Report

Dynamics of non linear robotic system

Homework assignment №3

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GitLab Link

The MatLab code you can find on this link:

https://github.com/rodosha98/DynamicsJacobian.git

Robot description

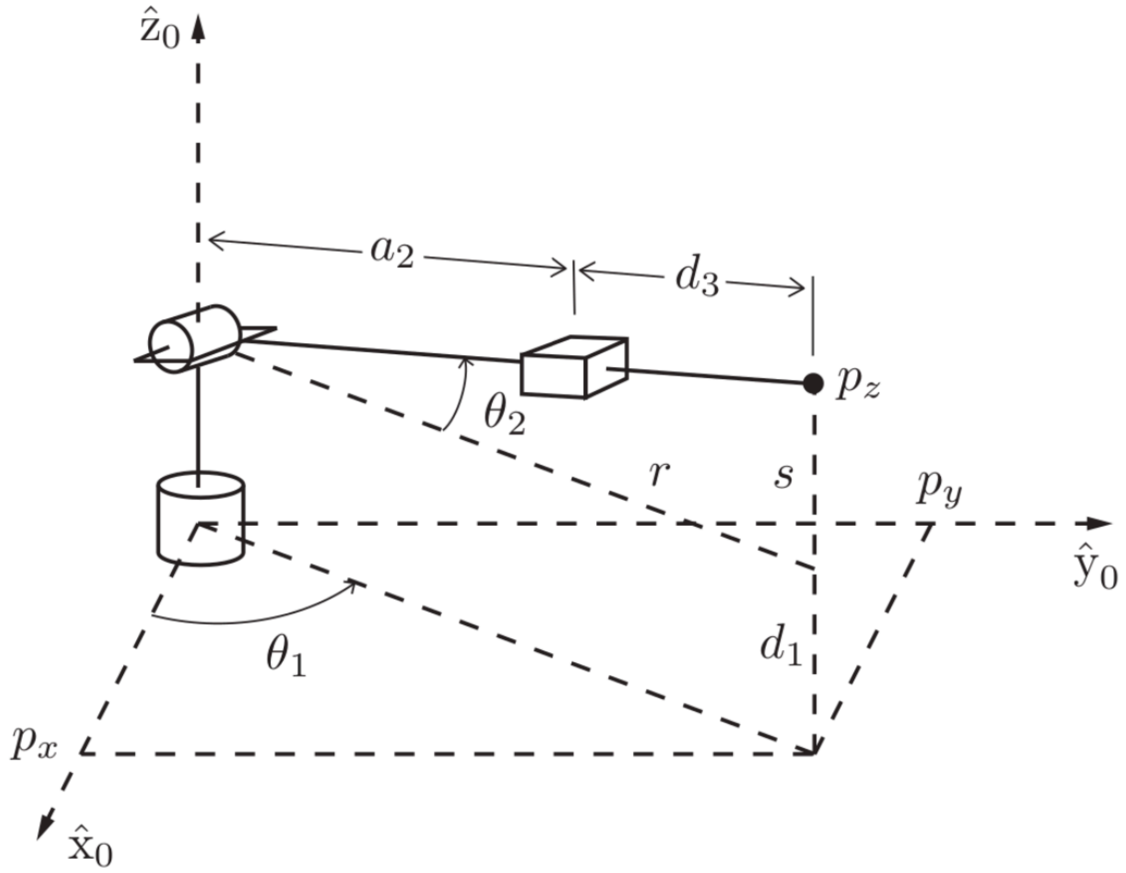

Figure 1 - Robot's scheme

The robot is comprised of 4 links (including ground) and 3 joints, 2 of them are rotational and the last one - translational. We have 3 joint variables: θ_1, θ_2 and d_3.

1. Forward kinematics

Joint variables are given. It's required to find operational variables.

