Model-based design

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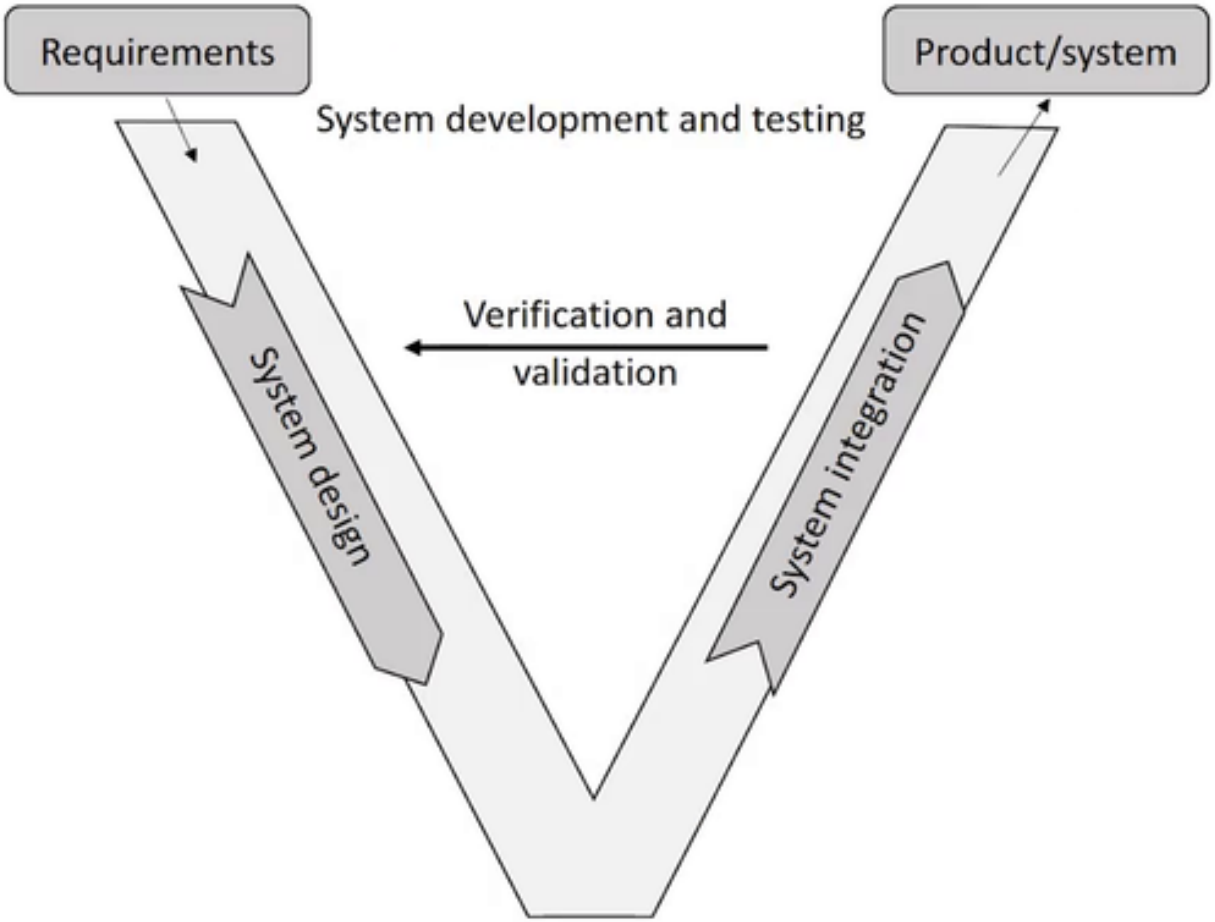
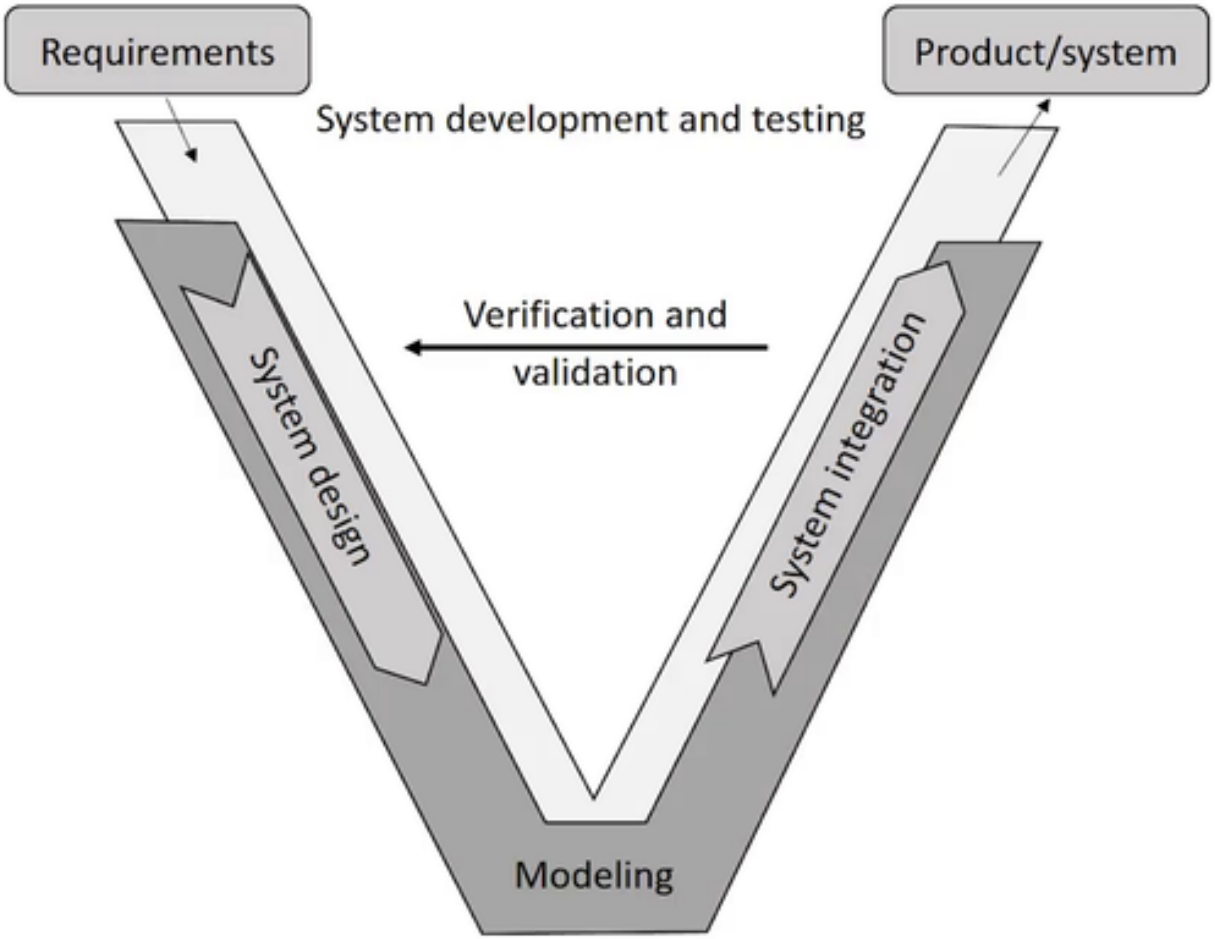
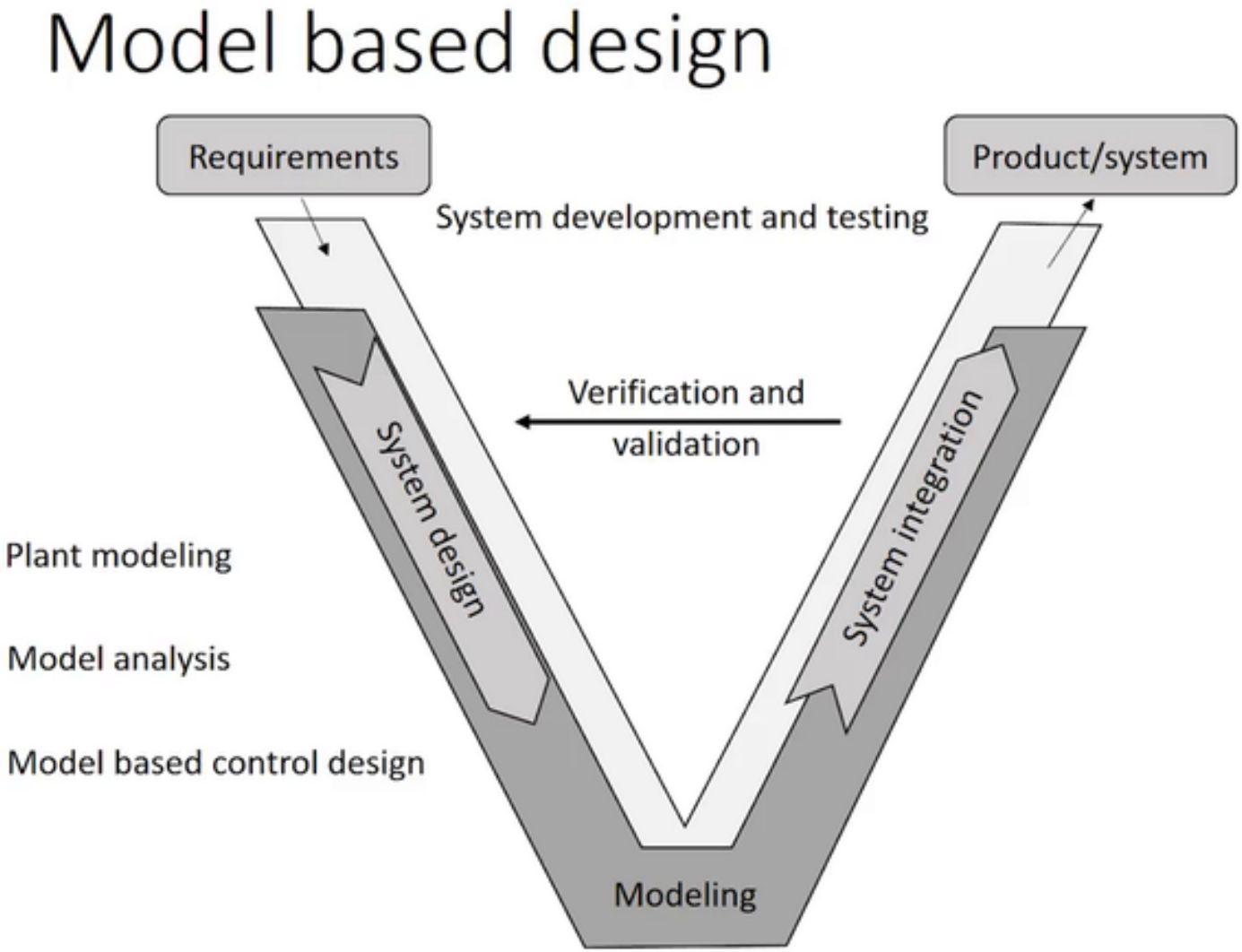
Summary

