Digital Image Processing (CSE/ECE 478)

Lecture-21: Image Compression (contd.)



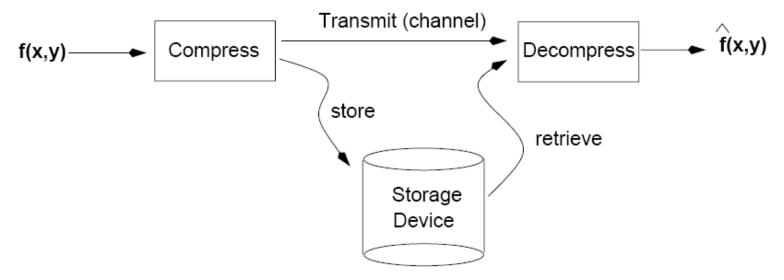
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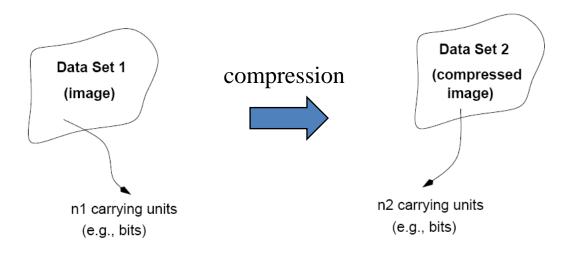
Image Compression

 Goal: Reduce amount of data required to represent a digital <u>image</u>



.. By leveraging redundancies in image data

Compression Ratio



Compression ratio: $C_R = \frac{n_1}{n_2}$

Relevant Data Redundancy

$$R_D = 1 - \frac{1}{C_R}$$

Example:

If
$$C_R = \frac{10}{1}$$
, then $R_D = 1 - \frac{1}{10} = 0.9$

(90% of the data in dataset 1 is redundant)

if
$$n_2 = n_1$$
, then $C_R = 1$, $R_D = 0$

if
$$n_2 \ll n_1$$
, then $C_R \to \infty$, $R_D \to 1$

Types of Redundancy

- (1) Coding Redundancy
- (2) Spatial/Temporal Redundancy
- (3) Psychovisual Redundancy

 Image compression attempts to reduce one or more of these redundancy types.

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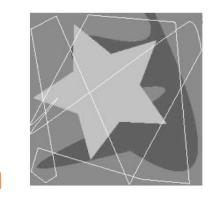
Coding - Definitions

- Code: a list of symbols (letters, numbers, bits etc.)
- Code word: a sequence of symbols used to represent some information (e.g., gray levels).
- Code word length: number of symbols in a code word.

Example: (binary code, symbols: 0,1, length: 3)

0: 000 4: 100 1: 001 5: 101 2: 010 6: 110 3: 011 7: 111

Coding Redundancy



Case 1: I(r_k) = constant length

Example:

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$
$r_0 = 0$	0.19	000	3
$r_1 = 1/7$	0.25	001	3
$r_2 = 2/7$	0.21	010	3
$r_3 = 3/7$	0.16	011	3
$r_4 = 4/7$	0.08	100	3
$r_5 = 5/7$	0.06	101	. 3
$r_6 = 6/7$	0.03	110	3
$r_7 = 1$	0.02	111	3

Average # of bits: $L_{avg} = E(l(r_k)) = \sum_{k=0}^{L-1} l(r_k)P(r_k)$

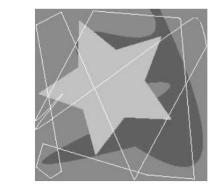
Total # of bits: NML_{avg}

Assume an image with L = 8

Assume $l(r_k) = 3$, $L_{avg} = \sum_{k=0}^{7} 3P(r_k) = 3 \sum_{k=0}^{7} P(r_k) = 3$ bits

Total number of bits: 3NM

Coding Redundancy (cont'd)



