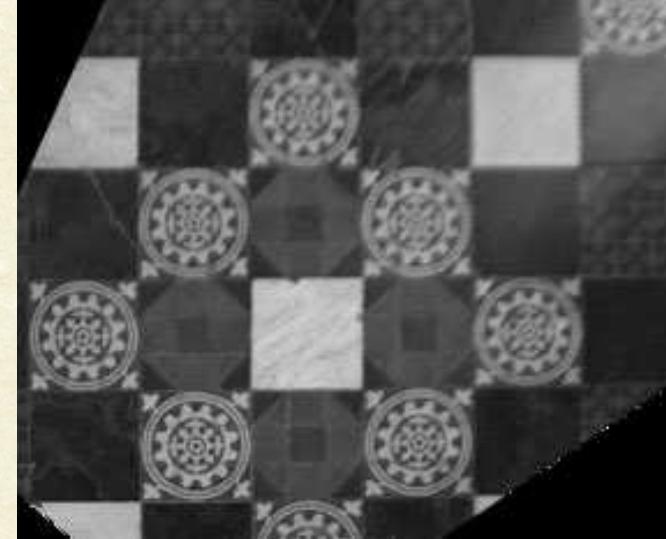




CS7.505: Computer Vision

Spring 2022: Image Rectification:
Recovering Structure from Single Image

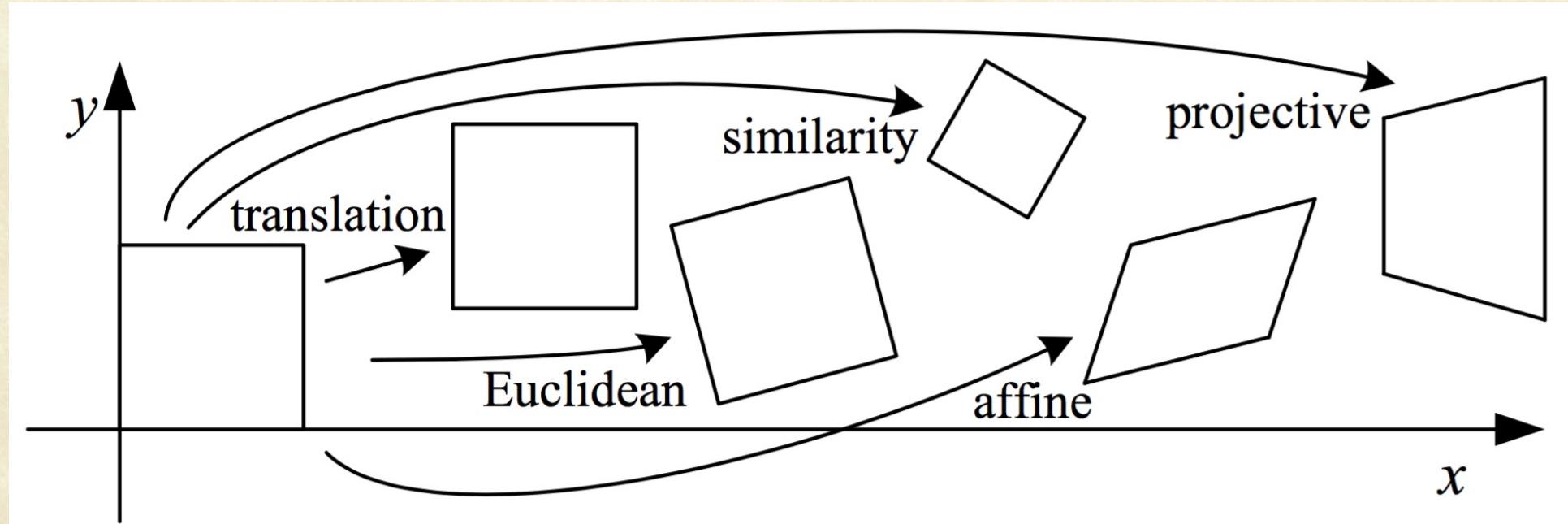


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Hierarchy of Transformations





Hierarchy of Transformations

- Translation (2)

$$\rightarrow \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$$

- Euclidean (3)
or Rigid Body

$$\begin{bmatrix} c_\theta & -s_\theta & t_x \\ s_\theta & c_\theta & t_y \end{bmatrix}$$

