Topological Phase Transition in non-Hermitian PD-Symmetric Waveguides

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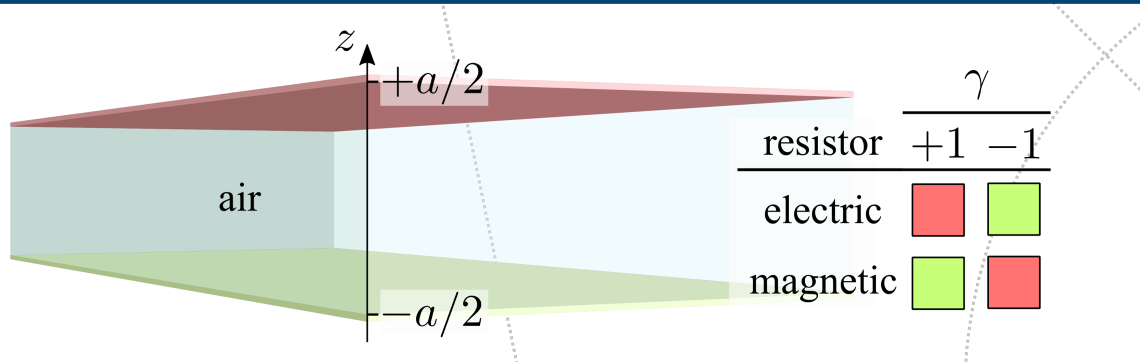
Introduction

Invariance under the action of the composition of the parity (P), time-reversal (T) and duality (T) operators has been established as a mechanism to sustain bidirectional backscattering-free waveguiding in optical structures [1]. Numerical evidence of this property first arose for combinations of PTD-symmetric parallel-plate waveguides (PPWs) with air confined within two perfect conductor walls, one electric (PEC) and other magnetic (PMC) [2]. Even though topological photonics is a privileged paradigm for one-way light transport, a proper topological characterization of those models is still lacking. We fill this gap by computing the spin Chern numbers of the band structures of bulk modes in generally PD-invariant PPWs that can also account for energy losses as the plates are relaxed to imperfect conductors. The resistivity parameter shared by the dual sheets controls the topological phases in the guide. We show that a transition occurs when the impedances of air and plates match.

1. PD-Symmetric PPW



- Two impedance sheets of negligible width, one electric and the other magnetic with resistivities

$$\varrho_e = \rho_e \tilde{Z}_0 \quad \text{and} \quad \varrho_m = \rho_m \tilde{Z}_0^{-1}$$

- PD-invariance renders the relative parameters equal

$$\rho_e = \rho_m \equiv \rho_r$$

- PTD-symmetric model [2] recovered when $\rho_r \to 0$.

