Report

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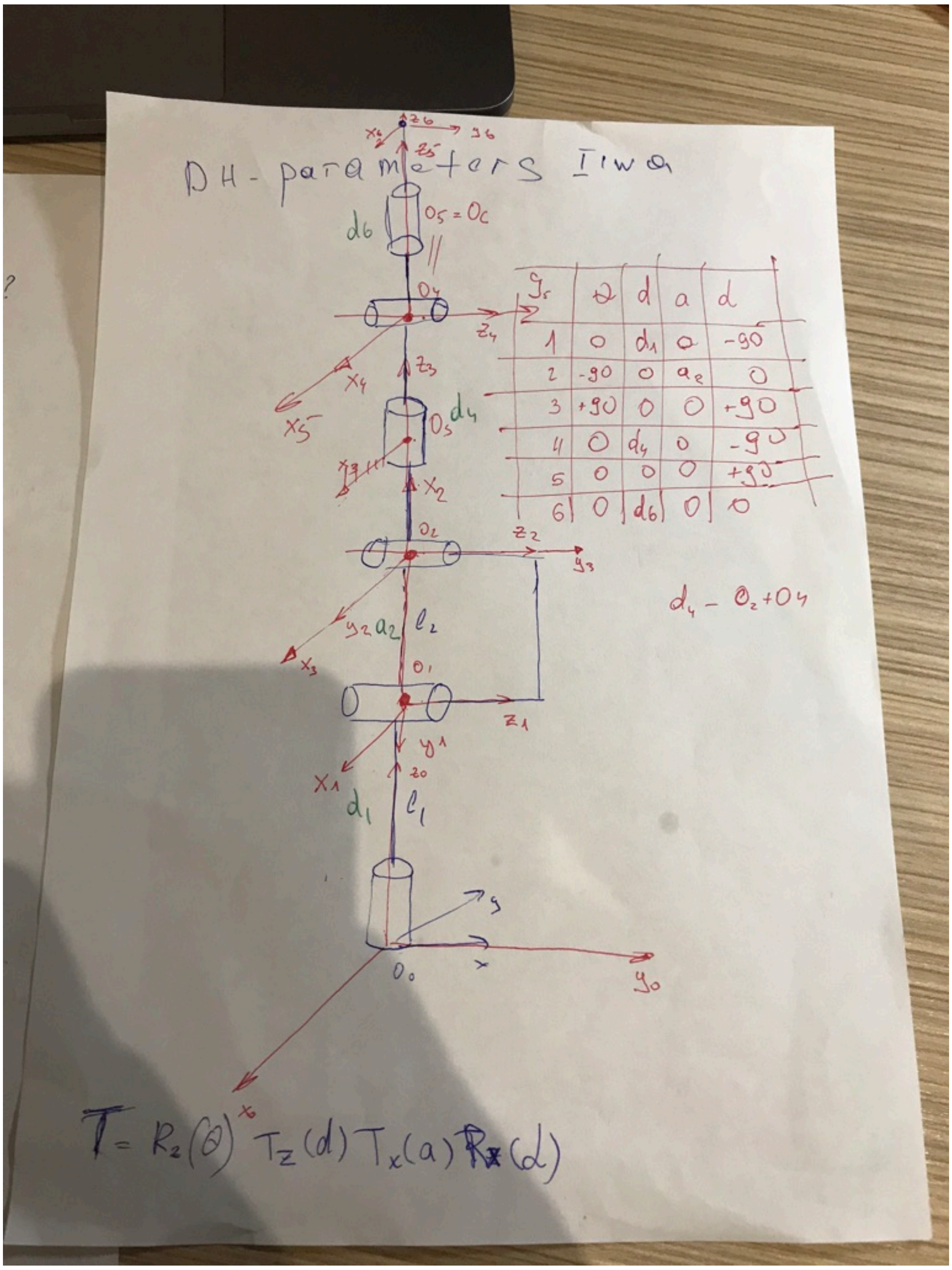
1. Direct Kinematics.

