Report

Dynamics of non linear robotic system

Homework assignment №4

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GitHub Link

The MatLab code you can find on this link:

https://github.com/rodosha98/DynamicsJacobian.git

Task 01

Initial parameters:

$q(0) = 1$ $\qquad$ $\dot{q}(0) = 0$

$q(2) = 4$ $\qquad$ $\dot{q}(2) = 0$

a. Polynomial position profile

General Expressions of polynomial(position and velocity):

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Taking into account initial conditions, there can be written 4 equations:

1) $a_0 = 1$
2) $a_0 + 2a_1 + 4a_2 + 8a_3 = 4$
3) $a_1 = 0$
4) $a_1 + 4a_2 + 12a_3 = 0$

I've solved this system of equations in Matlab, using the linear system solver. The result you can see on the figures 1.1 and 1.2.

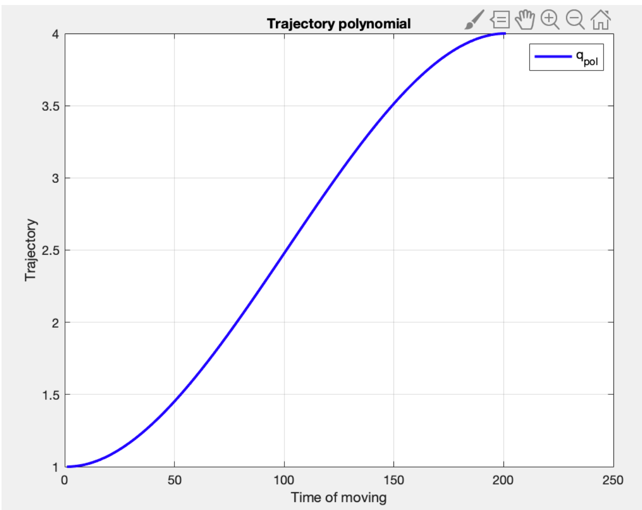


Figure 1.1 - 3 degree polynomial trajectory

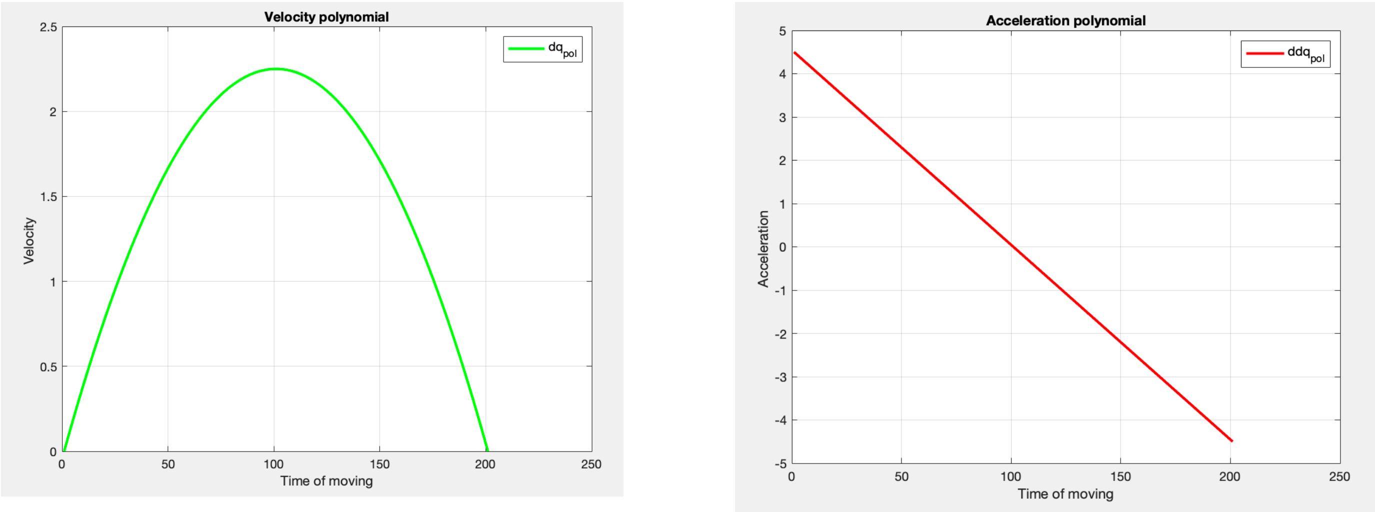


Figure 1.2 - Velocity and acceleration plots. Polynomial task.

As can be seen in the graphs, the initial conditions are satisfied, and also the speed has 2 degrees and the first is accelerated.

b.Trapezoidal profile

In trapezoidal profile we divide the task on 3 areas, corresponding velocity plot. Initial and finite areas are similar, and in the middle - constant velocity zone.

The first step is to define time moment t_c, where velocity becomes constant.