2. Band Structure

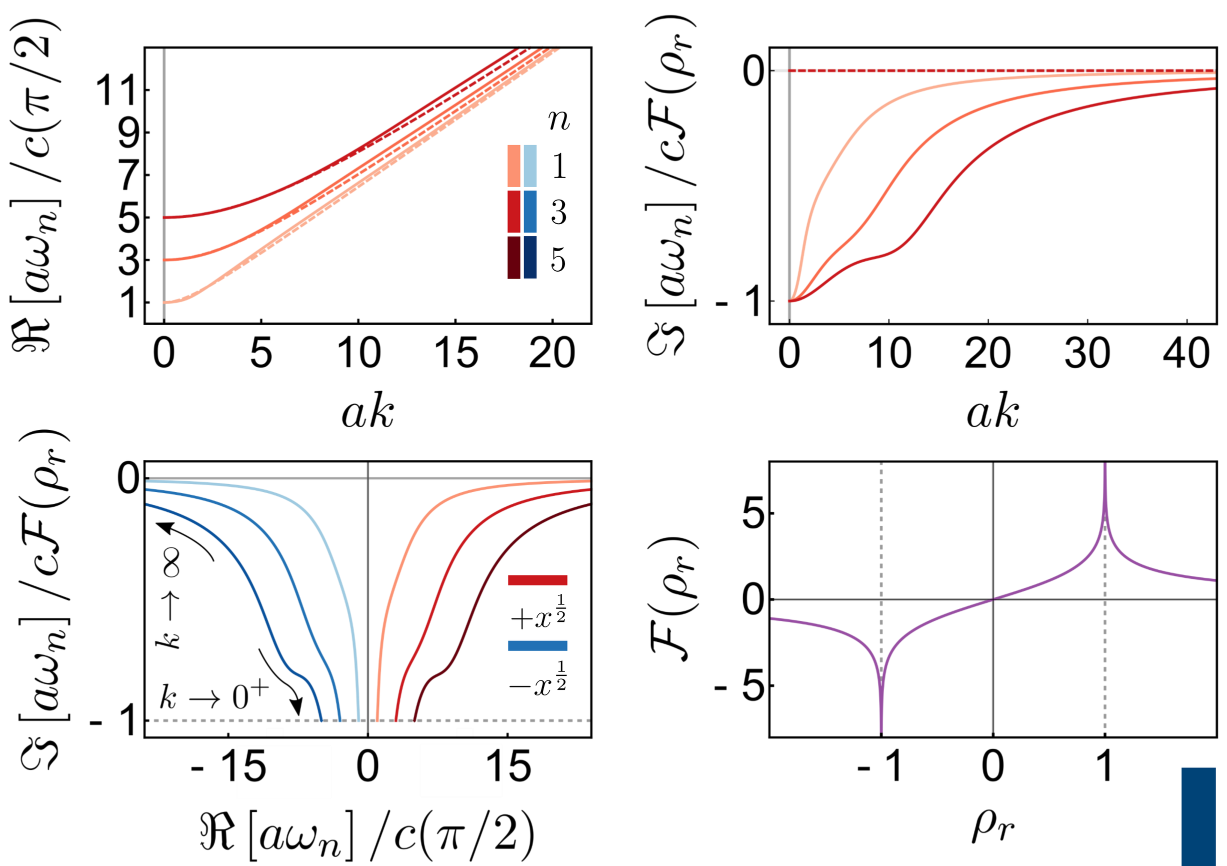
- Infinite propagating bulk modes indexed by a positive odd integer n and an xOy-in-plane wave vector $\boldsymbol{k}$.
- Oscillation frequency ω_n determined by $k = |\boldsymbol{k}|$ and transverse wavenumbers κ_n as

$$\omega_n = c\sqrt{k^2 + \kappa_n^2}$$

- Boundary conditions require

$$e^{2i\kappa_n a} = \frac{\omega_n + c\rho_r\kappa_n}{\omega_n - c\rho_r\kappa_n}\,\frac{\rho_r\omega_n + c\kappa_n}{\rho_r\omega_n - c\kappa_n}$$

- Solutions with $\rho_r = 0.6$ (solid) and $\rho_r = 0$ (dashed)



$$\mathcal{F}(\rho_r) = \mathrm{sgn}(\rho_r)\ln\left|\frac{|\rho_r|+1}{|\rho_r|-1}\right|$$

Scaling Function

Frequency Degenerate Singularities

- $k \to \infty,\ \rho_r \in \mathbb{R}$
- $k \to 0^+,\ \rho_r = \pm 1$

3. Spin Chern Numbers

- Maxwell's equations decouple under PD-symmetry and are solved by up and down, $\pm$, optical pseudospin states $\boldsymbol{\Psi}_\gamma^{\pm,n}(\boldsymbol{r},t;\rho_r) = \boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;\rho_r)e^{-i\omega t}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ with amplitudes

$$\boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;\rho_r) = \begin{bmatrix} \mp\left(e^{-i\kappa_n z} + h_n^\gamma e^{i\kappa_n z}\right) \\ -\frac{\epsilon_r\omega_n}{c_0\kappa_n}\left(e^{i\kappa_n z} - h_n^\gamma e^{-i\kappa_n z}\right) \\ \mp\frac{k}{\kappa_n}\left(e^{-i\kappa_n z} - h_n^\gamma e^{i\kappa_n z}\right) \end{bmatrix}$$

in the basis $(\hat{\boldsymbol{k}}, \hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}} = \hat{\boldsymbol{\phi}}, \hat{\boldsymbol{z}})$

where

$$h_n \equiv e^{i\kappa_n a}\frac{\omega_n - c\rho_r\kappa_n}{\omega_n + c\rho_r\kappa_n}$$