• Case 2: $I(r_k)$ = variable length

Table 6.1	Variable-Leng	th Coding Exa	mple	variable length	STATES
r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2 .
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	. 3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

$$C_R = \frac{n_1}{n_2}$$

$$C_R = \frac{3}{2.7} = 1.11 \text{ (about 10\%)}$$

$$R_D = 1 - \frac{1}{1.11} = 0.099$$

$$L_{avg} = \sum_{k=0}^{7} l(r_k)P(r_k) = 2.7 \text{ bits}$$

Total number of bits: 2.7NM

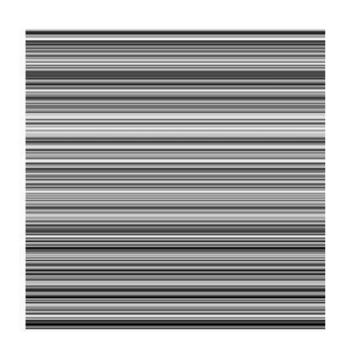
Coding redundancy occurs because least probable and most probable intensity values are represented using same length code. Most images do not have uniform intensity

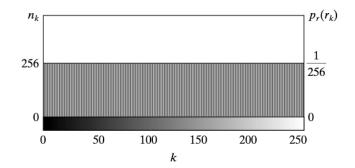
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Spatial Redundancy



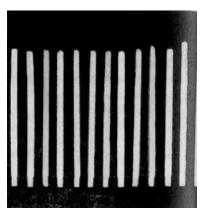


Run-length coding can be used to account for spatial redundancies

Spatial redundancy

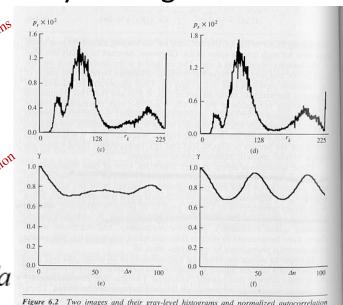
- Interpixel redundancy exists → pixel values are correlated
- i.e., a pixel value can be reasonably predicted by its neighbors





 $f(x) \circ g(x) = \int_{-\infty}^{\infty} f(x)g(x+a)da$

auto-correlation: f(x)=g(x)



Spatial and temporal redundancy







Types of Redundancy

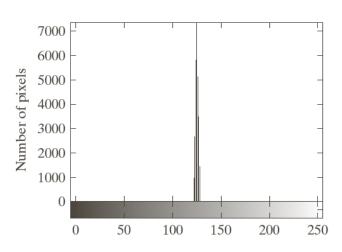
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Irrelevant information or perceptual redundancy

 Not all visual information is perceived by eye/brain, so throw away those that are not





Psychovisual redundancy (cont'd)

Example: quantization

256 gray levels



16 gray levels



C=8/4=2:1

16 gray levels + random noise



add a small pseudo-random number to each pixel prior to quantization

Information theory

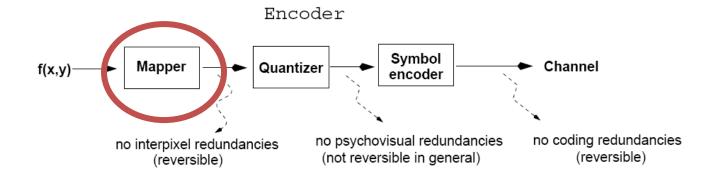
- Basic Premise: Generation of information can be treated as a probabilistic process defined over symbols.
- Symbol carrier of information
- Consider a symbol with an occurrence probability p.
- The amount of information contained in the symbol is defined as:

$$I = \log_2 \frac{1}{p}$$
 bits or $I = -\log_2 p$

Information theory: Shannon's theorem

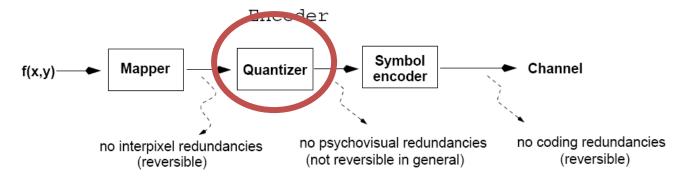
- Shannon's noiseless source coding theorem: For a discrete, memoryless, information source, the minimum bit rate required to encode a symbol on average is equal to the entropy of the source provides a lower bound on the bit rate for encoding statistically independent symbols
- In other words: we can't do better than the entropy
- For a memory information source (finite Markov), the bound might not hold

Image Compression Model



Mapper: transforms data to account for interpixel redundancies.

