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Digital Image Processing (CSE/ECE 478)

Lecture-19: Image Restoration

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degraded images

ideal image

Blurred image

- What caused the image to blur?
  - Camera: translation, shake, out-of-focus …
  - Environment: scattered and reflected light
  - Device noise: CCD/CMOS sensor and circuitry
  - Quantization noise
- Can we improve the image, or "undo" the effects?

Degradations

• original

• optical blur

• motion blur

• spatial quantization (discrete pixels)

• additive intensity noise

Examples (Optical Blur)

Lens Blur selfie, background focus

Photo by Rachel Been

Lens Blur selfie, foreground focus

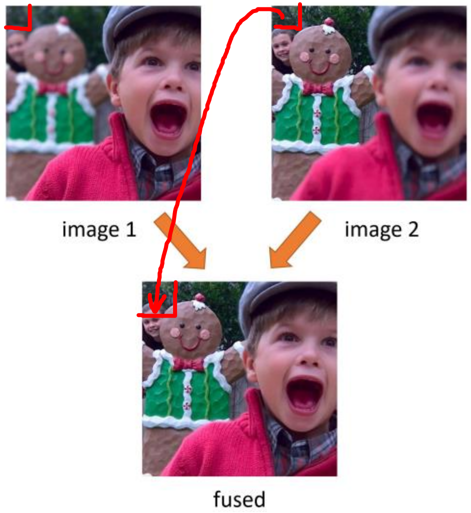
Limited depth of field

Solutions

Light field camera

This demonstrates the capability of changing the focal distance and depth of field after a photo is taken - Near focus (top), Far focus (middle), Full depth of field (bottom) - using the Lytro Illum light field camera software

https://en.wikipedia.org/wiki/Light_field_camera



Multifocus image fusion

Zhang, "Multi-focus Image fusion: A benchmark," arXiv 2020

Examples (Restoration from camera shake)

Figure 1: *Left*: An image spoiled by camera shake. *Middle*: result from Photoshop "unsharp mask". *Right*: result from our algorithm.

$$B = K * L + N$$

B: blurred image
K: blur kernel
L: blur-free image
N: sensor noise

1. Estimate blur kernel using gradient estimation
2. Perform deconvolution

Fergus et al. "Removing Camera shake from a Single Photograph," ACM Transactions on Graphics, 2006

Examples (Atmospheric conditions)

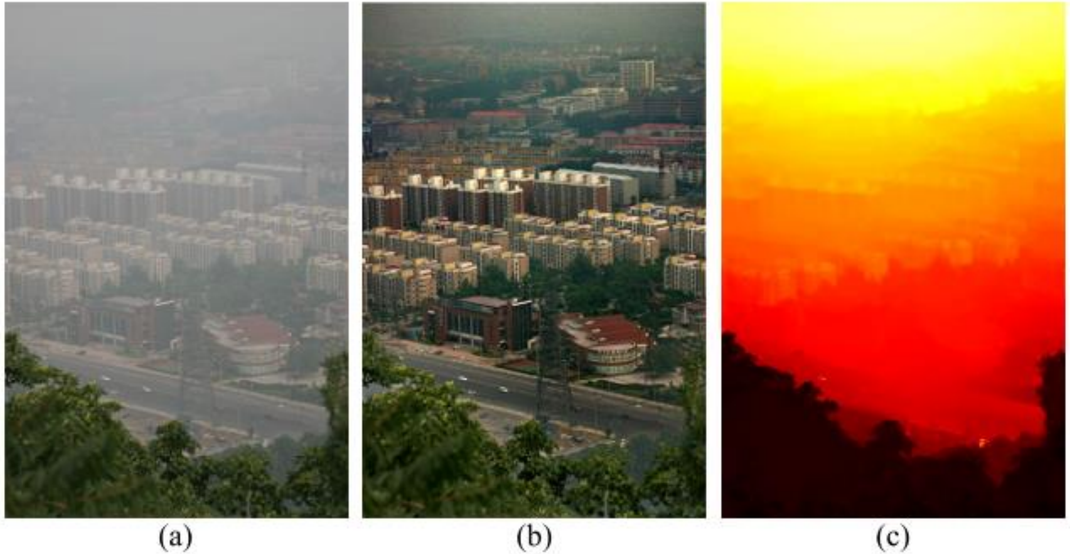


Figure 1. Haze removal using a single image. (a) input haze image. (b) image after haze removal by our approach. (c) our recovered depth map.

Dark channel prior is based on statistics of haze-free images

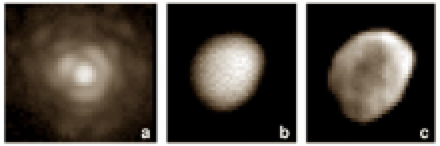
Dark pixels have very low intensity, these pixels accurately estimate haze transmission

He et al., “Single Image Haze Removal Using Dark Channel Prior,” CVPR 2009

image restoration

- started from the 1950s
- application domains
  - Scientific explorations
  - Legal investigations
  - Film making and archival
  - Image and video (de-)coding
  - …
  - Consumer photography

Example of image restoration
Asteroid Vesta



- related problem: image reconstruction in radio astronomy, radar imaging and tomography

[Banham and Katsaggelos 97]

Original image

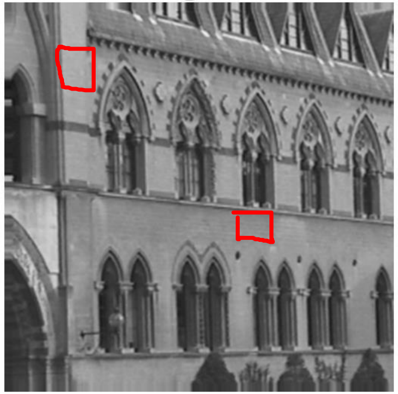
Blurred image

- Image enhancement: "improve" an image subjectively.
- Image restoration: remove distortion from image in order to go back to the "original" → objective process.

Degradation model

Model degradation as convolution with linear shift-invariant filter h(x,y)

