Report

Dynamics of non linear robotic system

Homework assignment №5

Newton-Euler approach

Student: Oleg Rodionov

GitLab Link

The MatLab code you can find on this link:

https://github.com/rodosha98/DynamicsHW5.git

1.Robot description

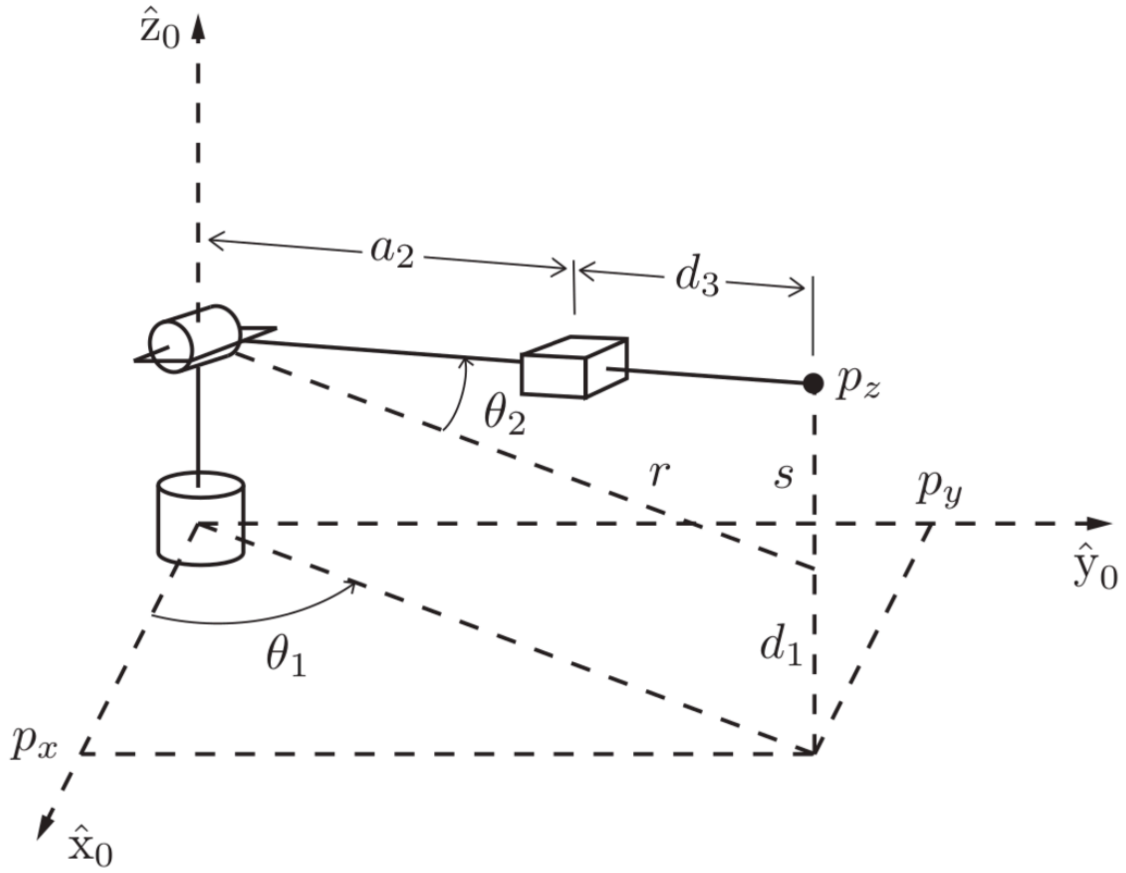

Figure 1 - Robot's scheme

The robot is comprised of 4 links (including ground) and 3 joints, 2 of them are rotational and the last one - translational. We have 3 joint variables: θ_1, θ_2 and d_3. RRP manipulator from homework 3.

