Non-Hermitian Photonic Topological Insulators

17th International Congress on Artificial Materials for Novel Wave Phenomena

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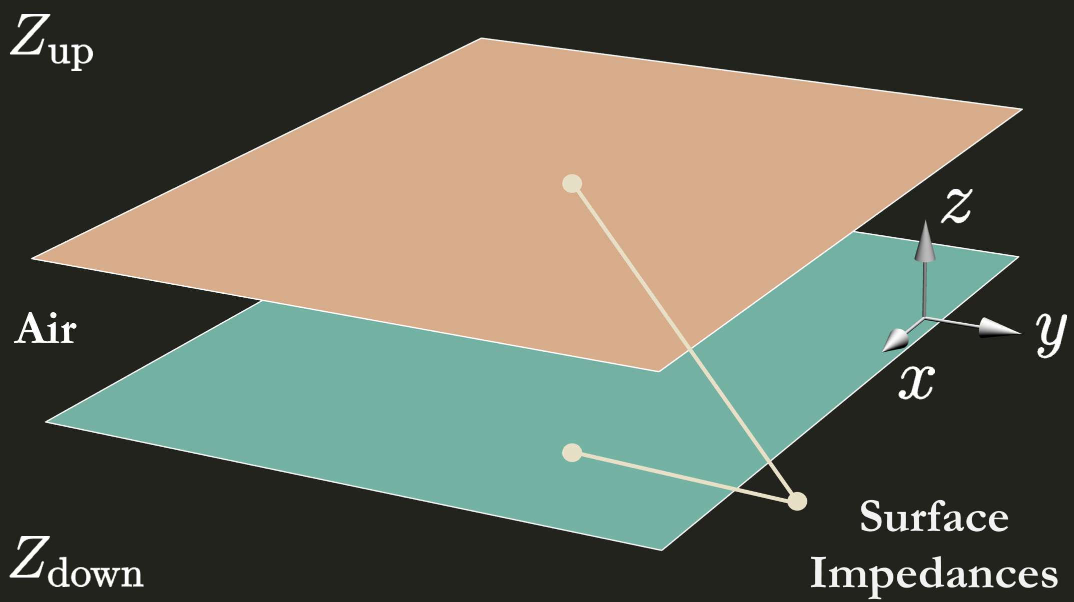
Reciprocal and $\mathcal{PD}$-Symmetric Waveguides



Parity Transformation $\mathcal{P}$

Reflection about xOy Plane

$$(x, y, z) \to (x, y, -z)$$

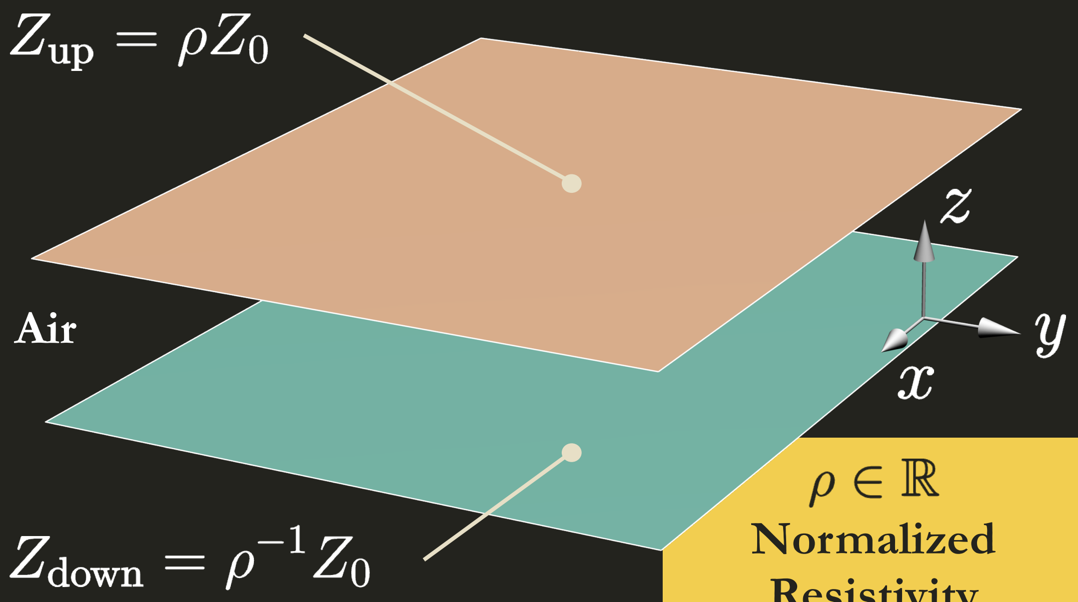
+

Duality Transformation $\mathcal{D}$

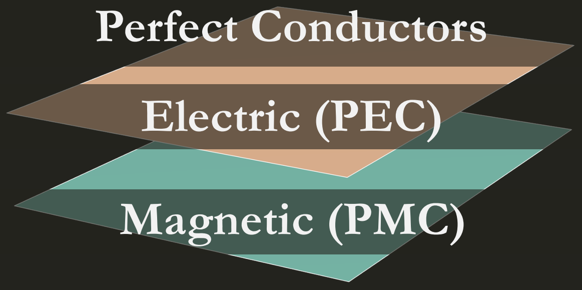
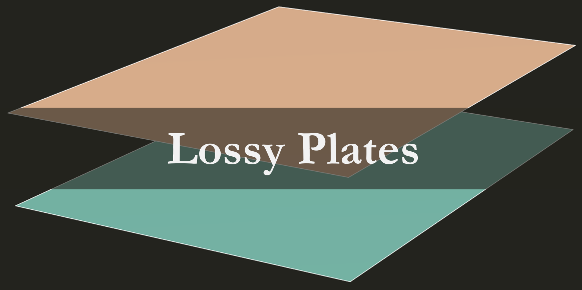
Response Exchange