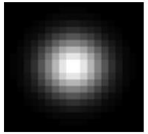
original



$f(x,y)$

$*$

$$\frac{1}{2\pi\sigma^2}e^{-r^2/2\sigma^2}$$



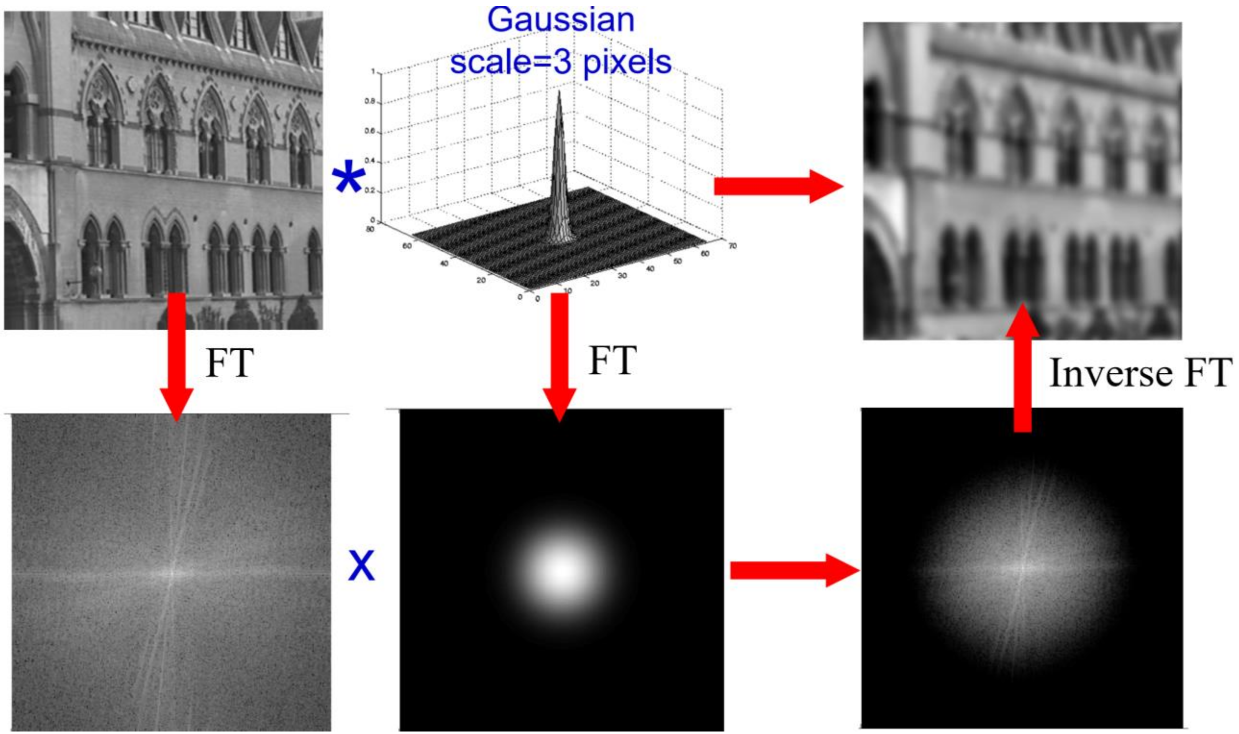
$h(x,y)$

blurred

$g(x,y)$

Out-of-focus blurring can be modeled using a Gaussian filter

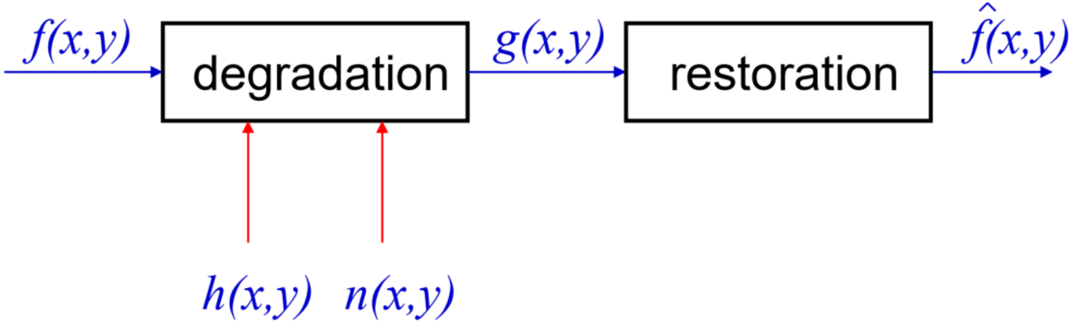
Limitations



Loss of information (blurring attenuates high spatial frequencies) and noise

Degradation model

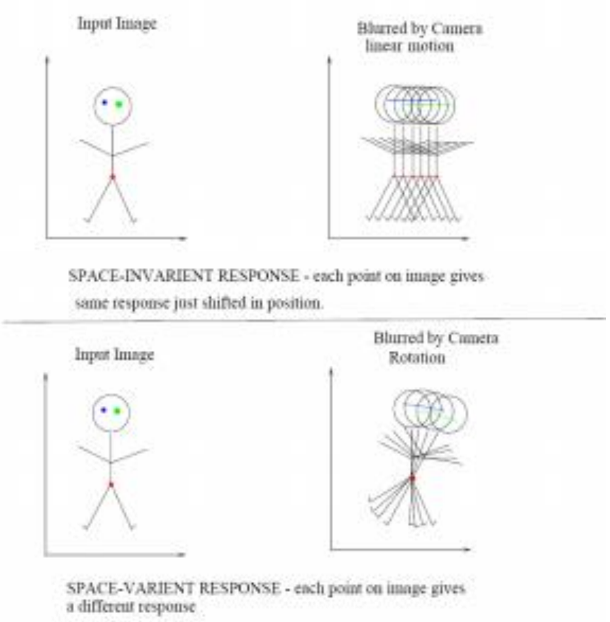
- $f(x,y)$ – image before degradation, 'true image'
- $g(x,y)$ – image after degradation, 'observed image'
- $h(x,y)$ – degradation filter
- $\hat{f}(x,y)$ – estimate of $f(x,y)$ computed from $g(x,y)$
- $n(x,y)$ – additive noise

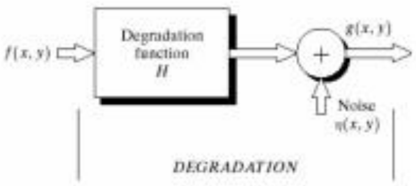


$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \Leftrightarrow \quad G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

Some assumptions…

- Noise
  - Independent of spatial location
    - Exception: periodic noise …
  - Uncorrelated with image
- Degradation function H
  - Linear
  - Position-invariant





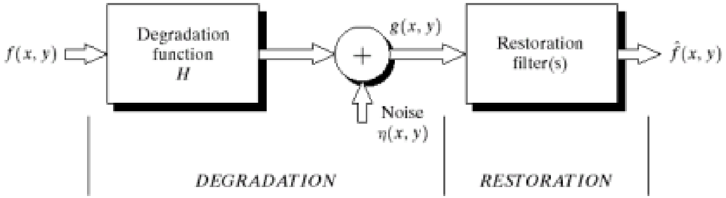
$$H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)\ if\ n(x,y) = 0$$

divide-and-conquer step #1: image degraded only by noise.

Noise based Degradation

- Assuming H is identity, model reduces to:



$$g(x,y) = f(x,y) + \eta(x,y)$$

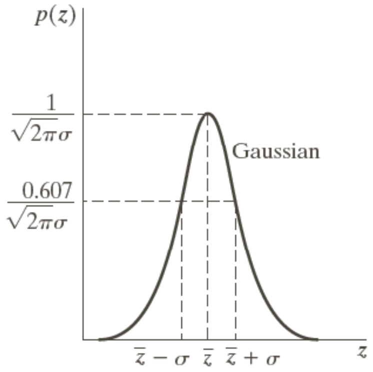
$$G(u,v) = F(u,v) + N(u,v)$$

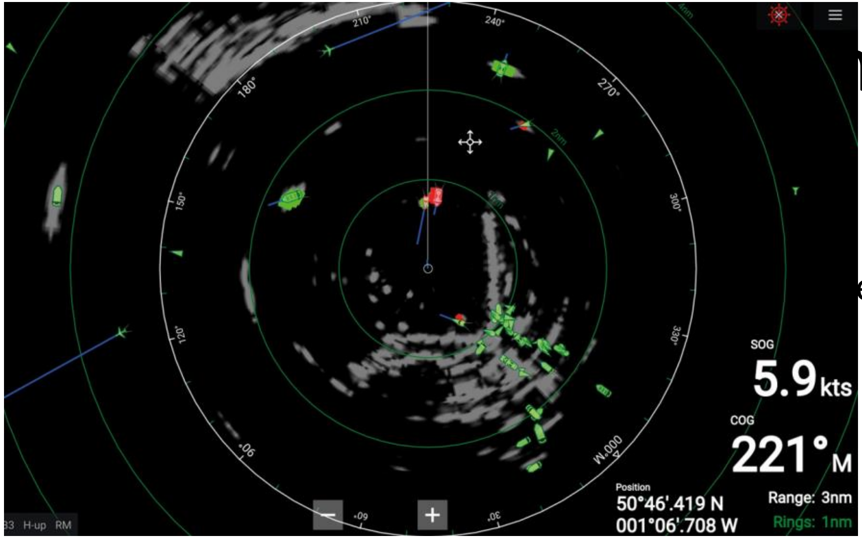
Consider noise as random variable following a probability density function

