Computers and Operations Research 91 (2018) 79–91

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Review

Vehicle routing with backhauls: Review and research perspectives

Çağrı Koç[a,*], Gilbert Laporte[b]

[a] Department of Management, Social Sciences University of Ankara, Ankara, Turkey
[b] CIRRELT, Canada Research Chair in Distribution Management and HEC Montréal, Montréal, Canada

ARTICLE INFO

Article history:
Received 20 April 2017
Revised 22 September 2017
Accepted 1 November 2017
Available online 7 November 2017

Keywords:
Vehicle routing
Backhauls
Survey
Research directions

ABSTRACT

In the Vehicle Routing Problem with Backhauls (VRPB), the customer set is partitioned into linehaul customers who require deliveries, and backhaul customers who require pickups. Both the linehaul customers and the backhaul customers must be visited contiguously, and all routes must contain at least one linehaul customer. All deliveries have to be loaded at the depot, and all pickups up have to be transported to the depot. This survey paper aims to comprehensively review the existing literature on VRPBs, including models, exact and heuristic algorithms, variants, industrial applications and case studies, with an emphasis on the recent literature. The paper contains several synthetic tables and proposes a number of promising research directions.

1. Introduction

The classical Vehicle Routing Problem (VRP) aims to construct routes for a fleet of homogeneous vehicles in order to serve a set of customers. Each customer is visited once by one vehicle, each vehicle route starts and ends at the depot, and some side constraints are satisfied. In the last decades, many variants have been intensively studied. For further details on the VRP and its variants, we refer the reader to Cordeau et al. (2007), Golden et al. (2008), Laporte (2009), and Toth and Vigo (2014). The VRP with Backhauls (VRPB) was introduced by Deif and Bodin (1984). In this problem, the customer set is partitioned into linehaul and backhaul customers. Each linehaul customer requires a delivery and each backhaul customer requires a pickup. The linehaul customers are first visited, followed by the backhaul customers. All deliveries have to be loaded at the depot, and all pickups up have to be transported to the depot. This type of partitioning is very frequent in distribution problems. The interest of not mixing pickup and deliveries on vehicle routes is typically to avoid rehandling the merchandise along the route since vehicles are often rear-loaded. A practical example arises in the grocery industry in which the supermarkets and shops are the linehaul customers and the grocery suppliers are the backhaul customers (see, e.g., Walmart, 2017). Another example can be found in drink distribution where the delivery of full bottles is followed by the collection of empty ones at the end of the route (see, e.g., Coca-Cola, 2017). Toth and Vigo (2002) provided a general overview of papers for the standard VRPB until 2002 with a focus on the computational performance of exact algorithms and heuristics. Parragh et al. (2008a; 2008b) comprehensively reviewed the pickup and delivery problems until 2007. In the first part of the survey (Parragh et al., 2008a), the authors reviewed the VRPB and its related variants, proposed a typology on the different problems with backhauls, and classified VRPB into four classes. In the first and second classes, customers are either delivery or pickup customers but cannot be both. In the third and fourth classes, each customer requires a delivery and a pickup. More specifically, in the first class, all linehauls are visited before backhauls. In the second class, any sequence, i.e., mixed, of linehauls and backhauls permitted. In the third class, customers demanding delivery and pickup service can be visited twice. In the fourth class, customers demanding both services have to be visited exactly once. The book chapter of Irnich et al. (2014) briefly reviews several VRPB papers published from 2002 to 2014, without providing comparisons of computational performance.

This paper presents a comprehensive and up-to-date review of the existing studies on the VRPB, with an emphasis on the recent literature. It includes a comparative analysis of the performance of the state-of-the-art heuristics for the standard problem. It also describes several extensions of the VRPB, as well as a number of industrial applications and case studies. It closes with some research directions.

The structure of this paper is as follows. The VRPB is defined, mathematical formulations are presented, and exact algorithms are reviewed in Section 2. An extended survey of heuristics and comparisons of the state-of-the-art heuristic algorithms on the standard VRPB are presented in Section 3. Other variants and exten-

* Corresponding author.
E-mail addresses: cagri.koc@asbu.edu.tr (Ç. Koç), Gilbert.Laporte@cirrelt.ca (G. Laporte).

https://doi.org/10.1016/j.cor.2017.11.003
0305-0548/

sions are reviewed in Section 4, while industrial applications and case studies are considered in Section 5. Section 6 provides a tabulated summary of the literature. Finally, Section 7 presents our concluding remarks and a number of future research perspectives.

2. Problem definition, mathematical models and exact algorithms

The VRPB is defined on a complete directed graph $G = (V, A)$. The node set V is partitioned into {0, L, B}, where node 0 is the depot, and the sets $L = \{1, \dots, n\}$ and $B = \{n+1, \dots, n+m\}$ correspond to linehaul and backhaul customers, respectively. The arc set A is defined as $\{(i, j): i, j \in V, i \neq j\}$. A nonnegative demand d_j to be delivered or collected is associated with each node $j \in L \cup B$. A given number of K homogeneous vehicles of capacity C are available at the depot. Let c_{ij} be the nonnegative cost associated with arc $(i, j) \in A$.

The VRPB aims to determine a set of vehicle routes such that (i) each vehicle route starts and ends at the depot; (ii) each vehicle performs exactly one route; (iii) each customer is visited by exactly one vehicle; (iv) in each route, the linehaul customers are first visited, followed by the backhaul customers; (v) for each route the total demands associated with the linehaul customers and with the backhaul customers do not exceed the vehicle capacity; (vi) the total traveled cost is minimized; and (vii) each route must contain at least one linehaul customer, while routes containing only backhaul customers are not allowed. The VRPB is NP-hard since it extends the VRP.

Two exact algorithms were developed for the standard VRPB which were reviewed in detail by Toth and Vigo (2002). But, we briefly present them here since they still represent the state-of-the-art methods.

2.1. Integer linear programming formulation

We first present the integer linear programming formulation of Toth and Vigo (1997). Let $\overline{G} = (\overline{V}, \overline{A})$ be a directed graph obtained from G by defining $\overline{V} = V$ and $\overline{A} = A_1 \cup A_2 \cup A_3$ where $\overline{A}$ is the set of arcs. More specifically, let $A_1 = \{(i, j) \in A : i \in L_0, j \in L\}$, $A_2 = \{(i, j) \in A : i \in B, j \in B_0\}$, and $A_3 = \{(i, j) \in A : i \in L, j \in B_0\}$, where $L_0 = L \cup \{0\}$ and $B_0 = B \cup \{0\}$. Let $\mathcal{F} = \mathcal{L} \cup \mathcal{B}$ where $\mathcal{L}$ and $\mathcal{B}$ are the all subsets of nodes in L and B, respectively. The minimum number of vehicles needed to serve all the customers in $S \subseteq \mathcal{F}$ is denoted by $r(S)$. Let $\Delta_i^+ = \{j : (i, j) \in \overline{A}, i \in V\}$ and $\Delta_i^- = \{j : (j, i) \in \overline{A}, i \in V\}$. Furthermore, let the binary variable x_{ij} be equal to 1 if and only if a vehicle travels on arc $(i, j) \in A$.

The integer linear programming formulation of Toth and Vigo (1997) for the VRPB is as follows:

$$\text{Minimize} \quad \sum_{(i,j) \in \overline{A}} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{i \in \Delta_j^-} x_{ij} = 1 \qquad j \in \overline{V} \backslash \{0\} \tag{2}$$

$$\sum_{j \in \Delta_i^+} x_{ij} = 1 \qquad i \in \overline{V} \backslash \{0\} \tag{3}$$

$$\sum_{i \in \Delta_0^-} x_{i0} = K \tag{4}$$

$$\sum_{j \in \Delta_0^+} x_{0j} = K \tag{5}$$

$$\sum_{j \in S} \sum_{i \in \Delta_j^- \backslash S} x_{ij} \geq r(S) \qquad S \in \mathcal{F} \tag{6}$$

$$\sum_{j \in S} \sum_{j \in \Delta_i^+ \backslash S} x_{ij} \geq r(S) \qquad S \in \mathcal{F} \tag{7}$$

$$x_{ij} \in \{0, 1\} \qquad (i, j) \in \overline{A}. \tag{8}$$

In this formulation, the objective function (1) minimizes the total cost. Constraints (2) and (3) define degree constraints for the customer nodes. Similarly, constraints (4) and (5) define degree constraints for the depot. Constraints (6) and (7) impose the connectivity and the capacity conditions. Finally, constraints (8) define the domains of the decision variables.

Toth and Vigo (1997) developed the first exact branch-and-bound algorithm which uses the integer linear programming formulation just presented, and computed an effective Lagrangian bound. Their method works on a model that transforms the VRPB into a directed VRP in which all the arcs connecting backhaul to linehaul customers are removed as a consequence of the precedence constraint. The Lagrangian bound was derived from combinatorial relaxations based on projections. Over the linehaul and the backhaul customer sets, the bound leads to the computation of directed trees. The exact algorithm was able to solve the Goetschalckx and Jacobs-Blecha (1989) instances which contain between 25 and 68 customers, and newly generated instances which contain between 21 and 100 customers.

2.2. Set partitioning formulation

We next present the set partitioning formulation of Mingozzi et al. (1999). Let $\overline{G} = G_L \cup G_B$, where $G_L = (L_0, A_1)$ and $G_B = (B_0, A_2)$ denote graphs induced by the linehaul and the backhaul customers, respectively. An elementary path P in G_L starting from the depot and in G_B ending at the depot is feasible if the total demand satisfies:

$$C_{min}^L \leq \sum_{j \in P} d_j \leq C \tag{9}$$

$$C_{min}^B \leq \sum_{j \in P} d_j \leq C \tag{10}$$

where C_{min}^L denotes the minimum total demand of linehaul customers of any feasible route, and C_{min}^B denotes the minimum total demand of backhaul customers of any feasible route. The C_{min}^L and C_{min}^B are computed as:

$$C_{min}^L = \max\left\{0, \sum_{j \in L} d_j - (K-1)C\right\} \tag{11}$$

$$C_{min}^B = \max\left\{0, \sum_{j \in B} d_j - (K-1)C\right\}. \tag{12}$$

The index set of all feasible paths in G_L is denoted by $\mathcal{L}$. Let $\mathcal{L}_i \subseteq \mathcal{L}$ denote the index set of all paths passing through customer $i \in L$. Let $\mathcal{L}_i^E \subseteq \mathcal{L}$ denote the index set of all paths ending at customer $i \in L$. Let $\mathcal{B}$ be the index set of all feasible paths in G_B. Let $\mathcal{B}_i \subseteq \mathcal{B}$ be the index set of all paths passing through customer $i \in \mathcal{B}$. Let $\mathcal{B}_i^S \subseteq \mathcal{B}$ be the index set of all paths starting from customer $i \in \mathcal{B}$. Furthermore, let $\overline{c}(l)$ be the total cost of path P_l $(l \in \mathcal{L} \cup \mathcal{B})$. Let x_l, $y_{l'}$, and ξ_{ij} be equal to 1 if and only if the paths $l \in \mathcal{L}$, $l' \in \mathcal{B}$ and the arc $(i, j) \in A_3$ are used, respectively.

