09.11.2021

Digital Image Processing (CSE/ECE 478)

Lecture-19b: Image Restoration

Ravi Kiran and Sudipta Banerjee

Center for Visual Information Technology (CVIT), IIIT Hyderabad

Mathematical Model of Image Degradation/Restoration

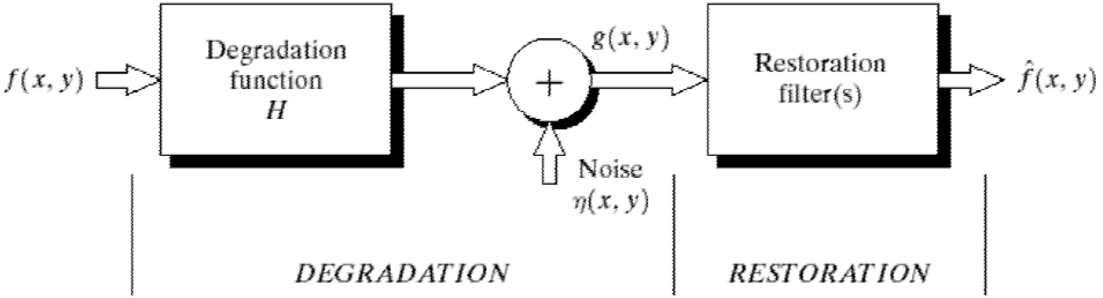


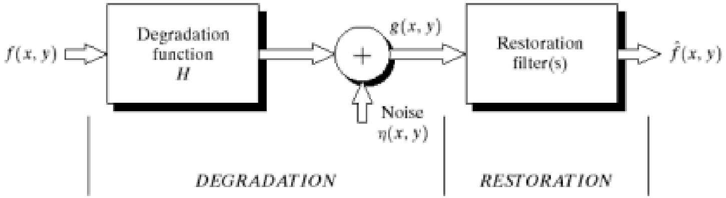
FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

Noise based Degradation

- Assuming H is identity, model reduces to:



$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

Noise Models

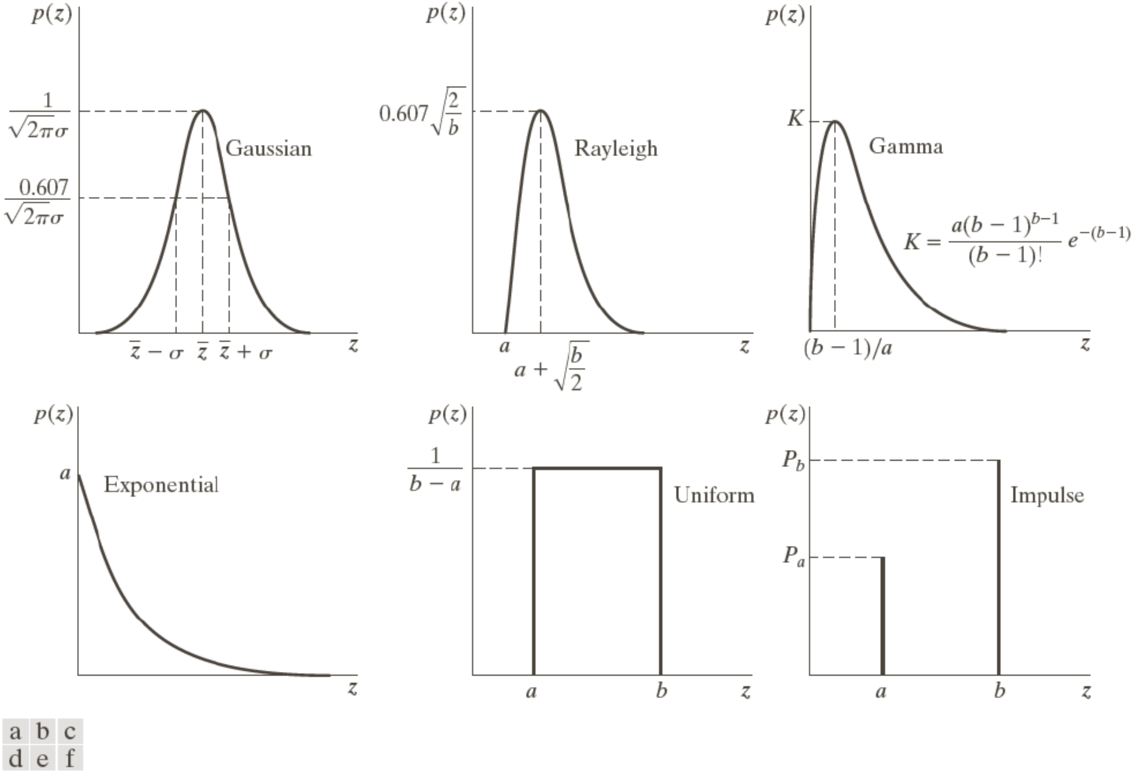
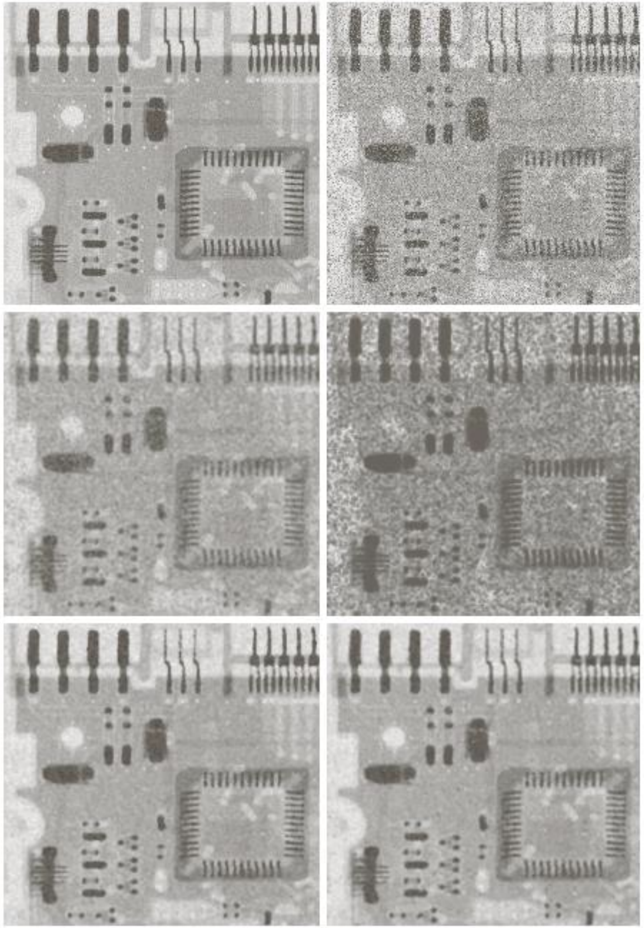