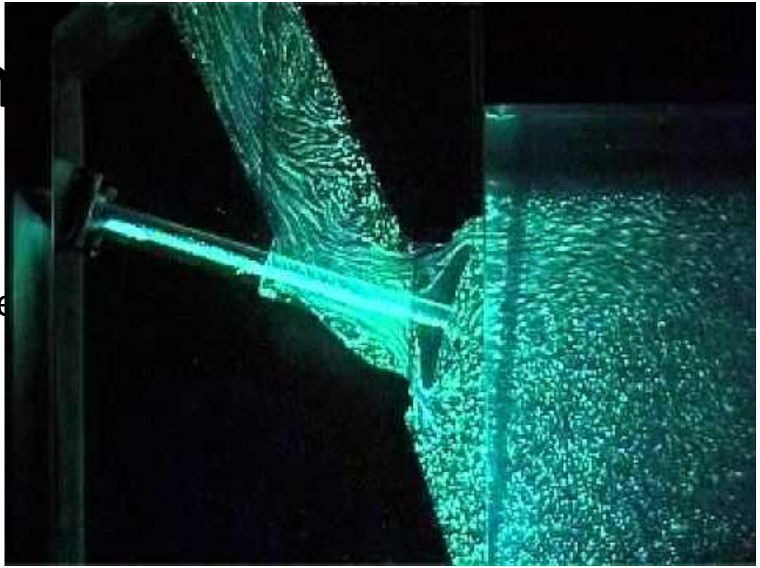
Noise Models

- Gaussian (normal) Noise
  - widely used due to mathematical convenience

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$







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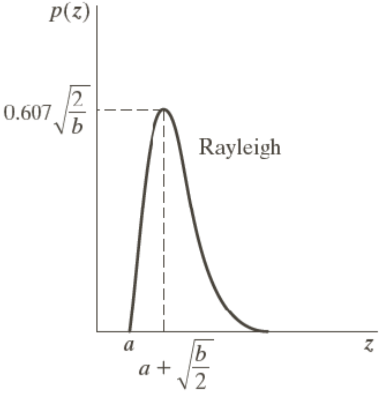
- Rayleigh Noise [Radar, Velocity images]



$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b/4}$ Variance: $\sigma^2 = \dfrac{b(4-\pi)}{4}$

Useful for modelling skewed histograms

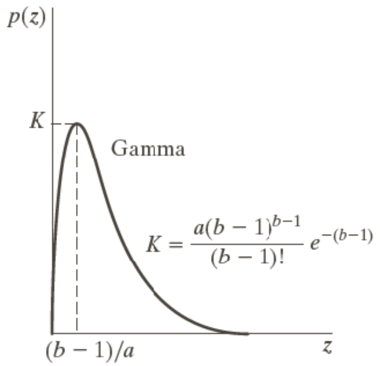
Noise Models

- Erlang (Gamma) Noise [Laser images]

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

b positive integer

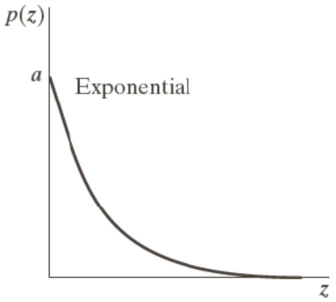


Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$

- Exponential Noise

$$p(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$a > 0$$

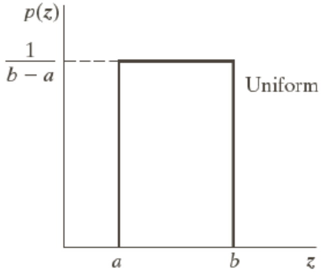


Noise Models

- Uniform Noise [quantization, most unbiased]

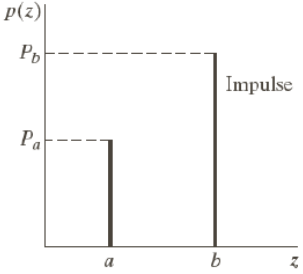
$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} = \frac{a+b}{2}$ Variance: $\sigma^2 = \frac{(b-a)^2}{12}$



- Impulse (salt-and-pepper) Noise [sync errors in digitization or transmission, sensor malfunction]

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



$$p(z) = P_s \; if \; z = 2^k - 1$$
$$p(z) = P_p \; if \; z = 0$$

$P_a = P_b \Rightarrow$ *unipolar* noise

Noise Models

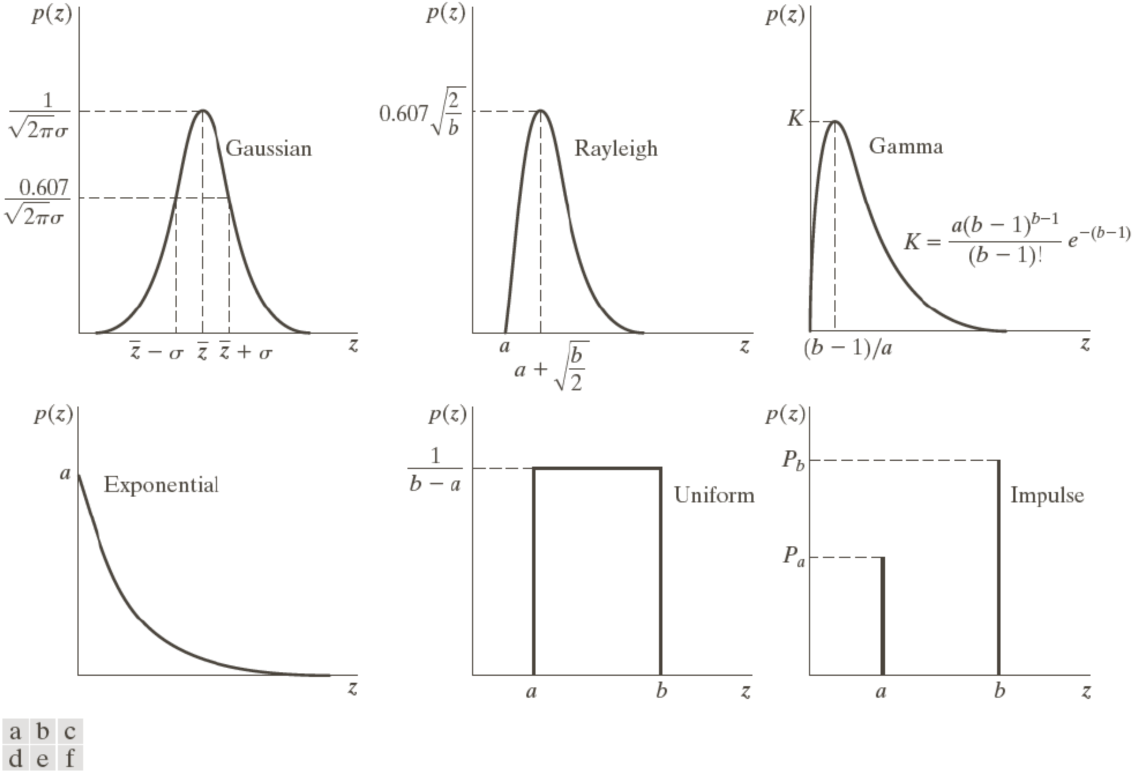


FIGURE 5.2 Some important probability density functions.

Modeling noise corruption

PDF	Noise model
Gaussian	Sensor noise caused by poor illumination and high/low temperature
Rayleigh	Characterizes noise phenomenon in range imaging
Exponential and gamma	Laser imaging
Impulse (salt and pepper)	Quick transients (faulty switching)
Uniform	Random number generators

