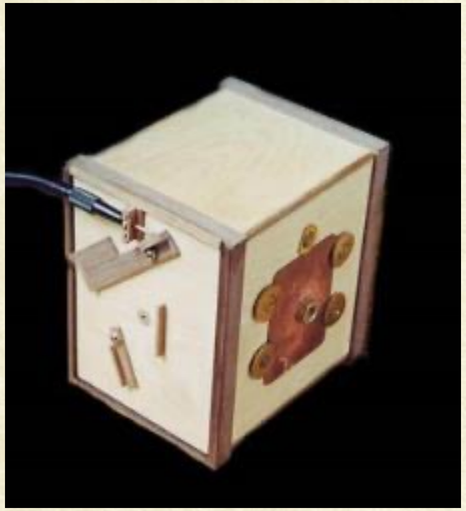
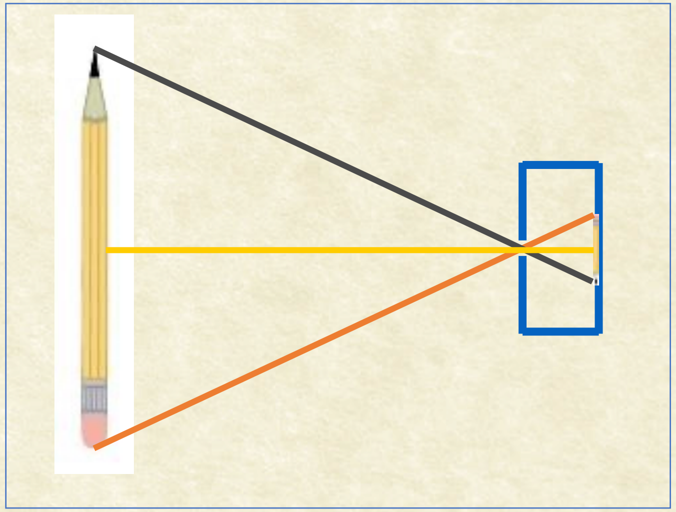
CS7.505: Computer Vision