Permittivity $\epsilon \rightleftarrows \mu$ Permeability

Reciprocal and $\mathcal{PD}$-Symmetric Waveguides



$\rho \in \mathbb{R}$
Normalized Resistivity

$\rho = 0$	$\rho \neq 0$
	
Conservative Hermitian Electrodynamics	Non-Conservative Non-Hermitian Electrodynamics

Reciprocal and $\mathcal{PD}$-Symmetric Waveguides

Symmetry-Induced Decoupling of Maxwell's Equations

$$\pm\frac{ic}{\epsilon(z)}\begin{pmatrix} 0 & \partial_z & \partial_y \\ -\partial_z & 0 & -\partial_x \\ \partial_y & -\partial_x & 0 \end{pmatrix}\cdot\mathbf{\Psi}^{\pm}(-z) = \omega\mathbf{\Psi}^{\pm}(z)$$

$$\mathbf{\Psi}^{\pm}(z) = \begin{pmatrix} E_x(z) \mp Z_0 H_x(-z) \\ E_y(z) \mp Z_0 H_y(-z) \\ E_z(z) \pm Z_0 H_z(-z) \end{pmatrix}$$

Non-Local Pseudospinors → **Electric Field** / **Parity-Transformed Magnetic Field** → **In Phase** $\mathbf{\Psi}^{+}$ | **Out of Phase** $\mathbf{\Psi}^{-}$

$\mathcal{PTD}$ Symmetry-Protected Scattering Anomaly in Optics

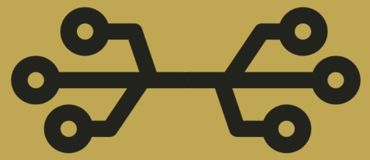
M. G. Silveirinha, Physical Review B 95 (2017).

$\mathcal{PD}$-Symmetry

$+$ Invariance under Time Reversal $\mathcal{T}$

Reflectionless Light Channels

System 1 System 2



N Modes Towards Junction

Antisymmetric Scattering Matrix

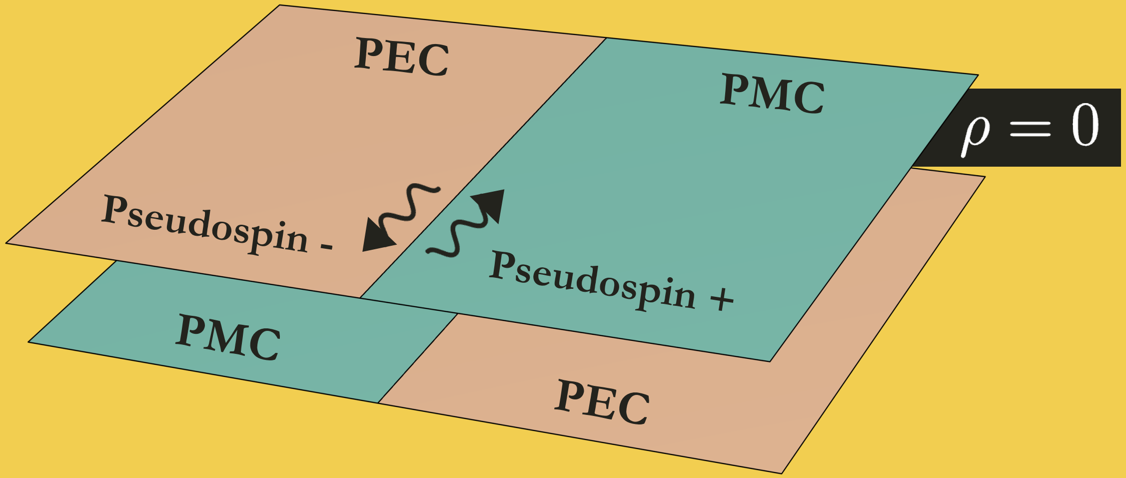
$\boldsymbol{S} = -\boldsymbol{S}^{\mathsf{T}}$

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{pmatrix}$$

$$\det \boldsymbol{S}_{11} = (-1)^N \det \boldsymbol{S}_{11} = 0 \text{ (odd } N\text{)}$$

$\mathcal{PTD}$ Symmetry-Protected Scattering Anomaly in Optics

W.-J. Chen, S.-J. Jiang, X.-D. Chen, B. Zhu, L. Zhou, J.-W. Dong, and C. T. Chan, Nature Communications 5 (2014).