Initial parameters:

$l_1 = 2$	$m_1 = 5$	Joint variables:	$q_1 = sin(t)$
$l_2 = 1$	$m_2 = 2$		$q_2 = 3cos(2t)$
$g_0 = 9.82$	$m_3 = 1$		$q_3 = sin(3t)$

2. Forward kinematics

Before Newton-Euler approach implementation we are needed to assign axes on each joint and prepare matrices.

Algebraic approach:

$T = R_z(\theta_1)T_z(l_1)R_x(\pi/2)R_z(q_2)T_x(l_2)R_y(\pi/2)T_z(q_3)$

Rotation matrices:

$R_1^0 = R_z(\theta_1)R_x(\pi/2)$

$R_2^1 = R_z(q_2)R_y(\pi/2)$

$R_3^2 = I$

And coordinate vectors d_1^0, d_2^1, d_3^2 from one to another frame as elements from homogeneous matrices.

Scheme of robot with axes you can see on the figure 2:

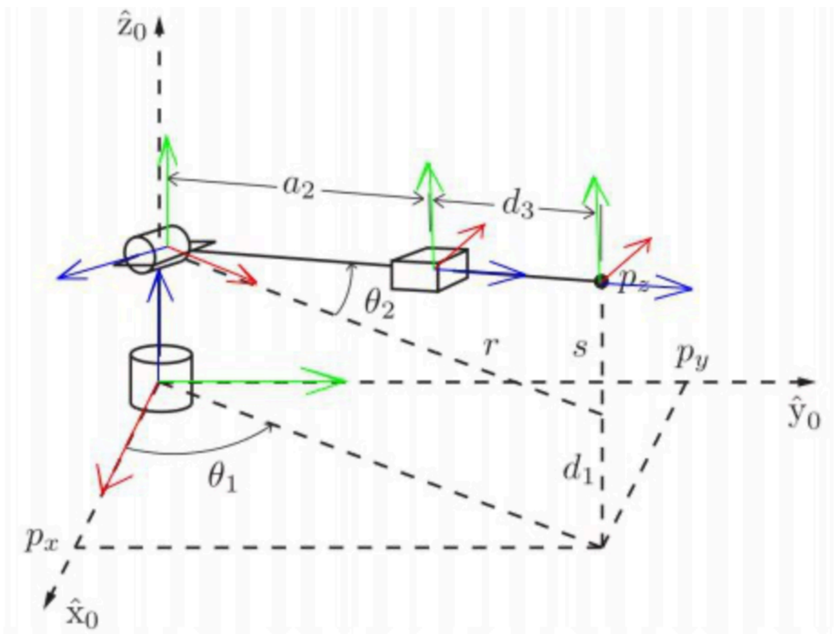

Figure 2 - Robot's scheme with axes

Also we need to find derivatives of q_1, q_2 and q_3 as a functions of time. (I've implemented this via matLab diff function.

3. Forward propagation

I've used Siciliano book(pages 282 - 292) and MatLab to implement Newton-Euler algorithm.

Here we need to define angular velocities, accelerations and linear velocities and accelerations of each link.

Initial conditions:

$\omega_0 = 0$ $\quad\quad a_0 = 0$

$\dot{\omega}_0 = 0$

0 is the zero link or base link.

The main formulas for this approach:

1.*Angular and linear velocities*

$$\boldsymbol{\omega}_i^i = \begin{cases} \boldsymbol{R}_i^{i-1T}\boldsymbol{\omega}_{i-1}^{i-1} & \text{for a } \textit{prismatic} \text{ joint} \\ \boldsymbol{R}_i^{i-1T}(\boldsymbol{\omega}_{i-1}^{i-1} + \dot{\vartheta}_i \boldsymbol{z}_0) & \text{for a } \textit{revolute} \text{ joint} \end{cases}$$

$$\dot{\boldsymbol{p}}_i = \begin{cases} \dot{\boldsymbol{p}}_{i-1} + \dot{d}_i \boldsymbol{z}_{i-1} + \boldsymbol{\omega}_i \times \boldsymbol{r}_{i-1,i} & \text{for a } \textit{prismatic} \text{ joint} \\ \dot{\boldsymbol{p}}_{i-1} + \boldsymbol{\omega}_i \times \boldsymbol{r}_{i-1,i} & \text{for a } \textit{revolute} \text{ joint.} \end{cases}$$

