

# On Edges, Templates, Texture

Lecture 9

Stella Yu

Slide Credit: Alyosha Efros

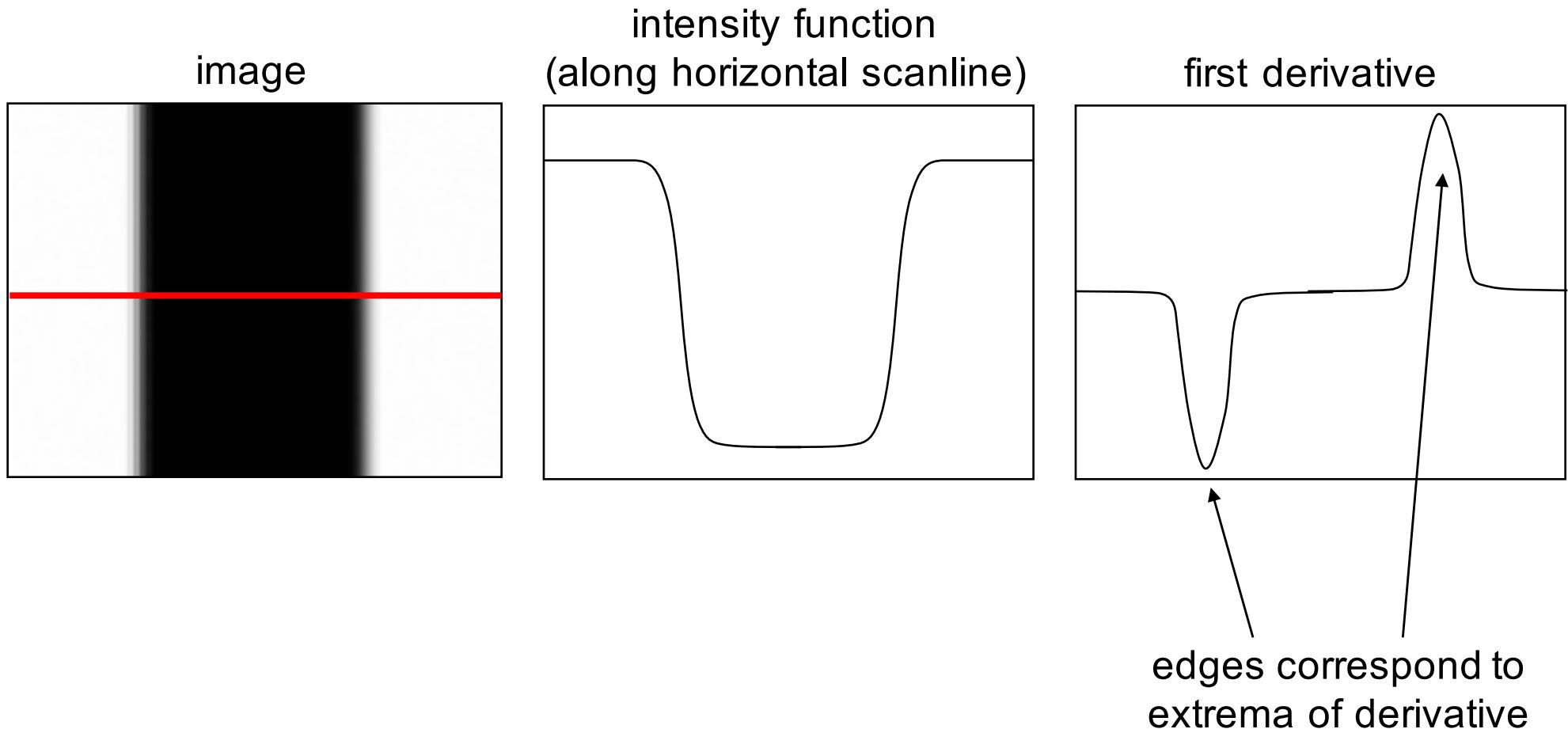
# Cinque Terre, May 2005



# Characterizing edges

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- An edge is a place of rapid change in the image intensity function



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Taking derivative by convolution

# Partial derivatives with convolution

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For 2D function  $f(x,y)$ , the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x + 1, y) - f(x, y)}{1}$$

To implement above as convolution, what would be the associated filter?

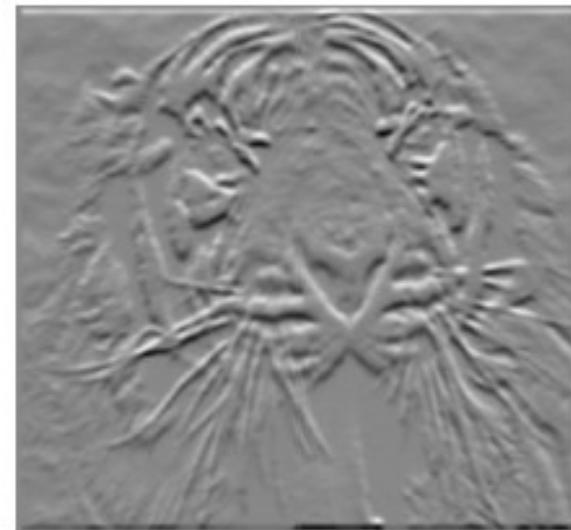
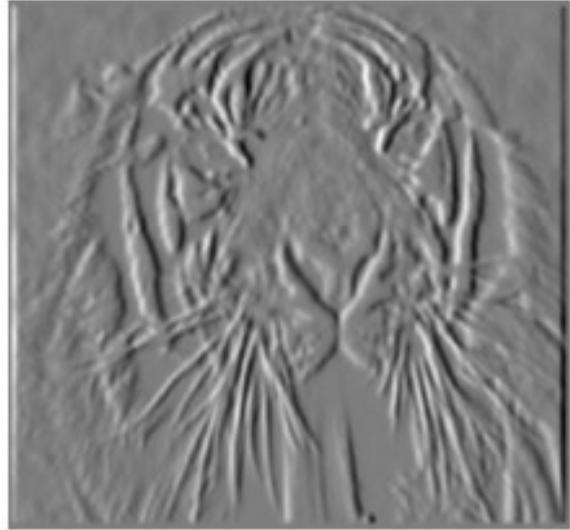
# Partial derivatives of an image

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$$\frac{\partial f(x, y)}{\partial x}$$

-1	1
----	---



$$\frac{\partial f(x, y)}{\partial y}$$

-1
1

or

1
-1

Which shows changes with respect to x?

# Finite difference filters

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Other approximations of derivative filters exist:

**Prewitt:**  $M_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$  ;  $M_y = \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$

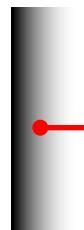
**Sobel:**  $M_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$  ;  $M_y = \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$

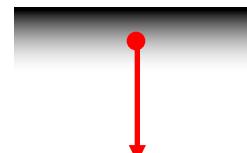
**Roberts:**  $M_x = \begin{array}{|c|c|} \hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array}$  ;  $M_y = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$

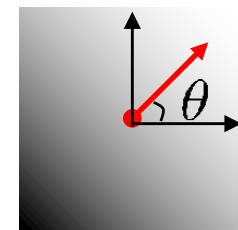
# Image gradient

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The gradient of an image:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$


$$\nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right]$$


$$\nabla f = \left[ 0, \frac{\partial f}{\partial y} \right]$$


$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

The gradient points in the direction of most rapid increase in intensity

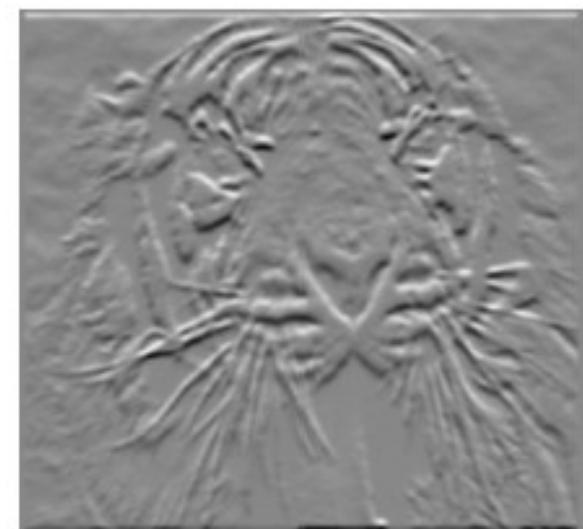
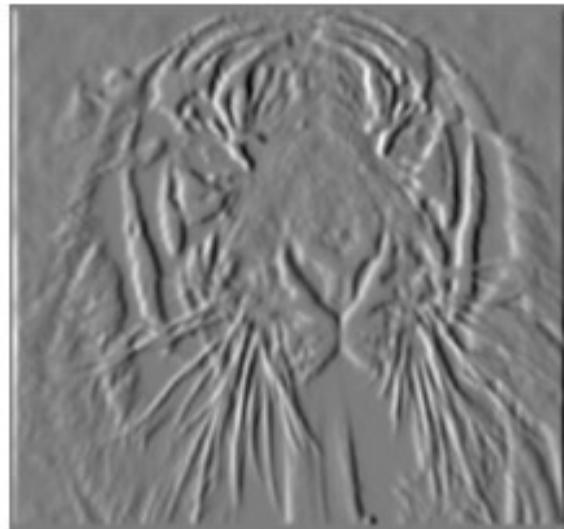
- How does this direction relate to the direction of the edge?

The gradient direction is given by  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}$$

# Image Gradient



$$\frac{\partial f(x, y)}{\partial x}$$

$$\frac{\partial f(x, y)}{\partial y}$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

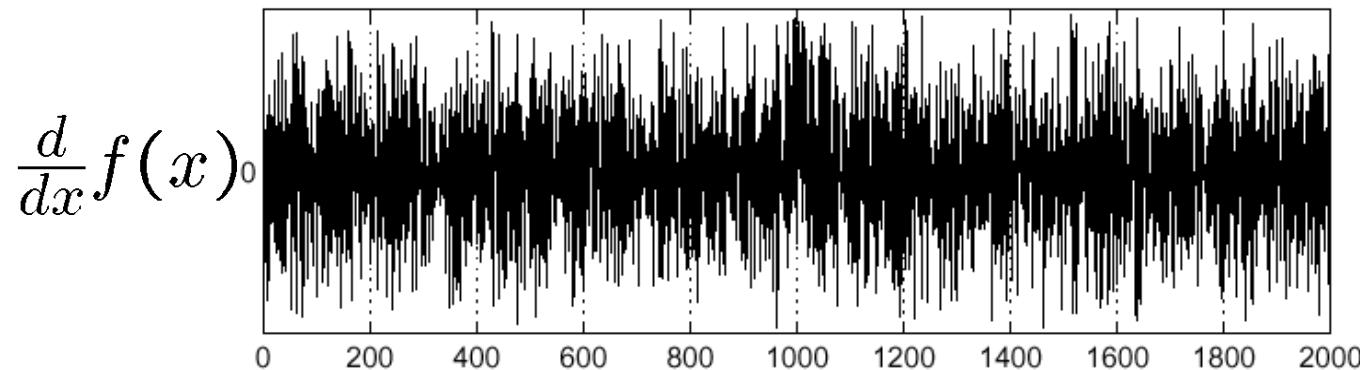
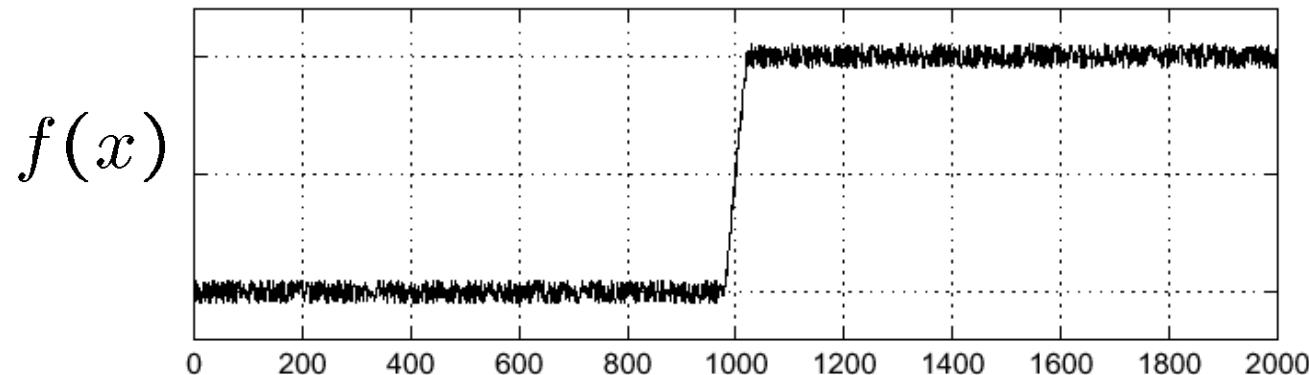


# Effects of noise

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Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

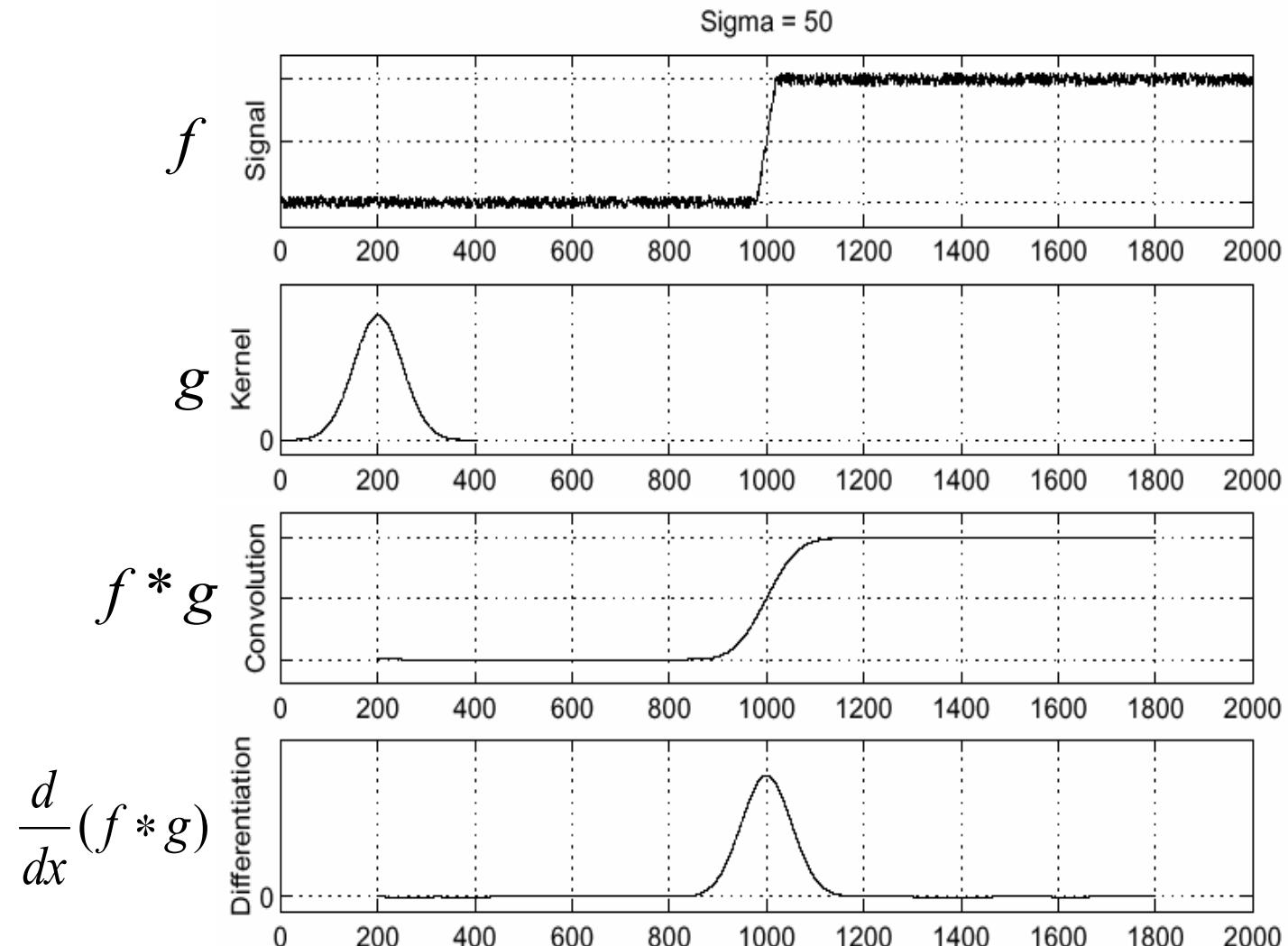


Where is the edge?

