03.09.2021

Digital Image Processing (CSE/ECE 478)

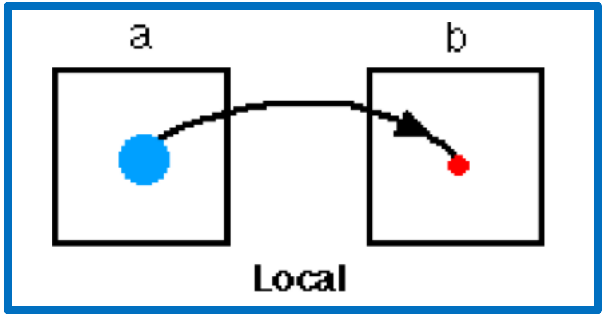
Lecture-6: Spatial Filtering

Center for Visual Information Technology (CVIT), IIIT Hyderabad
Ravi Kiran and Sudipta Banerjee

Announcements

- TAs
  - Indupuru Sai Manaswini Reddy
  - Jayant Duneja
  - M Kalyan Adithya
  - Fiza Husain
  - Anushree Korturti
  - Haripraveen Subramanian
- No classes next week

- **Neighborhood to Point**



What is Image Filtering?

- A process of transforming original image to create an output image with desirable properties

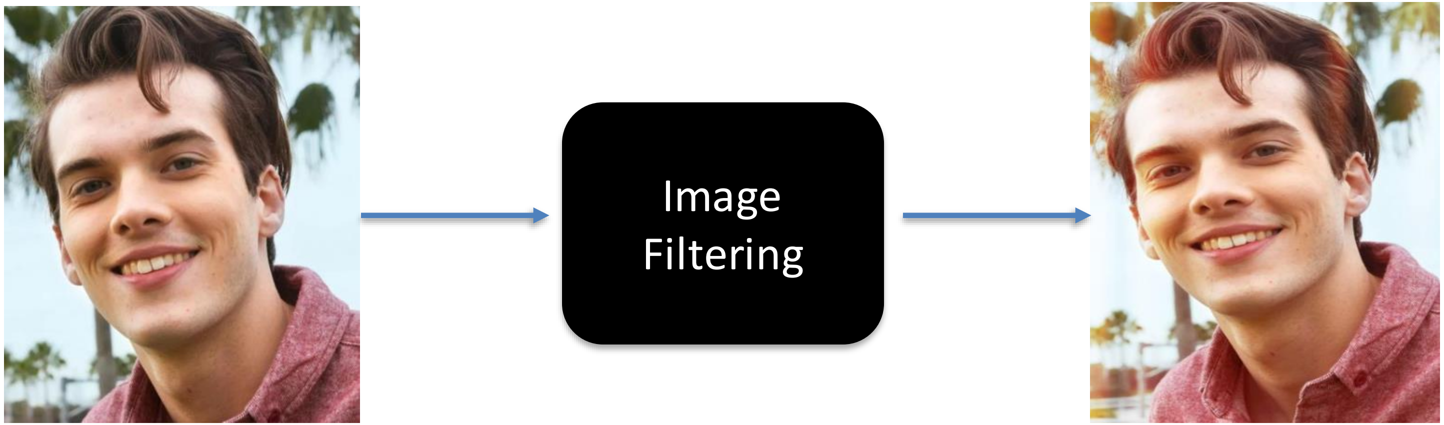


Image Courtesy: https://www.makeuseof.com/best-face-filter-apps-selfies/

What is Image Filtering?

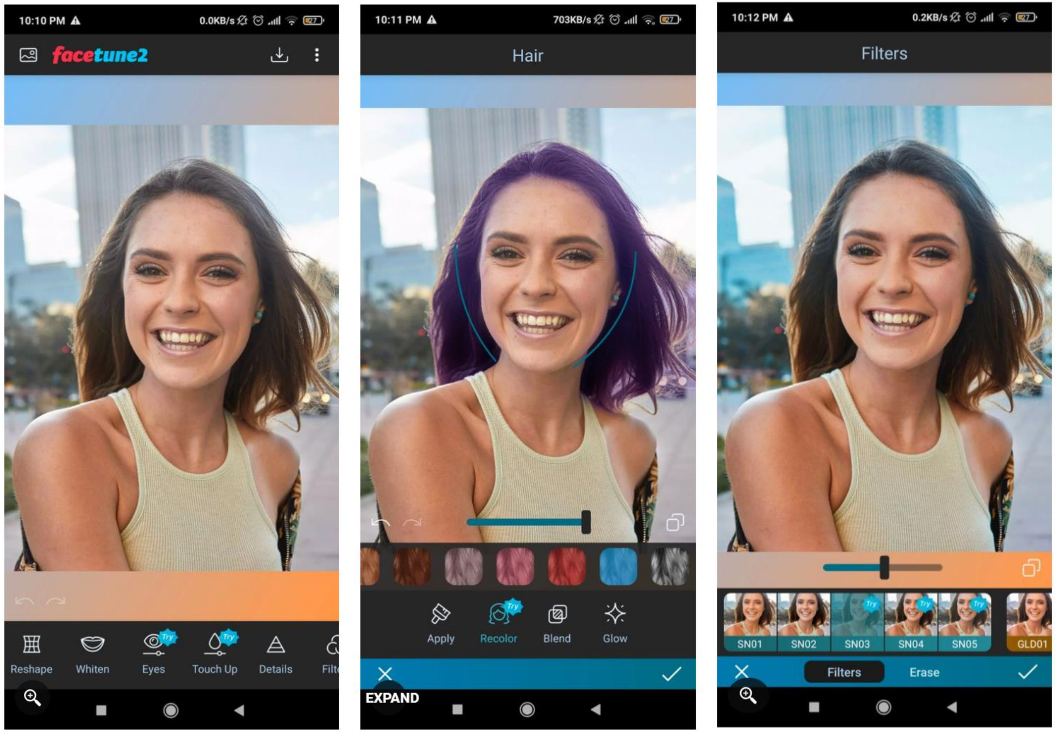
- The ‘black box’ has underlying mathematical properties
- Remember image is a 2-D signal (so signal processing theory applies)
- Filtering can be done in: Spatial domain or Frequency domain
  - Spatial domain: Directly on the pixels
  - Frequency domain: Apply Fourier transform and then perform filtering

$$y(t) = x(t) * h(t) \leftrightarrow Y(f) = X(f).H(f)$$

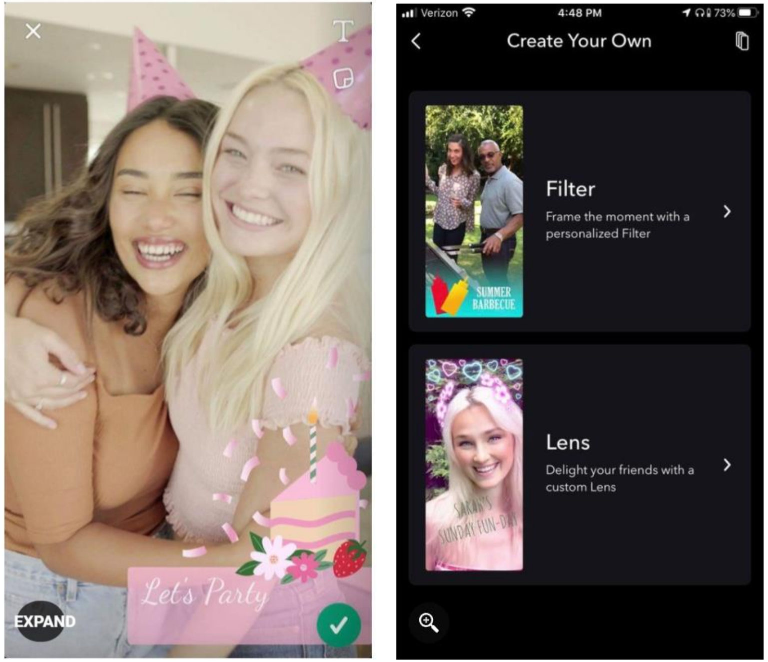
Prelim: Convolution operation in time domain is analogous to multiplication in frequency domain