Model based design

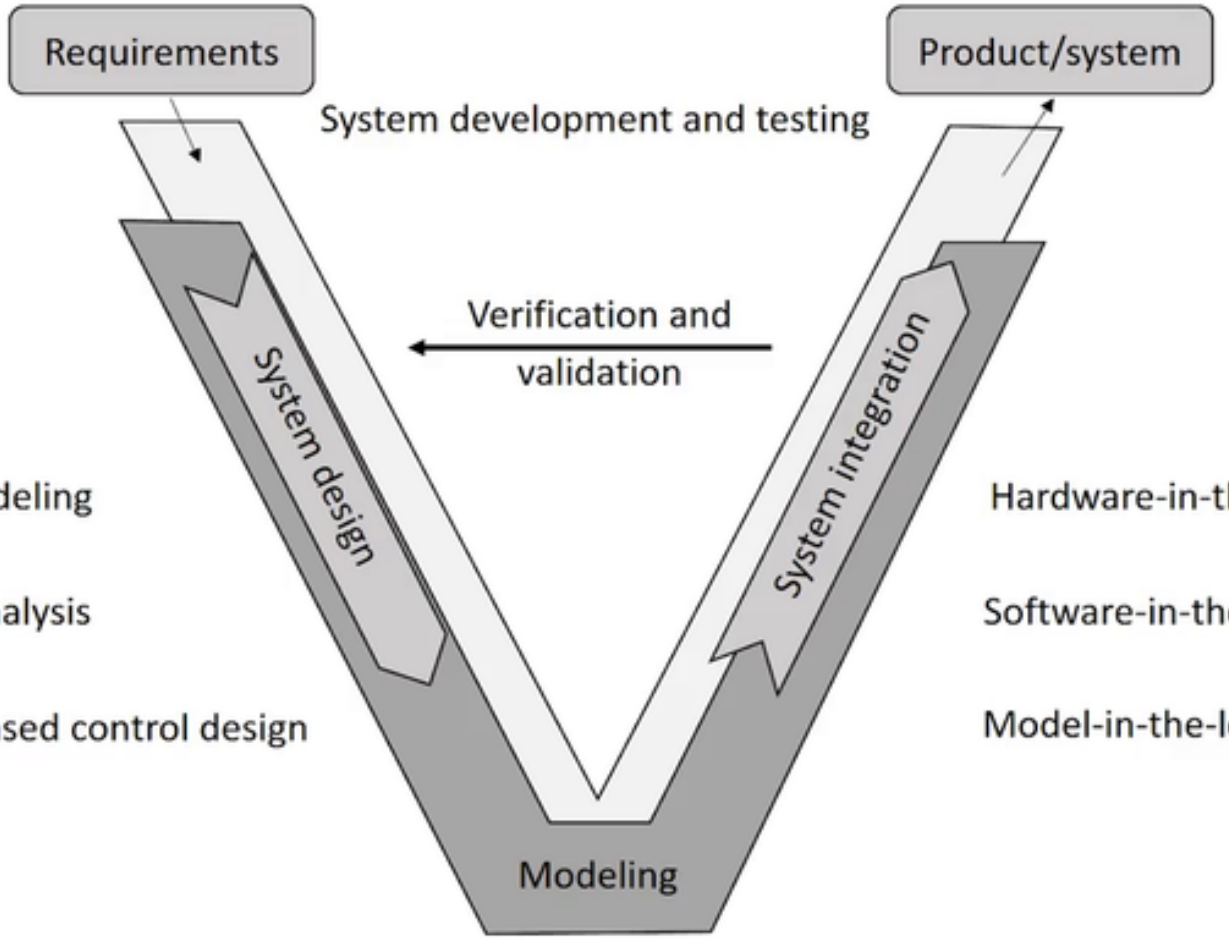


Model based design




Model based design
Requirements
Product/system
System development and testing
Verification and
validation
System design
System integration
Plant modeling
Model analysis
Model based control design
Modeling