The formulation of Mingozzi et al. (1999) for the VRPB is as follows:

$$\text{Minimize} \quad \sum_{l\in\mathcal{L}} \bar{c}_l x_l + \sum_{l\in\mathcal{B}} \bar{c}_l y_l + \sum_{(i,j)\in A_3} c_{ij}\xi_{ij} \tag{13}$$

subject to

$$\sum_{l\in\mathcal{L}_i} x_l = 1 \qquad i \in L \tag{14}$$

$$\sum_{l\in\mathcal{B}_j} y_l = 1 \qquad j \in B \tag{15}$$

$$\sum_{l\in\mathcal{L}_i^E} x_l - \sum_{j\in\mathcal{B}_0} \xi_{ij} = 0 \qquad i \in L \tag{16}$$

$$\sum_{l\in\mathcal{B}_j^S} y_l - \sum_{i\in\mathcal{L}} \xi_{ij} = 0 \qquad j \in B \tag{17}$$

$$\sum_{(i,j)\in A_3} \xi_{ij} = K \tag{18}$$

$$x_l \in \{0,1\} \qquad l \in \mathcal{L} \tag{19}$$

$$y_l \in \{0,1\} \qquad l \in \mathcal{B} \tag{20}$$

$$\xi_{ij} \in \{0,1\} \qquad (i,j) \in A_3. \tag{21}$$

In this formulation, the objective function (13) minimizes the total cost. Constraints (14) and (15) guarantee that each node will be visited by exactly one route. Constraints (16) ensure that the solution contains an arc of A_3 starting from node $i \in \mathcal{L}$ if a path in G_L ends at node i. Constraints (17) require the presence of an arc (i, j) with $i \in \mathcal{L}$ and $j \in \mathcal{B}$ if a path in G_B starts from customer j. Constraint (18) guarantees that the solution contains K routes by requiring K arcs of A_3. Finally, constraints (19)–(21) define the domains of the decision variables.

Mingozzi et al. (1999) developed the second exact method based on the set partitioning model just presented. It computes the lower bound by finding a feasible solution of the dual of the LP-relaxation of its integer program. This dual solution is obtained by combining two different bounding procedures. The exact algorithm uses the dual solution and a method for limiting the variables of the integer program so that the resulting problem can be solved by CPLEX. This method was also capable of solving all instances of the Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997) sets.

3. Heuristics on the standard VRPB

We first review classical heuristics in Section 3.1, local search heuristics in Section 3.2, population search heuristics in Section 3.3, and neural network heuristics in Section 3.4. We finally provide a computational comparison of recent metaheuristics on the standard VRPB in Section 3.5.

3.1. Classical heuristics

Until 2000, several classical heuristics mainly based on constructive structure have been developed for the standard VRPB.

Deif and Bodin (1984) developed the first heuristic for the VRPB, based on an extension of the well-known (Clarke and Wright, 1964) heuristic. This constructive algorithm first generates single-customer routes. It then iteratively combines routes by considering the saving that can be achieved by serving two customers on the same route instead of leaving them on separate routes. The method was tested on randomly generated instances with up to 300 customers.

In a later study, Goetschalckx and Jacobs-Blecha (1989) developed a heuristic based on the concept of space-filling curves of Bartholdi and Platzman (1982) which was first used for the solution of the planar Traveling Salesman Problem (TSP). The method partitions the two separate sequences of points to form feasible routes, each containing customers of only one type. According to the space-filling mapping, each linehaul route is merged with the nearest backhaul route to obtain the final set of routes. This constructive algorithm was tested on 57 generated instances with up to 150 customers. In general, worse results than those found by Deif and Bodin (1984) were obtained, but the method was faster for large instances. Goetschalckx and Jacobs-Blecha (1993), later proposed an extension of the cluster-first, route-second heuristic of Fisher and Jaikumar (1981). The method first determines K seed radials by iteratively solving a capacitated location-allocation problem, and then generates routes by considering the customers located close to the radial and sequencing the linehaul customers by increasing distance to the depot and the backhaul customers by decreasing distance. Generalized assignment problems are then solved heuristically to group the customers into K clusters. It finally determines the routes through a modified TSP insertion heuristic and postoptimization exchange procedures. The heuristic outperformed the previous methods on the Goetschalckx and Jacobs-Blecha (1989) instances. Toth and Vigo (1996) later presented an improved cluster-first, route-second heuristic which starts from an infeasible solution obtained from a Lagrangian relaxation of the problem and attempts to make it feasible through inter-route and intra-route arc exchanges. This algorithm, which was initially designed for the undirected VRPB, was later applied to the directed case (Toth and Vigo, 1999).

3.2. Local search metaheuristics

In the early 2000s, three tabu search metaheuristics were developed for the standard VRPB. The first is that of Osman and Wassan (2002) who proposed a reactive tabu search heuristic based on two route-construction heuristics, the saving-insertion and saving-assignment procedures, in order to quickly generate initial solutions. The reactive concept is used to speed up the several neighborhood structures for the intensification and diversification phases. The method combines several special data structures to efficiently manage the search. The method performed well on the Goetschalckx and Jacobs-Blecha (1989) and on the Toth and Vigo (1996) sets where it provided new best solutions for the large-size instances. The algorithm of Osman and Wassan (2002) was later combined with an adaptive memory programming scheme by Wassan (2007) who used it to balance the intensification and diversification processes. An adaptive memory scheme searches the unexplored regions of the solution space by maintaining a set of elite solutions. This integration brings robustness to the search process and results in early convergence. Forty-five new best-known solutions were obtained on benchmark instances. Another tabu search heuristic was developed by Brandão (2006), who initialized it with two constructive heuristics. The first one uses the solution of open VRP, where the vehicles are not required to return to the depot, on the two set of customers, and the other one uses a K-tree relaxation. The method outperformed the existing algorithms and yielded many new best-known solutions for large-size instances.

More recently, several other local search-based heuristics were developed for the standard VRPB. Thus, Ropke and

Pisinger (2006) put forward a unified adaptive large neighborhood search heuristic capable of solving the standard VRPB and five variants: the Mixed VRPB, the Multiple Depot Mixed VRPB, the VRPB with Time Windows, the Mixed VRPB with Time Windows, and the VRP with Simultaneous Deliveries and Pickups. The method also allows the modeling of other routing problems. It yielded state-of-the-art results on 338 instances, including 227 new best-known solutions.

Gajpal and Abad (2009) developed an ant colony system which combines route ants and vehicle ants while respecting the constraint on the maximum number of vehicles. The authors used the concept of Gambardella et al. (1999) which first minimizes the number of vehicles used and then the total tour length. The method is a cluster-first, route-second procedure and uses two types of ants to construct a solution. The first one assigns customers to vehicles while the second one constructs a route for a vehicle given the assigned customers. The heuristic achieved a good average performance and obtained five new best-known solutions.

Zachariadis and Kiranoudis (2012) developed a heuristic based on an efficient implementation of the variable length bone exchange local search. This algorithm investigates rich solution neighborhoods. Several local search moves are statically encoded by data structures that enable fast minimum retrieval, insertion and deletion capabilities. For diversification purposes, the concept of promises is introduced, which is a parameter-free mechanism based on the regional aspiration criterion generally used in tabu search implementations. The method was applied to 62 well-known VRPB benchmark instances and yielded a best-known solution for each of them.

Cuervo et al. (2014) proposed an iterated local search algorithm which allows intermediate infeasible solutions in order to diversify the search process. The algorithm explores a broad neighborhood structure at each iteration and uses a data structure to store information about the set of neighboring solutions. To control feasibility, a dynamic adjustment of the penalty costs are applied to infeasible solutions. Much better solutions were obtained compared with classical heuristics, but the method required substantially more computing time. More recently, Brandão (2016) described a deterministic iterated local search algorithm. The algorithm uses an intensification mechanism when it finds a better solution than the current one or identifies a new best solution. The method performed well on the classical benchmark instances.

3.3. Population search heuristics

Only one population search heuristic has been developed for the standard VRPB. Vidal et al. (2014) developed a unified genetic algorithm for different variants of the VRP, including the VRPB. Several local search operators such as split and crossover, and several diversification mechanisms are used which are problem-independent. To increase the quality of the local search phase, the authors proposed a unified route evaluation methodology based on two main procedures. The first evaluates the moves as a concatenation of known subsequences, and the second one is information preprocessing on subsequences. In addition, the algorithm also uses several other advanced mechanisms. It yielded many new best-known solutions on the Goetschalckx and Jacobs-Blecha (1989) instances, but it was not tested on other instances.

3.4. Neural networks heuristics

Ghaziri and Osman (2006) described a self-organizing feature maps algorithm based on unsupervised competitive neural networks. It is made up of three main procedures. In the first one, each solution consists of a certain number of nodes that are connected together and define the network architecture. In the second one, which is based on a winner-take-all principle, a customer is presented to the algorithm and the nearest neuron in terms of a certain distance is selected. The third one consists of identifying the way in which neurons adapt their states or positions. The routes are then improved by applying a 2-opt procedure. Very good results, including 14 new best-known solutions, were obtained on the Toth and Vigo (1996) instances.

3.5. Computational comparison of recent metaheuristics on the standard VRPB

To our knowledge, no exact algorithm has been developed since 2002, and Toth and Vigo (2002) provided a comparison of the current exact algorithms. Therefore, we only provide a comparative analysis of the heuristic algorithms.

