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A Unified Solution Framework for Multi-Attribute Vehicle Routing Problems†

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Abstract. Vehicle routing attributes are extra characteristics and decisions that complement the academic problem formulations and aim to properly account for real-life application needs. Hundreds of methods have been introduced in recent years for specific attributes, but the development of a single, general-purpose algorithm, which is both efficient and applicable to a wide family of variants remains a considerable challenge. Yet, such a development is critical for understanding the proper impact of attributes on resolution approaches, and to answer the needs of actual applications. This paper contributes towards addressing these challenges with a component-based design for heuristics, targeting multi-attribute vehicle routing problems, and an efficient general-purpose solver. The proposed Unified Hybrid Genetic Search metaheuristic relies on problem-independent unified local search, genetic operators, and advanced diversity management methods. Problem specifics are confined to a limited part of the method and are addressed by means of assignment, sequencing, and route-evaluation components, which are automatically selected and adapted and provide the fundamental operators to manage attribute specificities. Extensive computational experiments on 29 prominent vehicle routing variants, 42 benchmark instance sets and overall 1099 instances, demonstrate the remarkable performance of the method which matches or outperforms the current state-of-the-art problem-tailored algorithms.

Keywords: Vehicle routing, multiple attributes, general-purpose solver.

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1 Introduction

General-purpose solvers for combinatorial optimization are algorithms that can be used to address large classes of problem settings without requiring extensive adaptations, user involvement or expertise. The development of such solvers is critical to the understanding of the impact of problem characteristics on the performance of solution methods, as well as to the capability to efficiently address new problem settings and applications displaying particular sets of characteristic combinations. One thus aims for high-performance general-purpose solvers, achieving a subtle balance between generality of scope and specificity in exploiting particular problem characteristics, to identify high-quality solutions for the broadest set of problem settings possible within limited computation time. Such developments are very challenging. As illustrated by Wolpert (1997), generality may be paid for in terms of performance, while dedicated algorithms cannot address problem variants without extensive adaptation.

We focus on vehicle routing problems (VRPs), one of the major classes of combinatorial optimization problems with an extremely broad range of applications yielding a very large number of variants born of the requirement to manage a wide variety of characteristics and decisions, called *attributes* in Vidal et al. 2013b, to account for the particular customer, vehicle, driver, and network settings and to combine routing considerations with other tactical or strategic choices. The number of VRP attributes that need to be jointly considered is continuously increasing, yielding a considerable variety of *Multi-Attribute Vehicle Routing Problems* (*MAVRP*s).

The current state-of-the-art and knowledge does not offer the means to use exact solution methods for combinatorial optimization as general-purpose solvers for MAVRPs. Consequently, literally hundreds of papers were published recently, proposing supposedly different heuristic methods for VRP variants with diverse combinations of sets of attributes. As for the most general vehicle routing metaheuristics proposed in the literature (Cordeau et al. 1997, 2001, Ropke and Pisinger 2006a,b, Subramanian et al. 2013), they usually address a single difficult compound problem formulation including several variants as special cases, but still require extensive adaptation when the main problem settings is modified. The field thus lacks an efficient general-purpose MAVRP solver, and building one represents a considerable research challenge. Our objective is to address this challenge and propose a component-based heuristic framework and a general-purpose solver providing high performance in terms of solution quality and computational efficiency for a very broad and diverse set of multi-attribute vehicle routing problem settings. These new contributions may point to promising developments in related fields such as scheduling.

We thus introduce a component-based heuristic solution framework designed in accordance with problem structure and attribute specifics, as well as a Unified Hybrid Genetic Search (UHGS). Any unified method must ultimately account for the specific attributes, objectives, and constraints of the particular problem setting at hand. Yet, to achieve a high level of generality, these problem attributes are confined to restricted adaptive components. Thus, UHGS relies on unified problem-independent procedures: local search, crossover, Split algorithm and diversity management, while problem-specific strategies are restricted to a few modular components which take charge of assignment changes (e.g., of customers to depots or days), enumerations of sequencing alternatives, and route evaluations. These components are self-adapted in relation to the attributes of the problem at hand. Furthermore, to achieve high efficiency during local-improvement procedures, we propose a unified route evaluation methodology based on information preprocessing on sub-sequences, and move evaluations as a concatenation of known sub-sequences. This framework unifies and extends efficient pre-processing techniques which were previously used for different problems.

Extensive computational experiments demonstrate the remarkable performance of the resulting metaheuristic on the classical VRP as well as on MAVRP with multiple periods, multiple depots, vehicle-site dependencies, soft, multiple, and general time windows, backhauls, cumulative or load-dependent costs, simultaneous or mixed pickup and delivery, fleet mix, time dependency, service site choice, driving and working hour regulations, and many of their combinations. With a single implementation, parameter setting and termination criterion, UHGS matches or outperforms all current problem-tailored methods, from more than 180 articles, on 29 vehicle routing variants, 42 benchmark sets and a total of 1099 problem instances. Hence, it appears that generality does not necessarily impede efficiency for the considered problem classes.

The contributions of this work are the following: 1) A component-based heuristic design is proposed for multi-attribute vehicle routing problems, which efficiently isolates problem-specific adaptations from the generic framework; 2) A unified route-evaluation and local search framework, which builds and exploits information on sub-sequences through concatenation operations to efficiently explore neighborhoods. 3) A unified solution representation, *Split* algorithm, and genetic operators; 4) A UHGS which addresses a large set of variants with a single implementation and set of parameters, and yields solutions of exceptional quality on prominent VRP variants and benchmark instance sets.

This paper is structured as follows. Section 2 states the problem, reviews the main classes of general-purpose MAVRP solvers, and introduces the proposed component-based heuristic design. Section 3 details the unified local search and route-evaluation operators. Section 4 describes the UHGS. Computational experiments on a wide range of problems are reported in Section 5. Section 6 concludes.

2 Problem Statement and General Methodology

Vehicle routing problems have been studied for more than 50 years, serving as support for a vast literature, including numerous surveys (see Gendreau et al. 2008, Andersson et al. 2010, Vidal et al. 2013b, among others), books (Toth and Vigo 2002, Golden et al. 2008), and overall more than a thousand dedicated journal articles (Eksioglu et al. 2009). The research effort on the topic is still growing today, because of its major economic impact, the large difficulty of many settings, and the considerable variety of attributes combinations encountered in practice.

2.1 Vehicle routing problems, notations and attributes

The classical Capacitated Vehicle Routing Problem (CVRP) can be stated as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph with $|\mathcal{V}| = n + 1$ vertices, vertex $v_0 \in \mathcal{V}$ representing a depot, where a fleet of m identical vehicles with capacity Q is based, the other vertices $v_i \in \mathcal{V} \backslash \{v_0\}$ for $i \in \{1, \dots, n\}$ representing customers characterized by a demand for q_i units of product. Edges $(i, j) \in \mathcal{E}$ illustrate the possibility to travel from a customer v_i to a customer v_j for a cost d_{ij} (assimilated to the distance). The CVRP requires designing up to m cycles (vehicle routes) starting and ending at a depot v_0 in order to service each customer once.

Many VRP variants with *attributes* have emerged due to the requirements of practical applications. These particular versions aim at better accounting for customer requirements (e.g., time-dependent service costs, time windows, multiple planning periods), network and vehicle characteristics (multiple depots, congestion, heterogeneous fleet, vehicle-site dependencies), driver needs (working hour regulations, lunch breaks), or at better integrating the decisions in a tactical or strategic planning (inventory or location routing). The large variety of actual settings, characteristics and VRP attributes is addressed by a vast literature. For the sake of conciseness, a detailed literature review on all considered VRP variants is out of scope. Comprehensive surveys can be found in Gendreau et al. (2008), Golden et al. (2008), Andersson et al. (2010), and Vidal et al. (2013b).

As in Vidal et al. (2013b), three main categories of attributes are discerned in this paper. ASSIGN attributes are problem particularities requiring decisions on the assignment of customers to some globally constrained ASSIGN Attribute Resources (AARs), for example, depots, days or vehicle types. SEQ attributes are problem characteristics that explicitly impact the structure and geometry of the routes such as, backhaul trips, multiple trips, or multi-echelon attributes. Finally, EVAL attributes affect the way routes are evaluated. This latter class of attributes encompasses advanced route costs or feasibility evaluations, as well as the eventual optimization of additional decisions on routes (e.g., service dates, waiting times, packing of objects in the vehicle) when the sequence of visits if fixed. Each family of attributes thus impacts the resolution methodologies in a very different way.

2.2 General-purpose solution approaches for MAVRPs.

Three main approaches for achieving generality may be identified when analyzing the literature on general-purpose MAVRP solvers that we identify as *rich solvers* and *modeling and solution frameworks*, examined in this subsection, and *component-based frameworks*, which are the topic of the next one.

Rich solvers are designed to address a multi-attribute VRP formulation generalizing several variants associated to subsets of its attributes. Several well-known VRP heuristics are included in this category and are displayed in Table 1: the Unified Tabu Search (UTS; Cordeau et al. 1997, 2001, Cordeau and Laporte 2001, 2003, Cordeau et al. 2004), the Adaptive Large Neighborhood Search algorithm (ALNS; Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007), the Iterated Local Searches of Ibaraki et al. (2005, 2008) and Hashimoto et al. (2006, 2008) (ILS), and Subramanian et al. (2013) (ILS-SP), the latter being hybridized with integer programming components, and the exact integer programming approach of Baldacci and Mingozzi (2009), Baldacci et al. (2011a,b) (IPSP), based on a set partitioning formulation. Keeping in line with the focus of the paper on general-purpose algorithms, the table indicates for each method the largest subset of MAVRPs that was addressed in a **single implementation**, generally the one from the original paper. Most successful methodologies were extended later on to other variants, but separate developments were generally required. The subset of variants addressed by the general-purpose UHGS methodology we propose is also displayed for comparison purposes.

Table 1: Attributes addressed by some well-known rich VRP solvers

Type	Attribute	Acronym	UTS	ALNS	ILS	ILS-SP	IPSP	UHGS
ASSIGN	Multiple depots	MDVRP	X	X		X	X	X
	Multiple periods	PVRP	X				X	X
	Heterogeneous fleet	HVRP				X	X	X
	Site-dependent	SDVRP	X	X			X	X
	Split deliveries	VRPSD						
	Profits	TOP						X
SEQ	Multiple trips	MTVRP						
	Pickup & deliveries	VRPPD	X	X			X	
	Backhauls	VRPB		X				X
EVAL	Asymmetric	AVRP		X		X		X
	Open	OVRP		X		X		X
	Cumulative	CCVRP						X
	Load-dependent costs	LDVRP						X
	Simultaneous P.&D.	VRPSDP		X		X		X
	Mixed P.&D.	VRPSDP		X		X		X
	Vehicle Fleet Mix	VFMP				X	X	X
	Duration constraints	DurVRP	X		X			X
	Hard TW	VRPTW	X	X	X		X	X
	Soft TW	VRPSTW			X			X
	Multiple TW	VRPMTW			X			X
	General TW	VRPGTW			X			X
	Time-dep. travel time	TDVRP			X			X
	Flexible travel time	VRPFTT			X			X
	Lunch breaks	VRPLB						X
	Work hours reg	VRTDSP						X
	Service choice (Generalized VRP)	GVRP[2]						X

Hybrid Genetic Algorithms (HGA), with *giant-tour* solution representations and local search solution enhancements (Prins 2004), have proven their ability in addressing many MAVRPs (Labadi et al.

2008, Prins 2009, Ngueveu et al. 2010, Vidal et al. 2012a), as well as a large class of mixed node and arc routing problem variants (Prins and Bouchenoua 2005). We did not include them in this classification, however, because no unifying implementation of this class of methods has been proposed up to date, particular hard-coded implementations of solution representation, crossover, *Split*, and local search procedures being proposed for different MAVRPs. Generalizing these procedures to a wider range of variants is an important challenge that we address in this paper.