Illustration of Noise Models

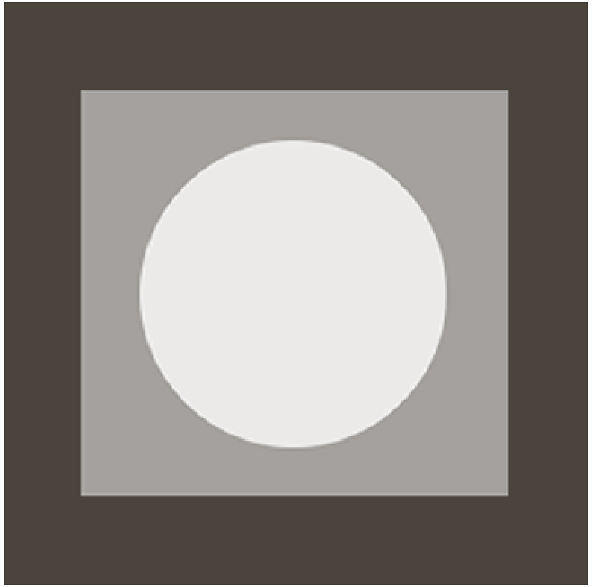
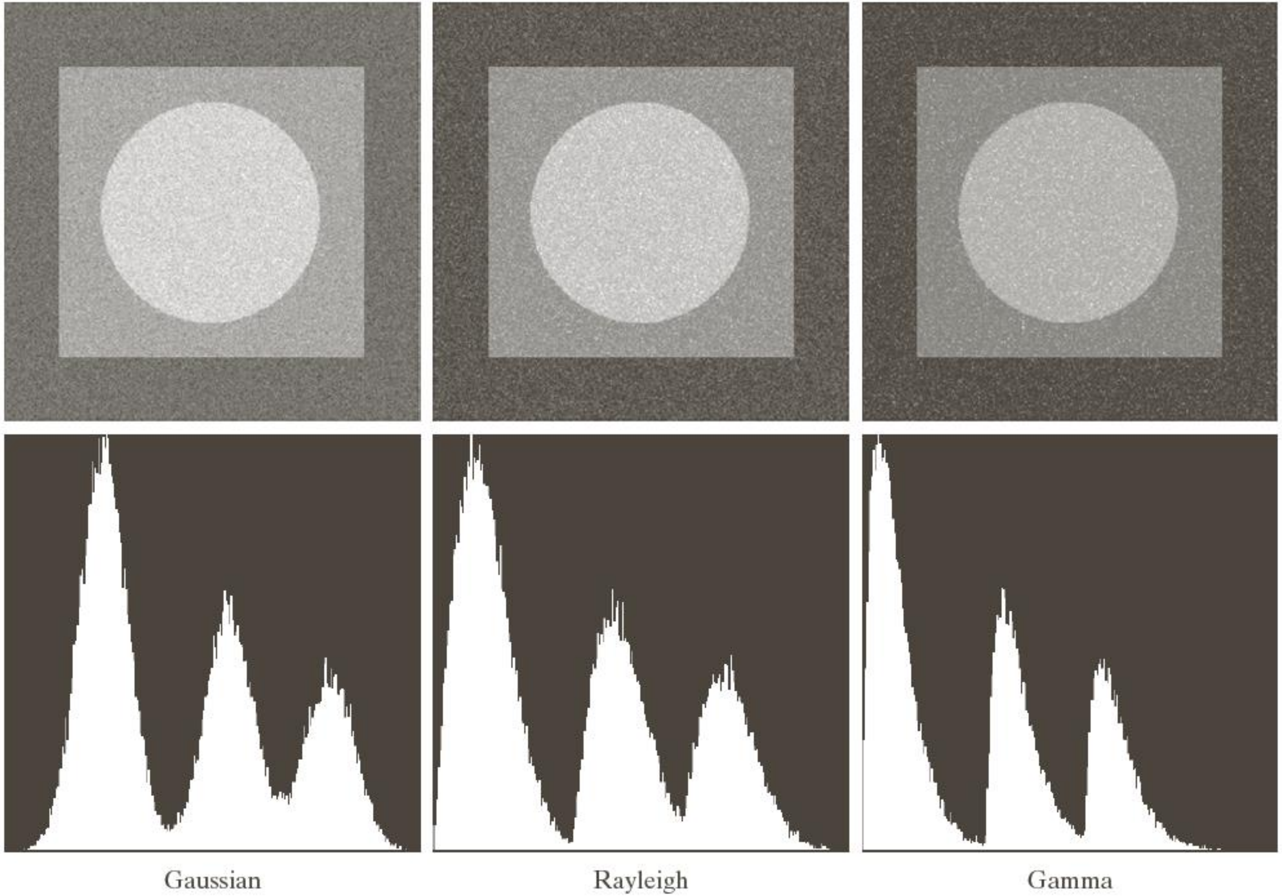


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Illustration of Noise Models



Noisy Images

Histograms

Illustration of Noise Models

Visually similar.

Not easy to determine noise model from appearance

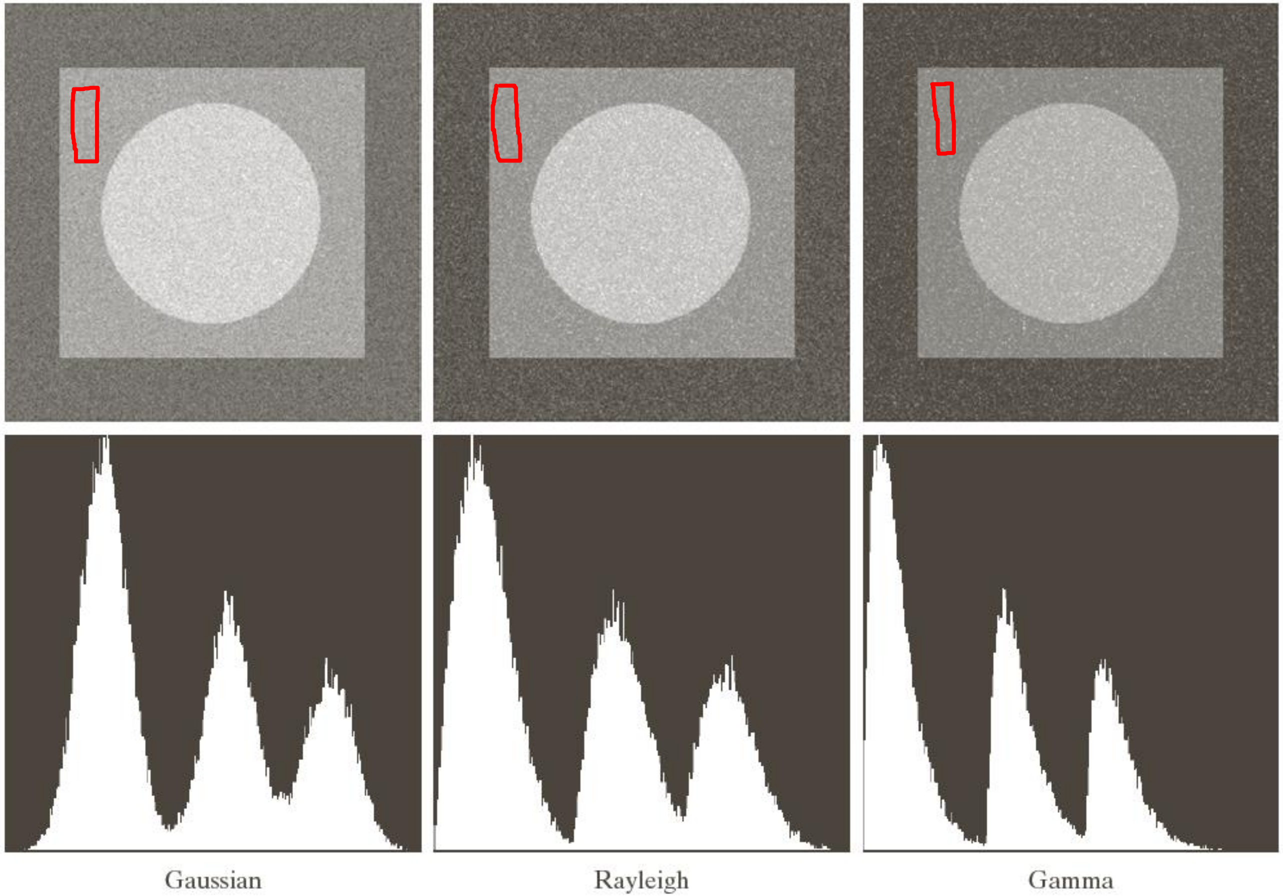
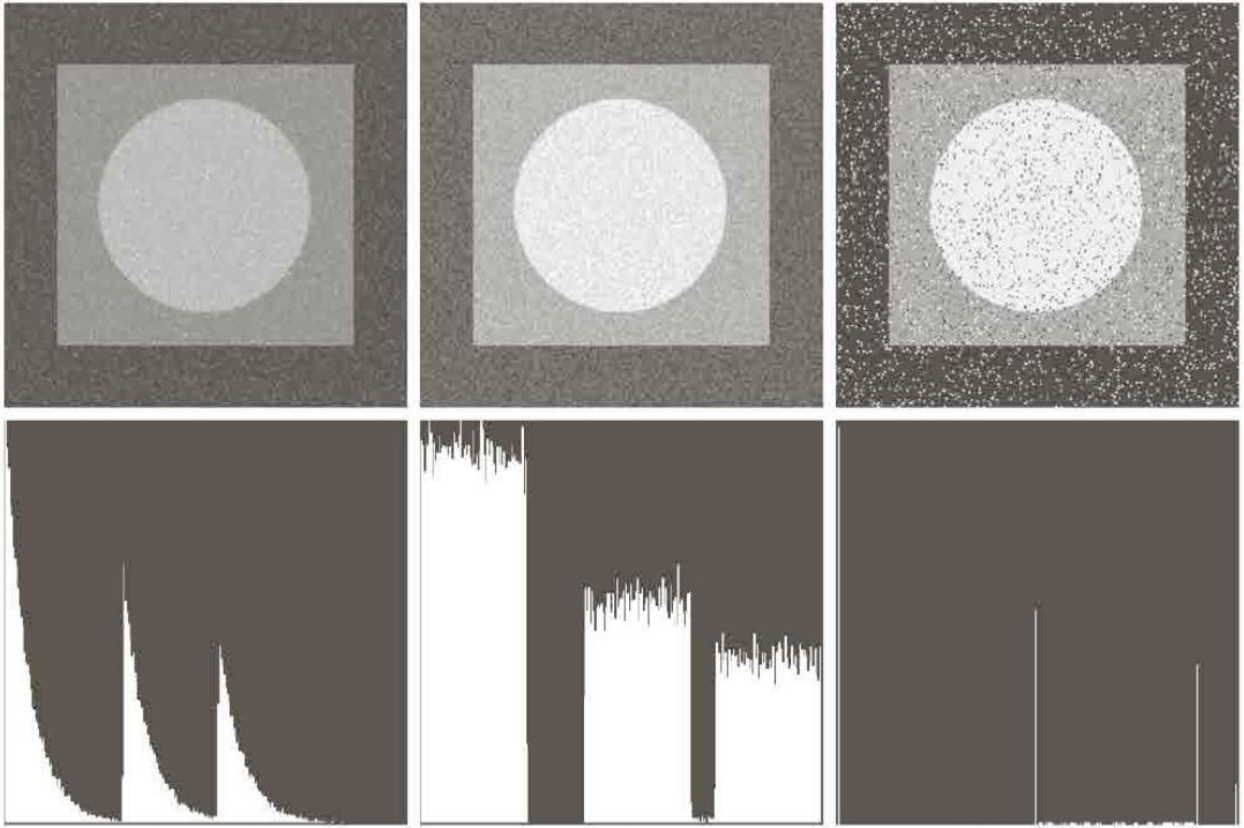