Source: S. Seitz

# Solution: smooth first

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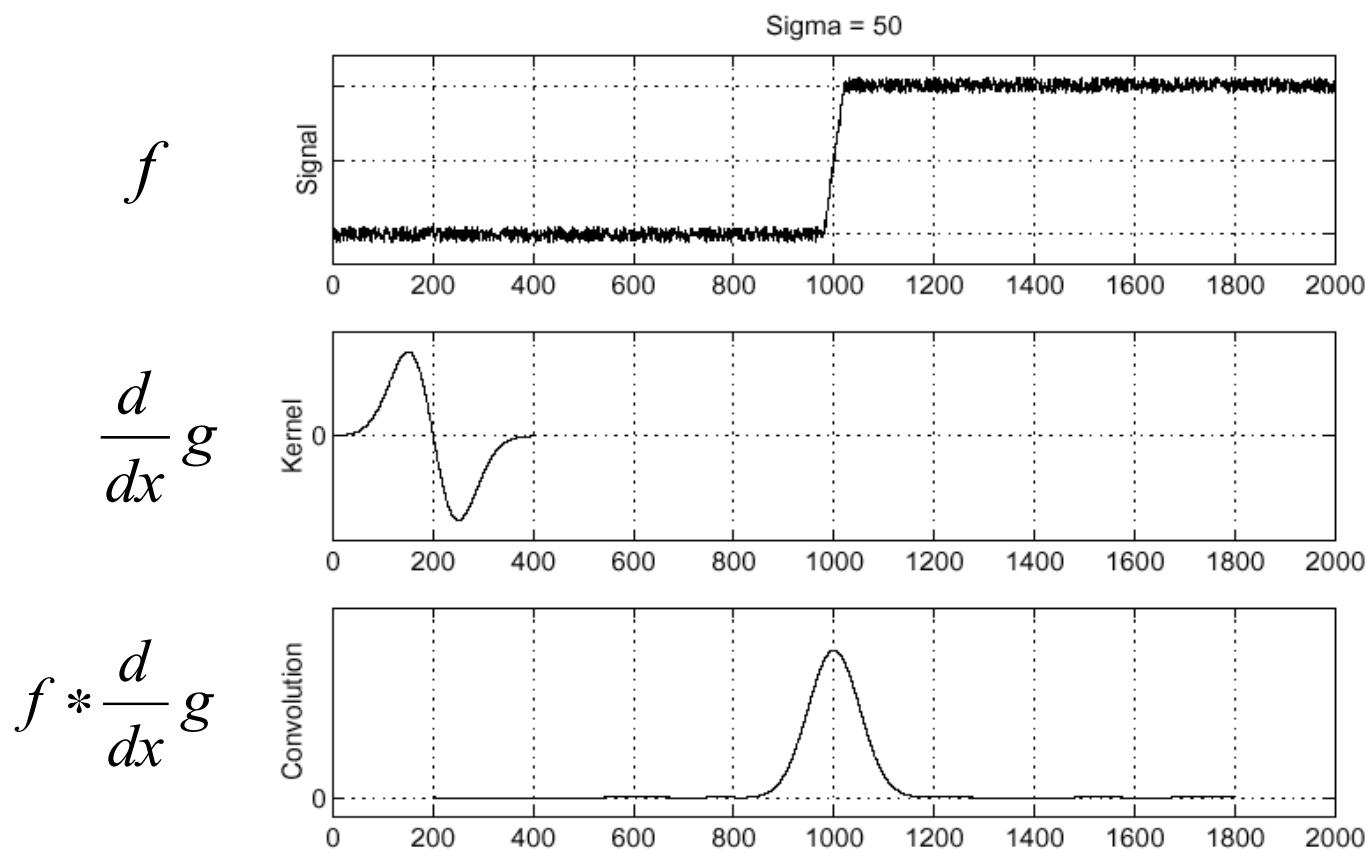
- To find edges, look for peaks in  $\frac{d}{dx}(f * g)$

Source: S. Seitz

# Derivative theorem of convolution

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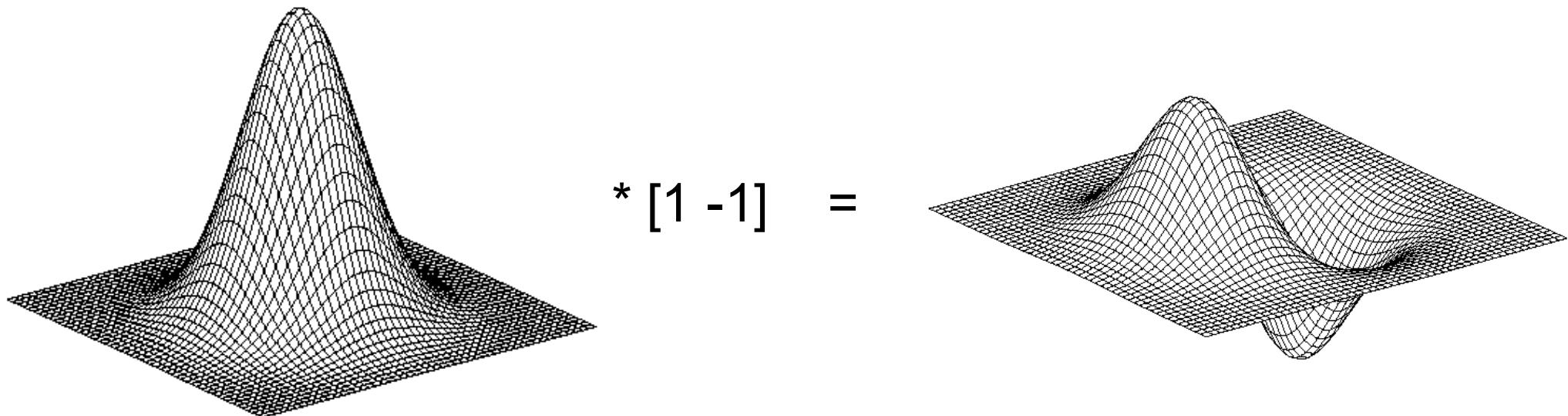
- Differentiation is convolution, and convolution is associative:  $\frac{d}{dx}(f * g) = f * \frac{d}{dx}g$
- This saves us one operation:



Source: S. Seitz

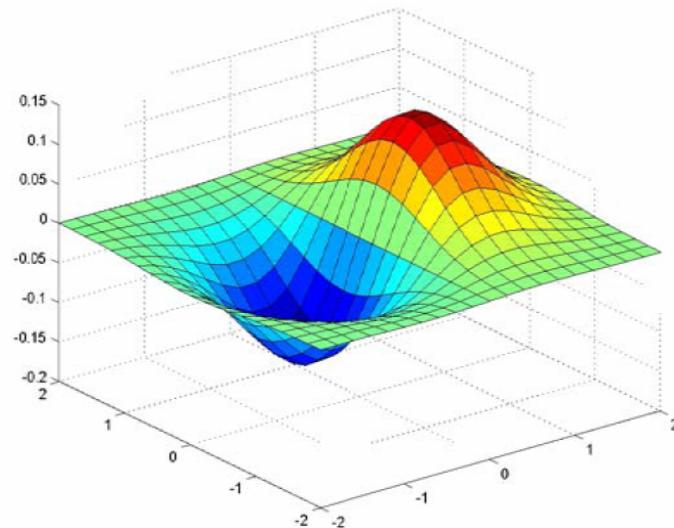
# Derivative of Gaussian filter

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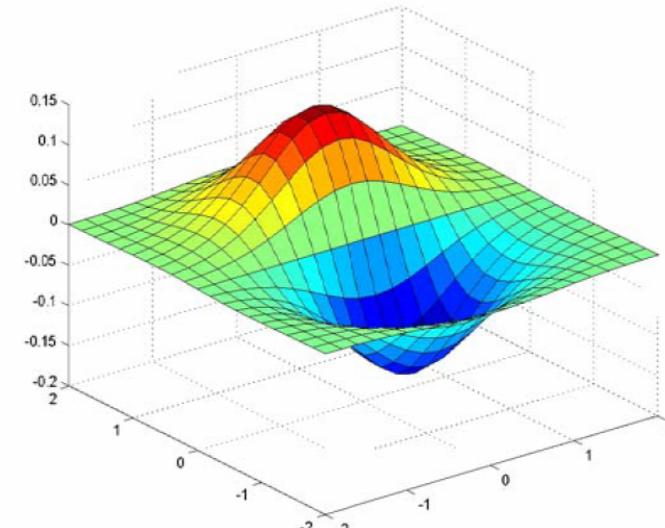


# Derivative of Gaussian filter

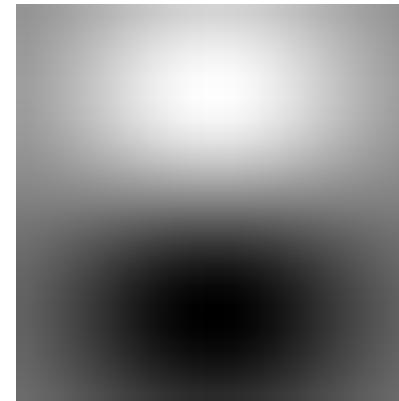
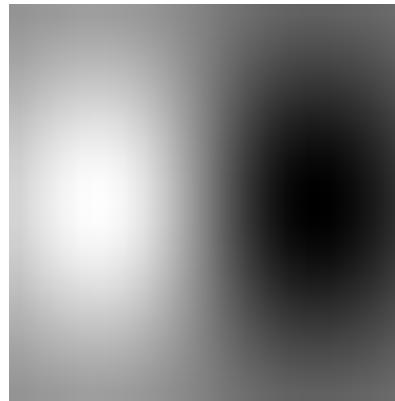
---



x-direction



y-direction



Which one finds horizontal/vertical edges?