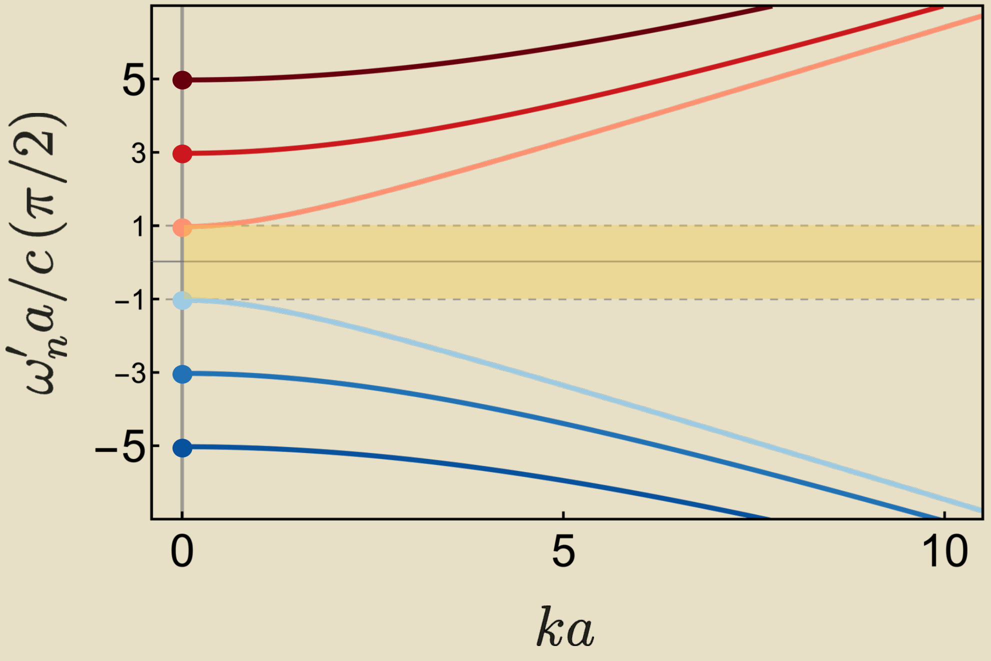
2 Chiral Edge Waves

Transport Protected by Pseudospin-Decoupling

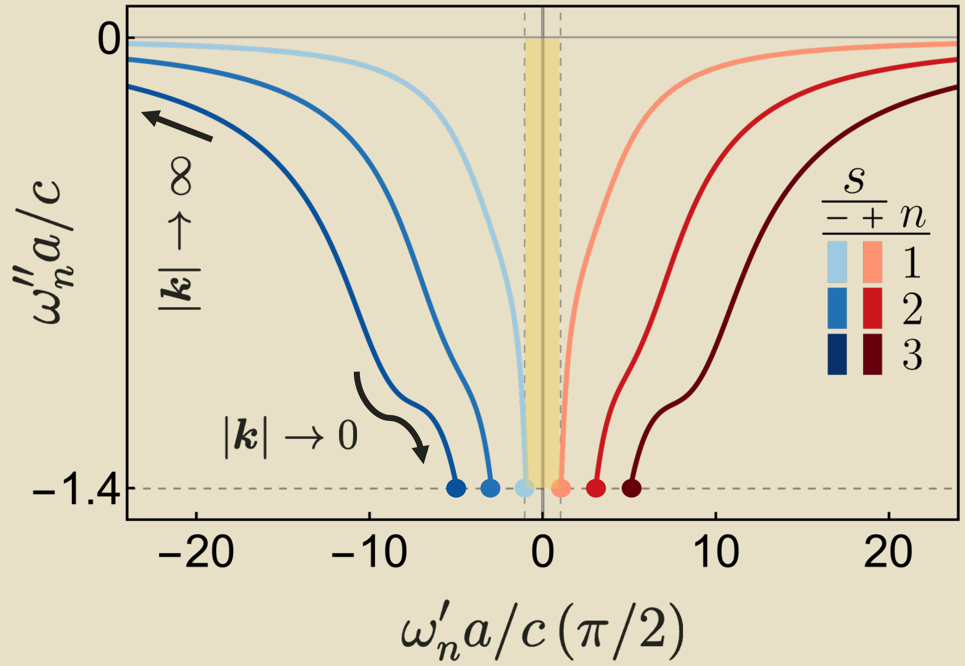
Dispersion Diagram of Bulk Waves

Waveguide Supports Infinite Number of Modes

$\rho = 0$



$\rho = 0.6$



Single Bandgap Between Positive and Negative Bands

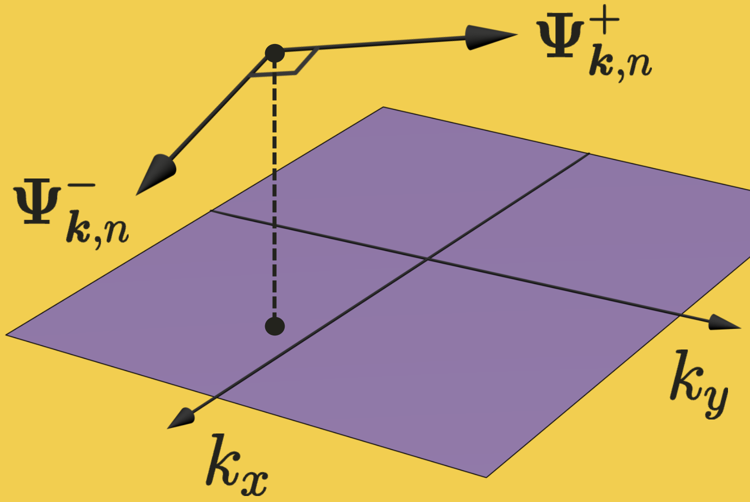
+

Separated Bands $\omega_n \neq \omega_m,\ \forall k \in [0, \infty[$

Suitable for Topological Characterization

Topological Characterization

Decoupled Pseudospinors $\Psi^{\pm}_{\boldsymbol{k},n} \in \mathbb{C}^3$



Wavevector Domain | Euclidean Plane

Base Manifold
$\mathbb{R}^2$

Typical Fiber
$U(1) \oplus U(1)$

⇩

Pair of Band Chern Numbers
$(\mathcal{C}^+_n, \mathcal{C}^-_n) \in \mathbb{Z} \times \mathbb{Z}$