Illustration of Noise Models



Exponential   Uniform   Salt & Pepper

Noisy Images

Histograms

x

y>>x

y

How to study system noise

- Case 1 : Imaging system available
  - Noise Calibration: Capture a set of 'flat' images (e.g. uniformly illuminated solid gray board)
  - Select the model with better statistical test scores (Akaike Information Criteria (AIC) or Likelihood Ratio Test (LRT))

$N(u,v)$

How to study system noise

- Case 2: Only images available
  - Estimate parameters of PDF from patches of constant background intensity
  - Compute mean and variance from intensity levels

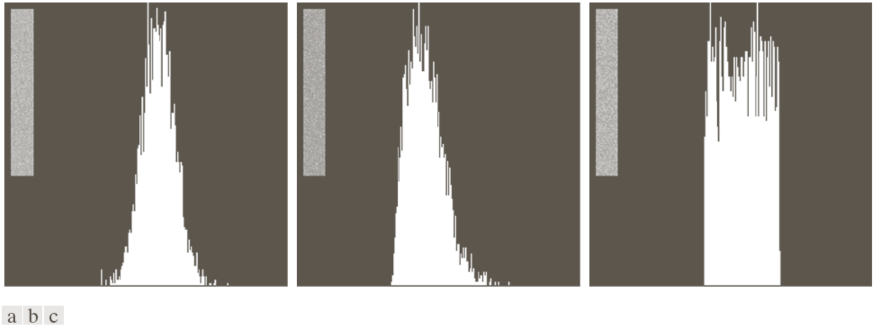
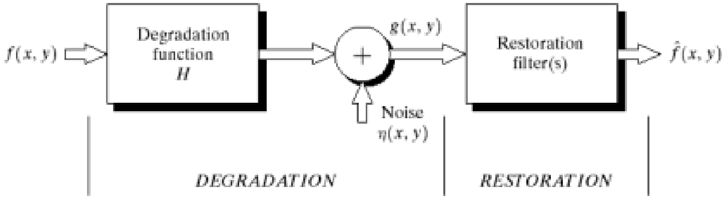


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in presence of noise only

- Assuming H is identity, model reduces to:



$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

Spatial filtering is preferred for denoising g(x,y)

Restoration (in presence of noise only)

- mean filters

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

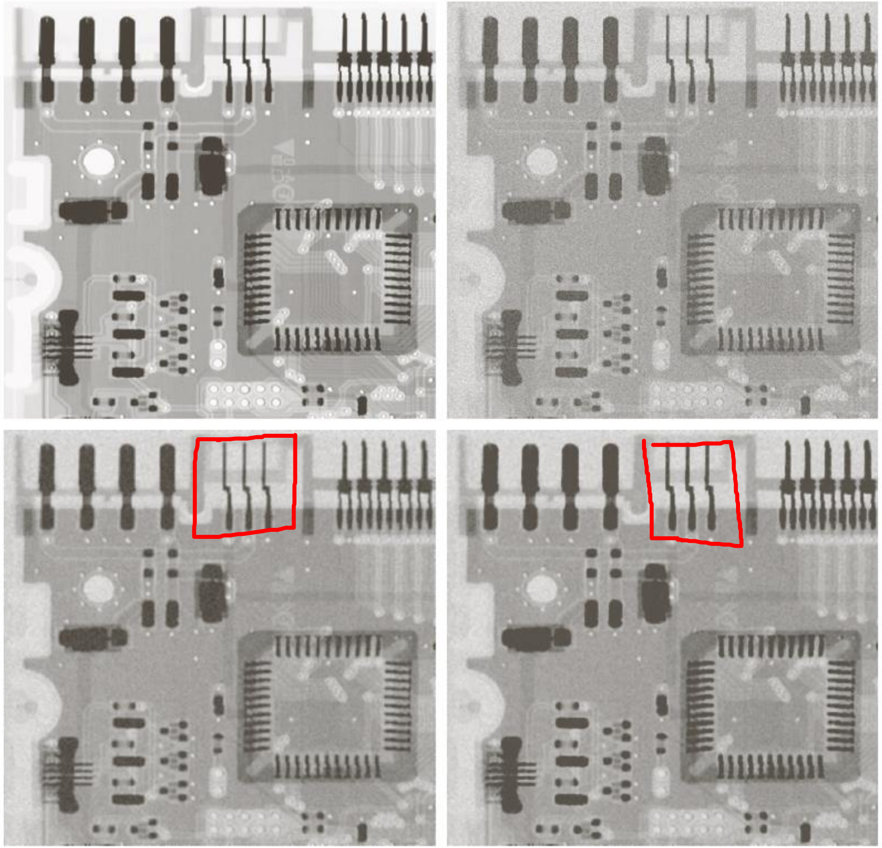
Geometric mean filter

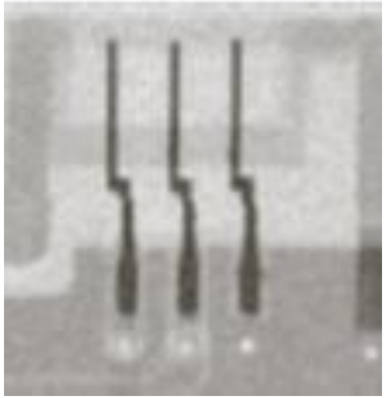
$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

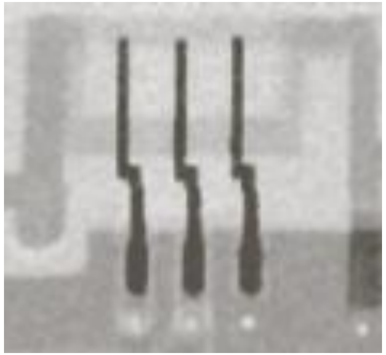
Restoration (in presence of noise only)

a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)







Restoration (in presence of noise only)