Algebraic approach:

$T = R_z(\theta_1) \bullet T_z(d_1) \bullet R_y(-\theta_2) \bullet T_x(a_2) \bullet T_x(d_3)$

The resulting matrix:

$$T = \begin{pmatrix} cos(\theta_1) \bullet cos(\theta_2) & -sin(\theta_1) & -cos(\theta_1) \bullet sin(\theta_2) & (a_2 + d3) \bullet cos(\theta_1) \bullet cos(\theta_2) \\ cos(\theta_2) \bullet sin(\theta_1) & cos(\theta_1) & -sin(\theta_1) \bullet sin(\theta_2) & (a_2 + d3) \bullet sin(\theta_1) \bullet cos(\theta_2) \\ sin(\theta_2) & 0 & cos(\theta_2) & d_1 + (a_2 + d_3) \bullet sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Operational space:

$p_x = (a_2 + d3) \bullet cos(\theta_1) \bullet cos(\theta_2)$

$p_y = (a_2 + d3) \bullet sin(\theta_1) \bullet cos(\theta_2)$

$p_z = d_1 + (a_2 + d3) \bullet sin(\theta_2)$

2. Inverse kinematics

Operational variables p_x, p_y, p_z are given.

$\theta_1 = atan2(p_y, p_x)$

$r^2 = p_x^2 + p_y^2$

$r = \sqrt{p_x^2 + p_y^2}$

$s = p_z - d_1$

$r^2 + s^2 = (a_2 + d_3)^2$

$d_3 = \sqrt{r^2 + s^2} - a_2$

$\theta_2 = atan2(s, r)$

Results:

Solution1:

$\theta_1 = atan2(p_y, p_x)$

$\theta_2 = atan2(s, r)$

$d_3 = \sqrt{r^2 + s^2} - a_2$

Solution2:

$\theta_1 = atan2(p_y, p_x) + \pi$

$\theta_2 = \pi - atan2(s, r)$

$d_3 = \sqrt{r^2 + s^2} - a_2$

3. Jacobians

Classical approach

To find Jacobian velocity matrix, that is needed to find partial derivatives of p_x, p_y, p_z :

$$J_v = \begin{pmatrix} \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial \theta_2} & \frac{\partial p_x}{\partial d_3} \\ \frac{\partial p_y}{\partial \theta_1} & \frac{\partial p_y}{\partial \theta_2} & \frac{\partial p_y}{\partial d_3} \\ \frac{\partial p_z}{\partial \theta_1} & \frac{\partial p_z}{\partial \theta_2} & \frac{\partial p_z}{\partial d_3} \end{pmatrix}$$

I was working in MatLab to estimate the derivatives and here the result:

$$J_v = \begin{pmatrix} -(a_2 + d_3) \bullet cos(\theta_2) \bullet sin(\theta_1) & -(a_2 + d_3) \bullet cos(\theta_1) \bullet sin(\theta_2) & cos(\theta_1) \bullet cos(\theta_2) \\ (a_2 + d_3) \bullet cos(\theta_1) \bullet cos(\theta_2) & -(a_2 + d_3) \bullet sin(\theta_1) \bullet sin(\theta_2) & sin(\theta_1) \bullet cos(\theta_2) \\ 0 & (a_2 + d3) \bullet cos(\theta_2) & sin(\theta_2) \end{pmatrix}$$

Jacobian of rotations defines by number of revolute joints(2). The first rotational axis z isn't moving, so it's part in Jacobian is (0; 0; 1). But the second is moving and we can esimate the projections of this axis on zero frame(figure2):