Computational testing for the VRPB is generally performed on the sets of Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997). Table 1 presents a summary of the comparison results of recent metaheuristics for the standard VRPB. For detailed comparison results, we refer the reader to the Appendix. The abbreviations of the papers used in the comparison are as follows: B06 for Brandão (2006), RP06 for Ropke and Pisinger (2006), GA09 for Gajpal and Abad (2009), ZK12 for Zachariadis and Kiranoudis (2012), CGSA14 for Cuervo et al. (2014), VCGP14 for Vidal et al. (2014), and finally B16 for Brandão (2016). In the table, the first column provides the references. The next columns present the instance sets of Goetschalckx and Jacobs-Blecha (1989) denoted as GJ89 and Toth and Vigo (1997) denoted as TV97. For each instance set, the first column shows the average cost. The second column shows the average percentage deviation (Dev (%)) from the value of the best-known solution (BKS) for each instance retrieved from the articles surveyed. The third column shows the average computation time in seconds (Time). We report the best running times over all runs. Scaled computation times for one reference computer would not be valid since the computers and programming languages used are not comparable. For this reason, we only provide the processor and CPU speed in the last two columns of the table.

For the GJ89, average deviations are 0.5% or less for all metaheuristics. In terms of solution quality, Vidal et al. (2014) and Gajpal and Abad (2009) are the top performers. For the TV97, average deviations are 0.75% or less for all metaheuristics. In terms of solution quality, Gajpal and Abad (2009) and Brandão (2006) are the top best.

4. Variants of the VRPB

In the last decades, a multitude of variants of the VRPB have received particular attention. We review them in this section. We also refer the reader to the review paper of Berbeglia et al. (2007) who extensively surveyed VRP with pickup and delivery problems, and proposed a classification scheme for these problems.

4.1. The mixed VRPB

The mixed VRPB was first studied by Wade and Salhi (2002) where backhauls are not restricted to be visited after all linehaul customers have been served. The authors presented an insertion-type constructive scheme which uses the Salhi and Rand (1987) greedy insertion heuristic. Experiments were performed on well-known VRPB instances and it was shown that savings can be achieved by incorporating a control on the mix of customers on routes.

Table 1
Average comparison of recent metaheuristics on the standard VRPB.

References	GJ89 instances			TV97 instances			Processor	CPU
	Cost	Dev (%)	Time	Cost	Dev (%)	Time		
B06	291305.70	0.26	66.84	702.50	0.27	24.36	Pentium III	500 MHz
RP06	291823.35	0.43	26.22	704.54	0.56	15.55	Pentium IV	1.5 GHz
GA09	290920.90	0.12	67.57	702.35	0.24	25.64	Intel Xeon	2.4 GHz
ZK12	291927.72	0.47	223.09	–	–	–	Intel Core 2	1.66 GHz
CGSA14	291170.20	0.21	86.75	703.52	0.41	13.29	Intel Core i7	2.93 GHz
VCGP14	290611.00	0.02	41.38	–	–	–	Opteron 250	2.4 GHz
B16	291154.18	0.21	30.50	705.88	0.75	14.40	Intel Core 2 Duo	2.0 GHz

Crispim and Brandão (2005) proposed a hybrid algorithm that combines tabu search and variable neighborhood descent for the mixed and simultaneous variants of the VRPB. In the simultaneous variant, customers may simultaneously send and receive goods. The authors allow the possibility for forbidding certain movements ring the search in order to increase diversification. The search is carried out within a first neighborhood structure by performing moves that are not tabu during a certain time. The heuristic obtained better solutions on all the instances of Salhi and Nagy (1999), which contain between 50 and 199 customers. Ropke and Pisinger (2006) later used their ALNS algorithm to solve the same problem. The authors performed experiments on the Nagy and Salhi (2005) instances, which contain between 50 and 199 customers, and obtained average cost reduction of 10% with respect to current heuristics, as well as 41 new best-known solutions over 42 instances. Tütüncü et al. (2009) described a decision support system and a visual interactive approach based on greedy randomized adaptive memory programming search to solve the VRPB and the mixed VRPB. Their method provides a flexible user interface to control the restriction of backhaul and linehaul customers to the required parts of the routes. It also allows users to investigate different solutions by observing the solution obtained and making changes. The method was applied by 18 undergraduate students on the Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997) instances and yielded competitive results including several new best-known solutions.

4.2. The multi-depot VRPB

The multi-depot VRPB was introduced by Salhi and Nagy (1999) who proposed a cluster insertion heuristic. In this problem, there are several depots and each of them has its own customers. The authors extended the idea of the 1-insertion procedure of Casco et al. (1988) to obtain a number of insertion heuristics. These include the possibility of inserting clusters of customers at the same time. This heuristic performed well on the Goetschalckx and Jacobs-Blecha (1989) VRPB instances, and new results on new instances derived from those of Gillett and Johnson (1976), which contain between 50 and 249 customers, for the multi-depot VRPB were presented. Ropke and Pisinger (2006) later used their ALNS algorithm to solve the multi-depot VRPB. These authors conducted experiments on the Salhi and Nagy (1999) instances and decreased the solution costs by up to 24%. They also obtained new best-known solutions for all instances at the expense of longer computation times.

More recently, Chávez et al. (2016) proposed a Pareto ant colony algorithm to solve a multi-objective variant of the multi-depot VRPB where the aim is to minimize distance, travel time and energy consumption. A random fixed speed between 30 km/h and 90 km/h was assigned to each arc, and the function considered by Bektaş and Laporte (2011) was used to compute energy consumption. The algorithm is based on the idea of Doerner et al. (2004) which uses three matrices of pheromones for each objective function. The method was tested on new instances based on those of Salhi and Nagy (1999).

4.3. The VRPB with time windows

The VRPB with time windows, where each customer has a specific time windows for delivery, has been extensively studied. Thangiah et al. (1996) introduced this problem and developed a route construction heuristic and various local search heuristics to improve an initial solution. The heuristic was tested on the 45 instances of Gélinas et al. (1992), which contain up to 100 customers. It yielded percentage gaps of up to 2.5% from the optimal solution values. It was also tested on 24 newly generated large-size instances involving up to 500 customers. Duhamel et al. (1997) later described a tabu search heuristic for the VRPB with time windows and customer precedence. The method first constructs a feasible solution by using an adapted version of the I1 insertion heuristic (Solomon, 1987). It then applies tabu search, local search, and improvement algorithms for intensification. It yielded 0.5% optimality gaps on the Gélinas et al. (1992) instances. Cho and Wang (2005) developed a threshold accepting heuristic combined with modified nearest neighbor and exchange procedures to solve the VRPB with time windows. This method is a variant of simulated annealing that uses a deterministic acceptance criterion to choose a neighbor solution and does not require the generation of random numbers and exponential functions as in simulated annealing. The authors compared several versions of their heuristic on new instances based on the Solomon (1987) VRPTW instances. Ropke and Pisinger (2006) later applied their ALNS algorithm to the VRPB with time windows and the mixed VRPB with time windows. For the former problem, the ALNS algorithm improved 10 out of the 15 solutions and reduced the number of vehicles needed for five of the Gélinas et al. (1992) instances. The method was also tested on the 24 (Thangiah et al., 1996) large-size instances and yielded new best-known solutions in all cases. For the mixed VRPB with time windows, the method was able to find better solutions on the Kontoravdis and Bard (1995) instances, compared to the previous heuristics.

Reimann and Ulrich (2006) studied three variants of the VRPB with time windows. In the first one, linehaul and backhaul customers can be separated and the problem is solved as two independent VRPTWs; the second variant is the standard VRPB with time windows, and the third one is the mixed VRPB with time windows. An ant colony optimization algorithm was developed to allow the possibility of optimizing the total route duration rather than total route length. The pheromone management was simplified, the solution construction was modified to take into account the possibility of serving backhauls prior to linehauls. Results on instances with up to 100 customers showed that the mixed version significantly outperforms the pure VRPB case.

The VRPB with time windows, with and without customer precedence, was studied by Zhong and Cole (2005) who developed a guided local search heuristic. The method first generates an initial infeasible solution, and then improves it by using a guided local search that applies a procedure called section planning to help reach feasibility. The method slightly improved some results on the Gélinas et al. (1992) instances. For the VRPB with time windows and without customer precedence, the method outperformed the Kontoravdis and Bard (1995) heuristics on their 27 instances.

Belmecheri et al. (2013) developed a particle swarm optimization algorithm for the mixed VRPB with heterogeneous fleet and time windows. This algorithm is based on a population of solutions where agents or particles change their positions in a multi-dimensional search space. The algorithm combines an ant colony optimization algorithm and several local search heuristics to improve solution quality. The Solomon (1987) VRPTW instances were modified to yield small-size instances with up to 20 vertices, and the heuristic provided competitive solutions compared with CPLEX.

Pradenas et al. (2013) introduced a greenhouse gas emissions function in the VRPB with time windows by considering the energy required for each route and by estimating the load and distance between customers. The authors used the method of Bektaş and Laporte (2011) to compute energy consumption, and described a scatter search heuristic. The algorithm is made up of five main stages: diversification generation, improvement, reference set update, subset generation, and solution combination. Cooperation between different transport companies is also considered to promote energy minimization while also incurring a cost increase. Experiments were conducted on the Gélinas et al. (1995) instances.

4.4. The VRPB with heterogeneous fleet

The heterogeneous fleet variant of the VRPB was first studied by Tavakkoli-Moghaddam et al. (2006) who described a memetic algorithm and presented a mathematical formulation. This problem is an extension of the VRPB in which one must additionally decide on the fleet composition. The heuristic combines several local search algorithms, and uses inter- and intra-route node exchanges as a part of the evolutionary algorithm. It is based on a simple conceptual framework and includes recombination and mutation procedures. The authors conducted experiments on new instances based on those of Goetschalckx and Jacobs-Blecha (1989) and on the Toth and Vigo (1999) homogeneous instances. Tütüncü (2010) studied the same problem and developed a new greedy randomized adaptive memory programming search. The method uses a visual interactive algorithm and is embedded within a visual decision support system. Competitive results were obtained on the classical heterogeneous VRP instances. New results on the heterogeneous VRPB are presented on generated instances with up to 100 nodes.

A new variant, the fleet size and mix VRPB, was introduced by Salhi et al. (2013), where the number of vehicles is considered to be unlimited. The authors proposed a heuristic algorithm based on a set partitioning formulation and presented several valid inequalities. Computational experiments were conducted on 36 instances with up to 100 nodes. The method yielded optimal solutions on small-size instances, and provided upper and lower bounds for large-size instances. Competitive solutions were obtained on the standard fleet size and mix VRP and on the fleet size and mix VRPB. Belloso et al. (2017b) later studied the same problem and proposed an iterative biased-randomized heuristic. The method solves a series of smaller instances of the homogeneous-fleet version of the problem and then uses them as partial solutions for the original problem. It also integrates three biased-randomized processes to improve the quality of the solutions. Experiments on Salhi et al. (2013) instances yielded 20 new best-known solutions, and the results on 36 new instances are presented.