- Topological band theory of continuous [3] and non-Hermitian [4] systems leads to the angular part of the Berry potential $\mathcal{A}_{\phi,\gamma}^{\pm,n}$ and the Chern numbers $\mathcal{C}_{\gamma,n}^{\pm}$ for **separable** frequency bands at $k \in]0,\infty[$.

Invariance under rotations about the z -axis

$$\mathcal{A}_{\phi,\gamma}^{\pm,n} = ik^{-1}\frac{\langle\boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;-\rho_r)|\partial_\phi\boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;\rho_r)\rangle}{\langle\boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;-\rho_r)|\boldsymbol{\Psi}_{\boldsymbol{k},\gamma}^{\pm,n}(z;\rho_r)\rangle}$$

$$\mathcal{C}_{\gamma,n}^{\pm} = \left[\lim_{k\to\infty} - \lim_{k\to 0^+}\right] k\,\mathcal{A}_{\phi,\gamma}^{\pm,n}(k)$$

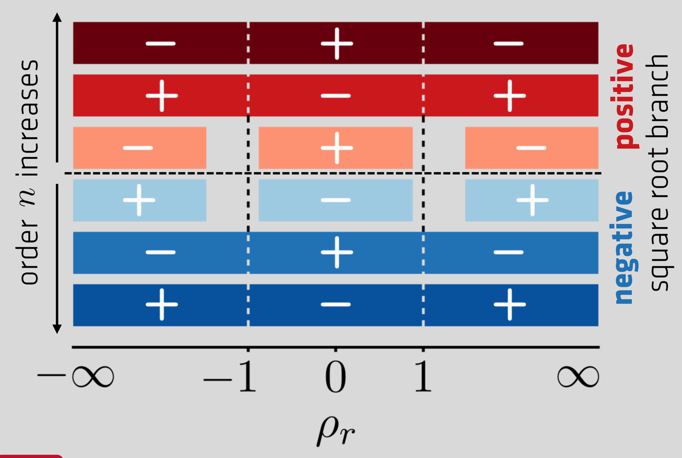
- Spin Chern numbers (invariants) $\mathcal{C}_{\gamma,n}^s \equiv (\mathcal{C}_{\gamma,n}^+ - \mathcal{C}_{\gamma,n}^-)/2$ for the $(\pm)$ square root branch read

$$\mathcal{C}_{\gamma,n}^s = (\pm)\gamma(-1)^{\frac{n-1}{2}}\mathrm{sgn}(\delta) \qquad \delta = 1 - |\rho_r|$$

