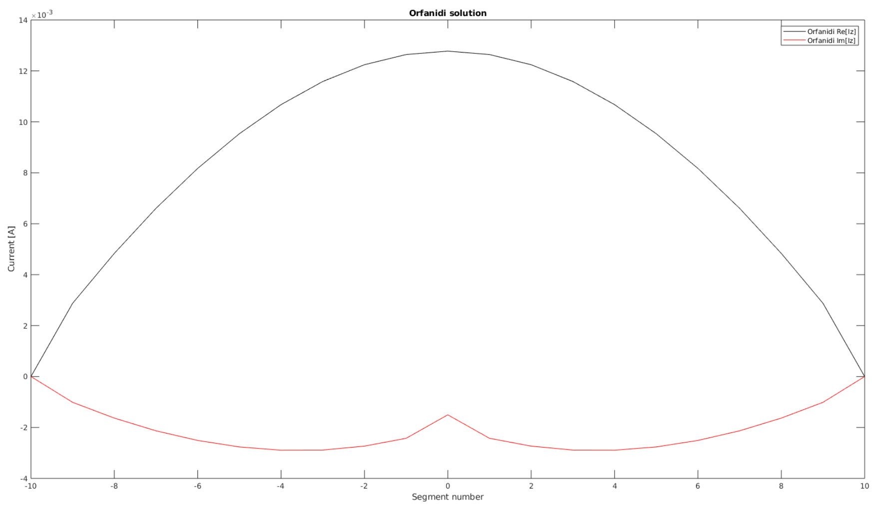

Figure 11.1: I computed by Orfanidi formula

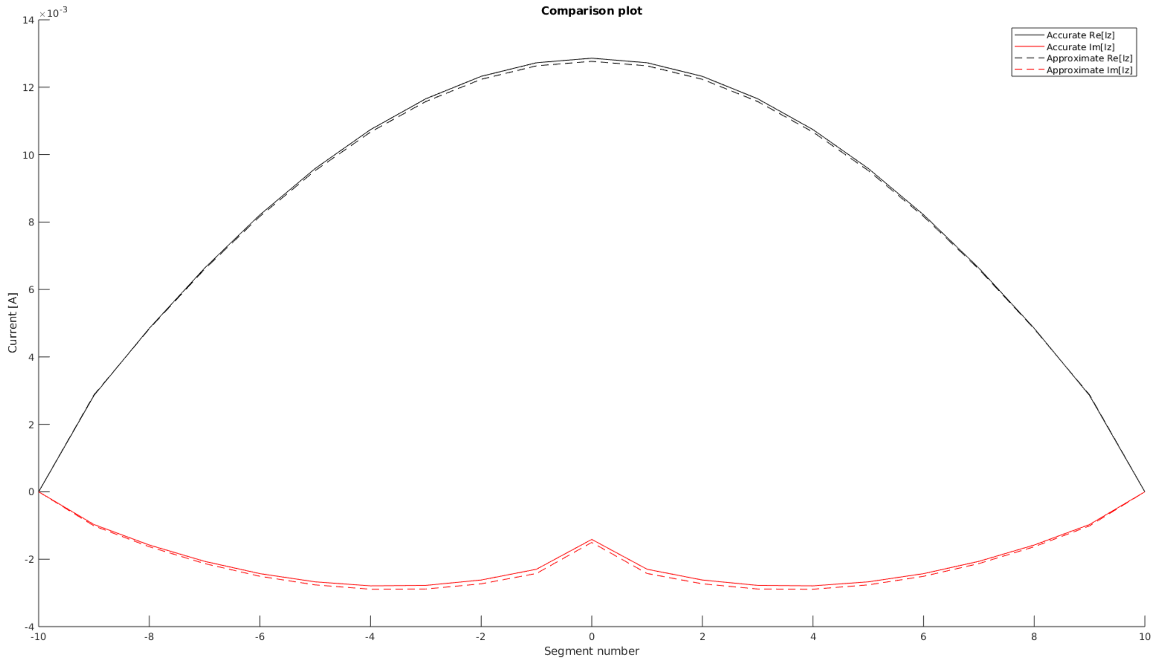

Figure 11.2: Comparison between approximate and accurate Green Kernel evaluating

Increase the number of nodes cause instability in the approximate Green Kernel solution, as you can see in the graph below for 301 nodes.