- Similarity (4)

$$\rightarrow \begin{bmatrix} 1+a & -b & t_x \\ b & 1+a & t_y \end{bmatrix}$$

$$\begin{bmatrix} 1+a & b & t_x \\ c & 1+d & t_y \end{bmatrix}$$

- Affine (6)

$$\rightarrow \begin{bmatrix} 1+h_{00} & h_{01} & h_{02} \\ h_{10} & 1+h_{11} & h_{12} \\ h_{20} & h_{21} & 1 \end{bmatrix}$$

- Projective (8)



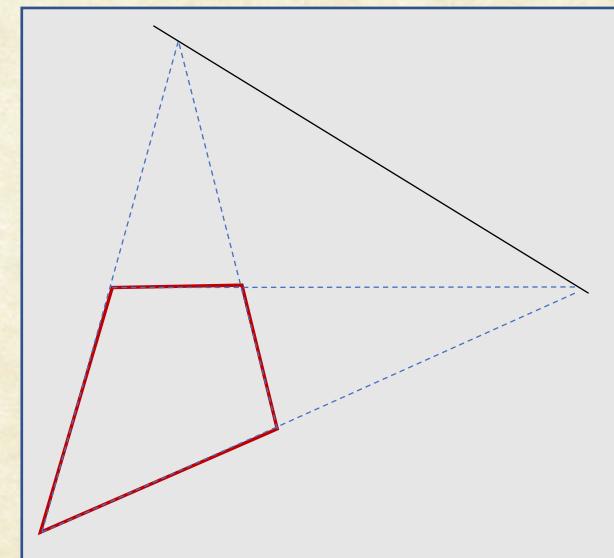
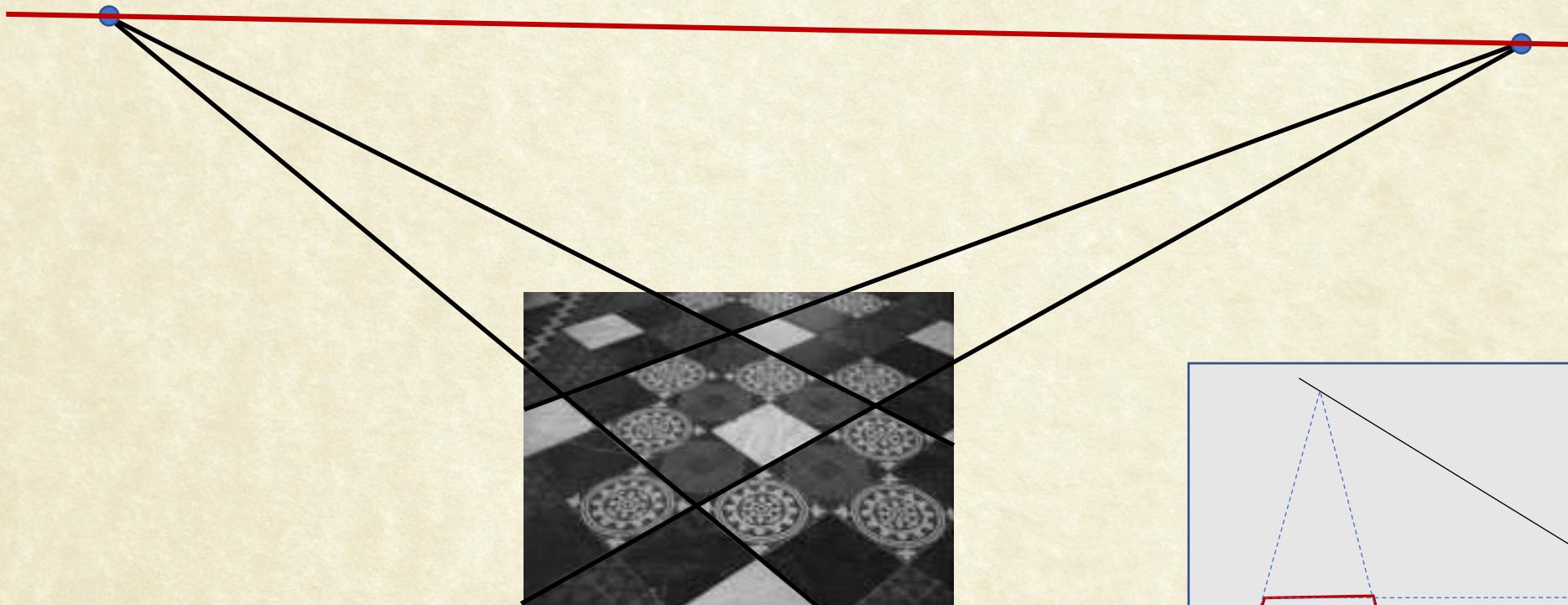
Invariants of Transformations

Property	Euclidean	Similarity	Affine	Projective
Length				
Angle				
Length Ratio				
Area Ratio				
Parallelism				
Centroid				
Ratio of len. ratio				
Collinearity				



Finding Line at Infinity

- Line at infinity can be found in the image from 2 sets of parallel lines.