FIGURE 5.2 Some important probability density functions.

Restoration (in presence of noise only)



original

Original + salt and pepper noise

Arithmetic mean filter

Geometric mean filter

Median filter

Alpha Trimmed filter

Estimation of degradation function

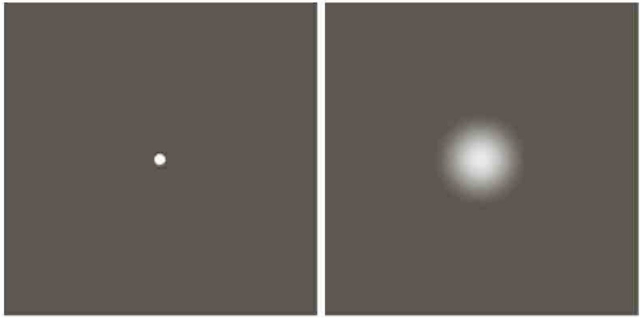
- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration
  - Mathematical modelling

Blind deconvolution

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



$$H(u,v) = \frac{G(u,v)}{A}$$

Recovering image (in presence of both Noise and degradation)

- Direct inverse filtering: Assuming H is known or obtained using any of the 3 methods:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \qquad \Rightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Even if we know the degradation function we cannot recover the un-degraded image!!

Two problems:

1. N(u,v) is a random function whose fourier transform is not known
2. If degradation has zero or small values → N(u,v)/H(u,v) will dominate

Weiner filter (Minimum Mean Square Error)

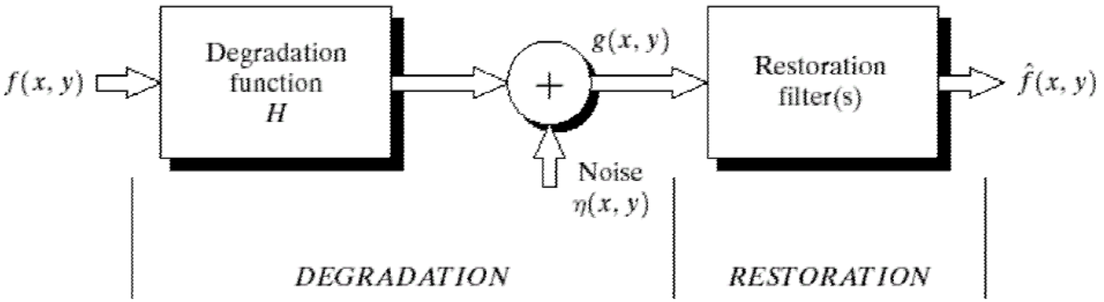


FIGURE 5.1 A model of the image degradation/restoration process.

Consider image and noise as random variables

$$e^2 = E\{(f - \hat{f})^2\}$$

Explicitly incorporates both degradation function and statistical characteristics of noise in restoration process

Assumption:

- Noise and image are uncorrelated

Weiner filter

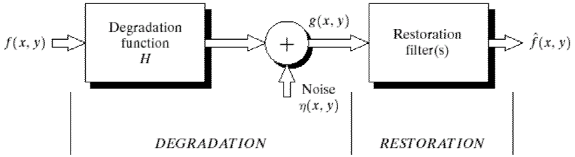


FIGURE 5.1 A model of the image degradation/restoration process.

$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function 'e' is given by:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$|H(u,v)|^2 = H^*(u,v)H(u,v)$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image