Spring 2022: Pinhole Camera Model



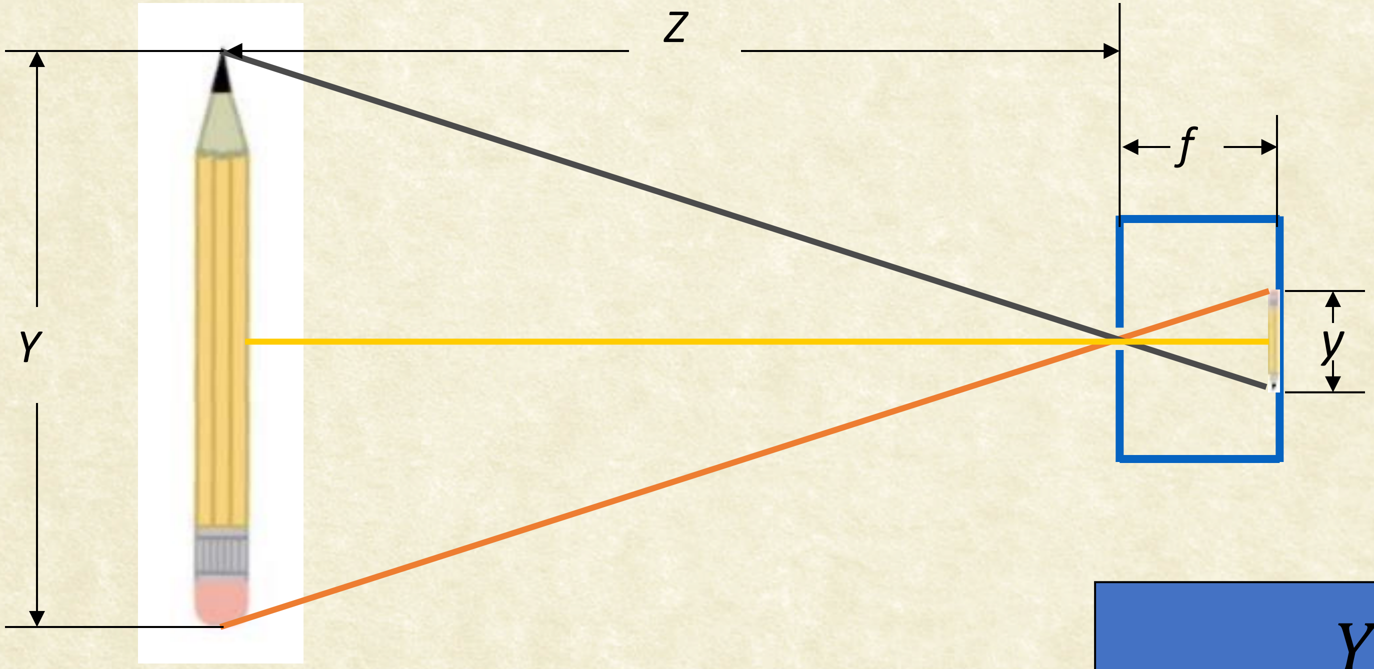


Anoop M. Namboodiri

Biometrics and Secure ID Lab, CVIT,

IIIT Hyderabad

The Pinhole Camera

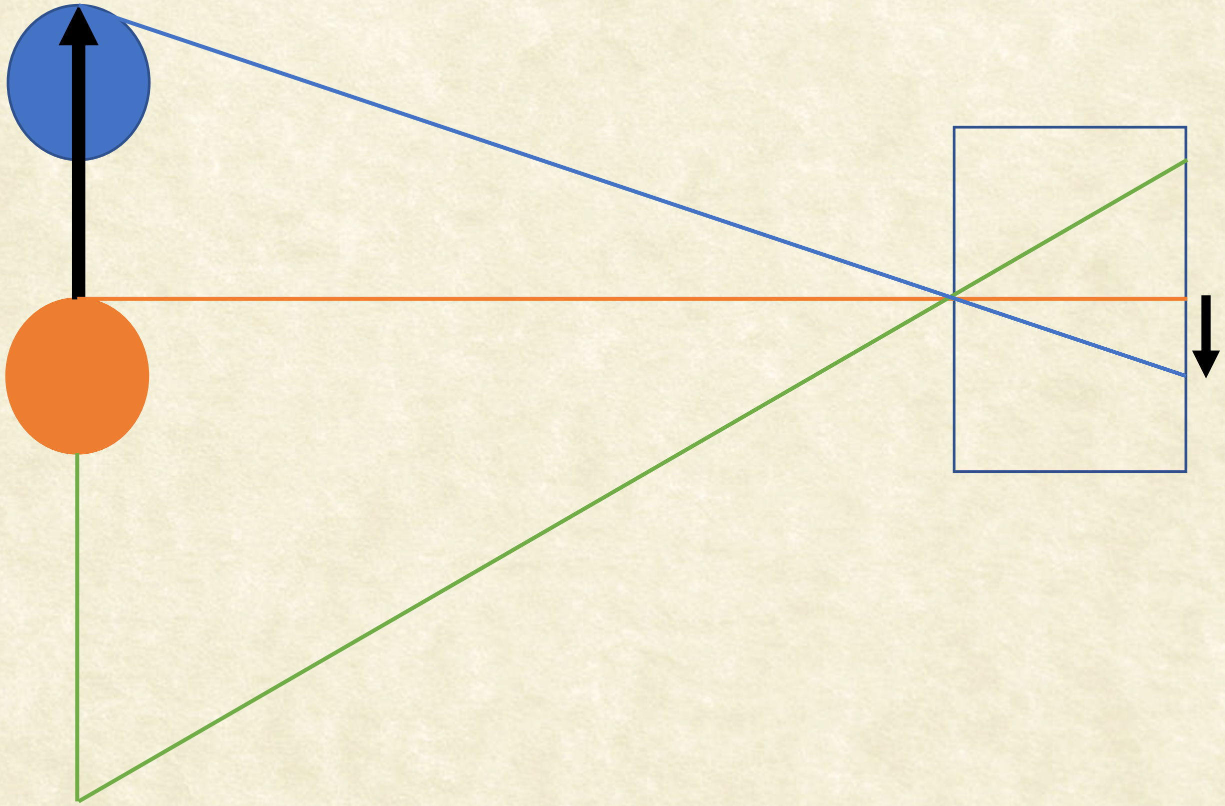


$$y = f\frac{Y}{Z}$$

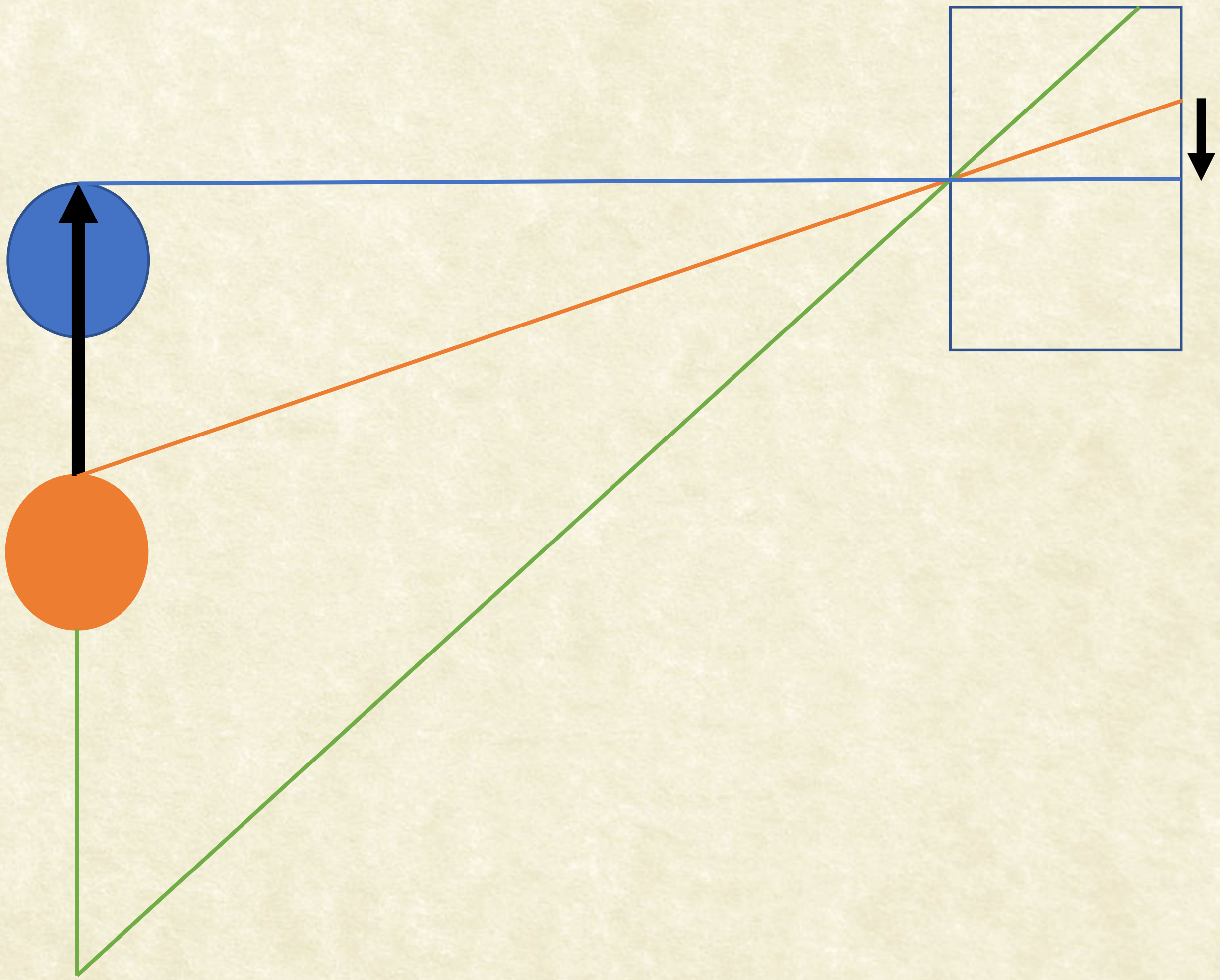
Problem

- You have a person who is **1.75m** tall standing at a distance of **7m** from a camera. The pinhole camera has a focal length of **20mm**. The sensor is 1cm tall and has a resolution of **4000x3000**.
  - Find the height of the person in pixels in the image.
  - If the camera is raised by **1m**, how much does the person move in the sensor (in pixels)?
  - How much does the Sun move in the above case
Note: Sun is **150 million kms** away (in pixels)?