Spatial Domain Filtering

Selfie Time…



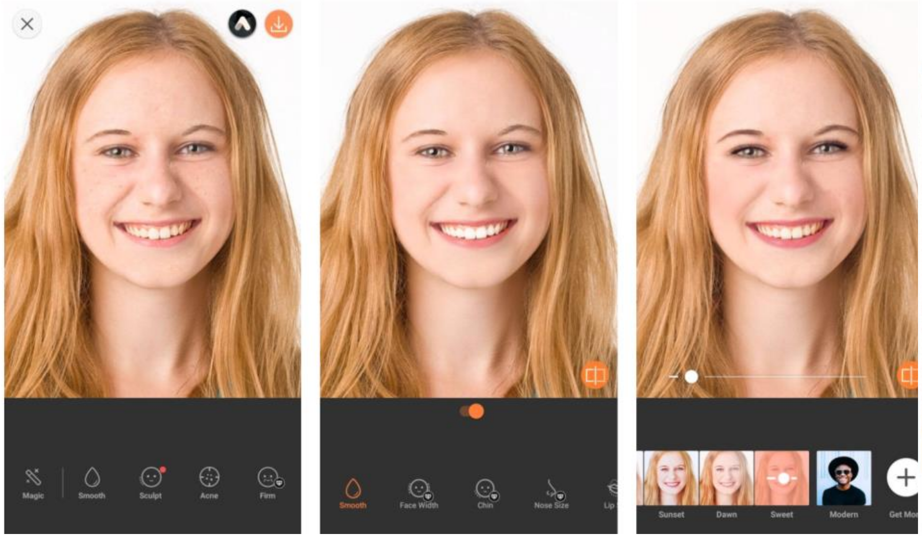
FaceTune2



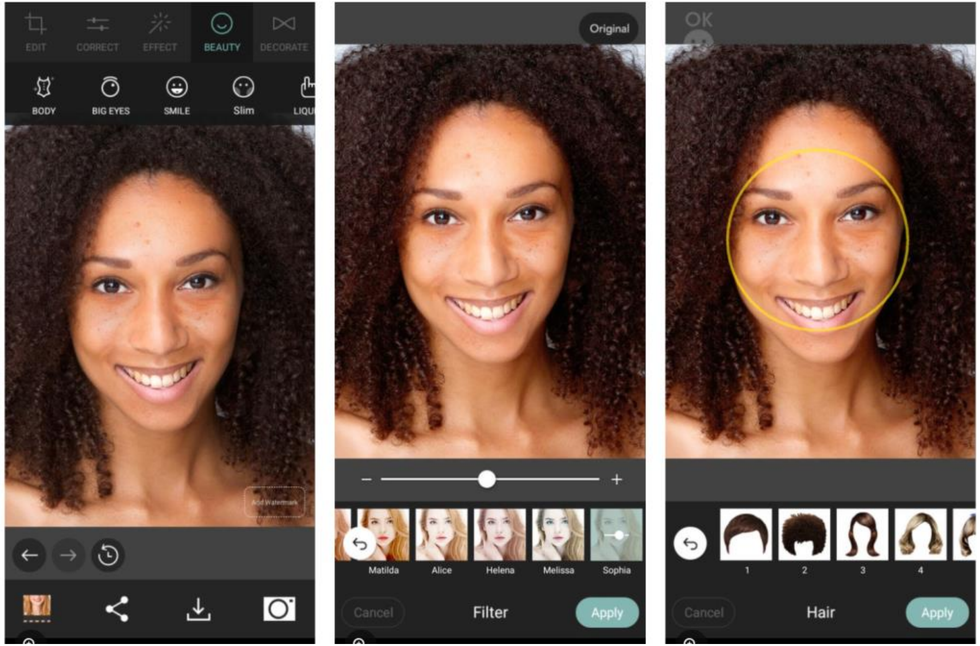
Snapchat

Image Courtesy: https://www.makeuseof.com/best-face-filter-apps-selfies/

Selfie Time…



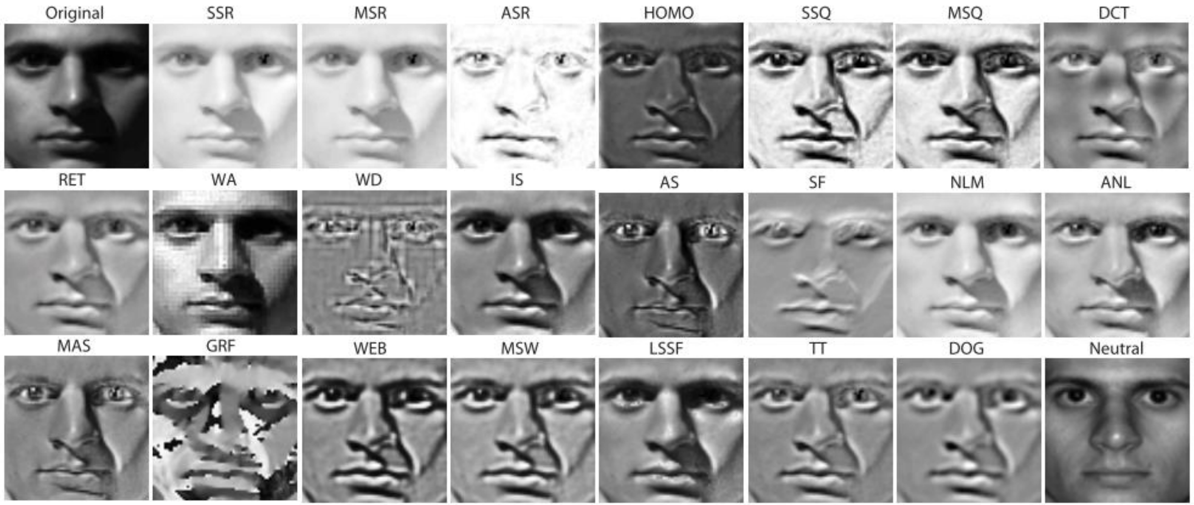
AirBrush



Cymera

Image Courtesy: https://www.makeuseof.com/best-face-filter-apps-selfies/

Filtering (Preprocessing)-Biometrics



INFace: MATLAB Toolbox for Illumination Invariant Face Recognition http://luks.fe.uni-lj.si/sl/osebje/vitomir/face_tools/INFace/

Jufei Xu et al., “Subspace-Based Discrete Transform Encoded Local Binary Patterns Representations for Robust Periocular Matching on NIST’s Face Recognition Grand Challenge,” IEEE Transactions on Image Processing, 2014

Filtering (Preprocessing)-Biometrics

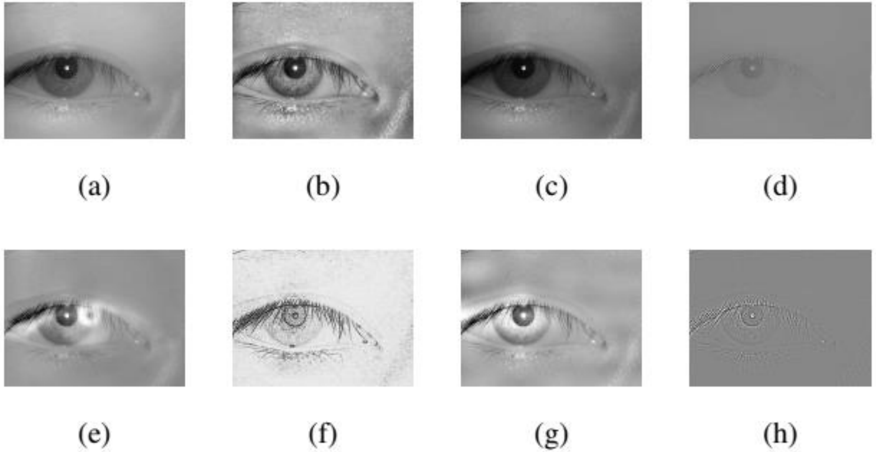
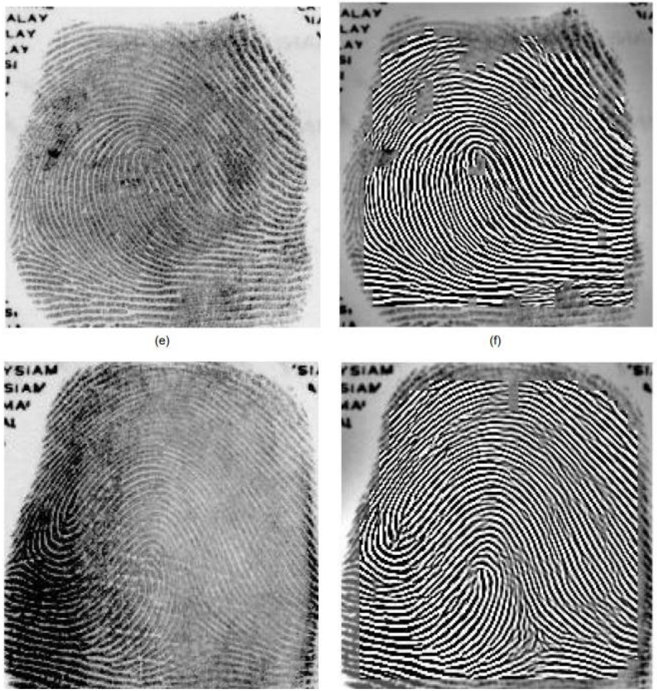