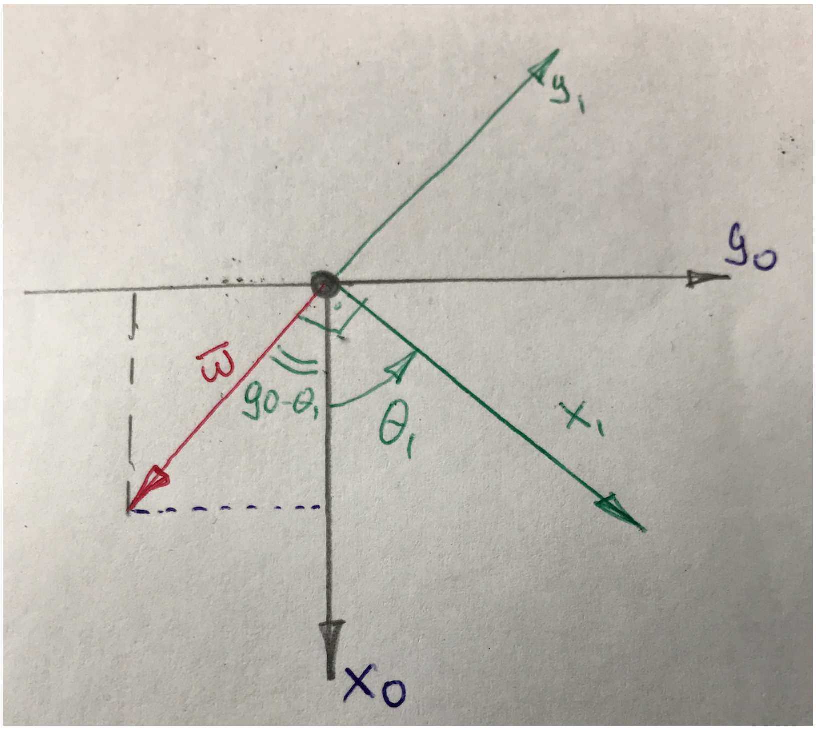

Figure 2 - Moving axis's vector

Projections on zero frame:

$$\omega_x = |\omega| \bullet cos(90 - \theta_1) = sin(\theta_1)$$

$$\omega_y = -|\omega| \bullet sin(90 - \theta_1) = -cos(\theta_1)$$

$\omega_z = 0$

Rotational Jacobian:

$$J_\omega = \begin{pmatrix} 0 & sin(\theta_1) & 0 \\ 0 & -cos(\theta 1) & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Geometric approach

Rules are given on the figure 3:

	Prismatic	Revolute
Linear	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(d_n^0 - d_{i-1}^0\right)$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Figure 3 - Geometric approach

We have 4 links, 3 joints(2 revolute, 1 prismatic). We are needed to assign the joint frames according to DH convention, but deriving DH parameters isn't required. You can see the frames on figure 4:

So now we can use previous knowledge and find Jacobian matrix:

$$R_0^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_1^0 = R_z(\theta_1) \bullet R_x(\pi/2)$$

$$R_2^0 = R_1^0 \bullet R_2^1 = R_1^0 \bullet R_z(\theta_2) \bullet R_y(\pi/2) \bullet R_z(\pi/2)$$

$$d_3^0 = O_3$$

$$d_2^0 = O_2$$

$$d_1^0 = O_1$$

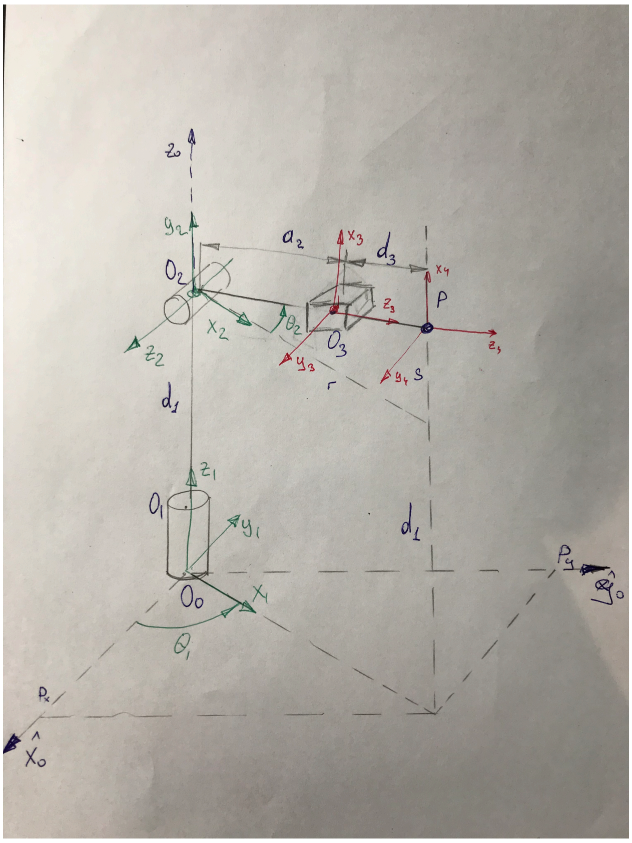

Figure 4 - Joint frames

Unit vector z:

$u_z = [0; 0; 1]$

$J_v^G = [(R_0^0 \bullet u_z) \times (d_3^0 - d_0^0), (R_1^0 \bullet u_z) \times (d_3^0 - d_1^0), (R_2^0 \bullet u_z)]$

