CS7.505: Computer Vision

Spring 2022: Two View Geometry

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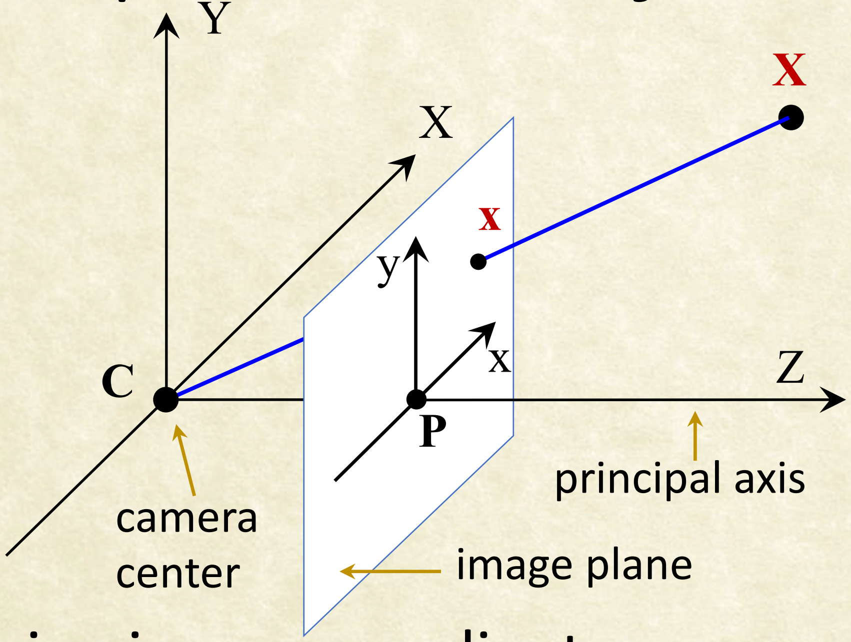
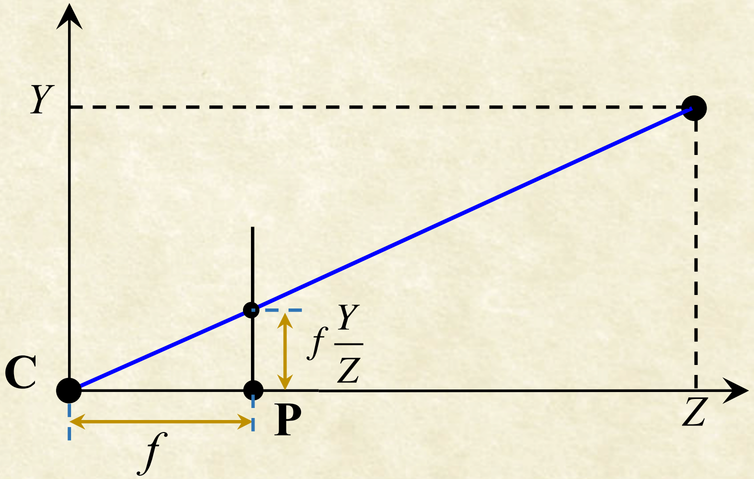
Perspective Projection





- Cartesian image coordinates:

$$x = f\frac{X}{Z}, \qquad y = f\frac{Y}{Z}$$

- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

General Camera Equation

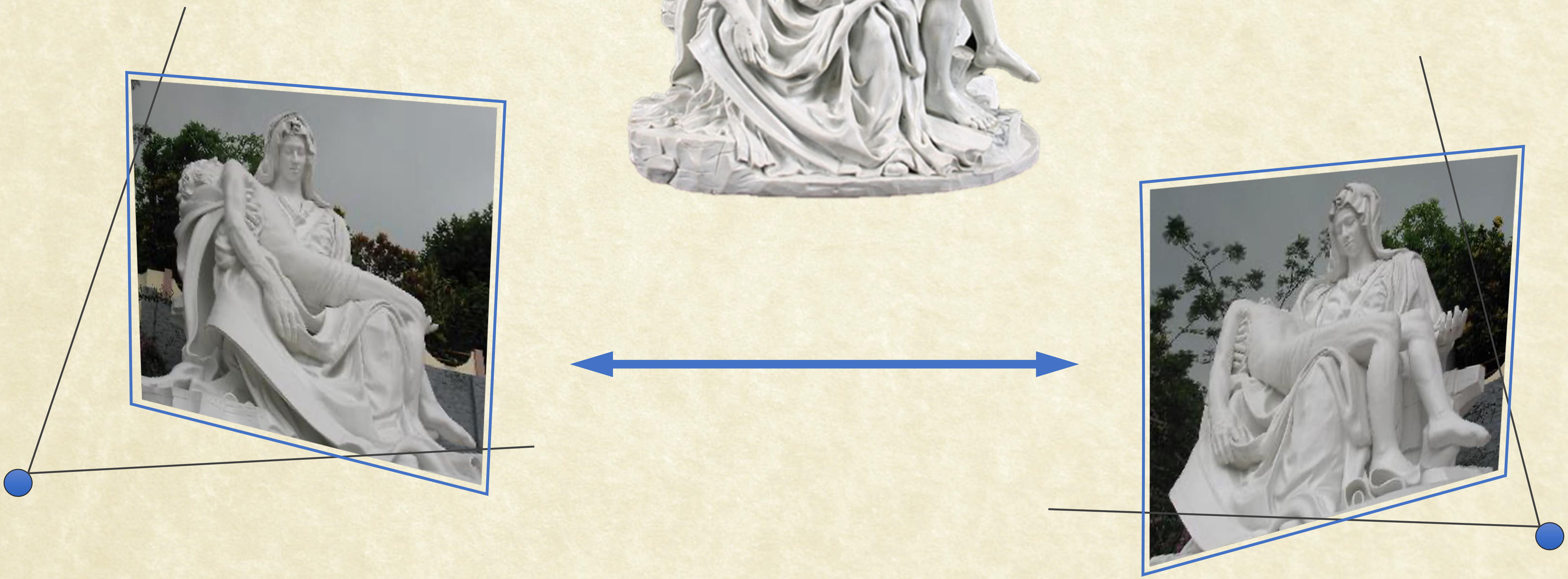
- Camera and world are related by: $\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$
- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$
  is given by: $\mathbf{x} = \mathbf{PX_w}$
- Camera matrix: $\mathbf{P} = [\mathbf{KR} \mid \mathbf{-KRC}] = [\mathbf{M} \mid \mathbf{p_4}]$
- Common K: General K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