Two critical points $\rho_r = \pm 1$

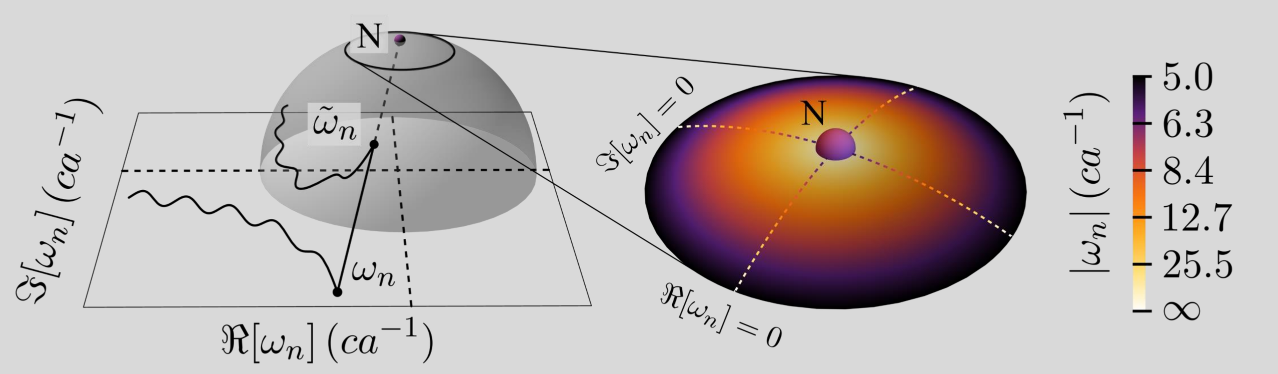
4. Topological Phase Transition

- Scheme of the topological phase diagram for the guide orientation $\gamma = 1$:



- Stereographic projection of the frequencies onto the Riemann sphere compactifies unbounded bands.

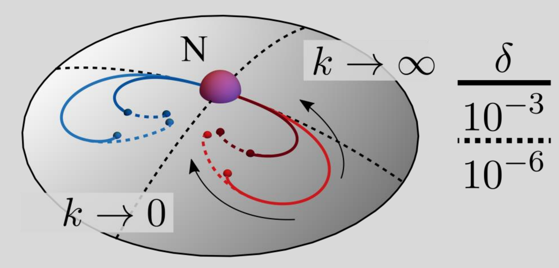
$$\omega_n \to \tilde{\omega}_n \equiv \left\{2\Re[\omega_n], 2\Im[\omega_n], |\omega_n|^2 - 1\right\}/\left(|\omega_n|^2 + 1\right)$$



Band structure near $\rho_r = 1$

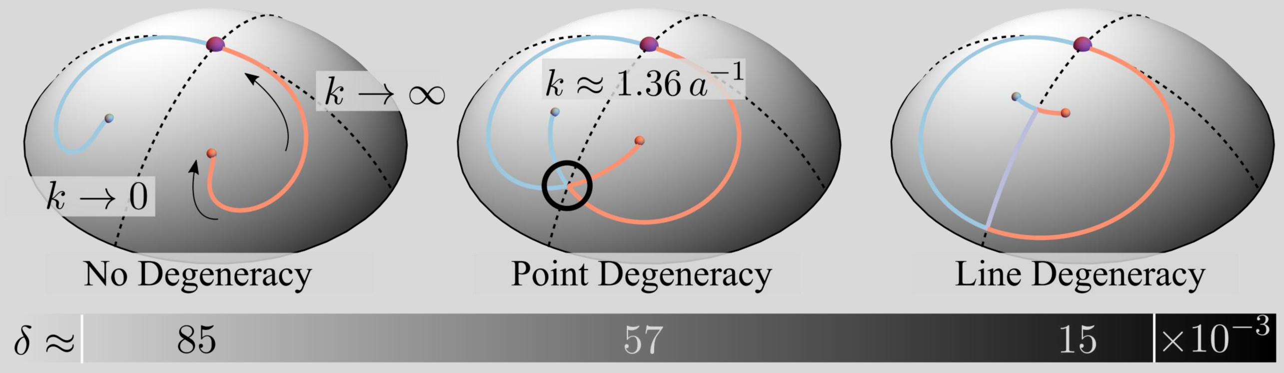
Higher-order Bands

- Invariants switch signs when bands touch N at $k \to 0^+$ and $\rho_r = 1$.



Lowest-order Bands

- Gap closing mediates phase transition. Undefined invariants in a neighbourhood of $\rho_r = 1$.



- Band structure can be easily deduced for other ρ_r values considering the symmetries

$$\omega_n(k;-\rho_r) = \overline{\omega}_n(k;\rho_r)$$
$$\omega_n(k;\rho_r) = \omega_n(k;\rho_r^{-1})$$

Watch the evolution of the band structure in the vicinity of the transition

References

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[4] H. Shen, B. Zhen, and L. Fu, Topological Band Theory for non-Hermitian Hamiltonians, Physical Review Letters 120, 10.1103/physrevlett.120.146402 (2018).

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