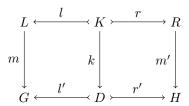
### Chapter 1

# DPO graph rewriting approach

Given a rule  $L \stackrel{l}{\longleftrightarrow} K \stackrel{r}{\rightarrowtail} R$  and a graph G, the DPO approach to graph rewriting is to commute the following diagram



in which both squares are pushouts.

The morphisms in this diagram are:

- ullet m: is a homomorphism detection algorithm. In the diagram, m is a single homomorphism, non-deterministically detected, but ideally, the algorithm should return all possible homomorphisms from L to G. It is calculated.
  - A second view of such morphism is that it is the set of arrows that maps  $a\mapsto b, a\in L, b\in G.$
- l, l', r, r' are all inclusions (supose that each one of them can be generically identified by  $\phi: S \to T$ ).  $\phi$  can be defined as the identities of the elements of the graph  $T \setminus S$ , and the inverse morphism  $\phi^{-1}: T \nrightarrow S$  is a partial morphism that excludes nodes and edges in  $T \setminus S$ .
- k, m': are morphisms derived from m. k has the arrows of m which the source is removed by  $l^{-1}$  removed, along with the target of such arrows. m' adds arrows to k, with source in the elements of  $R \setminus K$  and target in the elements of  $H \setminus D$ .

These are all structure preserving. Two conditions impede the transformation to be applied:

- dangling condition: If after the deletion of a node, an edge is left "dangling", i.e. the node deleted was the source or target of an edge that was not removed.
- $\bullet$  identification condition: if an element is both deleted and maintained.

### Chapter 2

# Representation and algorithms

#### 2.1 Graph

A graph is represented by two sets V of vertices and E of edges. Each element must have an numerical identity. Edges must contain also a pair of node identites that are the source and target of an edge.

A morphism between graphs is represented by two lists of pairs: node and edge identities. The first element is the source of the morphism and the second element is the target of the morphism. one of them can be  $\bot$  (Nothing), meaning that the element is created (when in the source) or deleted (when in the target). For the DPO approach, the source of each pair must be unique, except for  $\bot$ . This morphism allows to describe a span  $L \stackrel{l}{\leftarrow} K \stackrel{r}{\rightarrowtail} R$  as follows.

- L are all sources s that are mapped to  $\bot$  ( $s \mid s \mapsto \bot$ ), plus all that are maintained ( $s \mid s \mapsto s$ ).
- R are all targets t that are mapped from  $\bot$   $(t | \bot \mapsto t)$ , plus all that are maintained  $(t | t \mapsto t)$ .
- K are all item that are maintained  $(k \mid k \mapsto k)$ .
- l are all pairs that the target is  $\bot$   $((s, \bot) \mid s \mapsto \bot)$ .
- r are all pairs which the source if  $\bot ((\bot, t) | \bot \mapsto t)$ .