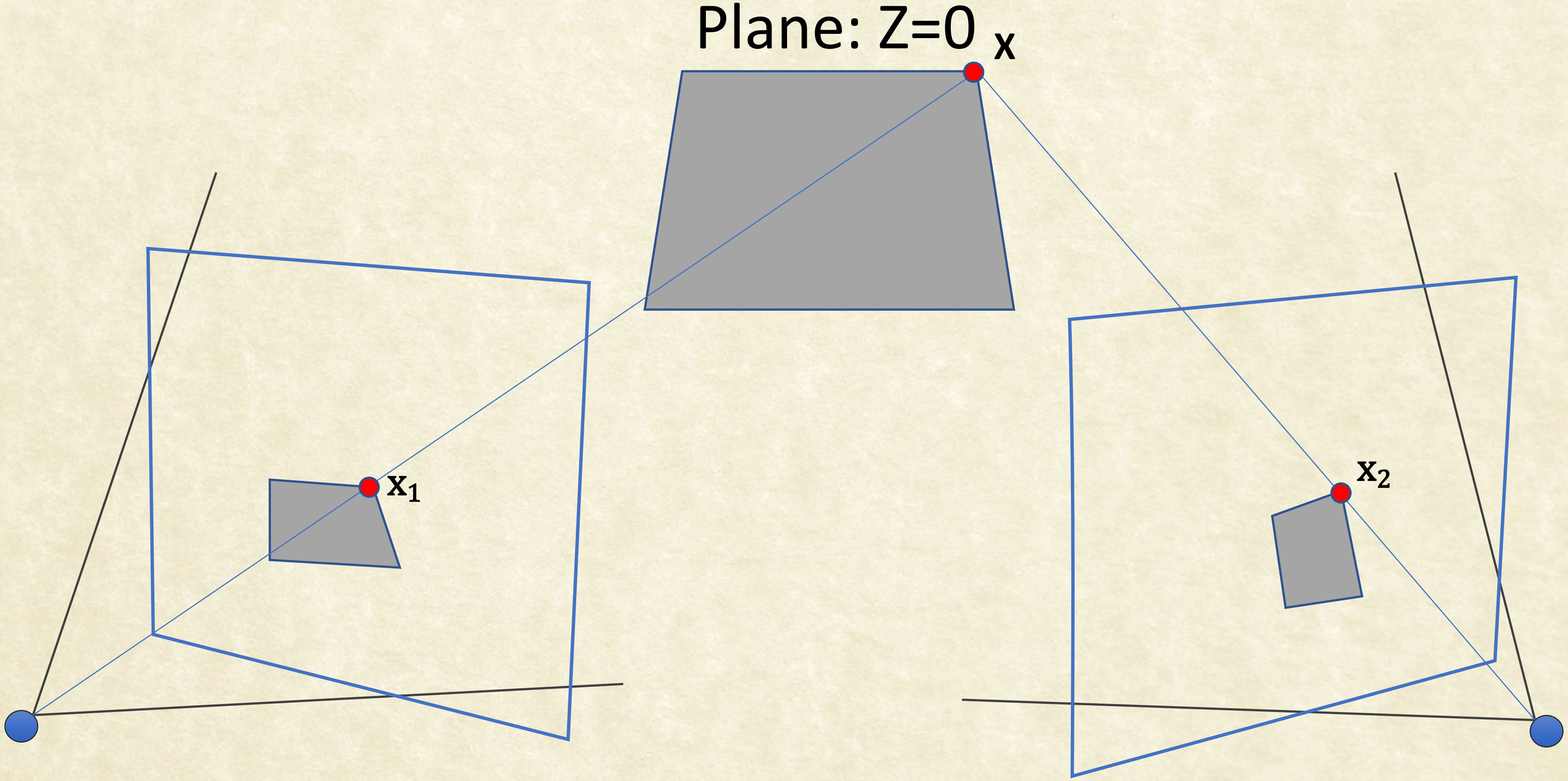
$$\boxed{\mathbf{x} = \mathbf{PX_w}}$$

Geometry of Two Views

- How are the two views related to the world and to each other?



Case 1: Planar World



What is the relation between $\mathbf{x}_1$ and $\mathbf{x}_2$?

Case 1: Planar World

- Projection equation of points on a plane:

$$\mathbf{x} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H}\mathbf{X},$$

where H is a 3 × 3 non-singular matrix

- Now if we consider two different views of the same world point, we get:

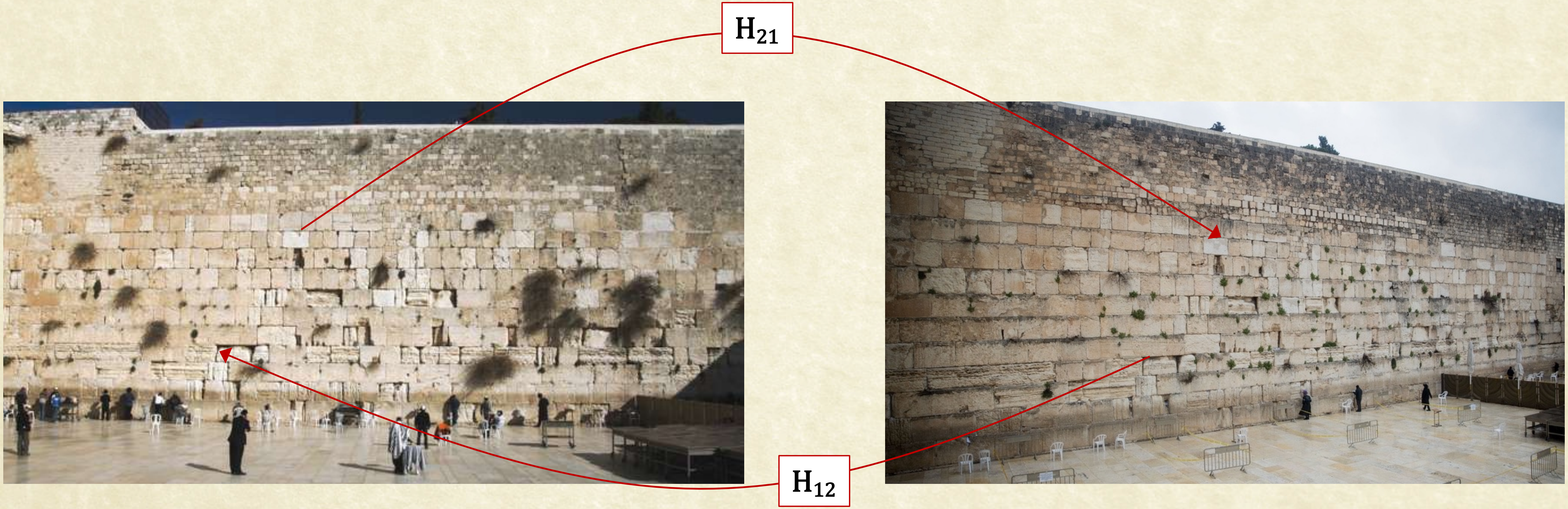
$$\mathbf{x}_1 = \mathbf{H}_1\mathbf{X} \qquad \mathbf{x}_2 = \mathbf{H}_2\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{H}_2\mathbf{H}_1{}^{-1}\mathbf{x}_1 = \mathbf{H}_{21}\mathbf{x}_1$$

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{12}\mathbf{x}_2; \qquad \mathbf{x}_2 = \mathbf{H}_{21}\mathbf{x}_1}$$