# MATLAB code

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```
im = im2double(imread(filemane));
g = fspecial('gaussian',15,2);
imagesc(g);
surfl(g);
gim = conv2(im,g,'same');
imagesc(conv2(im,[-1 1],'same'));
imagesc(conv2(gim,[-1 1],'same'));
dx = conv2(g,[-1 1],'same');
surfl(dx);
imagesc(conv2(im,dx,'same'));
```

# Example

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input image

# Compute Gradients (DoG)

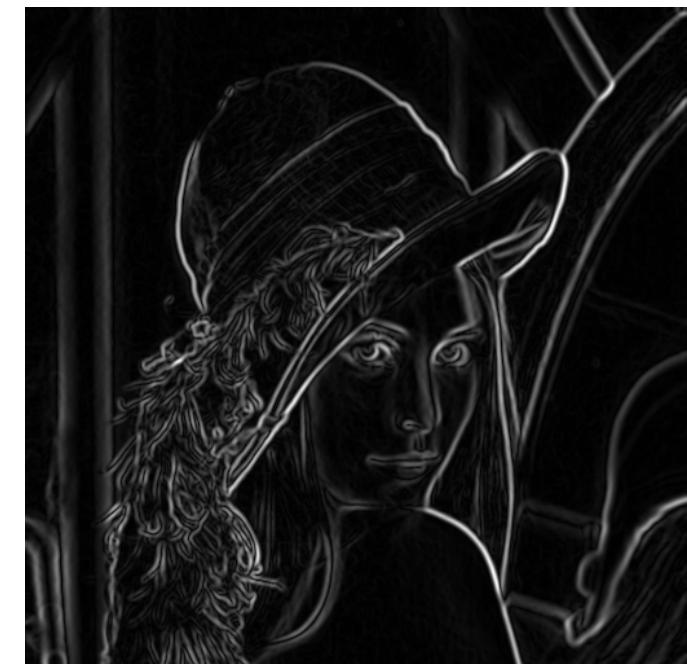
---



X-Derivative of  
Gaussian



Y-Derivative of  
Gaussian



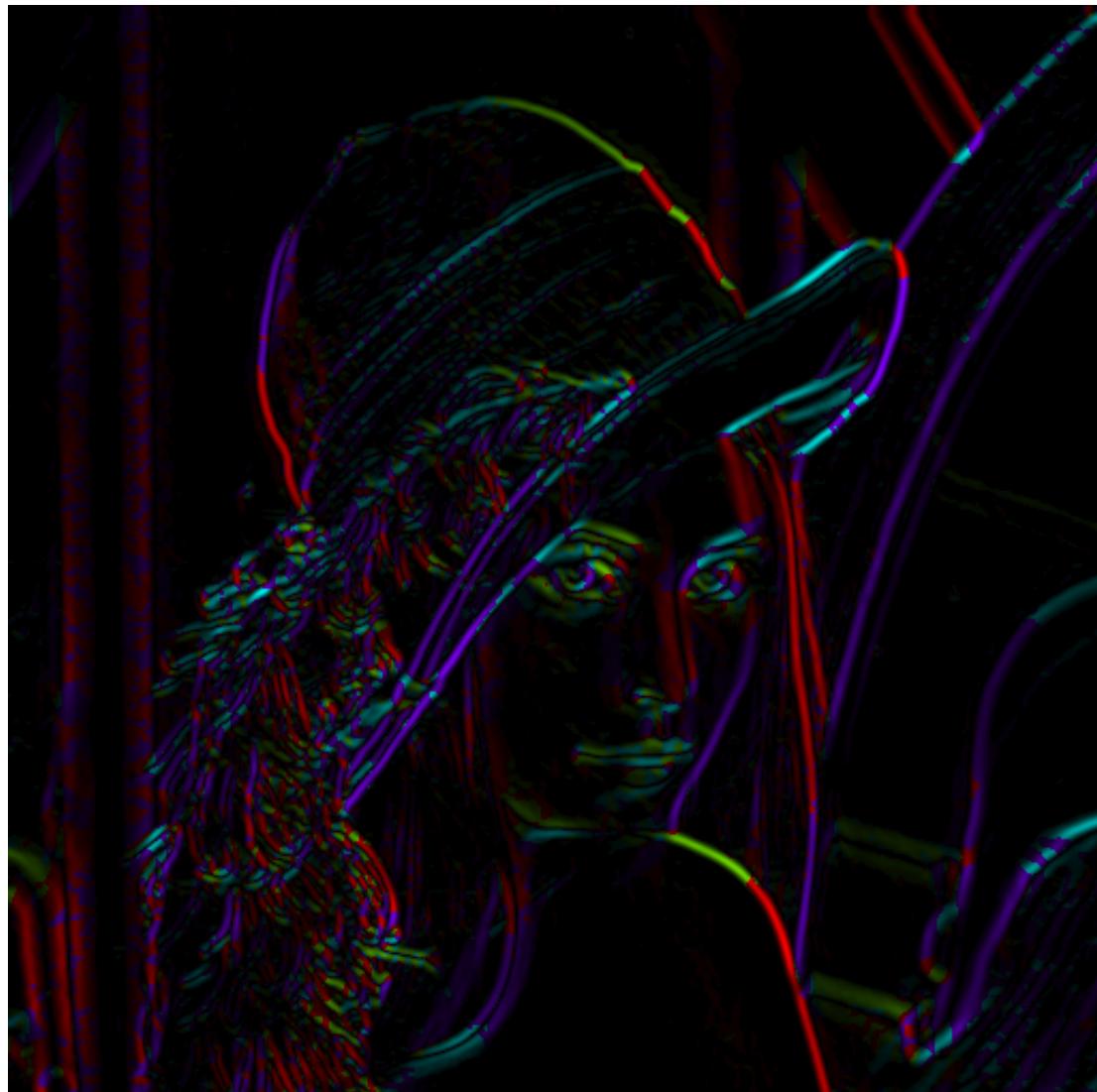
Gradient Magnitude

# Get Orientation at Each Pixel

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Threshold at minimum level

Get orientation

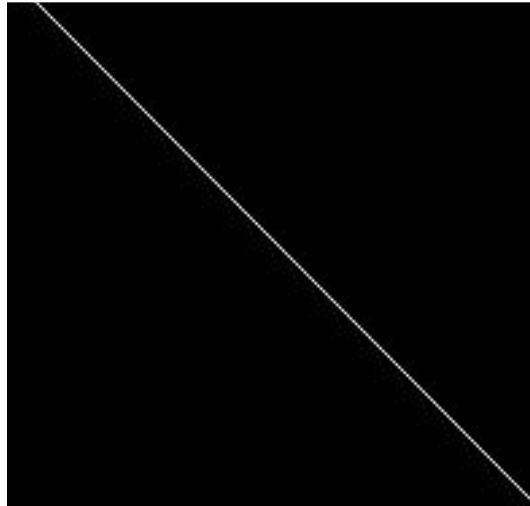


$\theta = \text{atan2}(-gy, gx)$

There is **ALWAYS** a tradeoff between smoothing and good edge localization!



Image with Edge



Edge Location

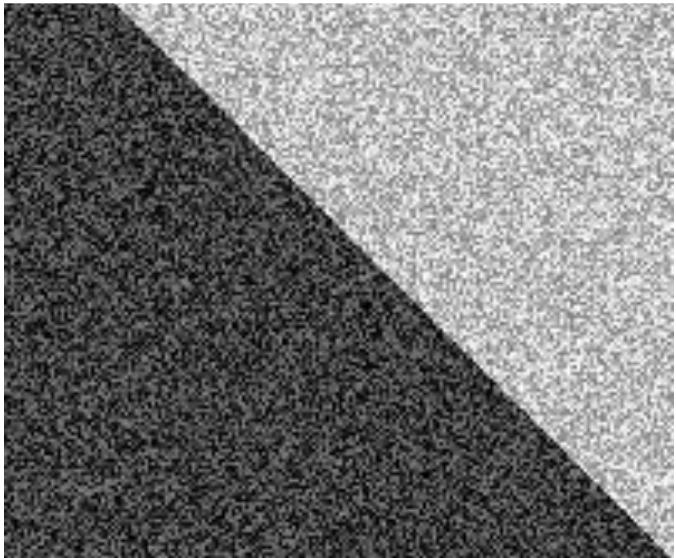
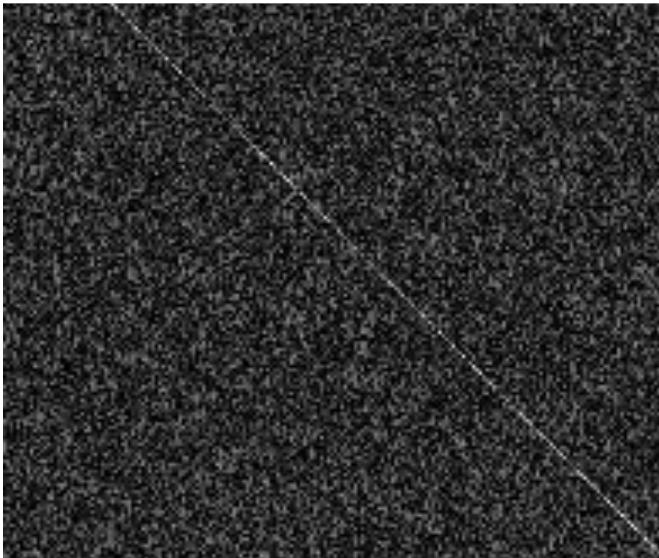
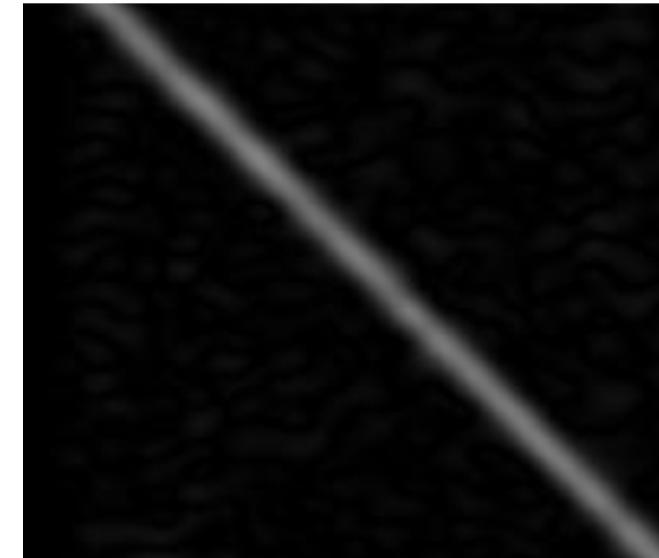


Image + Noise



Derivatives detect  
edge *and* noise



Smoothed derivative removes  
noise, but blurs edge

# The Canny edge detector

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norm of the gradient

# The Canny edge detector

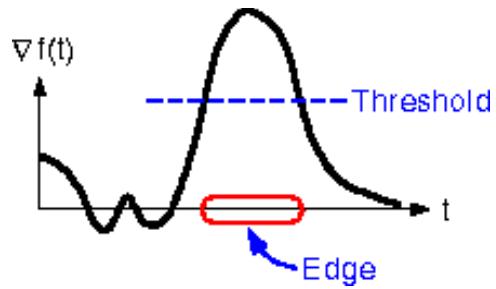
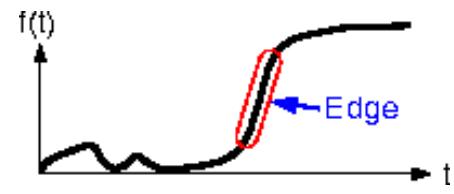
---



thresholding

# The Canny edge detector

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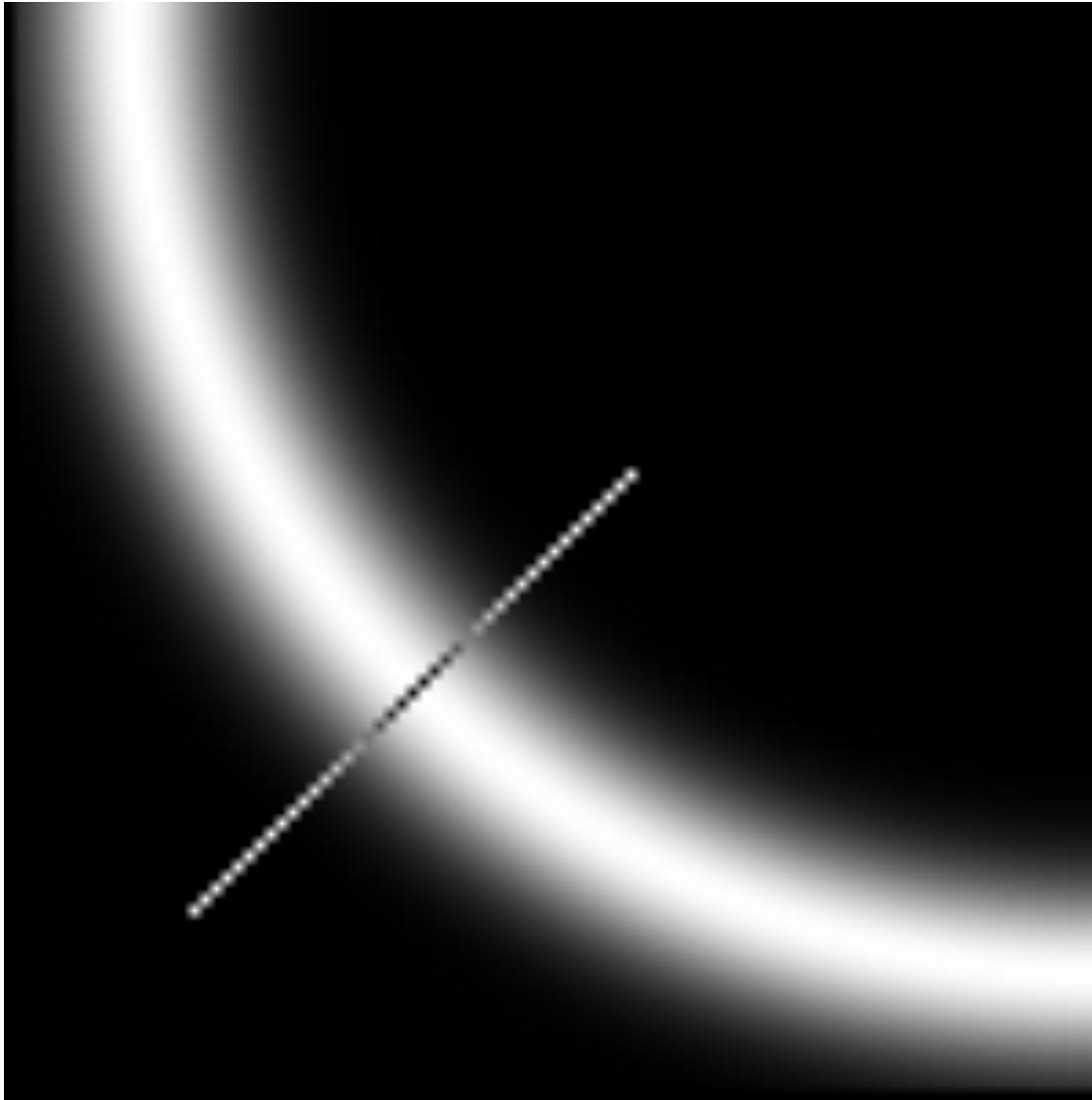


How to turn  
these thick  
regions of  
the gradient  
into  
curves?

thresholding

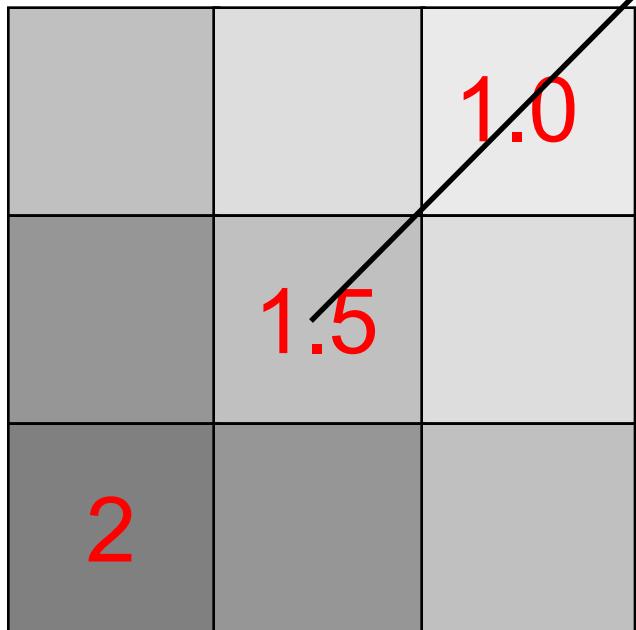
# Non-maximum suppression

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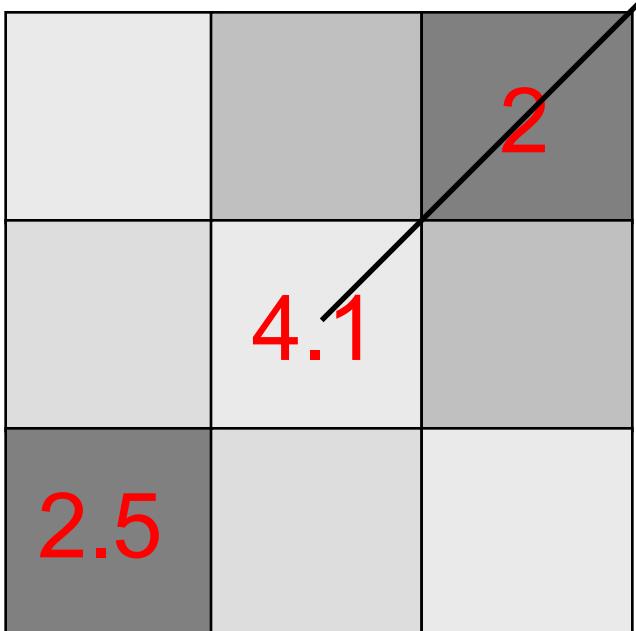


Check if pixel is local maximum along gradient direction, select single max across width of the edge