$J_\omega^G = [R_0^0 \bullet u_z, R_1^0 \bullet u_z, R_2^0 \bullet u_z]$

So, both classical and geometric approaches have given the same result:

$$J = \begin{pmatrix} -(a_2 + d_3) \bullet cos(\theta_2) \bullet sin(\theta_1) & -(a_2 + d_3) \bullet cos(\theta_1) \bullet sin(\theta_2) & cos(\theta_1) \bullet cos(\theta_2) \\ (a_2 + d_3) \bullet cos(\theta_1) \bullet cos(\theta_2) & -(a_2 + d_3) \bullet sin(\theta_1) \bullet sin(\theta_2) & sin(\theta_1) \bullet cos(\theta_2) \\ 0 & (a_2 + d3) \bullet cos(\theta_2) & sin(\theta_2) \\ 0 & sin(\theta_1) & 0 \\ 0 & -cos(\theta 1) & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

4. Singular cases

We have only 3 joint variables. To derive singular cases that is needed to consider only linear part of Jacobian(square matrix 3 by 3) and find determinant of this matrix. In this case we will have only positional singularities(there is no wrist).

Let’s denote:

$cos(\theta_1) = c_1; sin(\theta_1) = s_1$

$cos(\theta_2) = c_2; sin(\theta_2) = s_2$

$$J_v = \begin{pmatrix} -(a_2 + d_3) \bullet c_2 \bullet s_1 & -(a_2 + d_3) \bullet c_1 \bullet s_2 & c_1 \bullet c_2 \\ (a_2 + d_3) \bullet c_1 \bullet c_2 & -(a_2 + d_3) \bullet s_1 \bullet s_2 & c_2 \bullet s_1 \\ 0 & (a_2 + d3) \bullet c_2 & s_2 \end{pmatrix}$$

$$det(J_v) = -c_2 \bullet s_1 \bullet (a_2 + d_3) \bullet (-s_1 \bullet s_2^2 \bullet (a_2 + d_3) - c_2^2 \bullet s_1 \bullet (a_2 + d_3)) - (-c_1 \bullet s_2 \bullet (a_2 + d_3) \bullet (c_1 \bullet c_2 \bullet (a_2 + d_3) \bullet s_2) + c_1 \bullet c_2 \bullet (c_1 \bullet c_2^2 \bullet (a_2 + d_3)^2) =$$

$$= c_2 \bullet s_1^2 \bullet (a_2 + d_3)^2 + c_1^2 \bullet c_2 \bullet s_2^2 \bullet (a_2 + d_3)^2 + c_1^2 \bullet c_2^3 \bullet (a_2 + d_3)^2 = (a_2 + d_3)^2 \bullet (c_2 \bullet s_1^2 + c_1^2 \bullet c_2 \bullet (s_2^2 + c_2^2))$$

$det(J_v) = c_2 \bullet (a_2 + d_3)^2$

So, we have only 1 singular case: $c_2 = 0; \theta_2 = \pi/2 + \pi \bullet n$.

In this position links 1 and 2 becomes collinear and and directed up(manipulator is extended). We also can have singular case when $a_2 = -d_3$, but in the case r and s both equal to zero and, hence $p_x, p_y = 0; p_z = d_1$. It’s special case of the general position of the singularity described above.

5. Velocity of the tool frame

$\theta_1(t) = sin(t)$

$\theta_2(t) = cos(2t)$

$d_3(t) = sin(3t)$

We can write derivatives of this values:

$\dot{\theta}_1(t) = cos(t)$

$\dot{\theta}_2(t) = -2 \bullet sin(2t)$

$\dot{d}_3(t) = 3 \bullet sin(3t)$

Also we have already known the Jacobian and can estimate velocities using this formula:

$$\xi = J(q)\dot{q}$$

Results are represented by 2 plots of linear and angular velocities on figure 5 and figure 6 correspondingly:

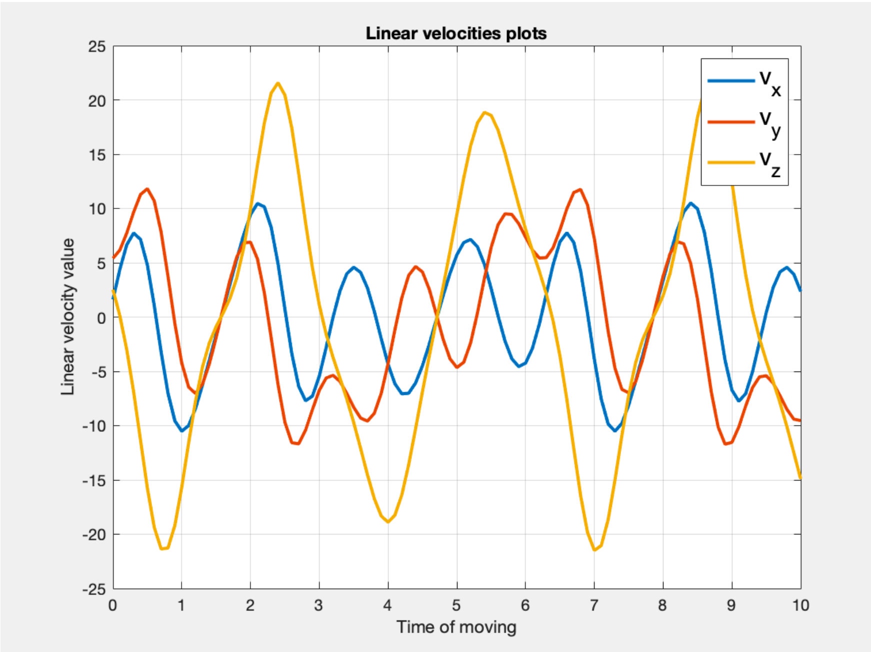

Figure 5 - Linear velocities

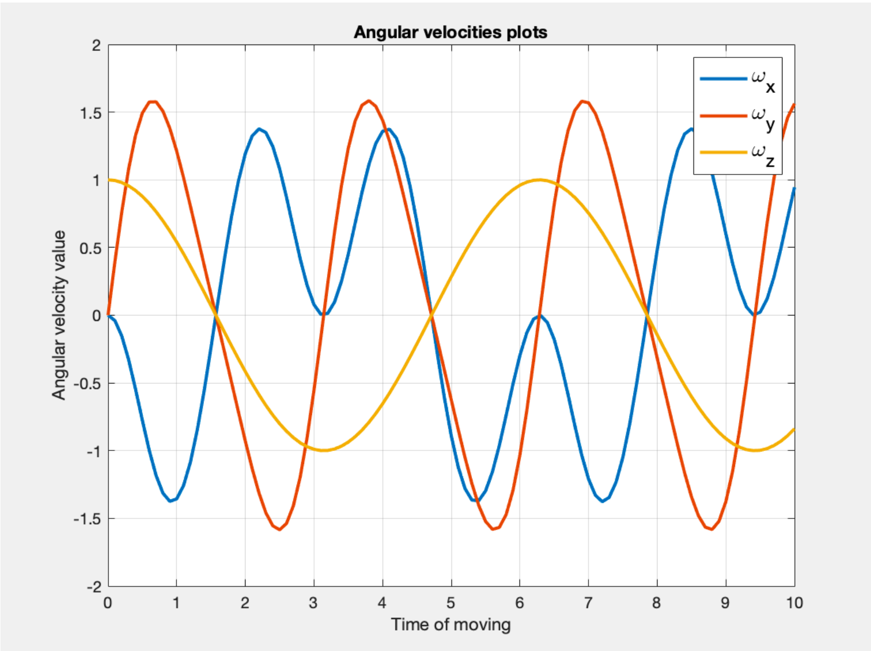

Figure 6 -Angular velocities

6. Feasible joint trajectory

Inverse kinematics solution

Because of we've already known the inverse kinematics solution. I've chosen parameters $d_1 = 20, a_2 = 10$. And we are getting 2 cases as we have 2 solutions of IK(figures 7 and 8) :