Berghida and Boukra (2016) developed quantum inspired harmony search algorithm with variable population size to solve the heterogeneous fixed fleet VRP with mixed backhauls and time windows. The method uses quantum principles to speed up the evolution process and variable population size.

4.5. Other VRPB variants and extensions

A number of other VRPB extensions have been studied, ranging from cases in which vehicle speed is time-dependent to cases containing realistic constraints relative to inventory decisions, multiple trips, etc. We now provide a brief overview of these studies.

Süral and Bookbinder (2003) introduced the single VRP with unrestricted backhauls where the aim is to minimize the total cost of a capacitated vehicle tour by choosing the best of optional backhaul opportunities depending on the revenue generated, and inserting them in a revised tour at an expense of deviations from the original tour. The problem could also be considered as a generalization of the TSP with arbitrary arc costs when a tour of a subset of nodes may be desirable. The problem is defined as a mixed integer model based on the classical Miller–Tucker–Zemlin (MTZ) subtour elimination constraints (Miller et al., 1960). Several improvement techniques are used, such as constraint disaggregation and coefficient improvement, a lifted version of the MTZ constraints, and valid inequalities. Solving the model yielded optimal results on instances with up to 30 customers within a reasonable computation time.

Hoff et al. (2009) studied lasso solution strategies for the VRP with pickups and deliveries where the same customer may require both a delivery and a pickup. In practice, vehicle may postpone the pickups until the vehicle has enough capacity. For example in the beer industry, vehicles first deliver bottles until the vehicle is partly unloaded, then performs both pickups and deliveries at the remaining customers, and finally pick up empty bottles from the first visited customers. This strategy gives rise to lasso-shaped solutions. In the same vein, Nagy et al. (2013) studied the VRPB with restricted mixing of deliveries and pickups, and in which the assumption that all deliveries are made before pickups is relaxed. The problem offers a compromise between the standard VRPB and the VRP with mixed pickups and deliveries, and allows a mixture of linehaul and backhaul goods only subject to a restriction defined as a percentage of the vehicle capacity. The authors formulated the problem and proposed a reactive tabu search heuristic extending that of Wassan et al. (2008). Small-size instances were solved by CPLEX and optimal solutions were obtained in most cases; large-size instances with up to 199 customers were solved by the heuristic.

Wang and Wang (2009) studied the time-dependent VRPB in which the travel speed of the vehicle is time-dependent, and proposed a two-phase heuristic. The first phase generates an initial solution by using an adapted version of the Clarke and Wright (1964) savings heuristic, while the second phase applies the reactive tabu search algorithm of Wassan (2006) to improve that solution. Several versions of the heuristic algorithm were tested on 21 instances.

Liu and Chung (2009) introduced a variant of the VRPB that considers inventory control decisions. They described a hybrid heuristic combining variable neighborhood search and tabu search. Three versions of the heuristic were compared on small-size instances with up to eight customers, and on large-size instances with up to 150 customers.

Belloso et al. (2015) studied the VRP with clustered backhauls where customers are either linehaul or backhaul but not both, and, in each route, the cluster of linehaul customers has to be served before the first backhaul customer can be visited. The authors developed a multi-start biased randomization heuristic. The algorithm combines a biased-randomized version of the savings heuristic of Clarke and Wright (1964), and several local search processes. A skewed probability distribution is used to randomize the savings list. Competitive results were obtained on the Goetschalckx and Jacobs-Blecha (1989) instances when compared with Zachariadis and Kiranoudis (2012).

Bortfeldt et al. (2015) introduced the VRP with clustered backhauls and with 3D loading constraints where each customer demand is given as a set of 3D rectangular items and several packing constraints are also considered, such as some concerning the stacking of boxes. The authors proposed a large neighborhood search and a variable neighborhood search to solve the problem, and applied them to the 95 new benchmark instances derived from the well-known (Goetschalckx and Jacobs-Blecha, 1989; Toth and Vigo, 1997) instances.

Belloso et al. (2017a) studied the VRP with clustered and mixed backhauls. The authors proposed a metaheuristic which integrates a biased-randomized version of the popular savings heuristic. To induce a oriented randomization effect on the savings list of routing edges, the method uses a skewed probability distribution. In order to improve a sequential order in delivery and pick-up activities, the proposed method assigns a penalty cost to edges connecting delivery customers with pick-up customers. The authors first solved the VRP with clustered backhauls on generated instances derived from classical instances, then solved the VRP with mixed backhauls.

García-Nájera et al. (2015) studied the multi-objective VRPB where the objective is to minimize the number of routes, travel cost, and uncollected backhauls. The authors developed a population search based algorithm called similarity-based selection multi-objective evolutionary algorithm. Competitive results were obtained on the Goetschalckx and Jacobs-Blecha (1989) instances with respect to the literature and results were presented for the newly introduced problem.

More recently, Wassan et al. (2016) introduced the multi-trip VRPB in which a vehicle may perform several trips within a given time period, and can also collect goods in each trip. The authors defined the problem as a mixed integer linear program and developed a two-level variable neighborhood search algorithm. The heuristic was embedded within a sequential variable neighborhood search, and a multi-layer local search procedure was applied for intensification and diversification purposes. A new benchmark set was generated based on those of Taillard et al. (1996) and Toth and Vigo (1999). The algorithm yielded good results when compared with the solutions found by CPLEX for small- and medium-size instances with up to 50 customers. The algorithm also obtained competitive results on two classical VRPB instances data sets.

5. Industrial applications and case studies

Several studies investigated and solved real-life VRPB problems. We review them in this section.

Yano et al. (1987) have proposed an exact algorithm for a special case of the VRPB in which the number of pickup and delivery customers in a route does not exceed four. The authors have modeled this particular VRPB as a set covering problem and developed a branch-and-bound algorithm for its solution. The method was implemented on a personal computer for retail stores with up to 50 nodes, where the amount of redistribution was reduced, and the transportation budget reduced by $450,000.

Cheung and Hang (2003) developed two multi-attribute label matching algorithms, a sequential vehicle-demand assignment algorithm and a simultaneous assignment algorithm, for the VRPB with time windows arising in a logistics company in Hong Kong. The method uses multi-attribute labels to describe the status of the vehicles for en-route times and locations on specific partial routes that originate from the depot. Furthermore, it uses the resource requirements and benefits of possible partial routes starting from different customer locations and terminating at the depot. Label matching feasibility conditions are used for diversification and intensification purposes. The authors first implemented their method on randomly generated instances based on those of Solomon (1987), and then on a real-data set with up to 122 customers, with varying percentages of backhauls. The sequential vehicle-demand assignment algorithm yielded better results than the simultaneous assignment algorithm, but required longer computational times. The total cost reduces when the percentage of backhauls increases.

Eguia et al. (2013) studied a green variant of the VRPB with time windows, heterogeneous fleet and different fuel types. The aim was to minimize internal costs (driver, energy, vehicle fixed, inspection, insurance, maintenance costs and toll costs), and external costs (social effects of transportation activities). The authors proposed a simple heuristic based on the Clarke and Wright (1964) savings heuristic, and applied it to an instance of the Spanish leading supermarket chain in the region of Huelva with 17 delivery points. The results of the heuristic are compared respect to the best solution value obtained with CPLEX where the average deviation is less than 1%.

Yu and Qi (2014) considered a variant of the VRPB faced by a large express delivery company in Hong Kong, with time window constraints, multiple-delivery and pick-up customers visits per day, multiple trips per vehicle, and a latency cost for each delivery. The company runs pick-up and delivery operations for packages, over a distribution network that contains a central hub and a set of transfer stations. The problem was modeled as a mixed integer linear program, and two tabu search algorithms were developed based on the record-to-record travel method of Dueck (1993). The method was applied to an instance containing transfer stations, one hub and 100-customer nodes. Significant cost savings were obtained with respect to the current solution, where the total cost savings range from 3% to 46%.

Yalcın and Erginel (2015) developed a fuzzy multi-objective programming heuristic for the VRPB containing three main phases: clustering, routing and local search. In the first phase, customers are assigned to vehicles by considering two objectives: minimizing the total distance and maximizing the total savings value. The second phase constructs feasible vehicle routes, and the final phase improves the current solution by using insertion and interchange operators. The algorithm provided competitive results on the Goetschalckx and Jacobs-Blecha (1989) instances, and was then applied to a logistics department of a ceramics firm in Turkey with 93 linehaul customers in 31 cities, and 27 backhaul customers in seven cities.

Dominguez et al. (2016) proposed a biased randomized large neighborhood search for the two-dimensional VRPB where customer demands consist of a set of rectangular items that cannot be stacked due to their weight, dimensions, or fragility. The study was motivated by the transportation activities of a medium-size Spanish company Opein. This firm provides industrial equipment to its customers, mostly in the construction field. It has to periodically deliver and pick up a large variety of industrial machinery, dumpers, compressors, energy-generation sets, and forklifts. To attain a high vehicle utilization, these items must be efficiently packed on the truck surface. The length and width of these items must be considered. The search is carried out within a large neighborhood search framework (Pisinger and Ropke, 2010). Packing heuristics are integrated in the routes construction, and the method is also enhanced with memory-based techniques for both the routing and the packing processes, improving its execution performance without penalizing the quality of the generated solutions. The method yielded significant savings of up 8.32% with respect to the current situation.

Table 2
Literature on the standard VRPB.