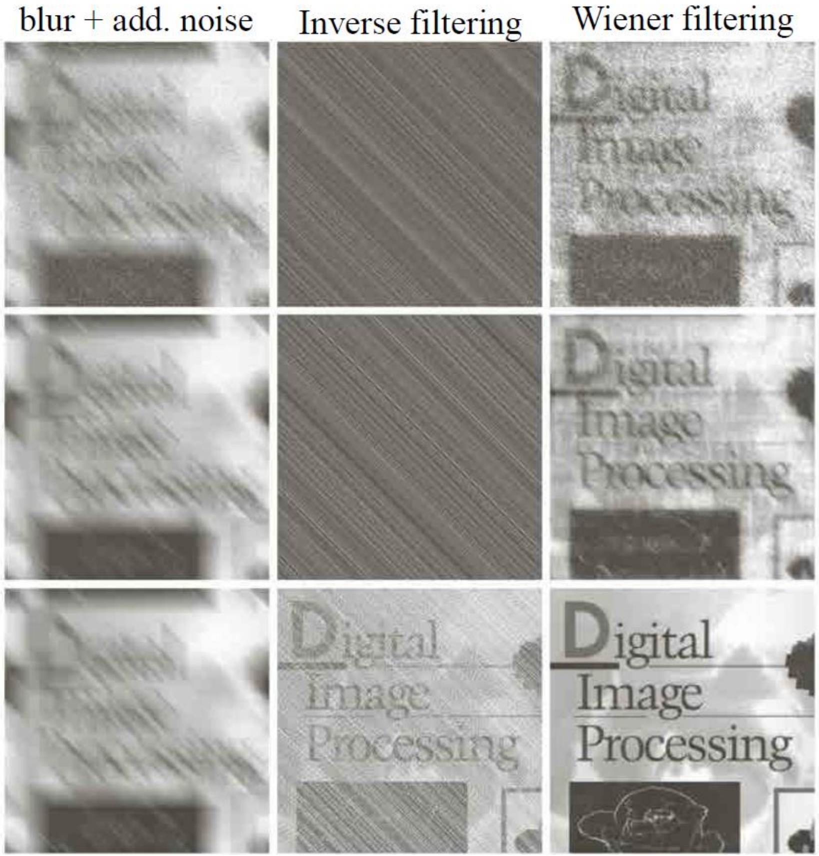
Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right]G(u,v) \qquad \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$SNR = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2 \Big/ \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2$$

Weiner filter



Reduced noise variance by one order of magnitude

Reduced noise variance by five orders of magnitude

Image Restoration

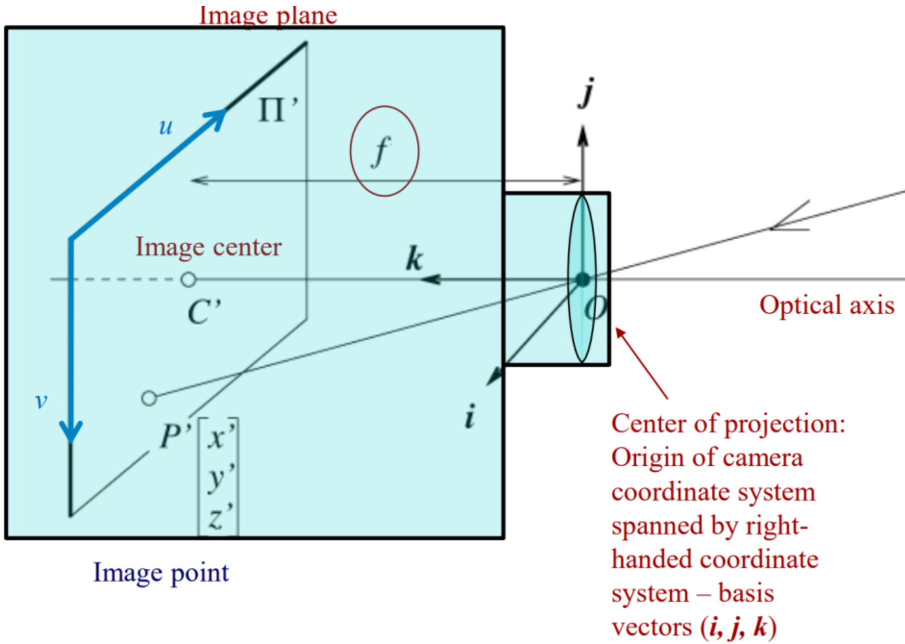
$$f \longrightarrow \boxed{H} \longrightarrow g$$

Inverse problems

Known	Problem type
H,g	Recovery
g	Blind recovery
g, H partially	Semi blind recovery
f,g	System identification

Geometric Distortion

Pinhole and the Perspective Projection



$$x' = f\frac{x}{z}$$

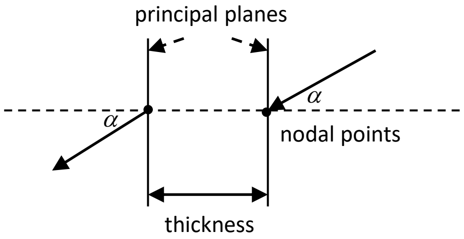
$$y' = f\frac{y}{z}$$

$$z' = f$$

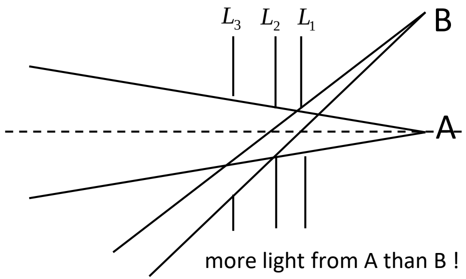
https://sites.cs.ucsb.edu/~yfwang/courses/cs181b/notes/9-projection-geometry.pdf

Common Lens Related Issues - Summary

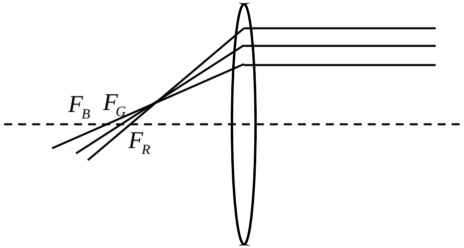
Compound (Thick) Lens



Vignetting



Chromatic Abberation



Lens has different refractive indices
for different wavelengths.

Radial and Tangential Distortion

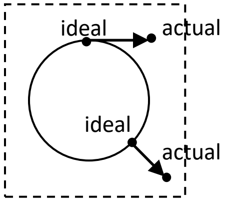


image plane

Lens Glare

•Happens when very bright sources are present in the scene.