Object Size vs Object Motion vs Camera Motion

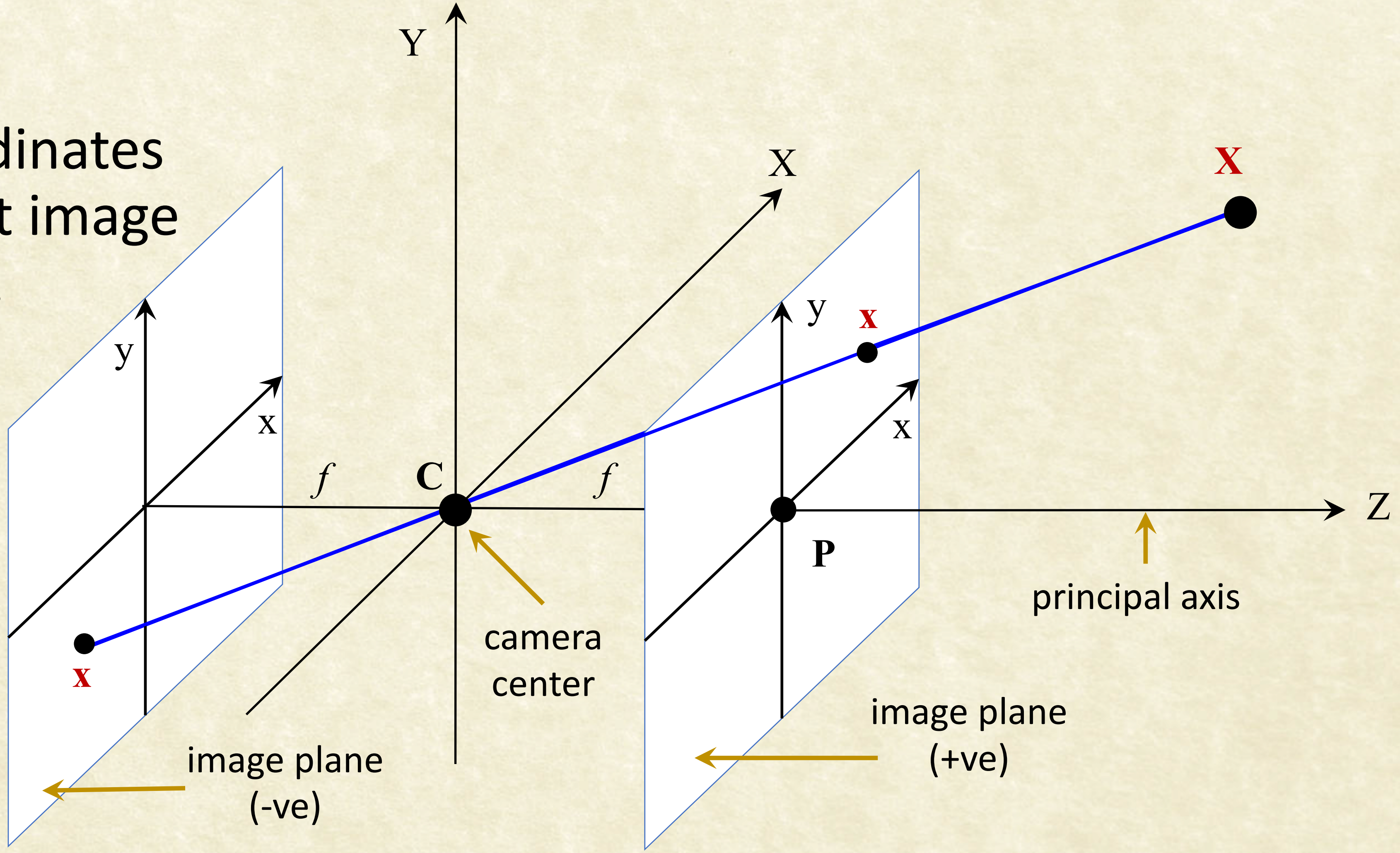


Object Size vs Object Motion vs Camera Motion

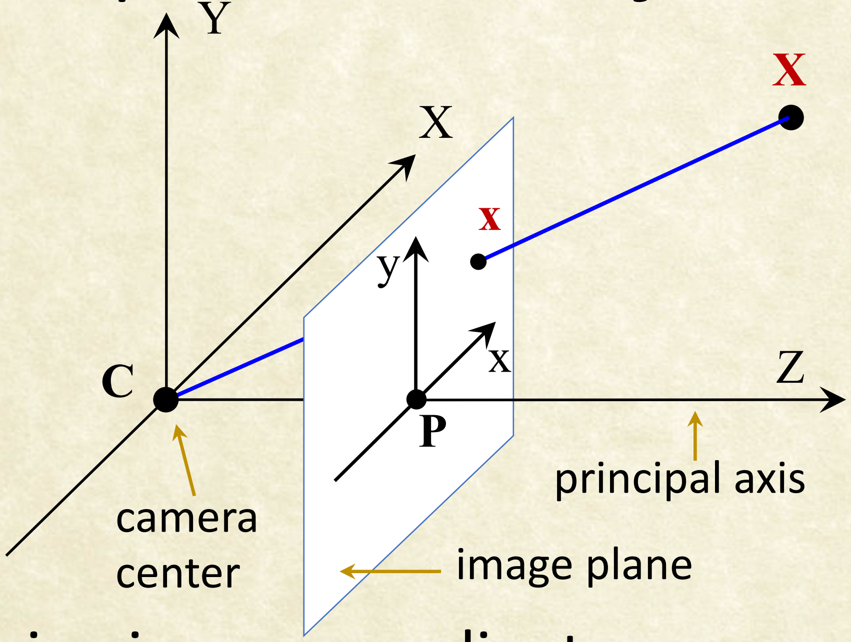