Initial parameters: q_i, q_f and additional parameter. There are needed additional assignments and we have to approaches:

1) Assign desired acceleration

Condition for acceleration:

$$|\ddot{q}_c| \geq \frac{4|q_f - q_i|}{t_f^2}$$

If equality, there will be no constant segment on profiles.(triangle)

Then:

$$t_c = \frac{t_f}{2} - \frac{1}{2}\sqrt{\frac{t_f^2\ddot{q}_c - 4(q_f - q_i)}{\ddot{q}_c}}$$

2) Assign desired constant velocity

Condition for velocity:

$$\frac{|q_f - q_i|}{t_f} < |\dot{q}_c| < \frac{2|q_f - q_i|}{t_f}$$

Time c and acceleration expressions:

$$t_c = \frac{q_i - q_f + \dot{q}_c t_f}{\dot{q}_C}$$

$$\ddot{q}_c = \frac{\dot{q}_c^2}{q_i - q_f + \dot{q}_c t_f}$$

I've implemented 2 this approach via flag. If flag is equal to 1 - that means velocity approach, 0 - acceleration. The program takes into account the boundary conditions for acceleration and velocities, so that a hint appears if they are set incorrectly. Both methods lead us to a piecewise given function of 3 polynomials.

$$f(x) = \begin{cases} q_i + 1/2\ddot{q}_c t^2 & 0 \le x \le t_c \\ q_i + \ddot{q}_c t_c (t - t_c/2) & t_c \le x \le t_f - t_c \\ q_f - 1/2\ddot{q}_c (t_f - t)^2 & t_f - t_c \le x \le t_f \end{cases}$$

I've chosen as an example $\dot{q}_c = 2$ The results are shown on figures 1.3 and 1.4

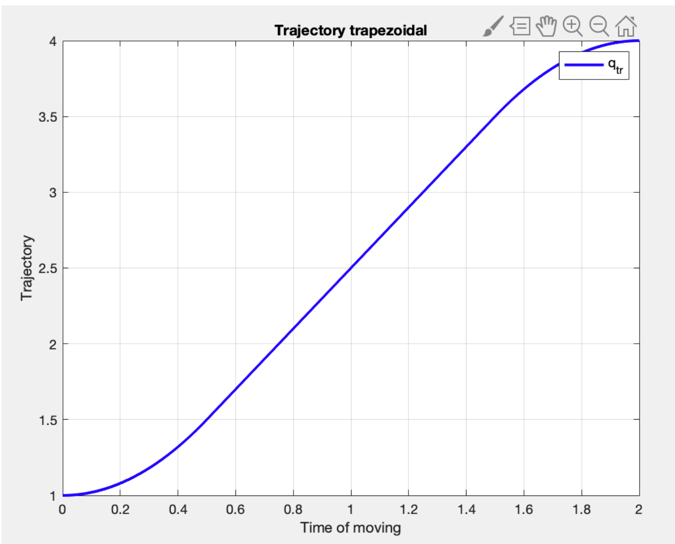
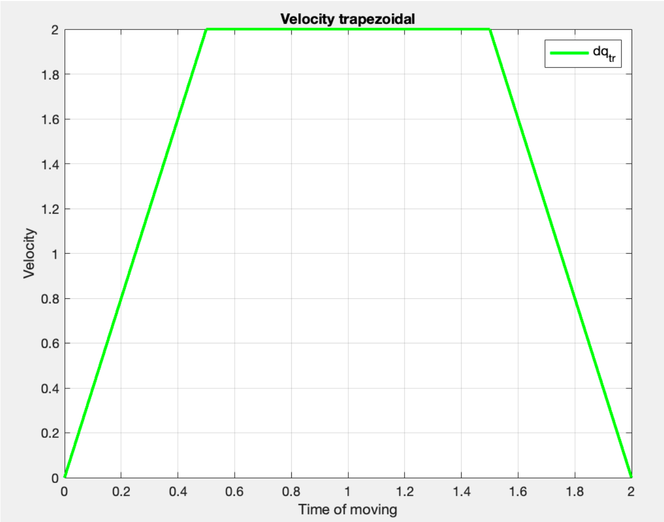