	References	Mathematical model	Solution method	Algorithm	Case study
1	Deif and Bodin (1984)		Heuristic	CH	
2	Goetschalckx and Jacobs-Blecha (1989)	•	Heuristic	CH	
3	Goetschalckx and Jacobs-Blecha (1993)		Heuristic	GA	
4	Toth and Vigo (1997)	•	Exact	BB, LB	
5	Mingozzi et al. (1999)	•	Exact	SP, DP, DA	
6	Toth and Vigo (1999)		Heuristic	CFRS, LR	
7	Osman and Wassan (2002)		Heuristic	TS	
8	Brandão (2006)		Heuristic	TS, CH	
9	Ghaziri and Osman (2006)		Heuristic	SOFM	
10	Ropke and Pisinger (2006)		Heuristic	ALNS	
11	Wassan (2007)		Heuristic	TS, AMP	
12	Gajpal and Abad (2009)		Heuristic	ACO	
13	Zachariadis and Kiranoudis (2012)		Heuristic	LS	
14	Cuervo et al. (2014)	•	Heuristic	ILS	
15	Vidal et al. (2014)		Heuristic	PS	
16	Yalcın and Erginel (2015)	•	Heuristic	FOM	•
17	Brandão (2016)		Heuristic	ILS	

Oesterle and Bauernhansl (2016) studied a mixed VRPB with heterogeneous fleet, time windows and manufacturing capacity arising in a German food company. The authors described a simple constructive heuristic and embedded it into an application that can run on all kinds of electronic devices such as smartphones and tablets. Road distances were extracted from the Google Web Services. The authors considered 12 small-size instances with up to 10 customers from a food company with a daily capacity of around 200 tons that covers a wide range of products such as fresh and processed meat, milk products and dry products. The authors compared the results of their heuristic with those obtained by CPLEX and obtained optimal solutions.

Wu et al. (2016) studied the mixed VRPB with heterogeneous fleet and time windows and developed a multi-attribute label procedure based on an ant colony algorithm. The main features of the ant colony system were used, such as swarm intelligence and searching robustness. The proposed algorithm obtained competitive results on the Solomon (1987) instances and on the VRP with heterogeneous fixed fleet instances of Brandão (2011) and Taillard (1999). The algorithm was then applied to a case study by solving an 84-node shuttle service problem for the Post Office in the Guangzhou province of China. The postal shuttles deliver packages to the post offices, and also collect packages which are sent to the mail distribution center. The authors used dispatch data from the Wusan distribution center to optimize the vehicle type, vehicle number, and driving paths. The method was able to decrease the total cost by up to 9% compared with the current routing plan.

Lin et al. (2017) studied the VRP with time windows, backhauling, order-dependent vehicle capacity, i.e., heterogeneous fleet, order loading and delivery restrictions, maximum number of stores per route, loading capacity at the warehouse, and maximum tour duration. The problem arised in Kroger, one of the largest grocery chain in the Cincinnati-Columbus region of Ohio. The number of stores ranges from 120 to 150. The authors developed a greedy randomized adaptive search procedure (GRASP) augmented with tabu search to provide solutions. The method is validated on seven real instances of Novoa et al. (2016) and obtained competitive results. Experiments on Kroger instances showed that cost reductions of $4887 per day or 5.58% per day on average, compared to the current solutions.

6. Summary

We now provide a tabulated summary of the existing literature on VRPBs. Fig. 1 contains a classification of the VRPB. Tables 2 and 3 present a summary of the reviewed publications for the standard VRPB, and its variants and case studies, respectively. For each reference, these tables show the problem types, solution method, whether a mathematical programming formulation was described and a case study was included ("•" for yes).

The abbreviations of side problems used in tables are as follows: clustered backhauls (CB), fixed number of customers (FC), green (GR), heterogeneous fleet (HF), inventory (IN), label matching (LM), last-in, first-out (LIFO), manufacturing capacity (MC), mixed (MX), mixed delivery and pickup (MDP), multi-depot (MD), multiple trip (MT), multiple delivery (MD), multi-objective (MO), restricted mixing (RM), time windows (TW), time-dependent (TD), two-dimensional (TD), unrestricted backhauls (UB), and loading capacity at the warehouse (LW).

The abbreviations of solution methods are as follows: branch-and-bound (BB), biasedÉrandomized metaheuristic (BR), dual approximation (DA), Lagrangian bound (LB), set covering (SC), set partitioning (SP), adaptive memory programming (AMP), adaptive large neighborhood search (ALNS), ant colony optimization (ACO), cluster-first route-second (CFRS), constructive heuristics (CH), fuzzy multi-objective programming (FMO), generalized assignment (GA), greedy randomized adaptive search procedure (GRASP), iterated local search (ILS), large neighborhood search (LNS), local search (LS), population search (PS), quantum inspired harmony search (QIHS), self-organizing feature maps (SOFM), scatter search (SS), simulated annealing (SA), simulation (SIM), tabu search (TS), threshold accepting (TA), variable neighborhood search (VNS), and valid inequalities (VI).

The following conclusions can be drawn from the tables:

1. The standard VRPB is the most widely studied version with 17 references.
2. In 14 references, the VRPB with time windows and its variants are studied.
3. In eight references, heterogeneous fleet and its variants are considered.
4. This is followed by the multi-depot and its variants studied in three references.
5. Two references consider green variants.
6. In nine references, industrial applications and case studies were conducted to investigate and solve real-life distribution problems.
7. For the standard VRPB, the most common solution methods are heuristics with 15 references, followed by only two exact methods.

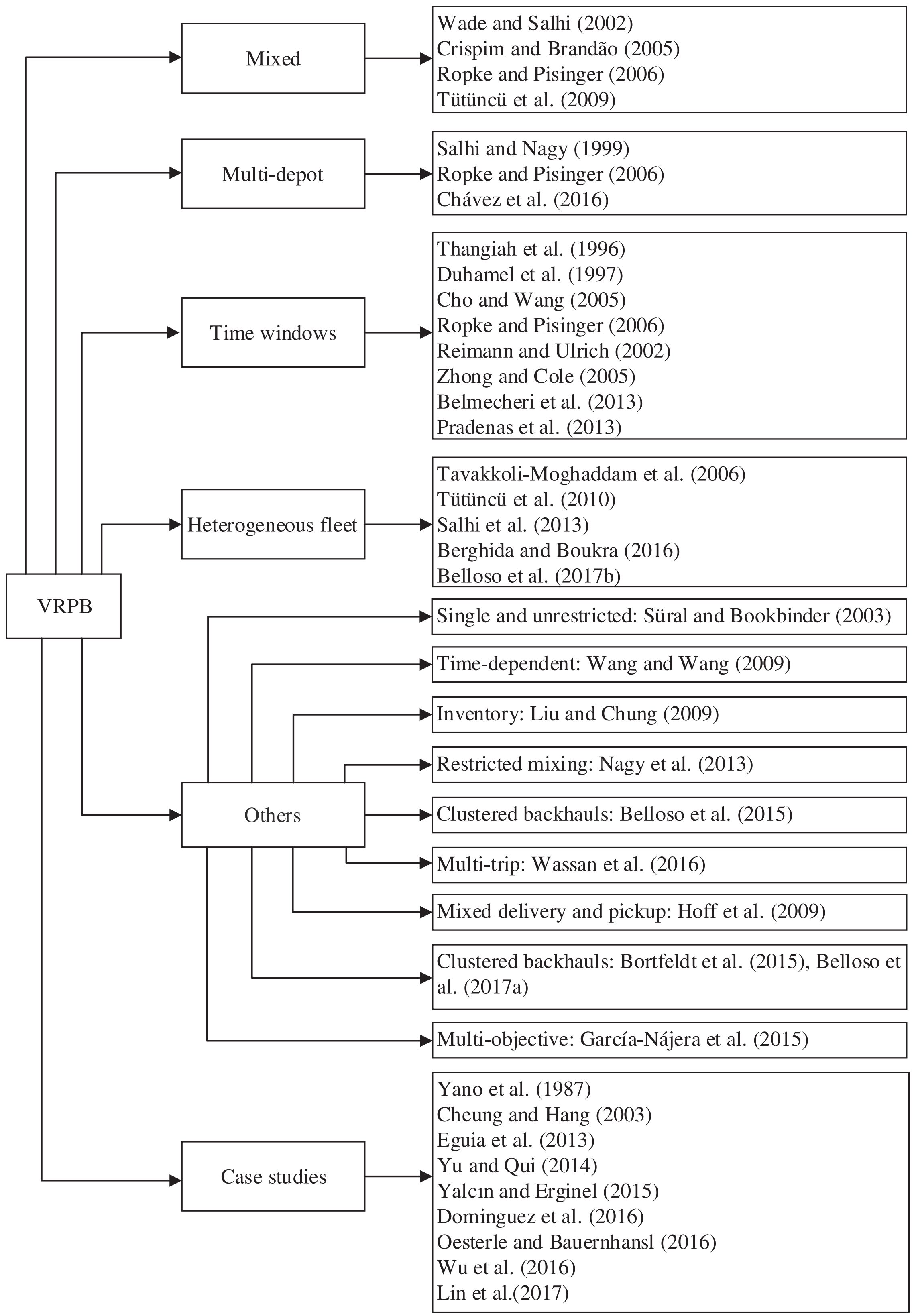

Fig. 1. A classification of the VRPB.

Table 3
Literature on the VRPB variants and case studies.

	References	Side problem	Mathematical model	Solution method	Algorithm	Case study
1	Yano et al. (1987)	FC		Exact	SC, BB	•
2	Thangiah et al. (1996)	TW		Heuristic	CH, LS	
3	Duhamel et al. (1997)	TW		Heuristic	TS, LS	
4	Salhi and Nagy (1999)	MD		Heuristic	CH	
5	Wade and Salhi (2002)	MX	•	Heuristic	CH	
6	Süral and Bookbinder (2003)	UB	•	Exact	MIP, VA	
7	Cheung and Hang (2003)	TW	•	Heuristic	LM	•
8	Cho and Wang (2005)	TW	•	Heuristic	TA	
9	Crispim and Brandão (2005)	MX		Heuristic	TS, VNS	
10	Zhong and Cole (2005)	TW		Heuristic	LS	
11	Reimann and Ulrich (2006)	MX, TW		Heuristic	ACO	
12	Ropke and Pisinger (2006)	MX, MD, TW		Heuristic	ALNS	
13	Tavakkoli-Moghaddam et al. (2006)	HF	•	Heuristic	PS	
14	Hoff et al. (2009)	MDP	•	Heuristic	TS	
15	Liu and Chung (2009)	IN	•	Heuristic	VNS, TS	
16	Tütüncü et al. (2009)	MX		Heuristic	AMP	
17	Wang and Wang (2009)	TD		Heuristic	CH, TS	
18	Tütüncü (2010)	HF		Heuristic	AMP	
19	Belmecheri et al. (2013)	MX, HF, TW	•	Heuristic	PSO, ACO, LS	
20	Eguia et al. (2013)	GR, HF, TW	•	Heuristic	CH	•
21	Nagy et al. (2013)	RM	•	Heuristic	TS	
22	Pradenas et al. (2013)	TW, GR	•	Heuristic	SS	
23	Salhi et al. (2013)	HF	•	Heuristic	SP, VA	
24	Yu and Qi (2014)	TW, MUD, MT	•	Heuristic	TS	•
25	Belloso et al. (2015)	CB		Heuristic	CH, LS	
26	Bortfeldt et al. (2015)	CB, 3D		Heuristic	LNS, VNS	
27	García-Nájera et al. (2015)	MO		Heuristic	PS	
28	Chávez et al. (2016)	MD, MO		Heuristic	ACO	
29	Dominguez et al. (2016)	TWD	•	Heuristic	LNS	•
30	Oesterle and Bauernhansl (2016)	MX, HF, TW, MC	•	Heuristic	CH	•
31	Wassan et al. (2016)	MT	•	Heuristic	VNS, LS	
32	Wu et al. (2016)	MX, HF, TW	•	Heuristic	ACO	•
33	Berghida and Boukra (2016)	MX, HF	•	Heuristic	QIHS	
34	Belloso et al. (2017a)	MX, CB		Heuristic	BR	
35	Belloso et al. (2017b)	HF		Heuristic	BR	
36	Lin et al. (2017)	TW, LIFO, HF, LW	•	Heuristic	GRASP, TS	•

8. Concerning the standard VRPB algorithms, the most common ones are constructive heuristics with four references, followed by tabu search with three references, and iterated local search with two references.
9. Concerning the VRPB variants, extensions, and case studies, the most common are constructive heuristics with six references, tabu search with seven references, local search with six references, ant colony optimization with four references, and variable neighborhood search with four references.