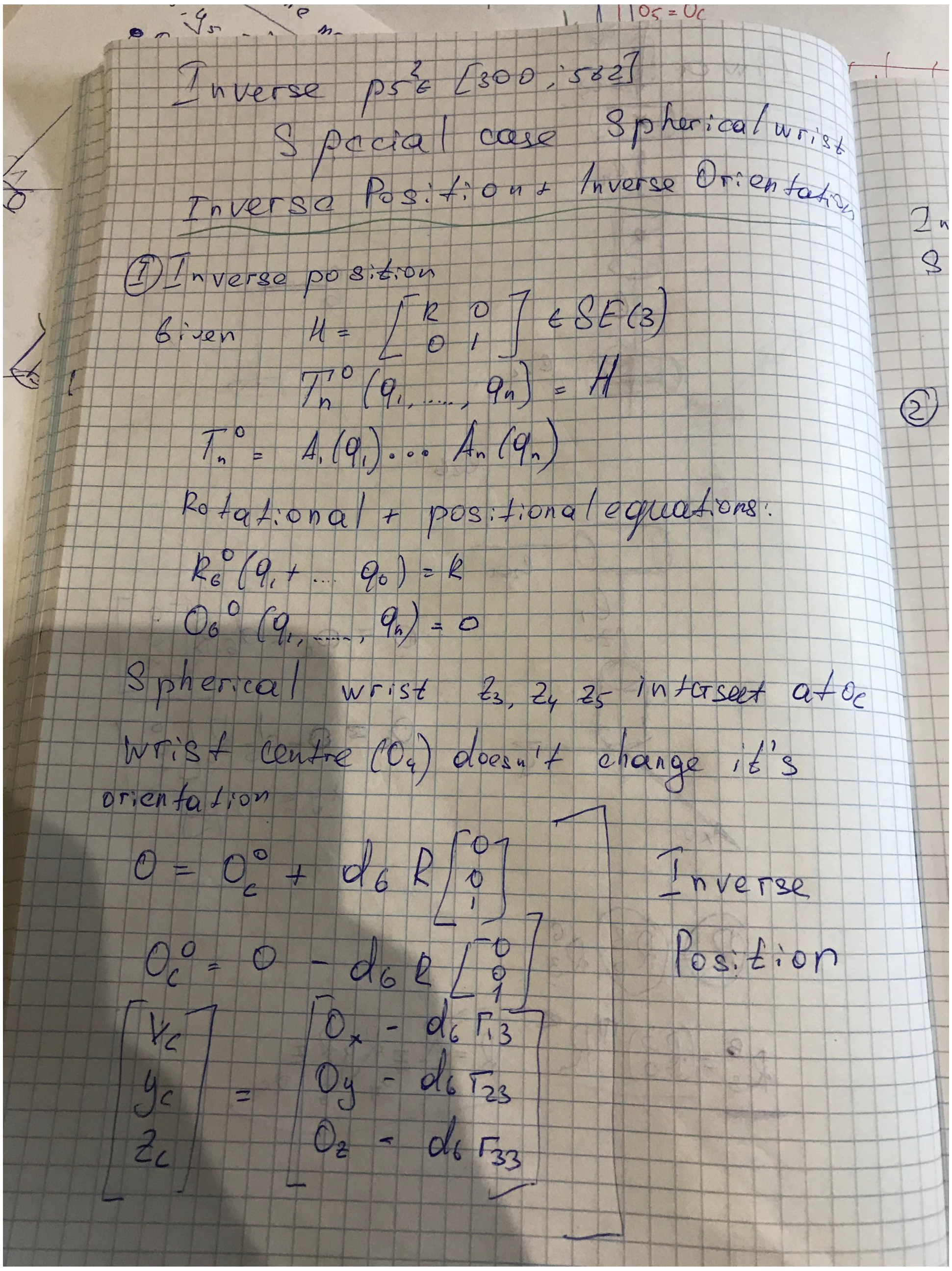
Find the DH - parameters:



In Matlab I also use algebraic approach to solve direct problem. There is a transformation matrix for this case.