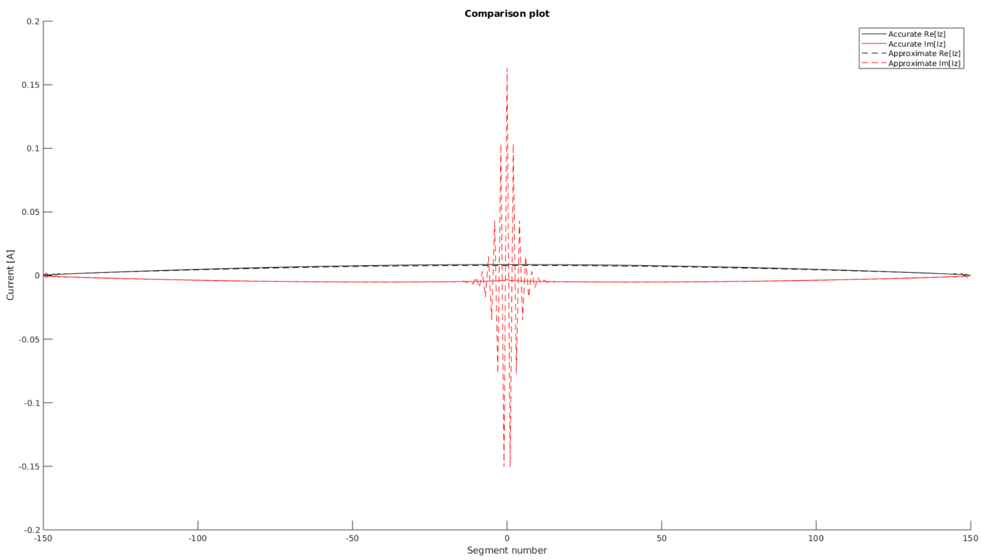

Figure 11.3: Instability of approximate solution

Computation time

Computation time is an essential aspect of the numerical modeling. Engineerings use very frequently these technique to simulate real physic problem before realize prototypes of products or to understand the behavior of an machine in its using. Simulation time is straightly linked to both computing capacity of the pc where the system is runned and to method used.

12.1 Parallel computing

Parallel computing is recent solution to reduce the computing time. That is the script command are not ran in sequential mode, as usual but each processor load a specific task of the code and compute them in the same time. This is very useful for example for for loop not strictly correlated each other and saves a lot of precious time. Matlab provide a Parallel Toolbox to implement easily these techniques in your coding.
Two main solution are available: local and cloud.

- Consist in exploiting all the available resources on your local machine running the script in parallel mode. Graphic cards allow a very good performance thanks to FPGA technology.

- Another solution are the cloud services that provide big cluster of server preconfigured where you can run script in batch or real mode. This solution is very interesting because allow to not have always an update hardware (with relative high cost) and very important computation potential. Matlab have some specific partners to setup easy the connection

Write a Matlab code to exploit parallel computing is not difficult, only some little attention in nested for loop are necessary. Also readopt an already written code is quite easy, like in the case below.
Some tests are done in local mode exploiting both the core of my pc. This graph represent the time used for FEM analysis proposed in test case number 2 (oscillating

function). The configuration used for this test is setup with the Matlab Parallel Toolbox. Only a for loop along all mesh nodes is changed. In the code the Matlab syntax expresssion for is substituted by $parfor$ to exploit parallel computing.