7. Conclusions and research perspectives

The Vehicle Routing Problem with Backhauls (VRPB) is rooted in the seminal paper of Deif and Bodin (1984) and has given rise to several important contributions especially in the last decade. A wide range of the VRPB variants have been studied and their industrial applications are encountered in many settings. We have presented a comprehensive and up-to-date review of the existing studies. We have comparatively analyzed the performance of the state-of-the-art heuristics for the standard VRPB. We have included references related to the standard VRPB, variants, extensions, industrial applications and case studies. We have classified the VRPB literature under several dimensions: the standard VRPB, the mixed VRPB, the multi-depot VRPB, the VRPB with time windows, the VRPB with heterogeneous fleet, and miscellaneous versions of VRPBs.

The following conclusions and future research perspectives are identified:

1. The standard VRPB instances of Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997) has been effectively solved by heuristics. However, it is our belief that further studies should focus on developing effective and powerful exact methods, such as branch-and-cut-and-price, to solve all available standard VRPB instances to optimality (see Poggi and Uchoa, 2014). In addition, it would also be effective to use matheuristic algorithms which are often used in vehicle routing and obtained by the interoperation of metaheuristics and mathematical programming techniques (see Archetti and Speranza, 2014; Boschetti et al., 2009; Koç et al., 2017).
2. Most of the research effort (36 papers) has focused on the study of several extensions of the VRPB such as multiple depots, multiple trips, two-dimensional loading, and time windows. We believe that there exist numerous research opportunities on these rich VRPB variants (see Lahyani et al., 2015).
3. Only one study considered the time-dependent version of the VRPB (Wang and Wang, 2009). In the context of the urban settings and in city logistics, it would be more realistic to consider time-dependencies on the standard VRPB and its variants. (see Bektaş et al., 2014; Gendreau et al., 2015).
4. No study has yet considered the continuous approximation model for the VRPB. Further studies should focus on developing effective continuous approximation models for the standard VRPB and its extensions (see Franceschetti et al., 2017; Jabali et al., 2012; Langevin et al., 1996).
5. In the last 15 years, there has been an increase in the study of stochastic versions of the VRP that deal with routing problems in which some of the key problem parameters are not known with certainty, and an increasing amount of data enabling the understanding of various stochastic phenomena. To our knowledge, no study has yet considered stochastic versions of the

VRPB. Stochastic extensions could be applied to the standard VRPB and to rich extensions of the VRPB (see Gendreau et al., 2016).

6. In the last decade, we have witnessed an increasing trend in electric VRP studies (see Juan et al., 2016; Koç and Karaoglan, 2016; Margaritis et al., 2016; Pelletier et al., 2016). To our knowledge, no electric vehicle version has yet been studied for the VRPB. We believe there exist numerous and meaningful research opportunities on electric VRPB extensions.
7. No exact algorithm has yet been proposed for the time windows extension of the VRPB (see Desaulniers et al., 2014; Kallehauge, 2008; Koç et al., 2015). This type of effective algorithms could be applied to the VRPB with time windows.
8. Most VRPB algorithms are heuristics (49 papers) since this problem is rather hard to solve. In the early years, simple interchange schemes are used, but later more advanced procedures are developed. However, the main methods are the constructive heuristics complemented with local search heuristics. Further studies should focus on hybrid algorithms such as combining population search and adaptive large neighborhood search or iterated local search (see Koç et al., 2014; Laporte et al., 2014; Vidal et al., 2014).

To conclude, it seems appropriate to say that the field of VRPBs is still have an opportunity of development. Given the advances in heuristic and exact algorithms for VRPs, as well as in computer technology, there is room for a very significant research effort on models and solution methods for VRPBs. We believe that this paper will encourage other researchers to conduct their studies on this rich research area.

Acknowledgments

The authors gratefully acknowledge funding provided by the Canadian Natural Sciences and Engineering Research Council under grant 2015-06189. Thanks are due to two referees for their valuable comments.

Appendix

Table A.1 provides a detailed computational results of comparison of recent metaheuristics on the standard VRPB where a entry with "*" indicates that the value is optimal.

Table A.1
Detailed comparison of recent metaheuristics on the standard VRPB.

Instance	BKS	B06	RP06	GA09	ZK12	CGSA14	VCGP14	B16
A1	229,886*	229,886	229,886	229,886	229,886	229,886	229,890	229,886
A2	180,119*	180,119	180,119	180,119	180,119	180,119	180,120	180,119
A3	163,405*	163,405	163,405	163,405	163,405	163,405	163,410	163,405
A4	155,796*	155,796	155,796	155,796	155,796	155,796	155,800	155,796
B1	239,080*	239,080	239,080	239,080	239,080	239,080	239,080	239,080
B2	198,048*	198,048	198,048	198,048	198,048	198,048	198,050	198,048
B3	169,372*	169,372	169,372	169,372	169,372	169,372	169,370	169,372
C1	249,448*	250,557	250,557	250,557	250,556	250,556	250,560	250,557
C2	215,020*	215,020	215,020	215,020	215,020	215,020	215,020	215,020
C3	199,346*	199,346	199,346	199,346	199,346	199,346	199,350	199,346
C4	195,366*	195,366	195,367	195,367	195,366	195,366	195,370	195,366
D1	322,530*	322,530	322,530	322,530	322,530	322,530	322,530	322,530
D2	316,709*	316,709	316,709	316,709	316,708	316,708	316,710	316,709
D3	239,479*	239,479	239,479	239,479	239,479	239,479	239,480	239,479
D4	205,832*	205,832	205,832	205,832	205,832	205,832	205,830	205,832
E1	238,880*	238,880	238,880	238,880	238,880	238,880	238,880	238,880
E2	212,263*	212,263	212,263	212,263	212,263	212,263	212,260	212,263
E3	206,659*	206,659	206,659	206,659	206,659	206,659	206,660	207,051
F1	263,173*	263,173	267,060	263,174	263,173	263,173	263,170	263,173
F2	265,213*	265,493	265,214	265,214	265,213	265,213	265,210	265,213
F3	241,120*	241,120	241,970	241,121	241,120	241,120	241,120	241,120
F4	233,861*	233,861	235,175	233,862	233,861	233,861	233,860	233,861
G1	306,306*	306,306	306,305	306,537	306,306	306,306	245,440	306,306
G2	245,441*	245,441	245,441	245,441	245,441	245,441	–	245,441
G3	229,507*	229,507	229,507	229,507	229,507	229,507	229,510	229,507
G4	232,521*	232,521	232,521	232,521	232,521	232,521	232,520	232,521
G5	221,730*	221,730	221,730	221,730	221,730	221,730	221,730	221,730
G6	213,457*	213,457	213,457	213,457	213,457	213,457	213,460	213,457
H1	268,933*	268,933	268,933	268,933	268,933	268,933	268,930	268,933
H2	253,365*	253,365	253,366	253,366	253,365	253,365	253,370	253,365
H3	247,449*	247,449	247,449	247,449	247,449	247,449	247,450	247,449
H4	250,221	250,221	250,221	250,221	250,221	250,221	250,220	250,221
H5	246,121*	246,121	246,121	246,121	246,121	246,121	246,120	246,121
H6	249,135*	249,135	249,135	249,135	249,135	249,135	249,140	249,135
I1	350,246	350,435	350,245	350,245	350,246	350,246	350,250	350,246
I2	309,944*	309,944	309,944	309,944	309,944	309,944	309,940	309,944
I3	294,507	294,507	294,507	294,507	294,507	294,507	294,510	294,507
I4	295,988	295,988	295,988	295,988	295,988	295,988	295,990	297,237
I5	301,226	301,236	301,236	301,236	301,236	301,236	301,240	302,380
J1	335,007	335,007	335,007	335,007	335,006	335,006	335,010	335,007
J2	310,417	310,793	310,417	310,417	310,417	310,417	310,420	310,417
J3	279,219	279,306	279,219	279,219	279,219	279,219	279,220	279,219
J4	296,533	296,860	296,533	296,533	296,533	296,533	296,530	297,615

(*continued on next page*)

Table A.1 (*continued*)

Instance	BKS	B06	RP06	GA09	ZK12	CGSA14	VCGP14	B16
K1	394,071	394,974	394,376	395,076	394,071	394,071	394,070	395,076
K2	362,130	363,829	362,130	362,130	362,130	362,130	362,130	365,754
K3	365,694	366,246	365,694	365,694	365,694	365,694	365,690	369,049
K4	348,950	351,345	348,949	349,870	348,950	348,950	348,950	350,304
L1	417,897	426,401	426,013	417,922	417,896	417,896	417,900	418,452
L2	401,228	402,152	401,229	401,248	401,228	401,228	401,230	401,965
L3	402,678	404,391	402,678	402,678	402,678	402,678	402,680	403,151
L4	384,637	384,999	384,636	384,636	384,636	384,636	384,640	385,552
L5	387,565	389,044	387,565	387,565	387,565	387,565	387,560	388,532
M1	398,593	400,384	398,914	398,730	398,593	398,593	398,590	401,447
M2	396,917	398,924	399,336	397,324	396,917	396,917	396,920	399,301
M3	375,695	377,433	377,212	377,329	375,696	375,696	375,700	377,979
M4	348,140	349,091	348,418	348,418	348,140	348,140	348,140	348,671
N1	408,101	409,531	410,789	408,101	408,101	408,101	408,100	408,690
N2	408,066	408,287	409,385	408,065	408,066	408,066	408,070	409,360
N3	394,338	394,338	394,338	394,338	394,338	394,338	394,340	394,547
N4	394,788	399,029	398,965	394,788	394,788	394,788	394,790	394,998
N5	373,477	376,522	373,476	373,723	373,476	373,476	373,480	378,985
N6	373,759	374,774	373,759	373,759	373,759	373,759	373,760	376,882
O1	526,261	–	–	–	–	478,348	–	484,000
O2	507,125	–	–	–	–	477,256	–	478,202
O3	491,935	–	–	–	–	457,294	–	459,495
O4	489,074	–	–	–	–	458,875	–	464,797
O5	479,438	–	–	–	–	436,974	–	442,378
O6	476,391	–	–	–	–	438,005	–	444,138

References

Archetti, C., Speranza, M.G., 2014. A survey on matheuristics for routing problems. EURO J. Comput. Optim. 2, 223–246.