Reading: http://www.dpreview.com

Vignetting

Vignetting appears as radial darkening towards frame corners

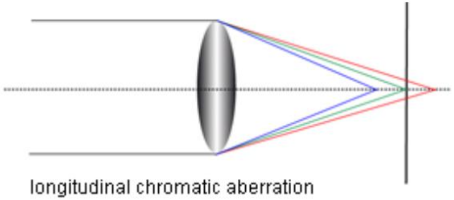
- Natural vignetting is caused due to light reaching different locations of camera sensor at different angles. Optical vignetting caused by shading from lens barrel
- Mechanical vignetting is due to physical obstruction in front of lens (filter rings)

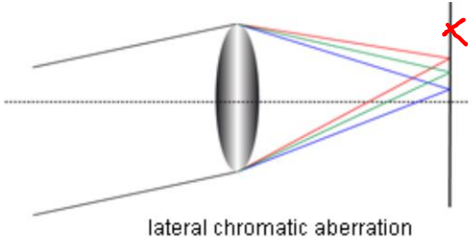
Courtesy: https://www.red.com/red-101/lens-vignetting

Chromatic Aberrations

longitudinal chromatic aberration (axial)

transverse chromatic aberration (lateral)





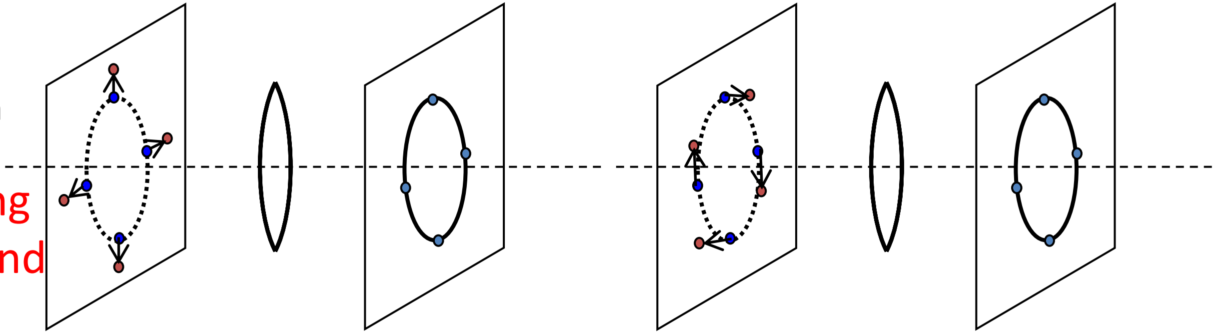
Refractive index of medium depends on wavelength. Lens is not able to focus all wavelengths at same focal point

Courtesy: https://www.image-engineering.de/library/technotes/750-longitudinal-and-lateral-chromatic-aberration

Geometric Lens Distortions

Radial distortion

Unequal bending of light (rays bend more near lens edges)

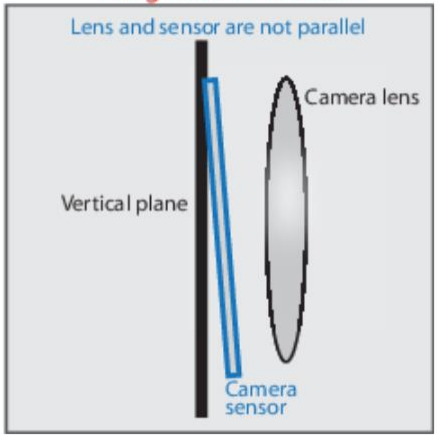


Tangential distortion

When lens and image screen or sensor are not parallel

Original | Barrel Distortion | Pincushion Distortion

Tangential Distortion



Courtesy: Filipo Bergamasco DAIS, Ca' Foscari University of Venice. Lecture on The Pinhole camera Model 2016/17

who said distortion is a bad thing?

blur ...

noise ...

geometric ...

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Digital Image Processing (CSE/ECE 478)