- Rectangle imaged as:



Affine Structure from Images

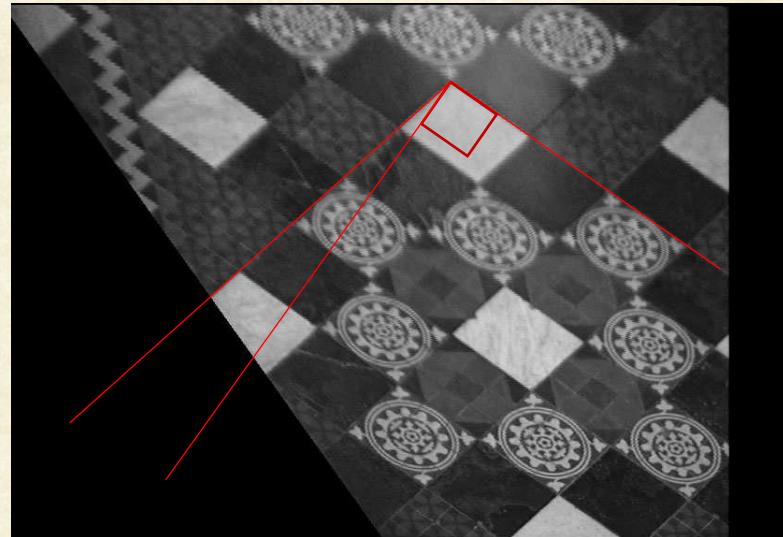
- Affine structure gives parallelism, ratio of areas, centroid, etc., and can be the basis of many decisions.
- Find l_∞ in image using parallel lines.
- Apply a transformation H that maps the line to $[0 \ 0 \ 1]^\top$

• $H_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$ sends points on $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$.

$$\text{So } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}^{-T} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Affine Rectification



- Parallel lines are parallel after rectification
- Angles are not restored (see right angles)



Circular Points

- Affinity maps l_∞ to itself. Conversely, any transformation that does that is an affine one
- General projectivity can map l_∞ to a finite line and vice versa
- A circle intersects l_∞ at **circular points**. Canonical (Euclidean) circle is: $x^2 + y^2 + dxw + eyw + fw^2 = 0$.
- Points on l_∞ have $w = 0$. Thus, $x^2 + y^2 = 0$.
- Circular points are given canonically by:

$$I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \quad \text{and} \quad J = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$$



Circular Points to Similarity

- Circular points are fixed under similarity

$$\begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} = \begin{bmatrix} s(\cos\theta - i\sin\theta) \\ s(\sin\theta + i\cos\theta) \\ 0 \end{bmatrix} = se^{-i\theta} \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix}$$

- Conversely, any transformation that fixes circular points is a similarity.
- Thus, a transformation \mathbf{H} that sends the circular points to their canonical form \mathbf{I} and \mathbf{J} leaves only a similarity transformation.

See: Serge Belongie: <https://cseweb.ucsd.edu/classes/sp04/cse252b/notes/lec06/lec6.pdf>