Figure 1.3-Trajectory plot. Polynomial task.



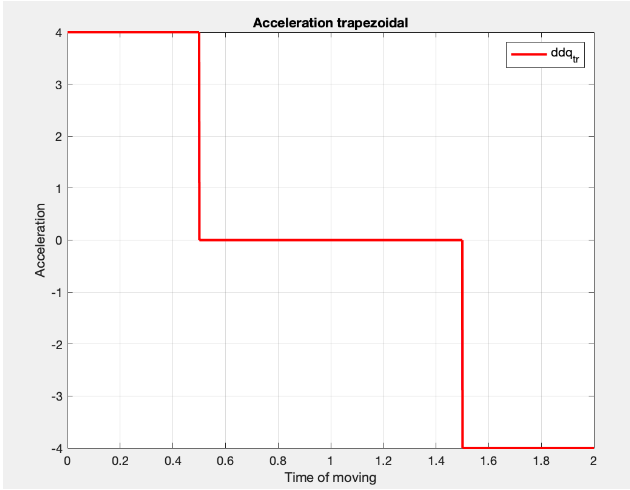


Figure 1.4 - Velocity and acceleration plots. Trapezoidal task.

Constant velocity corresponds to desired value.

c.Velocity profile of the type $\dot{q}(t) = a(b + sin(ct))$

I've solved this task by hands, because I didn't find the solver for this equations

Find the acceleration(derivative) and position(integral):

$$q(t) = \int (ab + asin(ct))dt = abt - \frac{a}{c}cos(ct) + c1$$

a, b, c - constants, they are unknown.

Equations, satisfying the initial conditions:

1) $q(0) = 1 \ \forall a, b, c;$

$c_1 = 1 + \frac{a}{c}$ - constant of integration

2) $\dot{q}(0) = 0 \quad ab = 0$

$a = 0$ or $b = 0$

3) $\dot{q}(2) = 0$

$ab + sin(2c) = 0$

ab is equal to 0

$sin(2c) = 0$

$c = \frac{\pi n}{2}$, n is integer , $n \neq 0$

$4) q(2) = 4$

$$2ab - \frac{a}{c}(cos(2c) - 1) + 1 = 4$$

Use known fact ab = 0:

$$cos2c = -\frac{3c}{a} + 1$$

$a \neq 0$, hence b = 0

Substitute c:

$$cos(\pi n) = -\frac{3\pi n}{2a} + 1$$

Now we can notice, than $cos(\pi n) = 1$ if n is even and -1, if n is odd. Let's consider these 2 cases:

1) $n = 2k$ - even. Then:

$1 = -\frac{3\pi k}{a} + 1$. Solution only if k is equal to 0, n = 0, c =0. But c can't be equal to zero. Therefore, we discard even n.