- mean filters

<u>Harmonic mean filter</u>

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise or Gaussian noise, but fails for pepper noise

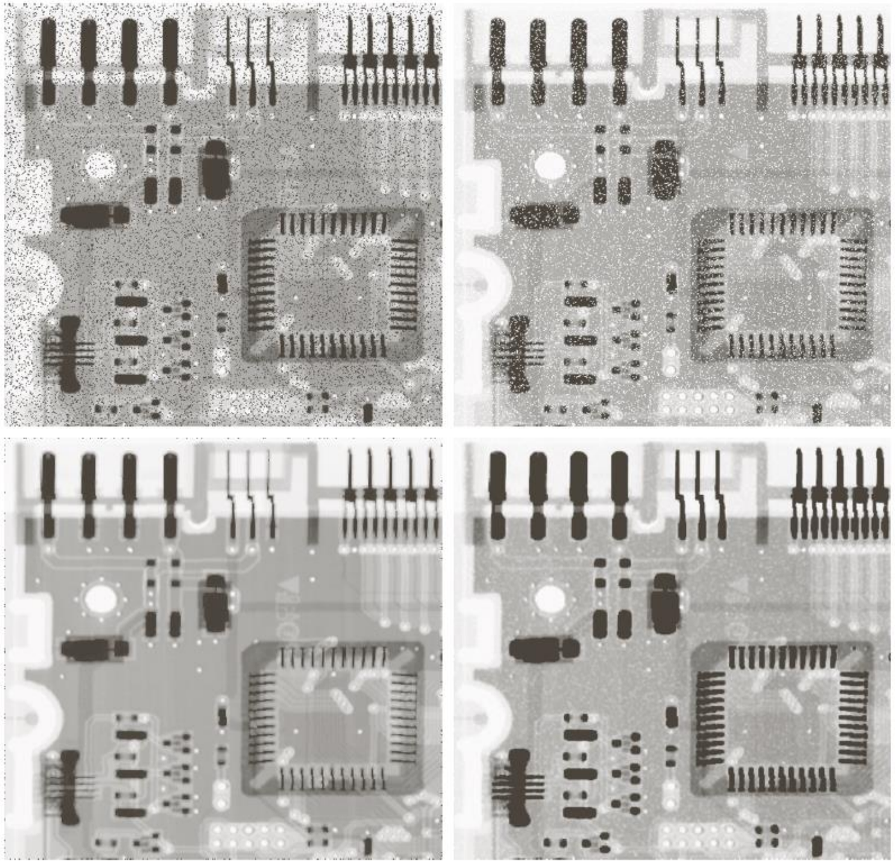
<u>Contraharmonic mean filter</u>

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$\sum 1 = mn$

Q = *order* of the filter
Good for salt-and-pepper noise.
Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$
NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

Restoration (in presence of noise only)



a b
c d

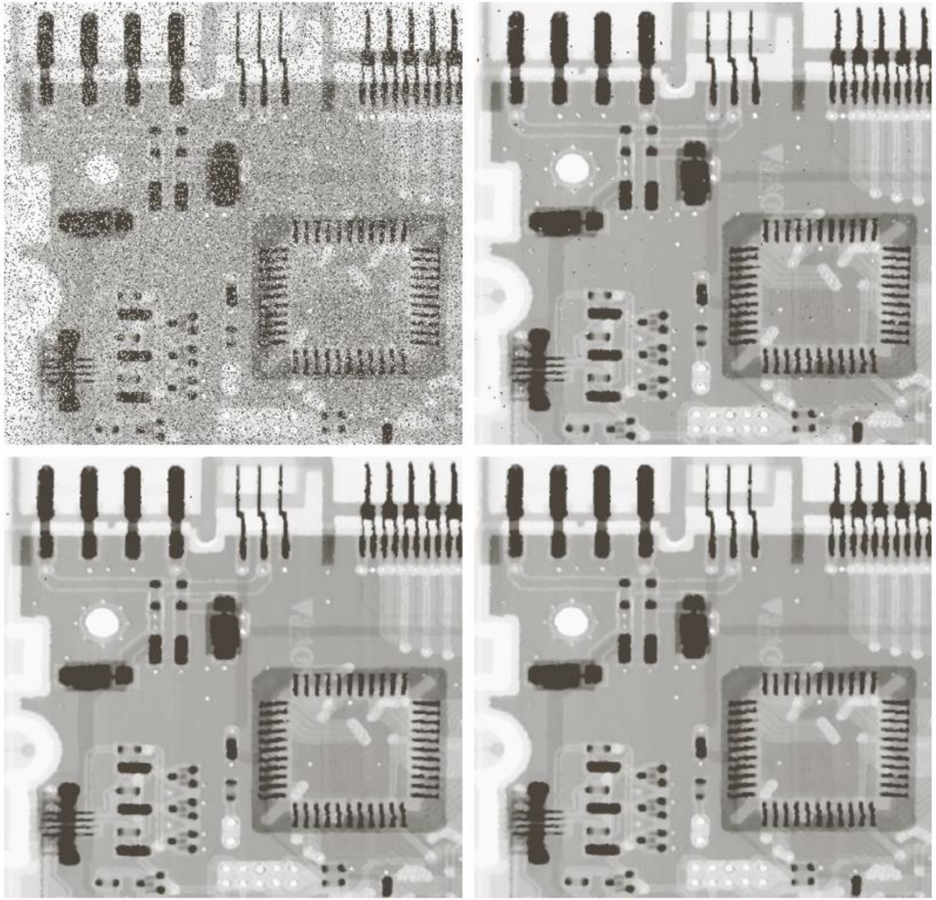
FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Restoration (in presence of noise only)

- Median filter

a	b
c	d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

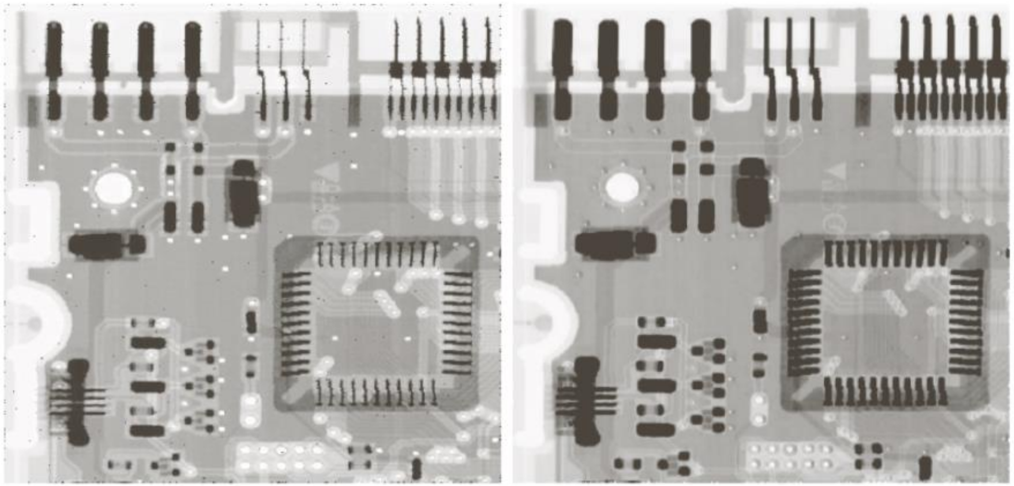


Restoration (in presence of noise only)

- Max, Min filters

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter Min filter

Restoration (in presence of noise only)

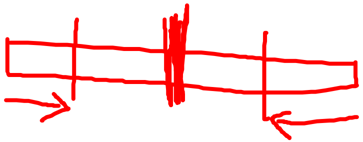
- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max\{g(s,t)\}_{(s,t)\in S_{xy}} + \min\{g(s,t)\}_{(s,t)\in S_{xy}}\right]$$