# Non-Local Maxima Suppression



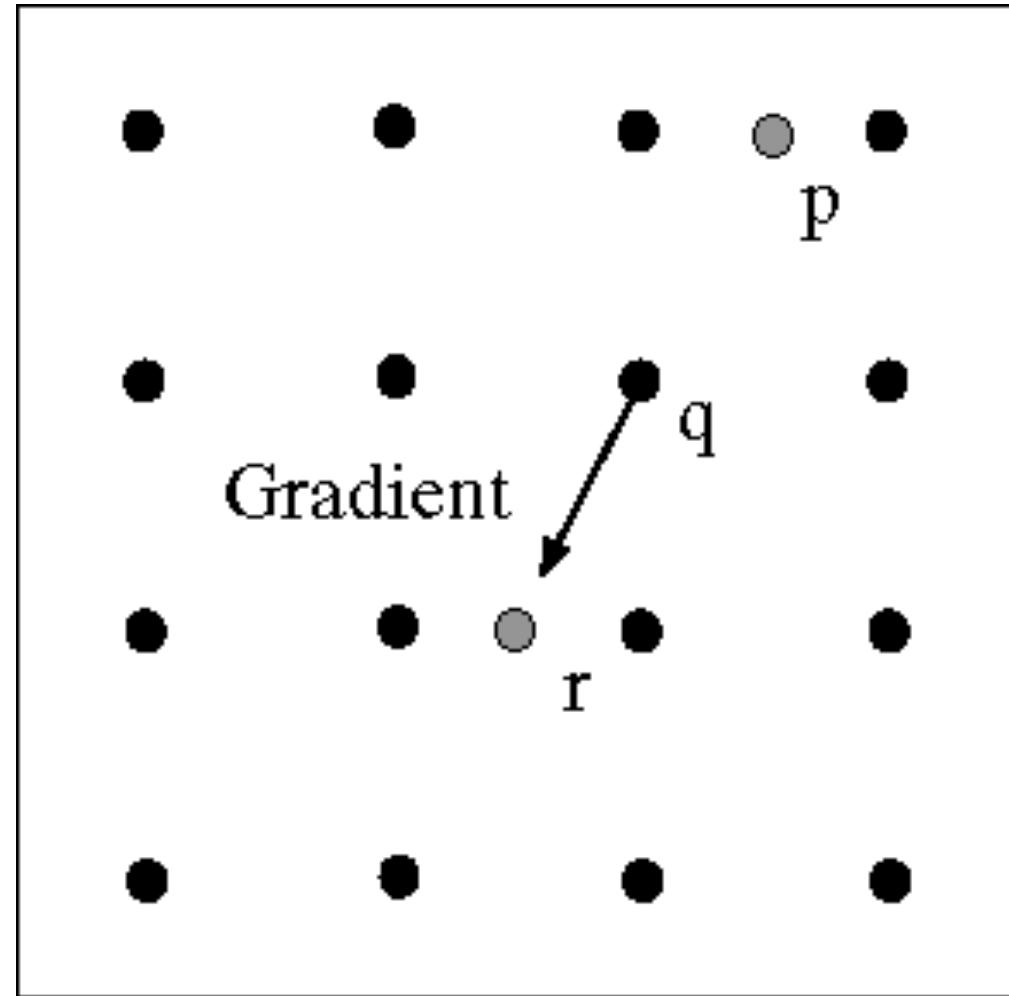
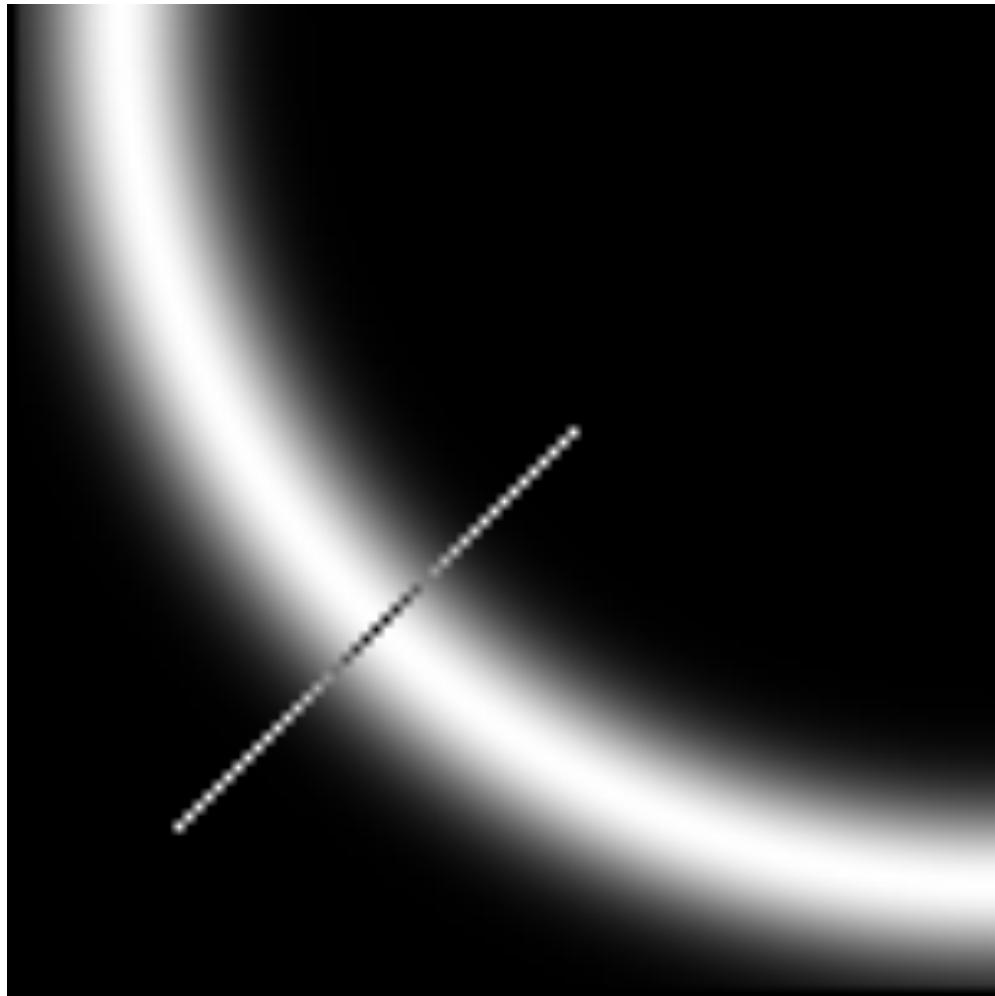
Gradient magnitude at center pixel  
is lower than the gradient magnitude  
of a neighbor *in the direction of the gradient*  
→ Discard center pixel (set magnitude to 0)



Gradient magnitude at center pixel  
is greater than gradient magnitude  
of all the neighbors *in the direction  
of the gradient*  
→ Keep center pixel unchanged

# Non-maximum suppression

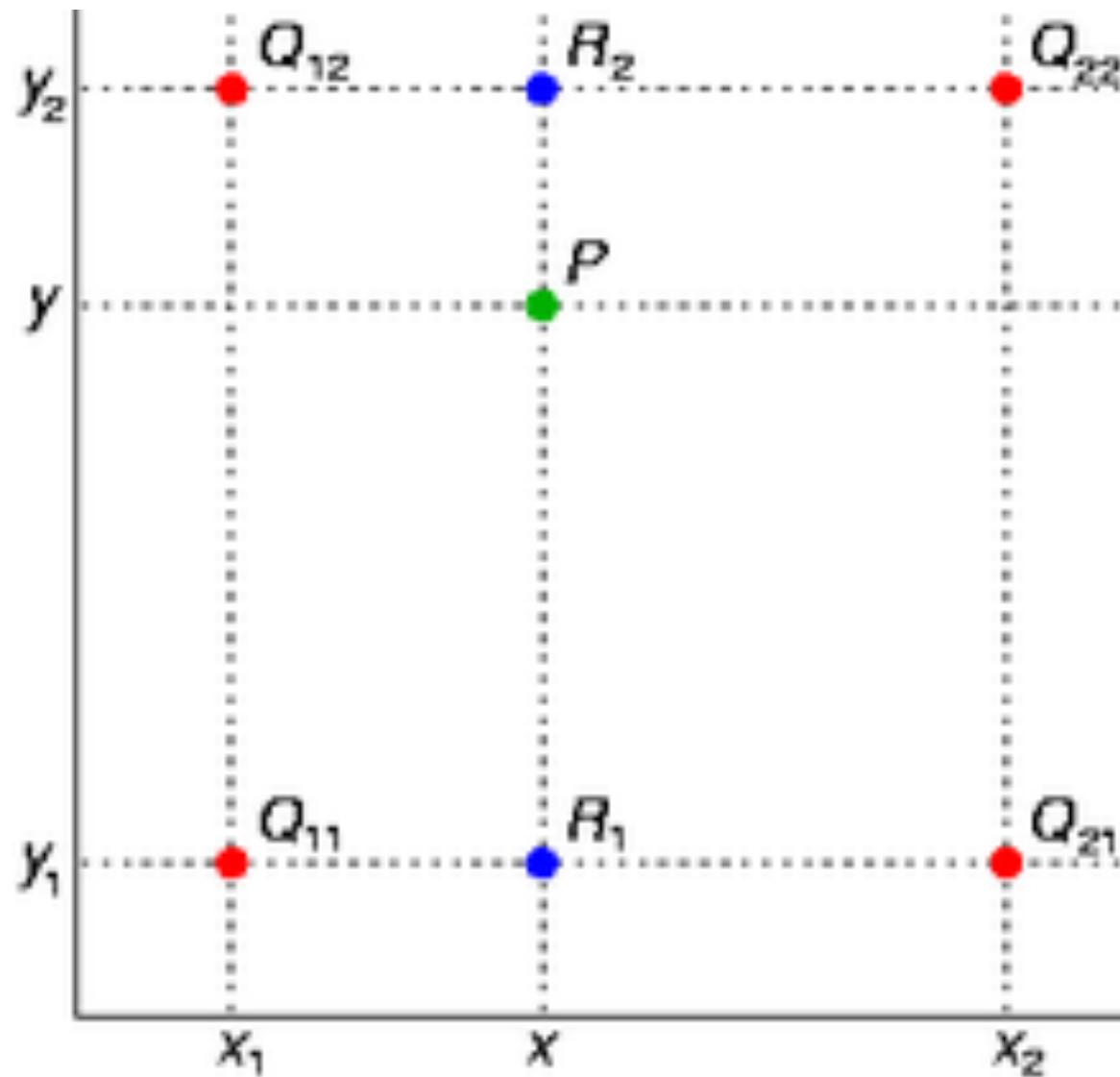
---



requires checking interpolated pixels p and r

# Bilinear Interpolation

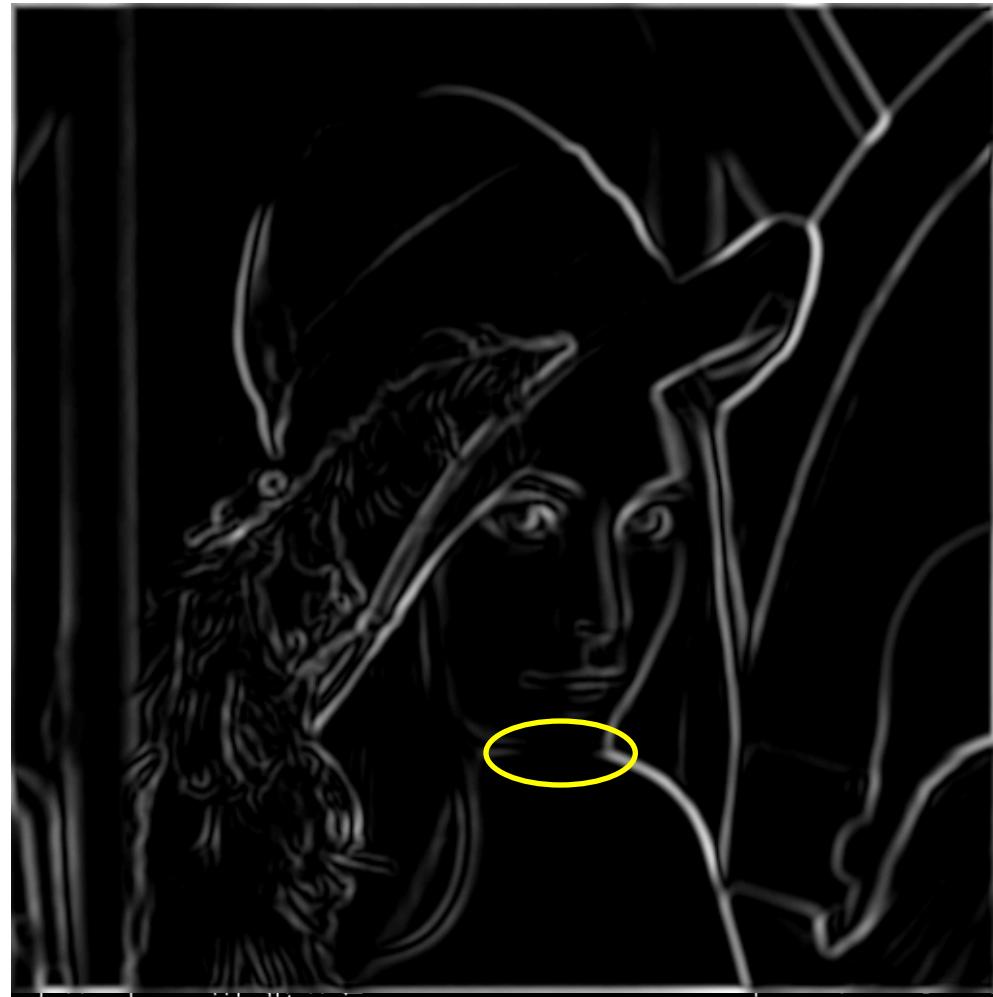
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[http://en.wikipedia.org/wiki/Bilinear\\_interpolation](http://en.wikipedia.org/wiki/Bilinear_interpolation)  
Help interp2

# The Canny edge detector

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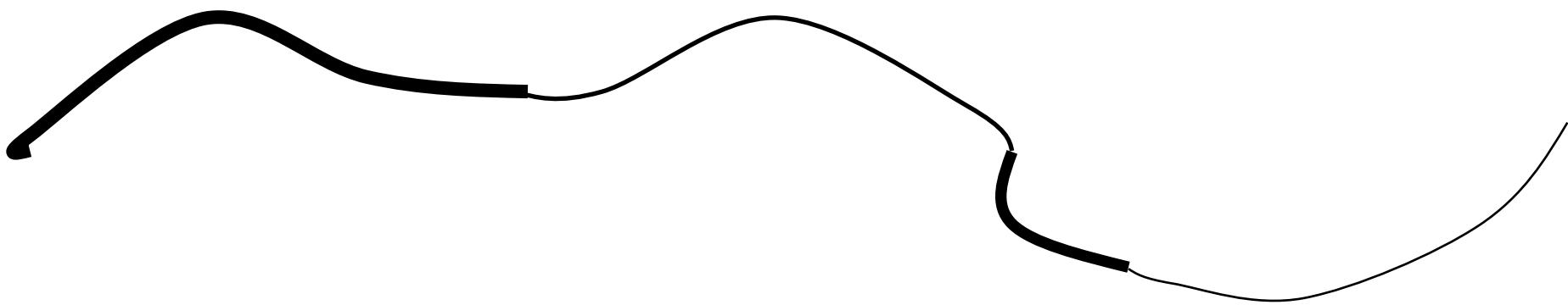


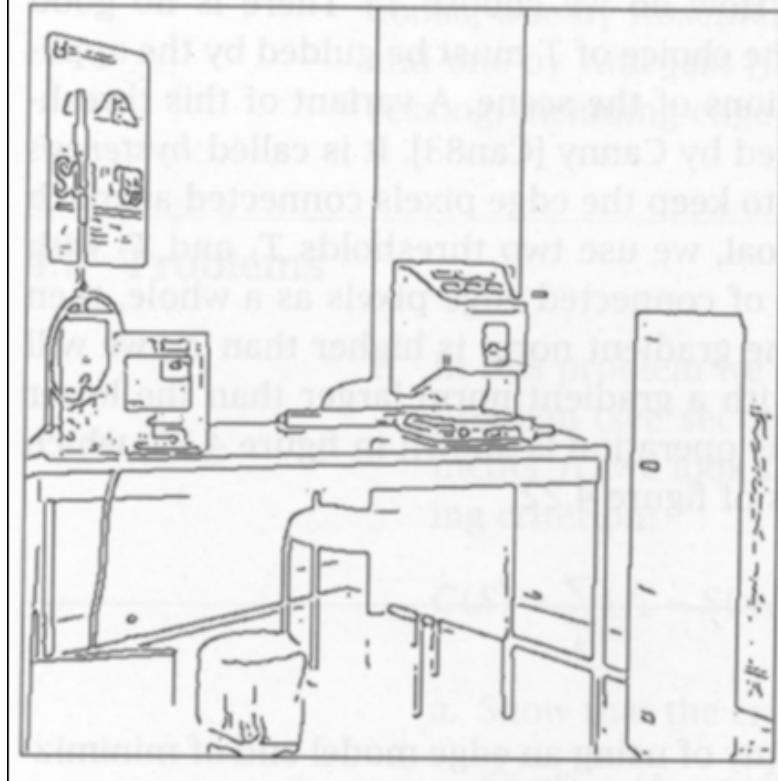
Problem:  
pixels along  
this edge  
didn't  
survive the  
thresholding

thinning  
(non-maximum suppression)

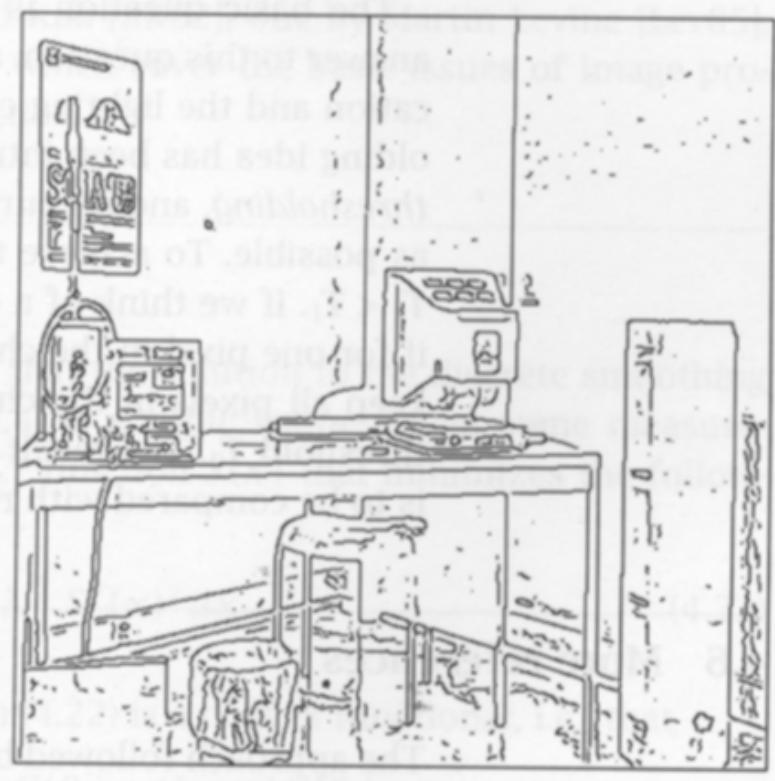
# Hysteresis thresholding

Use a high threshold to start edge curves, and a low threshold to continue them.





$T = 15$

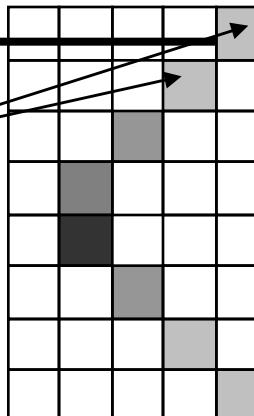


$T = 5$

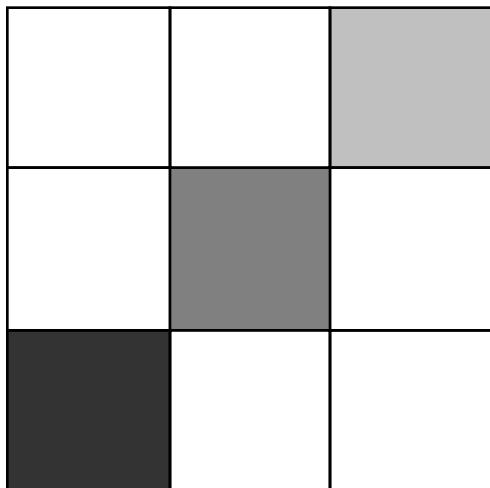
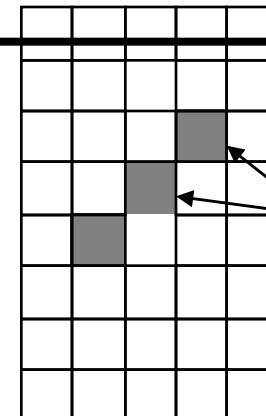
Two thresholds applied to gradient magnitude

# Hysteresis Thresholding

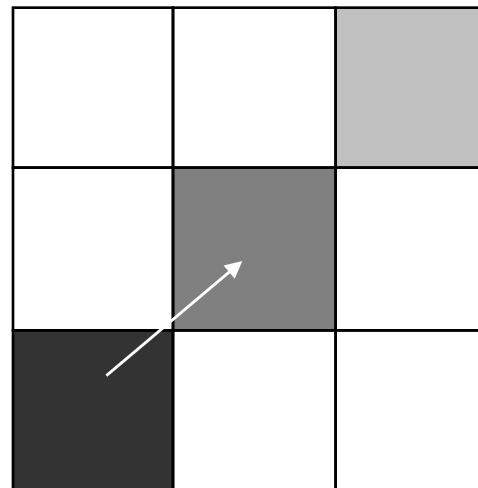
Weak pixels but connected



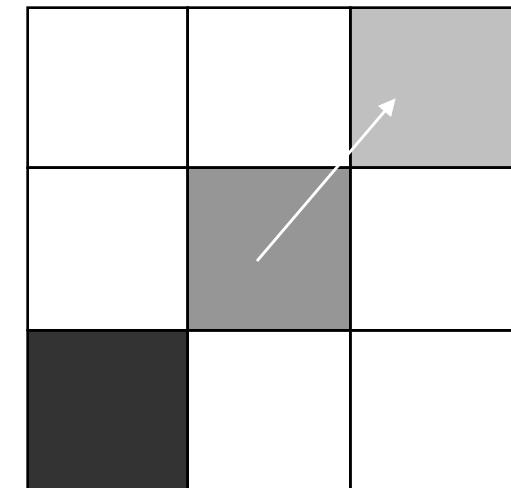
Weak pixels but isolated



Very strong edge response.  
Let's start here



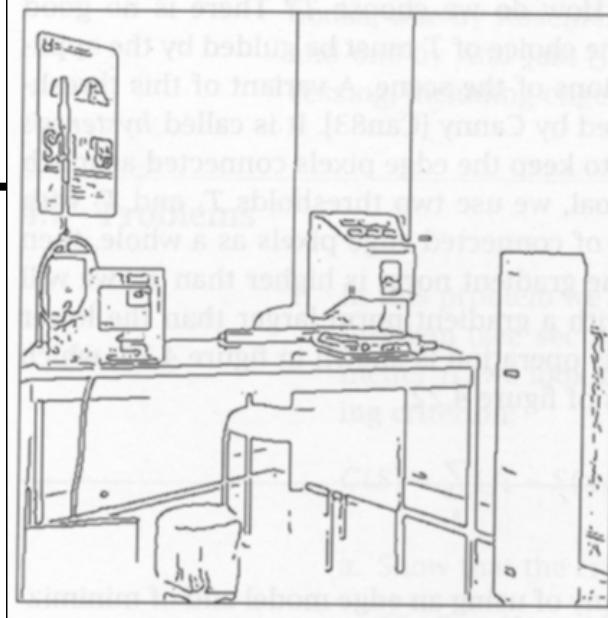
Weaker response but it is  
connected to a confirmed  
edge point. Let's keep it.



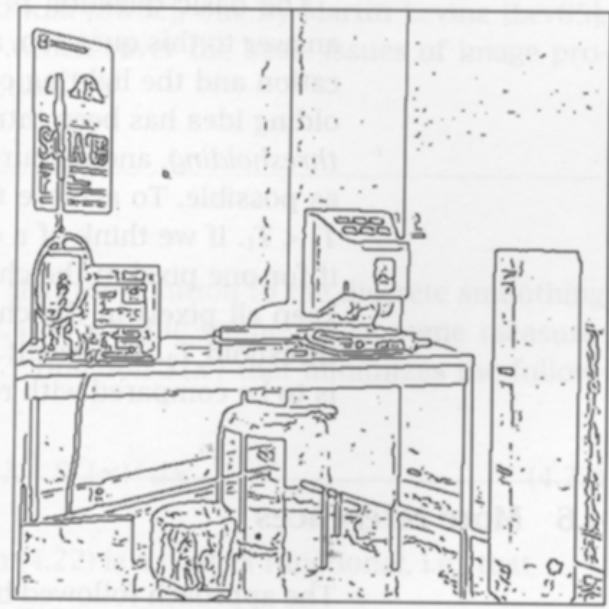
Continue....

Note: Darker squares illustrate stronger edge response (larger  $M$ )

$T=15$