Each rich solver included in Table 1 relies on a "rich" multi-attribute VRP formulation, a periodic VRP with time windows (UTS), a pick-up and delivery problem with time windows (ALNS), a VRP with general time windows, time-dependent, and flexible travel-times (ILS), or a heterogeneous pickup-and-delivery problem with time windows (ILS-SP). Yet, relying on such formulations to achieve generality presents two main limitations. First, problems become more intricate and difficult to address as the number of attributes one must consider simultaneously grows. Second, all the features of the general model are still present when particular variants, with less attributes, are considered, resulting in loss of efficiency through wasted computations induced by deactivated attributes and, sometimes, higher complexity for some algorithm components. The methodology we propose avoids these pitfalls.

Modeling and solution frameworks seek to capture the general properties of the attributes to transform them into machine-readable components. Thus, the framework of Desaulniers et al. (1998) formulates a number of classes of attributes as resources (e.g., load, distance, time), which are extended to successive customer visits through *resource extension functions (REFs)* subject to interval constraints. This framework was applied to various crew scheduling and routing problem variants, the resulting formulations being then solved efficiently by column generation (Desaulniers et al. 2005).

It is well known that the performance of many heuristics for MAVRPs is directly linked to the capability of efficiently evaluating new routes produced during the search. Hence, a large body of literature focuses on reducing the complexity of route evaluation in presence of difficult EVAL attributes (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997, Campbell and Savelsbergh 2004). These approaches share the common characteristic that they develop meaningful information on sub-sequences of successive visits (partial routes) to speed up evaluations of new routes. Using this methodology, time windows, simultaneous pickups and deliveries, and load-dependent costs attributes can be efficiently managed in the course of local searches, leading to notable gains in computational complexity.

Merging these two avenues of research, Irnich (2008b) considered forward and backward extension of resources, as well as the management of generalized resources extension functions on sub-sequences of visits to perform efficient route evaluations. This extended REF methodology was combined with *sequential search* concepts, leading to a unified solution approach (Irnich 2008a). Yet, strong properties on REFs inversion and generalization to segments are required for the framework to apply.

Finally, Puranen (2011) introduced a domain model able to express VRP variants and transform them into a routing metamodel workable by optimization methods. The routing metamodel is based on the concepts of *actors*, *activities*, *resources*, and *capabilities*. It exploits both the concept of resource extension functions, and a generalization called mapping-ordering constraints. The methodology covers the complete resolution process flow, from the domain model, to the routing metamodel and its resolution. However, few computational experiments were presented to demonstrate the capabilities of the approach.

2.3 Proposed component-based framework

As underlined in this review, a few unifying methodologies have been proposed for multi-attribute VRPs. However, these approaches are limited in the classes, properties and number of attributes they manage. Modeling and solution frameworks (Desaulniers et al. 1998, Irnich 2008a,b, Puranen 2011) do provide remarkable formalisms for many attributes, but in counterpart require strong properties to be efficiently applied, such as the existence of REFs which are invertible and generalizable to segments.

In this paper, we proposed a component-based heuristic framework designed in accordance with the problem structure. Any general-purpose solver must ultimately account for the specific attributes, objectives, and constraints of the particular problem setting at hand. In our approach, to achieve a high level of generality, the problem attributes are confined to small polymorphic (Meyer 1997) method components capable of adapting to the problem specifics. We create a library of basic attribute-dependent operator, out of which the algorithm can automatically select the necessary operators in accordance to the problem. Components are designed to offer the possibility to integrate attribute-specific strategies, opening the way to efficient route-evaluation procedures managing meaningful data on sequences.

Some related designs have been used in the combinatorial optimization literature to build general-purpose heuristic solvers or software libraries (e.g., Fink and Voss 2003, Cahon et al. 2004), hyper-heuristics (Burke et al. 2010), and cooperative methods (Crainic and Toulouse 2010). Component-based heuristic approaches are rare in the VRP literature (Du and Wu 2001, Groër et al. 2010). While polymorphism has been efficiently used to generate adaptable resolution strategies, i.e., configurable metaheuristics or local-search strategies, it has not yet provided the means to address the challenge of the broad variability in problem settings. Moreover, although hyper-heuristics and cooperative methods achieve more robust solving by making several basic methods adapt or cooperate, they are still dependent upon the availability of these basic problem-tailored methods.

We restrict this paper to the VRP class in order to keep the length of the paper within acceptable limits. Similarly to several other combinatorial optimization problems, MAVRPs present a particular structure combining decisions on assignment (and partitioning), sequencing, and fixed-sequence optimization and evaluation. Consequently, we identify three categories of attributes, defined relatively to their impact on the heuristic resolution: ASSIGN attributes requiring the assignment of routes and customers to global resources (depots, days, vehicle types), SEQ attributes determining the structure of the network and the sequences of visits, and EVAL attributes modifying the solution evaluations. We introduce three adaptive components, which account for these attributes, and which fulfill the following tasks:

- **Assignment.** Select and check the feasibility of customer and route re-assignments to different ASSIGN attribute resources (day, depot, vehicle type...);
- **Sequence choice.** Generate neighbor solutions with different sequence alternatives with regards to SEQ attributes;
- **Route evaluations.** Evaluate a fixed route and optimize side decisions related to EVAL attributes (timing or loading sub-problems).

We show in the next sections how these components can serve as building blocks for a wide range of general-purpose neighborhood- or population-based metaheuristics for MAVRPs. Section 3 first describes how route-evaluation components can lead to an efficient unified local search, and then Section 4 follows with a description of the proposed Unified Hybrid Genetic Search for MAVRPs.

3 Unified Local Search for Vehicle Routing Problems

Designing a general-purpose high-performance local search for MAVRPs is an important research challenge in itself. We therefore introduce first the methodology we propose to address this challenge, before proceeding to the complete UHGS framework. The emphasis is on *EVAL* attributes, which impact the heuristic resolution during route evaluations, such as loading constraints or timing aspects. In the proposed approach, these problem specifics are confined to *route-evaluation* components, which are adaptive problem-dependent elements of the methodology to perform route, move evaluation, and feasibility statements. Since high performance is sought, these components are designed to manage pre-processed information on sub-sequences during move evaluations. We first define these components, proceeding then to the corresponding route-evaluation operators and, finally, to the unified local search method.

3.1 Route-evaluation components

The route-evaluation components exploit the fact that any local-search move issued from a bounded number of edge exchanges and node relocations can be assimilated to a recombination of a bounded number of sequence of visits from an incumbent solution (Kindervater and Savelsbergh 1997, Vidal et al. 2011). As illustrated in Figure 1, an inter-route RELOCATE move of a sequence of visits $[\sigma_r(u), \dots, \sigma_r(v)]$ next to a visit $\sigma_{r'}(w)$ yields two recombined routes $\rho = [\sigma_r(1), \dots, \sigma_r(u-1)] \oplus [\sigma_r(v+1), \dots, \sigma_r(|r|)]$ and $\rho' = [\sigma_{r'}(1), \dots, \sigma_{r'}(w)] \oplus [\sigma_r(u), \dots, \sigma_r(v)] \oplus [\sigma_{r'}(w+1), \dots, \sigma_{r'}(|r'|)]$, $\oplus$ denoting the concatenation operator.

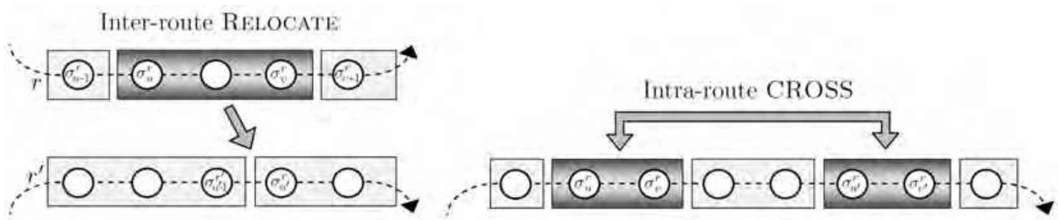

Figure 1: Moves assimilated to recombinations of sequences

We thus introduce in Table 2 five operations of route-evaluation components. The first three operations, called INIT(σ), FORW(σ), and BACK(σ) provide the means to initialize and build the re-optimization information on sequences by forward and backward concatenation of single visits, respectively. Within a local search, they can be used during a pre-processing phase to build the information on sub-sequences. The evaluation of new sequences made of a concatenation of several sub-sequences is then performed by using an evaluator, which takes advantage from the previously developed information on sub-sequences. Two evaluators, EVAL2(σ_1, σ_2) or EVALN($\sigma_1, \dots, \sigma_n$), are presented. The former considers the concatenation of two segments, while the latter allows for any number of segments. The reasons for designing two different operations relate to the fact that all attributes do not allow for an efficient EVALN operation and thus, in some well-defined settings, the algorithm must rely on EVAL2 and construction operations to perform route evaluations (Section 3.3).

Table 2: Route-evaluation components

Data construction:	
INIT(σ)	Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit.
FORW(σ)	Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence σ and vertex v_i.
BACK(σ)	Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex v_i and sub-sequence σ.
Route evaluations:	
EVAL2(σ_1, σ_2)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \sigma_2$.
EVALN($\sigma_1, \dots, \sigma_n$)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \cdots \oplus \sigma_n$.

The route-evaluation component provides the basic structure to obtain state-of-the-art local search procedures for all *EVAL* attributes. It relies on a library of route-evaluation operators, specific to each attribute, which are selected automatically by the method relatively to the problem specification.

Route-evaluation operators for different attributes are presented in Section 3.2. A unified local search based on these operators is presented in Section 3.3.

3.2 Route-evaluation operators for several attributes

Route-evaluation operators are specific to each attribute, but always respect the five-operations scheme described in Section 3.1. Three cases of attributes arise:

- For the first case, some type of information on s, including cost characterization, is efficiently computable by induction on the concatenation operation, such that a single equation can serve as the basis for all operations. Such a situation corresponds in the framework of Irnich (2008b) to the case of REFs that are invertible and generalizable to segments. Among the MAVRPs that can be managed in this way, we find the VRP with capacity, distance constraints, backhauls, cumulative costs, hard (eventually multiple) time windows, simultaneous deliveries and pickups, or lunch breaks.

- In the second case, which includes soft time windows and time-dependent travel times, among others, the structure of the re-optimization information is more complex and FORW(σ) or BACK(σ) operations may become more computationally expensive than quick concatenation evaluations. In addition, EVALN may not be available in all cases.

- Finally, a more advanced role may be given to the route-evaluation operator for some MAVRPs. These operators can indeed assume the optimization of additional decisions on visit locations within groups of customers (case of the generalized VRP), explicitly determine the break times placement for drivers (VRP with truck driver schedule regulations), or position the objects in the vehicle (VRP with loading constraints). Bi-directional shortest path procedures, tree search methods, or integer programming components are then potentially employed in the operators.

To illustrate the different cases, we now describe route-evaluation operators for several important attributes. Some of these operators are derived from past works and hereby unified in our framework. In addition, new operators are introduced for some other problems, e.g. VRP with cumulative costs, load-dependent costs, simultaneous deliveries and pickups, lunch breaks, service site choices, and hours of service regulations.

Capacity and distance. The classical CVRP is perhaps the simplest setting for which information preprocessing is frequently used. Indeed, it is natural to manage for each sub-sequence σ its partial load $Q(\sigma)$ and partial distance $D(\sigma)$ to speed-up the load constraint checks and distance computations. Equations (1) and (2) enable to compute these quantities by induction on the concatenation operation, and provide the means to perform both FORW, BACK and EVALN in $O(1)$ time. It is also worth noting that other globally constrained resources accumulated on arcs or vertices on the routes can be managed in the same way (see Irnich 2008b).