Lecture-20: Image Compression

Ravi Kiran and Sudipta Banerjee

Center for Visual Information Technology (CVIT), IIIT Hyderabad

amazon prime video

hotstar

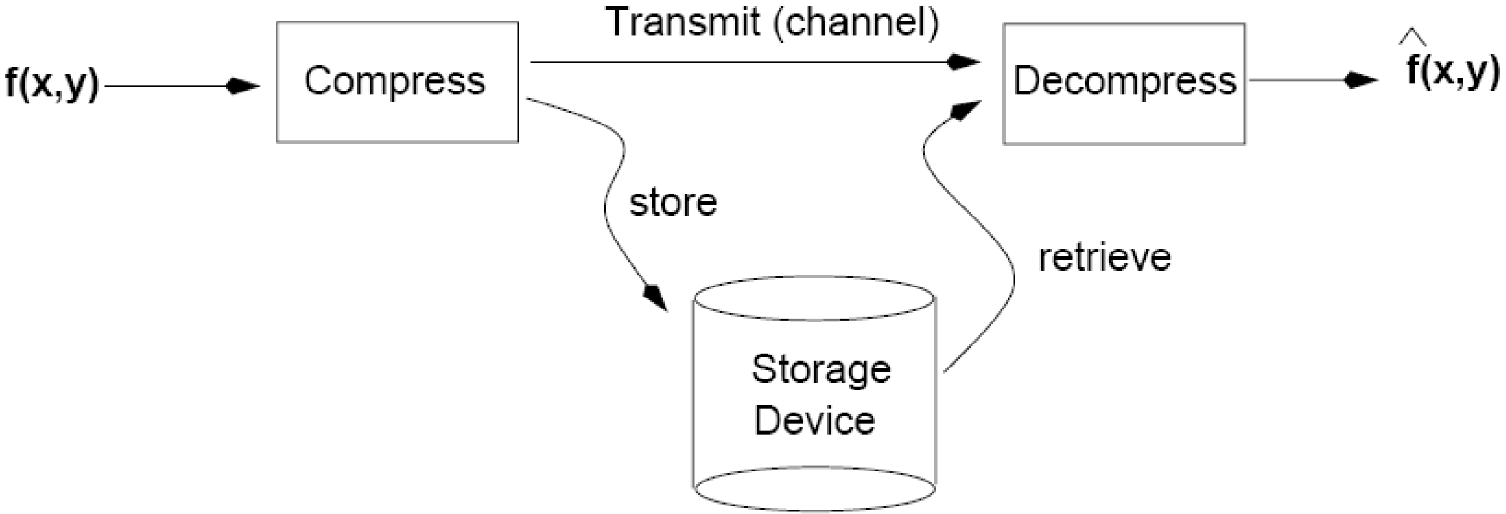
NETFLIX

Data Compression

- Data compression aims to reduce the amount of data while preserving as much information as possible.

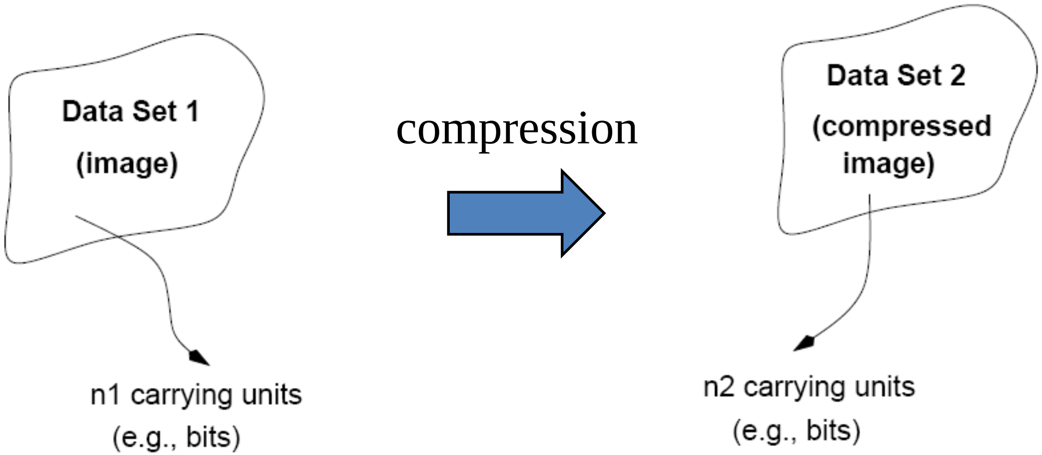
Image Compression

- Goal: Reduce amount of data required to represent a digital image (signal).



.. By leveraging redundancies in image data

Compression Ratio



Compression ratio: $$C_R = \frac{n_1}{n_2}$$

Relative Data Redundancy

$$R_D = 1 - \frac{1}{C_R}$$

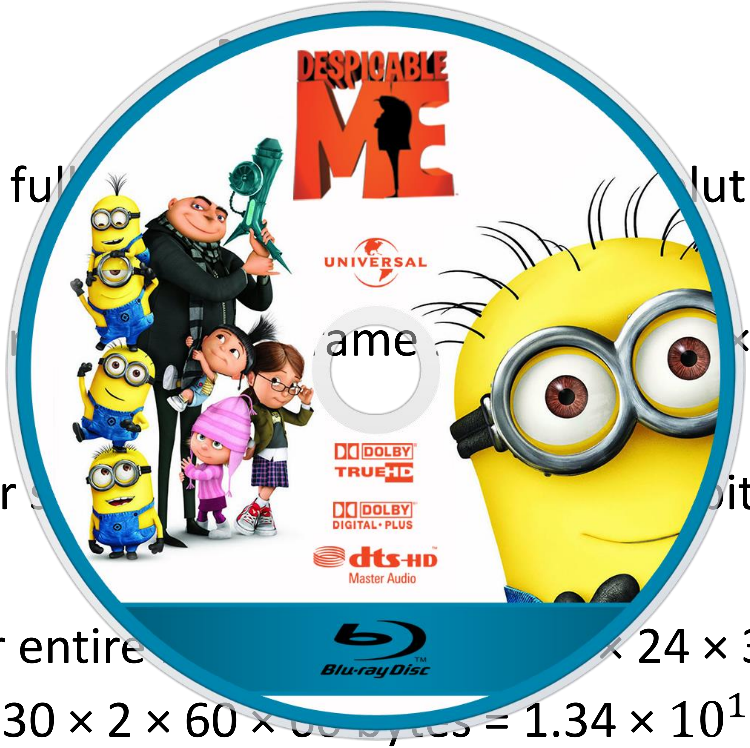
Example:

If $C_R = \frac{10}{1}$, then $R_D = 1 - \frac{1}{10} = 0.9$

(90% of the data in dataset 1 is redundant)

if $n_2 = n_1$, then C_R=1, R_D=0

if $n_2 \ll n_1$, then $C_R \to \infty$, $R_D \to 1$

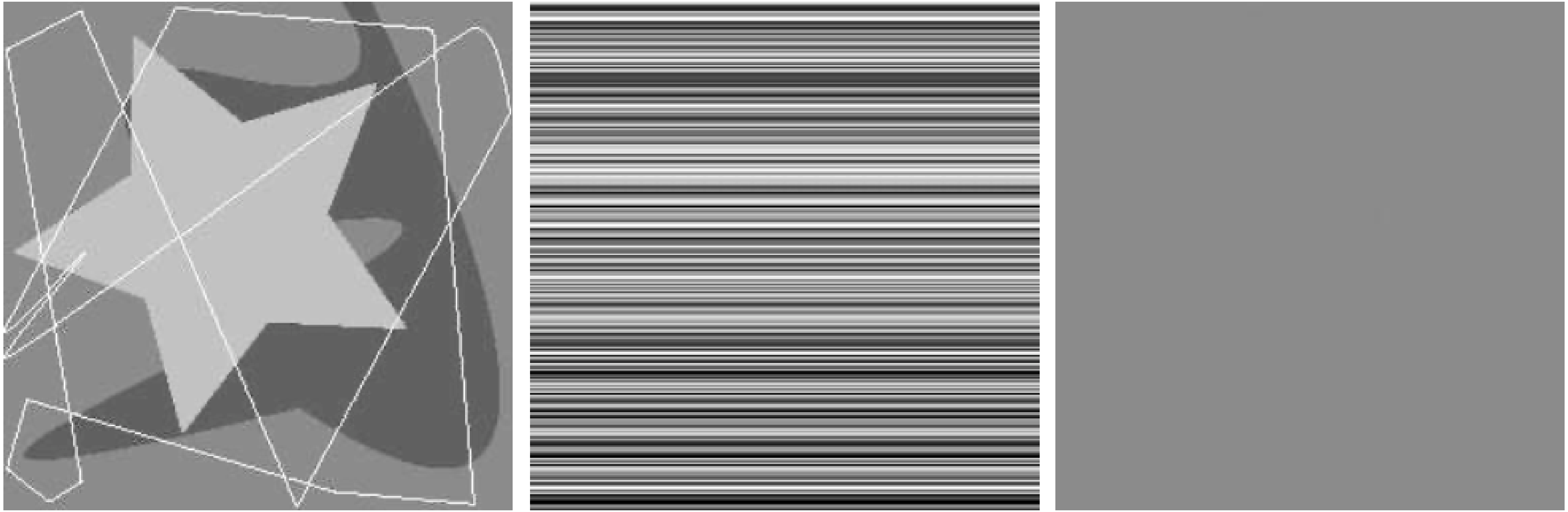


- Consider a 2 hour, ful … lution@30fps)
- The storage space r … ame … × 24 bits = 6.22 MB
- Space required per s … bits
- Space required for entire … × 24 × 30 × 2 × 60 × 60 bits = 1920 × 1080 × 3 × 30 × 2 × 60 × … = 1.34 × 10^{12} bytes= **1340 GB**
- To put it on a 25 GB blu ray disc: required compression factor = **53.6**

Types of Redundancy

(1) Coding Redundancy
(2) Spatial/Temporal Redundancy
(3) Psychovisual Redundancy

- Image compression attempts to reduce one or more of these redundancy types.



a b c

FIGURE 8.1 Computer generated $256 \times 256 \times 8$ bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)

Types of Redundancy

(1) Coding Redundancy

(2) Spatial/Temporal Redundancy

(3) Psychovisual Redundancy

- Image compression attempts to reduce one or more of these redundancy types.

Coding - Definitions

- **Code:** a list of symbols (letters, numbers, bits etc.)
- **Code word:** a sequence of symbols used to represent some information (e.g., gray levels).
- **Code word length**: number of symbols in a code word.

Example: (binary code, symbols: 0,1, length: 3)

0: 000	4: 100
1: 001	5: 101
2: 010	6: 110
3: 011	7: 111

Optimal Information Coding

Assume an $\boldsymbol{M} \times \boldsymbol{N}$ image

r_k: Discrete random variable in the range [0,L-1], i.e., the k-th intensity level

$l(r_k)$: no.of bits used to represent each value of r_k

$p_r(r_k)$: probability of r_k (each r_k occurs with probability $p_r(r_k)$)

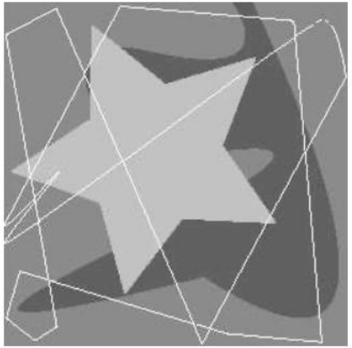
$p_r(r_k) = \frac{n_k}{MN}, k = 0,1,2,\dots,L-1$ (L is the no. of intensity values and n_k is the number of times that k^{th} intensity level appears in the image)

$$E(X) = \sum x.P(X = x)$$

Average no. of bits required to represent each pixel is $L_{avg} = \sum_{k=0}^{L-1} l(r_k)p_r(r_k)$

Total number of bits to represent the image = MNL_{avg}

Coding Redundancy



- <u>Case 1:</u> $l(r_k)$ = constant length

Example:

r_k	$p_r(r_k)$	*Code 1*	$l_1(r_k)$
$r_0 = 0$	0.19	000	3
$r_1 = 1/7$	0.25	001	3
$r_2 = 2/7$	0.21	010	3
$r_3 = 3/7$	0.16	011	3
$r_4 = 4/7$	0.08	100	3
$r_5 = 5/7$	0.06	101	3
$r_6 = 6/7$	0.03	110	3
$r_7 = 1$	0.02	111	3

Average # of bits: $L_{avg} = E(l(r_k)) = \sum_{k=0}^{L-1} l(r_k)P(r_k)$

Total # of bits: NML_{avg}

Assume fixed–length code

Assume $l(r_k) = 3$, $L_{avg} = \sum_{k=0}^{7} 3P(r_k) = 3\sum_{k=0}^{7} P(r_k) = 3$ bits