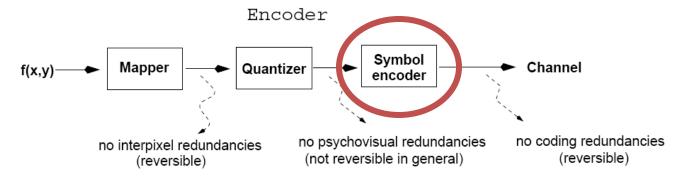
Image Compression Model (cont'd)



•

 Quantizer: quantizes the data to account for psychovisual redundancies.

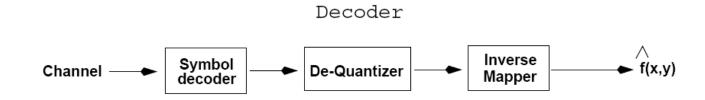
Image Compression Model (cont'd)



•

 Symbol encoder: encodes the data to account for coding redundancies.

Image Compression Models (cont'd)



• The decoder applies the inverse steps.

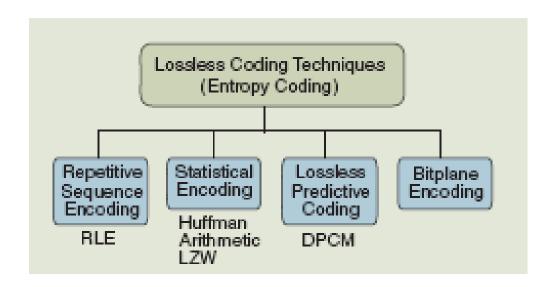
• Note that quantization is irreversible in general.

Lossless Compression

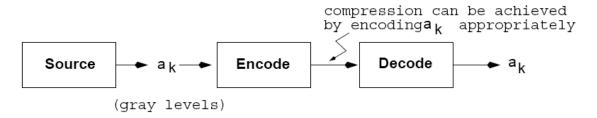


$$e(x, y) = \hat{f}(x, y) - f(x, y) = 0$$

Taxonomy of Lossless Methods



Huffman Coding (addresses coding redundancy)



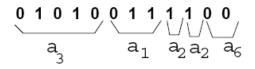
- A variable-length coding technique.
- Source symbols are encoded one at a time!
 - There is a one-to-one correspondence between source symbols and code words.
- Optimal code minimizes code word length per source symbol.

Huffman Coding (cont'd)

Forward Pass

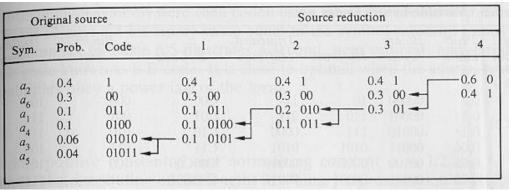
- 1. Sort probabilities per symbol
- 2. Combine the lowest two probabilities
- 3. Repeat *Step2* until only two probabilities remain.

Original source			Source re	eduction	
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0 .6
a_2 a_6	0.3	0.3	0.3	0.3 -	0.4
a_1	0.1	0.1	► 0.2 →	0.3	0.4
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				



Backward Pass

Assign code symbols going backwards



Huffman Coding (cont'd)

L_{avg} assuming Huffman coding:

$$L_{avg} = E(l(a_k)) = \sum_{k=1}^{6} l(a_k)P(a_k) =$$

$$3x0.1 + 1x0.4 + 5x0.06 + 4x0.1 + 5x0.04 + 2x0.3 = 2.2$$
 bits/symbol

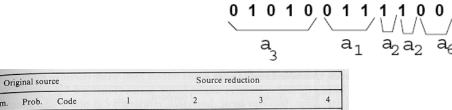
6 symbols, we need a 3-bit code

• $(a_1: 000, a_2: 001, a_3: 010, a_4: 011, a_5: 100, a_6: 101)$

$$L_{avg} = \sum_{k=1}^{6} l(a_k)P(a_k) = \sum_{k=1}^{6} 3P(a_k) = 3 \sum_{k=1}^{6} P(a_k) = 3 \text{ bits/symbol}$$

Huffman Coding/Decoding

- Coding/Decoding can be implemented using a look-up table.
- Decoding can be done unambiguously.