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{1}$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{2}$$

Cumulative costs. The Cumulative VRP (CCVRP) is based on a different objective seeking the minimization of the sum of arrival times to customers. Evaluating the cost of a route subject to some modifications requires more advanced methods than for the classical CVRP, since arrival times to many customers in the route are impacted. Still, evaluations remain manageable in amortized $O(1)$ operations for several families of classical local search neighborhoods (Ngueveu et al. 2010). Vidal et al. (2011) and Silva et al. (2012) show that three types of information on sub-sequences are sufficient to efficiently evaluate route costs: the duration $D(\sigma)$ to perform the sequence of visits σ, the cumulative cost $C(\sigma)$ when starting at time 0, thus representing the cost of the sequence, and the delay cost $W(\sigma)$ for each unit of time delay in the starting date. For a sequence σ_0 containing a single vertex, the information can be initialized by setting $D(\sigma_0) = 0$ as no travel time is performed, $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ when the vertex is a customer, otherwise $W(\sigma_0) = 0$. Equations (3-5) then enable to compute this information by induction on the concatenation operation, thus allowing to efficiently implement all route-evaluation operations.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{3}$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2) \tag{4}$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2) \tag{5}$$

Load-dependent costs. The fuel consumption f_{ij} of a vehicle is estimated in Xiao et al. (2012) to grow linearly with the load q_{ij} on a segment, and thus $f_{ij} = (f_1 q_{ij} + f_2)d_{ij}$, where f_1 represents the fuel cost per mile and unit of load, and f_2 stands for the base cost per mile. We propose an efficient evaluation of fuel consumption on a route which involves the computation of cumulated demand $Q(\sigma)$, distance $D(\sigma)$, and the load-factor $F(\sigma)$ (load-times-distance) on sequences. The fuel consumption $C(\sigma)$ can be derived from this information since $C(\sigma) = f_1 F(\sigma) + f_2 D(\sigma)$. For a sequence σ_0 containing a single vertex v_i, $Q(\sigma_0) = q_i$, $D(\sigma_0) = 0$, and $F(\sigma_0) = 0$. Furthermore, Equations (6-8) enable to compute these values by induction on larger sub-sequences, leading to route evaluations in $O(1)$ time.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{6}$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{7}$$

$$F(\sigma_1 \oplus \sigma_2) = F(\sigma_1) + Q(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + F(\sigma_2) \tag{8}$$

Backhauls. In the VRP with Backhauls (VRPB), to each customer is either associated a delivery quantity $q_i \neq 0$ of a product, or a pickup quantity $p_i \neq 0$ of a different product. The capacity of the vehicle is limited to Q product units. Furthermore, a structural route constraint is imposed, pick-up customers being necessarily serviced at the end of the route, after at least one delivery customer. This structural constraint can be modeled directly in the distance matrix by setting $c_{ij} = +\infty$ if vertex v_i corresponds to a pickup customer and v_j is a delivery customer, and by setting the distance from the depot $c_{0j} = +\infty$ for any pickup customer v_j. Evaluating the routes then requires checking the load constraints and summing up the distances. Three types of information are developed on sequences σ to that extent: the partial distance $D(\sigma)$, the total delivery quantity $Q^{\mathrm{D}}(\sigma)$, and the total pickup quantity $Q^{\mathrm{P}}(\sigma)$. Since the two types of products are never jointly in the vehicle because of structural route constraints, checking load feasibility on a sequence involves simply to check whether $Q^{\mathrm{D}}(\sigma) \leq Q$ and $Q^{\mathrm{P}}(\sigma) \leq Q$. Hence, both $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$ can be independently evaluated as previously described in Equation (1) to perform route evaluations.

Simultaneous deliveries and pickups. The VRP with simultaneous deliveries and pickups (VRPSDP) also involves two different products to be respectively delivered and picked-up. In contrast with the VRPB, no structural constraint is imposed on the routes, and a vertex can require both a delivery and a pick-up. As the vehicle can now contain both types of products simultaneously, load feasibility must be ensured at each vertex of the trip. To address this case, we introduce a re-optimization method based on three types of information on sub-sequences: $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$, the sum of deliveries and pick-ups on the sequence σ, respectively, and $Q^{\mathrm{MAX}}(\sigma)$, the maximum load in the vehicle while processing the sequence σ when starting with an initial load of $Q^{\mathrm{D}}(\sigma)$. These values can be computed by induction on the concatenation operation using Equations (9-11), leading to efficient constant time route-evaluation operations.

$$Q^{\mathrm{P}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{P}}(\sigma_1) + Q^{\mathrm{P}}(\sigma_2) \tag{9}$$
$$Q^{\mathrm{D}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{D}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2) \tag{10}$$
$$Q^{\mathrm{MAX}}(\sigma_1 \oplus \sigma_2) = \max\{Q^{\mathrm{MAX}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2), Q^{\mathrm{MAX}}(\sigma_2) + Q^{\mathrm{P}}(\sigma_1)\} \tag{11}$$

Another variant of VRPSDP has been addressed in Kindervater and Savelsbergh (1997), where a single commodity was considered and products picked-up at a location could be used to service further customers in the route, leading to different equations.

Time windows and duration constraints. The VRP with hard time windows (VRPTW) imposes interval constraints $[e_i, l_i]$ on arrival dates to each customer v_i, as well as service durations s_i (by default $s_0 = 0$). Waiting time is allowed on the route. The VRPTW is the first variant on which information on sub-sequences was managed and exploited (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997). These authors proposed to characterize any sub-sequence with four types of information: a feasibility statement $F(\sigma)$, the sum of travel and service times $T(\sigma)$, the earliest possible completion time for the sequence of visits $E(\sigma)$, and the latest feasible starting date $L(\sigma)$. For a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$, and $F(\sigma_0) = true$. Equations (12-15) enable then to compute by induction the information for a concatenation of sequences.

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2) \tag{12}$$
$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{13}$$
$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{14}$$
$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \ \wedge \ F(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{15}$$

When the departure date of the vehicle is not fixed, starting dates have an influence on the total waiting time on the route. The minimum duration for the route can still be obtained from the previous information as $DUR(\sigma) = \max\{E(\sigma_i) - L(\sigma_i), T(\sigma_i)\}$. Route evaluations are thus manageable in $O(1)$ time.

Lunch breaks and depot choices. Lunch breaks appear in several real-life applications (Sahoo et al. 2005, Bostel et al. 2008), but have been the focus of only moderate attention in the literature. Let the VRPTW with lunch breaks (VRPTWLB) be defined as a VRPTW variant such that for any non-empty route a single break of duration s_{LB} must be taken between $[e_{\mathrm{LB}}, l_{\mathrm{LB}}]$ at one dedicated location v^{LB} chosen in a set of potential locations $\mathcal{V}^{\mathrm{LB}}$. Let also the variant with flexible breaks (VRPTWFB) represent the case where the location of the break is unconstrained. In the following, we introduce route-evaluation operators to account for lunch placement choices in the VRPTWFB.

Any sub-sequence σ can be characterized by two sets of information: a data set $T(\sigma)$, $E(\sigma)$, $L(\sigma)$, $F(\sigma)$, characterizing the time windows as in Equations (12-15) when no break has been taken in the sub-sequence, and another data set $E'(\sigma)$, $L'(\sigma)$, $F'(\sigma)$, characterizing the case where a break is taken *somewhere* between the first and the last visit of σ. By definition, $T'(\sigma) = T(\sigma) + s_{\mathrm{LB}}$ for any σ. Initially, for a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = true$. Furthermore, breaks are exclusively taken inside the sequence and thus, a sequence made of a single visit should not include a break, such that $E'(\sigma_0) = +\infty$, $L'(\sigma_0) = 0$ and $F'(\sigma_0) = false$. Computing $T(\sigma_1 \oplus \sigma_2)$, $E(\sigma_1 \oplus \sigma_2)$, $L(\sigma_1 \oplus \sigma_2)$, $F(\sigma_1 \oplus \sigma_2)$ can be done as previously with Equations (12-15). Computing their counterparts with breaks by induction comes to select a best case out of three: the break is either taken during σ_1 (Case 1), between σ_1 and σ_2 (Case 2), or during σ_2 (Case 3). These computations are displayed in Equations (17-27).

$$E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case i}} | F'_{\text{case i}} = true\} \cup +\infty) \tag{16}$$
$$L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case i}} | F'_{\text{case i}} = true\} \cup -\infty) \tag{17}$$
$$F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case 1}} \vee F'_{\text{case 2}} \vee F'_{\text{case 3}} \tag{18}$$
$$E'_{\text{case 1}} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{19}$$
$$E'_{\text{case 2}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\} \tag{20}$$
$$E'_{\text{case 3}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T'(\sigma_2), E'(\sigma_2)\} \tag{21}$$
$$L'_{\text{case 1}} = \min\{L'(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T'(\sigma_1)\} \tag{22}$$
$$L'_{\text{case 2}} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - s_{\text{LB}} - T(\sigma_1)\} \tag{23}$$
$$L'_{\text{case 3}} = \min\{L(\sigma_1), L'(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{24}$$
$$F'_{\text{case 1}} = F'(\sigma_1) \wedge F(\sigma_2) \wedge (E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{25}$$
$$F'_{\text{case 2}} = F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) \leq l_{\text{LB}}) \wedge (E(\sigma_1) + s_{\text{LB}} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{26}$$
$$F'_{\text{case 3}} = F(\sigma_1) \wedge F'(\sigma_2) \wedge (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L'(\sigma_2)) \tag{27}$$

It is worth mentioning that a similar methodology can be used to adjust dynamically, within the evaluation of the routes, the break location choices in the VRPTWLB case, as well as the choice and placement of depot visits in a multi-depot setting (Vidal et al. 2012b). Integrating these decisions in the evaluation operators enables to combine the placement or assignment features within local search moves, and considerably reduce the combinations of choices to be worked out in the remaining parts of the method.

Soft and general time windows. For all previously-mentioned attributes, constant-size characteristic data was available for the segments, as well as general concatenation equations (segment REFs in the terminology of Irnich 2008b). However, several MAVRPs fall outside of this class. This is the case for the VRP with soft time windows (VRPSTW), which allows penalized late arrivals to customers, and, more generally, for the generalization of the VRPTW where the service cost $c_i(t_i)$ of each customer v_i is a piecewise linear function of the service time. For this latter variant, the placement of departure times and waiting times, and thus the determination of a good schedule for a fixed route, makes for a non-trivial *timing problem with separable time-dependent processing costs* (Vidal et al. 2011) and no judicious $O(1)$ size data structure is known to characterize the sub-sequences and their exact cost when concatenated together.

In this case, the route-evaluation information can be developed as a set of piecewise functions (Hendel and Sourd 2006, Ibaraki et al. 2005, 2008). Each sub-sequence is characterized by a function $F(\sigma)(t)$ representing the minimum cost to service the sequence σ while arriving at the last customer before time t, and $B(\sigma)(t)$ stating the minimum cost of servicing σ after time t.

For a sequence $\sigma_0 = (v_i)$ with a single vertex, $F_{\sigma_0}(t) = \min_{x \leq t} c_i(x)$ and $B_{\sigma_0}(t) = \min_{x \geq t} c_i(x)$. The construction operator FORW relies on forward dynamic programming (Equation 28) to build explicitly the information for the concatenation of a sequence σ with a vertex v_i. In reverse, BACK is based on backward dynamic programming (Equation 29). Equation (30) provides the cost $Z^*(\sigma_1 \oplus \sigma_2)$ of the concatenated sequence $\sigma_1 \oplus \sigma_2$ when $F(\sigma_1)(t)$ and $B(\sigma_2)(t)$ are available, thus leading to an efficient EVAL2 operation.