Total number of bits: $3NM$

Coding Redundancy (cont'd)

- <u>Case 2:</u> $l(r_k)$ = variable length

$$C_R = \frac{n_1}{n_2}$$

Table 6.1 Variable-Length Coding Example

variable length

r_k	$p_r(r_k)$	*Code 1*	$l_1(r_k)$	*Code 2*	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

$$L_{avg} = \sum_{k=0}^{7} l(r_k)P(r_k) = 2.7 \text{ bits}$$

Total number of bits: 2.7MN

$$C_R = \frac{3}{2.7} = 1.11 \text{ (about 10\%)}$$

$$R_D = 1 - \frac{1}{1.11} = 0.099$$

Coding redundancy occurs because same no. of bits are assigned to least and most probable intensity values

Types of Redundancy

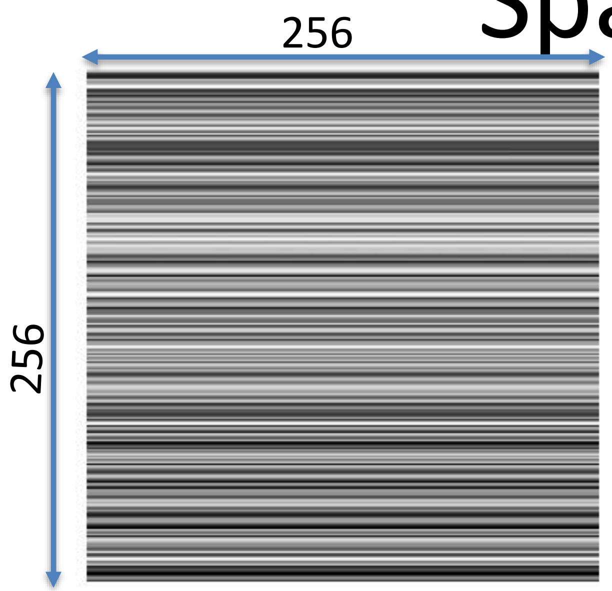
(1) Coding Redundancy

(2) Spatial/Temporal Redundancy

(3) Psychovisual Redundancy

- Image compression attempts to reduce one or more of these redundancy types.

Spatial Redundancy



- All 256 intensities are equiprobable
- Pixels are independent in vertical direction
- Pixels are maximally correlated in horizontal direction, each row has constant intensity

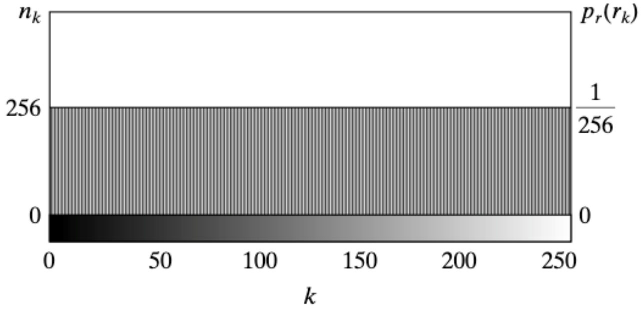
Run-length pairs:

- Each run-length pair specifies start of new intensity and the no. of consecutive pixels that have that intensity (I, n_I)

8-bit code r_0 to r_{255}

Row1: (23,256)
Row2: (240,256)

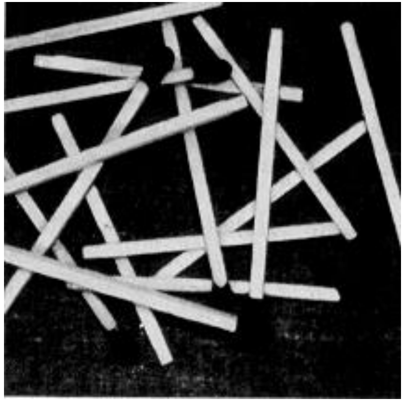
Row256: (10,256)

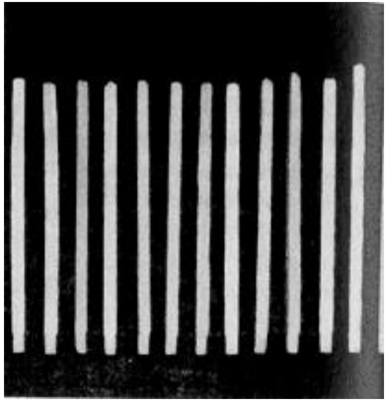


$$\text{Compression ratio} = \frac{256 \times 256 \times 8}{(256+256) \times 8} = \frac{2^{16}}{2^9} = 128:1$$

Spatial redundancy

- Interpixel/Intra-image redundancy exists ➔ pixel values are correlated
- i.e., a pixel value can be reasonably predicted by its neighbors





histograms

auto-correlation

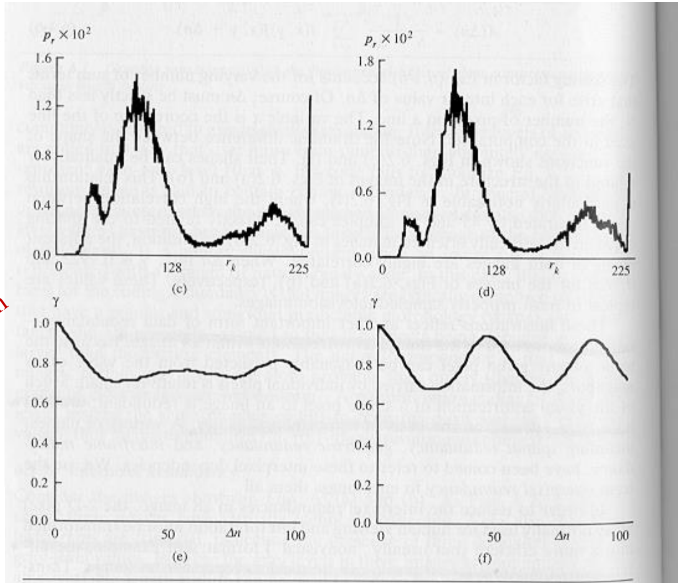


Figure 6.2 Two images and their gray-level histograms and normalized autocorrelation