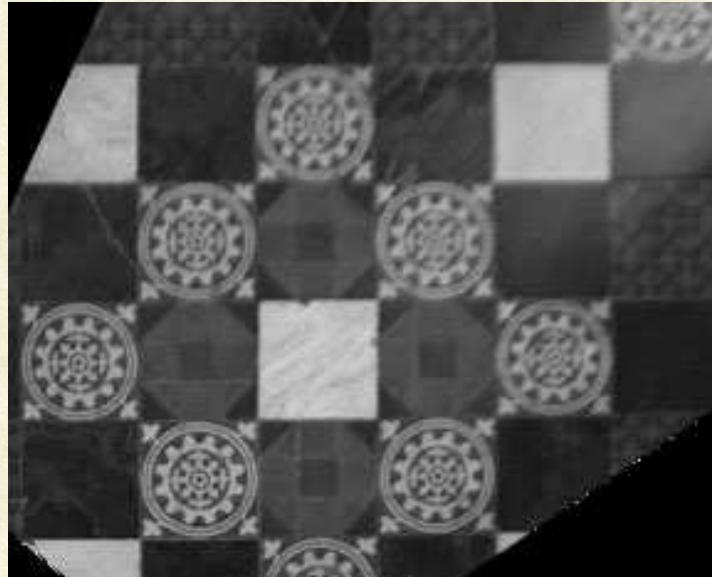


Metric Structure from Images

- Identify circular points in the image. This can be done by finding a world circle in the image as a conic, finding the I_∞ in the image and finding their intersection
- Map one circular point to I and the other to J. The transformation H that does it metric rectifies the image.
- I_∞ gives affine structure, the circle gives metric structure. 
- Can be done using 2 non-parallel orthogonal line pairs instead of a circle or 5 orthogonal line pairs from projective!



Metric Rectification



- Resulting image is a scale away from actual image



CS7.505: Computer Vision



Spring 2022: 3D Reconstruction:
Recovering Structure from Single/Multiple Images



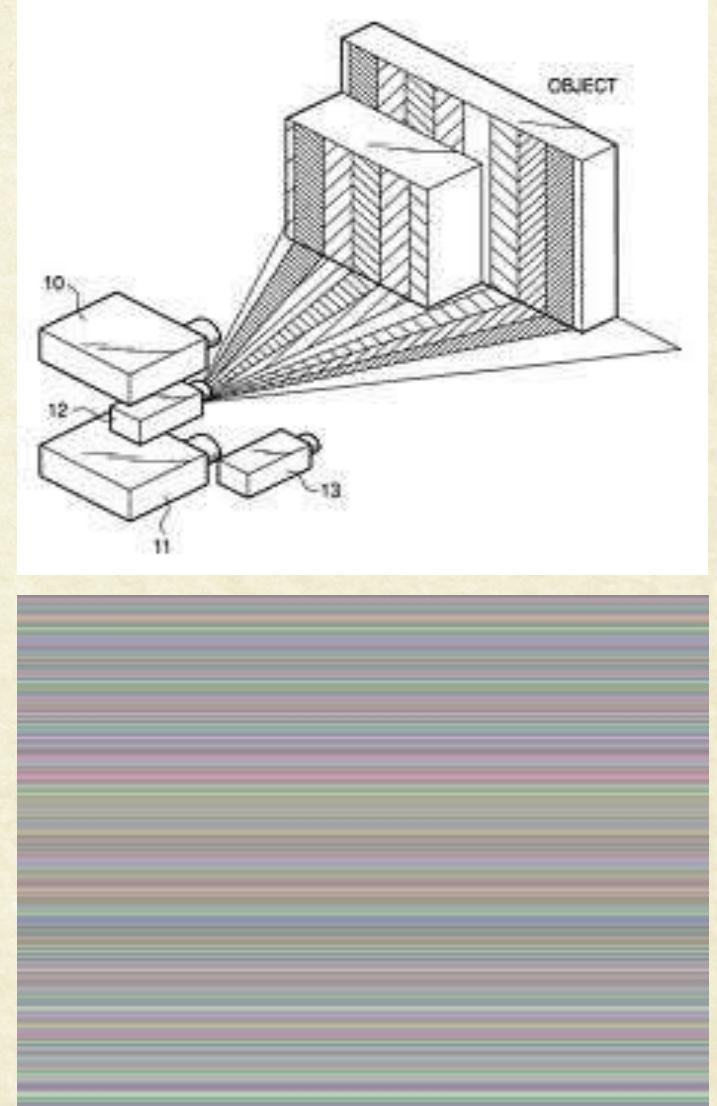
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Structured Lighting

- Finding correspondences is hard by itself
- Can we help it by projecting patterns onto the world?
- Structured Lights!
- Light-strip range finders, etc.
- Combination of sinusoids sometimes to get dense matches
- Active vision, as it changes the appearance
- The light projected need not be in the visible spectrum