Fig. 4: An example of an NIR iris image subjected to seven illumination normalization schemes. (a) Original, (b) CLAHE, (c) Gamma correction, (d) Homomorphic filtering, (e) MSR, (f) SQI, (g) DCT normalization and (h) DoG.[4]

Banerjee et al., “Impact of Photometric Transformations on PRNU Estimation Schemes: A Case Study Using Near Infrared Ocular Images,” IAPR International Workshop on Biometrics and Forensics, 2018

Enhancement-Biometrics



Hong et al., “Fingerprint Image Enhancement: Algorithm and Performance Evaluation,” IEEE Transactions on Pattern Analysis and Machine Intelligence, 1998

Mean/Average Filter (Smoothing)

```
M = 3
For each valid location [x,y] in S
        a ← Average of intensities in a M x M neighborhood centered on [x,y]
        D[x,y] = round(a)
```

120	190	140	150	200
17	21	30	8	27
89	123	150	73	56
10	178	140	150	18
190	14	76	69	87

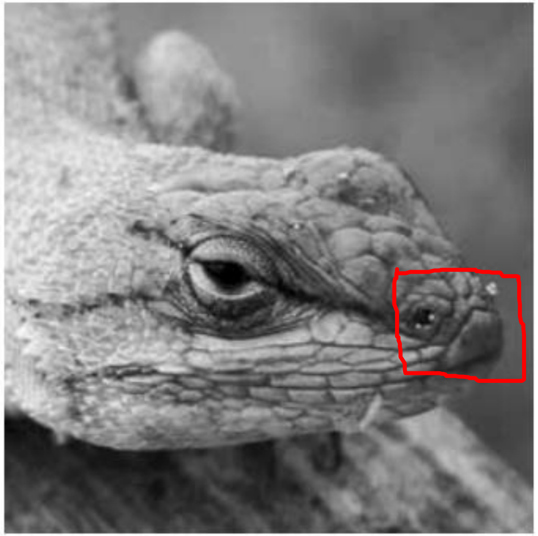
x

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

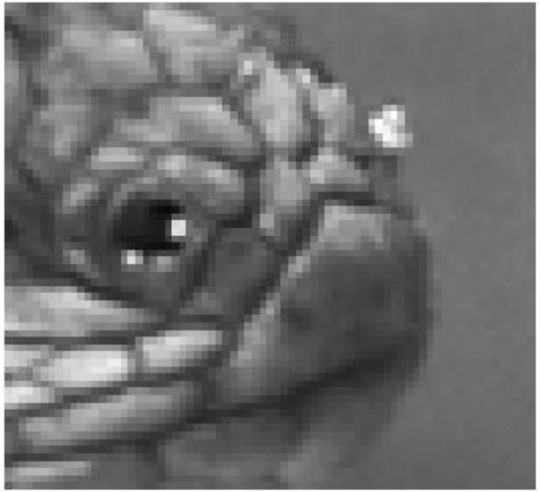
=

	98			

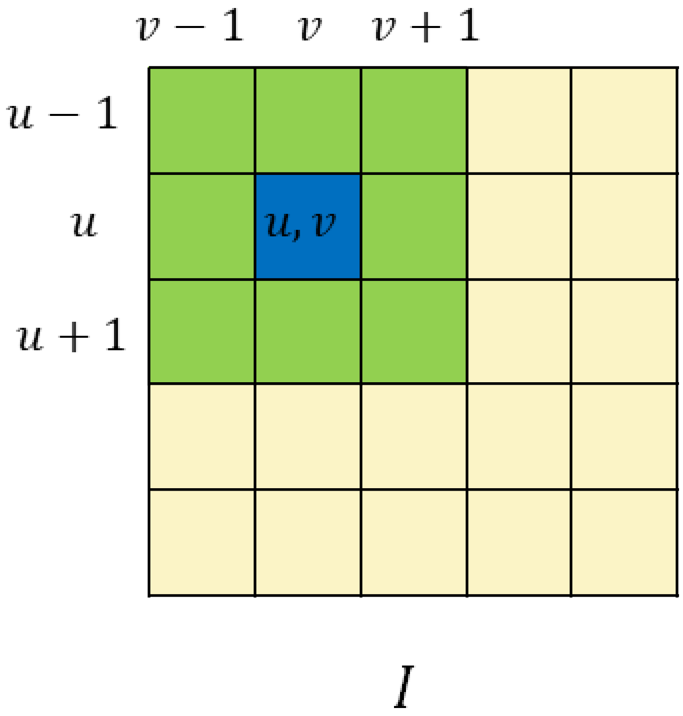
http://199.116.233.101/index.php?title=File:Animated-kernels.gif



$$1/9 * \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$



Mean/Average Filter



Note: Coefficients sum to 1

H → 3 x 3 Weight Mask / Kernel / Filter

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

$$I'(u,v) \leftarrow \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

- $H(i,j)$

Effect of Mask Size

Original Image

[3x3]

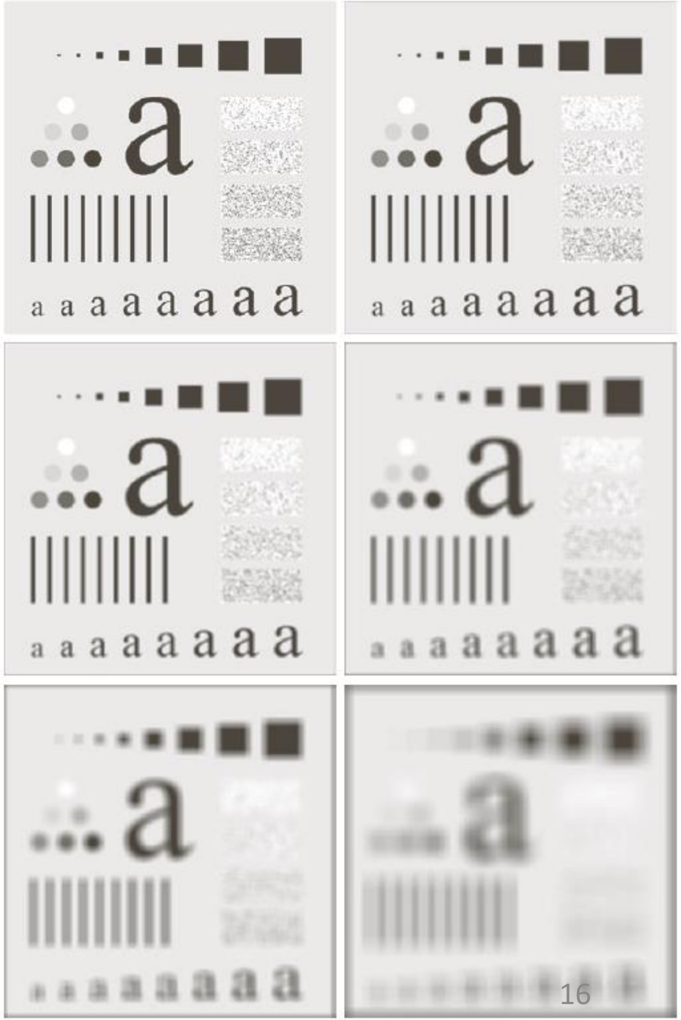
[5x5]

[7x7]

Square averaging filter

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively.'
squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a	b
c	d
e	f