Where: ϑ - joint variable q

$\dot{p}$ - linear velocity

$r_{i-1,i}$ - vector from i-1 frame to I frame

We should take into account, that frames are changing, so we need to multiply our omegas and velocities on R_i^{i-1}. For linear velocity that is also needed to pre-multiply terms, except term with cross product.

I' ve implemented plots for each link separately. In the git I've pushed all of them, but here I will provide some.(For link 1)

Angular and linear velocity for link 1 you can see on figures 3 and 4.

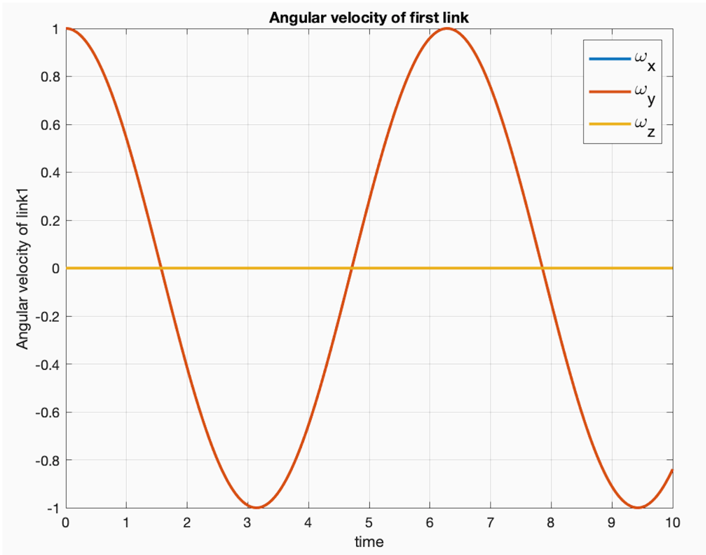

Figure 3 - Angular velocity for link 1

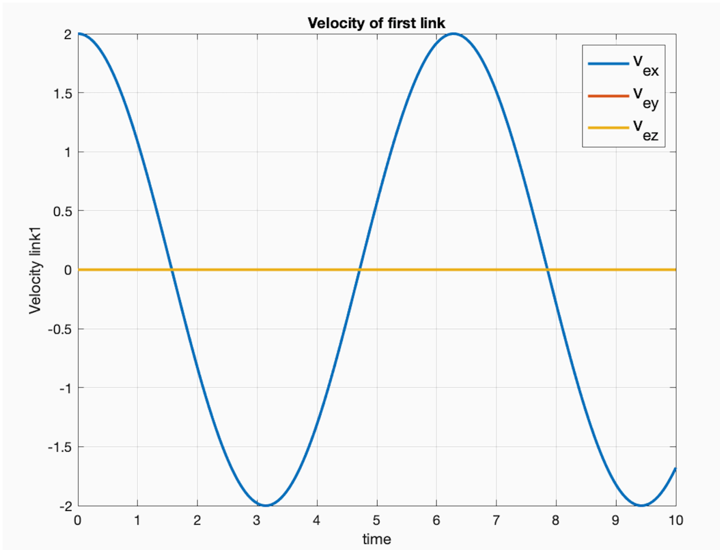

Figure 4 - Linear velocity for link 1

2. *Angular and linear accelerations*

We are needed accelerations of centre of mass of each link to further calculations, it's used in the second Newton's law.