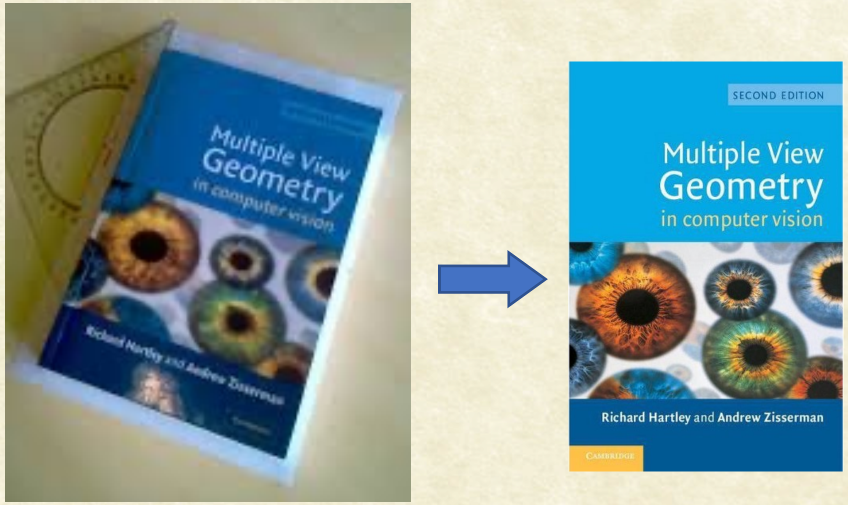
Planar Homography

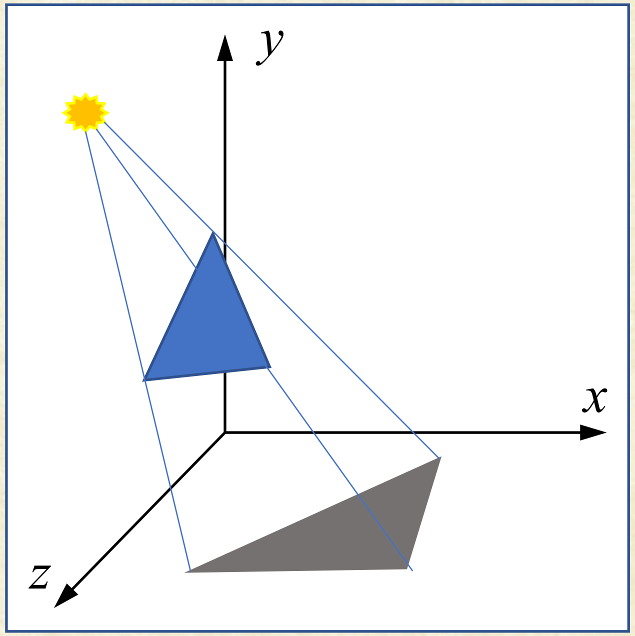
- Given two images of a planar world, every pixel in an image can be computed from the other.   Its location is given by $\mathbf{x}_a = \mathbf{H}\mathbf{x}_b$