Bartholdi, J., Platzman, L., 1982. An $o(n \log n)$ planar traveling salesman heuristic based on spacefilling curves. Oper. Res. Lett. 1, 121–125.

Bektaş, T., Laporte, G., 2011. The pollution-routing problem. Transp. Res. Part B 45, 1232–1250.

Bektaş, T., Repoussis, P.P., Tarantilis, C.D., 2014. Dynamic vehicle routing problems. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications. In: MOS-SIAM Series on Optimization, Philadelphia. MOS-SIAM, pp. 299–347.

Belloso, J., Juan, A.A., Faulin, J., 2017b. An iterative biased-Érandomized heuristic for the fleet size and mix vehicle-routing problem with backhauls. Int. Trans. Oper. Res. (in press).

Belloso, J., Juan, A.A., Faulin, J., Serrano, A., 2015. Using multi-start biased randomization of heuristics to solve the vehicle routing problem with clustered backhauls. Lect. Not. Manag. Sci. 7, 15–20.

Belloso, J., Juan, A.A., Martinez, E., Faulin, J., 2017a. A biased-Érandomized metaheuristic for the vehicle routing problem with clustered and mixed backhauls. Networks 69, 241–255.

Belmecheri, F., Prins, C., Yalaoui, F., Amodeo, L., 2013. Particle swarm optimization algorithm for a vehicle routing problem with heterogeneous fleet, mixed backhauls, and time windows. J. Intell. Manuf. 24, 775–789.

Berbeglia, G., Cordeau, J.-F., Gribkovskaia, I., Laporte, G., 2007. Static pickup and delivery problems: a classification scheme and survey. TOP Off. J. Span. Soc. Stat. Oper. Res. 15, 1–31.

Berghida, M., Boukra, A., 2016. Quantum inspired algorithm for a VRP with heterogeneous fleet mixed backhauls and time windows. Int. J. Appl. Metaheuristic Comput. 7, 18–38.

Bortfeldt, A., Hahn, T., Männel, D., Mönch, L., 2015. Hybrid algorithms for the vehicle routing problem with clustered backhauls and 3D loading constraints. Eur. J. Oper. Res. 243, 82–96.

Boschetti, M.A., Maniezzo, V., Roffilli, M., Röhler, A.B., 2009. Matheuristics: optimization, simulation and control. In: Proceedings of the 2009 International Workshop on Hybrid Metaheuristics. Springer, Berlin Heidelberg, pp. 171–177.

Brandão, J., 2006. A new tabu search algorithm for the vehicle routing problem with backhauls. Eur. J. Oper. Res. 173, 540–555.

Brandão, J., 2011. A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. Comput. Oper. Res. 38, 140–151.

Brandão, J., 2016. A deterministic iterated local search algorithm for the vehicle routing problem with backhauls. TOP Off. J. Span. Soc. Stat. Oper. Res. 24, 445–465.

Casco, D.O., Golden, B.L., Wasil, E.A., 1988. Vehicle routing with backhauls: models, algorithms, and case studies. In: Golden, B.L., Assad, A.A. (Eds.), Vehicle Routing: Methods and Studies. Elsevier, Amsterdam, pp. 127–147.

Chávez, J.J., Escobar, J.W., Echeverri, M.G., 2016. A multi-objective pareto ant colony algorithm for the multi-depot vehicle routing problem with backhauls. Int. J. Ind. Eng. Comput. 7, 35–48.

Cheung, R.K., Hang, D.D., 2003. Multi-attribute label matching algorithms for vehicle routing problems with time windows and backhauls. IIE Trans. 35, 191–205.

Cho, Y.J., Wang, S.D., 2005. A threshold accepting meta-heuristic for the vehicle routing problem with backhauls and time windows. J. East. Asia Soc. Transp. Stud. 6, 3022–3037.

Clarke, G., Wright, J.W., 1964. Scheduling of vehicles from a central depot to a number of delivery points. Oper. Res. 12, 568–581.

Coca-Cola, 2017. http://www.coca-cola.com/global/glp.html (accessed 03.01.2017).

Cordeau, J.-F., Laporte, G., Savelsbergh, M.W.P., Vigo, D., 2007. Vehicle routing. In: Barnhart, C., Laporte, G. (Eds.), Transportation, Handbooks in Operations Research and Management Science. Elsevier, Amsterdam, pp. 367–428.

Crispim, J., Brandão, J., 2005. Metaheuristics applied to mixed and simultaneous extensions of vehicle routing problems with backhauls. J. Oper. Res. Soc. 56, 1296–1302.

Cuervo, D.P., Goos, P., Sörensen, K., Arráiz, E., 2014. An iterated local search algorithm for the vehicle routing problem with backhauls. Eur. J. Oper. Res. 237, 454–464.

Deif, I., Bodin, L.D., 1984. Extension of the clarke and wright algorithm for solving the vehicle routing problem with backhauling. In: Proceedings of the Babson Conference on Software Uses in Transportation and Logistics Management, Babson Park, MA.

Desaulniers, G., Madsen, O.B.G., Ropke, S., 2014. The vehicle routing problem with time windows. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications. In: MOS-SIAM Series on Optimization, Philadelphia, pp. 119–159.

Doerner, K.F., Gutjahr, W.J., Hartl, R.F., Strauss, C., Stummer, C., 2004. Pareto ant colony optimization: a metaheuristic approach to multiobjective portfolio selection. Ann. Oper. Res. 131, 79–99.

Dominguez, O., Guimarans, D., Juan, A.A., de la Nuez, I., 2016. A biased-randomised large neighbourhood search for the two-dimensional vehicle routing problem with backhauls. Eur. J. Oper. Res. 255, 442–462.

Dueck, G., 1993. New optimization heuristics: the great deluge algorithm and the record-to-record travel. J. Comput. Phys. 104, 86–92.

Duhamel, C., Potvin, J.-Y., Rousseau, J.-M., 1997. A tabu search heuristic for the vehicle routing problem with backhauls and time windows. Transp. Sci. 31, 49–59.

Eguia, I., Racero, J., Molina, J.C., Guerrero, F., 2013. Environmental issues in vehicle routing problems. In: Sustainability Appraisal: Quantitative Methods and Mathematical Techniques for Environmental Performance Evaluation. Springer, Berlin Heidelberg, pp. 215–241.

Fisher, M.L., Jaikumar, R., 1981. A generalized assignment heuristic for the vehicle routing problem. Networks 11, 109–124.

Franceschetti, A., Jabali, O., Laporte, G., 2017. Continuous approximation models in freight distribution management. TOP Off. J. Span. Soc. Stat. Oper. Res. 25, 413–433.

Gajpal, Y., Abad, P.L., 2009. Multi-ant colony system (MACS) for a vehicle routing problem with backhauls. Eur. J. Oper. Res. 196, 102–117.

Gambardella, L.M., Taillard, E.D., Agazzi, G., 1999. MACS-VRPTW: a multiple ant colony system for vehicle routing problems with time windows. In: Corne, D., Dorigo, M., Glover, F. (Eds.), New Ideas in Optimization. McGraw-Hill, London, pp. 63–76.

García-Nájera, A., Bullinaria, J.A., Gutiérrez-Andrade, M.A., 2015. An evolutionary approach for multi-objective vehicle routing problems with backhauls. Comput. Ind. Eng. 81, 90–108.

Gélinas, S., Desrochers, M., Desrosiers, J., Solomon, M. M., 1992. Vehicle routing with backhauling. GERAD Technical Report G-92-13, Montréal.