So we get this formulas for accelerations:

$$\dot{\boldsymbol{\omega}}_i^i = \begin{cases} \boldsymbol{R}_i^{i-1T}\dot{\boldsymbol{\omega}}_{i-1}^{i-1} & \text{for a } \textit{prismatic} \text{ joint} \\ \boldsymbol{R}_i^{i-1T}(\dot{\boldsymbol{\omega}}_{i-1}^{i-1} + \ddot{\vartheta}_i \boldsymbol{z}_0 + \dot{\vartheta}_i \boldsymbol{\omega}_{i-1}^{i-1} \times \boldsymbol{z}_0) & \text{for a } \textit{revolute} \text{ joint} \end{cases}$$

$$\ddot{\boldsymbol{p}}_i^i = \begin{cases} \boldsymbol{R}_i^{i-1T}(\ddot{\boldsymbol{p}}_{i-1}^{i-1} + \ddot{d}_i \boldsymbol{z}_0) + 2\dot{d}_i \boldsymbol{\omega}_i^i \times \boldsymbol{R}_i^{i-1T} \boldsymbol{z}_0 & \\ \quad + \dot{\boldsymbol{\omega}}_i^i \times \boldsymbol{r}_{i-1,i}^i + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i-1,i}^i) & \text{for a } \textit{prismatic} \\ \boldsymbol{R}_i^{i-1T} \ddot{\boldsymbol{p}}_{i-1}^{i-1} + \dot{\boldsymbol{\omega}}_i^i \times \boldsymbol{r}_{i-1,i}^i & \\ \quad + \boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i-1,i}^i) & \text{for a } \textit{revolute} \end{cases}$$

Angular and linear accelerations for link 1 you can see on figures 5 and 6.

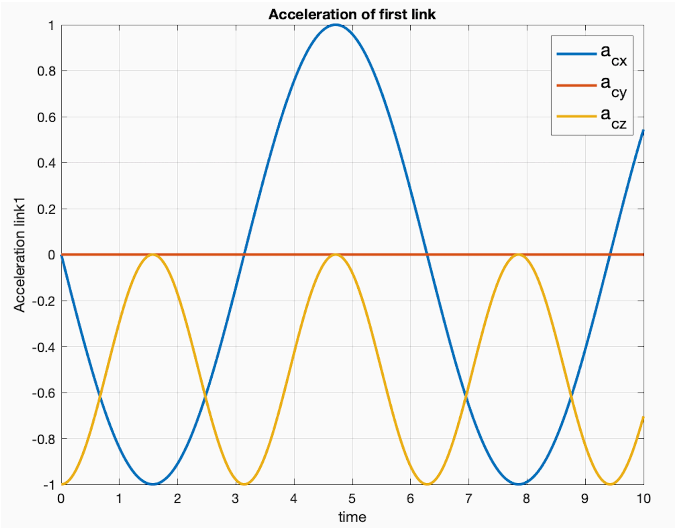

Figure 5- Linear acceleration for centre mass of link 1

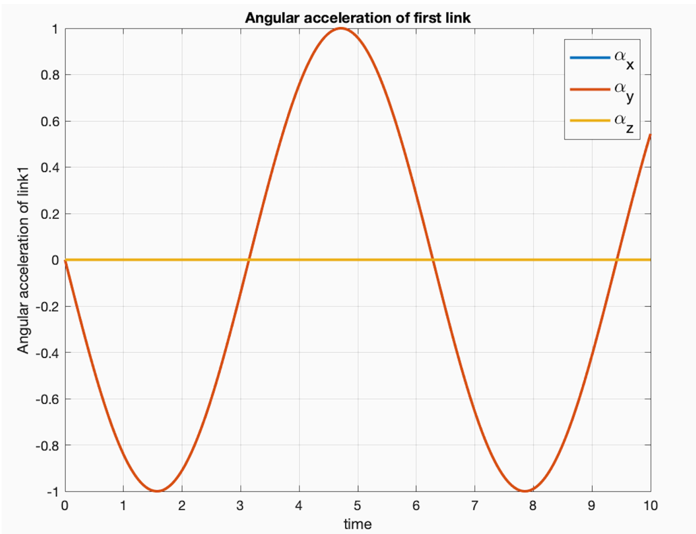

Figure 6 - Angular acceleration of link 1

So, here we define angular accelerations, than acceleration of end of each link, and using plane motion equation and previous knowledge about velocities, seek the accelerations of centre mass.

