Written assignment

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Q) Given

$$\text{transition probability} = A = \begin{array}{c|ccc} & u_1 & u_2 & u_3 \\ u_1 & 0.1 & 0.4 & 0.5 \\ u_2 & 0.6 & 0.2 & 0.2 \\ u_3 & 0.3 & 0.4 & 0.3 \end{array}$$

$$\text{emission probability} = B = \begin{array}{c|ccc} & R & G & B \\ u_1 & 0.3 & 0.5 & 0.2 \\ u_2 & 0.1 & 0.4 & 0.5 \\ u_3 & 0.6 & 0.1 & 0.3 \end{array}$$

Initial probabilities $= \pi_1 = \pi_2 = \pi_3 = 0.33$

A1) $P(RRGG/\lambda)$

forward method:

i) initialization.

$$\alpha_1(u_1) = \pi_{u_1} b_{u_1}('R') = 0.33 \times 0.3$$
$$= 0.099$$

$$\alpha_1(u_2) = \pi_{u_2} b_{u_2}('R') = 0.33 \times 0.1 = 0.033$$

$$\alpha_1(u_3) = \pi_{u_3} b_{u_3}('R') = 0.33 \times 0.6 = 0.198$$

ii) Induction.

$$\alpha_{t+1}(u_j) = \left[\sum_{i=1}^{N} \alpha_t(u_i)\, a_{ij}\right] b_{u_j}(O_{t+1})$$

$$\alpha_2(u_1) = [\alpha_1(u_1)a_{11} + \alpha_1(u_2)a_{21} + \alpha_1(u_3)a_{31}] \times b_{u_1}('R')$$

$$= [0.099 \times 0.1 + 0.033 \times 0.6 + 0.198 \times 0.3] \cdot 0.3$$

$$= [0.0099 + 0.0198 + 0.0594] \times 0.3$$

$$= 0.02673$$

similarly,

$\alpha_2(u_2) = 0.01254$

$\alpha_3(u_3) = 0.0693$, and

$\alpha_3(u_1) = 0.0154935$	$\alpha_4(u_1) = 0.00623$
$\alpha_3(u_2) = 0.016368$	$\alpha_4(u_2) = 0.00438$
$\alpha_3(u_3) = 0.0036663$	$\alpha_4(u_3) = 0.00121$

iii) Termination

$$P(O|\lambda) = P('RRGG'|\lambda)$$

$$= \sum_{i=1}^{N} \alpha_4(u_i)$$

$$= \alpha_4(u_1) + \alpha_4(u_2) + \alpha_4(u_3)$$

$$= 0.00623 + 0.00437 + 0.00121$$

$$= \boxed{0.011822}$$

A2) given the sequence, find the least state sequence.

using viterbi algorithm,

1) Initialization.

$$\delta_1(u_i) = \pi_{u_i} b_{u_i}(O_1) \quad 1 \leq i \leq N$$

$$\psi_1(u_i) = 0$$

((cont)

$\delta_1(u_1) = \pi_{u_1} \times b_{u_1}('R')$

$= 0.33 \times 0.3$

$= 0.099.$

Similarly,

$\delta_1(u_2) = 0.33 \times 0.1 = 0.033$

$\delta_1(u_3) = 0.33 \times 0.6 = 0.198.$

$\psi_1(u_1) = \psi_1(u_2) = \psi_1(u_3) = 0$

2) Recursion

$$\delta_t(u_j) = \max_{1 \le i \le N} [\delta_{t-1}(u_i)\, a_{ij}]\, b_{a_j}(O_t)$$

$2 \le t \le T$

$1 \le i \le N.$

$$\psi_t(u_j) = \underset{1 \le i \le N}{\arg\max} (\delta_{t-1}(u_i)\, a_{ij})$$

$\delta_2(u_1) = \max[\delta_1(u_1) \times a_{11} \times b_1('R'),$

$\delta_1(u_2) \times a_{21} \times b_1('R'),$

$\delta_1(u_3) \times a_{31} \times b_1('R')]$

(second 'R' in 'RRG'

$= \max [[0.099 \times 0.1 \times 0.3] \times$
$[0.033 \times 0.6 \times 0.3]$
$0.198 \times 0.3 \times 0.3]$

$= \max [0.00297, 0.00594, 0.01782)$

$= 0.01782.$ $\psi_2(u_1) = 3$

Similarly, computing other values

$\delta_2(u_2) = 0.00792$ $\psi_2(u_2) = 3$

$\delta_2(u_3) = 0.03564.$ $\psi_2(u_3) = 3$

$\delta_3(u_1) = 0.005346$

$\delta_3(u_2) = 0.0057024$

$\delta_3(u_3) = 0.0010692$

$\delta_4(u_1) = 0.001710$

$\delta_4(u_2) = 0.000855$

$\delta_4(u_3) = 0.000267$

$\psi_3(u_1) = 3$

$\psi_3(u_2) = 3$

$\psi_3(u_3) = 3$

$\psi_4(u_1) = 2$

$\psi_4(u_2) = 1$

$\psi_4(u_3) = 1$

3) Termination

$$p^* = \max_{1 \le i \le N} [\delta_T(u_i)]$$

$$= \max[\delta_4(u_1), \delta_4(u_2), \delta_4(u_3)]$$

$$= \max[0.00171072, 0.000855, 0.000267]$$

$$= 0.00171072.$$

$$\therefore \boxed{p^* = 0.00171072}$$

$$q_4^* = \operatorname{argmax}_{1 \le i \le N} [\delta_T(u_i)]$$

$$= 1 \qquad \boxed{q_4^* = 1}$$

path sequence =

$$q_t^* = \psi_{t+1}(q_{t+1}^*)$$

$$q_3^* = \psi_4(q_4^*)$$

$$= \psi_4[1]$$

$$= 2$$

Similarly,

$$q_2^* = \psi_3(q_3^*) = 3$$

$$q_1^* = \psi_2[q_2^*] = 3$$

$\therefore$ best sequence $= q_1^* q_2^* \cdots q_T^*$

$= q_1^* q_2^* q_3^* q_4^*$

$\boxed{= 3\,3\,2\,1}$