Model based design



Plant modeling

Model analysis

Model based control design

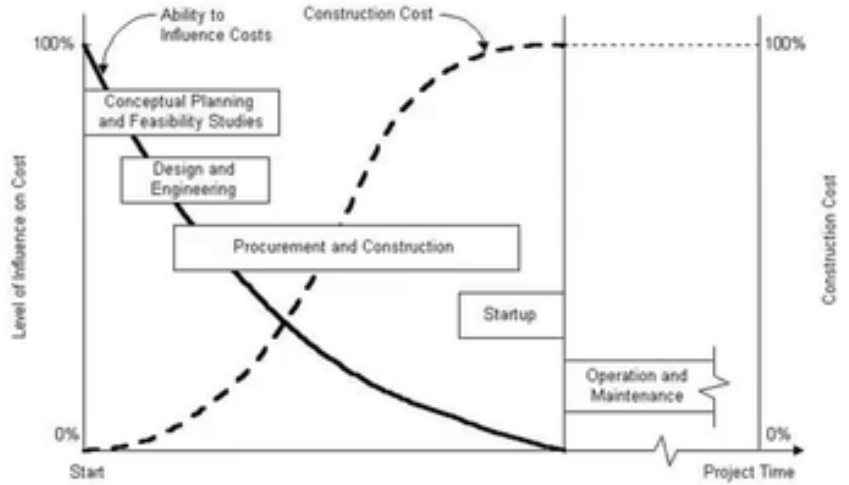
Hardware-in-the-loop (HIL)

Software-in-the-loop (SIL)

Model-in-the-loop (MIL)

Why model based design?

- **The main: productivity, quality**
- To gain better understanding
- Less error-prone
- Less sensitive to changes in technology
- More cost effective
- Up-to-date documentation
- Captures domain knowledge
- Difficult to do trial and error experiments on some things
- Difficult to compare, meaningful validation



https://theconstructor.org/construction/strategic-construction-project-planning-programming/16044/

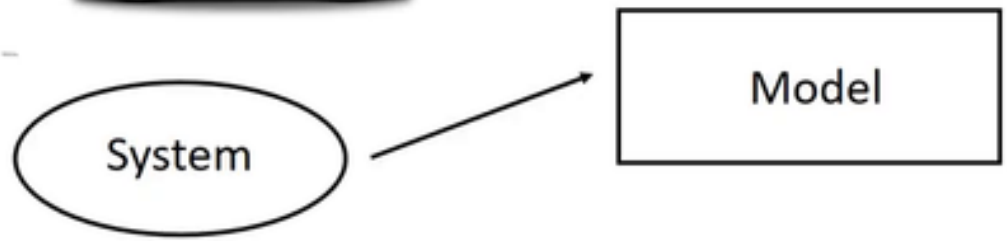
Modeling of physical systems



System

- *A system* is a piece of the reality


Modeling of physical systems



- *A system* is a piece of the reality
- A *model* is a simplified description of the reality

Modeling of physical systems





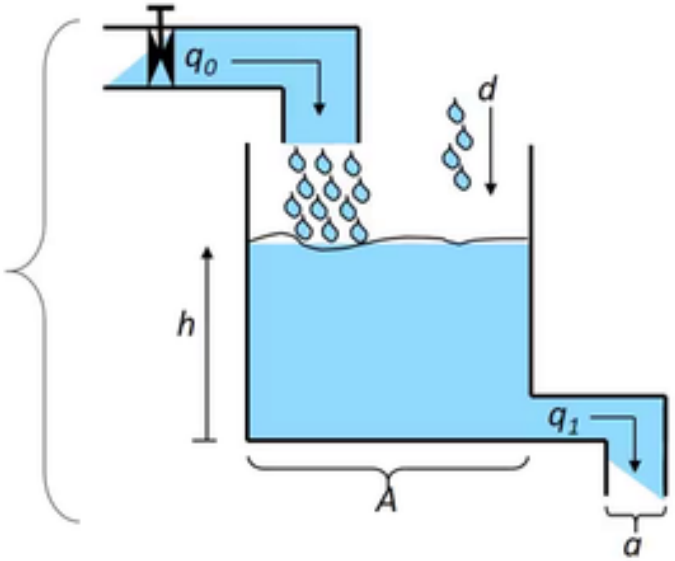
System → Model → Analysis, Simulation, Prediction, Design

- *A system* is a piece of the reality
- A *model* is a simplified description of the reality

Modeling of physical systems

Models can be of different kinds:

- Drawings, schematics



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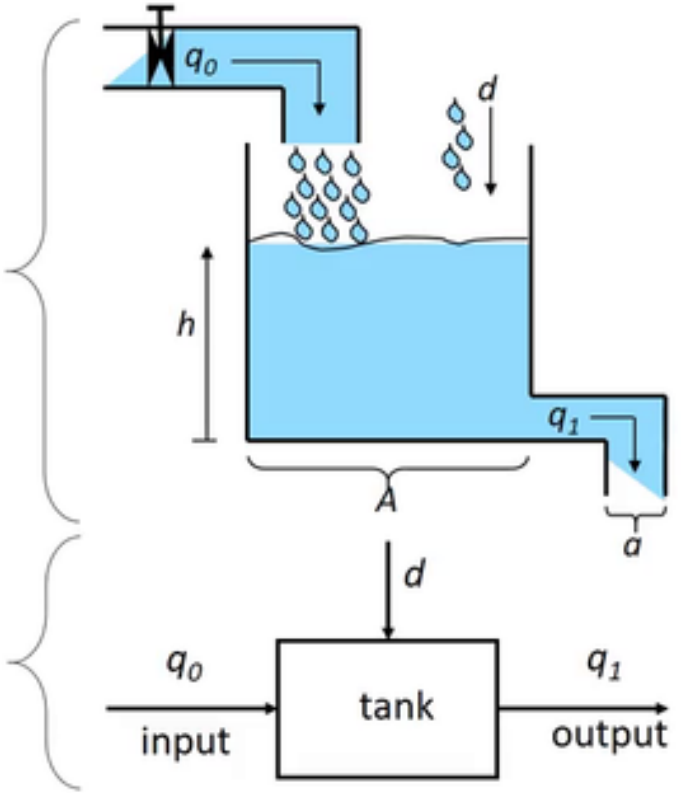
Models can be of different kinds:

- Drawings, schematics
- Block scheme, flow chart

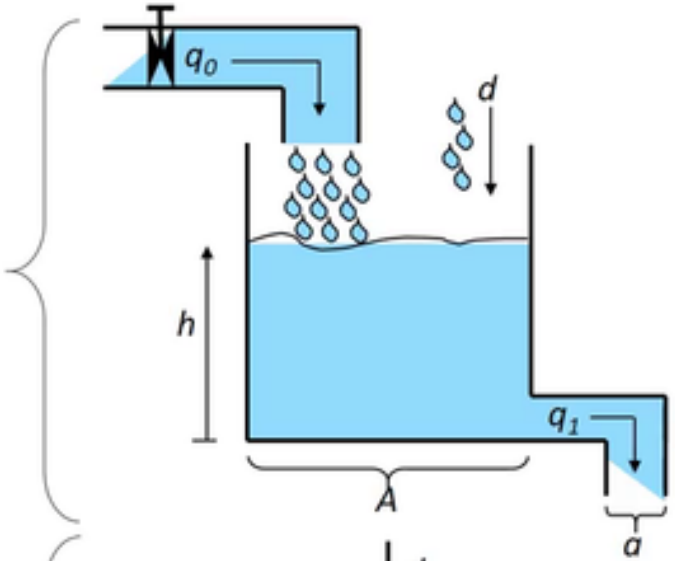


Modeling of physical systems

Models can be of different kinds:

- Drawings, schematics
- Block scheme, flow chart
- Mathematical models

We are interested in mathematical models of dynamical systems



$$A\frac{dh}{dt} = q_0 + d - a\sqrt{2gh}$$

$$q_1 = a\sqrt{2gh}$$

Modeling of dynamic systems

Our focus: mathematical models of physical (dynamic) systems, in particular linear, time invariant models (LTI) in continuous or discrete time:

- Differential- and difference equations

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_2y(t) = b_1u(t)$$

$$y(k) + a_1y(k-1) + a_2y(k-2) = b_1u(k)$$

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$$Y(s) = \frac{b_1}{s^2 + a_1s + a_2}U(s)$$

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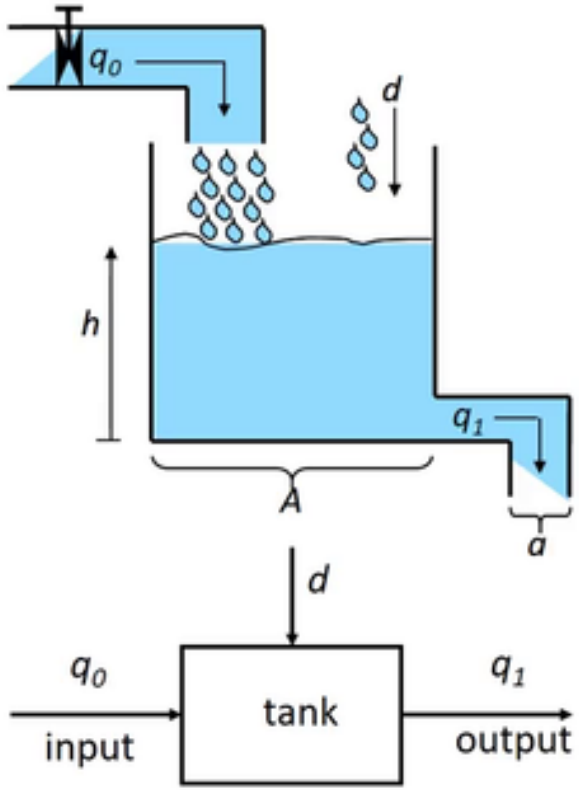
- State-space models

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Why are we doing models?

- Compact representation
- Simpler
- Cost effective
- Safer
- No other alternative, the system might not exist



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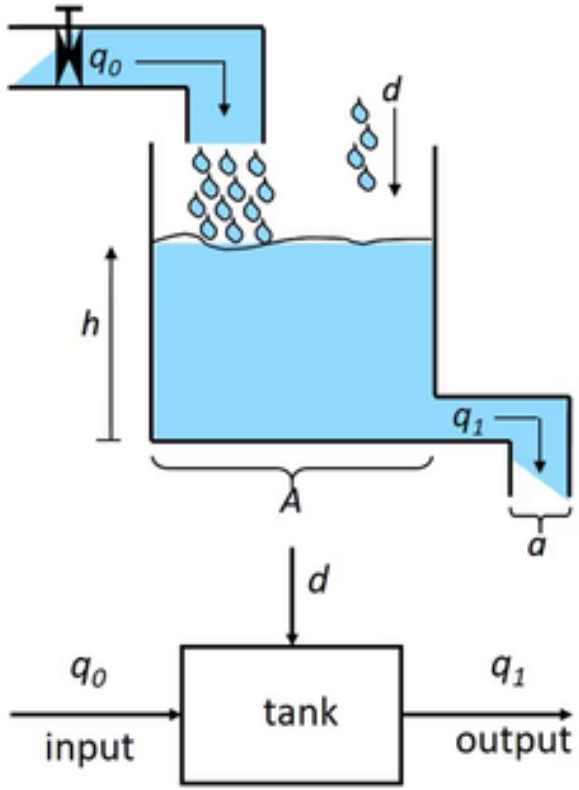
How do we build models?

- Physical relations
- Empirical knowledge
- Data



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Three phase method

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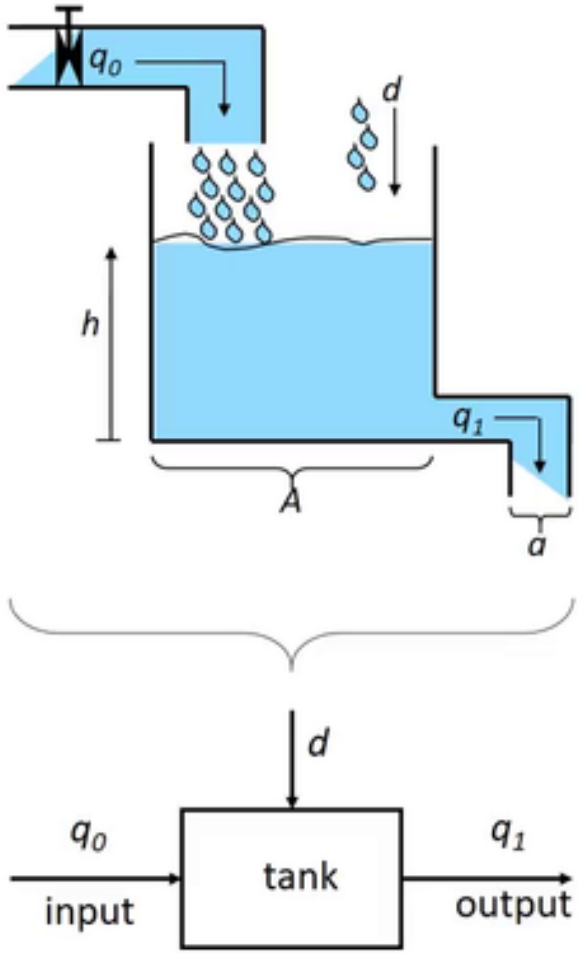

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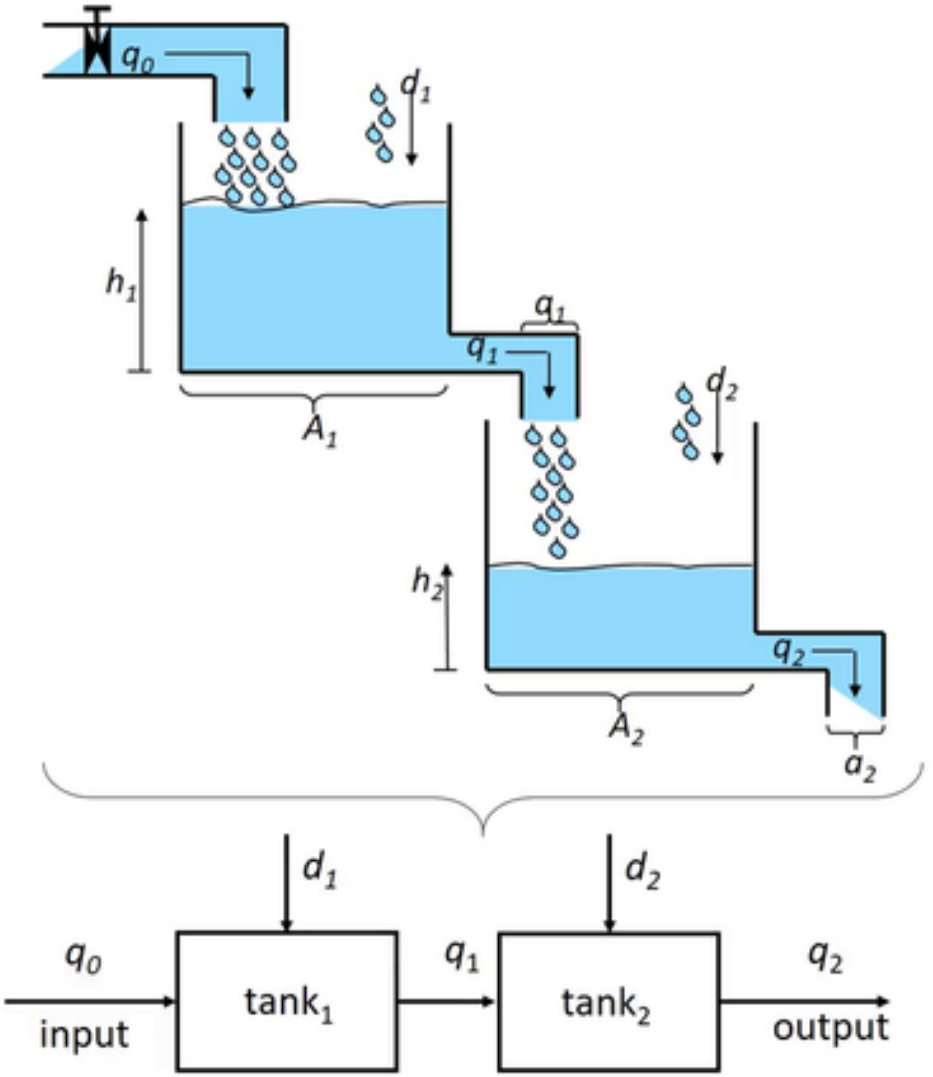