$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\} \tag{28}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t} \{c_i(x) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\} \tag{29}$$

$$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\} \tag{30}$$

In our implementations, the data structures $F(\sigma)(t)$ and $B(\sigma)(t)$ are managed as linked lists of function pieces characterized by interval, origin value and slope. The data construction operations $\textsc{Forw}(\sigma)$, $\textsc{Back}(\sigma)$ and $\textsc{Eval2}$ work in $O(\Sigma_i \xi(c_i))$ time, $\xi(c_i)$ representing the number of pieces of a piecewise cost function c_i. However the $\textsc{EvalN}$ operation is not efficiently manageable. In the particular case where all functions $c_i(t)$ are convex, more advanced implementations based either on heaps (Hendel and Sourd 2006) or on search trees (Ibaraki et al. 2008) achieve a complexity of $O(\log \Sigma_i \xi(c_i))$ for both $\textsc{Eval2}$ and $\textsc{EvalN}$.

Other time features. The literature contains various other EVAL attributes related to time, such as duration constraints, multiple time windows, time-dependent trip durations, flexible travel times, and minimum and maximum intervals of time between pairs of services. We refer to Vidal et al. (2011) for a comprehensive review and analysis of state-of-the-art algorithms for the underlying *timing* sub-problems for route evaluations, and their incremental resolution during local searches. These approaches were used to generate UHGS route-evaluation operators for the related problems with time characteristics.

Service site choices. In the Generalized Vehicle Routing Problem (GVRP), each request v_i is associated to a set (group) of λ_i alternative locations $L_i = \{l_{i1}, \ldots, l_{i|\lambda_i|}\}$. Exactly one location of each group must be serviced. As illustrated in Baldacci et al. (2009), the GVRP is relevant for several practical applications and directly generalizes other variants of vehicle routing.

The most recent metaheuristics for this problem (Moccia et al. 2012) conduct local search on the order of groups, and iteratively solve shortest path problems to optimally choose the best customer sequence within the groups. Building upon these concepts, we propose efficient route evaluation operators for the GVRP, storing for each sequence of groups some auxiliary data to speed-up the shortest path computations during the search. In this case, the information to be stored for a sequence σ is the shortest path $S(\sigma)[i,j]$ between the i^{th} location of $\sigma(1)$ and the j^{th} location of $\sigma(|\sigma|)$, where $i \in \{1, \ldots, \lambda_{\sigma(1)}\}$ and $j \in \{1, \ldots, \lambda_{\sigma(|\sigma|)}\}$. For a sequence $\sigma_0 = (v_i)$ containing a single service, $S(\sigma_0)[x,x] = 0$ for any $x \in \{1, \ldots, \lambda_i\}$ and $S(\sigma_0)[x,y] = +\infty$ if $x \neq y$. Equation (31) enables then to develop this information on larger sub-sequences by induction on the concatenation operation. This approach is closely related to the Floyd-Warshall algorithm and to bi-directional dynamic programming concepts.

$$\begin{aligned} S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} & S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j] \\ & \forall i \in \{1, \ldots, \lambda_{\sigma_1(1)}\}, \forall j \in \{1, \ldots, \lambda_{\sigma_2(|\sigma_2|)}\} \end{aligned} \tag{31}$$

Equation (31) provides the means to perform efficiently in $O(\lambda^2)$ operations all route-evaluation operations, λ standing for the maximum number of locations associated to a service. This complexity is notably better than the complexity of computing each shortest path from scratch, which would be $O(n_r \lambda^2)$ operations for a route containing n_r services.

Hours of service regulations. Governments worldwide impose complex regulations on truck-driver schedules to limit the amount of work and driving within intervals of time and impose a minimum frequency and duration for break and rest periods. Because of their large impact on driving times, these regulations should be accounted for when optimizing the routes, leading to combined vehicle routing and truck-driver scheduling problems (VRTDSP). However, even checking the existence of a feasible placement of breaks for a fixed sequence of visits makes for a highly complex problem which is known to be solvable in a quadratic time for United States hours of service regulations (Goel and Kok 2012), while no polynomial algorithm is known for many other cases, with European Union, Canadian, and Australian rules.

Despite this high complexity, most efficient methods for the VRTDSP integrate break scheduling feasibility checks directly in the local search (Prescott-Gagnon et al. 2010, Goel and Vidal 2013), and thus during each route evaluation. In the proposed methodology, these break-scheduling procedures are used inside the route-evaluation operators. A set of schedule alternatives is maintained for each sub-sequence of consecutive visits. The schedule information is extended to larger sub-sequences by appending new driving and break activities at the end of the schedules, and selecting only a relevant subset by means of dominance relationships. Our current implementation is exclusively based on forward operators, and thus EVAL2(σ_1,σ_2) is performed by iteratively completing the schedule of σ_1 with services of σ_2.

Summary. As reviewed in this section, efficient route-evaluation operators relative to different VRP attributes may require to develop radically different information on sequences, and use more or less complex evaluation procedures. Still, all previously-mentioned approaches respect the same five-operations scheme, based on the forward or backward propagation of labels (or, generally, of any information to characterize the sequences), and the evaluation of the concatenation of two or more sequences using the information developed on sequences. As shown in the following, this library of route-evaluation operators provides the means to create a general-purpose state-of-the-art local search for many MAVRPs.

3.3 Unified local search procedure

The route-evaluation component can be used to efficiently explore any neighborhood with moves involving a bounded number of edge exchanges and node relocations, since all these moves can be evaluated as a recombination of partial sub-sequences from the incumbent solution. The resulting Unified Local Search (ULS) is illustrated in Algorithm 1. To efficiently evaluate moves, ULS manages information on sub-sequences of consecutive visits (and reverse sub-sequences in presence of moves that impact the route orientation), using the INIT, FORW, and BACK route construction operations. This information is built during a pre-processing phase at the beginning of the local search, and is then updated whenever any route is modified. Moves are then evaluated by means of EVAL2 and EVALN.

In the specific implementation of this paper, the neighbor solutions issued from moves are explored in random order, using the acceptance criterion of Vidal et al. (2013a) and terminating whenever no improving move can be found in the whole neighborhood. As in Vidal et al. (2013a), the classical 2-OPT*, and 2-OPT neighborhoods are used, as well as the inter-route and intra-route CROSS and I-CROSS neighborhoods, restricted to sub-sequences of length smaller than $L_{max} = 2$ and including relocate moves as special cases. Only moves involving *neighbor vertices* in terms of distance and time characteristics are attempted, leading to a neighborhood size of $O(L_{max}^2 \Gamma n)$ instead of $O(L_{max}^2 n^2)$, where Γ stands for the number of neighbor vertices per vertex (Toth and Vigo 2003, Vidal et al. 2013a).

It should be noted that all *inter-route* moves such as CROSS, I-CROSS and 2-OPT*, require either EVAL2(σ_1, σ_2) or EVALN($\sigma_1, \sigma_L, \sigma_2$), where σ_L is a sequence of size bounded by L_{max}. When no efficient EVALN is available, in presence of attributes such as soft and general time windows for example, this first family of *inter-route* moves can still be evaluated efficiently as EVALN($\sigma_1, \sigma_L, \sigma_2$)

Algorithm 1 Unified local search based on route-evaluation operators

```
Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on sub-sequences using the INIT, FORW and BACK operators.
while some improving moves exist in the neighborhood N do
   for each move μ_i in N do
      for each route r_j^μ produced by the move do
         Determine the k sub-sequences [σ_1, ..., σ_k] that are concatenated to produce r_j^μ
         if k = 2, then NEWCOST(r) = EVAL2(σ_1,σ_2)
         else if k > 2, then NEWCOST(r) = EVALN(σ_1,...,σ_k)
      end for
      if ACCEPTCRITERIA(μ_i) then perform the move μ and update the re-optimization data on
      for each route r_j^μ using the INIT, FORW and BACK operators.
   end for
end while
```

can be replaced by less than L_{max} successive calls to Forw to yield the information on $\sigma' = \sigma_1 \oplus \sigma_L$, with a final call to Eval2(σ', σ_2). *Intra-route* Cross and I-Cross and 2-opt moves require calling EvalN on a set of 3 to 5 sub-sequences. If no efficient EvalN is available, the same reasoning for replacement can still be used, but in this case the number of necessary calls to Forw becomes linear in the route size since the size of intermediate sub-sequences is not bounded. However, since intra-route moves are usually in minority, this increased number of operations did not impact the method speed.

The good combination of route-evaluation operators is automatically determined relatively to the problem attributes according to the component-based framework of Section 2.3, and thus the route-evaluation operators allow to use advanced move evaluation techniques which were until now considered as problem-specific in a unified framework for MAVRPs. The resulting unified local search is efficient and applicable to many VRP variants. It can be extended into any generic neighborhood-based metaheuristic such as tabu search, iterated local search, or variable neighborhood search. Relatively to the recent advances in genetic algorithms and diversity management for vehicle routing, we opted to combine this procedure with the approach of Vidal et al. (2012a) to obtain a Unified Hybrid Genetic Search (UHGS). Such integration requires addressing several additional challenges, related to the design of a generic solution representation, genetic operators, and population management methods. The next section explains how to address them.

4 Unified Hybrid Genetic Search

The proposed UHGS is an extension of the Hybrid Genetic Search with Advanced Diversity Control of Vidal et al. (2012a), and aims to address MAVRPs in a unified manner by means of the proposed component-based design. The method stands out from previous works since all its elements (solution representation, genetic operators, local searches) are fully generic and detached from the attributes of the problem, relying on the subset of adaptive *assignment* and *route-evaluation* components to make the interface with problem-specific knowledge (Section 2.3). Note that in this work, only single-echelon problems with a route structure, e.g., a single sequence, are addressed, thus allowing to rely on a unique *sequencing component* based on standard VRP neighborhoods (Section 3.3). This Section briefly recalls the general structure of UHGS, then details in turn the main elements of the unified method.

UHGS combines four main optimization methodologies: 1) hybridization of genetic algorithms with local search procedures; 2) the use of penalized infeasible solutions, managed through two distinct sub-populations during the search; 3) a solution representation *without trips delimiters* (Prins 2004) with an optimal *Split* procedure for delimiter computation; 4) an advanced population management method with *diversity-and-cost objective* for solution evaluation.