Original source			Source reduction			7/10				
Sym.	Prob.	Code	Sill	I are have		2		3	4	1
a ₂ a ₆ a ₁ a ₄ a ₃ a ₅	0.4 0.3 0.1 0.1 0.06 0.04	1 00 011 0100 01010 01011	0.4 0.3 0.1 0.1 - 0.1	1 00 011 0100 0101	0.4 0.3 —0.2 0.1	1 00 010 011	0.4 0.3 — 0.3	1 00 01	0.6 0.4	0

Ori	iginal sou	rce
Sym.	Prob.	Code
<i>a</i> -	0.4	1
a_2 a_6	0.3	00
a_1	0.1	011
a.	0.1	0100
a.	0.06	01010
a ₄ a ₃ a ₅	0.04	01011

Arithmetic (or Range) Coding (addresses coding redundancy)

- The main weakness of Huffman coding is that it encodes source symbols one at a time.
- Arithmetic coding encodes sequences of source symbols together.
 - There is no one-to-one correspondence between source symbols and code words.
- Slower than Huffman coding but can achieve better compression.

Arithmetic Coding (cont'd)

 A sequence of source symbols is assigned to a sub-interval in [0,1) which can be represented by an arithmetic code, e.g.:



• Start with the interval [0, 1); a sub-interval is chosen to represent the message which becomes smaller and smaller as the number of symbols in the message increases.

Arithmetic Coding (cont'd)

Encode message: $\alpha_1 \alpha_2 \alpha_3 \alpha_3 \alpha_4$

Probability
0.2
0.2
0.4
0.2

1) Start with interval [0, 1)

0 1

2) Subdivide [0, 1) based on the probabilities of α_i

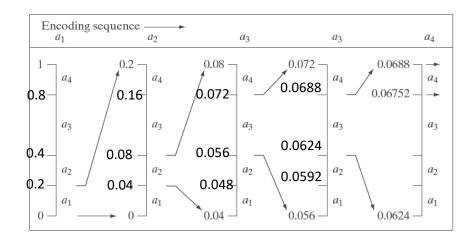
0				\vdash
	a_1	a_2	a_3	a_4

Initial Subinterval
[0.0, 0.2) [0.2, 0.4) [0.4, 0.8)
[0.8, 1.0)

3) Update interval by processing source symbols

Example

Source Symbol	Probability	Initial Subinterval
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)



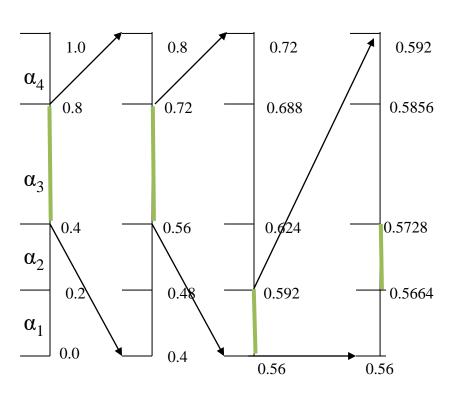
Encode

 $\alpha_1 \; \alpha_2 \; \alpha_3 \; \alpha_3 \; \alpha_4$



[0.06752, 0.0688)

Arithmetic Decoding



Decode 0.572

(code length=4)



 $\alpha_3 \alpha_3 \alpha_1 \alpha_2$

LZW (Lempel-Ziv-Welch) Coding (addresses interpixel redundancy)

- Requires no prior knowledge of symbol probabilities.
- Assigns fixed length code words to variable length symbol sequences.
 - There is no one-to-one correspondence between source symbols and code words.
- Included in GIF, TIFF and PDF file formats

LZW Coding

 A codebook (or dictionary) needs to be constructed.

• Initially, the first 256 entries of the dictionary are assigned to the gray levels 0,1,2,..,255 (i.e., assuming 8 bits/pixel)

Consider a 4x4, 8 bit image 39 39 126 126 39 39 126 126 39 39 126 126 39 39 126 126

Initial Dictionary

Dictionary Location	Entry
0	0
1	1
255	255
256	
511	-

LZW Coding (cont'd)

As the encoder examines image pixels, gray level sequences (i.e., blocks) that are not in the dictionary are assigned to a new entry.