Illustrations of Noisy Images

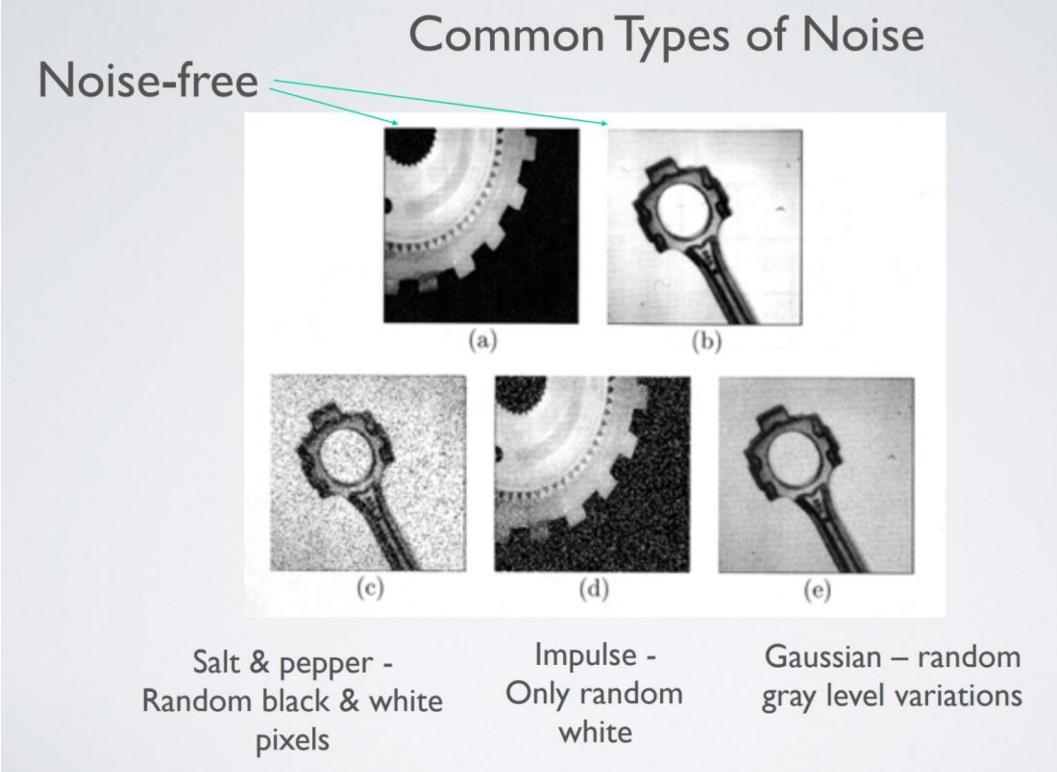
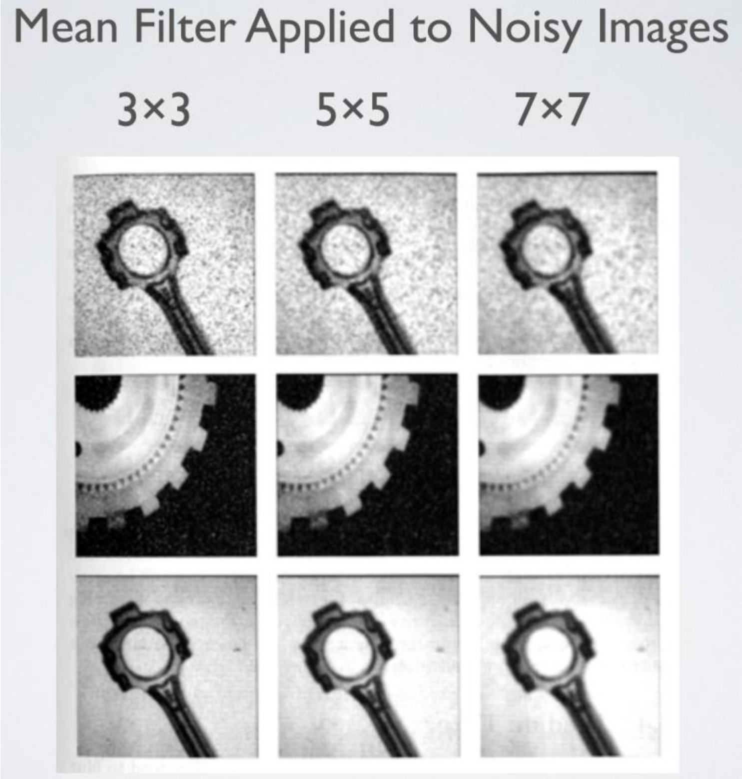
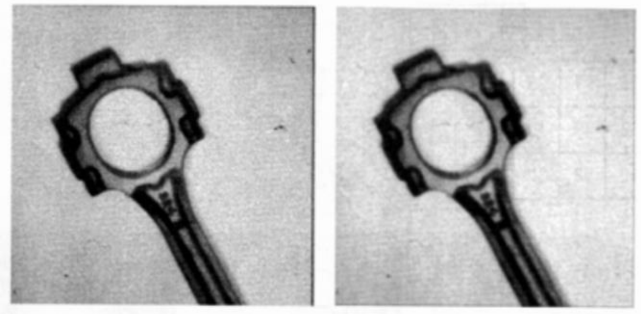


Image Courtesy: http://ece.uprm.edu/~mtoledo/web/6088/F2018/Slides/6_Filtering.pdf

Effect of Mean Filter on Noisy Images

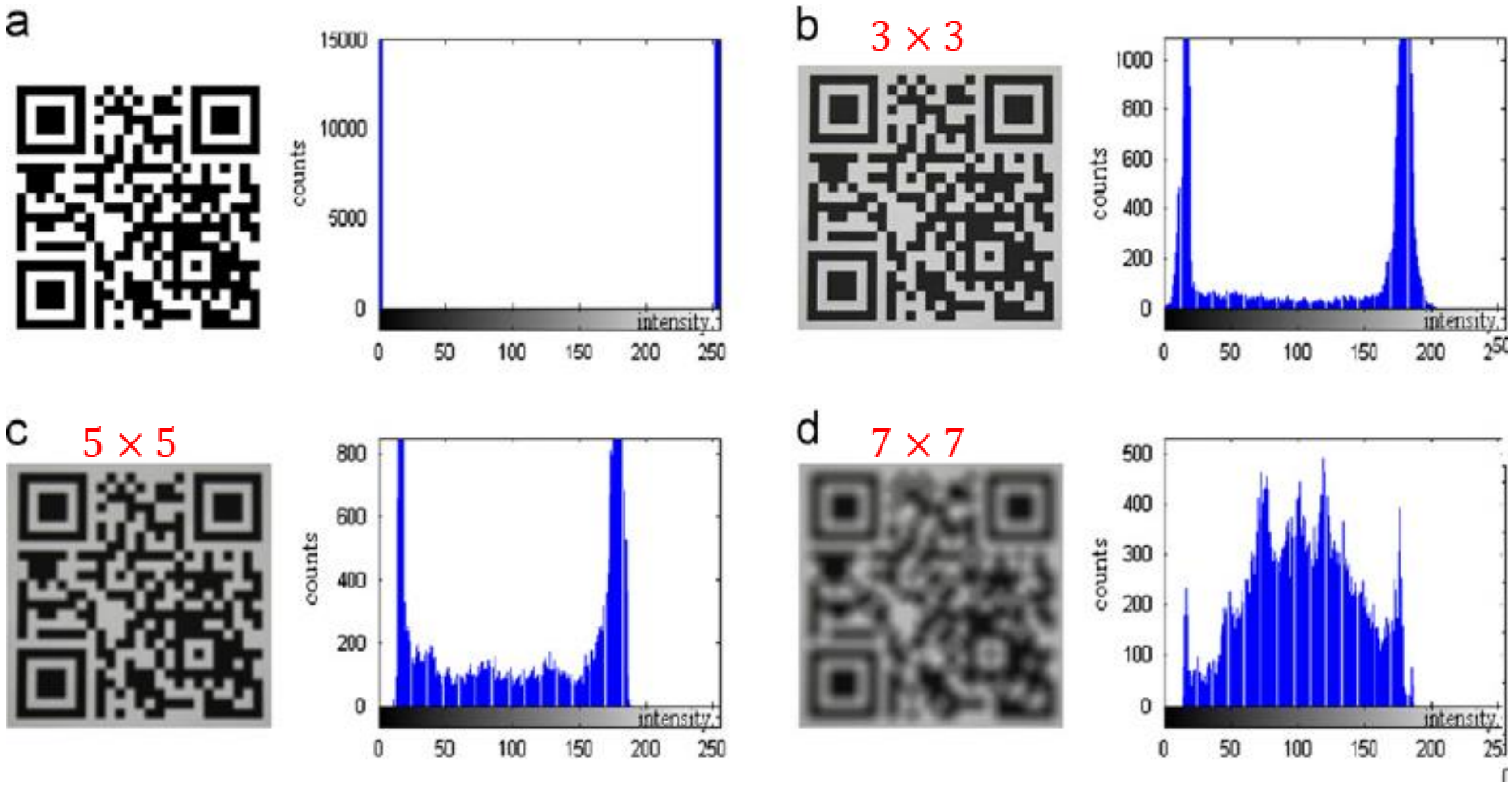




Can sometimes remove too much detail…

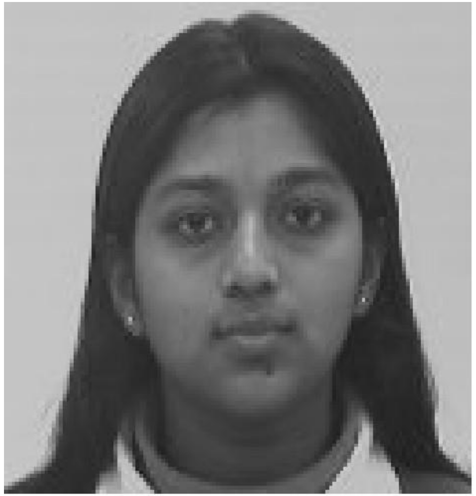
Image Courtesy: http://ece.uprm.edu/~mtoledo/web/6088/F2018/Slides/6_Filtering.pdf