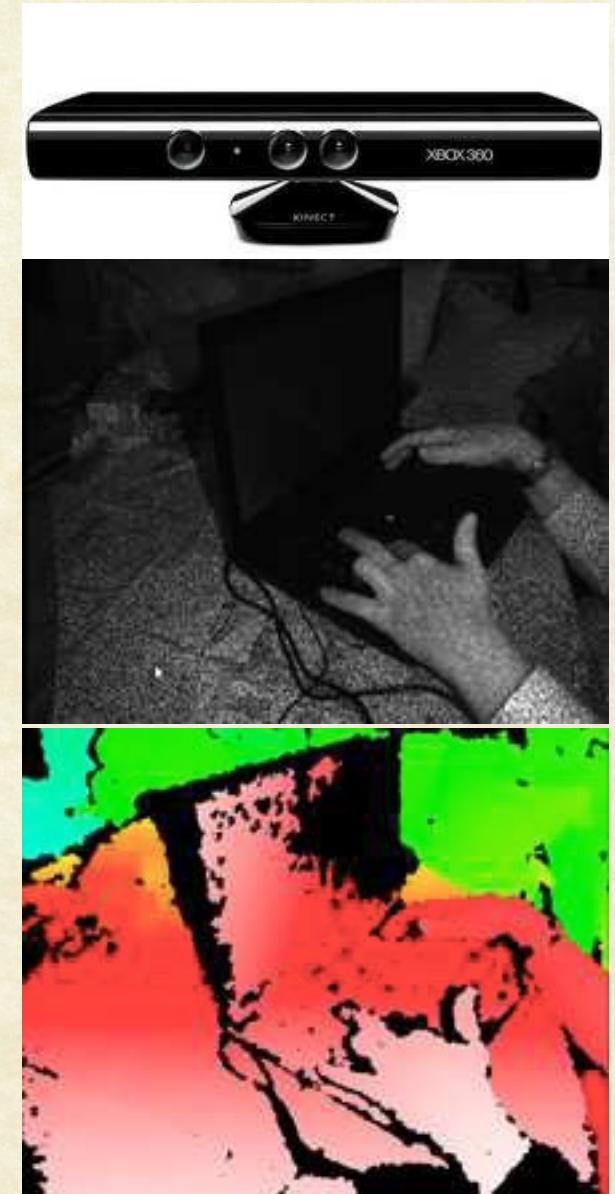




Xbox Kinect

IR-based range sensor for Xbox

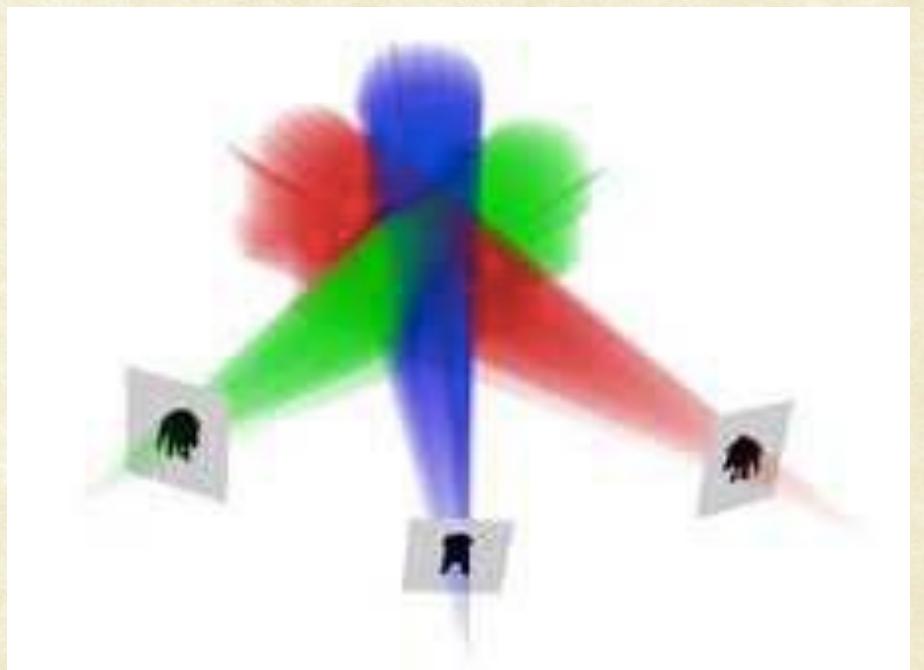
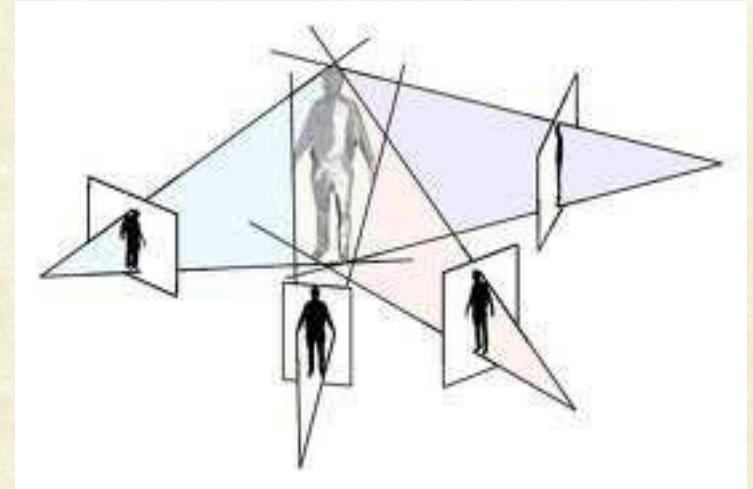
- Aligned depth and RGB images at 640×480
- Original goal: Interact with games in full 3D
- Computer vision happy with real-time depth and image
 - Games, HCI, etc
 - Action recognition
 - Image based modelling of dynamic scenes
- Fastest selling electronic appliance ever!!
- Other products that use PrimeSense sensor