39	39	126	126
39	39	126	126
39	39	126	126
39	39	126	126

Dictionary Location	Entry
0	0
1	1
255	255
256	39-39
511	_

- Is 39 in the dictionary.....Yes
 What about 39-39.....No
 - * Add 39-39 at location 256

Example

39 39 126 126 39 39 126 126 39 39 126 126 39 39 126 126

CR = empty

repeat

P=next pixel

CS=CR+P

If CS is found:

- (1) No Output
- (2) CR=CS

else:

- (1) Output D(CR)
- (2) Add CS to D
- (3) CR=P

Concatenated Sequence: CS = CR + P

(CR)

(P)

Currently Recognized Sequence	Pixel Being Processed	Encoded Output	Dictionary Location (Code Word)	Dictionary Entry
	39	/ \		
39	39	39	256	39-39
39	126	39	257	39-126
126	126	126	258	126-126
126	39	126	259	126-39
39	39	1 1		
39-39	126	256	260	39-39-126
126	126	1 1		
126-126	39	258	261	126-126-39
39	39	1 1		
39-39	126	1 1		
39-39-126	126	260	262	39-39-126-126
126	39	1 /		
126-39	39	259	263	126-39-39
39	126	\ /		
39-126	126	257	264	39-126-126
126		126		

Decoding LZW

- Use the dictionary for decoding the "encoded output" sequence.
- The dictionary need not be sent with the encoded output.
- Can be built on the "fly" by the decoder as it reads the received code words.

Run-length coding (RLC) (addresses interpixel redundancy)

• Reduce the size of a repeating string of symbols (i.e., runs):

$$111110000001 \rightarrow (1,5)(0,6)(1,1)$$

a a a b b b b b c c \rightarrow (a,3) (b, 6) (c, 2)

- Encodes a run of symbols into two bytes: (symbol, count)
- Can compress any type of data but cannot achieve high compression ratios compared to other compression methods.

Combining Huffman Coding with Run-length Coding

 Assuming that a message has been encoded using Huffman coding, additional compression can be achieved using run-length coding.

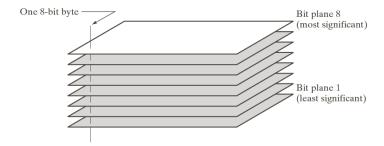
0 1 0 1 0 0 1 1 1 1 0 0

e.g., (0,1)(1,1)(0,1)(1,0)(0,2)(1,4)(0,2)

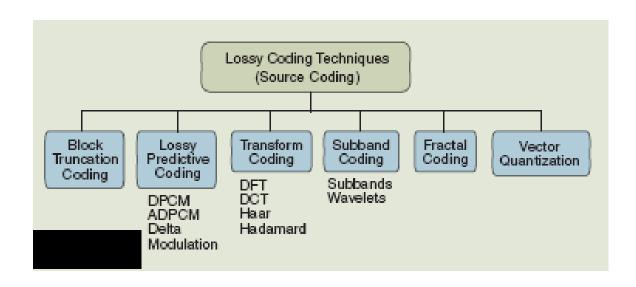
Bit-plane coding

(addresses interpixel redundancy)

- Process each bit plane individually.
- (1) Decompose an image into a series of binary images.
- (2) Compress each binary image (e.g., using run-length coding)

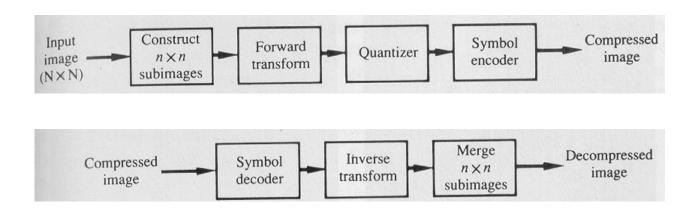


Lossy Methods - Taxonomy



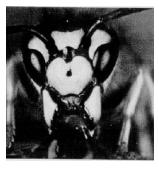
Lossy Compression

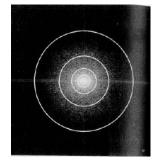
 Transform the image into some other domain to reduce interpixel redundancy.