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Conservation laws (balance equation):

- Mass balance [kg]
- Force balance [kgm/s^2 = N] (Newton's law)
- Torque balance [kgm^2/s^2 = Nm] (Newton)
- Voltage balance [V] (Kirchhoff's voltage law)
- Current flows [A] (Kirchhoff's current law)
- Volume flows [m^3/s]
- Energy flows [J/s = W]
- ...

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Constitutive relations (relate variables of different kind):

- Ohm's law: U=RI (voltage and current)
- Ideal gas law: pV=nRT (pres, vol and temp)
- Hooke's law: F=kx (force and distance)
- Air resistance: $F=bv^2$ (force and velocity)
- …

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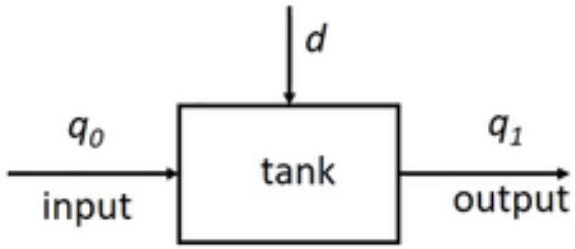

Conservation laws (balance equation):

What is it that changes?

change of volume per time instant, "accumulation"

Three phase method

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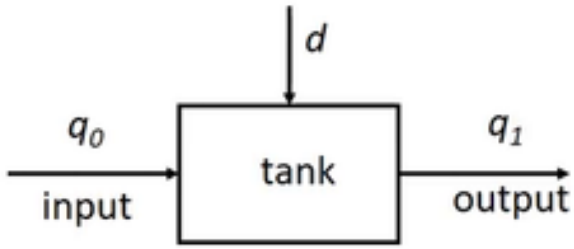

Conservation laws (balance equation):

What is it that changes? What is causing this change?

[change of volume per time instant, "accumulation"]

[supplied, input]

[consumed, output]

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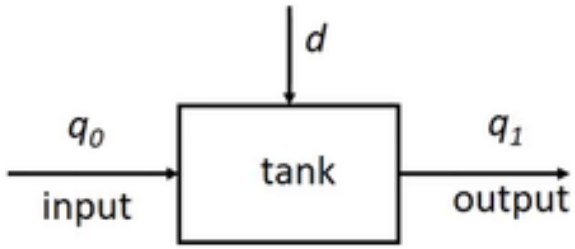

Conservation laws (balance equation):

What is it that changes? What is causing this change?

[change of volume per time instant, "accumulation"] = [supplied, input] - [consumed, output]

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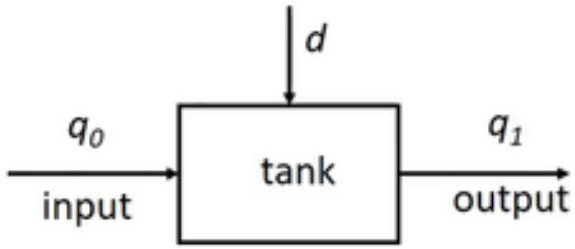

Conservation laws (balance equation):

- Volume flows [m^3/s]

$$\frac{dV}{dt} = q_0 + d - q_1$$

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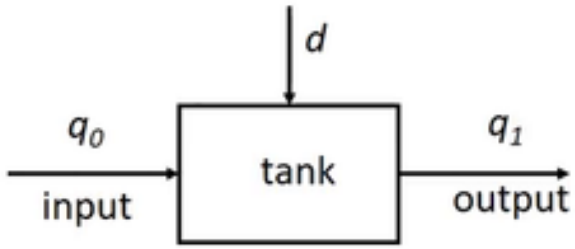

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Constitutive relations:

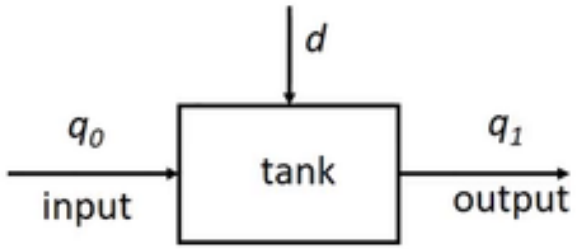
$$V = Ah$$

Volume and level

$$q_1 = a\sqrt{2gh}$$

Bernoullis equation

Three phase method



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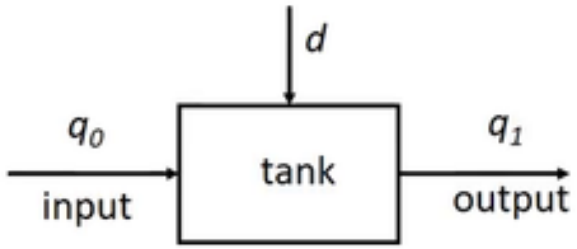

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$$\frac{dV}{dt} = q_0 + d - q_1$$

$$q_1 = a\sqrt{2gh}$$

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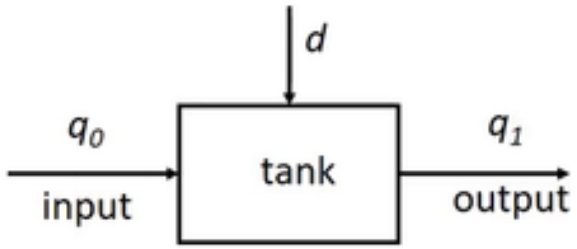
$$\dot{x} = \frac{dx}{dt} = f(x, u, d)$$

$$y = h(x, u, d)$$

Three phase method



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$$\frac{dV}{dt} = q_0 + d - q_1$$

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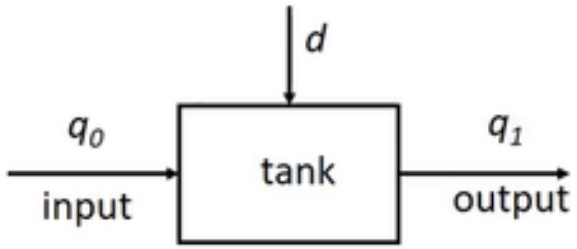
Choose state variables:

- What is changing? V or h

Three phase method



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Choose state variables: V

$$\frac{dV}{dt} =$$

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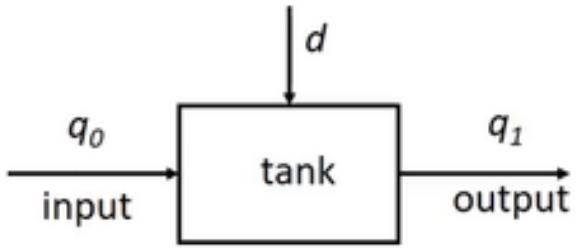