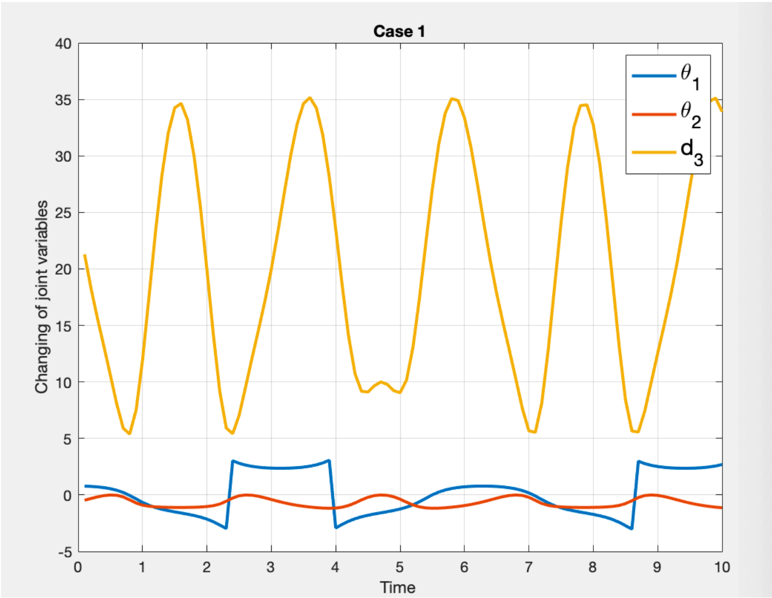

Figure 7 - Joint variables trajectory on solution 1

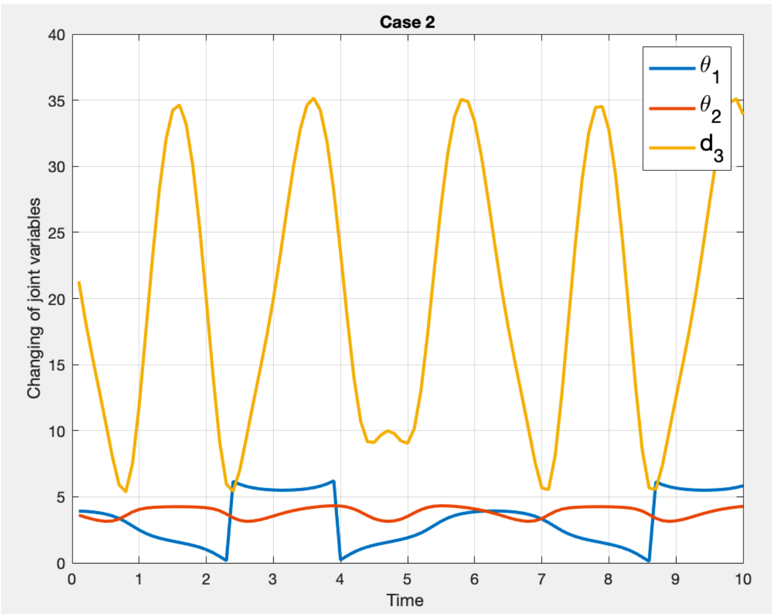

Figure 8 - Joint variables trajectory on solution 2

Differential kinematics approach

That is numerical approach. The main idea is to find derivative of joint variable vector:

$$v = J \bullet \dot{q}$$

$$\dot{q} = J^{-1} \bullet v$$

$$q = \int_0^t J^{-1} \bullet v + q_0$$

q_0 - initial condition;

$p_x(t) = 2 \bullet a_2 \bullet sin(t)$

$p_y(t) = 2 \bullet a_2 \bullet cos(2t)$

$p_z(t) = d_1 \bullet sin(3t)$

Velocities:

$\dot{p}_x(t) = 2 \bullet a_2 \bullet cos(t)$

$\dot{p}_y(t) = -4 \bullet a_2 \bullet sin(2t)$

$\dot{p}_z(t) = 3 \bullet d_1 \bullet cos(3t)$

Next task is a numerical integration. And here a lot depends on the method chosen. We can compare results with previous graphs and judge about precision of the current method. That is also required to assign initial conditions, which I've chosen from IK solution.

The first variant is Euler integration. It's simple iterative method, the main formula is:

$$q_{k+1} = q_k + \dot{q}_k \cdot \Delta t$$

The results you can see on the figure 9.