Best for Uniform or Gaussian noise

- Alpha trimmed filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

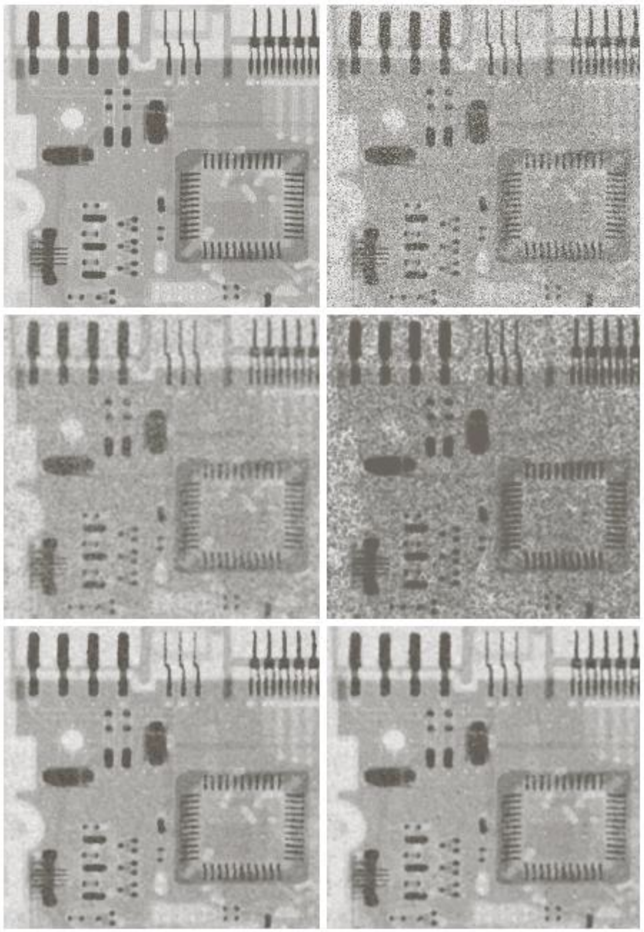


Where g_r represents the image *g* in which the *d/2* lowest and *d/2* highest intensity values in the neighbourhood S_{xy} were deleted. If d=0 (arithmetic mean), if d=mn-1 (median filter). Best for removing combination of salt and pepper and Gaussian noises

Restoration (in presence of noise only)

original

Original + salt and pepper noise



Arithmetic mean filter

Geometric mean filter

Median filter

Alpha Trimmed filter

Adaptive, Local Noise Reduction Filter

Given the corrupted image $g(x, y)$, find $f(x, y)$.

Conditions:

(a) σ^2_η is zero (Zero-noise case)
- Simply return the value of $g(x, y)$.

(b) If σ^2_L is higher than σ^2_η
- Could be edge and should be preserved.
- Return value close to $g(x, y)$.

(c) If $\sigma^2_L = \sigma^2_\eta$
- when the local area has similar properties with the overall image.
- Return arithmetic mean value of the pixels in S_{xy}.

σ_n^2 should be known apriori

General expression:

$$\hat{f}(x,y) = g(x,y) - \underbrace{\frac{\sigma_\eta^2}{\sigma_L^2}}_{\text{Assumed} < 1}[g(x,y) - m_L]$$

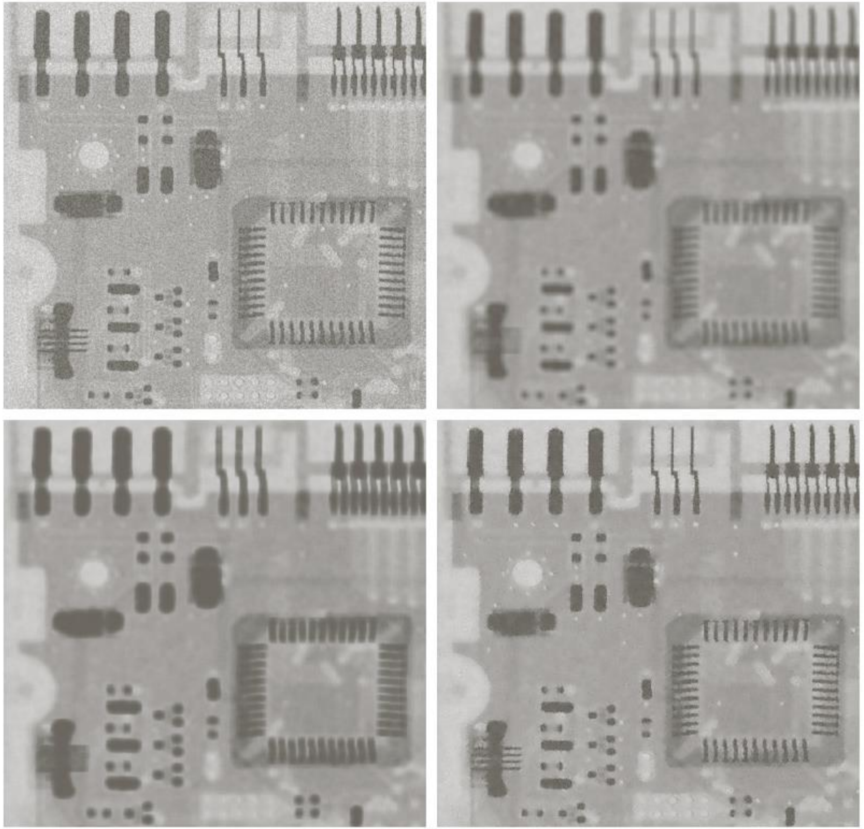
σ_L^2 - Local variance of the local region

m_L - Local Mean

σ_η^2 -Variance of overall noise

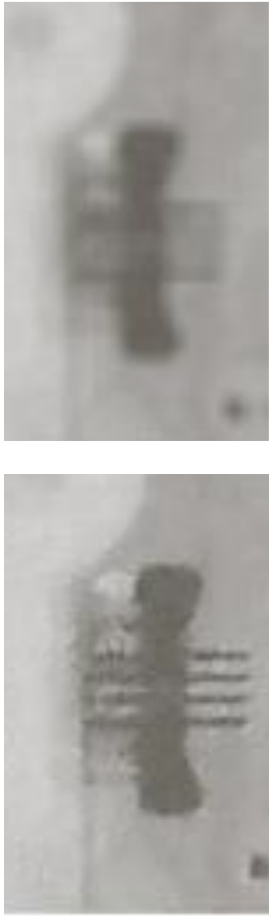
g(x,y)- Pixel value at the position (x,y)

Adaptive mean filtering



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Restoration (in presence of noise only – Periodic noise)

- Band pass/reject

a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

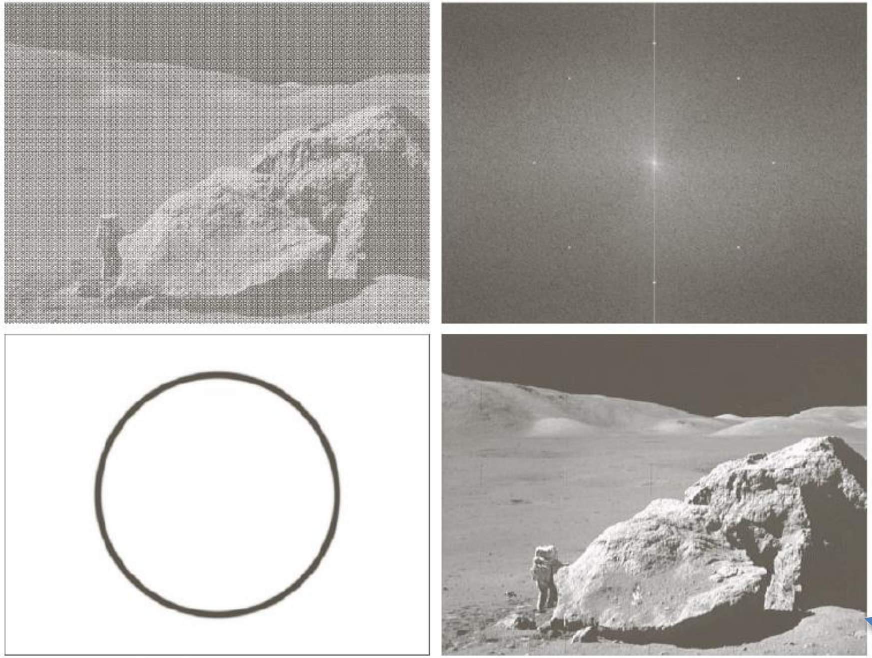
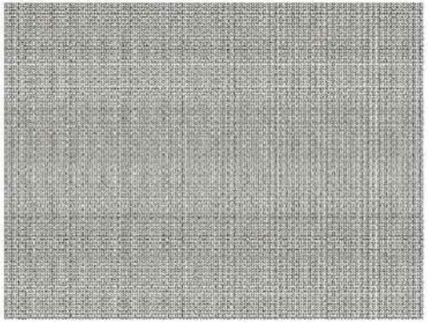