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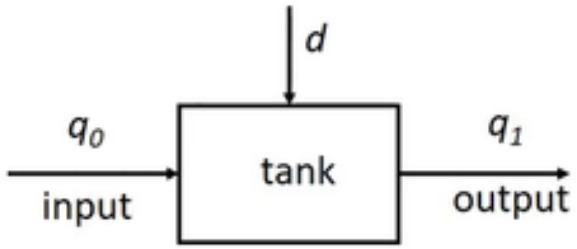

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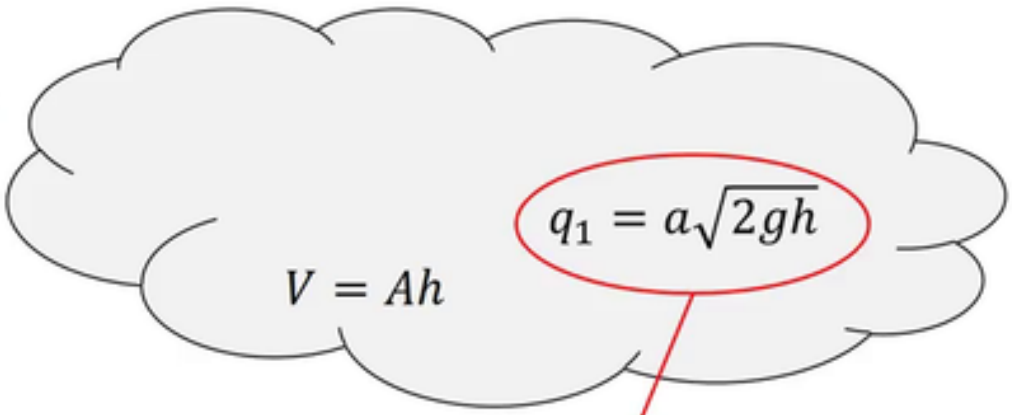

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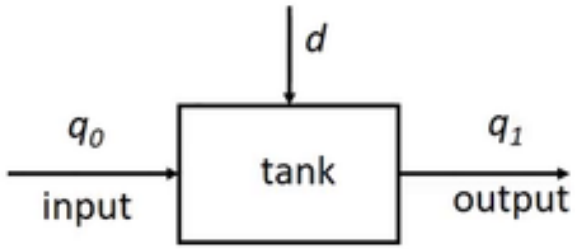

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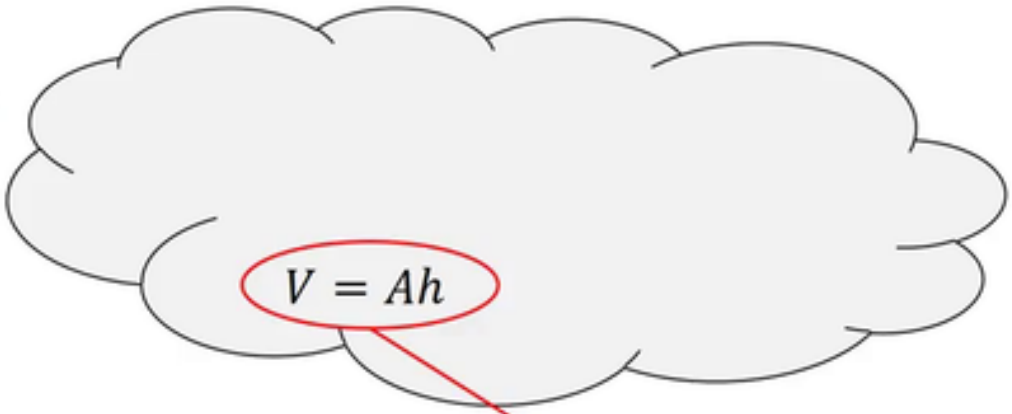

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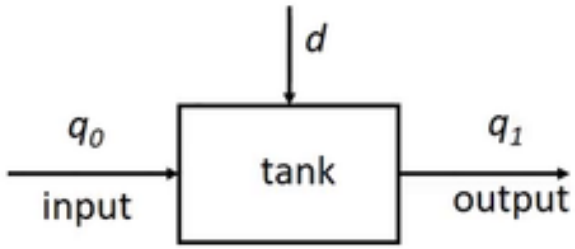

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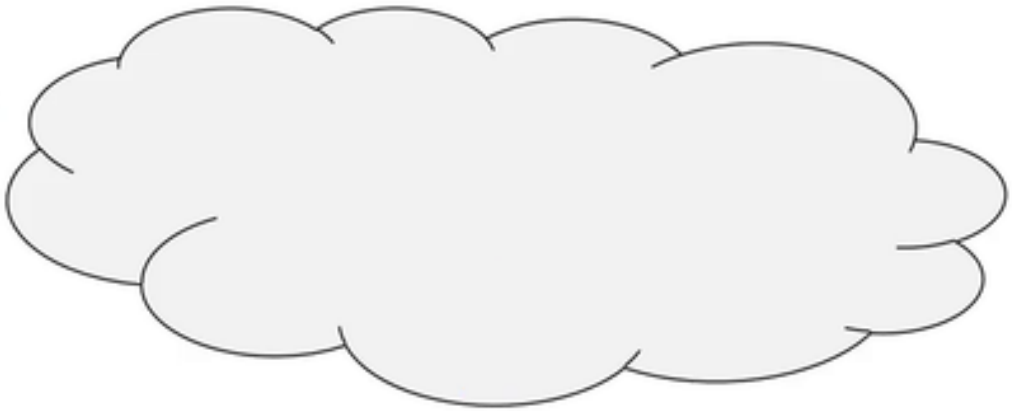
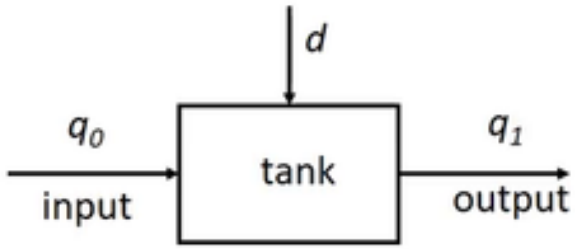


Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2g\,V/A}$$

Three phase method



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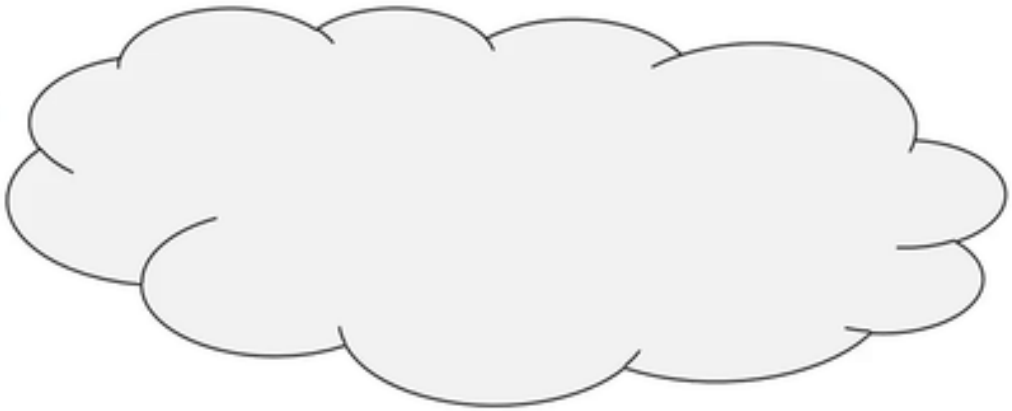

Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2g\,V/A}$$

$$q_1 = a\sqrt{2gh}$$

Three phase method

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$$\begin{cases} \dfrac{dV}{dt} = q_0 + d - a\sqrt{2gV/A} \\ q_1 = a\sqrt{2gV/A} \end{cases}$$

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Variable change: $x = V, u = q_0$ *and* $y = q_1$

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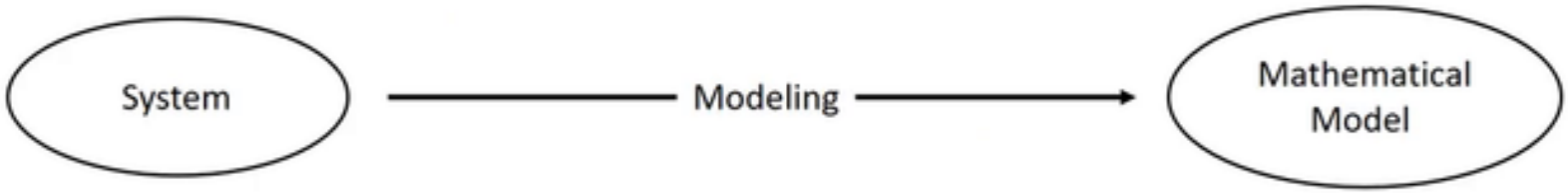
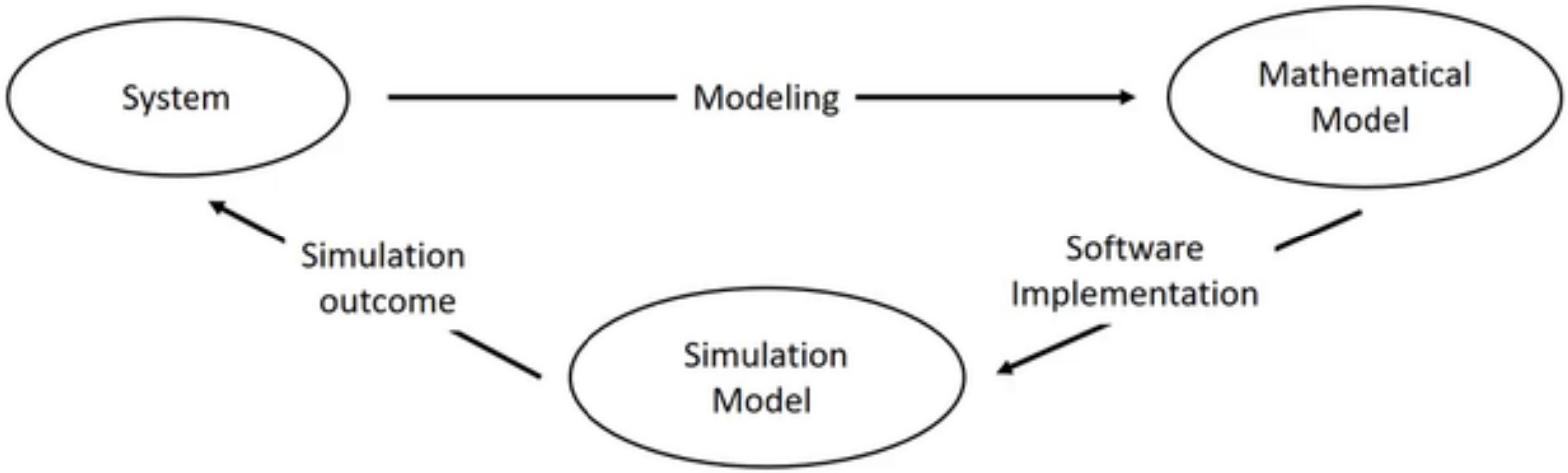
Variable change: $x = V, u = q_0$ *and* $y = q_1$

$$\begin{cases} \dot{x} = \dfrac{dx}{dt} = u + d - a\sqrt{2g\,x/A} \\ y = a\sqrt{2g\,x/A} \end{cases}$$

Model verification and validation

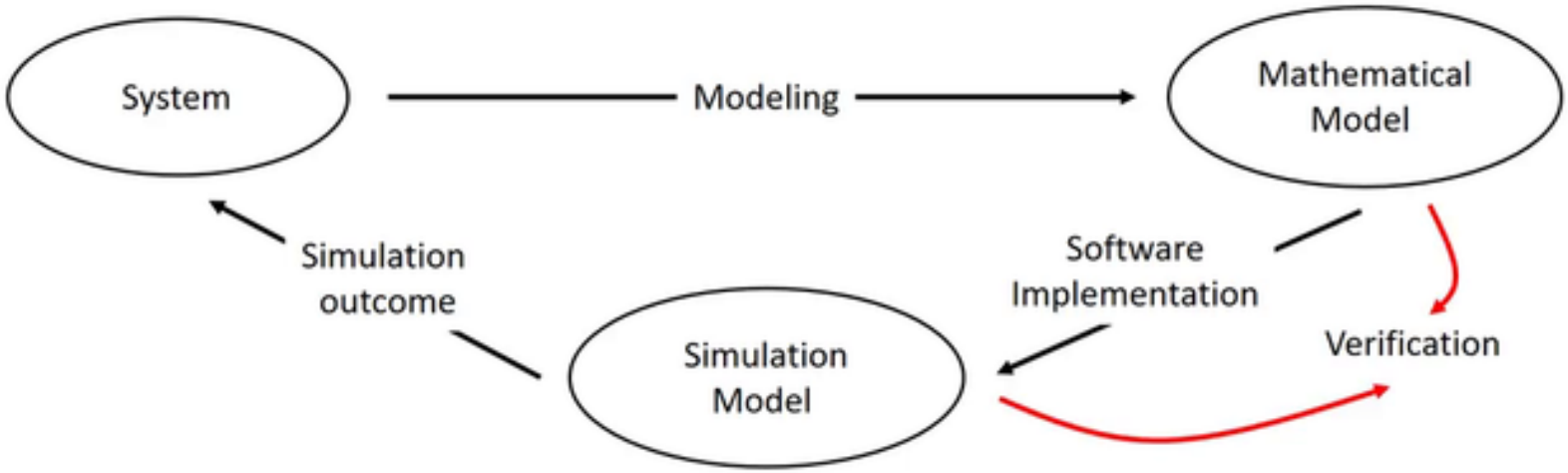


Model verification and validation



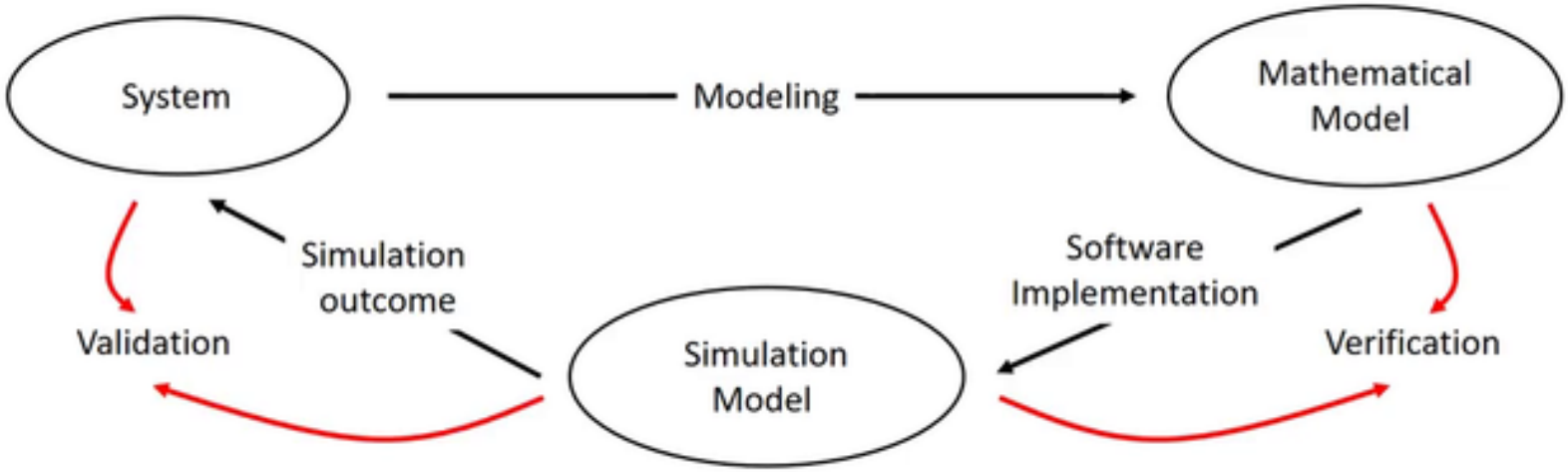
Model verification and validation

- Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.