$T=5$



Hysteresis  
thresholding



Hysteresis  
 $T_h=15$   $T_l = 5$

# Recap: Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression:**
  - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding (hysteresis):**
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

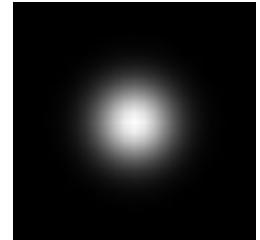
MATLAB: `edge(image, 'canny');`

J. Canny, [A Computational Approach To Edge Detection](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

# Review: Smoothing vs. derivative filters

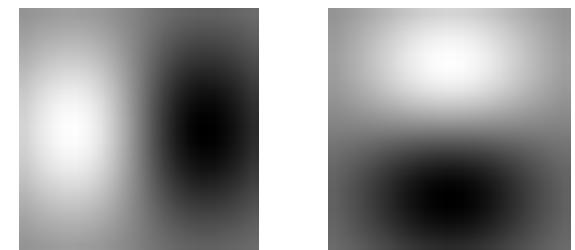
## 1. Smoothing filters

- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One**: constant regions are not affected by the filter



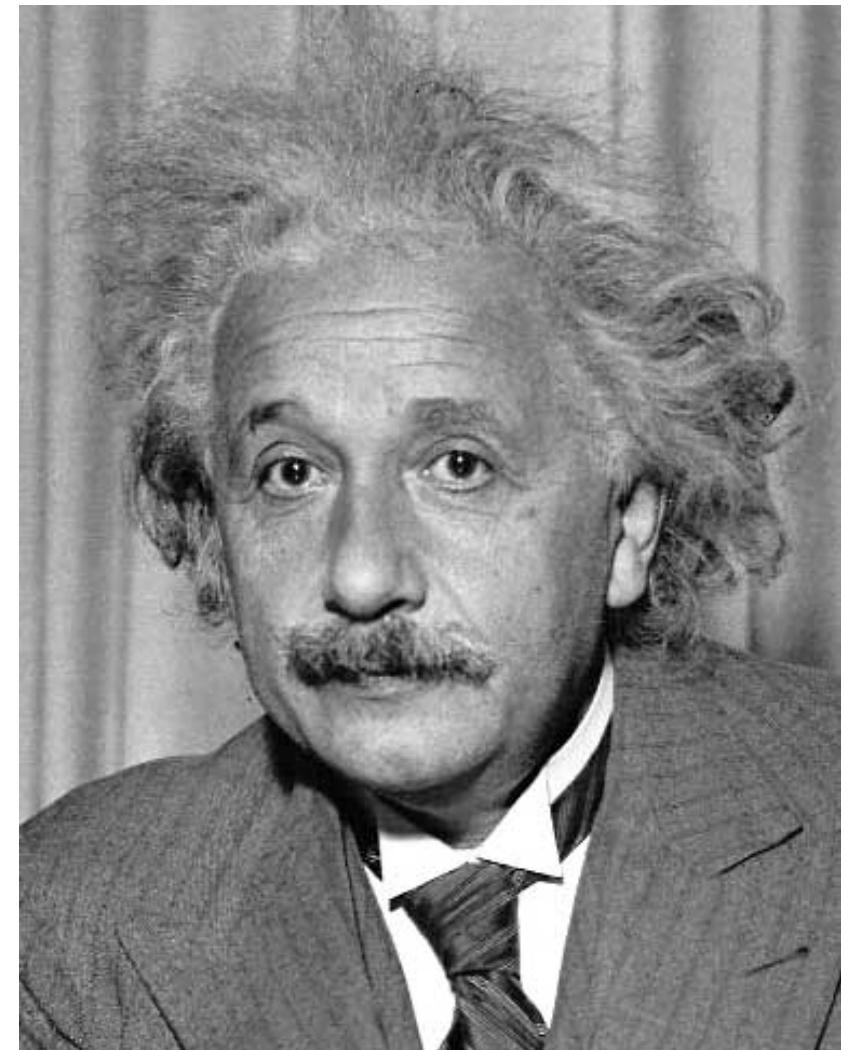
## 2. Derivative filters

- E.g. Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero**: no response in constant regions
- High absolute value at points of high contrast



### 3. Filtering as template matching

- Goal: find  in image
- Main challenge: What is a good similarity or distance measure between two patches?

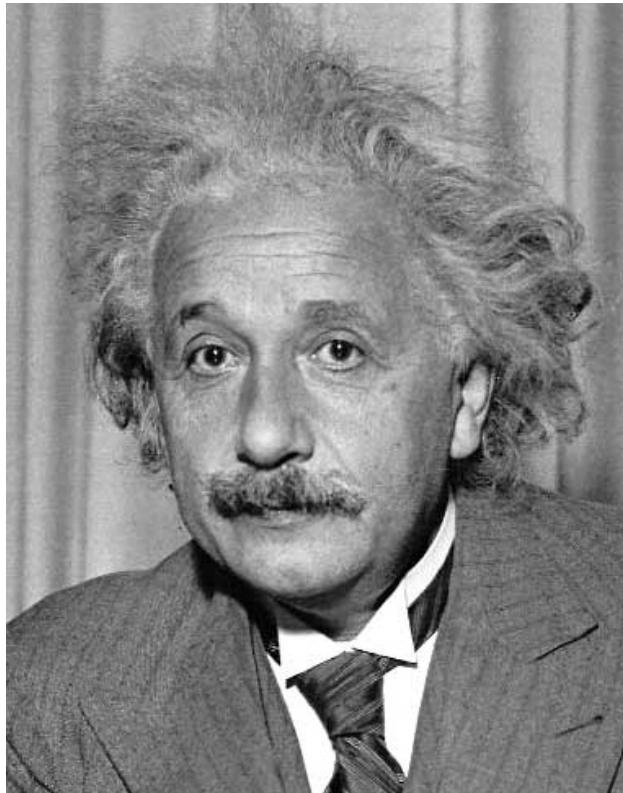


# Matching with filters

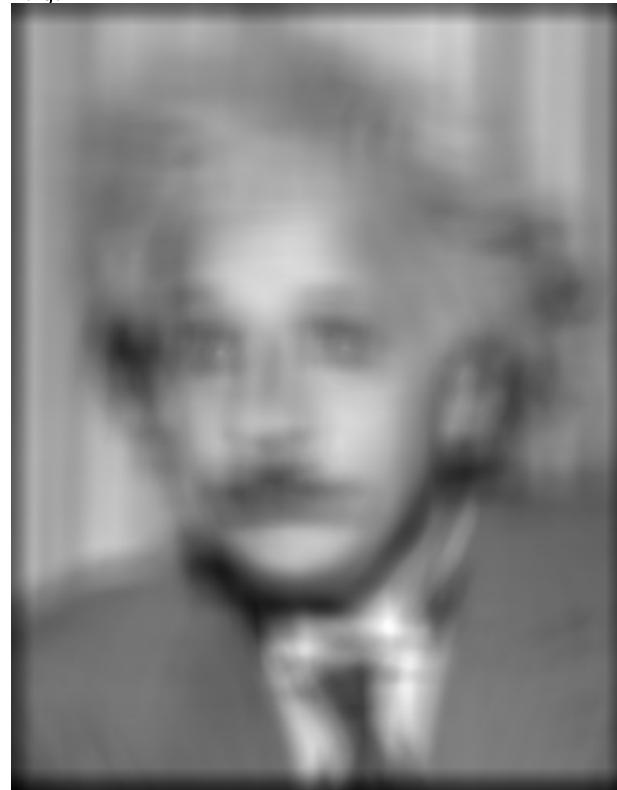
- Goal: find  in image
- Method 0: filter the image with eye patch

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

$f$  = image  
 $g$  = filter



Input



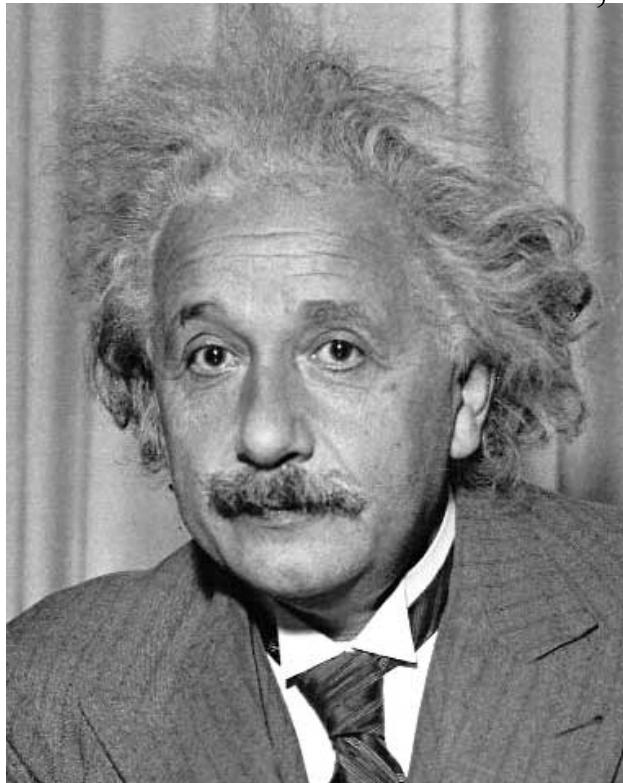
Filtered Image

What went wrong?

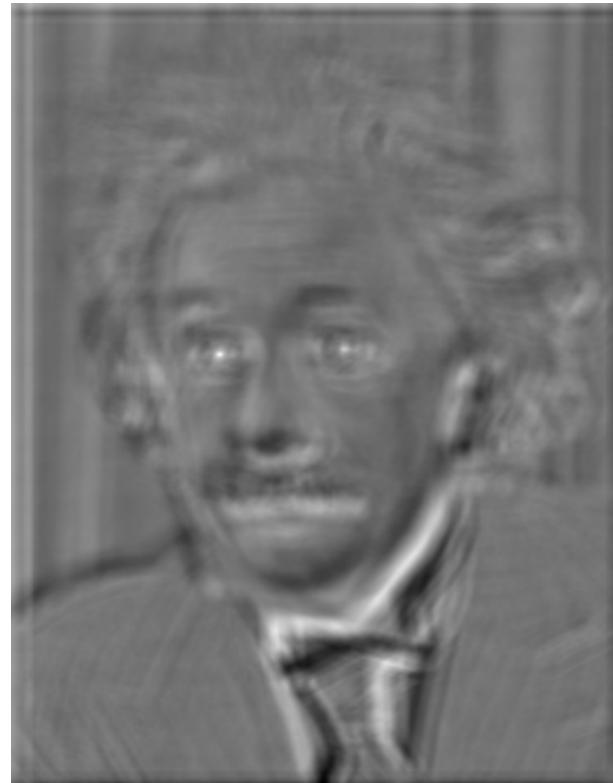
# Matching with filters

- Goal: find  in image
- Method 1: filter the image with zero-mean eye

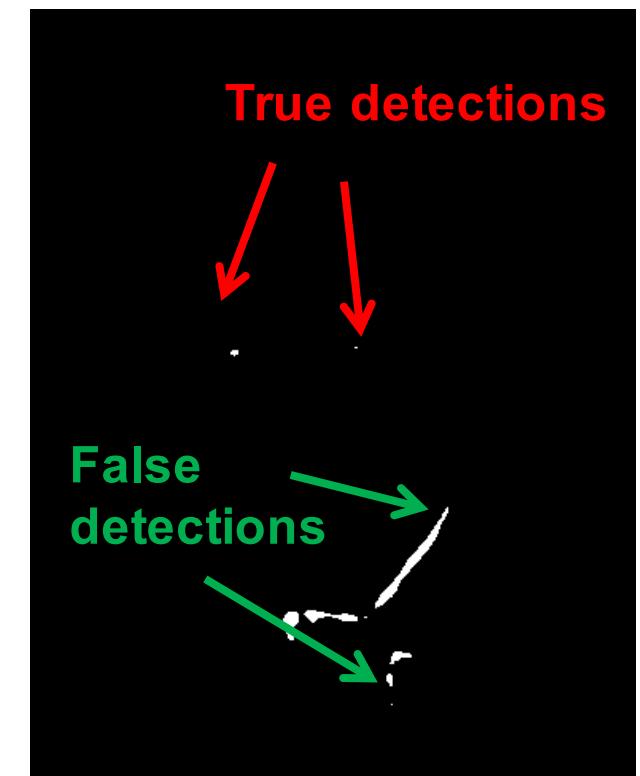
$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) \underbrace{(f[m+k, n+l])}_{\text{mean of template } g}$$



Input



Filtered Image (scaled)

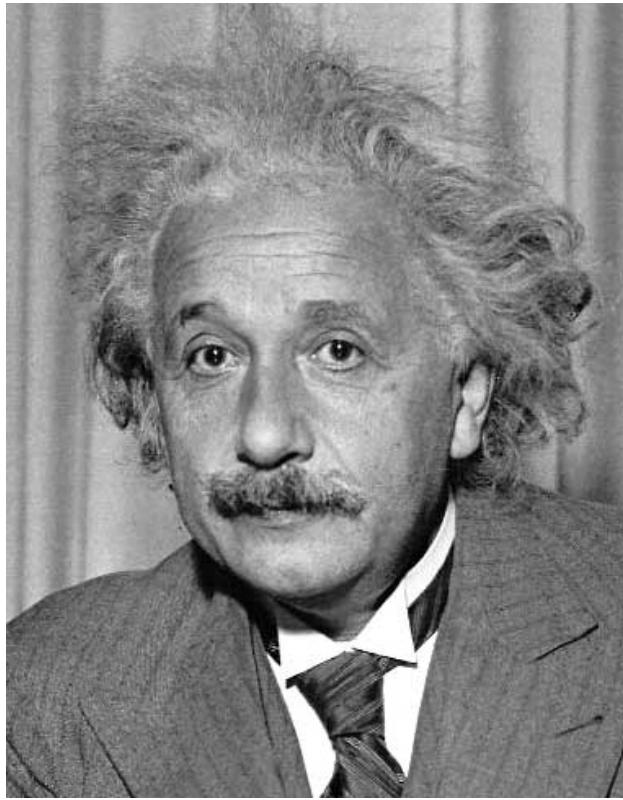


Thresholded Image

# Matching with filters

- Goal: find  in image
- Method 2: SSD (L2-norm)

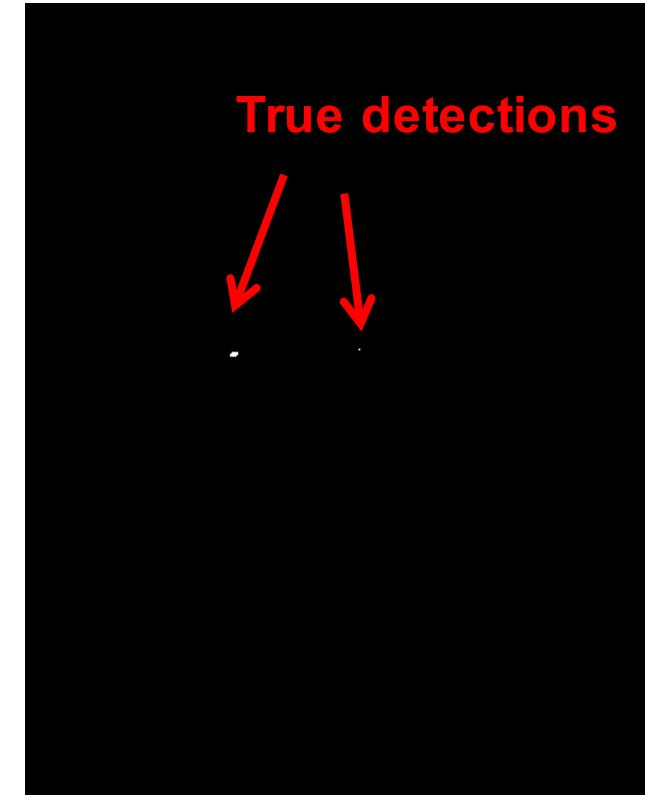