Averaging – a histogram perspective



https://www.researchgate.net/figure/Histograms-of-barcode-images-with-different-levels-of-blur-a-a-synthetic-QR-code-image_fig3_224201499

Repeated Averaging Using Same Filter



Before

After

After repeated averaging

NOTE: Can get the <u>effect</u> of larger filters by smoothing repeatedly with smaller filters

Weighted Averaging

$$I'(u,v) = \frac{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} I(u+i, v+j) \cdot H(i,j)}{\sum_{(j=-a)}^{a} \sum_{(i=-b)}^{b} H(i,j)}$$

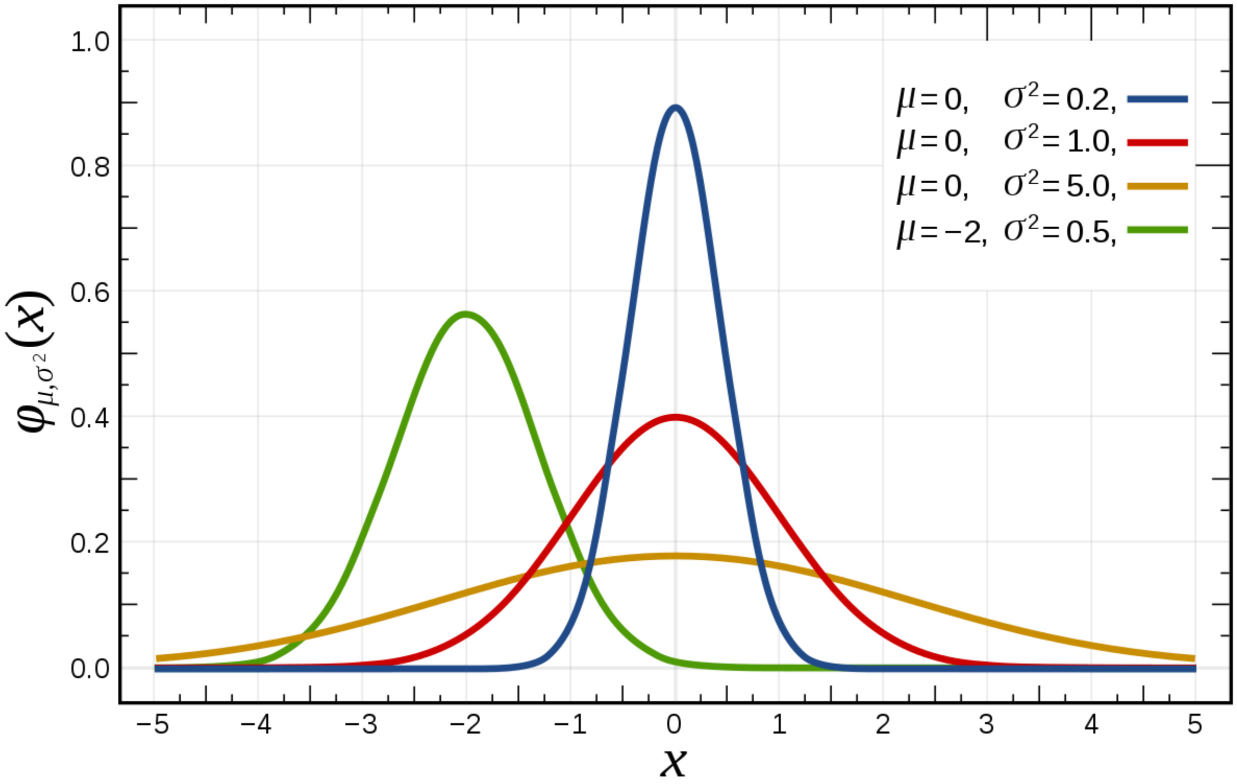
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Standard average

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Weighted average

Gaussian Function (1-D)



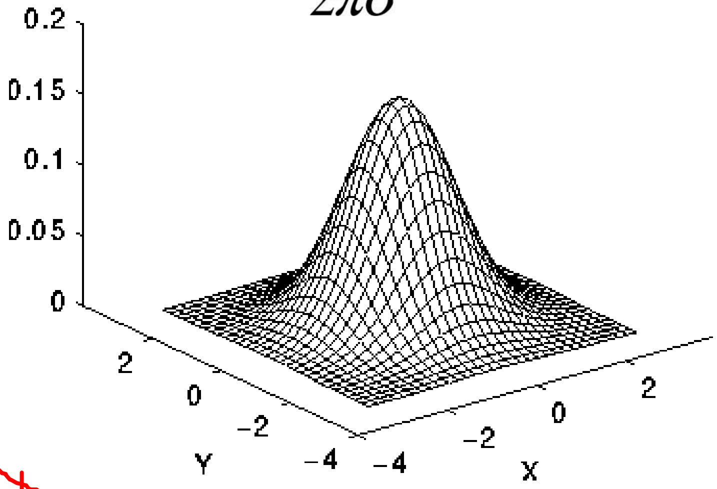
$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

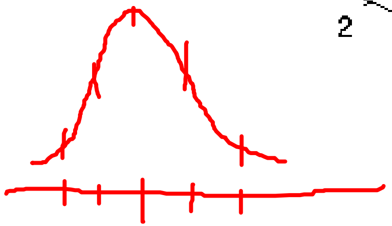
μ: *Mean*, σ: *Standard deviation*

Gaussian Smoothing

- Mask weights are samples of a zero-mean 2-D Gaussian

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$





$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

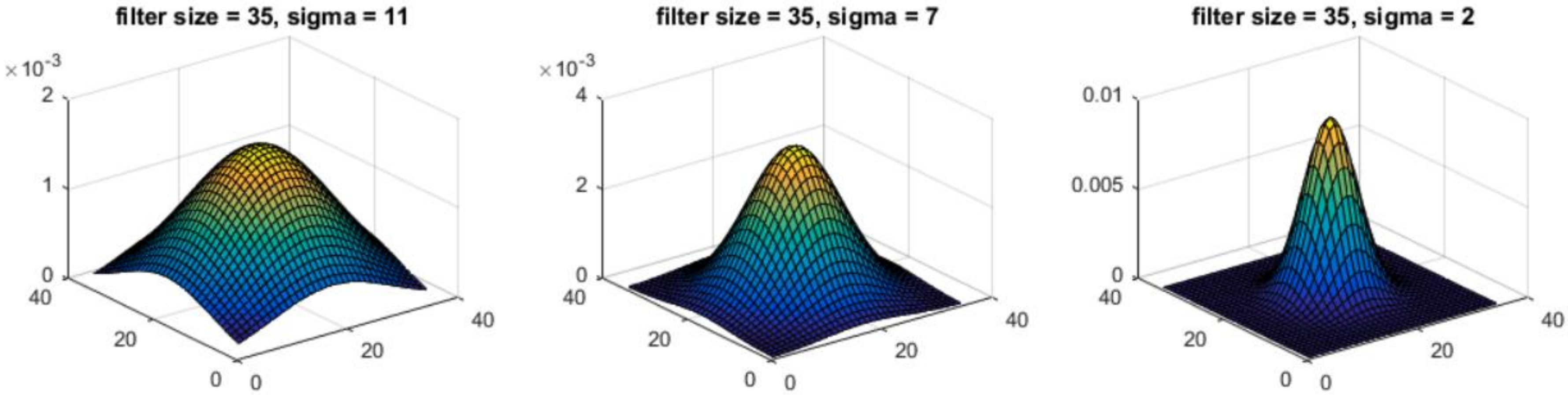
5×5 Gaussian filter, σ=1.0

*This is an approximation

Matrix Source: https://www.cs.auckland.ac.nz/compsci373s1c/PatricesLectures/Gaussian%20Filtering_1up.pdf
https://homepages.inf.ed.ac.uk/rbf/HIPR2/gsmooth.htm