Electromagnetic Reciprocity
$\mathcal{C}^-_n = -\mathcal{C}^+_n$

⬇

$\mathbb{Z}$-Valued Topological Invariant
$\mathcal{C}^+_n$

Topological Characterization

H. Shen, B. Zhen, and L. Fu, Physical Review Letters <u>120</u> (2018).

M. G. Silveirinha, Physical Review B <u>92</u> (2015).

Topological Band Theory of Non-Hermitian Systems

Berry Potential

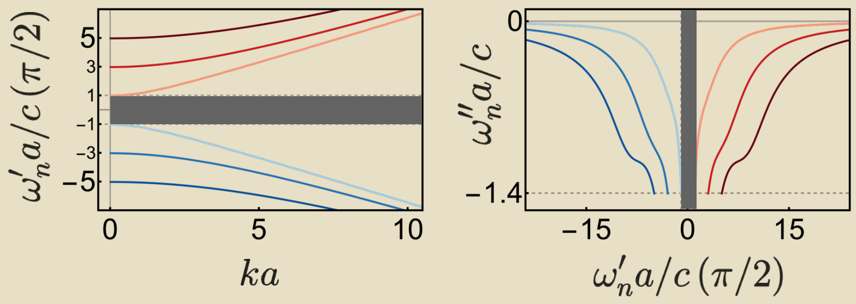
$$\mathcal{A}^{\pm}_{\boldsymbol{k},n} = i\left\langle \boldsymbol{\Psi}^{\pm}_{\boldsymbol{k},n}(-\rho) \middle| \nabla_{\boldsymbol{k}} \boldsymbol{\Psi}^{\pm}_{\boldsymbol{k},n}(\rho) \right\rangle$$

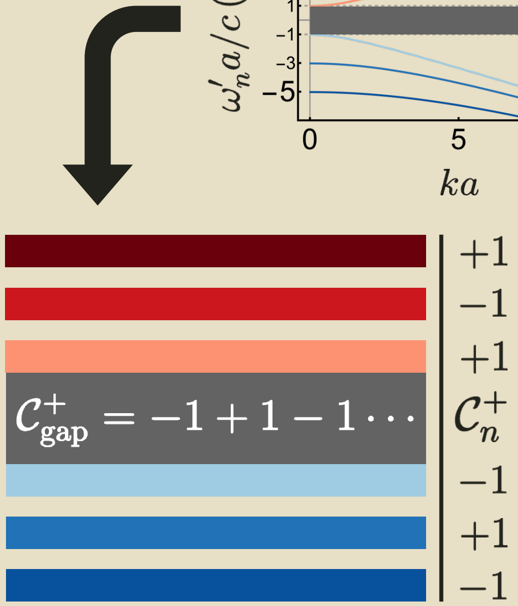
Condition Met for Integer $\mathcal{C}^{+}_{n}$

$$|\boldsymbol{k}|^2 \left[\nabla_{\boldsymbol{k}} \times \mathcal{A}^{+}_{\boldsymbol{k},n}\right]_z \to 0$$

when $|\boldsymbol{k}|^2 \to \infty$

Ordered Scheme of Bands in the Dispersion Diagram





Band Chern Numbers

Alternate

Gap Chern Number

Non-Convergent Series

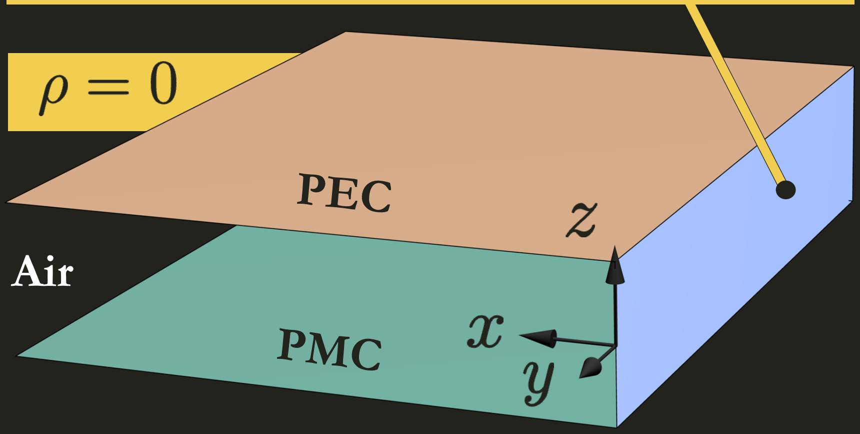
Topological Characterization

Bulk-Edge Correspondence holds for

$$\mathcal{C}^{+}_{\mathbf{gap}} = -1 + 1 - 1 \cdots ?$$

Chiral Transport via Infinitely Many Edge Waves

Lateral Opaque Wall



(as in Corrugated Conducting Plates)