$$h[m, n] = \sum_{k,l} (g[k, l] - f[m + k, n + l])^2$$



Input



1 -  $\sqrt{\text{SSD}}$



Thresholded Image

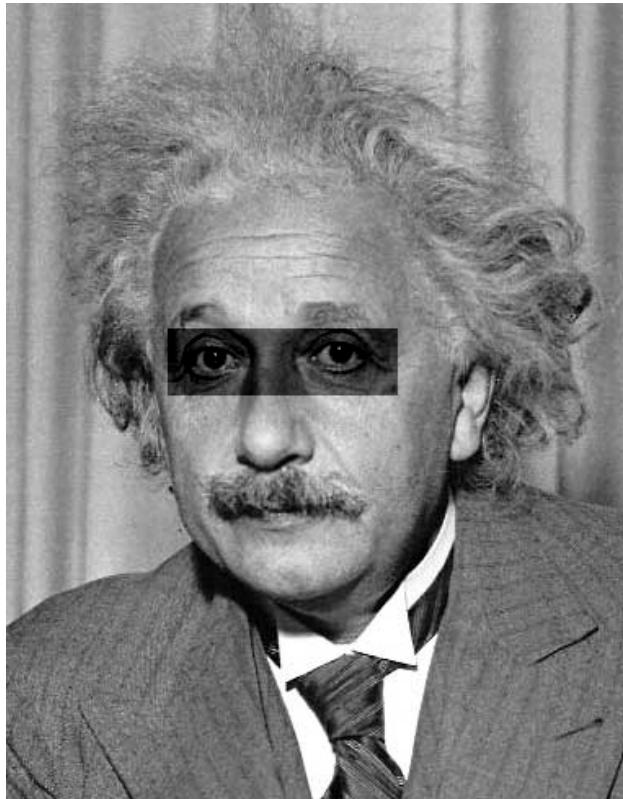
True detections

# Matching with filters

- Goal: find  in image
- Method 2: SSD

What's the potential downside of SSD?

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$



Input



1- sqrt(SSD)

# Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation

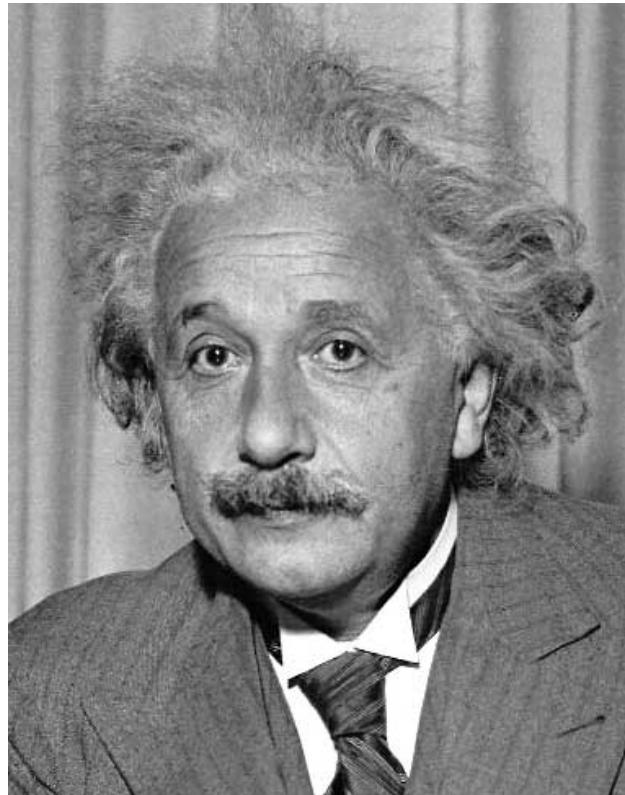
$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m+k, n+l] - \bar{f}_{m,n})}{\left( \sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m+k, n+l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

mean template  
↓  
 $\sum_{k,l}$   
mean image patch  
↓

Matlab: `normxcorr2(template, im)`

# Matching with filters

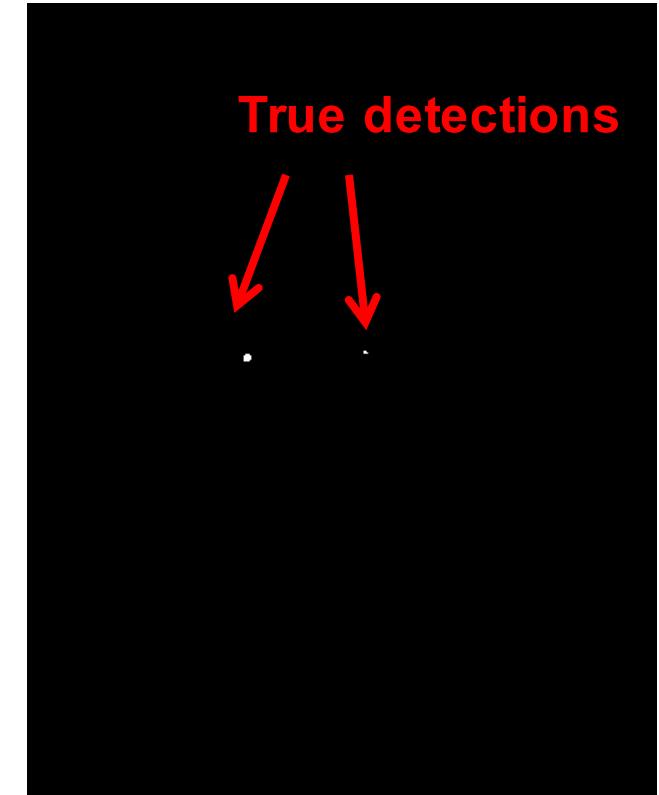
- Goal: find  in image
- Method 3: Normalized cross-correlation



Input



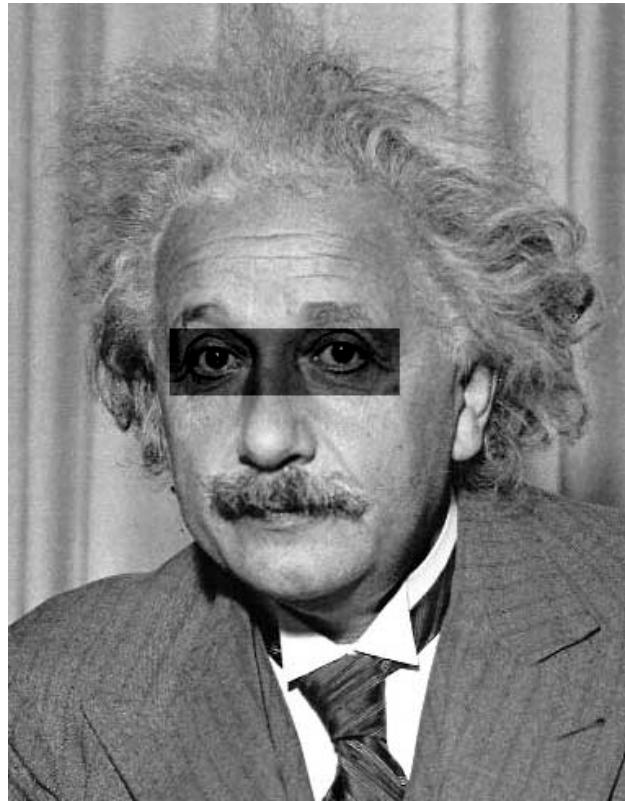
Normalized X-Correlation



True detections  
Thresholded Image

# Matching with filters

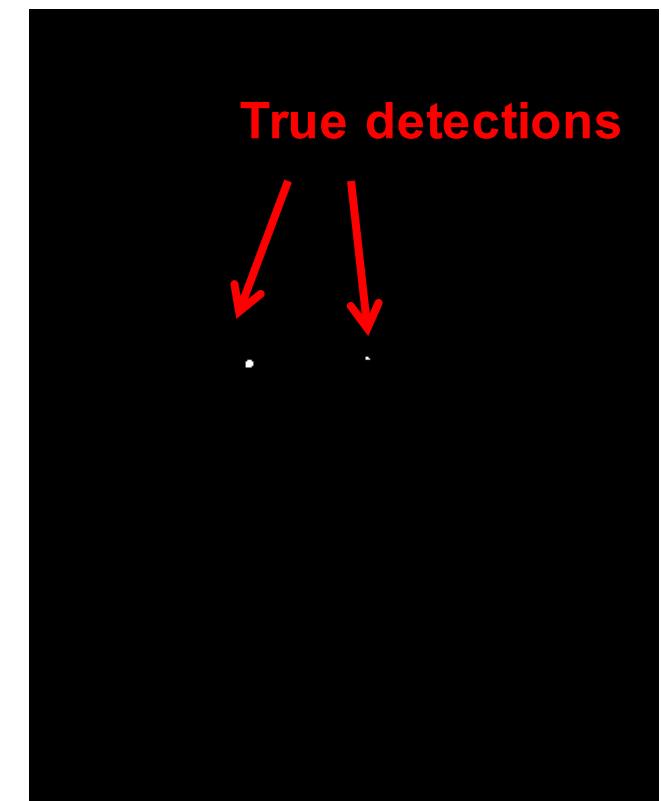
- Goal: find  in image
- Method 3: Normalized cross-correlation



Input



Normalized X-Correlation



True detections  
Thresholded Image

# Q: What is the best method to use?

A: Depends

- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast

# Object recognition

## Is it really so hard?

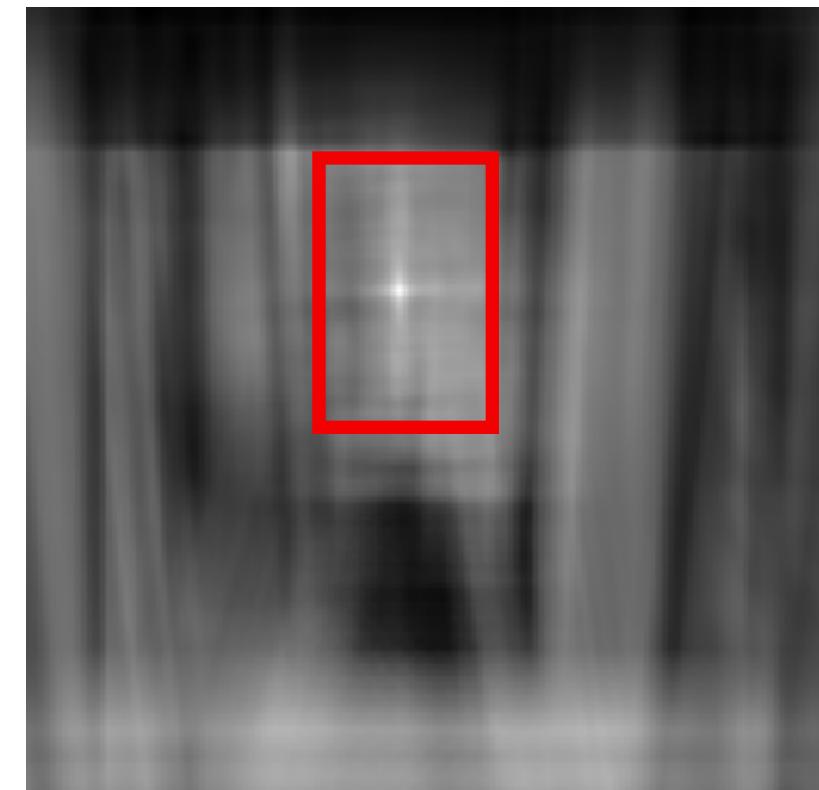
This is a chair



Find the chair in this image



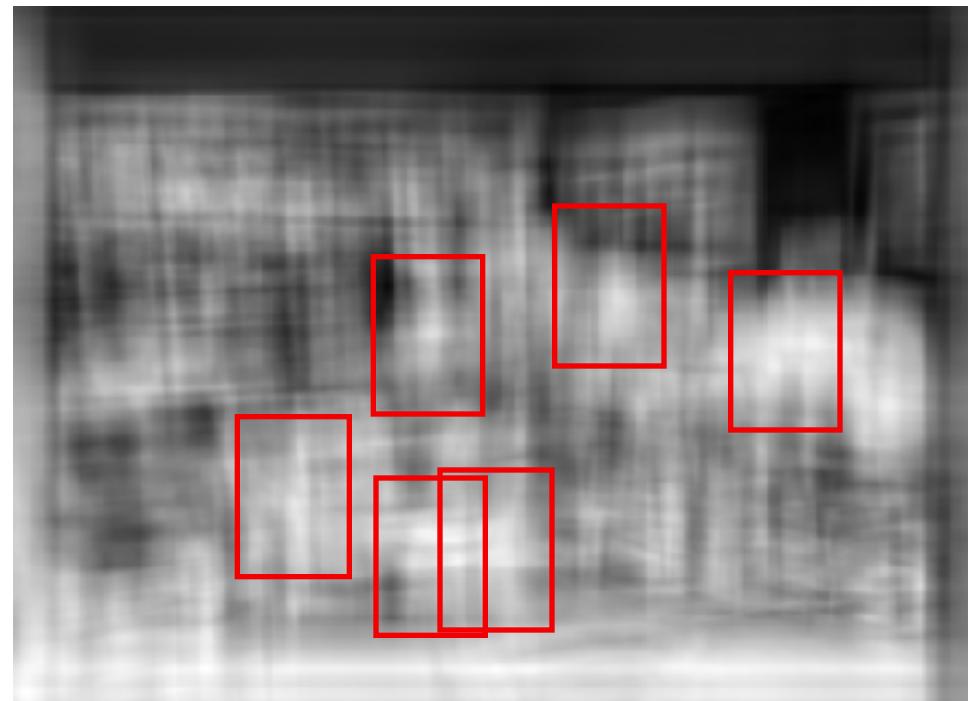
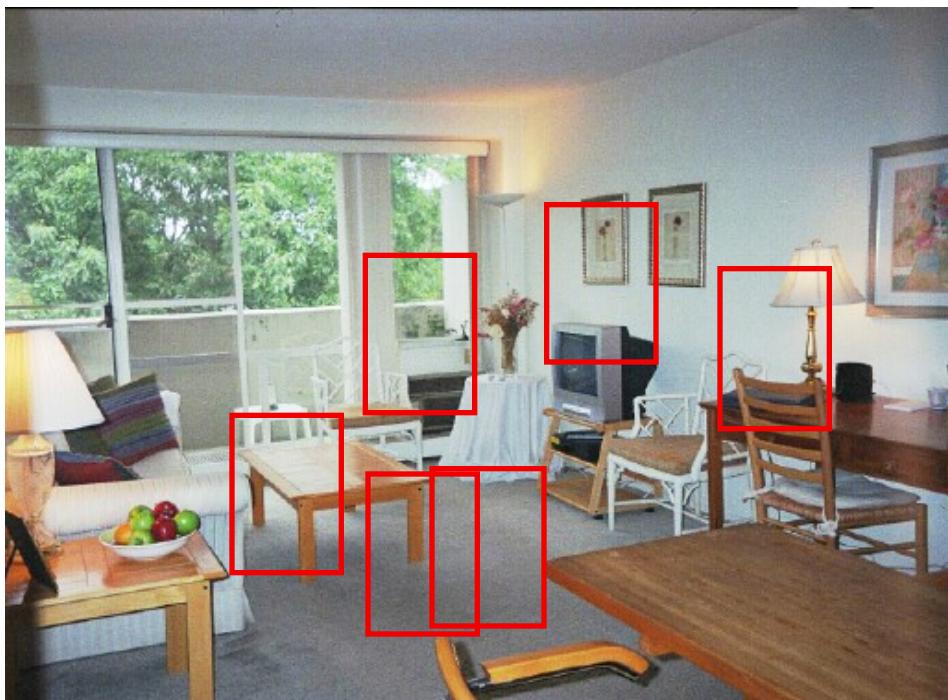
Output of normalized correlation





# Object recognition Is it really so hard?

Find the chair in this image



# Recognition: Instance vs. Category

- Instance recognition:
  - “Find me this particular chair again”
  - Often simple template matching works OK
    - Even better with many small templates, a.k.a. feature descriptors (later in the course)
