Non-Hermitian Photonic Topological Insulators

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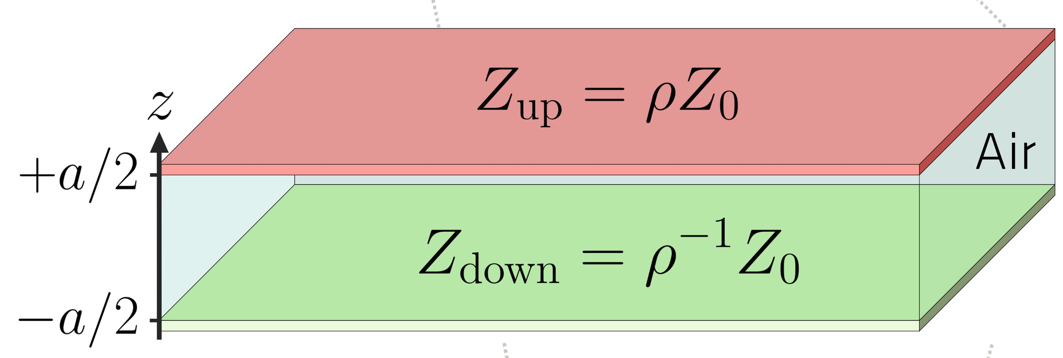
Introduction

Optical structures invariant under the composition of parity ($\mathcal{P}$), time-reversal ($\mathcal{T}$) and duality ($\mathcal{D}$) transformations may host reflectionless propagation channels for light [1]. Here, we show that this scattering anomaly protected by $\mathcal{PTD}$-symmetry has a nontrivial topological origin that is robust against dissipative effects. To illustrate so, we characterize the topology of non-Hermitian $\mathcal{PD}$-symmetric parallel-plate waveguides (PPWs) formed by electric-like and magnetic-like walls. We find that the gap topological charge of these systems is determined by a non-convergent sum of integers. Interestingly, we show that this result manifests itself physically as an infinite number of decoupled light modes propagating along the edge of the lossless PPW. When the impedance of the top plate matches that of air, the energy of the guided modes is resonantly dissipated. We reveal that this effect is related to a topological phase transition and to the formation of exceptional points.

1. PD-Symmetric PPW



- Two parallel plates with surface impedances Z_{up} and Z_{down}.
- $\mathcal{PD}$-invariance requires $\boxed{Z_{\text{up}} Z_{\text{down}} = Z_0^2}$
- Surface impedances determined by a resistivity parameter $\rho \in \mathbb{R}$.
- $\mathcal{PTD}$-symmetric guide [2] recovered when $\rho = 0$: plates become perfect electric (PEC) and magnetic (PMC) conductors.

2. Symmetry-Induced Decoupling

- Maxwell's equations decouple in the spectral domain for $\mathcal{PD}$-symmetric platforms.

Schrödinger-like equations for separate pseudospinors

$$\boxed{\mathcal{H}^{\pm} \mathbf{\Psi}^{\pm} = \omega \mathbf{\Psi}^{\pm}}$$

- Nonlocal pseudospinors given by

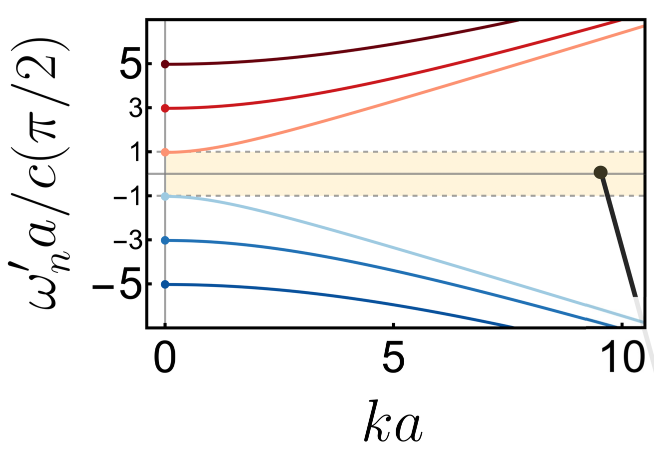
$$\boxed{\mathbf{\Psi}^{\pm}(z) = \boldsymbol{E}(z) \pm \mathcal{P} \cdot \boldsymbol{H}(-z) Z_0}$$