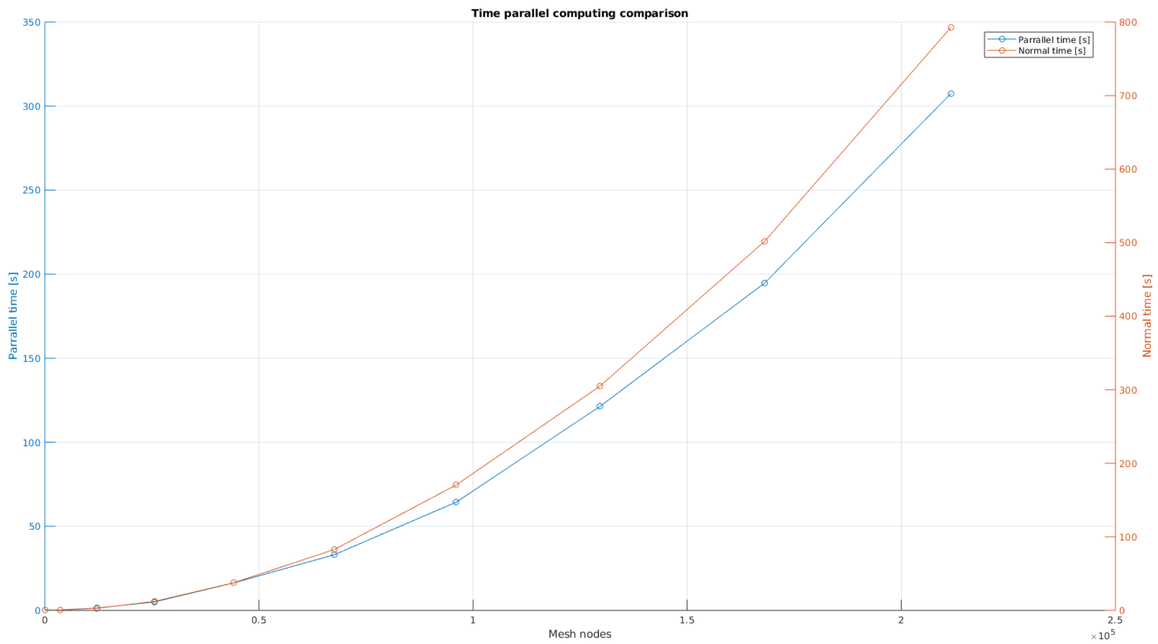

Figure 12.1: Comparison of computation time between normal and parallel mode

I also tried to setup a cloud server HPC in order to have a comparison with a strong computing resources but it wasn't possible for technical mismatching with our department IT configuration for the verification of Matlab license. It would be very interesting evaluate the time of computing in this cluster. For example the simplest configuration allow to use 28 cores.

Another interesting test to do is try to exploit FPGA technology using an Nvidia card graphic (only these are now well supported cards by Matlab). They allow to have an important parallel resources in local machine with a small cost and configuration.

12.2 Computation approaches

Obviously computation time is strictly dependent on the method used to resolve your simulation, dimension of shape consider, step size of discretion mesh and many other parameters evaluete case by case.
Other test is done about the different approaches. For example a comparison between two different methods analyzed in test case number 10 BEM and FEM for the same oscillating function use different times to compute, as you can see in the graph below.

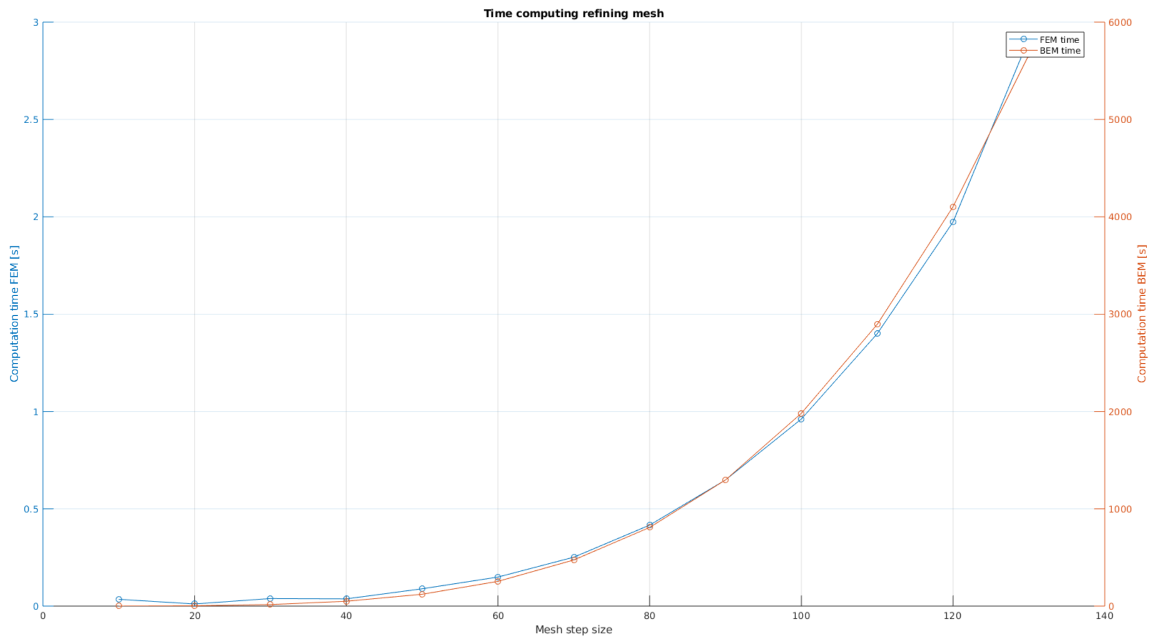

Figure 12.2: Comparison of computation time between FEM and BEM