Leontovich Boundary Condition ◀ Anisotropic Surface Impedance

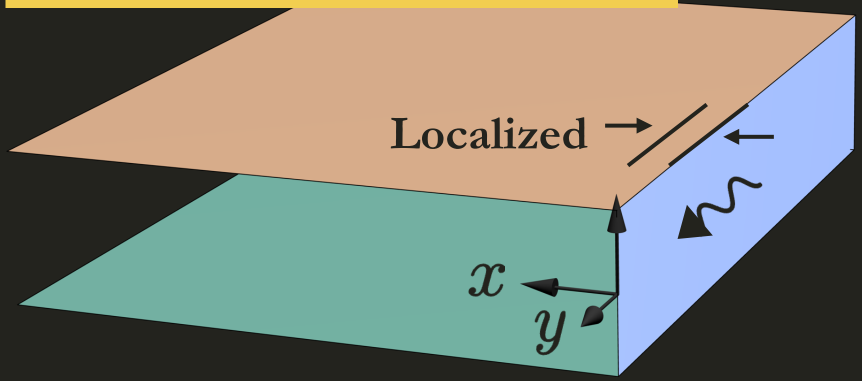
$$\boldsymbol{Z} \cdot (\hat{\boldsymbol{x}} \times \boldsymbol{H}_{\tan}) = \boldsymbol{E}_{\tan}$$

$$\boldsymbol{Z} = Z_y\, \hat{\boldsymbol{y}} \otimes \hat{\boldsymbol{y}} + Z_z\, \hat{\boldsymbol{z}} \otimes \hat{\boldsymbol{z}}$$

Reactive Wall with $Z_y Z_z = Z_0^2$ ▶ Dispersion of Edge Waves

Chiral Transport via Infinitely Many Edge Waves

$$\boldsymbol{\Psi}_n^{\pm} = \boldsymbol{\Psi}_{\boldsymbol{k},n}^{\pm} e^{-i\omega_n t} e^{ik_y y} e^{-\alpha_x x}$$



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Leontovich Boundary Condition ◀ **Anisotropic Surface Impedance**

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Reactive Wall with $Z_y Z_z = Z_0^2$ ▶ **Dispersion of Edge Waves**

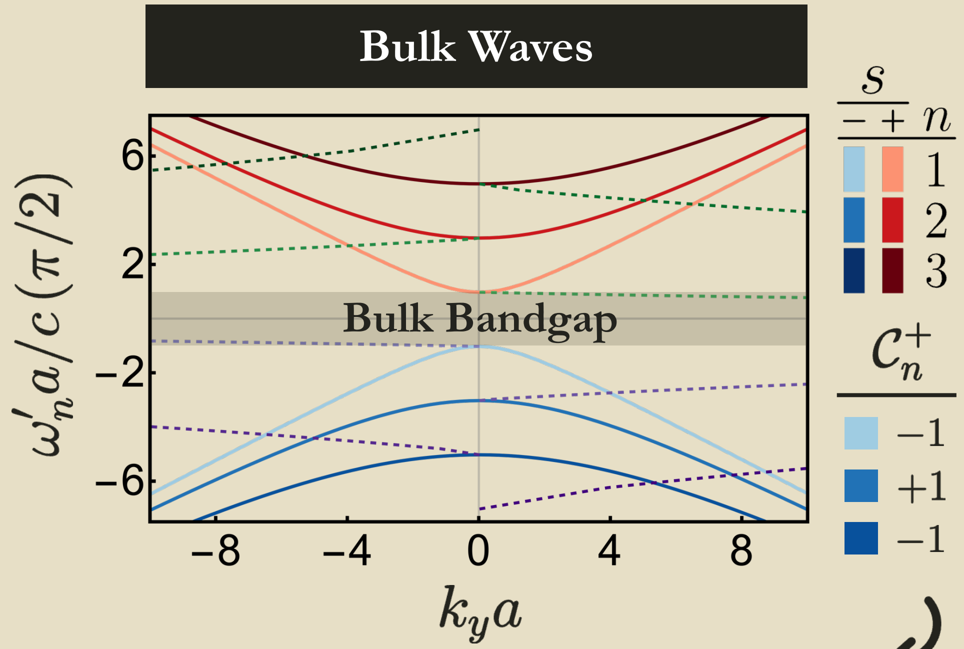
$$\frac{\omega_n^2/c^2 - \kappa_n^2}{\kappa_n} = i\frac{Z_z}{Z_0}\left(\pm(-1)^n k_y - \frac{\omega_n}{c\kappa_n} s \times \sqrt{k_y^2 + \kappa_n^2 - \omega_n^2/c^2}\right)$$