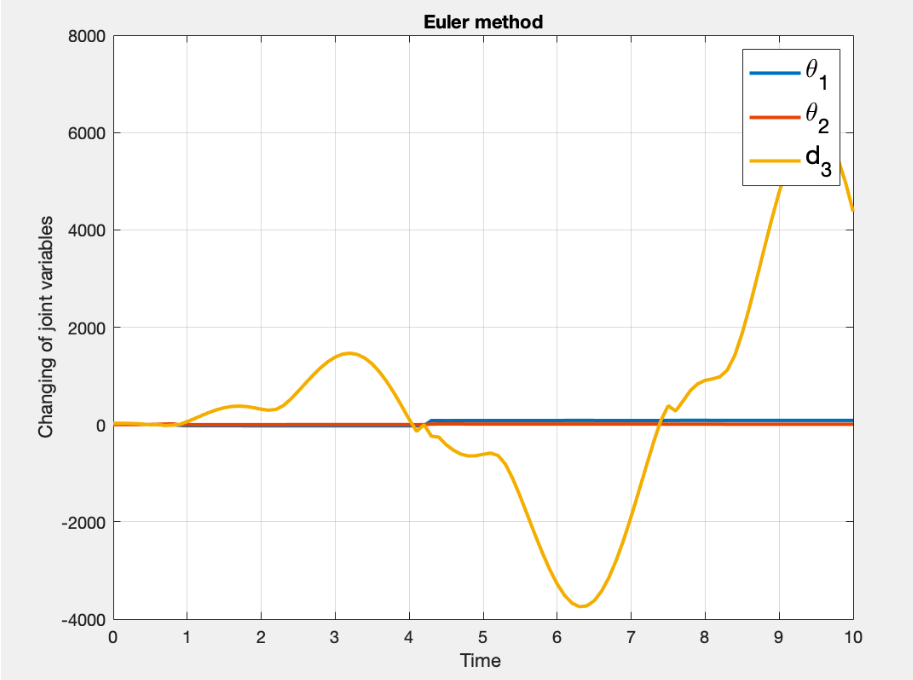

Figure 9 - Euler method solution

Comparing with plots on figure 7, which corresponds to the desired position, you can notice, that Euler method gives too much error and not satisfy requirements in this case. So, I've chosen another method(Runge-Kutta of 5 order, ode45). The results you can see on the figure 10 and it's much better and similar to the real one. Also the error is palpable because of chosen values of d_1 and a_2.

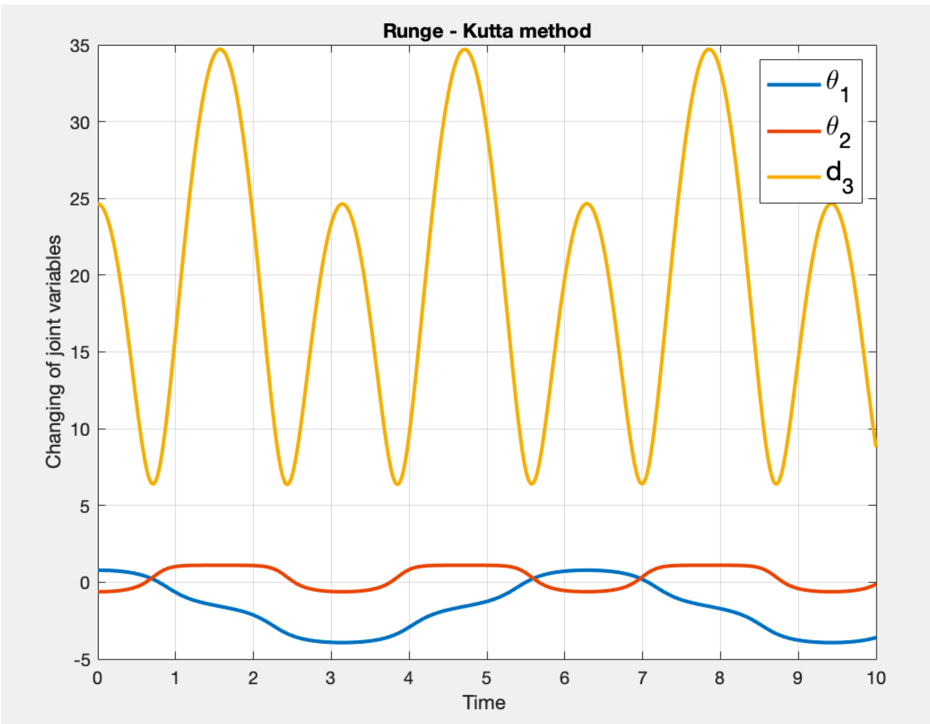

Figure 10 - Runge - Kutta algorithm results