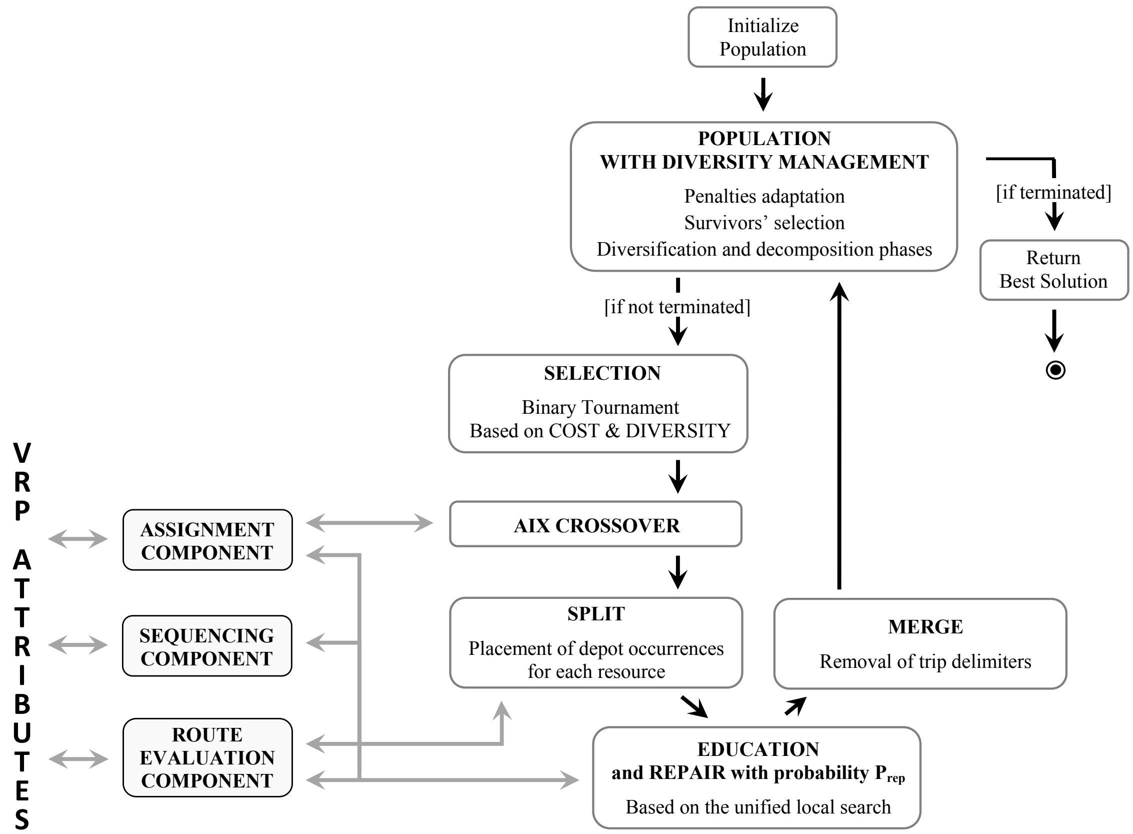

Figure 2: UHGS structure and relationships with problem attributes

The structure of UHGS is illustrated in Figure 2. UHGS iteratively selects two individuals in the merged sub-populations to serve as input of a crossover operator, yielding a single offspring. After going through the *Split* procedure to compute trip delimiters, the offspring is *Educated* by means of a local search, *Repaired* with probability P_{rep} when infeasible, and transfered to the suitable sub-population. Each sub-population is managed separately to trigger a *Survivor Selection* procedure when reaching a maximum size. *Diversification procedures* and *decomposition phases* are regularly used to further enhance the diversity and intensify the search around elite solution characteristics. The algorithm terminates when It_{max} successive iterations (individual generations) have been performed without improving the best solution, or when a time limit T_{max} is reached.

4.1 Solution representation and Unified Split

MAVRPs generally involve two levels of decisions relative to the assignment of customer services to some ASSIGN Attribute Resources (AARs), and the optimization of routes for each AAR. In accordance with this problem structure, solutions are represented in the course of UHGS as a collection of giant tours without explicit mention of visits to the depot. Each giant tour corresponds to a different combination of AAR, for example, a (vehicle type/day) couple in a heterogeneous periodic VRP. Problems without ASSIGN attributes lead to only one AAR, and thus to a solution representation as a single giant tour.

To extract a full solution out of an individual and its giant tours, we introduce in Algorithm 2 a fully generic *Split* procedure for MAVRPs based on route-evaluation components. For any giant tour $\tau = (\tau_1, \ldots, \tau_\nu)$ containing ν customers, the splitting problem is assimilated to a shortest path problem on a directed acyclic auxiliary graph $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ includes $\nu + 1$ nodes notated 0 to ν. Any arc $a_{ij} \in \mathcal{A}$ with $i < j$ represents the route originating from the depot, visiting customers σ_{i+1} to σ_j, and returning. The cost of each arc is set to the cost of the associated route.

Algorithm 2 Generic Split

1: **for** each node $i \in \{1, \ldots, \nu\}$ **do**
2: $\quad$ $SeqData(\sigma) = \textsc{Init}(\{v_0\})$ //*Initialize with depot vertex*
3: $\quad$ **for** each node $j \in \{i+1, \ldots, \min(i+\bar{r}, \nu)\}$ **do**
4: $\quad\quad$ $SeqData(\sigma) = \textsc{Forw}(\sigma,\{\tau_j\})$ //*Append a new customer to the route end*
5: $\quad\quad$ $\phi(a_{ij}) = \textsc{Eval2}(\sigma,\{v_0\})$ //*Evaluate the route*
6: $\quad$ **end for**
7: **end for**
8: Solve the shortest path problem on $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc a_{ij}
9: Return the set of routes associated to the set of arcs of the shortest path

All arc costs can be computed by calling $\mathcal{O}(\nu^2)$ times the Forw and Eval2 operators (Lines 1-5 of Algorithm 2). Setting a maximum value $\bar{r}$ on the number of customers in a route enables to reduce this number of calls to $\mathcal{O}(\nu\bar{r})$. Once this pre-processing is achieved, the shortest path is solved by means of m iterations of the Bellman-Ford algorithm (see Cormen et al. 2001) in presence of a fleet size limit to m. If no limit on the fleet size is imposed, a shortest path based on the topological order of indexes is used. The final complexity of the proposed unified *Split* algorithm is $\mathcal{O}([m+\xi(\textsc{Forw})+\xi(\textsc{Eval2})]\nu\bar{r})$, where $\xi(\textsc{Forw})$ and $\xi(\textsc{Eval2})$ represent respectively the complexity of Forw and Eval2. It should be noted that this unified Split algorithm is applicable to all VRP variants mentioned in Section 3.2.

4.2 Evaluation of individuals

An individual p in UHGS is evaluated relatively to its *feasibility*, *cost*, and *contribution to the population diversity*. Define the *penalized cost* $\phi_{\mathcal{P}}^{\text{COST}}(p)$ of p as the sum on all routes of total distance and penalized excesses relatively to load and other constraint violations of N_{ATT} EVAL attributes. For any route σ with distance $\varphi^{\text{D}}(\sigma)$, load excess $\varphi^{\text{Q}}(\sigma)$, and excesses $\varphi^{\text{E}_i}(\sigma)$ for $i \in \{1, \ldots, N_{\text{ATT}}\}$ relatively to EVAL attributes, the penalized cost $\phi(r)$ is given by Equation (32), where ω^{Q} and ω^{E_i} for $i \in \{1, \ldots, N_{\text{ATT}}\}$ represent the associated penalty coefficients. The set of excesses $\varphi^{\text{E}_i}(\sigma)$ depends upon the EVAL attributes of the problem, and can include the excess of pickup load (variants of VRPB), excess in duration (variants of DurVRP), time-window relaxations in the sense of Nagata et al. (2010) or service lateness (VRTDSP, TDVRP). Penalty coefficients are adapted during the search relatively to the proportion of feasible individuals as in Vidal et al. (2012a, 2013a).

$$\phi(r) = \varphi^{\text{D}}(r) + \omega^{\text{Q}}\varphi^{\text{Q}}(r) + \sum_{i=1}^{N_{\text{ATT}}} \omega^{\text{E}_i}\varphi^{\text{E}_i}(\sigma) \tag{32}$$

Define the *diversity contribution* $\phi_{\mathcal{P}}^{\text{DIV}}(p)$ of an individual p as its average distance with the μ^{CLOSE} most similar individuals in the sub-population. The Hamming distance on assignment decisions is used in presence of ASSIGN attributes, while in the other case, the broken pairs distance (Prins 2009) is automatically used to measure the proportion of common edges. Finally, Equation (33) states the *biased fitness* $f_{\mathcal{P}}(p)$ of an individual p in the sub-population $\mathcal{P}$ as a weighted sum of its penalized cost rank $f_{\mathcal{P}}^{\text{COST}}(p)$ and its rank $f_{\mathcal{P}}^{\text{DIV}}(p)$ relative to its diversity contribution. This trade-off between diversity and cost is balanced by parameter μ^{ELITE} and was shown to play an essential role in the performance of the method.

$$f_{\mathcal{P}}(p) = f_{\mathcal{P}}^{\text{COST}}(p) + \left(1 - \frac{\mu^{\text{ELITE}}}{|\mathcal{P}|}\right) f_{\mathcal{P}}^{\text{DIV}}(p) \tag{33}$$

4.3 Iterative generation of new individuals

Two parent individuals are iteratively selected during the course of UHGS, by binary tournament within the merged feasible and infeasible sub-populations, to serve as input to the crossover and produce a single offspring. An Assignment and Insertion Crossover (AIX) is then used for problems involving at least one ASSIGN attribute, otherwise the simple Ordered Crossover (OX) is applied (see Prins 2004).

AIX is a generalization of the PIX crossover of Vidal et al. (2012a). It first decides for each of the n_{AAR} ASSIGN Attribute Resources whether the genetic material of p_1, p_2 or both parents is transmitted, and then inherits the genetic material according to the rules of Vidal et al. (2012a). At each tentative insertion of a visit into the offspring, the *assignment* component (Section 2.3) verifies whether this inheritance of a customer still allows for completion of a feasible assignment relatively to ASSIGN attributes. The considered visit is not transmitted to the offspring if this property in not fulfilled. Finally, as the previous constraints can lead to an incomplete offspring, missing visits to customers are inserted in turn in a random order to the best location relatively to the penalized route cost. Good insertion procedures require the knowledge of the depot occurrences, and thus this final step of AIX is completed after using the unified *Split* algorithm (Section 4.1).

Any offspring issued from the crossover is improved by means of an *Education* operator based on two local searches. First, the local search procedure of Section 3.3 is applied independently for each AAR to perform Route Improvements (RI). Second, an Assignment Improvement (AI) procedure is designed to optimize on assignment decisions. Finally, RI is applied a last time. The AI procedure is the generalization of the *pattern improvement* procedure of Vidal et al. (2012a). AI tentatively removes all services to a customer, and chooses the best combination of insertion locations in the AARs for reinsertion. AI relies on the *assignment* operator to list the tentative combinations of resource assignments, and evaluates each insertion position by means of the *route-evaluation* operator. Any insertion is thus assimilated to a call to $\textsc{EvalN}(\sigma_1, \sigma_0, \sigma_2)$ where σ_0 contains a single vertex. Customer re-assignments are exhaustively tried in random order, the best re-assignment position being systematically chosen. AI stops when no improving re-assignment can be found.

The solutions issued from *Education* are directly accepted in the appropriate sub-population relatively to their feasibility. Furthermore, any infeasible solution with penalty (see Section 4.2) is *Repaired* with probability P_{rep}. The *Repair* operator temporarily increases the penalty coefficients by a factor of 10 and calls *Education* to redirect the search towards feasible solutions.

4.4 Population management and search guidance

Sub-populations are independently managed to always contain between μ^{MIN} and $\mu^{\text{MIN}} + \mu^{\text{GEN}}$ individuals, by triggering a *survivor selection* phase each time a sub-population reaches a maximum size of $\mu^{\text{MIN}} + \mu^{\text{GEN}}$. *Survivor selection* consists in iteratively removing μ^{GEN} times the worst individual with regards to the biased fitness of Section 4.2, privileging first the removal of *clone* individuals with null distance to at least another individual. To regularly introduce new genetic material, these population management mechanisms are completed by diversification phases (Vidal et al. 2013a, 2012a) which take place after each It_{div} successive iterations without improvement of the best solution, and consist in retaining the best $\mu/3$ individuals and replacing the others by new individuals. Decomposition phases are also triggered after every It_{dec} iterations as in Vidal et al. (2013a). This advanced population management procedure, coupled with the diversity-based evaluation of individuals, plays a main role in the success of the overall algorithm.

5 Computational Experiments

Extensive computational experiments were conducted on a wide range of MAVRPs to assess the performance of the general-purpose UHGS relatively to the best problem-tailored algorithms for each setting. Both "academic" problems and rich multi-attribute VRPs have been addressed in these studies. A single parameter setting, the same as in Vidal et al. (2013a, 2012a), was used for all the experiments in order to examine the applicability of the method without extensive problem-tailored parameter customization. The termination criteria was set to ($It_{max} = 5000$; $T_{max} = 30min$) to compare with other authors in similar time. Problems requiring fleet minimization were solved by iteratively decrementing the fleet size limit and running UHGS until no feasible solution can be found. The algorithm was implemented in C++ and run on Opteron 250 2.4GHz and Opteron 275 2.2GHz processors.