Example: Fourier Transform

$$f(x,y) = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u,v) e^{\frac{j2\pi(ux+vy)}{N}}, \quad x,y=0,1,...,N-1$$





Note that the magnitude of the FT decreases, as *u*, *v* increase!

 $K \ll N$

$$\hat{f}(x,y) = \frac{1}{N} \sum_{u=0}^{K-1} \sum_{v=0}^{K-1} F(u,v)e^{\frac{j2\pi(ux+vy)}{N}}, \quad x,y=0,1,...,N-1$$

$$\sum_{x,v} (\hat{f}(x,y) - f(x,y))^2 \text{ is very small } !!$$

Transform Selection

$$f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T(u,v)h(x,y,u,v)$$

- T(u,v) can be computed using various transformations, for example:
 - DFT
 - DCT (Discrete Cosine Transform)
 - KLT (Karhunen-Loeve Transformation) or Principal Component Analysis (PCA)
- JPEG using DCT for handling interpixel redundancy.

DCT (Discrete Cosine Transform)



[proposed by Nasir Ahmed, T. Natarajan, K.R.Rao (1972)]

Forward:
$$C(u, v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) cos(\frac{(2x+1)u\pi}{2N}) cos(\frac{(2y+1)v\pi}{2N}),$$

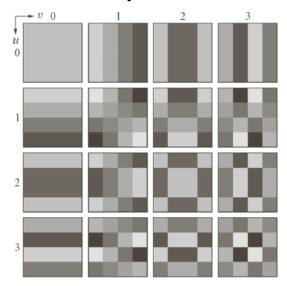
$$u, v=0,1,...,N-1$$

Inverse:
$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \alpha(u)\alpha(v)C(u, v)cos(\frac{(2x+1)u\pi}{2N})cos(\frac{(2y+1)v\pi}{2N}),$$

 $x, y=0,1,...,N-1$

$$\alpha(u) = \begin{cases} \sqrt{1/N} & \text{if } u=0\\ \sqrt{2/N} & \text{if } u>0 \end{cases} \quad \alpha(v) = \begin{cases} \sqrt{1/N} & \text{if } v=0\\ \sqrt{2/N} & \text{if } v>0 \end{cases}$$

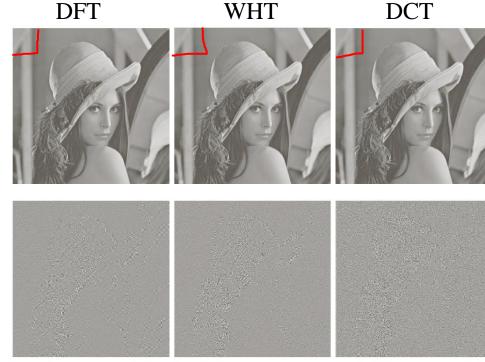
• Basis functions for a 4x4 image (i.e., cosines of different frequencies).



Using 8 x 8 sub-images yields 64 coefficients per sub-image.

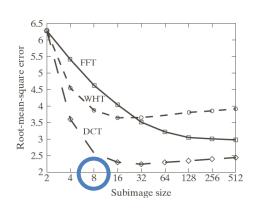
Reconstructed images by truncating 50% of the coefficients

More compact transformation

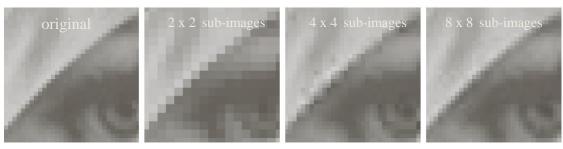


RMS error: 2.32 1.78 1.13

• Sub-image size selection:



Reconstructions



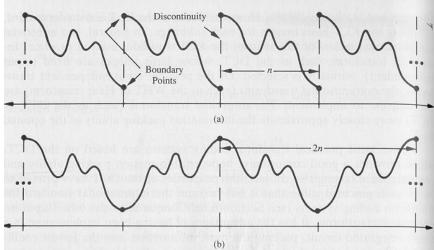
 DCT minimizes "blocking artifacts" (i.e., boundaries between subimages do not become very visible).