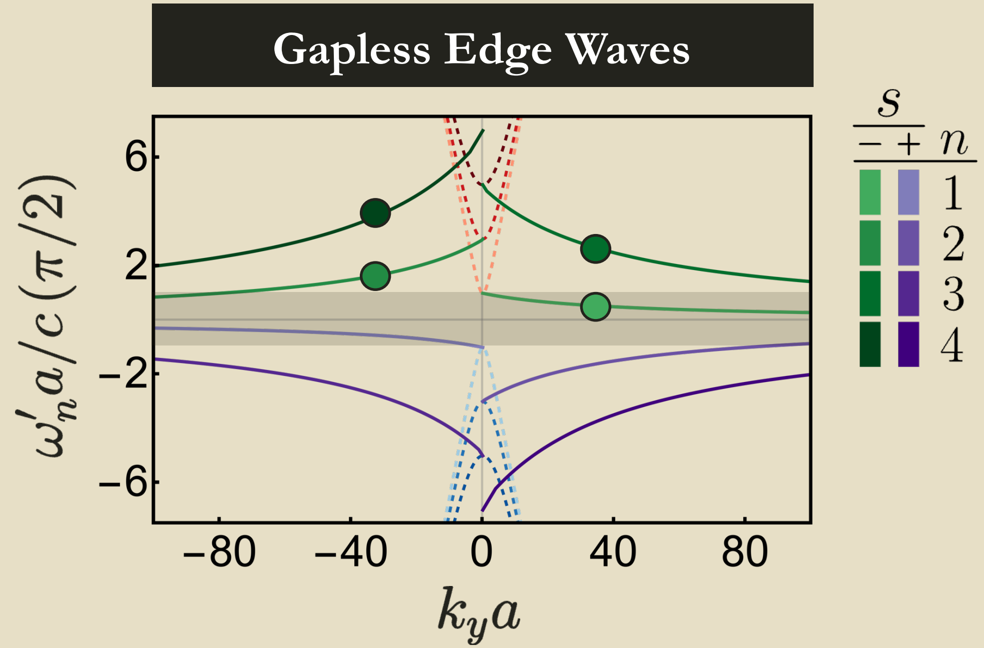
$$+ \to - \qquad k_y \to -k_y$$

Reciprocity

$$\mathcal{C}_n^- = -\mathcal{C}_n^+$$

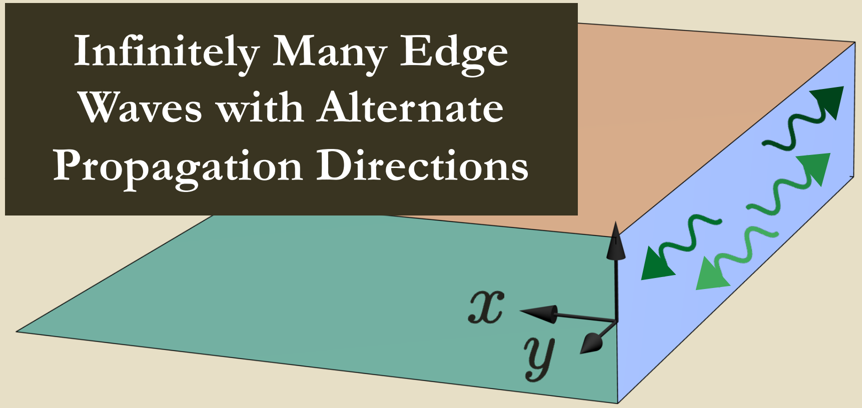
Chiral Transport via Infinitely Many Edge Waves





$$\mathcal{C}^+_{\text{gap}} = \sum_{n\in\mathbb{N}}^{s=-} \mathcal{C}^+_n = -1+1-1+\dots \quad \Leftrightarrow$$