Table 3: List of acronyms for benchmark instances and methods

Benchmark instances:					
B11	Bektas et al. (2011)	G84	Golden (1984)	LS99	Liu and Shen (1999)
CGL97	Cordeau et al. (1997)	G09	Goel (2009)	MG06	Montané and Galvão (2006)
CL01	Cordeau and Laporte (2001)	GDDS95	Gélinas et al. (1995)	SD88	Solomon and Desrosiers (1988)
CMT79	Christofides et al. (1979)	GH99	Gehring and Homberger (1999)	SN99	Salhi and Nagy (1999)
F94	Fisher (1994)	GJ89	Goetschalckx and J.-B. (1989)		
FTV94	Fischetti et al. (1994)	GWKC98	Golden et al. (1998)		
State-of-the-art algorithms:					
B10	Belhaiza (2010)	KTDHS12	Kritzinger et al. (2012)	RT10	Repoussis and Tarantilis (2010)
BDHMG08	Bräysy et al. (2008)	MB07	Mester and Bräysy (2007)	RTBI10	Repoussis et al. (2010)
BER11	Bektas et al. (2011)	MCR12	Moccia et al. (2012)	RTI09a	Repoussis et al. (2009a)
BLR11	Balseiro et al. (2011)	NB09	Nagata and Bräysy (2009)	RTI09b	Repoussis et al. (2009b)
BPDRT09	Bräysy et al. (2009)	NBD10	Nagata et al. (2010)	SDBOF10	Subramanian et al. (2010)
CM12	Cordeau and M. (2012)	NPW10	Ngueveu et al. (2010)	SPUO12	Subramanian et al. (2012)
F10	Figliozzi (2010)	P09	Prins (2009)	SUO13	Subramanian et al. (2013)
FEL07	Fu et al. (2007)	PBDH08	Polacek et al. (2008)	XZKX12	Xiao et al. (2012)
GA09a	Gajpal and Abad (2009b)	PDDR10	Prescott-Gagnon et al. (2010)	ZTK10	Zachariadis et al. (2010)
GA09b	Gajpal and Abad (2009a)	PR07	Pisinger and Ropke (2007)	ZK10	Zachariadis and Kiranoudis (2010)
GG11	Groër et al. (2011)	PR08	Pirkwieser and Raidl (2008)	ZK11	Zachariadis and Kiranoudis (2011)
HDH09	Hemmelmayr et al. (2009)	RP06	Ropke and Pisinger (2006a)	ZK12	Zachariadis and Kiranoudis (2012)
ISW09	Imran et al. (2009)	RL12	Ribeiro and Laporte (2012)		

Table 3 displays the list of acronyms for the benchmark instances and methods used in the comparative analysis. Tables 4 and 5 compare the results of UHGS with the current best methods in the literature for each problem class taken separately. Columns (1-4) indicate the variant considered, the origin of the benchmark instances, the number n of customers in these instances, and the objective function ("C" standing for distance, "D" for duration, i.e., the time elapsed between departure and return, "T" for travel time, "F" for fleet size, "TW" for time-window violations). Hierarchical objectives are presented by decreasing order of priority, separated with the sign "/". The last column reports, for each state-of-the-art method, the gap of an average or single run with respect to the current Best Known Solutions (BKS), the gap of the best solution produced by the method, the average run time to achieve these results (for parallel methods, the computation time on a single CPU is reported in italics as well as the number of CPUs), and the type of processor used. The algorithm yielding the best result quality, for each benchmark instances set and problem class, is indicated in boldface. Table 4 also includes the results from previous HGSADC applications on the PVRP, MDVRP, CVRP, and VRTDSP with European Union regulations (Vidal et al. 2013a, 2012a, Goel and Vidal 2013) since UHGS works identically when instantiated on these problems. Detailed results are reported in an Electronic Companion, and can also be downloaded at http://w1.cirrelt.ca/∼vidalt/en/VRP-resources.html.

Table 4: Performance analysis on several VRP variants with various objectives

Variant	**Bench.**	***n***	**Obj.**	**State-of-the-art methods**				
				Author	Avg.%	Best%	T(min)	CPU
CVRP	CMT79	[50,199]	C	GG11	—	+0.03%	*2.38*	8×Xe 2.3G
				MB07	+0.03%	—	2.80	P-IV 2.8G
				UHGS*	**+0.02%**	**+0.00%**	11.90	Opt 2.4G
CVRP	GWKC98	[200,483]	C	GG11	—	+0.29%	*5.00*	8×Xe 2.3G
				NB09	+0.27%	+0.16%	21.51	Opt 2.4G
				UHGS*	**+0.15%**	**+0.02%**	71.41	Opt 2.4G
AVRP	FTV94	[33,70]	C	SUO13	+0.02%	+0.00%	2.24	Xe 2.93G
				UHGS	**+0.00%**	**+0.00%**	0.30	Opt 2.2G
VRPB	GJ89	[25,200]	C	ZK12	+0.38%	+0.00%	1.09	T5500 1.67G
				GA09a	+0.09%	+0.00%	1.13	Xe 2.4G
				UHGS	**+0.01%**	**+0.00%**	0.99	Opt 2.4G
CCVRP	CMT79	[50,199]	C	NPW10	+0.74%	+0.28%	5.20	Core2 2G
				RL12	+0.37%	+0.07%	2.69	Core2 2G
				UHGS	**+0.01%**	**-0.01%**	1.42	Opt 2.2G
CCVRP	GWKC98	[200,483]	C	NPW10	+2.03%	+1.38%	94.13	Core2 2G
				RL12	+0.34%	+0.07%	21.11	Core2 2G
				UHGS	**-0.14%**	**-0.23%**	17.16	Opt 2.2G
VRPSDP	SN99	[50,199]	C	SDBOF10	+0.16%	+0.00%	*0.37*	256×Xe 2.67G
				ZTK10	—	+0.11%	—	T5500 1.66G
				UHGS	**+0.01%**	**+0.00%**	2.79	Opt 2.4G
VRPSDP	MG06	[100,400]	C	SDBOF10	+0.30%	+0.17%	*3.11*	256×Xe 2.67G
				UHGS	+0.20%	+0.07%	12.00	Opt 2.4G
				SUO13	**+0.08%**	**+0.00%**	7.23	I7 2.93G
VRPMDP	SN99	[50,200]	C	GA09b	+0.41%	+0.22%	0.85	Xe 2.4G
				SUO13	+0.09%	+0.00%	2.97	I7 2.93G
				UHGS	**+0.06%**	**+0.00%**	2.46	Opt 2.2G
VFMP-F	G84	[20,100]	C	ISW09	—	+0.07%	8.34	P-M 1.7G
				SPUO12	+0.12%	+0.01%	0.15	I7 2.93G
				UHGS	**+0.04%**	**+0.01%**	1.13	Opt 2.4G
VFMP-V	G84	[20,100]	C	ISW09	—	+0.02%	8.85	P-M 1.7G
				SPUO12	+0.17%	+0.00%	0.06	I7 2.93G
				UHGS	**+0.03%**	**+0.00%**	0.85	Opt 2.4G
VFMP-FV	G84	[20,100]	C	P09	—	+0.02%	0.39	P4M 1.8G
				UHGS	+0.01%	+0.00%	0.99	Opt 2.4G
				SPUO12	**+0.01%**	**+0.00%**	0.13	I7 2.93G
LDVRP	CMT79	[50,199]	C	XZKX12	+0.48%	+0.00%	1.3	NC 1.6G
				UHGS	**-0.28%**	**-0.33%**	2.34	Opt 2.2G
LDVRP	GWKC98	[200,483]	C	XZKX12	+0.66%	+0.00%	3.3	NC 1.6G
				UHGS	**-1.38%**	**-1.52%**	23.81	Opt 2.2G
PVRP	CGL97	[50,417]	C	HDH09	+1.69%	+0.28%	3.09	P-IV 3.2G
				UHGS*	+0.43%	+0.02%	6.78	Opt 2.4G
				CM12	**+0.24%**	**+0.06%**	*3.55*	64×Xe 3G
MDVRP	CGL97	[50,360]	C	CM12	+0.09%	+0.03%	*3.28*	64×Xe 3G
				SUO13	+0.07%	+0.02%	11.81	I7 2.93G
				UHGS*	**+0.08%**	**+0.00%**	5.17	Opt 2.4G
GVRP	B11	[16,262]	C	BER11	+0.06%	—	0.01	Opt 2.4G
				MCR12	+0.11%	—	0.34	Duo 1.83G
				UHGS	**+0.00%**	**-0.01%**	1.53	Opt 2.4G

Table 5: Performance analysis on several VRP variants with various objectives (continued)

Variant	Bench.	n	Obj.	State-of-the-art methods Author	Avg.%	Best%	T(min)	CPU
OVRP	CMT79 &F94	[50,199]	F/C	RTBI10	0%/+0.32%	—	9.54	P-IV 2.8G
				SUO13	—/+0.16%	0%/+0.00%	2.39	I7 2.93G
				UHGS	**0%/+0.11%**	**0%/+0.00%**	1.97	Opt 2.4G
OVRP	GWKC 98	[200,480]	F/C	ZK10	0%/+0.39%	0%/+0.21%	14.79	T5500 1.66G
				SUO13	0%/+0.13%	0%/+0.00%	64.07	I7 2.93G
				UHGS	**0%/-0.11%**	**0%/-0.19%**	16.82	Opt 2.4G
VRPTW	SD88	100	F/C	RTI09	0%/+0.11%	0%/+0.04%	17.9	Opt 2.3G
				UHGS*	0%/+0.04%	0%/+0.01%	2.68	Xe 2.93G
				NBD10	**0%/+0.02%**	**0%/+0.00%**	5.0	Opt 2.4G
VRPTW	HG99	[200,1000]	F/C	RTI09b	—	+0.16%/+3.36%	270	Opt 2.3G
				NBD10	+0.20%/+0.42%	+0.10%/+0.27%	21.7	Opt 2.4G
				UHGS*	**+0.18%/+0.11%**	**+0.08%/-0.10%**	141	Xe 2.93G
OVRPTW	SD88	100	F/C	RTI09a	+0.89%/+0.42%	0%/+0.24%	10.0	P-IV 3.0G
				KTDHS12	0%/+0.79%	0%/+0.18%	10.0	Xe 2.67G
				UHGS	**+0.09%/-0.10%**	**0%/-0.10%**	5.27	Opt 2.2G
VRPBTW	RP06	100	F/C	RP06	+0.39%/—	0%/0%	1.90	P-IV 1.5G
				UHGS	**-0.36%/+0.45%**	**-0.61%/+0.72%**	4.10	Opt 2.2G
TDVRPTW	SD88	100	F/C	KTDHS12	+1.56%	0%	10.00	Xe 2.67G
				UHGS	**-2.22%**	**-2.48%**	14.40	Opt 2.2G
VFMPTW	LS99	100	D	BDHMG08	—	+0.59%	10.15	Ath 2.6G
				RT10	+0.22%	—	16.67	P-IV 3.4G
				UHGS	**-0.15%**	**-0.24%**	4.58	Opt 2.2G
VFMPTW	LS99	100	C	BDHMG08	—	+0.25%	3.55	Ath 2.6G
				BPDRT09	—	+0.17%	0.06	Duo 2.4G
				UHGS	-0.38%	-0.49%	4.82	Opt 2.2G
PVRPTW	CL01	[48,288]	C	PR08	—	+1.75%	—	Opt 2.2G
				CM12	+1.10%	+0.76%	*11.3*	64×Xe 3G
				UHGS*	**+0.63%**	**+0.22%**	32.7	Xe 2.93G
MDVRPTW	CL01	[48,288]	C	PBDH08	—	+1.37%	147	P-IV 3.6G
				CM12	+0.36%	+0.15%	*6.57*	64×Xe 3G
				UHGS*	**+0.19%**	**+0.03%**	6.49	Xe 2.93G
SDVRPTW	CL01	[48,288]	C	B10	+2.23%	—	2.94	Qd 2.67G
				CM12	+0.62%	+0.36%	*5.60*	64×Xe 3G
				UHGS*	**+0.36%**	**+0.10%**	5.48	Xe 2.93G
VRPSTW (type 1, α=100)	SD88	100	F/TW/C[1]	F10	0%	—	9.69	P-M 1.6G
				UHGS	**-3.05%**	**-4.42%**	18.62	Opt 2.2G
VRPSTW (type 1, α=1)	SD88	100	C+TW	KTDHS12	+0.62%	+0.00%	10.0	Xe 2.67G
				UHGS	**-0.13%**	**-0.18%**	5.82	Opt 2.2G
VRPSTW (type 2, α=100)	SD88	100	F/TW/C[1]	FEL07	0%	—	5.98	P-II 600M
				UHGS	**-13.91%**	**-13.91%**	41.16	Opt 2.2G
VRPSTW (type 2, α=1)	SD88	100	C+TW	**UHGS**	**+0.26%**	**0%**	29.96	Opt 2.2G
MDPVRPTW	New	[48,288]	C	**UHGS**	**+0.77%**	**0%**	16.89	Opt 2.2G
VRTDSP (E.U. rules)	G09	100	F/C	PDDR10	0%/0%	0%/0%	88	Opt 2.3G
				UHGS*	**-0.56%/-0.54%**	**-0.85%/-0.70%**	228	Xe 2.93G

∗ These results are introduced Vidal et al. (2013a, 2012a) and Goel and Vidal (2013).
[1] For the sake of brevity, only the fleet size is reported in this Table.