- Category recognition:
  - “find me all chairs”
  - Templates don’t work. Why?
  - Focus on things that might be invariant across the category
  - Hugely dependent on human vision

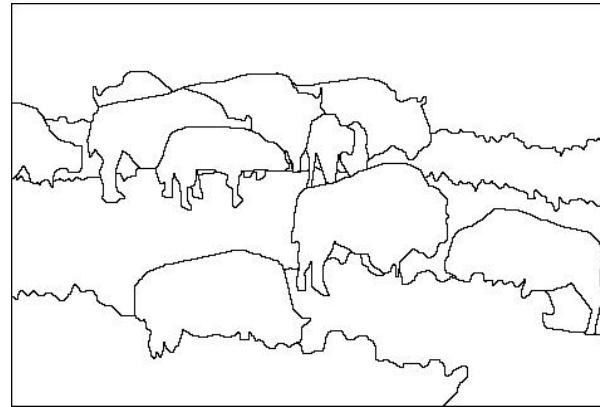
# Edge detection is just the beginning...

---

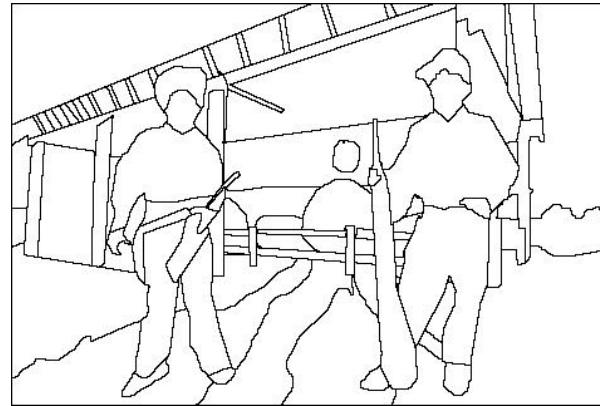
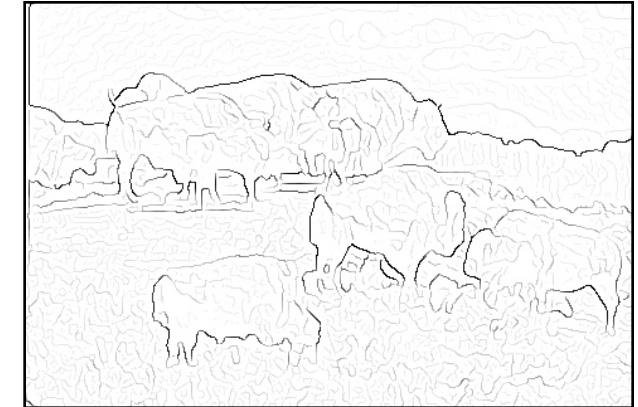
image



human segmentation



gradient magnitude



Berkeley segmentation database:

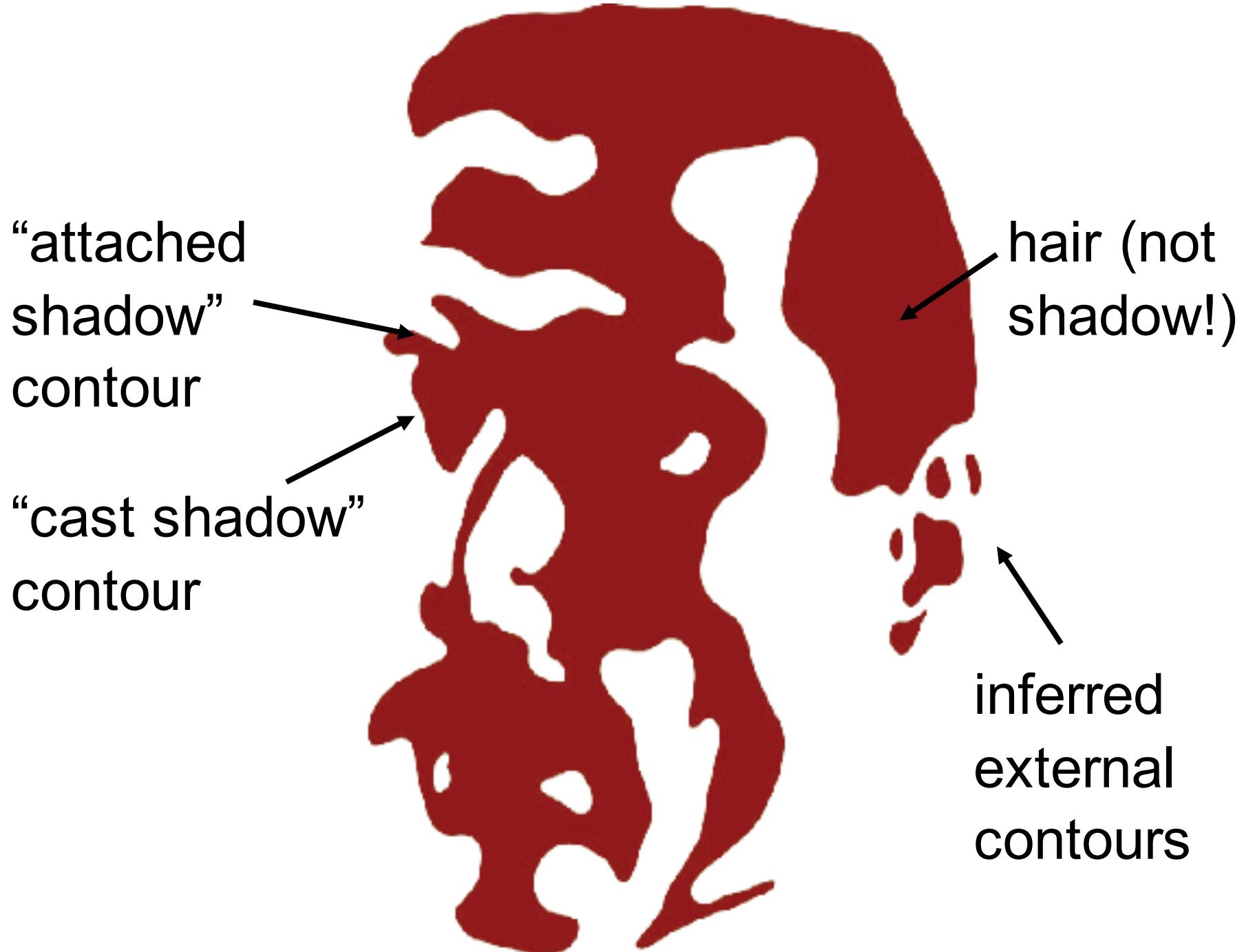
<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/>



two-tone images













# What is Texture?

- Texture depicts spatially repeating patterns
- Many natural phenomena are textures



radishes



rocks



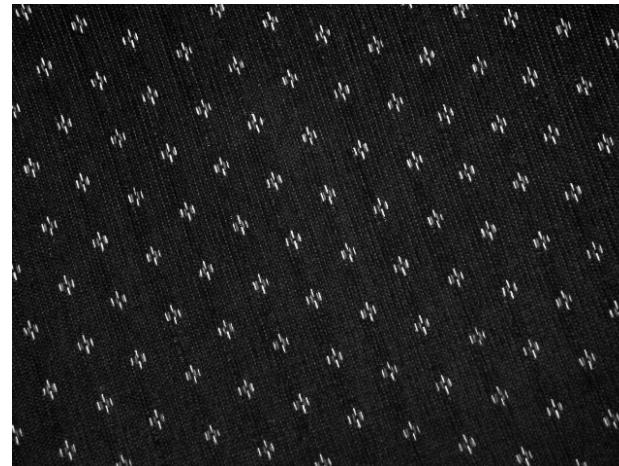
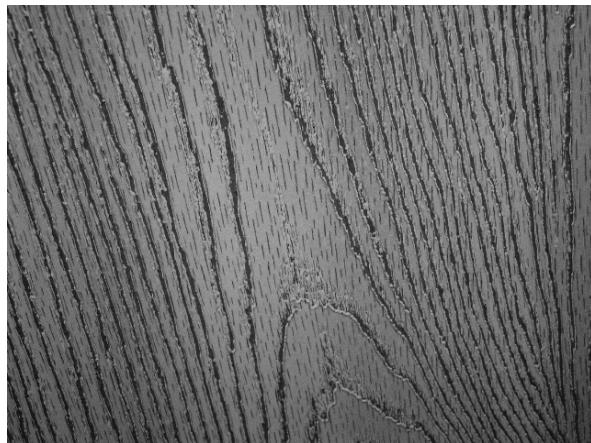
yogurt

# Texture as “stuff”



Source: Forsyth

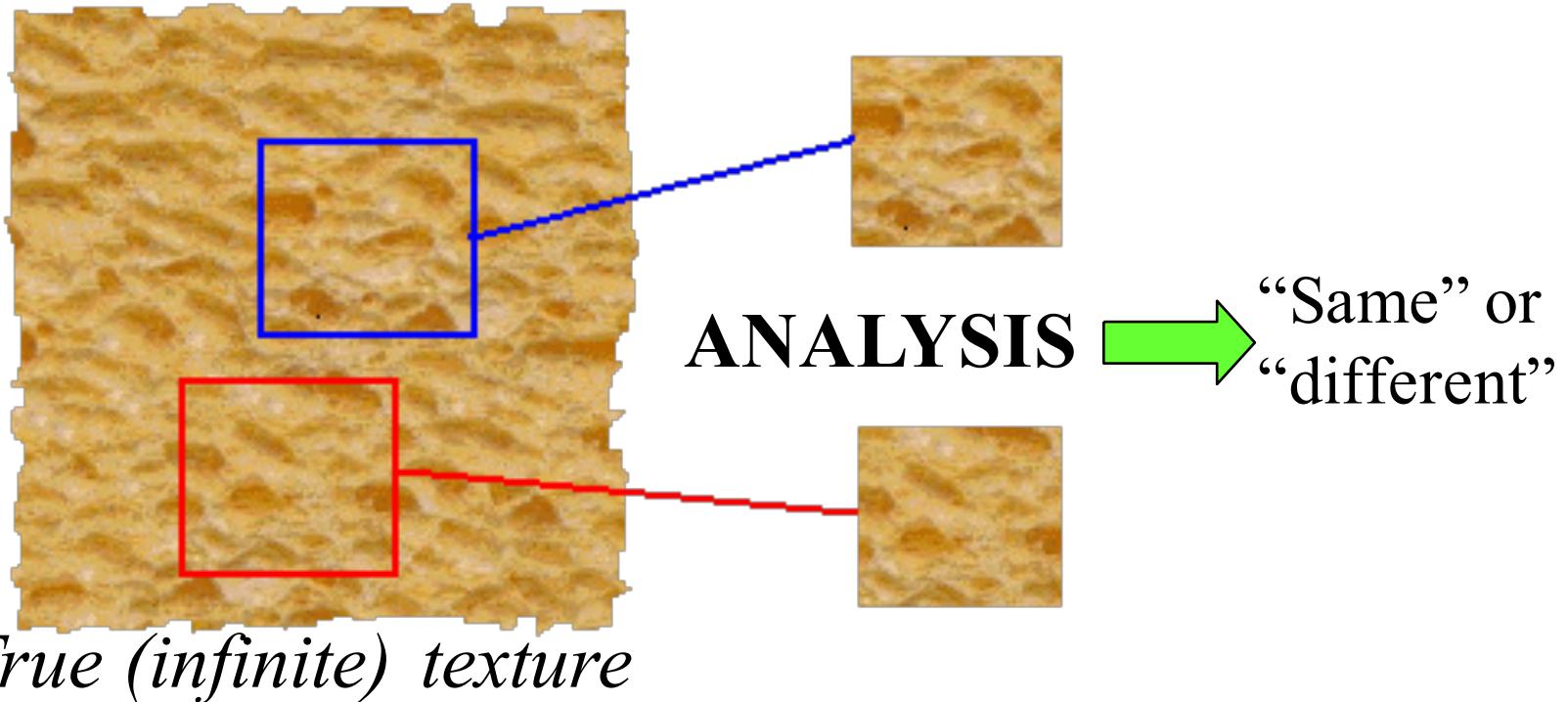
# Texture and Material



# When are two textures similar?

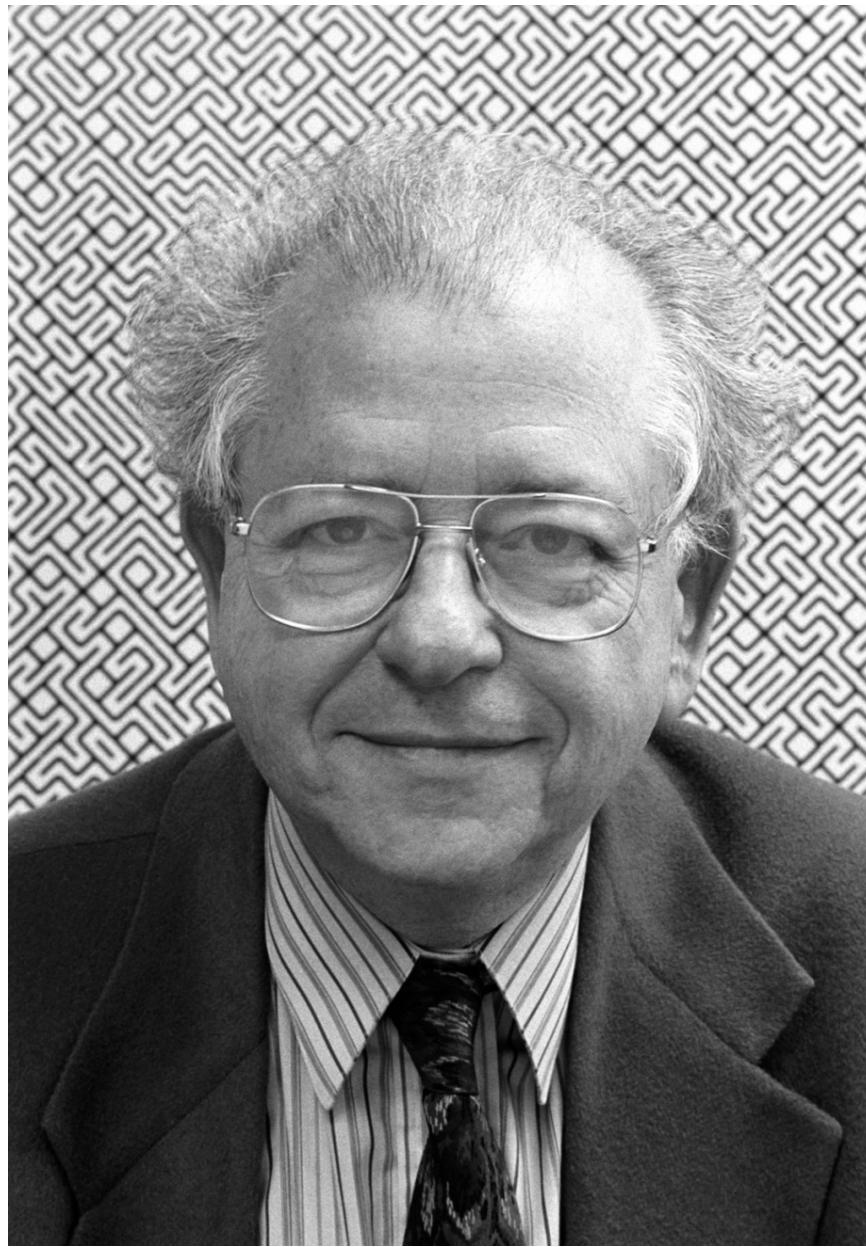


# Texture Analysis

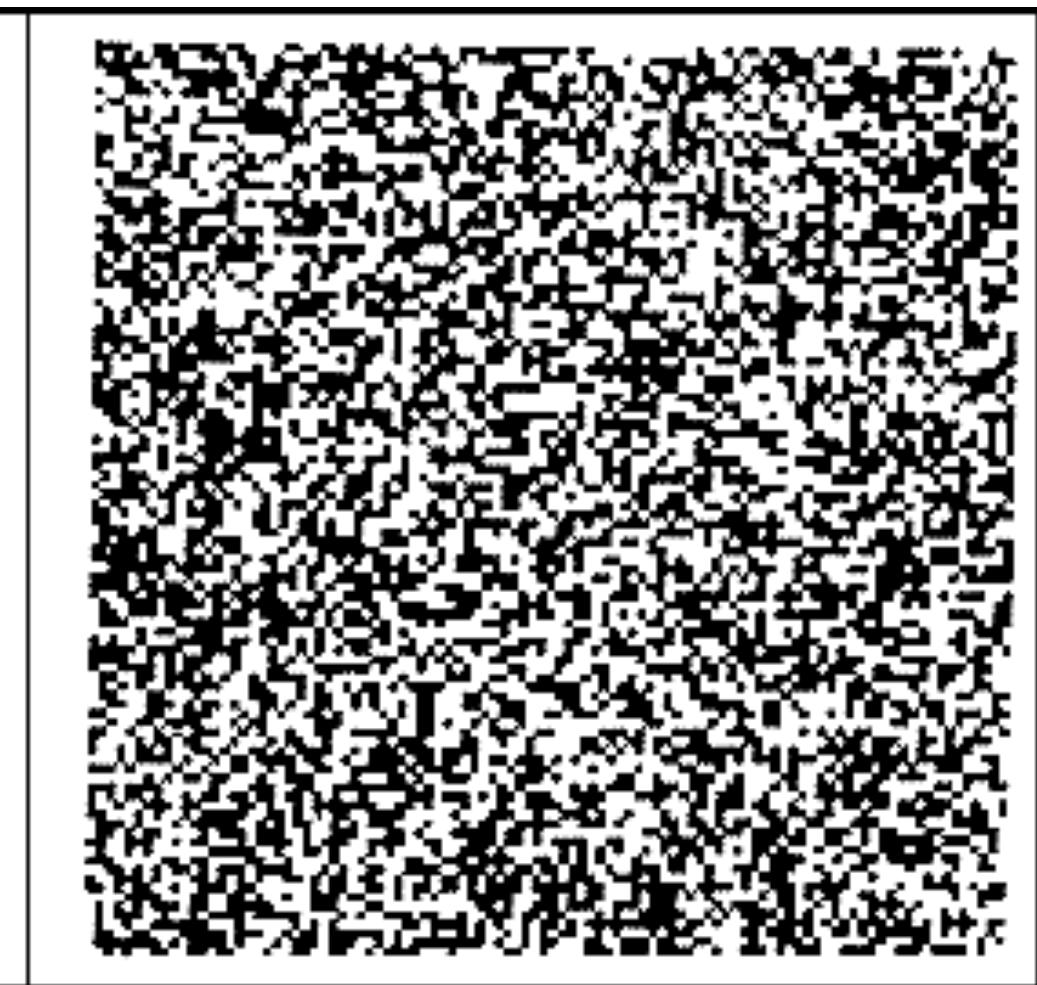
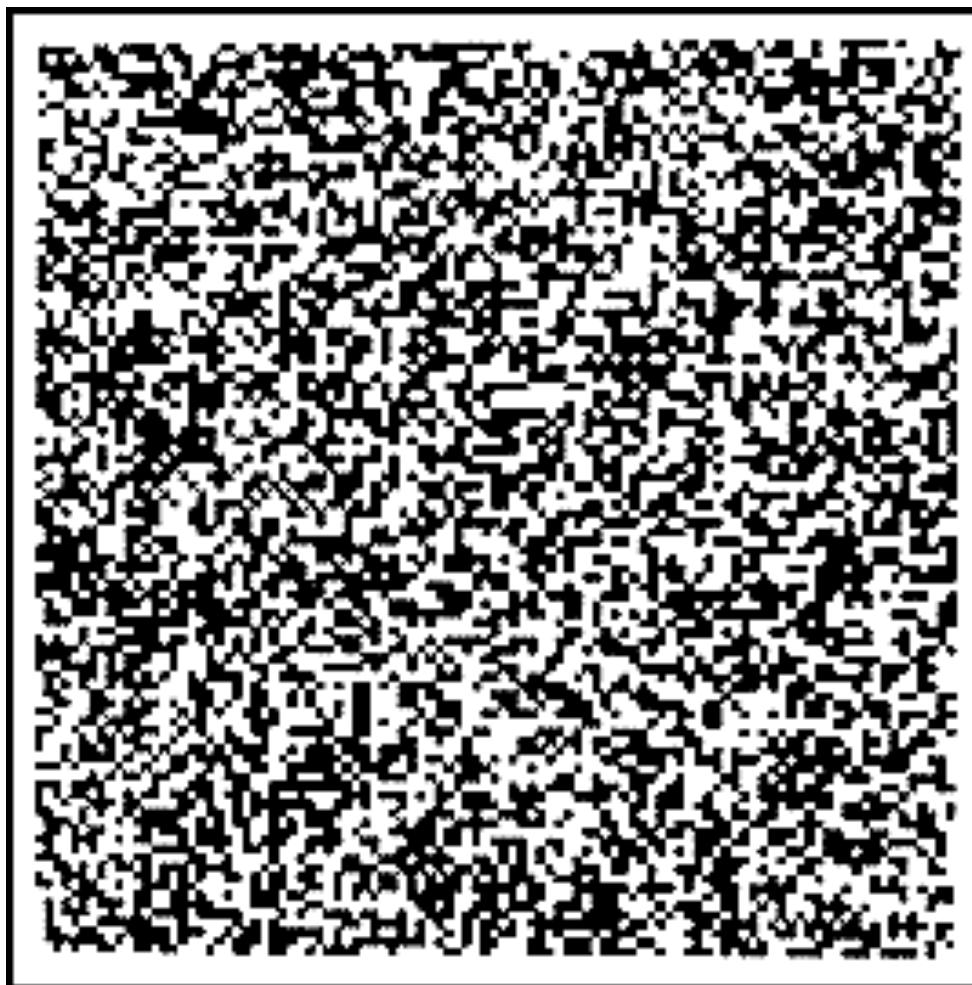


Compare textures and decide if they're made of the same “stuff”.

# Béla Julesz, father of texture

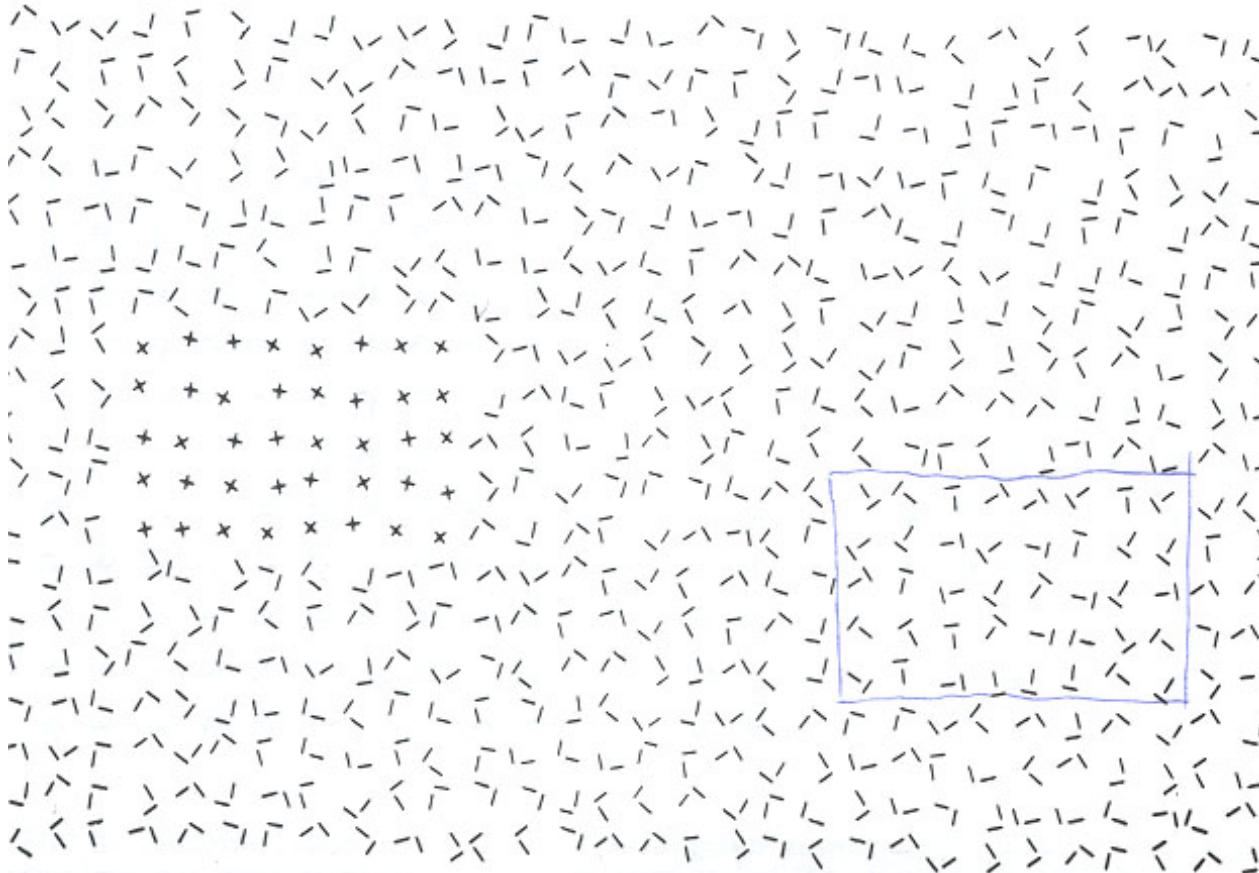


# Random Dot Stereograms



# Texton Discrimination (Julesz)

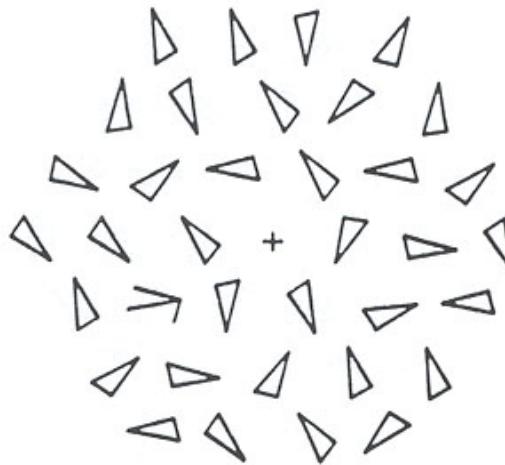
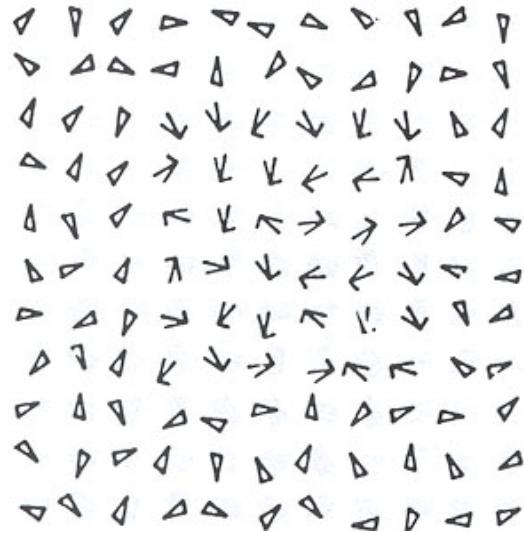
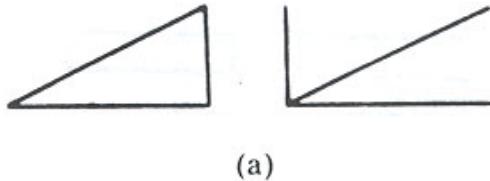
---



Human vision is sensitive to the difference of some types of elements and appears to be “numb” on other types of differences.

# Search Experiment I

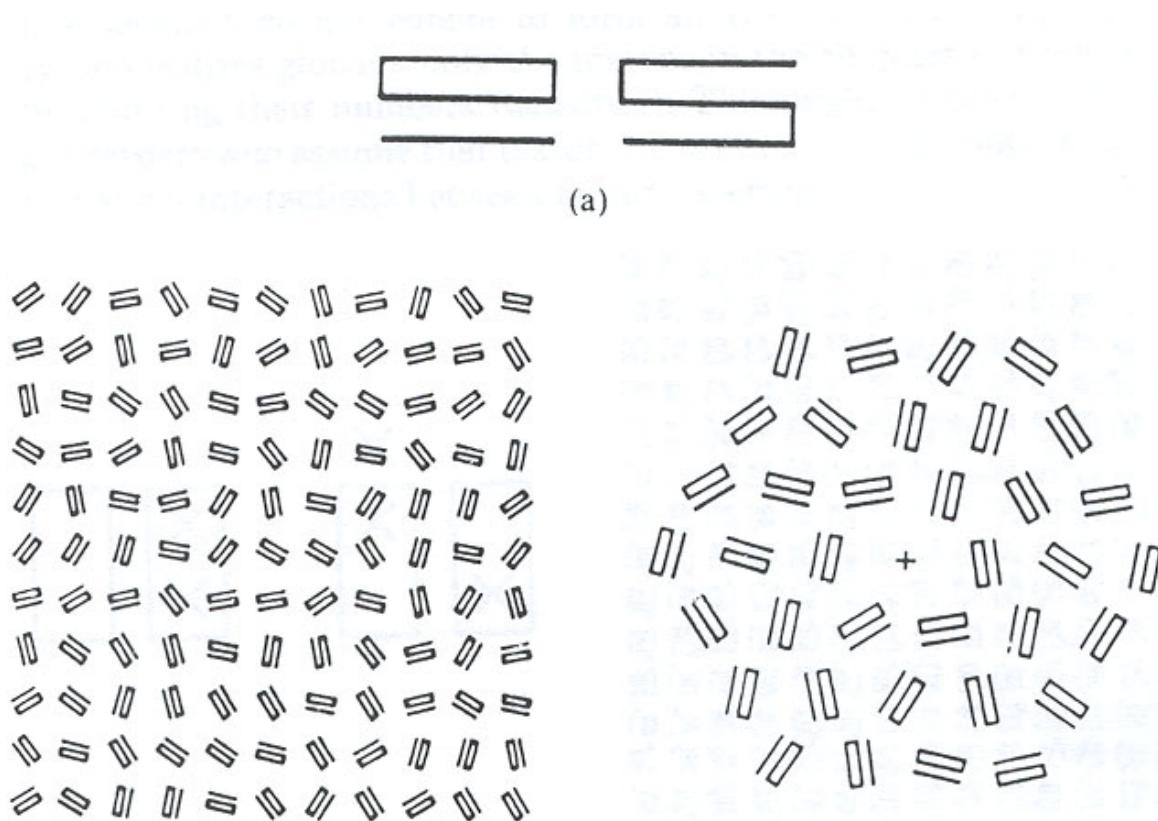
---



The subject is told to detect a target element in a number of background elements.  
In this example, the detection time is independent of the number of background elements.

# Search Experiment II

---



In this example, the detection time is proportional to the number of background elements.  