2) $n = 2k + 1$ -odd Then:

$$-1 = -\frac{3\pi(2k + 1)}{2a} + 1$$

We get this set of solutions:

$$a = \frac{3\pi(2k + 1)}{4}$$

$$b = 0$$

$$c = \frac{\pi(2k + 1)}{2}$$

k now is a parameter, that we assign. And in fact it defines frequency of curves.

The results for k=2 are shown on the figures 1.5 - 1.7. They are satisfied to initial conditions and frequency of oscillations depends on k. So we get infinite number of solutions, but many of them we can't implement in real life, because of frequency and amplitudes.

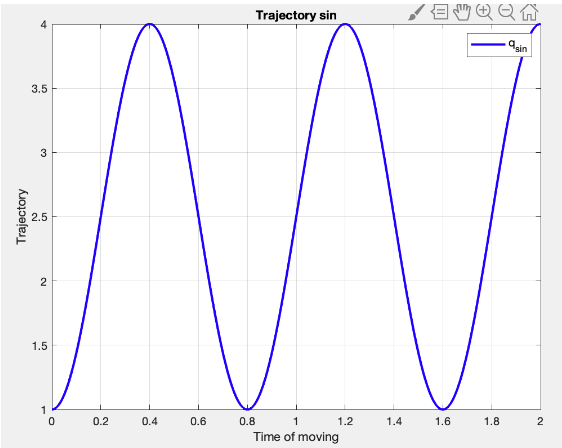


Figure 1.5 - Trajectory plot. Sinusoidal task, k =2.

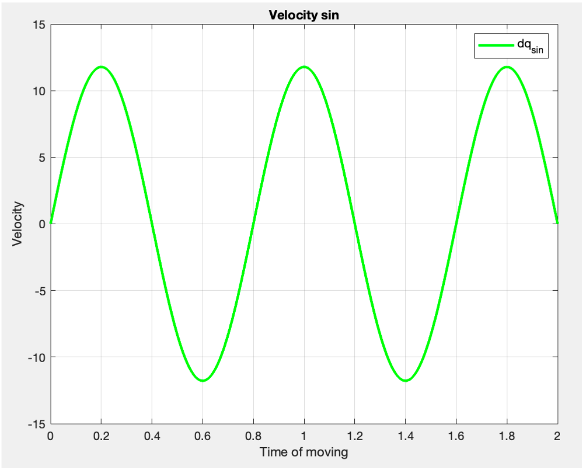


Figure 1.6 - Velocity plot. Sinusoidal task, k =2.

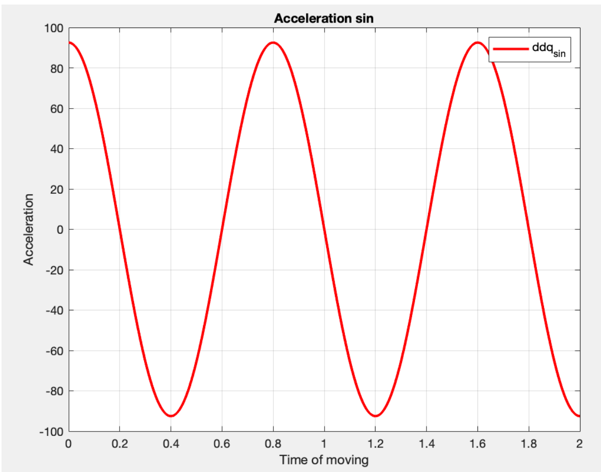


Figure 1.7 - Acceleration plot. Sinusoidal task, k =2.

Task 02

Initial parameters:

$q(0) = 1$ $\qquad\qquad$ $\dot{q}(0) = 0$ $\qquad\qquad$ $\ddot{q}(0) = 0$

$q(2) = 4$

$q(4) = 0$ $\qquad\qquad$ $\dot{q}(4) = 0$ $\qquad\qquad$ $\ddot{q}(4) = 0$

a. Polynomial position profile

Now we have 7 constrains, hence we should use polynomial with degree 6.

