Chapter 1

DPO graph rewriting approach

Given a rule $L \overset{l}{\leftarrowtail} K \overset{r}{\rightarrowtail} R$ and a graph G, the DPO approach to graph rewriting is to commute the following diagram

$$
\begin{tikzcd}
L \arrow[d, "m"'] & K \arrow[l, tail, "l"'] \arrow[r, tail, "r"] \arrow[d, "k"'] & R \arrow[d, "m'"'] \\
G & D \arrow[l, tail, "l'"'] \arrow[r, tail, "r'"] & H
\end{tikzcd}
$$

in which both squares are pushouts.

The morphisms in this diagram are:

- m: is a homomorphism detection algorithm. In the diagram, m is a single homomorphism, non-deterministically detected, but ideally, the algorithm should return all possible homomorphisms from L to G. It is calculated.

  A second view of such morphism is that it is the set of arrows that maps $a \mapsto b, a \in L, b \in G$.

- l, l', r, r' are all inclusions (supose that each one of them can be generically identified by $\phi : S \rightarrow T$). ϕ can be defined as the identities of the elements of the graph $T \backslash S$, and the inverse morphism $\phi^{-1} : T \nrightarrow S$ is a partial morphism that excludes nodes and edges in $T \backslash S$.

- k, m': are morphisms derived from m. k has the arrows of m which the source is removed by l^{-1} removed, along with the target of such arrows. m' adds arrows to k, with source in the elements of $R \backslash K$ and target in the elements of $H \backslash D$.

These are all structure preserving. Two conditions impede the transformation to be applied:

- *dangling condition*: If after the deletion of a node, an edge is left "dangling", i.e. the node deleted was the source or target of an edge that was not removed.
- *identification condition*: if an element is both deleted and maintained.

Chapter 2

Representation and algorithms

2.1 Graph

A graph is represented by two sets V of vertices and E of edges. Each element must have an numerical identity. Edges must contain also a pair of node identites that are the source and target of an edge.

A morphism between graphs is represented by two lists of pairs: node and edge identities. The first element is the source of the morphism and the second element is the target of the morphism. one of them can be $\perp$ (`Nothing`), meaning that the element is created (when in the source) or deleted (when in the target). For the DPO approach, the source of each pair must be unique, except for $\perp$. This morphism allows to describe a span $L \overset{l}{\hookleftarrow} K \overset{r}{\rightarrowtail} R$ as follows.

- L are all sources s that are mapped to $\perp$ $(s \mid s \mapsto \perp)$, plus all that are maintained $(s \mid s \mapsto s)$.
- R are all targets t that are mapped from $\perp$ $(t \mid \perp \mapsto t)$, plus all that are maintained $(t \mid t \mapsto t)$.
- K are all item that are maintained $(k \mid k \mapsto k)$.
- l are all pairs that the target is $\perp$ $((s, \perp) \mid s \mapsto \perp)$.
- r are all pairs which the source if $\perp$ $((\perp, t) \mid \perp \mapsto t)$.