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Very difficult to get result of this quality via spatial domain filtering using small convolutional masks

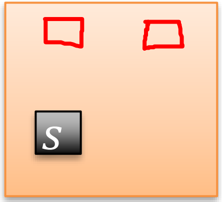
Estimation of degradation function

- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration
  - Mathematical modelling

Blind deconvolution

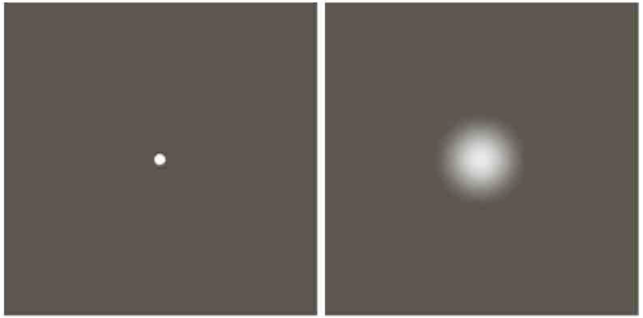
H(u,v)

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



$$H(u,v) = \frac{G(u,v)}{A}$$

Motion Blur

- Exposure
- If amount of light hitting the sensor changes significantly over exposure period → Motion Blur
- Causes (one or more of)
  - Camera motion
  - Subject motion

Estimation by Modeling (uniform motion blurring)

g(x,y) is the blurred image caused due to uniform linear motion between the image and sensor during acquisition

T is the exposure period

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

$$G(u,v) = F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$G(u,v) = F(u,v)H(u,v)$$

Estimation by Modeling (uniform motion blurring)

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

putting, $x_0(t) = at/T$ and $y_0(t) = bt/T$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\, e^{-j\pi(ua+vb)}$$

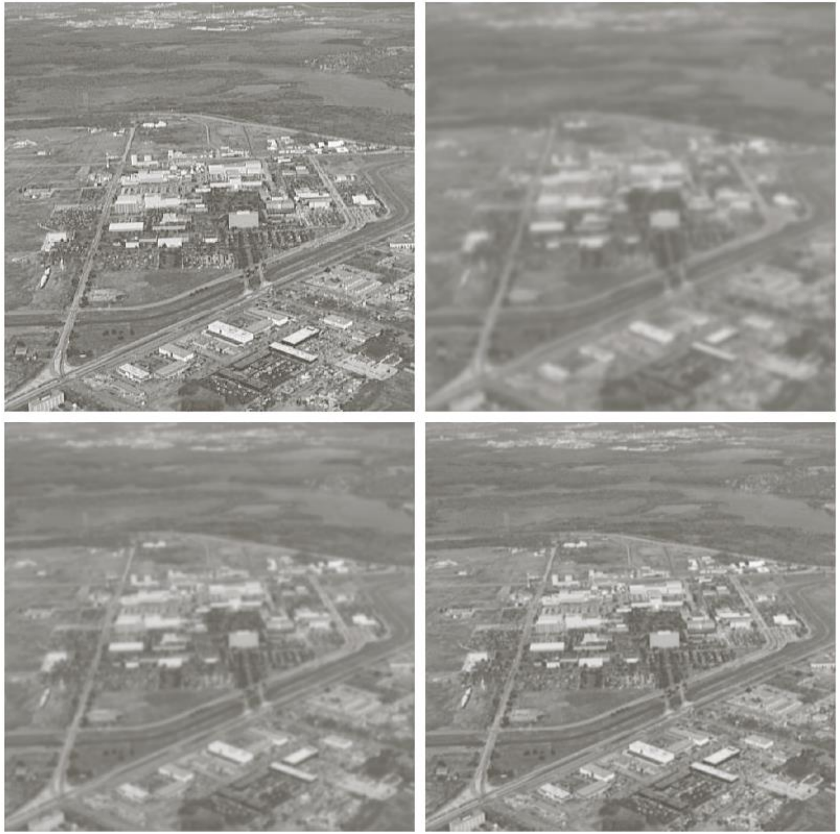
a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Estimation by Modeling (atmospheric turbulence)

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

Model of Image Degradation/Restoration

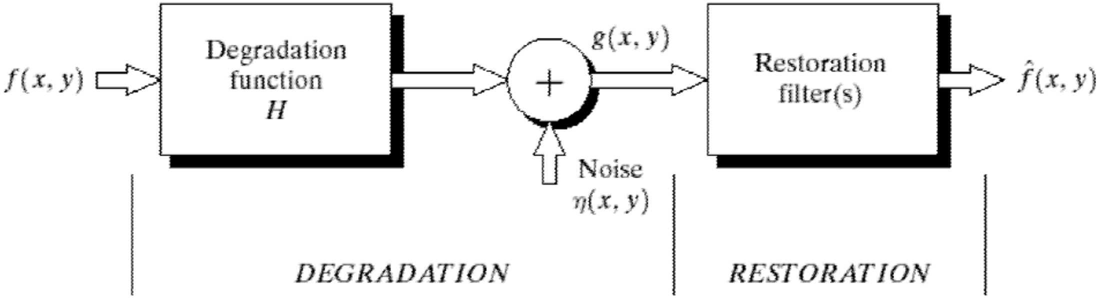


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

Recovering image (in presence of both Noise and degradation)

- Direct inverse filtering: Assuming H is known or obtained using any of the 3 methods:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \qquad \Rightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Even if we know the degradation function we cannot recover the un-degraded image!!

Two problems:

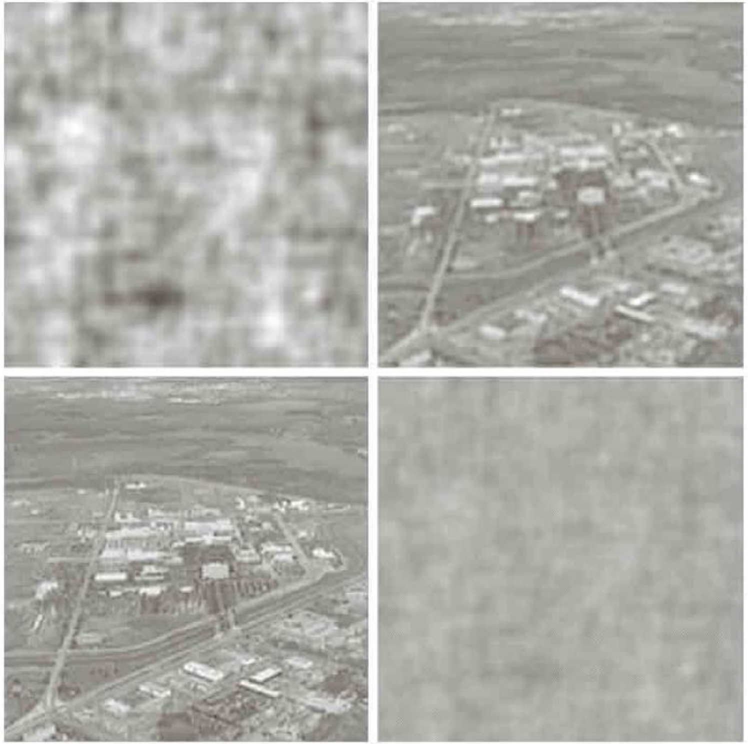
1. N(u,v) is a random function whose fourier transform is not known
2. If degradation has small values → N(u,v)/H(u,v) will dominate

Recovering image (in presence of both Noise and degradation)

Degraded Image
(with known model)

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



No explicit provision for handling noise!

References

- http://www.robots.ox.ac.uk/~az/lectures/ia/lect3.pdf
- https://www.ece.iastate.edu/~namrata/EE528_Spring07/ImageRestoration1.pdf
- http://www.ee.columbia.edu/~xlx/ee4830/notes/lec7.pdf
- DIP Ch. 5 (G&W)