As reported in Tables 4 and 5, UHGS produces high-quality solutions for all problems and benchmark sets, from pure academic problems such as the CVRP to a large variety of VRP variants and rich settings. The average gap to the BKS ever found is in all cases smaller than +0.78%, the worst case being achieved on the multi-depot periodic VRP (MDPVRPTW). The average standard deviation of solutions, measured separately for each single objective problem, ranges between 0.002% (GVRP) and 0.66% (MDPVRPTW), thus showing that the metaheuristic produces high-quality solutions in a consistent manner.

From a solution-quality standpoint, our general-purpose method matches or outperforms all problem-tailored approaches, often specifically calibrated for the considered instances, on all 29 problems and 42 benchmark sets. For a more fair comparison with some parallel methods, such as CM12 which relies on 64 CPU, one should consider the "best" results of UHGS and multiply the reported CPU time by 10, therefore assessing the CPU effort and solution quality of a parallel independent multi-search with 10 UHGS.

The average run time remains in most cases smaller than 10 minutes for average-sized problems (100 to 200 customers), being thus adequate for daily or weekly planning. We opted for a single termination criteria for all problems to avoid any form of problem-specific calibration. The proposed UHGS is thus sometimes slower than other algorithms, sometimes faster. In average, its run time is comparable to the others. Overall, 1046 BKS out of 1099 have been either retrieved or improved during these experiments, and 549 BKS out of 1099 have been strictly improved.

6 Conclusions and Perspectives

A new component-based heuristic framework and a Unified Hybrid Genetic Search (UHGS) have been introduced to address a large variety of difficult VRP variants. The heuristic framework strongly exploits the MAVRP structure, using adaptive assignment, sequencing, and route-evaluation components to tackle problem specifics. These components are used in UHGS to build unified local-search improvement, *Split*, and genetic operators. The remarkable performance of the resulting metaheuristic has been demonstrated on a wide range of problems. On 29 VRP variants and 42 sets of benchmark instances, UHGS matches or outperforms the current state-of-the-art problem-tailored algorithms. Overall, 1046 of the 1099 best known solutions have been either retrieved or improved. It appears that the proposed heuristic framework is particularly efficient for dealing with MAVRPs and, furthermore, that generality does not necessarily play against performance for the considered classes of VRP variants.

The research avenues are numerous. This new general-purpose solution framework can serve as a testing ground to compare different heuristic strategies on a wide range of structurally different problems instead of a single one. In addition, the method can be extended towards a wider variety of attributes, multi-objective and stochastic settings. It has not been tested, up to date, on problems for which the evaluation of routes are interdependent, e.g., VRP with synchronization, nor on problems with SEQ attributes with one-to-one deliveries or multiple echelons. Finally, further studies could focus on the combinatorial aspect of attribute combinations, the ability to build operators for compound problems out of simpler ones, and the impact of attribute combinations on local-search complexity and performance.

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References

Andersson, H., A. Hoff, M. Christiansen, G. Hasle, A. Lokketangen. 2010. Industrial aspects and literature survey: Combined inventory management and routing. *Computers & Operations Research* **37**(9) 1515–1536.

Baldacci, R., E. Bartolini, G. Laporte. 2009. Some applications of the generalized vehicle routing problem. *Journal of the Operational Research Society* **61**(7) 1072–1077.

Baldacci, R., E. Bartolini, A. Mingozzi, A. Valletta. 2011a. An Exact Algorithm for the Period Routing Problem. *Operations Research* **59**(1) 228–241.

Baldacci, R., A. Mingozzi. 2009. A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming* **120**(2) 347–380.

Baldacci, R., A. Mingozzi, R.W. Calvo. 2011b. An exact method for the capacitated location-routing problem. *Operations Research* **59**(5) 1284–1296.

Balseiro, S.R., I. Loiseau, J. Ramonet. 2011. An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows. *Computers & Operations Research* **38**(6) 954–966.

Bektas, T., G. Erdogan, S. Ropke. 2011. Formulations and branch-and-cut algorithms for the generalized vehicle routing problem. *Transportation Science* **45**(3) 299–316.

Belhaiza, S. 2010. Hybrid Variable Neighborhood - Tabu Search Algorithm for the Site Dependent Vehicle Routing Problem with Time Windows. Tech. rep., GERAD, Montreal, Canada.

Bostel, N., P. Dejax, P. Guez, F. Tricoire. 2008. Multiperiod planning and routing on a rolling horizon for field force optimization logistics. Bruce Golden, S. Raghavan, Edward Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*, vol. 43. Springer, New York, 503–525.

Bräysy, O., W. Dullaert, G. Hasle, D. Mester, M. Gendreau. 2008. An Effective Multirestart Deterministic Annealing Metaheuristic for the Fleet Size and Mix Vehicle-Routing Problem with Time Windows. *Transportation Science* **42**(3) 371–386.

Bräysy, O., P.P. Porkka, W. Dullaert, P.P. Repoussis, C.D. Tarantilis. 2009. A well-scalable metaheuristic for the fleet size and mix vehicle routing problem with time windows. *Expert Systems with Applications* **36**(4) 8460–8475.

Burke, E.K., M. Hyde, G. Kendall, G. Ochoa, E. Özcan, J.R. Woodward. 2010. A classification of hyper-heuristic approaches. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, 449–468.

Cahon, S., N. Melab, E.-G. Talbi. 2004. ParadisEO: A framework for the reusable design of parallel and distributed metaheuristics. *Journal of Heuristics* **10**(3) 357–380.

Campbell, A.M., M. Savelsbergh. 2004. Efficient insertion heuristics for vehicle routing and scheduling problems. *Transportation science* **38**(3) 369–378.

Christofides, N., A. Mingozzi, P. Toth. 1979. The vehicle routing problem. N. Christofides, A. Mingozzi, P. Toth, C. Sandi, eds., *Combined Optimization*. Wiley, Chichester, UK, 315–338.

Cordeau, J.-F., M. Gendreau, G. Laporte. 1997. A tabu search heuristic for periodic and multi-depot vehicle routing problems. *Networks* **30**(2) 105–119.

Cordeau, J.-F., G. Laporte. 2001. A tabu search algorithm for the site dependent vehicle routing problem with time windows. *INFOR* **39**(3) 292–298.

Cordeau, J.-F., G. Laporte. 2003. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. *Transportation Research Part B: Methodological* **37**(6) 579–594.

Cordeau, J.-F., G. Laporte, A. Mercier. 2001. A unified tabu search heuristic for vehicle routing problems with time windows. *Journal of the Operational Research Society* **52**(8) 928–936.

Cordeau, J.-F., G. Laporte, A. Mercier. 2004. Improved tabu search algorithm for the handling of route duration constraints in vehicle routing problems with time windows. *Journal of the Operational Research Society* **55**(5) 542– 546.

Cordeau, J.-F., M. Maischberger. 2012. A Parallel Iterated Tabu Search Heuristic for Vehicle Routing Problems. *Computers & Operations Research* **39**(9) 2033–2050.

Cormen, T.H., C. Stein, R.L. Rivest, C.E. Leiserson. 2001. *Introduction to Algorithms*. McGraw-Hill Higher Education.

Crainic, T.G., M. Toulouse. 2010. Parallel Meta-heuristics. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, Boston, MA, 497–541.

Desaulniers, G., J. Desrosiers, I. Ioachim, M.M. Solomon, F. Soumis, D. Villeneuve. 1998. A unified framework for deterministic time constrained vehicle routing and crew scheduling problems. T.G. Crainic, G. Laporte, eds., *Fleet Management and Logistics*. Kluwer Academic Publishers, Boston, MA, 129–154.

Desaulniers, G., J. Desrosiers, M.M. Solomon, eds. 2005. *Column generation*. Springer.

Du, T.C., J.-L. Wu. 2001. Using object-oriented paradigm to develop an evolutional vehicle routing system. *Computers in Industry* **44**(3) 229–249.

Eksioglu, B., A.V. Vural, A. Reisman. 2009. The vehicle routing problem: A taxonomic review. *Computers & Industrial Engineering* **57**(4) 1472–1483.

Figliozzi, M.A. 2010. An iterative route construction and improvement algorithm for the vehicle routing problem with soft time windows. *Transportation Research Part C: Emerging Technologies* **18**(5) 668–679.

Fink, A., S. Voss. 2003. HOTFRAME: A Heuristic Optimization Framework. S. Voss, D.L. Woodruff, eds., *Optimization Software Class Libraries*. Operations Research/Computer Science Interfaces Series, Vol. 18, Springer, 81–154.

Fischetti, M., P. Toth, D. Vigo. 1994. A branch-and-bound algorithm for the capacitated vehicle routing problem on directed graphs. *Operations Research* **42**(5) 846–859.

Fisher, M.L. 1994. Optimal solution of vehicle routing problems using minimum k-trees. *Operations Research* **42**(4) 626–642.

Fu, Z., R. Eglese, L.Y.O. Li. 2007. A unified tabu search algorithm for vehicle routing problems with soft time windows. *Journal of the Operational Research Society* **59**(5) 663–673.

Gajpal, Y., P. Abad. 2009a. An ant colony system (ACS) for vehicle routing problem with simultaneous delivery and pickup. *Computers & Operations Research* **36**(12) 3215–3223.

Gajpal, Y., P. Abad. 2009b. Multi-ant colony system (MACS) for a vehicle routing problem with backhauls. *European Journal of Operational Research* **196**(1) 102–117.

Garcia, B.-L. 1996. Une approche générique des méthodes par amélioration itérative. Application à la résolution de problèmes d'optimisation dans les réseaux. Ph.D. thesis, Laboratoire d'Informatique de l'Université Blaise Pascal, Clermont-Ferrand, France.

Gehring, H., J. Homberger. 1999. A parallel hybrid evolutionary metaheuristic for the vehicle routing problem with time windows. *Proceedings of EURO-GEN'99*. 57–64.

Gélinas, S., M. Desrochers, J. Desrosiers, M.M. Solomon. 1995. A new branching strategy for time constrained routing problems with application to backhauling. *Annals of Operations Research* **61**(1) 91–109.

Gendreau, M., J.Y. Potvin, O. Bräysy, G. Hasle, A. Lokketangen. 2008. Metaheuristics for the vehicle routing problem and its extensions: A categorized bibliography. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, New York, 143–169.

Goel, A. 2009. Vehicle Scheduling and Routing with Drivers' Working Hours. *Transportation Science* **43**(1) 17–26.

Goel, A., L. Kok. 2012. Truck driver scheduling in the United States. *Transportation Science* **46**(3) 317–326.