Gélinas, S., Desrochers, M., Desrosiers, J., Solomon, M.M., 1995. A new branching strategy for time constrained routing problems with application to backhauling. Ann. Oper. Res. 61, 91–109.
Gendreau, M., Ghiani, G., Guerriero, E., 2015. Time-dependent routing problems: a review. Comput. Oper. Res. 64, 189–197.
Gendreau, M., Jabali, O., Rei, W., 2016. Future research directions in stochastic vehicle routing. Transp. Sci. 50, 1163–1173.
Ghaziri, H., Osman, I.H., 2006. Self-organizing feature maps for the vehicle routing problem with backhauls. J. Sched. 9, 97–114.
Gillett, B.E., Johnson, J.G., 1976. Multi-terminal vehicle dispatch algorithm. Omega (Westport) 4, 711–718.
Goetschalckx, M., Jacobs-Blecha, C., 1989. The vehicle routing problem with backhauls. Eur. J. Oper. Res. 42, 39–51.
Goetschalckx, M., Jacobs-Blecha, C., 1993. The vehicle routing problem with backhauls: properties and solution algorithms. Georgia Institute of Technology, Atlanta Technical report mhrc-tr-88-13.
Golden, B.L., Raghavan, S., Wasil, E.A., 2008. The Vehicle Routing Problem: Latest Advances and New Challenges. Springer, New York.
Hoff, A., Gribkovskaia, I., Laporte, G., Lø kketangen, A., 2009. Lasso solution strategies for the vehicle routing problem with pickups and deliveries. Eur. J. Oper. Res. 192, 755–766.
Irnich, S., Schneider, M., Vigo, D., 2014. Four variants of the vehicle routing problem. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications. In: MOS-SIAM Series on Optimization, pp. 241–271. Philadelphia
Jabali, O., Gendreau, M., Laporte, G., 2012. A continuous approximation model for the fleet composition problem. Transp. Res. Part B 46, 1591–1606.
Juan, A.A., Mendez, C.A., Faulin, J., de Armas, J., Grasman, S.E., 2016. Electric vehicles in logistics and transportation: a survey on emerging environmental, strategic, and operational challenges. Energies 9, 1–21.
Kallehauge, B., 2008. Formulations and exact algorithms for the vehicle routing problem with time windows. Comput. Oper. Res. 35, 2307–2330.
Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2014. The fleet size and mix pollution-routing problem. Transp. Res. Part B 70, 239–254.
Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2015. A hybrid evolutionary algorithm for heterogeneous fleet vehicle routing problems. Comput. Oper. Res. 64, 11–27.
Koç, Ç., Jabali, O., Laporte, G., 2017. Long-haul vehicle routing and scheduling with idling options. J. Oper. Res. Soc. (in press).
Koç, Ç., Karaoglan, I., 2016. The green vehicle routing problem: a heuristic based exact solution approach. Appl. Soft Comput. 39, 154–164.
Kontoravdis, G., Bard, J.F., 1995. A GRASP for the vehicle routing problem with time windows. ORSA J. Comput. 7, 10–23.
Lahyani, R., Khemakhem, M., Semet, F., 2015. Rich vehicle routing problems: from a taxonomy to a definition. Eur. J. Oper. Res. 241, 1–14.
Langevin, A., Mbaraga, P., Campbell, J.F., 1996. Continuous approximation models in freight distribution: an overview. Transp. Res. Part B 30, 163–188.
Laporte, G., 2009. Fifty years of vehicle routing. Transp. Sci. 43, 408–416.
Laporte, G., Ropke, S., Vidal, T., 2014. Heuristics for the vehicle routing problem. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods and Applications. In: MOS-SIAM Series in Optimization, Philadelphia, pp. 87–116.
Lin, S., Bard, J.F., Jarrah, A.I., Zhang, X., Novoa, L.J., 2017. Route design for last-in, first-out deliveries with backhauling. Transp. Res. Part C 76, 90–117.
Liu, S.C., Chung, C.H., 2009. A heuristic method for the vehicle routing problem with backhauls and inventory. J. Intell. Manuf. 20, 29–42.
Margaritis, D., Anagnostopoulou, A., Tromaras, A., Boile, M., 2016. Electric commercial vehicles: practical perspectives and future research directions. Res. Transp. Bus. Manag. 18, 4–10.
Miller, C.E., Tucker, A.W., Zemlin, R.A., 1960. Integer programming formulations and traveling salesman problems. J. Assoc. Comput. Mach. 7, 326–329.
Mingozzi, A., Giorgi, S., Baldacci, R., 1999. An exact method for the vehicle routing problem with backhauls. Transp. Sci. 33, 315–329.
Nagy, G., Salhi, S., 2005. Heuristic algorithms for single and multiple depot vehicle routing problems with pickups and deliveries. Eur. J. Oper. Res. 162, 126–141.
Nagy, G., Wassan, N.A., Salhi, S., 2013. The vehicle routing problem with restricted mixing of deliveries and pickups. J. Sched. 16, 199–213.
Novoa, L.J., Jarrah, A.I., Bard, J.F., Lin, S., Zhang, X., 2016. Optimizing Daily Service Routes for Major Grocery Chains Working paper. The George Washington University, Washington, DC.
Oesterle, J., Bauernhansl, T., 2016. Exact method for the vehicle routing problem with mixed linehaul and backhaul customers, heterogeneous fleet, time window and manufacturing capacity. Procedia CIRP (Collège International pour la Recherche en Productique) 41, 573–578.
Osman, I.H., Wassan, N.A., 2002. A reactive tabu search metaÉheuristic for the vehicle routing problem with backÉhauls. J. Sched. 5, 263–285.
Parragh, S.N., Doerner, K.F., Hartl, R.F., 2008a. A survey on pickup and delivery problems. part I: transportation between customers and depot. J. Betr. 58, 21–51.
Parragh, S.N., Doerner, K.F., Hartl, R.F., 2008b. A survey on pickup and delivery problems. Part II: transportation between pickup and delivery locations. J. Betr. 58, 81–117.
Pelletier, S., Jabali, O., Laporte, G., 2016. Goods distribution with electric vehicles: review and research perspectives. Transp. Sci. 50, 3–22.
Pisinger, D., Ropke, S., 2010. Large neighborhood search. In: Gendreau, M., Potvin, J.-Y. (Eds.), Handbook of Metaheuristics. Springer, New York, pp. 399–419.
Poggi, M., Uchoa, E., 2014. New exact algorithms for the capacitated vehicle routing problem. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications. In: MOS-SIAM Series on Optimization, Philadelphia, pp. 59–86.
Pradenas, L., Oportus, B., Parada, V., 2013. Mitigation of greenhouse gas emissions in vehicle routing problems with backhauling. Expert Syst. Appl. 40, 2985–2991.
Reimann, M., Ulrich, H., 2006. Comparing backhauling strategies in vehicle routing using ant colony optimization. Cent. Eur. J. Oper. Res. 14, 105–123.
Ropke, S., Pisinger, D., 2006. A unified heuristic for a large class of vehicle routing problems with backhauls. Eur. J. Oper. Res. 171, 750–775.
Salhi, S., Nagy, G., 1999. A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. J. Oper. Res. Soc. 50, 1034–1042.
Salhi, S., Rand, G., 1987. Improvements to vehicle routing heuristics. J. Oper. Res. Soc. 38, 293–295.
Salhi, S., Wassan, N.A., Hajarat, M., 2013. The fleet size and mix vehicle routing problem with backhauls: formulation and set partitioning-based heuristics. Transp. Res. Part E 56, 22–35.
Solomon, M.M., 1987. Algorithms for the vehicle routing and scheduling problem with time window constraints. Oper. Res. 35, 254–265.
Süral, H., Bookbinder, J.H., 2003. The single-vehicle routing problem with unrestricted backhauls. Networks 41, 127–136.
Taillard, E.D., 1999. A heuristic column generation method for the heterogeneous fleet vehicle routing problem. RAIRO (Recherche Opérationnelle/Oper. Res.) 33, 1–14.
Taillard, E.D., Laporte, G., Gendreau, M., 1996. Vehicle routing with multiple use of vehicles. Eur. J. Oper. Res. 47, 1065–1070.
Tavakkoli-Moghaddam, R., Saremi, A.R., Ziaee, M.S., 2006. A memetic algorithm for a vehicle routing problem with backhauls. Appl. Math. Comput. 181, 1049–1060.
Thangiah, S.R., Potvin, J.-Y., Sun, T., 1996. Heuristic approaches to vehicle routing with backhauls and time windows. Comput. Oper. Res. 23, 1043–1057.
Toth, P., Vigo, D., 1996. Heuristic algorithms for the vehicle routing problem with backhauls. In: Bianco, L., Toth, P. (Eds.), Advanced Methods in Transportation Analysis. Springer-Verlag, Berlin, pp. 585–608.
Toth, P., Vigo, D., 1997. An exact algorithm for the vehicle routing problem with backhauls. Transp. Sci. 31, 372–385.
Toth, P., Vigo, D., 1999. A heuristic algorithm for the symmetric and asymmetric vehicle routing problems with backhauls. Eur. J. Oper. Res. 113, 528–543.
Toth, P., Vigo, D., 2002. VRP with backhauls. In: Toth, P., Vigo, D. (Eds.), The Vehicle Routing Problem. SIAM, Philadelphia, pp. 195–224.
Toth, P., Vigo, D., 2014. Vehicle routing: problems, methods, and applications. In: Toth, P., Vigo, D. (Eds.). MOS-SIAM Series on Optimization, Philadelphia.
Tütüncü, G.Y., 2010. An interactive GRAMPS algorithm for the heterogeneous fixed fleet vehicle routing problem with and without backhauls. Eur. J. Oper. Res. 201, 593–600.
Tütüncü, G.Y., Carreto, C.A., Baker, B.M., 2009. A visual interactive approach to classical and mixed vehicle routing problems with backhauls. Omega (Westport) 37, 138–154.
Vidal, T., Crainic, T.G., Gendreau, M., Prins, C., 2014. A unified solution framework for multi-attribute vehicle routing problems. Eur. J. Oper. Res. 234, 658–673.
Wade, A.C., Salhi, S., 2002. An investigation into a new class of vehicle routing problem with backhauls. Omega (Westport) 30, 479–487.
Walmart, 2017. https://www.walmart.com/ (accessed 03.01.2017).
Wang, Z., Wang, Z., 2009. A novel two-phase heuristic method for vehicle routing problem with backhauls. Comput. Math. Appl. 57, 1923–1928.
Wassan, N., Wassan, N., Nagy, G., Salhi, S., 2016. The multiple trip vehicle routing problem with backhauls: formulation and a two-level variable neighbourhood search. Comput. Oper. Res. 78, 454–467.
Wassan, N.A., 2006. A reactive tabu search for the vehicle routing problem. J. Oper. Res. Soc. 57, 111–116.
Wassan, N.A., 2007. Reactive tabu adaptive memory programming search for the vehicle routing problem with backhauls. J. Oper. Res. Soc. 58, 1630–1641.
Wassan, N.A., Nagy, G., Ahmadi, S., 2008. A heuristic method for the vehicle routing problem with mixed deliveries and pickups. J. Sched. 11, 149–161.
Wu, W., Tian, Y., Jin, T., 2016. A label based ant colony algorithm for heterogeneous vehicle routing with mixed backhaul. Appl. Soft Comput. 47, 224–234.
Yalcın, G.D., Erginel, N., 2015. Fuzzy multi-objective programming algorithm for vehicle routing problems with backhauls. Expert Syst. Appl. 42, 5632–5644.
Yano, C.A., Chan, T.J., Richter, L.K., Cutler, T., Murty, K.G., McGettigan, D., 1987. Vehicle routing at quality stores. Interfaces (Providence) 17, 52–63.
Yu, M., Qi, X., 2014. A vehicle routing problem with multiple overlapped batches. Transp. Res. Part E 61, 40–55.
Zachariadis, E.E., Kiranoudis, C.T., 2012. An effective local search approach for the vehicle routing problem with backhauls. Expert Syst. Appl. 39, 3174–3184.
Zhong, Y., Cole, M.H., 2005. A vehicle routing problem with backhauls and time windows: a guided local search solution. Transp. Res. Part E 41, 131–144.