General Expressions of polynomial(position and velocity):

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 + 6a_6 t^5$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4$$

Taking into account initial conditions, there can be written 7 linear equations as in the previous task and I've obtain the results, shown on the figures 2.1 and 2.2.

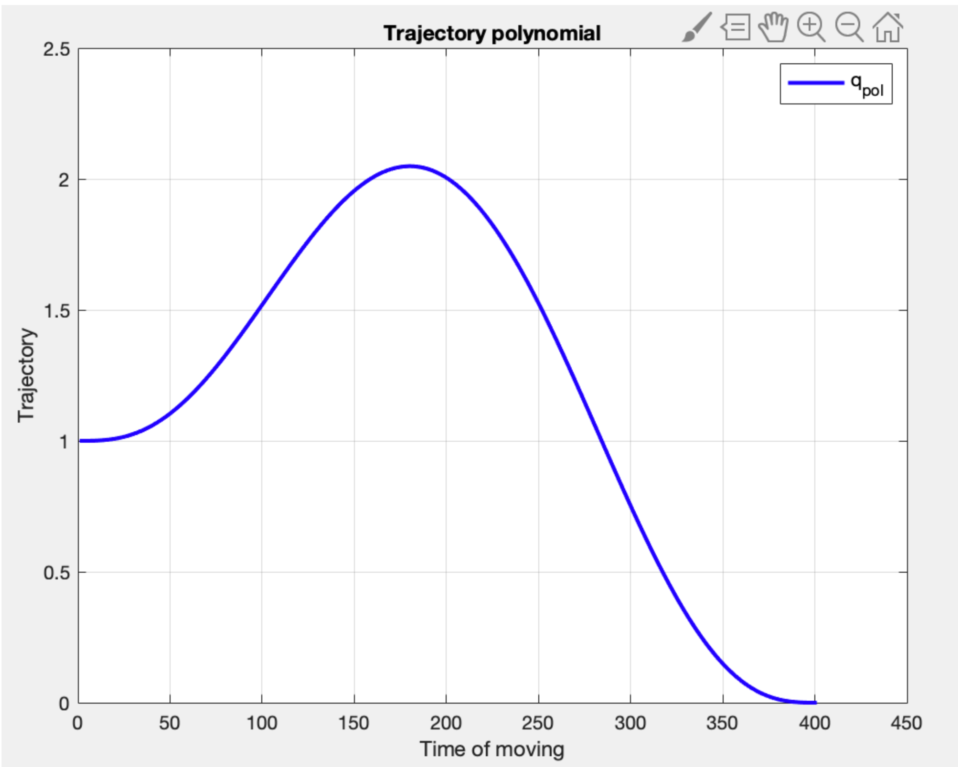


Figure 2.1 - 6 degree polynomial trajectory

As you can see, graphs satisfy the initial conditions.