Electric Field and **P-Transformed Magnetic Field** → **in** / **out** $[\pm]$ **of phase**

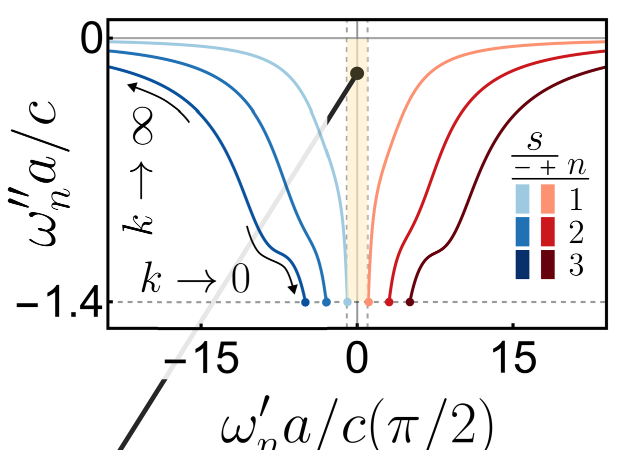
3. Band Structure of Bulk Waves

- Waveguide supports infinite number of modes.
- Frequency bands $\omega_n = \omega_n' + i\omega_n''$ depend on ρ and $k = |\boldsymbol{k}|$ (continuous rotational symmetry).
- Particle-hole symmetry in photonics gives negative $(s = -)$ and positive $(s = +)$ branches.

$\boxed{\rho = 0}$ Real-Valued Bands **Hermitian Physics**



$\boxed{\rho = 0.6}$ Complex-Valued Bands **Non-Hermitian Physics**

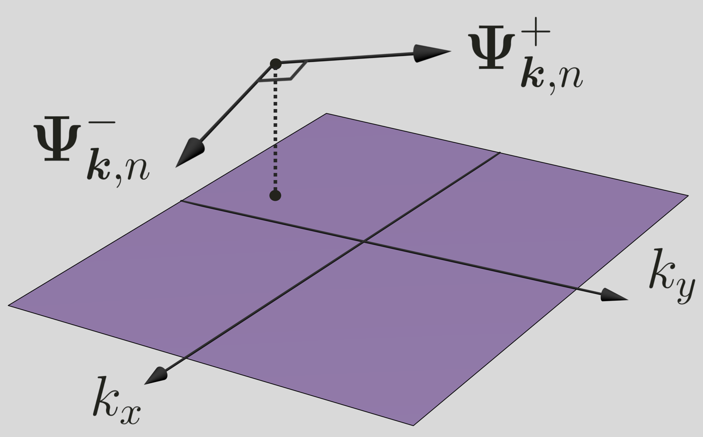


- For ρ not too close to 1:

Bands Separated From Each Other (except when $k = \infty$) + **Single Bandgap Between Positive and Negative Frequencies**

4. Topological Characterization

Momentum Space **Base Manifold** $\mathbb{R}^2$ | Gauges for Decoupled Modes **Typical Fiber** $U(1) \oplus U(1)$



- Unbounded momentum plane can be compactified because Berry curvature decays sufficiently fast when $k \to \infty$ [3].

Pair of Band Chern Numbers (one for each pseudospin sector)

$$\boxed{(\mathcal{C}_n^+, \mathcal{C}_n^-) \in \mathbb{Z} \times \mathbb{Z}}$$

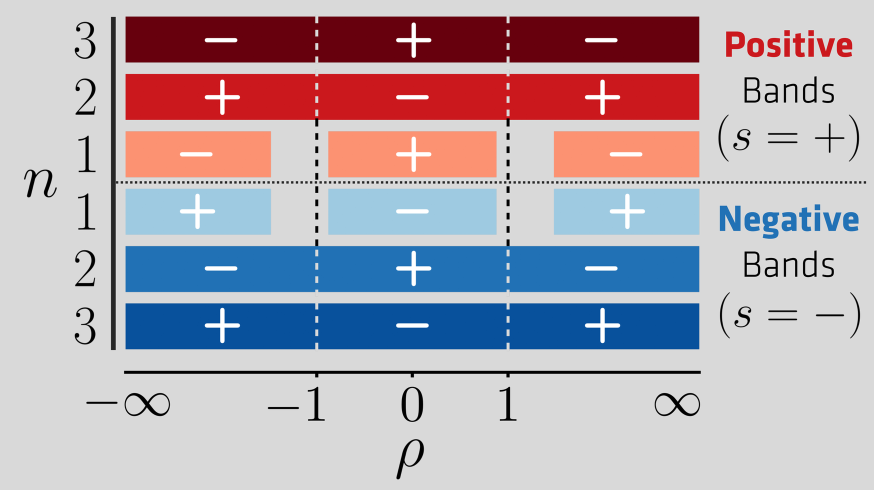
- Lorentz reciprocity implies that light transport is bidirectional and balanced.