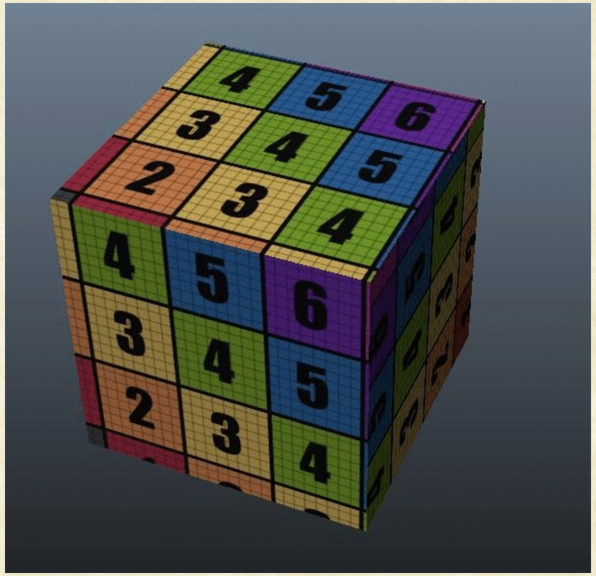


Planar Homography: Applications

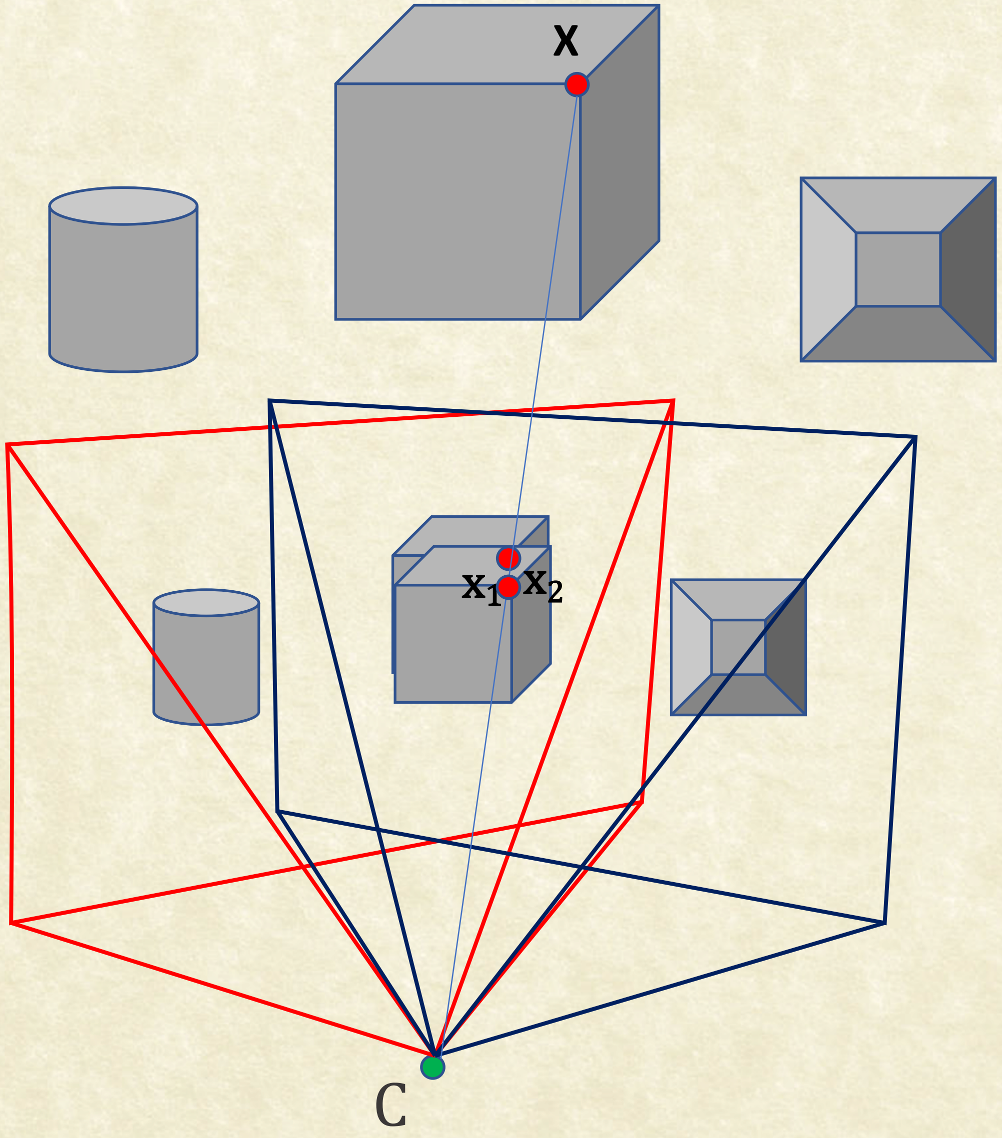
- Removing perspective distortion
- Rendering planar textures
- Rendering planar shadows
- Estimating Camera Pose; AR







Case 2: Same Camera Center



Arbitrary world

What is the relation between $\mathbf{x}_1$ and $\mathbf{x}_2$?

Case 2: Same Camera Center

- Projection equation for two cameras with same C:

$$\mathbf{x}_1 = \mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{K}_2\mathbf{R}_2\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{x}_1$$

$$= \mathbf{H}_{21}\mathbf{x}_1$$

where H is a 3 × 3 non-singular matrix

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{12}\mathbf{x}_2; \quad \mathbf{x}_2 = \mathbf{H}_{21}\mathbf{x}_1}$$

Homography: Applications

- Image Mosaicing
- Detecting camera translation
- Multi-frame Super-resolution

Questions?

Cross Product: A Recap

- Consider $\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$; and Let $\widehat{\mathbf{A}} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix}$
- $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{bmatrix} A_y B_z - B_y A_z \\ B_x A_z - A_x B_z \\ A_x B_y - B_x A_y \end{bmatrix}$
- $\widehat{\mathbf{A}}\mathbf{B} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} A_y B_z - B_y A_z \\ B_x A_z - A_x B_z \\ A_x B_y - B_x A_y \end{bmatrix}$
- Note: The cross product, AxB or ÂB is a vector perpendicular to both A and B

Questions?