2. Inverse Kinematics

We have special case - spherical wrist.And we can solve this problem in 2 steps:
1) Find the position of the centre of the wrist.
2) Find the orientation of the wrist


Inverse ps [300;582]

Special case Spherical wrist

Inverse Position + Inverse Orientation

① Inverse position

Given $H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix} \in SE(3)$

$$T_n^0(q_1, \dots, q_n) = H$$

$$T_n^0 = A_1(q_1) \cdots A_n(q_n)$$

Rotational + positional equations:

$$R_6^0(q_1 + \dots q_6) = R$$

$$O_6^0(q_1, \dots, q_n) = o$$

Spherical wrist z_3, z_4, z_5 intersect at o_c

Wrist centre (O_4) doesn't change it's orientation

$$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$o_c^0 = o - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

Inverse Position

Введите текст

$R = R_3^0 \cdot R_6^3$

$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$

Inverse position - simple

Singular cases ($x=0$, $y=0$) $atan2\left(\frac{0}{0}\right)$

Infinite number of solutions

(2) Inverse orientation

For the spherical wrist that problem is to find a set of Euler angles

Z-Y-Z

ϕ, θ, ψ

$q_4 = \phi$

$q_5 = \theta$

$q_6 = \psi$

$\cancel{q_4 = atan2(}R$

$R_{30} = $ C,

$$\begin{bmatrix} c_4 c_5 c_6 - s_4 s_6, & -c_6 s_4 - c_4 c_5 s_6, & c_4 s_5 \\ c_4 s_6 + c_5 c_6 s_4, & c_4 c_6 - c_5 s_4 s_6, & s_4 s_5 \\ -c_6 s_5, & s_5 s_6, & c_5 \end{bmatrix}$$

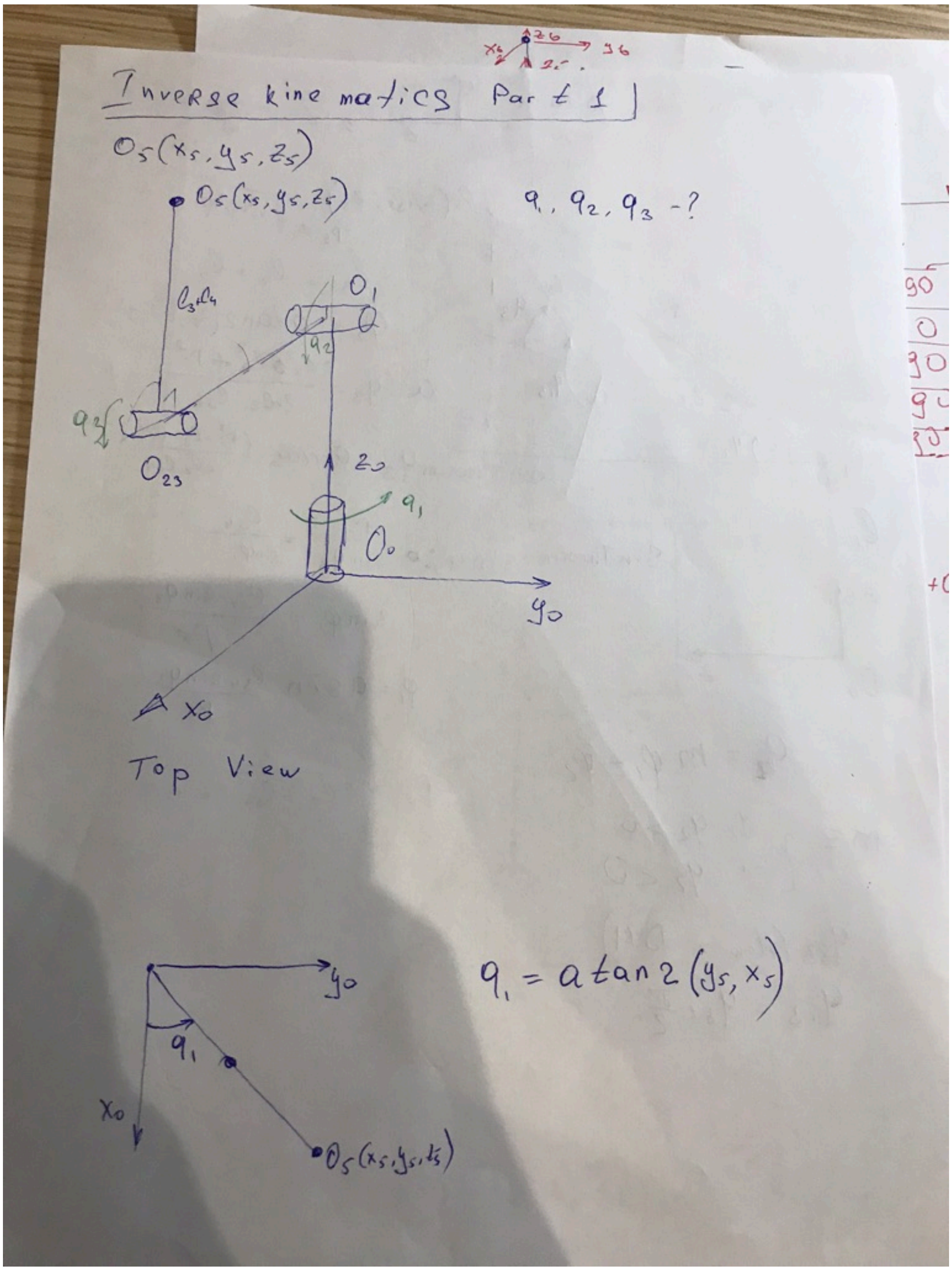
$$q_4 = atan2\left(\frac{s_4 s_5}{c_4 s_5}\right) = atan\left(\frac{R_{13}}{R_{23}}\right)$$