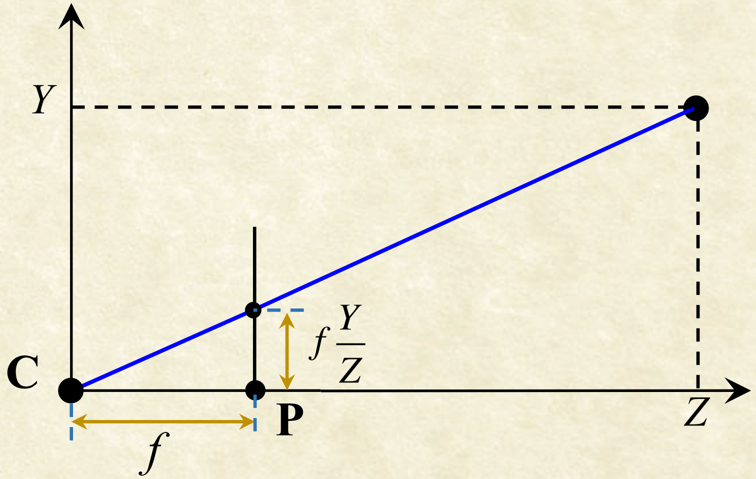
Perspective Projection

- Inverted Image
- Multiply image coordinates with -1 to get upright image
- Equivalent to placing the image plane in front of C