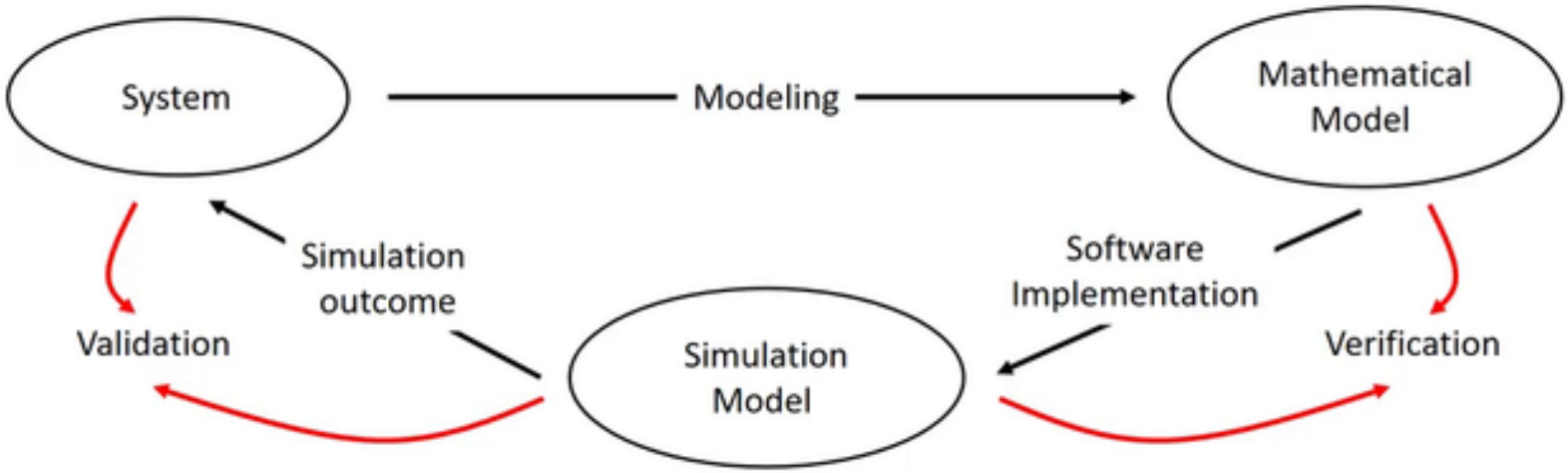
Model verification and validation

- Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.
- Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.



Model verification and validation

- All models have a limited domain of validity.
- Be aware of the model's (lack of) accuracy.



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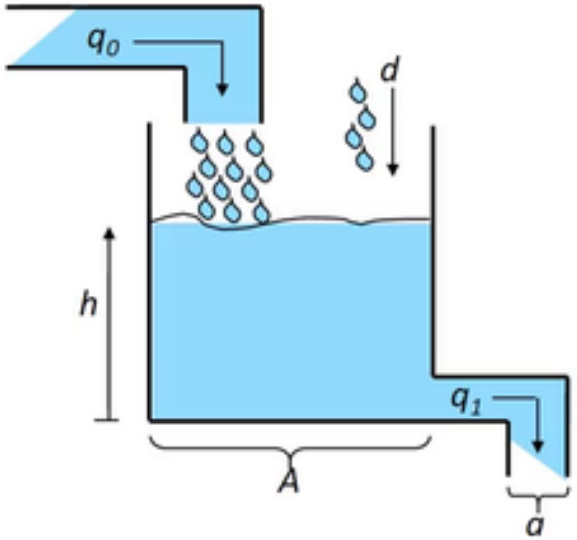

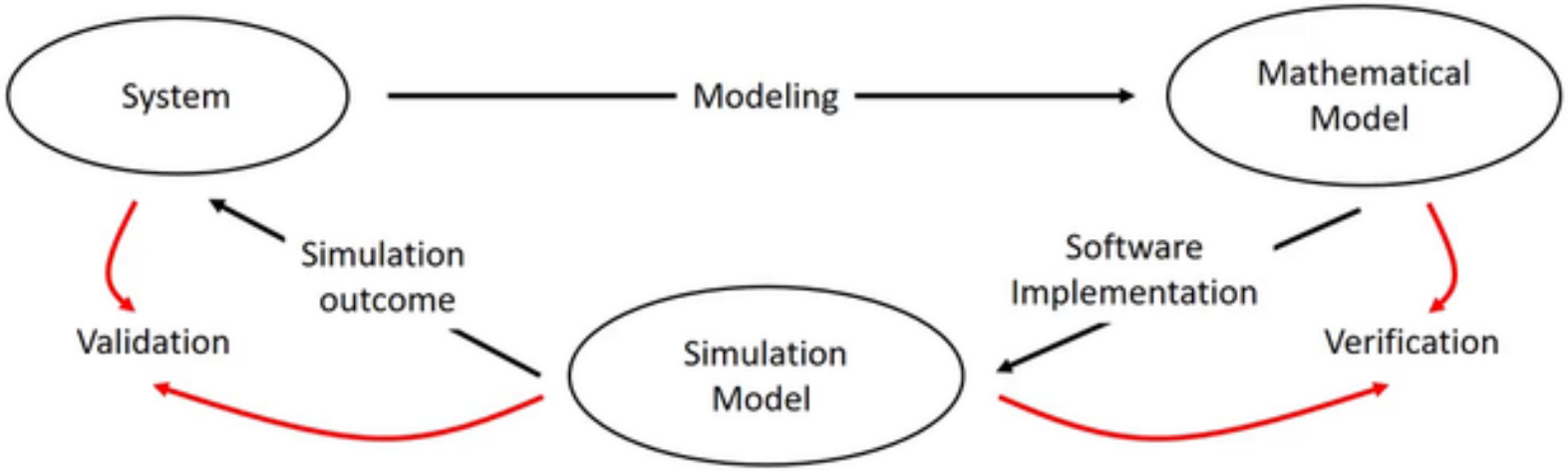

Model Verification and Validation

"All models are wrong but some are useful."

George Box, *Robustness in the strategy of scientific model building*, in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics, Academic Press, pp. 201–236, 1979

"Models and simulations can never replace observations and experiments – but they constitute an important and useful complement."

Lennart Ljung and Torkel Glad, *Modeling and Identification of Dynamic Systems*, Studentliteratur, 2016

Bibliography

- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 3.
- Karmopp, Dean et al. *Systems Dynamics: Modeling, Simulation, and Control of Mechatronic Systems*. Fith Edition. John Wiley & Sons, Inc. 2012. Chapters 1 y 2.1.