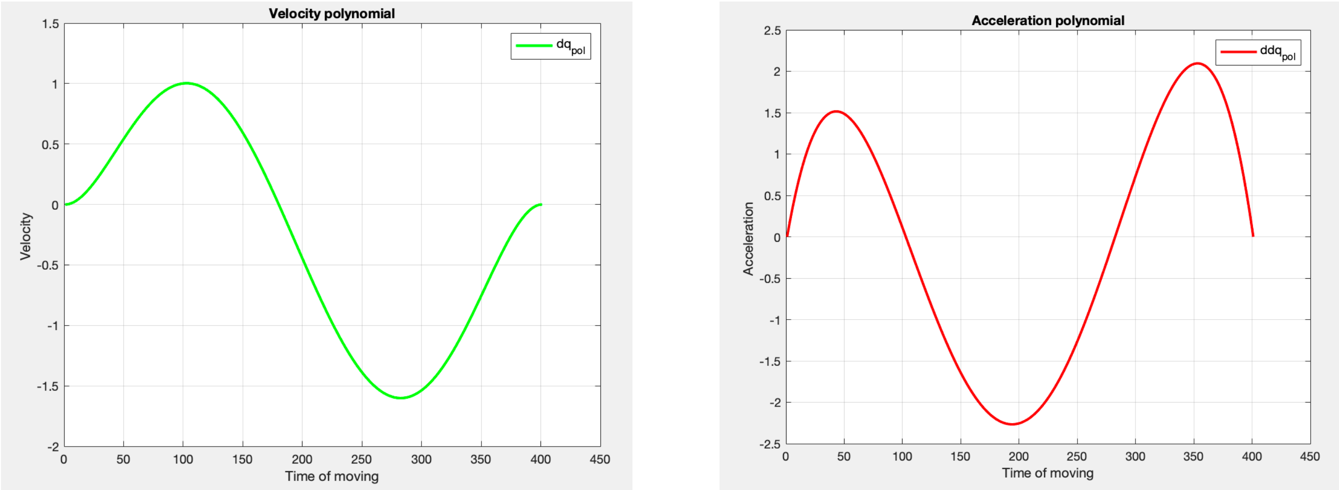


Figure 2.2 - 6 degree polynomial velocity and acceleration

b.Trapezoidal profile

We will use the same approaches as in previous trapezoidal task, but now we have 3 points(initial, intermediate and finite). And we will obtain two trapezoids profiles of velocities(one positive and one negative) I've implemented this via for loop in Matlab with 2 iterations and connect 2 arrays. You can assign first and second constant velocities or accelerations. The results are shown on the figures 2.3 and 2.4.