Perspective Projection





- Cartesian image coordinates: $x = f\dfrac{X}{Z}, \qquad y = f\dfrac{Y}{Z}$

- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

Basic Camera Equation

A pinhole camera projects a 3D point $\mathbf{X_c}$ in camera coords to an image point $\mathbf{x}$ via the 3x4 camera matrix $\mathbf{P}$ as:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_c = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I}|\mathbf{0}]\mathbf{X}_c = \mathbf{K}[\mathbf{I}|\mathbf{0}]\mathbf{X_c},$$

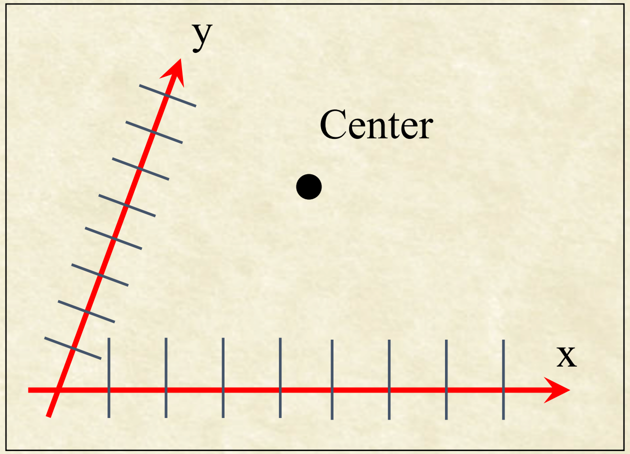
where $\mathbf{K}$ is the internal camera calibration matrix.

Note that:
- The camera is at the origin
- Z is the Camera or Optical axis
- Principal Point: Center of the image
- Focal length in pixel units
- Orthogonal image axes with uniform scale

A General Camera

Image center at (x_0, y_0), Non-orthogonal axes with skew s, and different scales for axes with focal lengths, α_x and α_y.