Visual Hull

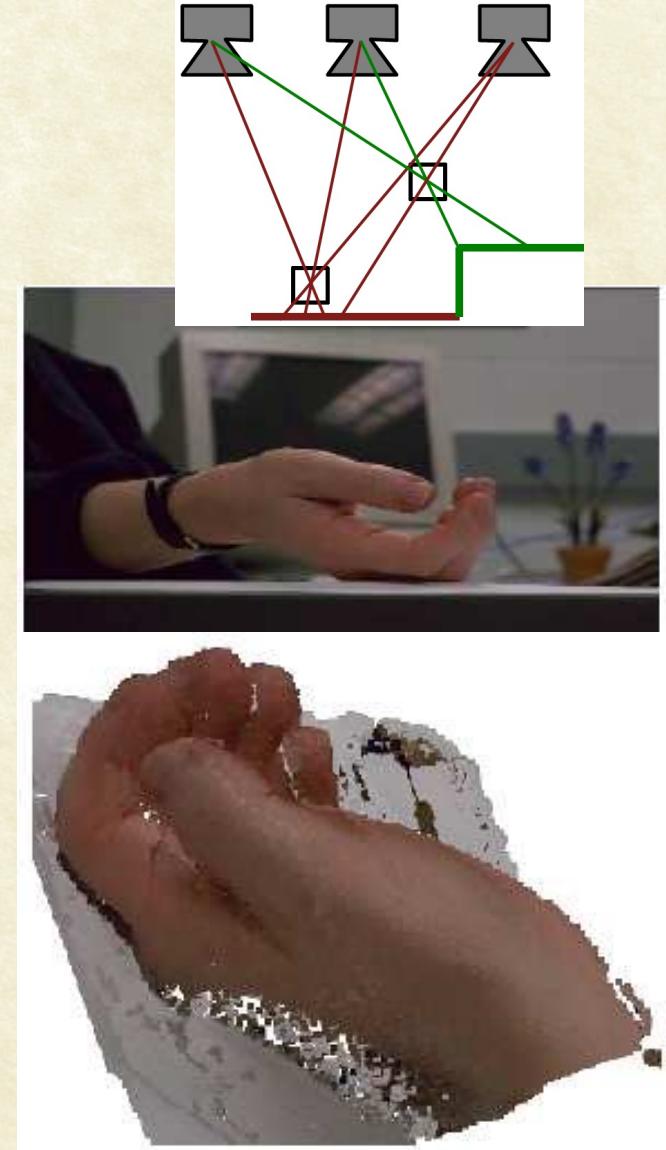
- Object silhouette represents a generalized cone with the camera centre as the apex
- Intersect these cones for multiple views in the 3D space
- Visual Hull, like convex hull
- Cannot get fine details like concavities
- Gives a very good, approximate shape, without scene modification!
- Generalized Visual Hull: RGBD Sensors





Space Carving

- Reason directly in a volumetric voxel space
- If a voxel is filled, it projects to similar colours in all cameras
- If a voxel is empty, its projections will have different appearances
- Colour consistency: filled or empty?
- Assume all filled initially; carve out empty ones by going over the images, guessing visibility, etc.





Questions?