Gaussian Smoothing – Effect of sigma

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$



Gaussian Smoothing – Effect of sigma

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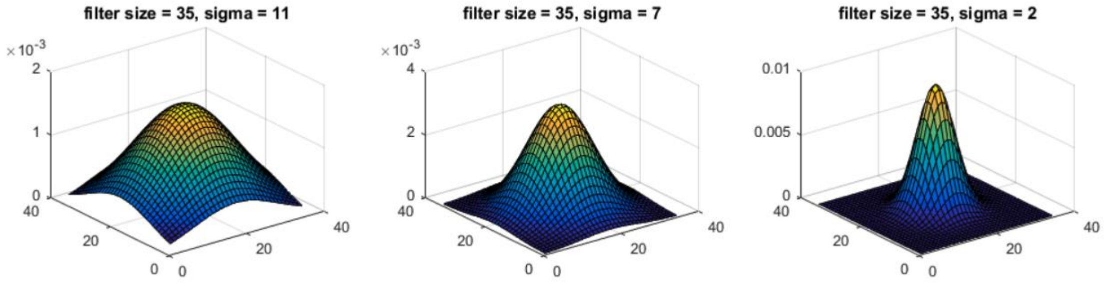
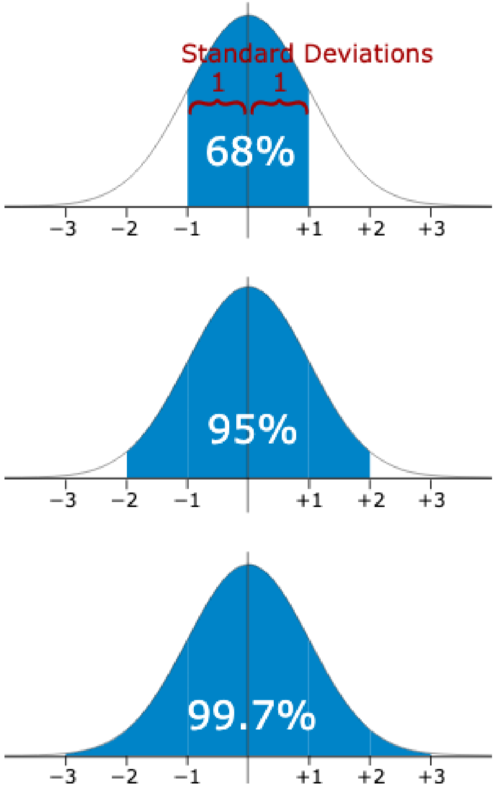




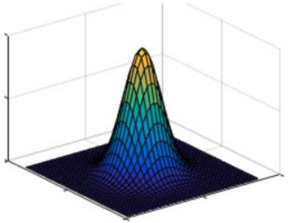
Gaussian Smoothing – Effect of sigma

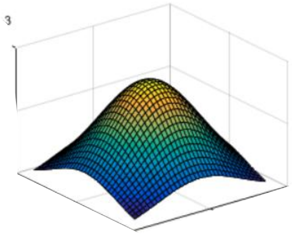
$$G(x,y)=\frac{1}{2\pi\sigma^2}\exp\{-(x^2+y^2)/2\sigma^2\}$$

Original Image
(Sigma 0)

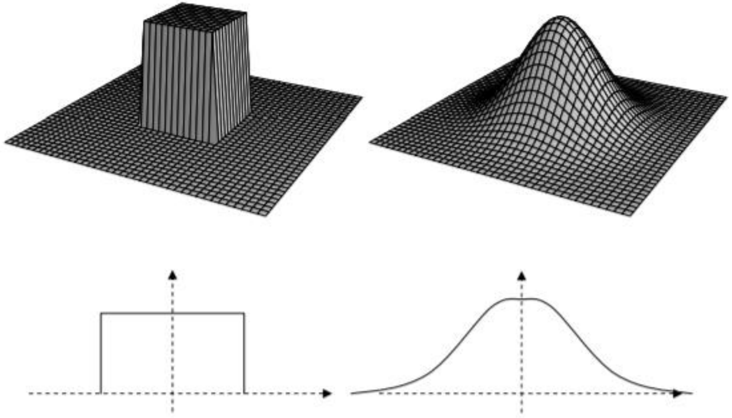
Gaussian Blur
(Sigma 0.7)

Gaussian Blur
(Sigma 2.8)



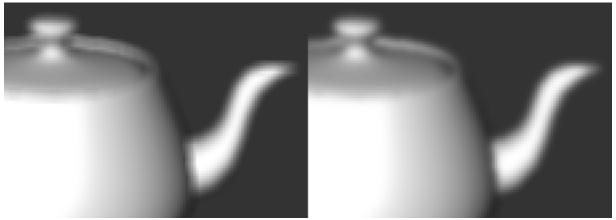


Averaging vs Gaussian filters



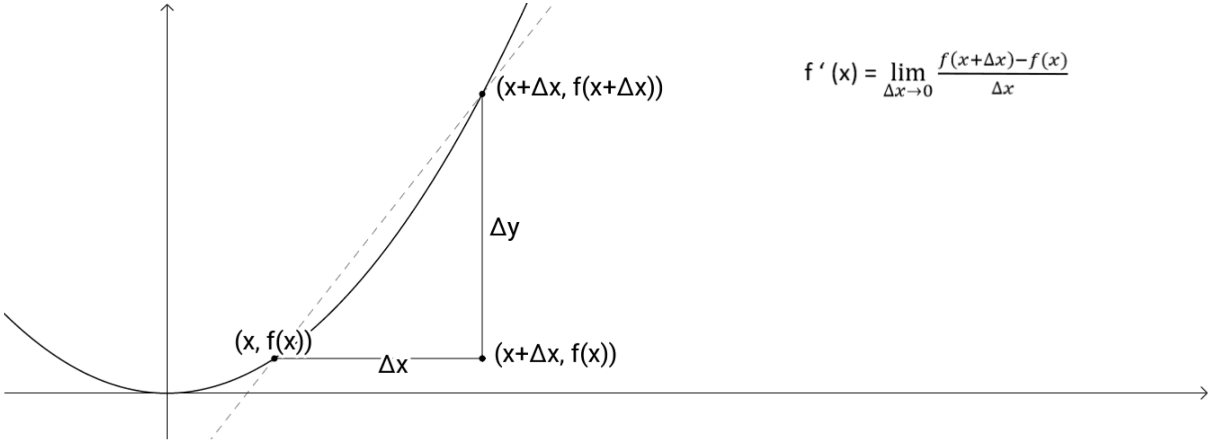
0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0