DFT
has n-point periodi

has n-point periodicity

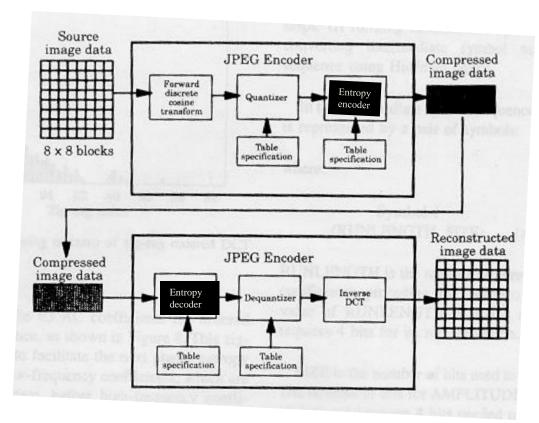
DCT

has 2n-point periodicity



JPEG Compression

Accepted as an international image compression standard in 1992.



JPEG - Steps

1. Divide image into 8x8 subimages/bocks.

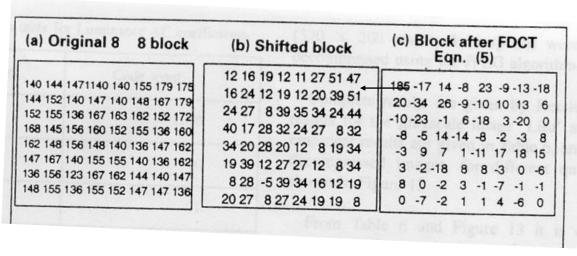
For each subimage do:

- 2. Shift the gray-levels in the range [-128, 127]
- 3. Apply DCT \rightarrow 64 coefficients

1 DC coefficient: F(0,0)

63 AC coefficients: F(u,v)

Example



[-128, 127]

(non-centered spectrum)

JPEG Steps

4. Quantize the coefficients (i.e., reduce the amplitude of coefficients that do not contribute a lot).

$$C_q(u, v) = Round[\frac{C(u, v)}{Q(u, v)}]$$

Q(u,v): quantization table

Example

Quantization Table Q[i][j]

```
for i=0 to n;

for j=0 to n;

Q[i,j]= 1 + (1+i+j)*quality;

end j;

end i;
```

(d) Quantization table (quality = 2)

3 5 7 9 11 13 15 17 5 7 9 11 13 15 17 19 7 9 11 13 15 17 19 21 9 11 13 15 17 19 21 23 11 13 15 17 19 21 23 25 13 15 17 19 21 23 25 27 15 17 19 21 23 25 27 29 17 19 21 23 25 27 29 31

$$1 \le quality \le 25$$



(best - low compression)



(worst - high compression)

Example (cont'd)

(c) Block after FDCT Eqn. (5)

185 -17 14 -8 23 -9 -13 -18 20 -34 26 -9 -10 10 13 6 -10 -23 -1 6 -18 3 -20 0 -8 -5 14 -14 -8 -2 -3 8 -3 9 7 1 -11 17 18 15 3 -2 -18 8 8 -3 0 -6 8 0 -2 3 -1 -7 -1 -1 0 -7 -2 1 1 4 -6 0

(d) Quantization table (quality = 2)

3 5 7 9 11 13 15 17 5 7 9 11 13 15 17 19 7 9 11 13 15 17 19 21 9 11 13 15 17 19 21 23 11 13 15 17 19 21 23 25 13 15 17 19 21 23 25 27 15 17 19 21 23 25 27 29 17 19 21 23 25 27 29 31

Quantization

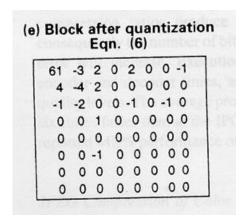


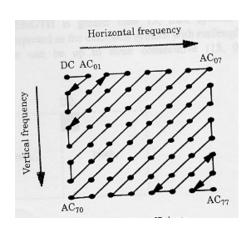
(e) Block after quantization Eqn. (6)

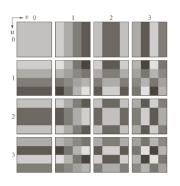
JPEG Steps (cont'd)

5. Order the coefficients using zig-zag ordering

- Creates long runs of zeros (i.e., ideal for run-length encoding)







(f) Zig-zag sequence

JPEG Steps (cont'd)

- **6.** Encode coefficients using variable length encoding:
 - 6.1 Form "intermediate" symbol sequence.
 - 6.2 Encode "intermediate" symbol sequence into a binary sequence.