We can distinguish Centrifugal acceleration for each joint in $\boldsymbol{\omega}_i^i \times (\boldsymbol{\omega}_i^i \times \boldsymbol{r}_{i-1,i}^i)$ term of

For link 3 there is also Coriolis acceleration is included as a result of relative motion of prismatic joint and external movement of links 1 and 2. Plots of this terms for link 3 you can see on the figures 7 and 8.

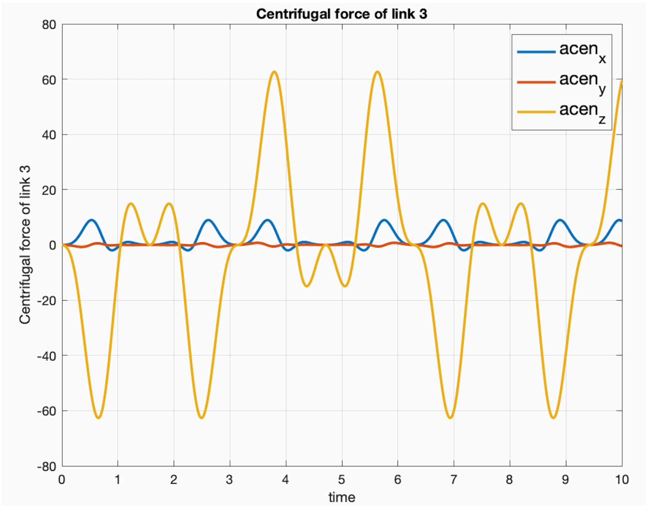

Figure 7 - Centrifugal acceleration of link 3

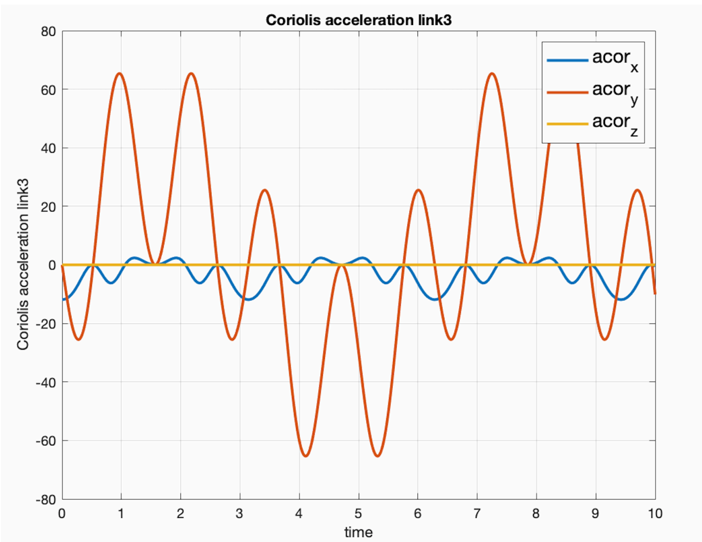

Figure 8 - Coriolis acceleration of link 3

3. Backward propagation

After we know all accelerations, we can define all the forces and momentums for each link, starting from the last one and initial conditions $f_4 = 0$ and $\mu_4 = 0$

$$\begin{aligned}\boldsymbol{f}_i^i &= \boldsymbol{R}_{i+1}^i \boldsymbol{f}_{i+1}^{i+1} + m_i \ddot{\boldsymbol{p}}_{C_i}^i \\ \boldsymbol{\mu}_i^i &= -\boldsymbol{f}_i^i \times (\boldsymbol{r}_{i-1,i}^i + \boldsymbol{r}_{i,C_i}^i) + \boldsymbol{R}_{i+1}^i \boldsymbol{\mu}_{i+1}^{i+1} + \boldsymbol{R}_{i+1}^i \boldsymbol{f}_{i+1}^{i+1} \times \boldsymbol{r}_{i,C_i}^i \\ &\quad + \bar{\boldsymbol{I}}_i^i \dot{\boldsymbol{\omega}}_i^i + \boldsymbol{\omega}_i^i \times (\bar{\boldsymbol{I}}_i^i \boldsymbol{\omega}_i^i)\end{aligned}$$

Here the main formulas:

We need to define Inertia matrices I as a simple matrix of the string.