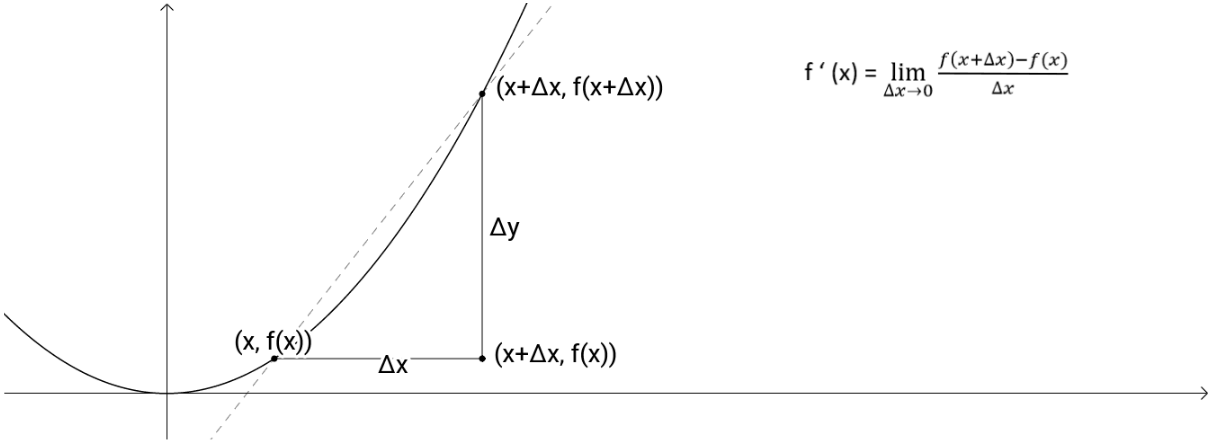
Smoother intensity transitions

Recap: Derivatives

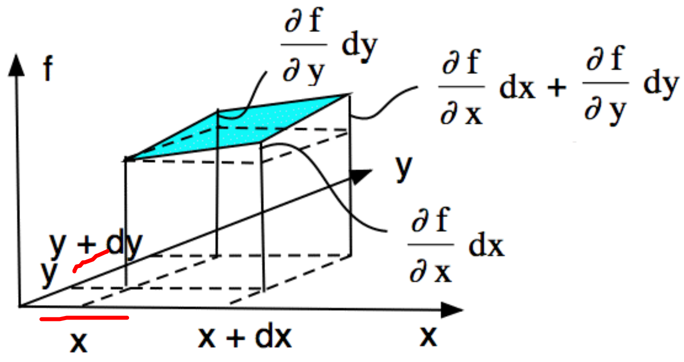


$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Recap: Derivatives



f ' (x) = $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$



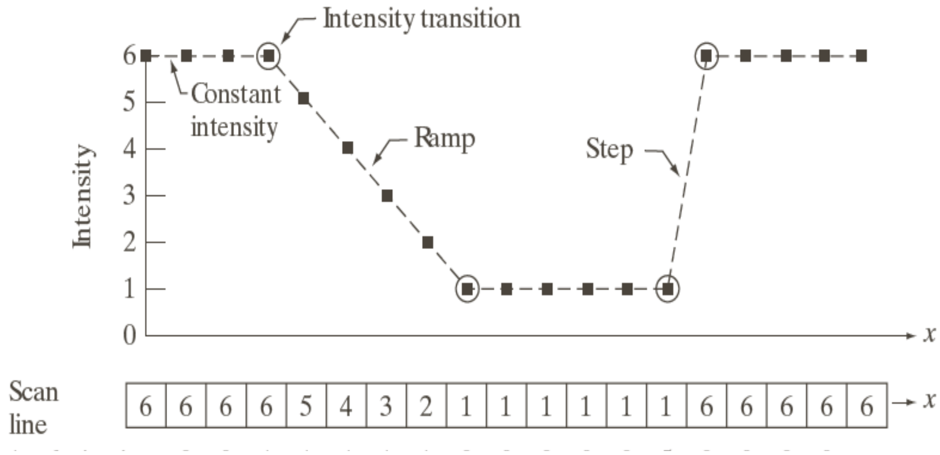
$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$

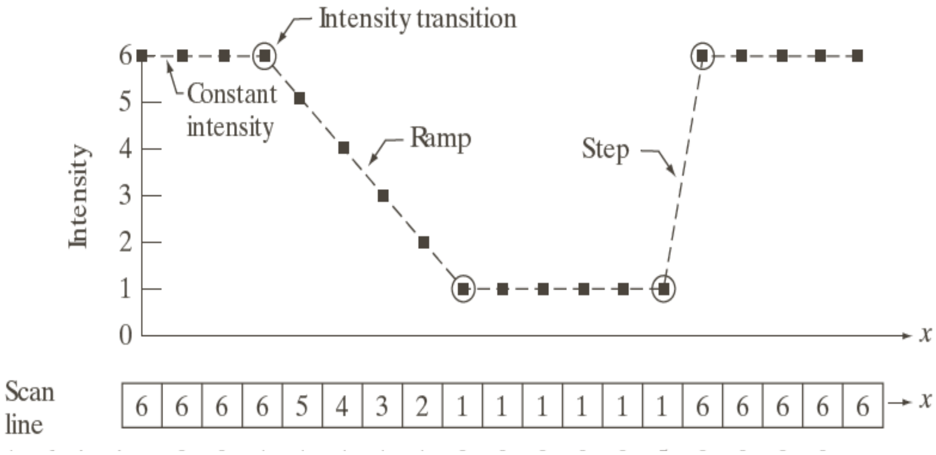


-1	1
f(x,y)	f(x+1,y)

First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$



Second Derivative (Digital Approximation)

$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$

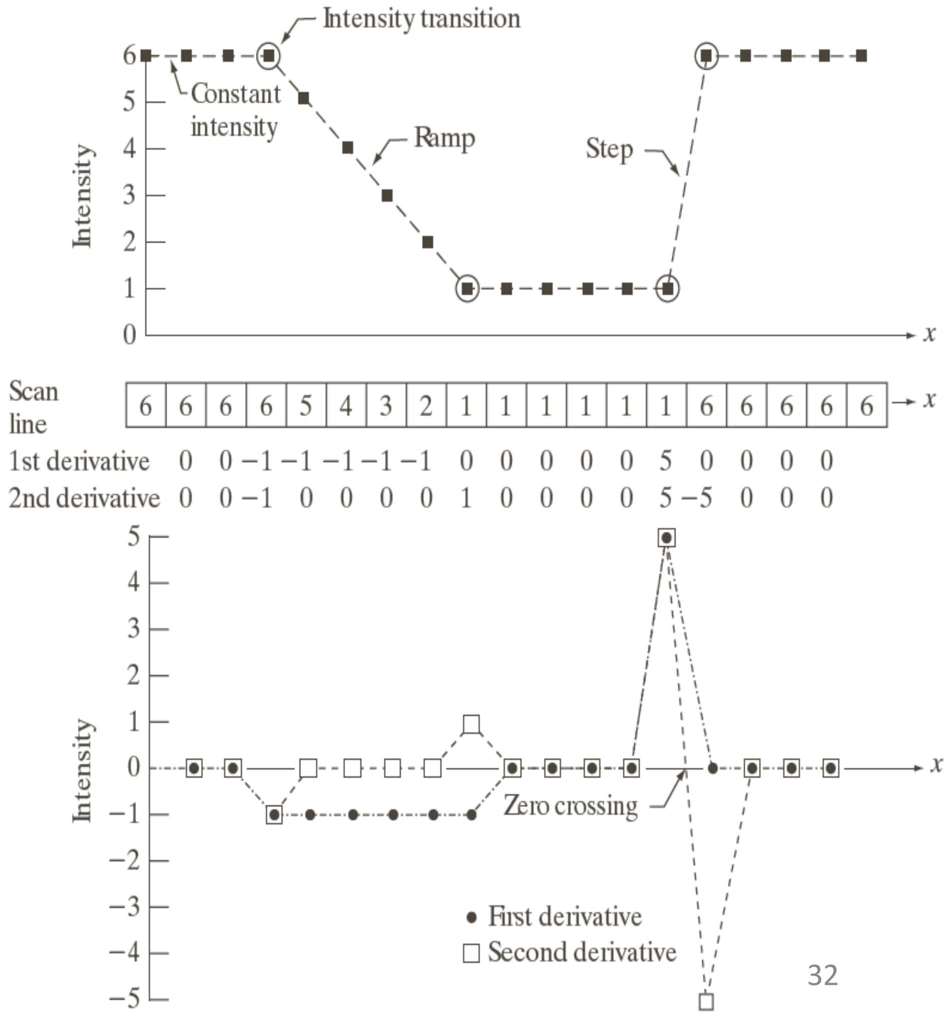
First Derivative (Digital approximation)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$

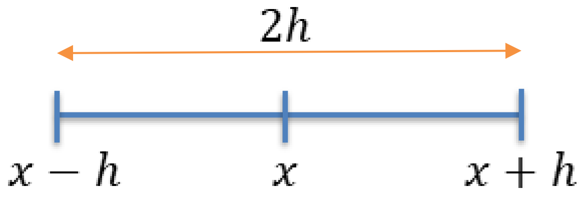
Second Derivative (Digital Approximation)

$$\frac{\partial^2 f(x,y)}{\partial x^2} \sim (f[x+1,y] - f[x,y]) - (f[x,y] - f[x-1,y])$$



Alt: Derivative as symmetric Difference

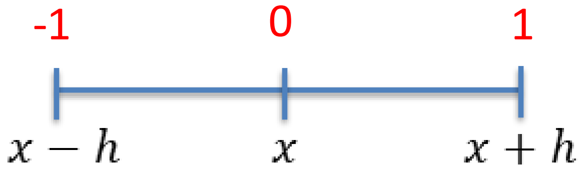
$$\frac{\partial f(x,y)}{\partial x} \sim f[x+1,y] - f[x,y]$$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(x) = \lim_{h \to 0} \frac{1.f(x+h) + 0.f(x) - 1.f(x-h)}{2h}$$

$$f'(x) = \lim_{h \to 0} \frac{-1.f(x-h) + 0.f(x) + 1.f(x+h)}{2h}$$



$$\frac{f(x+h,y)-f(x-h,y)}{2h} \Rightarrow \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

x-derivative

$$\frac{f(x,y+h)-f(x,y-h)}{2h} \Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