$$q_5 = atan\left(\frac{\sqrt{(R_{13} + R_{23})^2}}{R_{33}}\right)$$

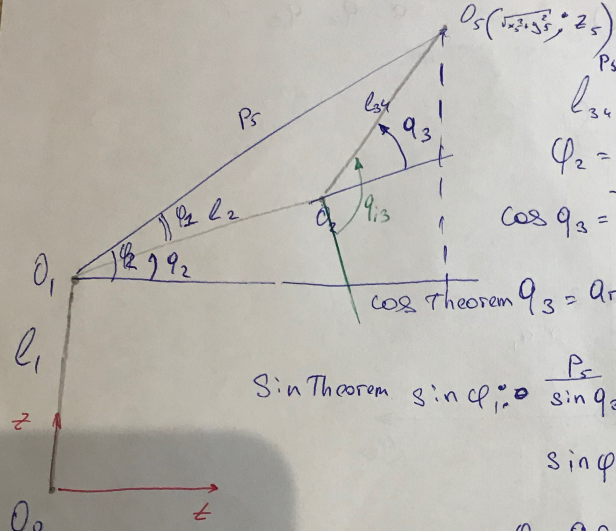
$$q_6 = atan\left(\frac{R_{32}}{-R_{31}}\right)$$

Singularities cases will be when x = 0 and y =0; There will be a infinite number of solutions;

Part 1 in my case



View in the plane z, t ; $t = \sqrt{x^2+y^2}$



$p_5 = \sqrt{x_5^2+y_5^2+(z_5-l_1)^2}$

$l_{34} = l_3 + l_4$

$\varphi_2 = \operatorname{atan2}\left(z_5; \sqrt{x_5^2+y_5^2}\right)$

$$\cos q_3 = \frac{-l_2^2 - l_{34}^2 + p_5^2}{2 \cdot l_2 \cdot l_{34}}$$

cos Theorem $q_3 = \arccos\left(\frac{p_5^2 - l_2^2 - l_{34}^2}{2 l_2 \cdot l_{34}}\right)$

Sin Theorem $\sin\varphi_1$: $\frac{p_5}{\sin q_3} = \frac{l_{34}}{\sin\varphi_1}$

$$\sin\varphi_1 = \frac{l_{34} \cdot \sin q_3}{p_5}$$

$$\varphi_1 = \operatorname{asin} \frac{l_{34} \sin q_3}{p_5}$$

$$q_2 = m\varphi_1 + \varphi_2$$

$$p_5 > \sqrt{l_2^2 + l_{34}^2} = 10\sqrt{2}$$

$$m = \begin{cases} 1, & q_2 > 0 \\ -1 & q_2 < 0 \end{cases}$$

q_{i3} (How in DH)

$$q_{i3} = q_3 + \frac{\pi}{2}$$