Infinitely Many Edge Waves with Alternate Propagation Directions

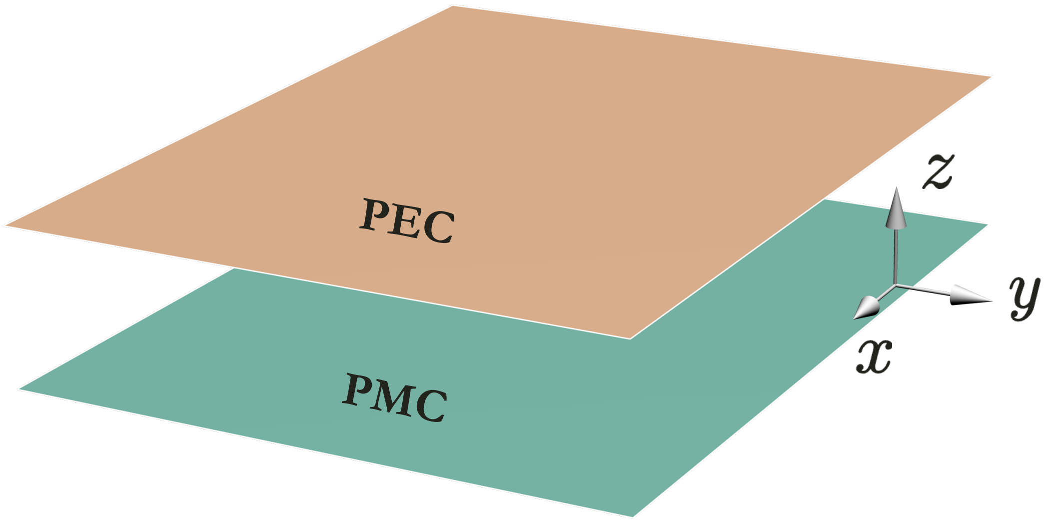


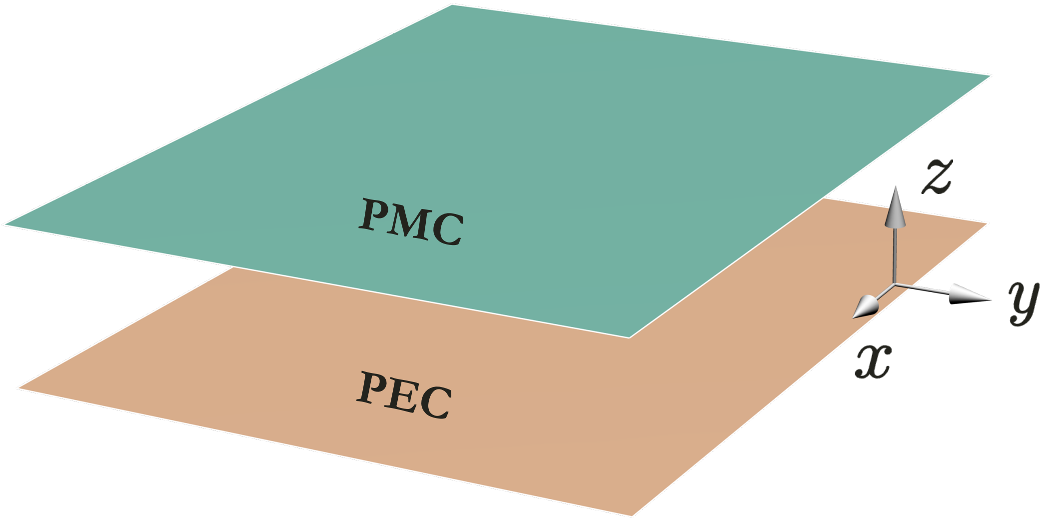
$$\mathcal{C}^+_{\text{gap}} = \sum_{n\in\mathbb{N}}^{s=-} \mathcal{C}^+_n = -1+1-1+\dots$$

Infinite

Alternate

Topological Phase Transition



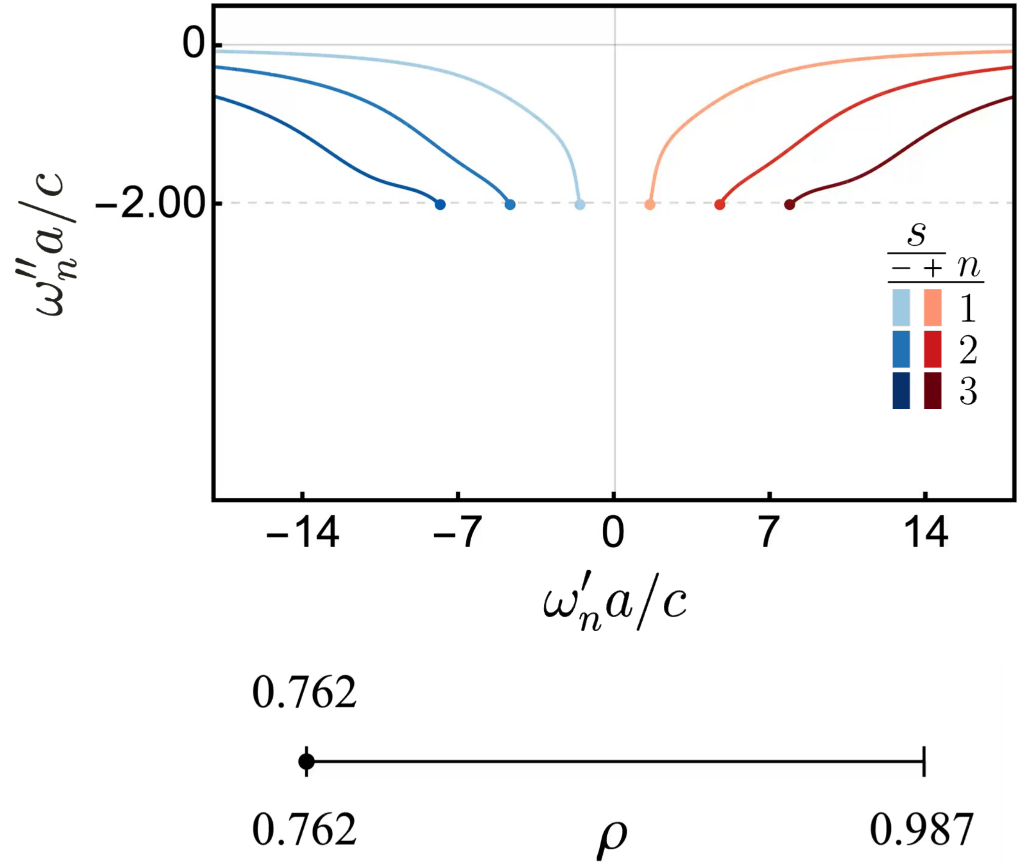


$\rho = 0$	$\rho = \infty$
$\mathcal{C}^{+}_{\text{gap}} = -1 + 1 - 1 + \dots$	$\mathcal{C}^{+}_{\text{gap}} = +1 - 1 + 1 - \dots$

Topological Numbers Switch Sign

Band Structure near $\rho = 1$

Locus of Frequency Bands in Complex Plane $\omega_n = \omega_n' + i\omega_n''$



Bandgap Closes

$$0.94 < \rho < 0.94^{-1}$$

First-Order Bands Coalesce

$$\omega_{n=1}^{s=+} = \omega_{n=1}^{s=-}$$

$+$

(Electromagnetic Fields Match)