And thus suggests that the subject is doing element-by-element scrutiny.

# Heuristic (Axiom) I

---

Julesz then conjectured the following axiom:

Human vision operates in two distinct modes:

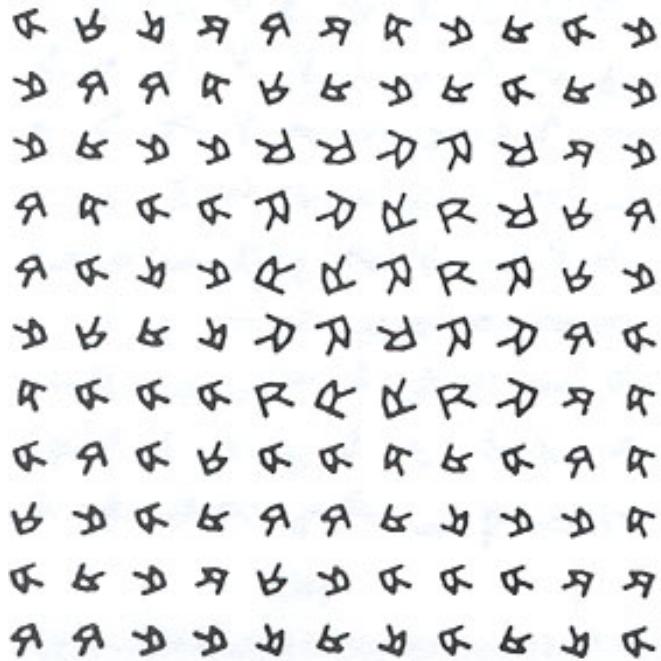
1. Preattentive vision

parallel, instantaneous (~100--200ms), without scrutiny,  
independent of the number of patterns, covering a large visual field.

2. Attentive vision

serial search by focal attention in 50ms steps limited to small aperture.

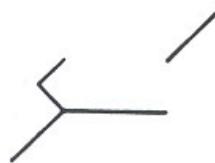
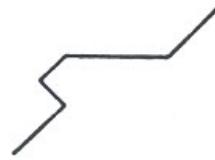
# Examples



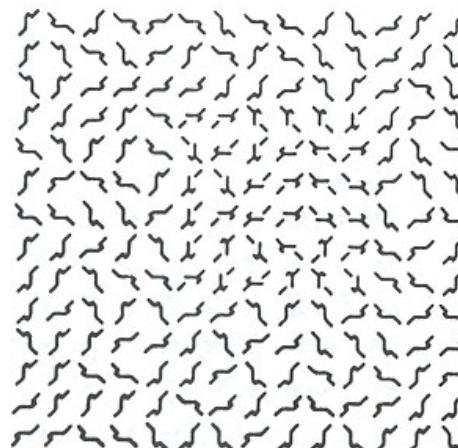
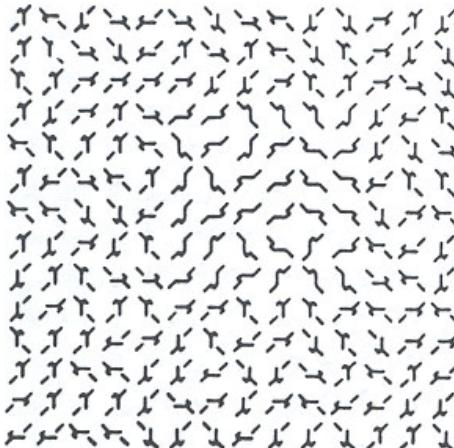
Pre-attentive vision is sensitive to size/width, orientation changes

# Examples

---



(a)



Sensitive to number  
of terminators

Left: fore-back  
Right: back-fore

See previous examples  
For cross and terminators

# Heuristic (Axiom) II

---

Textons are the fundamental elements in preattentive vision, including

1. Elongated blobs

- rectangles, ellipses, line segments with attributes  
color, orientation, width, length, flicker rate.

2. Terminators

- ends of line segments.

3. Crossings of line segments.

But it is worth noting that Julesz's conclusions are largely based on ensemble of artificial texture patterns. It was infeasible to synthesize natural textures for controlled experiments at that time.

# Julesz Conjecture

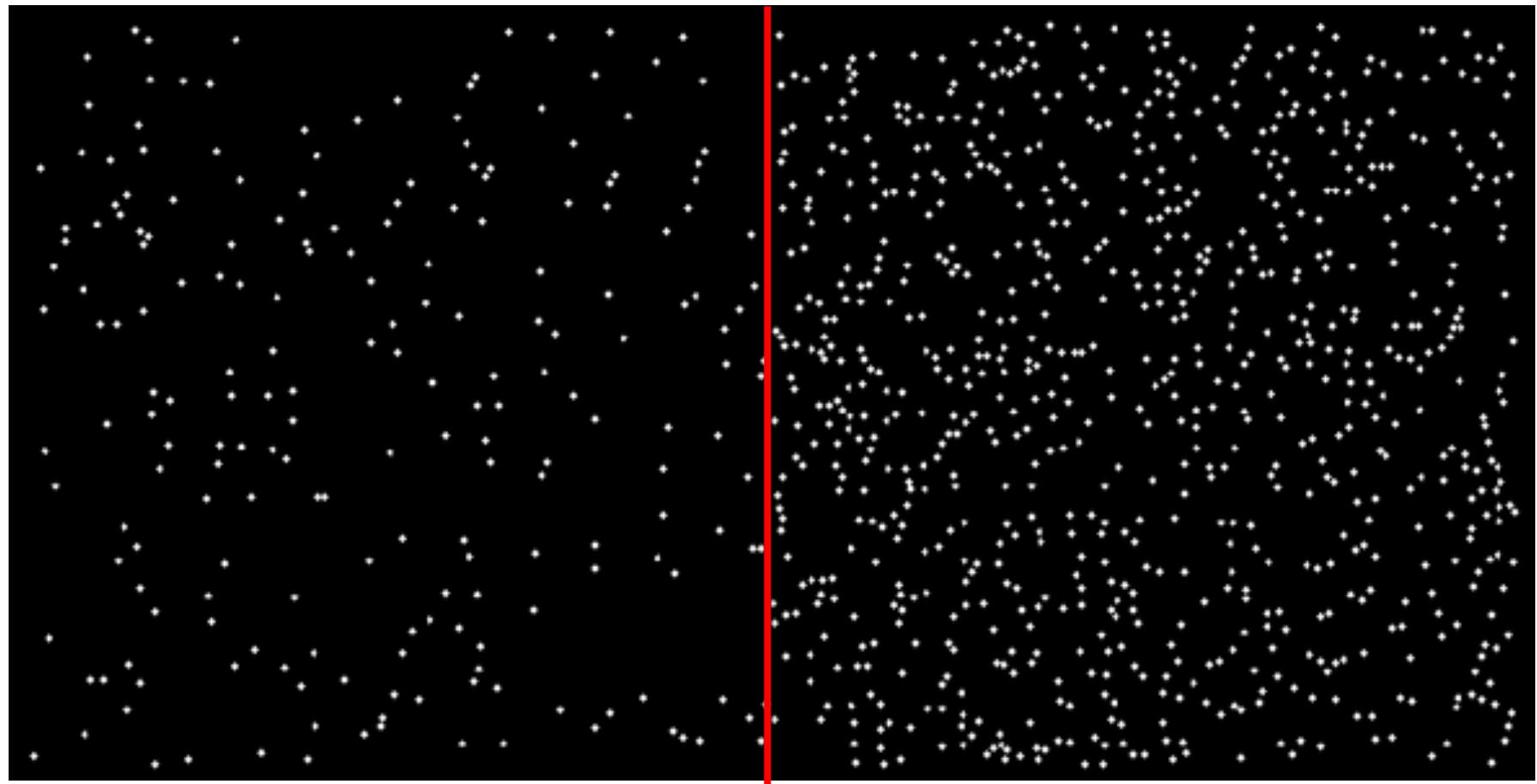
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*Textures cannot be spontaneously discriminated if they have the same first-order and second-order statistics and differ only in their third-order or higher-order statistics.*

(later proved wrong)

# 1<sup>st</sup> Order Statistics

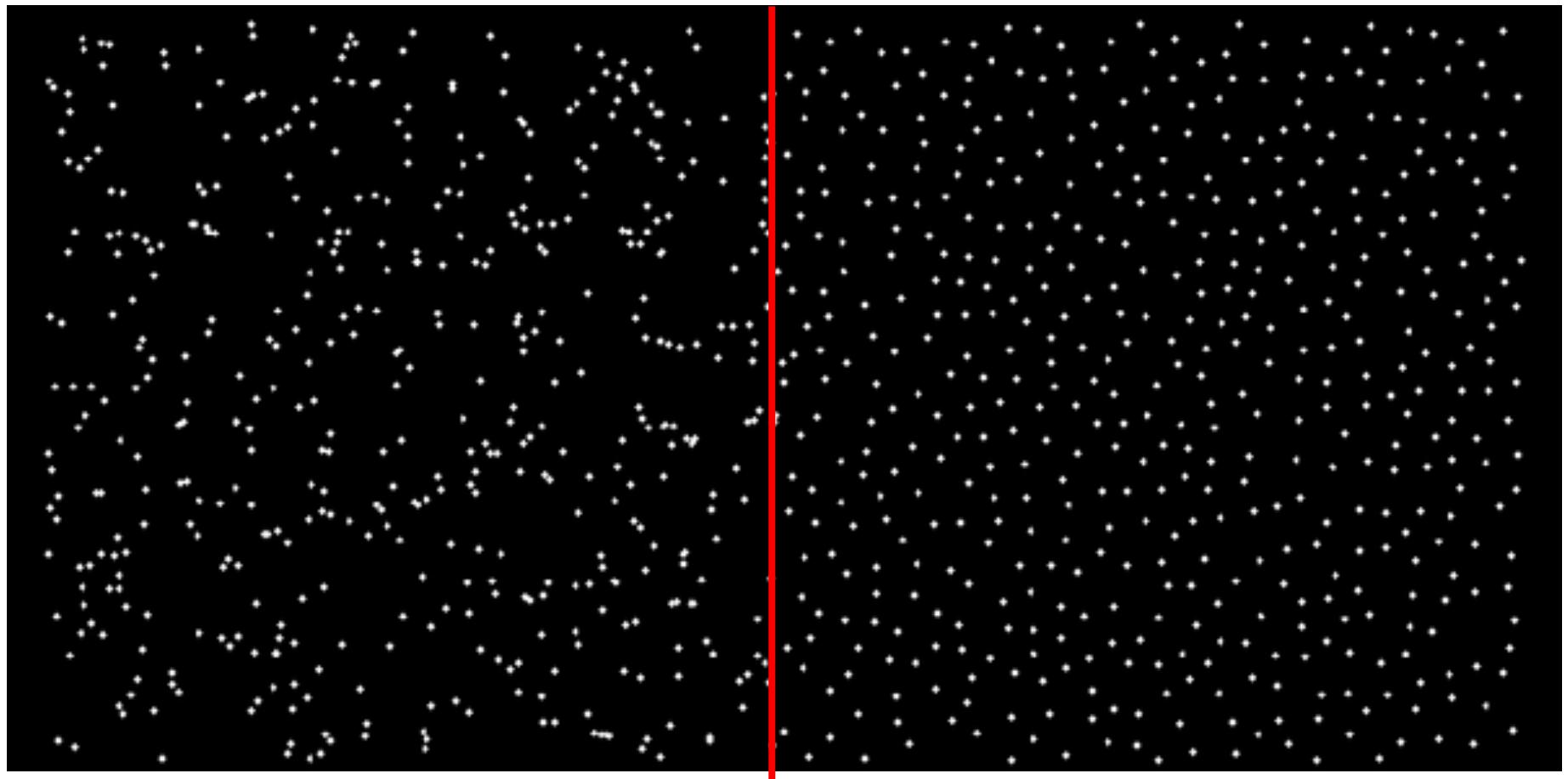
---



5% white

20% white

# 2<sup>nd</sup> Order Statistics



10% white