And generalised force

$$\tau_i = \begin{cases} \boldsymbol{f}_i^{iT} \boldsymbol{R}_i^{i-1T} \boldsymbol{z}_0 + k_{ri} I_{m_i} \dot{\boldsymbol{\omega}}_{m_i}^{i-1T} \boldsymbol{z}_{m_i}^{i-1} \\ \quad + F_{vi} \dot{d}_i + F_{si} \operatorname{sgn}(\dot{d}_i) \\ \boldsymbol{\mu}_i^{iT} \boldsymbol{R}_i^{i-1T} \boldsymbol{z}_0 + k_{ri} I_{m_i} \dot{\boldsymbol{\omega}}_{m_i}^{i-1T} \boldsymbol{z}_{m_i}^{i-1} \\ \quad + F_{vi} \dot{\vartheta}_i + F_{si} \operatorname{sgn}(\dot{\vartheta}_i) \end{cases}$$

But here we use only first terms from both equations (bottom is for revolute and upper id for prismatic joint)

The examples of force, moment for link 1 and generalised forces for all robot (links 1-3) you can see on the figures 9 -11.

Also for correct calculations there are needed to take into account gravity term. It's plot is provided on the figure 12.

Amplitude of the plots highly depends on the initial parameters and conditions.

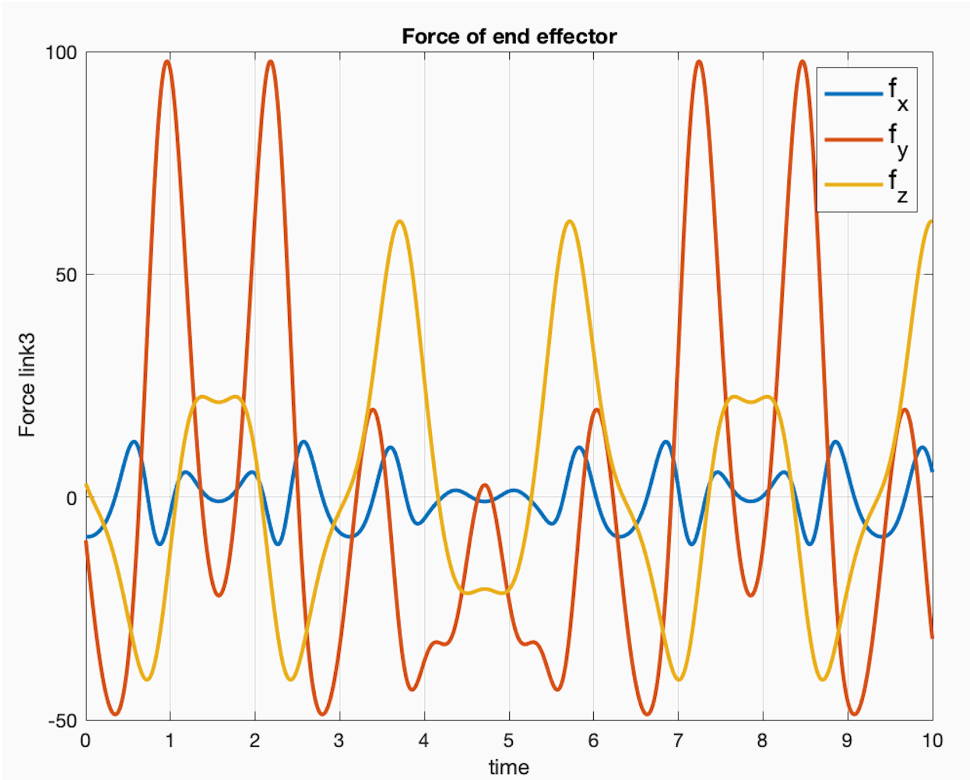

Figure 9 - Forces on link 1

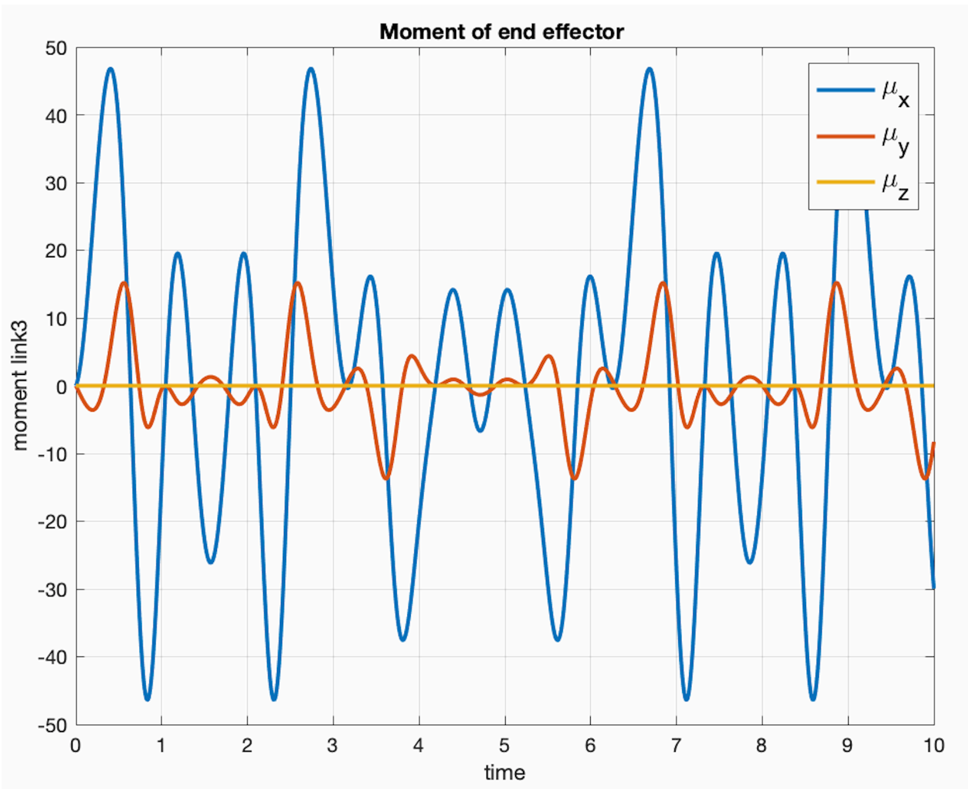

Figure 10 - Moments on link 1

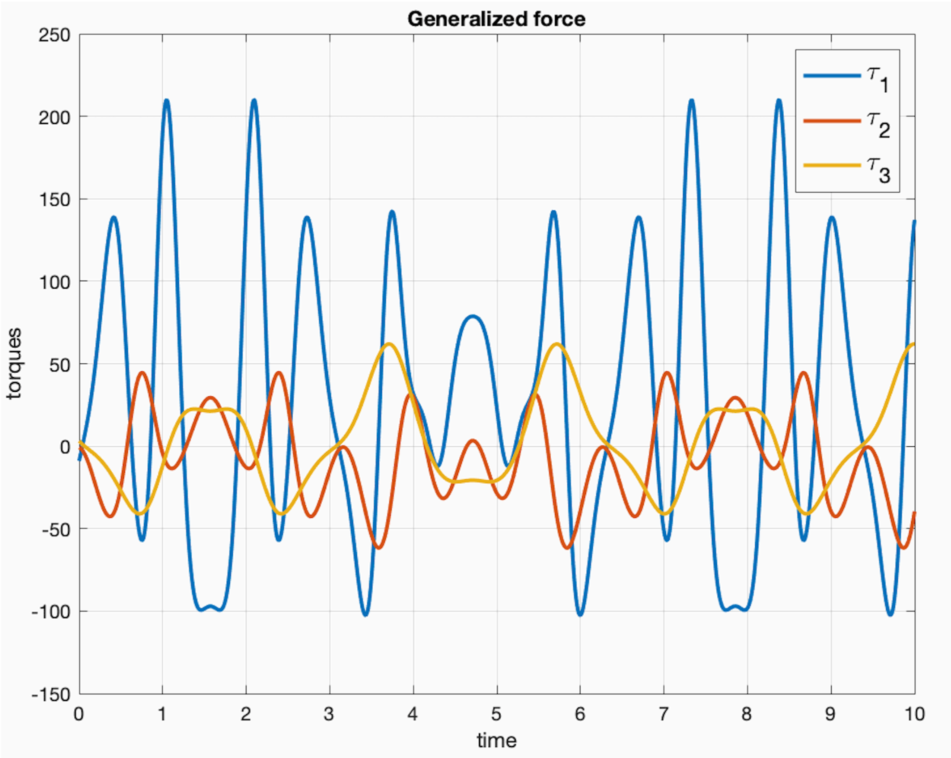

Figure 11 - Torques on 3 RRP robot manipulator

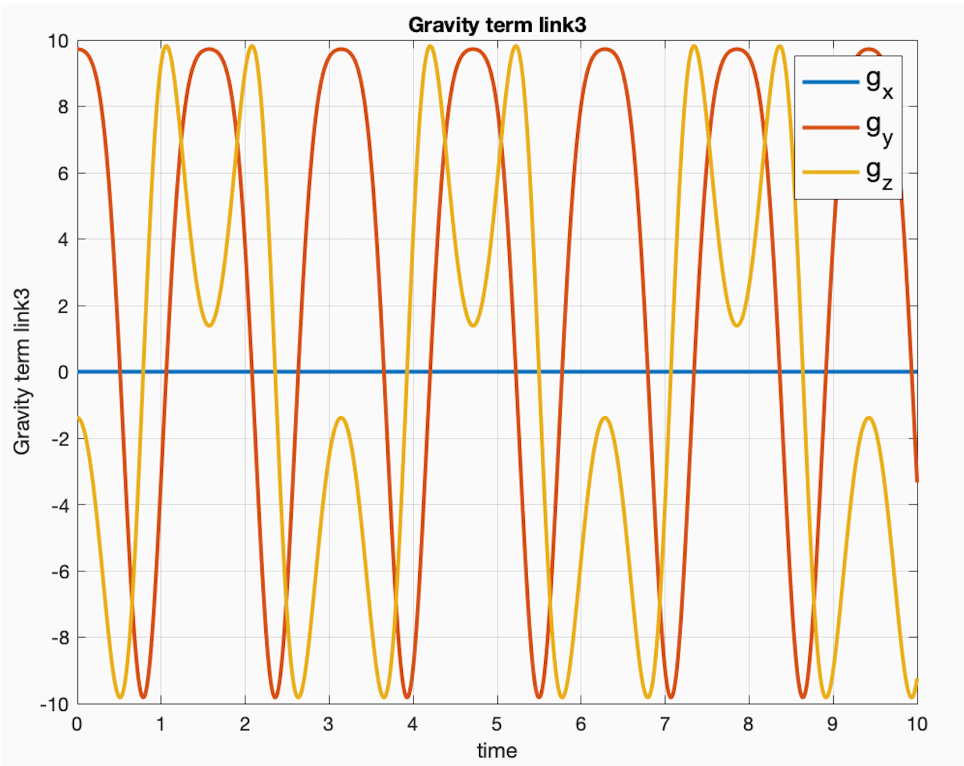

Figure 12 - Gravity term of link 1