$$f(x) \circ g(x) = \int_{-\infty}^{\infty} f(x)g(x+a)da$$

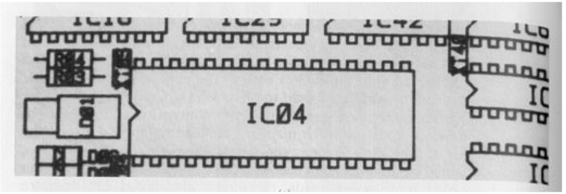
auto-correlation: f(x)=g(x)

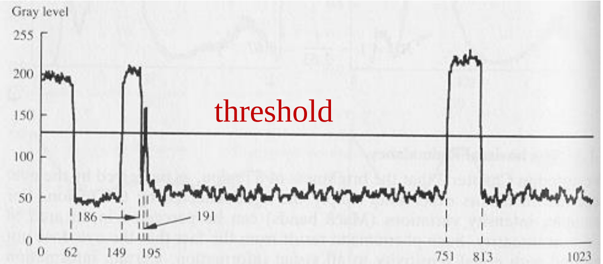
Interpixel redundancy (cont'd)

- To reduce interpixel redundancy, some kind of transformation must be applied on the data (e.g., thresholding, DFT, DWT)

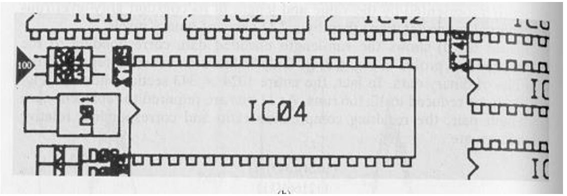
Example:

original





thresholded



11 ……………0000……………………..11…..000…..

Run-length encoding:

(1,63) (0,87) (1,37) (0,5) (1,4) (0, 556) (1,62) (0,210)

Using 11 bits/pair: (1+10) bits/pair

88 bits are required (compared to 1024 !!)

Spatial and temporal redundancy

frame t

frame t+1

Inter-frame redundancy

Spatial and temporal redundancy

Types of Redundancy

(1) Coding Redundancy

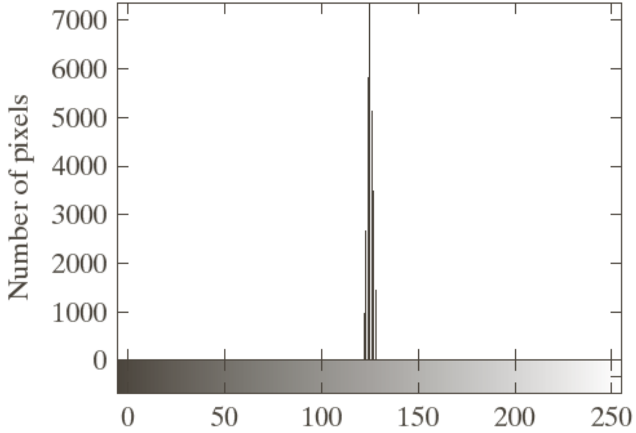
(2) Spatial/Temporal Redundancy

(3) Psychovisual Redundancy

- Image compression attempts to reduce one or more of these redundancy types.

Irrelevant information or perceptual redundancy

- Not all visual information is perceived by eye/brain, so throw away those that are not



Maybe use the average intensity alone (single 8-bit value!)

Psychovisual redundancy (cont'd)

Example: quantization

256 gray levels

16 gray levels

16 gray levels + random noise

C=8/4 = 2:1

add a small pseudo-random number to each pixel prior to quantization

Information theory

- Basic Premise: Generation of information can be treated as a probabilistic process defined over symbols.

- Symbol - carrier of information
- Consider a symbol with an occurrence probability p.
- The amount of information contained in the symbol is defined as:

$$I = \log_2 \frac{1}{p} \text{ bits} \quad \text{or} \quad I = -\log_2 p$$

Information theory: Entropy

- Consider a statistically-independent source that contains L possible symbols {s,i=0,1,2,…,L-1}
- With corresponding occurrence probabilities defined as {p_i, i=0,1,2,…,L−1}
- **Entropy**

$$H = -\sum_{i=0}^{L-1} p_i \log_2 p_i$$

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

H=-[0.25xlog(0.25)+0.47xlog(0.47)+0.25 xlog(0.25)+0.03xlog(0.03)=1.66 bits/pixel

Information theory: Shannon's noiseless coding theorem

- Shannon's lossless source coding theorem: For a discrete, memoryless information source, the minimum bit rate required to encode a symbol on average is equal to the entropy of the source

- In other words: entropy provides a **lower bound** on compression that can be achieved when coding statistically independent symbols

- For correlated pixels, we can leverage spatial/temporal redundancy to get fewer average bits/pixel. If current output depends on finite past outputs, we get a Markov or finite memory source

Reference

- Ch 5 for Image Restoration and Ch 8 for Image Compression, G&W textbook