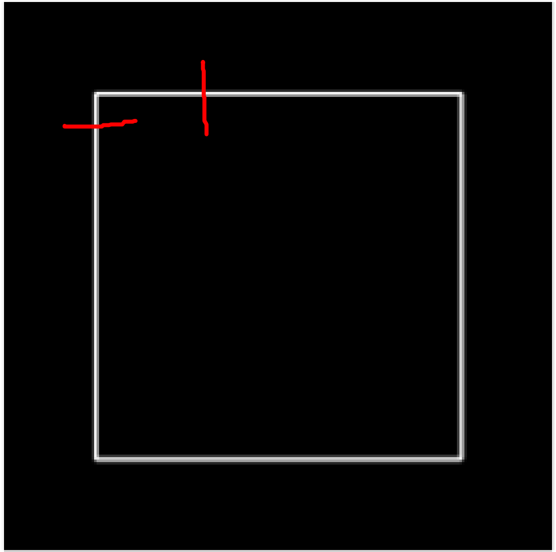
y-derivative

Image Gradient and Edges

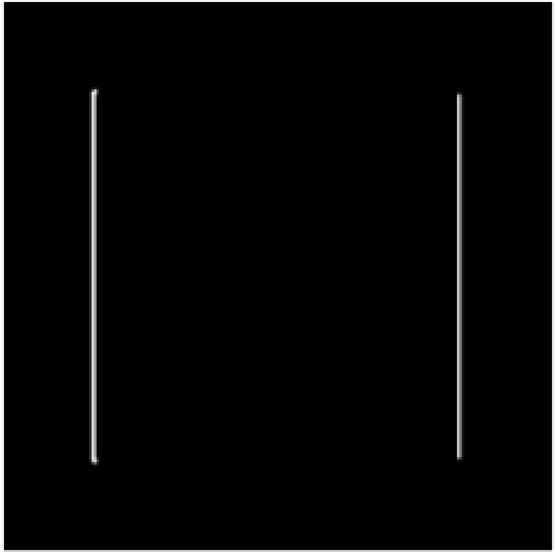
0 0 255 0 0

-1 0 1 -1 0 1 -1 0 1

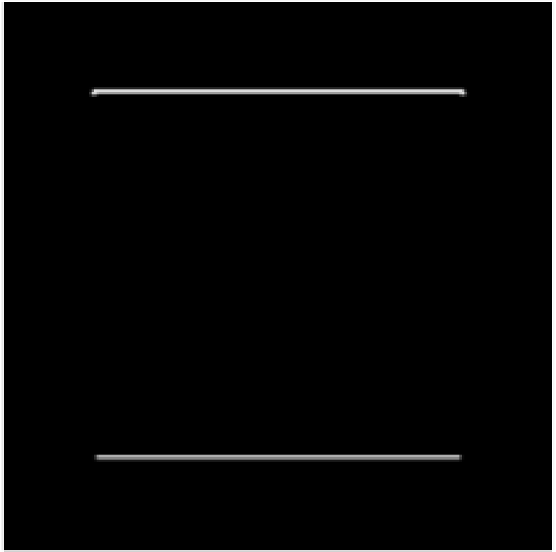
255 0 -255



Image



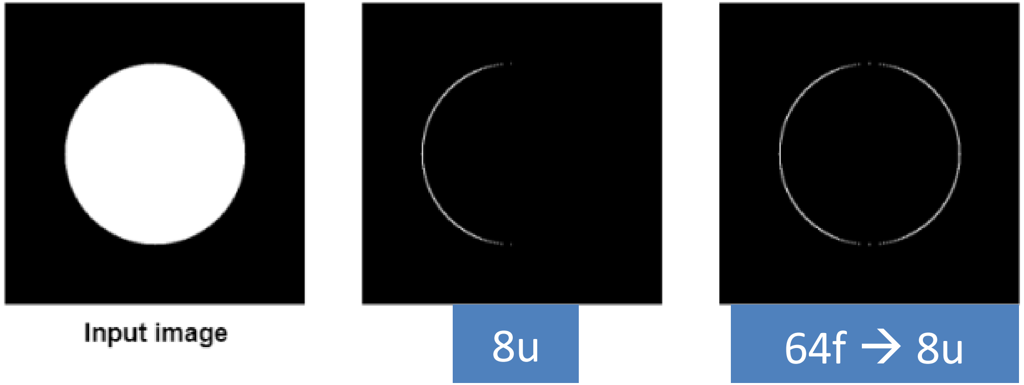
Gradient in x



Gradient in y

https://i1.wp.com/theailearner.com/wp-content/uploads/2019/05/gradxy.png?resize=768%2C280&ssl=1

Edge ‘Image’



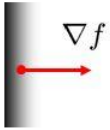
https://i0.wp.com/theailearner.com/wp-content/uploads/2019/05/Sobel64.png?w=731&ssl=1

Image gradient

The gradient of an image:

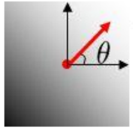
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity

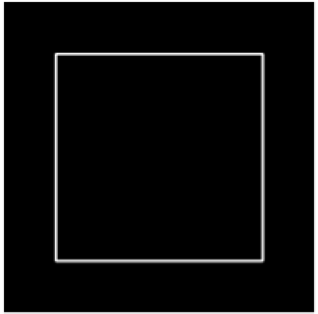


$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

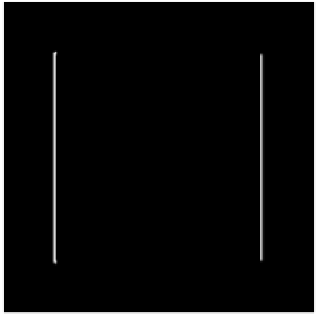
$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

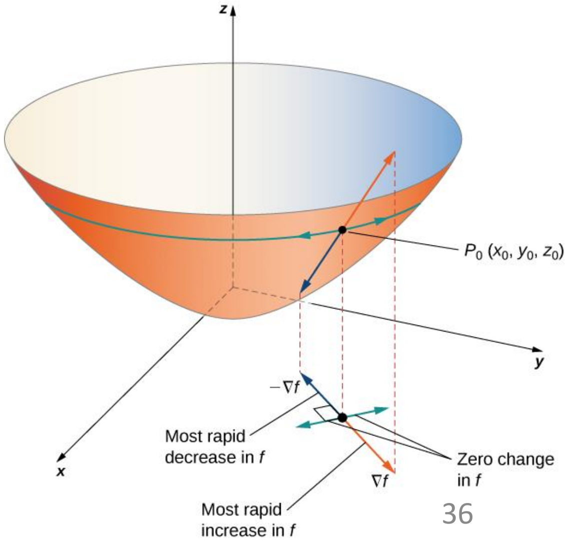


Image



Gradient in x

Gradient in y



Prewitt Edge Filter

Dr. Prewitt

https://nihrecord.nih.gov/sites/recordNIH/files/pdf/1984/NIH-Record-1984-03-13.pdf

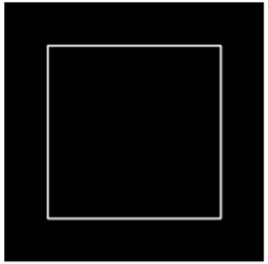
-1	0	+1
-1	0	+1
-1	0	+1

G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

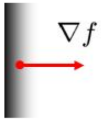
Edge is perpendicular to gradient



Image

Gradient in x

Gradient in y



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

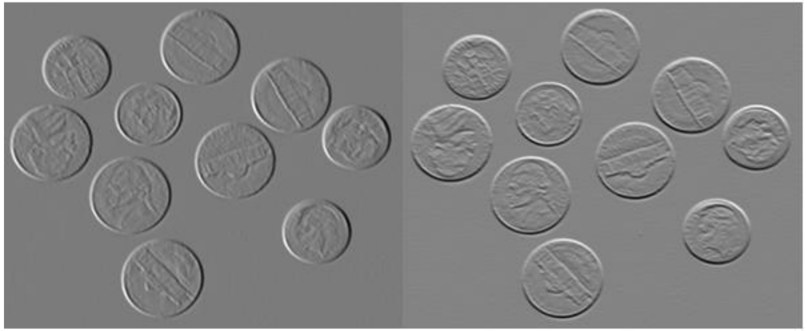
-1	0	+1
-1	0	+1
-1	0	+1

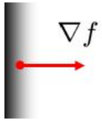
G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

Edge is perpendicular to gradient





$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$

$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

-1	0	+1
-1	0	+1
-1	0	+1

G_x

+1	+1	+1
0	0	0
-1	-1	-1

G_y

Reference

- Read from Sec 3.4 to Sec 3.6 from G&W