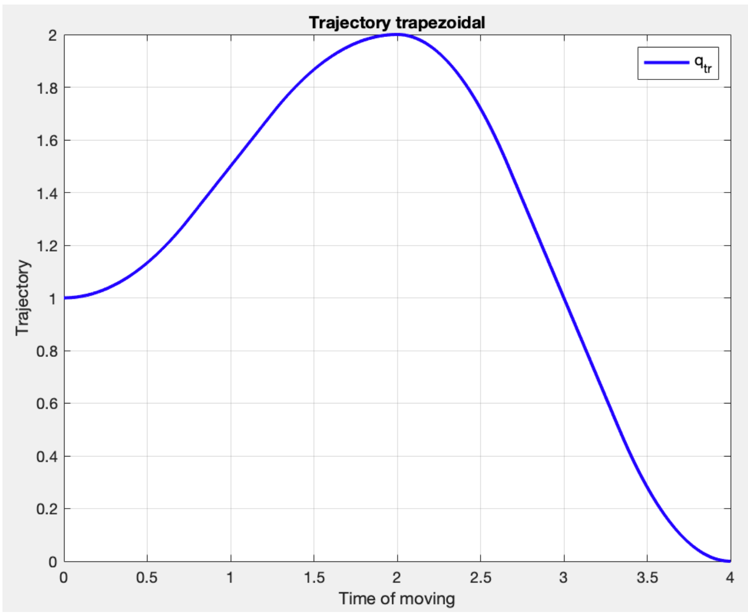


Figure 2.3 - Trapezoidal trajectory

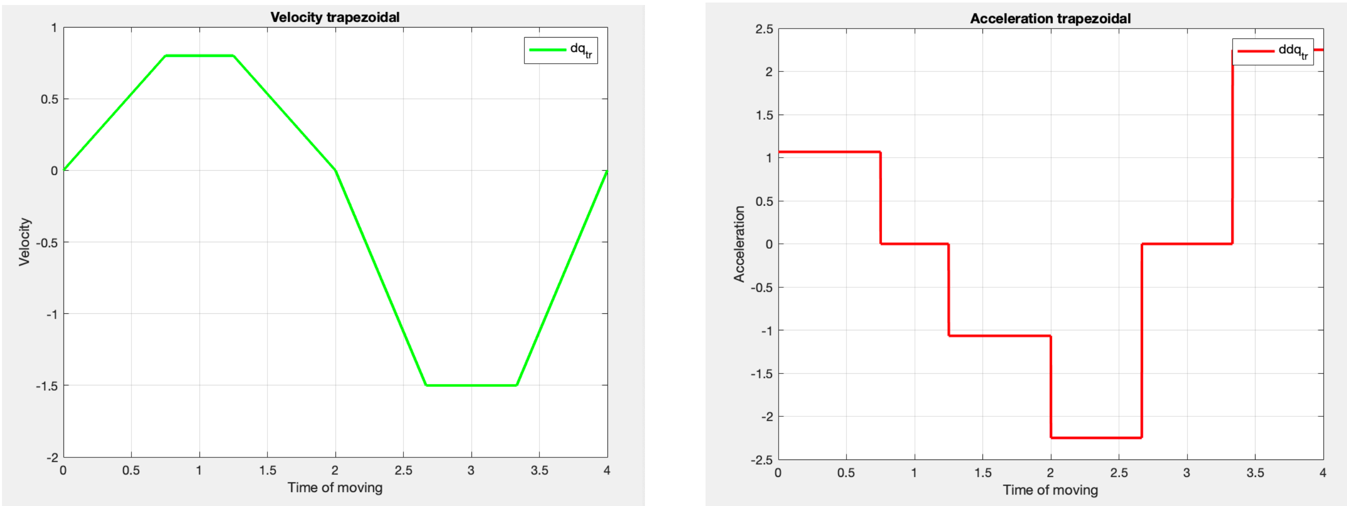


Figure 2.4 - Trapezoidal velocities and accelerations

It is worth noting that the initial conditions are not fully met, because of initial and finite accelerations. But in this case we can assume, that the are zero in t = 0 and t = 4, but then abruptly change their meaning for a small period of time. Otherwise, we will not be able to make a trapezoid with linear faces on the profile, we will have to increase the degree of polynomials and then the trapezoid will be at the acceleration.

c.Splines

I've implemented the simple spline approach. We have one knot (intermediate point t = 2). So we will divide 6th order polynomial on 2 splines 3d and 5th order polynomial respectively. We can fit only one 3d order polynomial on 4 constraints: (initial position, velocity and acceleration) and intermediate position only. After that we can obtain intermediate point's velocity and acceleration and fit 5th degree polynomial on 6 constraints(3 in intermediate point and 3 in finite).

The results you can see on figures 2.5, 2.6 and 2.7.