Band Chern numbers with opposite sign → $\boxed{\mathcal{C}_n^- = -\mathcal{C}_n^+}$

- Topological band theory of continuous [3] and non-Hermitian [4] systems gives

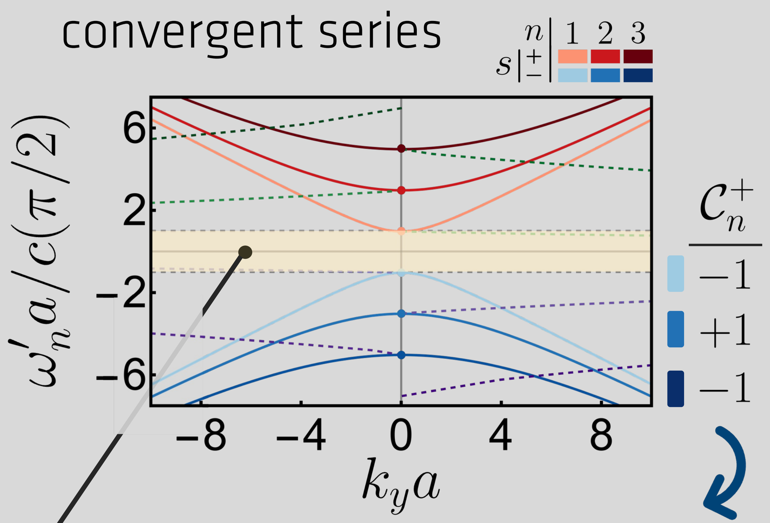
$$\boxed{\mathcal{C}_n^+ = s \times (-1)^{n+1} \text{sgn}(1 - |\rho|)}$$

Invariant alternates between +1 and -1 with n



5. Light Transport along PEC-PMC Waveguide's Edge

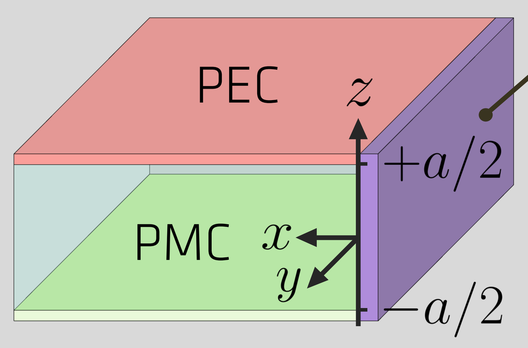
- Topological charge of gap in PEC-PMC waveguide is a non-convergent series



$$\mathcal{C}_{\text{gap}}^+ = \sum_{n \in \mathbb{N}}^{s=-} \mathcal{C}_n^+ = -1 + 1 - 1 + \ldots \Leftrightarrow$$

Infinitely Many Edge Modes with Alternate Propagation Directions

- Guide terminated by wall with surface impedance

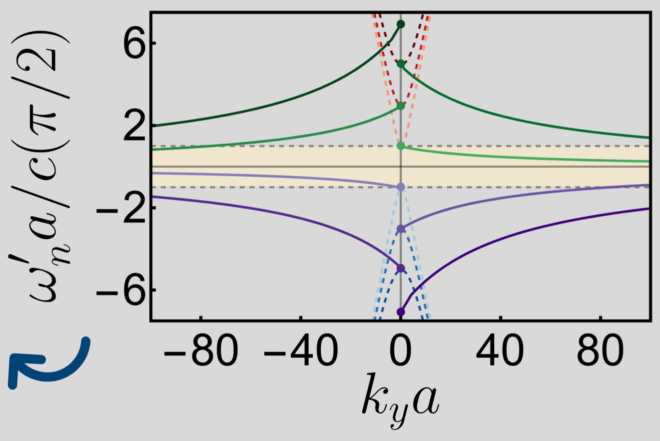


$$\boxed{\boldsymbol{Z} = Z_y \hat{\boldsymbol{y}} \otimes \hat{\boldsymbol{y}} + Z_z \hat{\boldsymbol{z}} \otimes \hat{\boldsymbol{z}}}$$