Pseudospinors Align

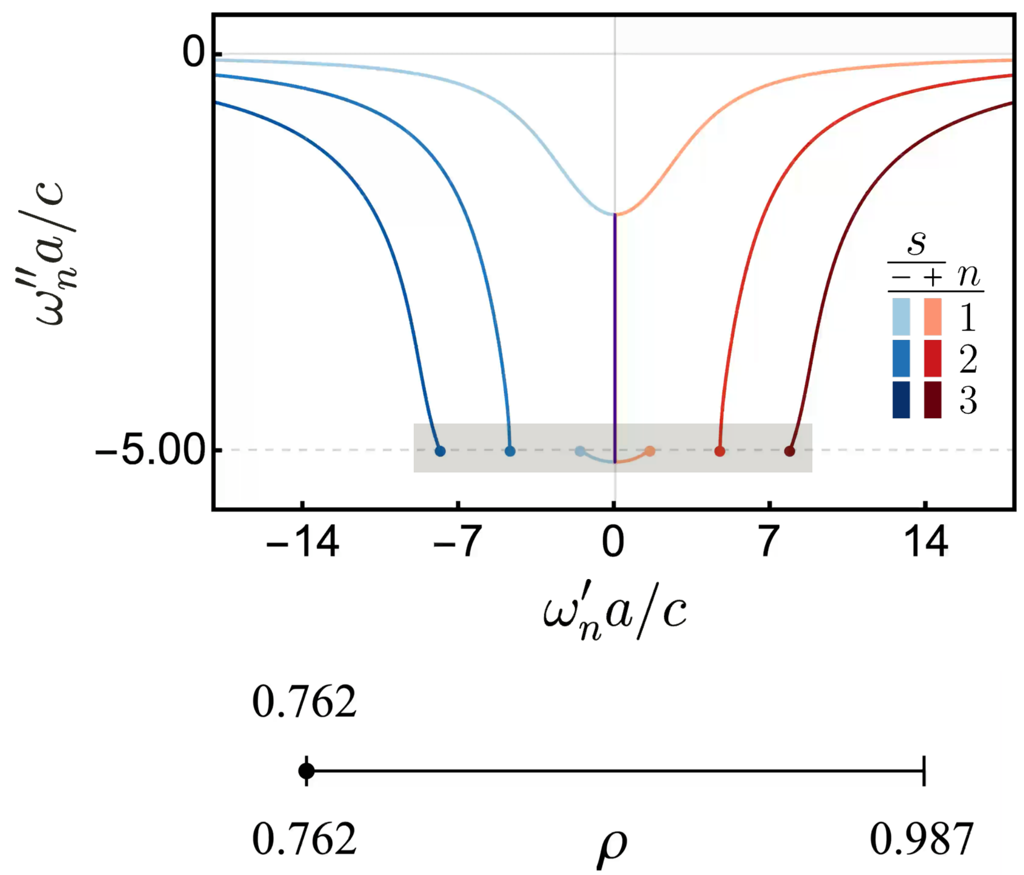
$$\mathbf{\Psi}_{n=1}^{\pm,s=+} \parallel \mathbf{\Psi}_{n=1}^{\pm,s=-}$$

Exceptional Point Degeneracies

M.-A. Miri and A. Alù, Science 363 (2019).

Band Structure near $\rho = 1$

Locus of Frequency Bands in Complex Plane $\omega_n = \omega_n' + i\omega_n''$



Impedance of top plate matches that of air when $\rho = 1$

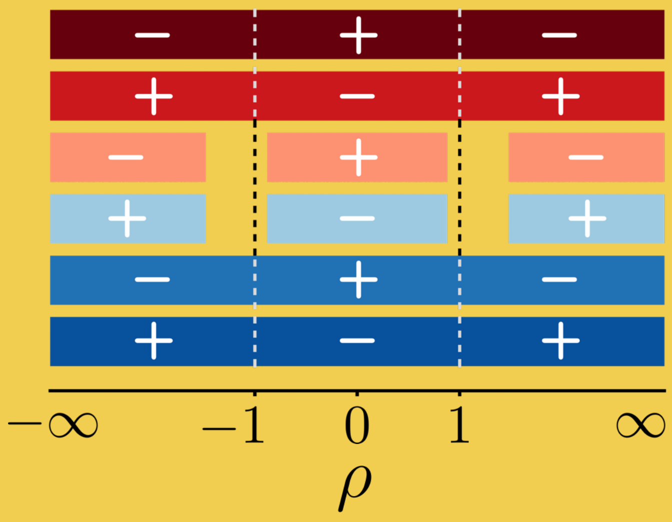
Resonant Absorption at $k = 0$

$$\omega_n'' \approx c a^{-1} \ln(\rho - 1)$$

Conclusions

- Scattering anomaly in PTD-symmetric platforms has a topological origin that is robust against dissipative effects.
- Gap topological charge of PD-symmetric waveguides is a non-convergent sum of integers.
- Bulk-edge correspondence can hold despite the ill-defined topology. Infinite number of decoupled chiral modes propagate along the edge of the guide.
- A topological phase transition occurs with the formation of exceptional points.
- At the exceptional points, there is resonant absorption of energy. Lifetime of guided modes approaches zero.

Phase Diagram $\mathcal{C}_n^+$



$$\mathcal{C}^+_{\text{gap}} = \sum_{n\in\mathbb{N}}^{s=-} \mathcal{C}_n^+ = -1+1-1+\dots$$

Thank you!

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