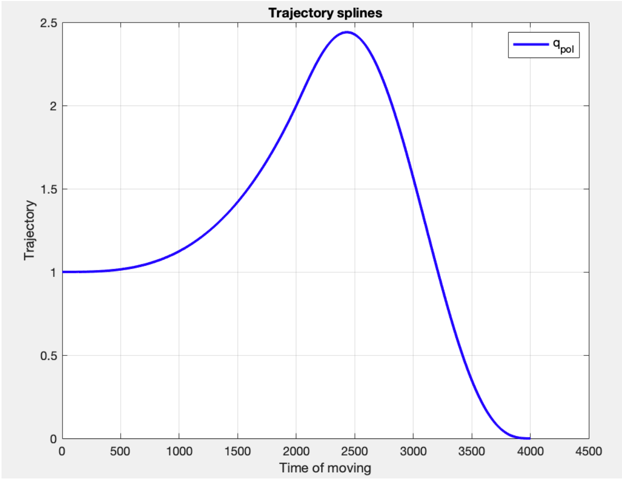


Figure 2.5 - Spline trajectory

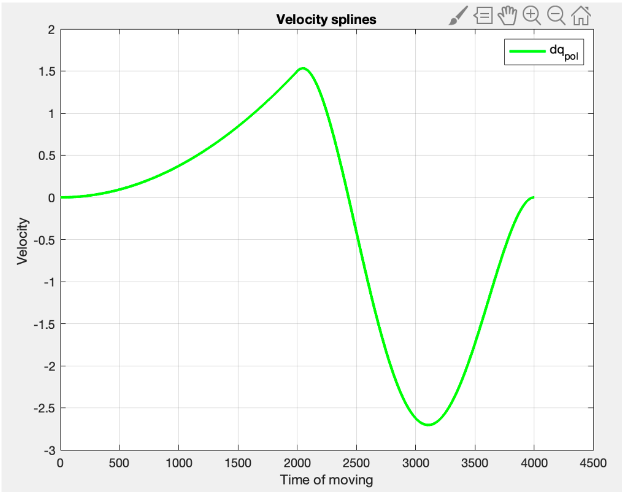


Figure 2.6 - Spline velocity

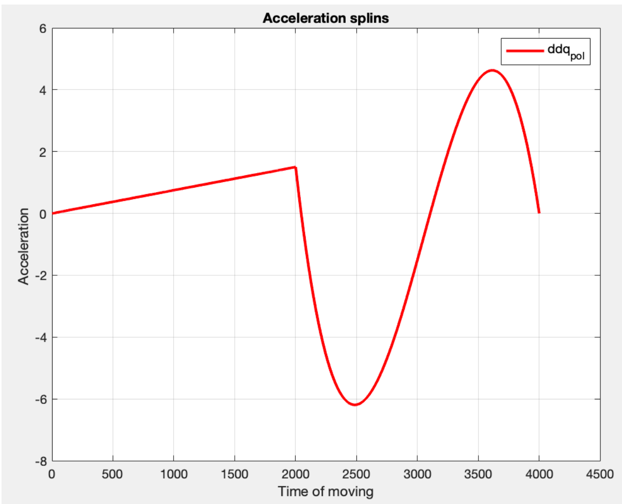


Figure 2.7- acceleration

Splines allow to decrease the polynomial order, but need to think about the junction. Because of it, we’re obtaining good trajectory plot, medium velocity and bad acceleration. In acceleration case we have acute angle at the junction, which means that there is no next derivative. This is the problem of the splines, but it depends on the task.

Comparison and conclusions

Maximum and minimum velocities and accelerations are given in the table 1.

Table1 Results

Tasks	Min velocity	Max velocity	Min acceleration	Max acceleration
Polinomial3	0	2.25	-4.5	4.5
Trapezoidal1	0	2	-4	4
Trapez1(Equal accelerations with polynomial3)	0	1.9	-4.5	4.5
Sinusoidal	-11.8	11.8	-92.5	92.5
Sinusoidal(simplest case k=0 n=1)	0	2.356	-3.7	3.7
Polinomial6	-1.6	1	-2.27	2.09
Trapezoidal2	-1.5	0.8	-2.25	2.25
Trapez2 (Equal accelerations with polynomial3)	-1.15	0.531	-4.5	4.5
Splines	-2.7	1.53	-6.19	4.625

Conclusions: from my point of view, we can judge the energy consumption of the method by the speed values: the greater the velocities at the same accelerations, the more energy the drive needs to spend in order to set the joint in motion.

The most energy efficient methods are spline methods, but they are difficult to put into practice. Especially if we use a lot of restrictions, small speed limits are obtained, but the degree of the polynomial is very large.

In practice, the trapezoidal profile is better, but we lose a little in efficiency and also have problems with the infinite third derivative.

In the case of a sinusoid, we get even lower efficiency even in the simplest case, and if you increase the frequency, then the speed and, accordingly, the energy expended increases very much.

Splines can reduce the degree of polynomial, but at the same time there is a risk of losing much in efficiency, so splines need to be selected.