Reactive wall with $\boxed{Z_y Z_z = Z_0^2}$

$\mathcal{PTD}$-Symmetric Structure

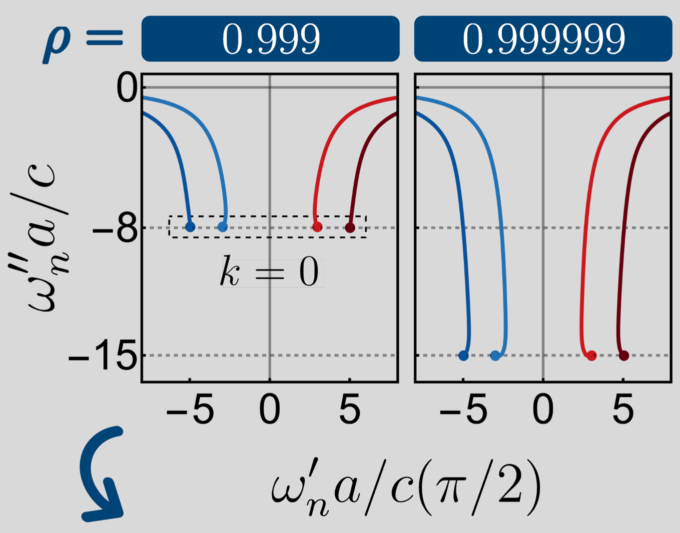
- Gapless edge waves



6. Resonant Absorption

- Near $\rho = 1$, some bands intersect, and all diverge along the imaginary frequency axis.

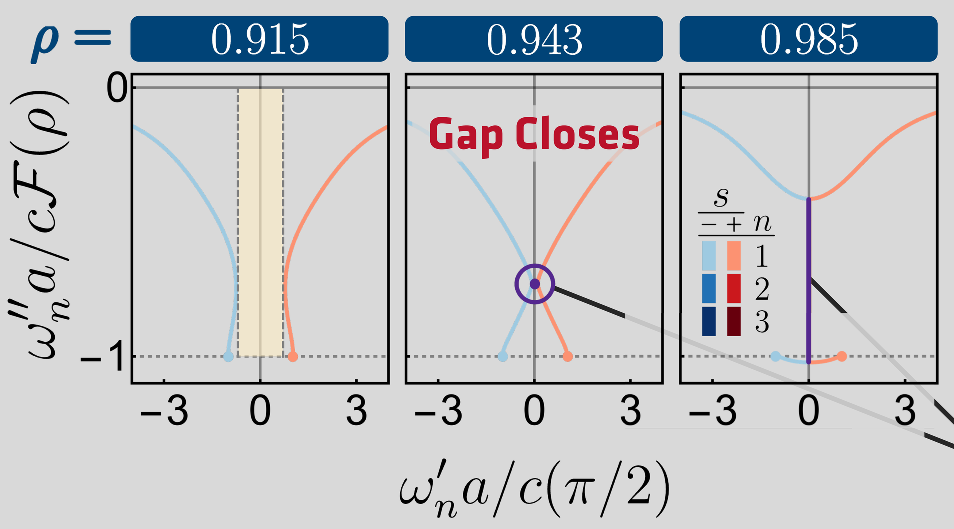
Higher-order Bands



Absorption Rate Grows Logarithmically (top plate impedance matches that of air)

$$\boxed{\omega_n'' \approx ica^{-1} \ln(\rho - 1)} \quad \boxed{k = 0}$$

Lowest-order Bands



Topological Phase Transition (after gap reopens as ρ increases)

$$\boxed{\mathcal{C}_n^+ \to -\mathcal{C}_n^+}$$

Frequencies Coalesce

$$\boxed{\omega_{n=1}^{s=+} = \omega_{n=1}^{s=-}}$$

+

Pseudospinors Align

$$\boxed{\mathbf{\Psi}_{n=1}^{\pm, s=+} \parallel \mathbf{\Psi}_{n=1}^{\pm, s=-}}$$

↓

Exceptional Points

See the evolution of the band structure near the transition

References

1. M. G. Silveirinha, Physical Review B **95** (2017).
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