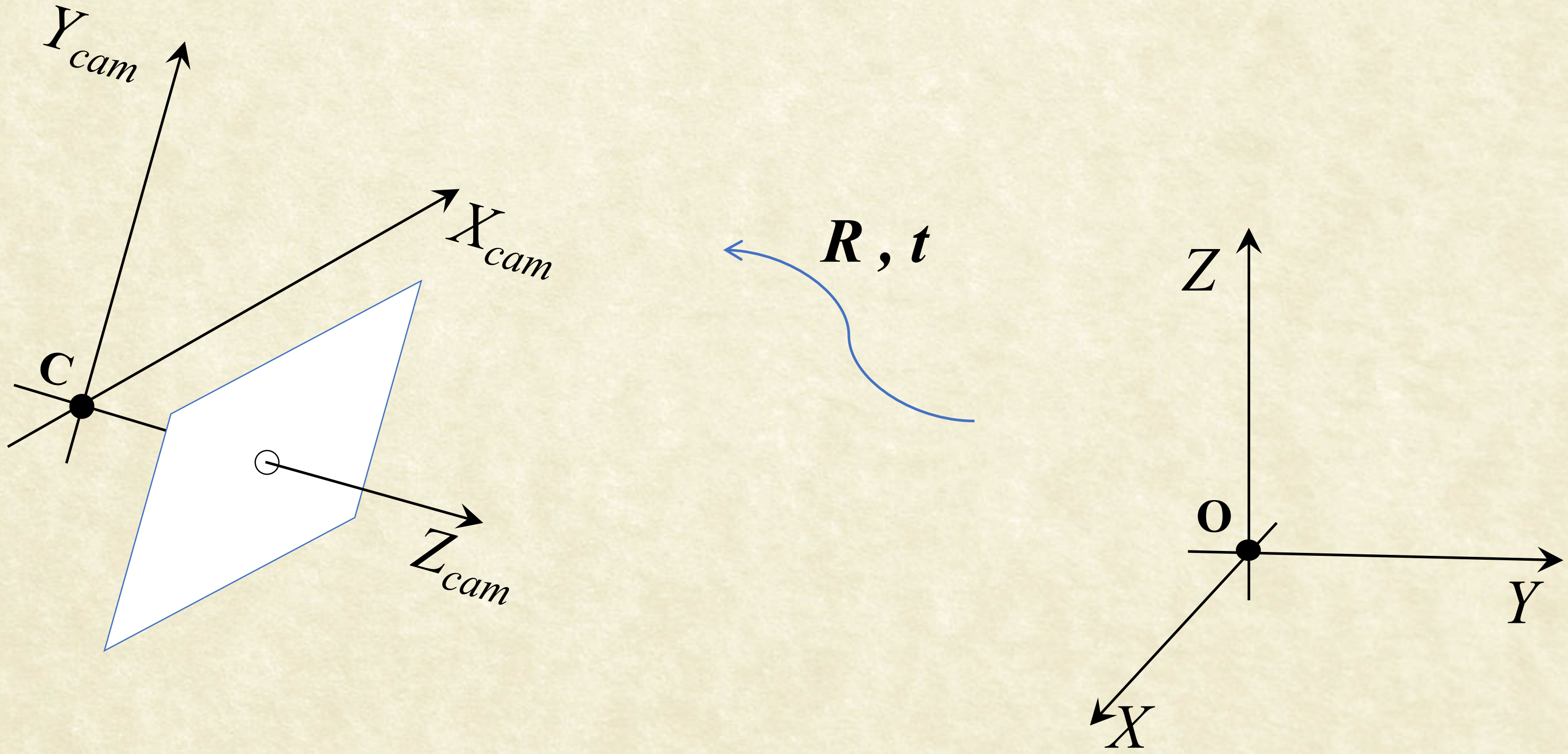
$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$



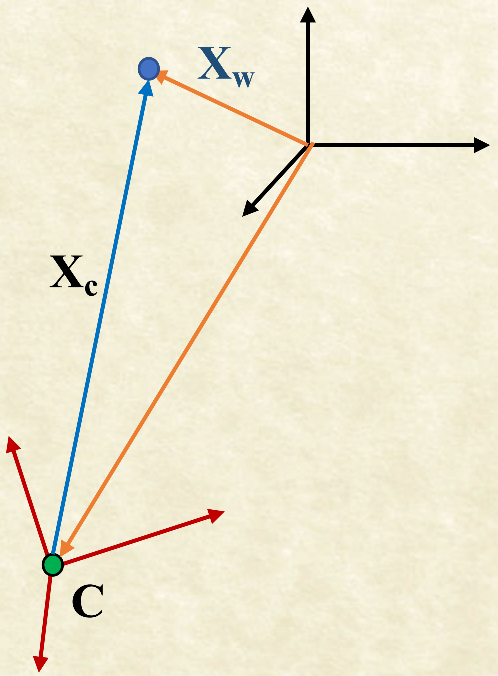
K an upper triangular matrix with 5 degrees of freedom.

Moving the Camera from Origin

- General Setting: Camera is not at origin and Z is not the optical axis.
- Camera is at a point **C** in world coordinates. The world to camera rotation is **R** (rows of R are X_{cam} Y_{cam} Z_{cam} in world coordinates).



General Camera Equation



- In cartesian coordinates, $\mathbf{X_c = R\,(X_w - C) = RX_w - RC}$
- In homogeneous co-ordinates:

$$\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$$

- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ given by:
  - $\mathbf{x = K[\,I \mid 0\,]\,X_c = K\,[\,R \mid -RC\,]\,X_w}$
- $\mathbf{x = PX_w}$; camera matrix $\mathbf{P = [KR \mid -KRC] = [M \mid p_4]}$
- Common K: $\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$ General K: $\begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

Questions?