Final Symbol Sequence

(g) Intermediate synbol sequence

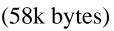
$$(6)(61),(0,2)(-3),(0,3)(4),(0,1)(-1),(0,3)(-4),(0,2)(2),(1,2)(2),(0,2)(-2),\\ (5,2)(2),(3,1)(1),(6,1)(-1),(2,1)(-1),(4,1)(-1),(7,1)(-1),(0,0)$$



(e) Encoded bit sequence (total 98 bits)

What is the effect of the "Quality" parameter?







(21k bytes)



(8k bytes)

lower compression

higher compression

$$1 \le quality \le 25$$

Results using JPEG compression

file size 45853 bytes compression ratio: 12.9



Fine details have been lost.

Image has an artificial "blocky" pattern superimposed on it.

Artifacts will affect the performance of fingerprint recognition.

Results using WSQ compression

file size 45621 bytes compression ratio: 12.9



Fine details are better preserved.

No "blocky" artifacts.

JPEG

- JPEG compression exploits two observations:
 - #1: Human eyes don't see color (chrominance) quiteas well as brightness (luminance) → Compress in a colordominant space (RGB --> YCbCr)
 - #2: Human eyes can't distinguish high frequency changes in image intensity (downsample (Cb,Cr), quantize, DCT)

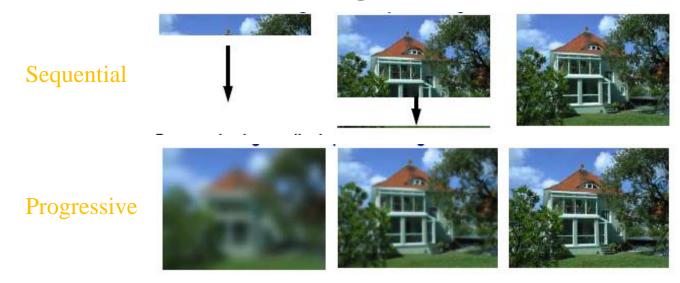
JPEG Modes

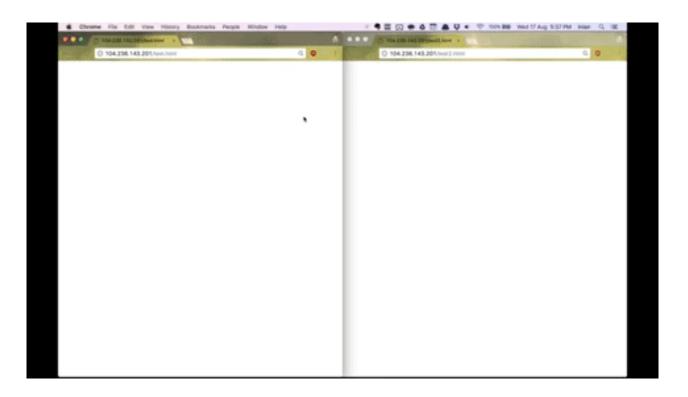
- JPEG supports several different modes
 - Sequential Mode
 - Progressive Mode
 - Hierarchical Mode
 - Lossless Mode

- The default mode is "sequential"
 - Image is encoded in a single scan (left-to-right, top-to-bottom).

Progressive JPEG

 Image is encoded in multiple scans, in order to produce a quick, rough decoded image when transmission time is long.





JPEG at 0.125 bpp (enlarged)



JPEG2000 at 0.125 bpp



Other popular formats

- GIF \rightarrow lossy
- PNG → lossless
- Video
 - MPEG etc. (exploit temporal redundancy also)

Reference

• Ch 8, G&W textbook