Goel, A., T. Vidal. 2013. Hours of service regulations in road freight transport: an optimization-based international assessment. *Transportation Science, Forthcoming* .

Goetschalckx, M., C. Jacobs-Blecha. 1989. The vehicle routing problem with backhauls. *European Journal of Operational Research* **42**(1) 39–51.

Golden, B. 1984. The fleet size and mix vehicle routing problem. *Computers & Operations Research* **11**(1) 49–66.

Golden, B., S. Raghavan, E. Wasil, eds. 2008. *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer, New York.

Golden, B.L., E.A. Wasil, J.P. Kelly, I.M. Chao. 1998. The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results. T.G. Crainic, G. Laporte, eds., *Fleet management and logistics*. Kluwer Academic Publishers, Boston, MA, 33–56.

Groër, C., B. Golden, E. Wasil. 2010. A Library of Local Search Heuristics for the Vehicle Routing Problem. *Mathematical Programming Computation* **2**(2) 79–101.

Groër, C., B. Golden, E. Wasil. 2011. A Parallel Algorithm for the Vehicle Routing Problem. *INFORMS Journal on Computing* **23**(2) 315–330.

Hashimoto, H., T. Ibaraki, S. Imahori, M. Yagiura. 2006. The vehicle routing problem with flexible time windows and traveling times. *Discrete Applied Mathematics* **154**(16) 2271–2290.

Hashimoto, H., M. Yagiura, T. Ibaraki. 2008. An iterated local search algorithm for the time-dependent vehicle routing problem with time windows. *Discrete Optimization* **5**(2) 434–456.

Hemmelmayr, V.C., K.F. Doerner, R.F. Hartl. 2009. A variable neighborhood search heuristic for periodic routing problems. *European Journal of Operational Research* **195**(3) 791–802.

Hendel, Y., F. Sourd. 2006. Efficient neighborhood search for the one-machine earliness-tardiness scheduling problem. *European Journal of Operational Research* **173**(1) 108–119.

Ibaraki, T., S. Imahori, M. Kubo, T. Masuda, T. Uno, M. Yagiura. 2005. Effective Local Search Algorithms for Routing and Scheduling Problems with General Time-Window Constraints. *Transportation Science* **39**(2) 206–232.

Ibaraki, T., S. Imahori, K. Nonobe, K. Sobue, T. Uno, M. Yagiura. 2008. An iterated local search algorithm for the vehicle routing problem with convex time penalty functions. *Discrete Applied Mathematics* **156**(11) 2050–2069.

Ichoua, S., M. Gendreau, J.Y. Potvin. 2003. Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research* **144**(2) 379–396.

Imran, A., S. Salhi, N.A. Wassan. 2009. A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* **197**(2) 509–518.

Irnich, S. 2008a. A Unified Modeling and Solution Framework for Vehicle Routing and Local Search-Based Metaheuristics. *INFORMS Journal on Computing* **20**(2) 270–287.

Irnich, S. 2008b. Resource extension functions: properties, inversion, and generalization to segments. *OR Spectrum* **30** 113–148.

Kindervater, G.A.P., M.W.P. Savelsbergh. 1997. Vehicle routing: Handling edge exchanges. E.H.L. Aarts, J.K. Lenstra, eds., *Local Search in Combinatorial Optimization*. Princeton Univ Pr, 337–360.

Kritzinger, S., F. Tricoire, K.F. Doerner, R.F. Hartl, T. Stützle. 2012. A Unified Framework for Routing Problems with Fixed Fleet Size. Tech. rep., Johannes Kepler University, Linz, Austria.

Labadi, N., C. Prins, M. Reghioui. 2008. A memetic algorithm for the vehicle routing problem with time windows. *RAIRO Operations Research* **42**(3) 415–431.

Liu, F.-H., S.-Y. Shen. 1999. The fleet size and mix vehicle routing problem with time windows. *Journal of the Operational Research Society* **50**(7) 721–732.

Mester, D., O. Bräysy. 2007. Active-guided evolution strategies for large-scale capacitated vehicle routing problems. *Computers & Operations Research* **34**(10) 2964–2975.

Meyer, B. 1997. *Object-oriented software construction*. Prentice Hall, Englewood Cliffs, USA.

Moccia, L., J.-F. Cordeau, G. Laporte. 2012. An incremental tabu search heuristic for the generalized vehicle routing problem with time windows. *Journal of the Operational Research Society* **63**(2) 232–244.

Montané, F.A.T., R.D. Galvão. 2006. A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service. *Computers & Operations Research* **33**(3) 595–619.

Nagata, Y., O. Bräysy. 2009. Edge Assembly-Based Memetic Algorithm for the Capacitated Vehicle Routing Problem. *Networks* **54**(4) 205–215.

Nagata, Y., O. Bräysy, W. Dullaert. 2010. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research* **37**(4) 724–737.

Ngueveu, S.U., C. Prins, R. Wolfler Calvo. 2010. An effective memetic algorithm for the cumulative capacitated vehicle routing problem. *Computers & Operations Research* **37**(11) 1877–1885.

Pirkwieser, S., G.R. Raidl. 2008. A variable neighborhood search for the periodic vehicle routing problem with time windows. *Proceedings of the 9th EU Meeting on Metaheuristics for Logistics and Vehicle Routing, Troyes, France* .

Pisinger, D., S. Ropke. 2007. A general heuristic for vehicle routing problems. *Computers & Operations Research* **34**(8) 2403–2435.

Polacek, M., S. Benkner, K.F. Doerner, R.F. Hartl. 2008. A cooperative and adaptive variable neighborhood search for the multi depot vehicle routing problem with time windows. *BuR-Business Research.* **1**(2) 207–218.

Prescott-Gagnon, E., G. Desaulniers, M. Drexl, L.-M. Rousseau. 2010. European Driver Rules in Vehicle Routing with Time Windows. *Transportation Science* **44**(4) 455–473.

Prins, C. 2004. A Simple and Effective Evolutionary Algorithm for the Vehicle Routing Problem. *Computers & Operations Research* **31**(12) 1985–2002.

Prins, C. 2009. Two memetic algorithms for heterogeneous fleet vehicle routing problems. *Engineering Applications of Artificial Intelligence* **22**(6) 916–928.

Prins, C., S. Bouchenoua. 2005. A memetic algorithm solving the VRP, the CARP, and more general routing problems with nodes, edges and arcs. W. Hart, N. Krasnogor, J. Smith, eds., *Recent advances in memetic algorithms*. Studies in Fuzziness and Soft Computing, Springer, 65–85.

Puranen, T. 2011. Metaheuristics Meet Metamodels - A Modeling Language and a Product Line Architecture for Route Optimization Systems. Ph.D. thesis, University of Jyväskylä, Finland.

Repoussis, P.P., C.D. Tarantilis. 2010. Solving the Fleet Size and Mix Vehicle Routing Problem with Time Windows via Adaptive Memory Programming. *Transportation Research Part C: Emerging Technologies* **18**(5) 695–712.

Repoussis, P.P., C.D. Tarantilis, O. Bräysy, G. Ioannou. 2010. A hybrid evolution strategy for the open vehicle routing problem. *Computers & Operations Research* **37**(3) 443–455.

Repoussis, P.P., C.D. Tarantilis, G. Ioannou. 2009a. An Evolutionary Algorithm for the Open Vehicle Routing Problem with Time Windows. F.B. Pereira, J. Tavares, eds., *Bio-inspired Algorithms for the Vehicle Routing Problem*. Studies in Computational Intelligence, Springer, 55–75.

Repoussis, P.P., C.D. Tarantilis, G. Ioannou. 2009b. Arc-Guided Evolutionary Algorithm for the Vehicle Routing Problem With Time Windows. *IEEE Transactions on Evolutionary Computation* **13**(3) 624–647.

Ribeiro, G.M., G. Laporte. 2012. An Adaptive Large Neighborhood Search Heuristic for the Cumulative Capacitated Vehicle Routing Problem. *Computers & Operations Research* **39**(3) 728–735.

Ropke, S., D. Pisinger. 2006a. A unified heuristic for a large class of Vehicle Routing Problems with Backhauls. *European Journal of Operational Research* **171**(3) 750–775.

Ropke, S., D. Pisinger. 2006b. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science* **40**(4) 455–472.

Sahoo, S., S. Kim, B.-I. Kim, B. Kraas, A. Popov Jr. 2005. Routing Optimization for Waste Management. *Interfaces* **35**(1) 24–36.

Salhi, S., G. Nagy. 1999. A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. *Journal of the Operational Research Society* **50**(10) 1034–1042.

Savelsbergh, M.W.P. 1985. Local Search in Routing Problems with Time Windows. *Annals of Operations Research* **4**(1) 285–305.

Savelsbergh, M.W.P. 1992. The Vehicle Routing Problem with Time Windows: Minimizing Route Duration. *ORSA Journal on Computing* **4**(2) 146–154.

Silva, M.M., A. Subramanian, T. Vidal, L.S. Ochi. 2012. A simple and effective metaheuristic for the Minimum Latency Problem. *European Journal of Operational Research* **221**(3) 513–520.

Solomon, M.M., J. Desrosiers. 1988. Time Window Constrained Routing and Scheduling Problems. *Transportation Science* **22**(1) 1–13.

Subramanian, A., L.M.A. Drummond, C. Bentes, L.S. Ochi, R. Farias. 2010. A parallel heuristic for the Vehicle Routing Problem with Simultaneous Pickup and Delivery. *Computers & Operations Research* **37**(11) 1899–1911.

Subramanian, A., P.H.V. Penna, E. Uchoa, L.S. Ochi. 2012. A Hybrid Algorithm for the Heterogeneous Fleet Vehicle Routing Problem. *European Journal of Operational Research* **221**(2) 285–295.

Subramanian, A., E. Uchoa, L.S. Ochi. 2013. A hybrid algorithm for a class of vehicle routing problems. *Computers & Operations Research, forthcoming* .

Toth, P., D. Vigo, eds. 2002. *The vehicle routing problem*. Society for Industrial Mathematics, Philadelphia, PA, USA.

Toth, P., D. Vigo. 2003. The Granular Tabu Search and Its Application to the Vehicle-Routing Problem. *INFORMS Journal on Computing* **15**(4) 333–346.

Vidal, T., T.G. Crainic, M. Gendreau, N. Lahrichi, W. Rei. 2012a. A Hybrid Genetic Algorithm for Multidepot and Periodic Vehicle Routing Problems. *Operations Research* **60**(3) 611–624.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2011. A Unifying View on Timing Problems and Algorithms. Tech. rep., CIRRELT.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2012b. Implicit Depot Assignments and Rotations in Vehicle Routing Heuristics. Tech. rep., CIRRELT.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2013a. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. *Computers & Operations Research* **40**(1) 475–489.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2013b. Heuristics for Multi-Attribute Vehicle Routing Problems: A Survey and Synthesis. *European Journal of Operational Research, Forthcoming* .

Wolpert, D.H. 1997. No free lunch theorems for optimization. *IEEE Transactions on Evolutionary Computation* **1**(1) 67–82.

Xiao, Y., Q. Zhao, I. Kaku, Y. Xu. 2012. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research* **39**(7) 1419–1431.

Zachariadis, E.E., C.T. Kiranoudis. 2010. An open vehicle routing problem metaheuristic for examining wide solution neighborhoods. *Computers & Operations Research* **37**(4) 712–723.

Zachariadis, E.E., C.T. Kiranoudis. 2011. A local search metaheuristic algorithm for the vehicle routing problem with simultaneous pick-ups and deliveries. *Expert Systems with Applications* **38**(3) 2717–2726.

Zachariadis, E.E., C.T. Kiranoudis. 2012. An effective local search approach for the Vehicle Routing Problem with Backhauls. *Expert Systems with Applications* **39**(3) 3174–3184.

Zachariadis, E.E., C.D. Tarantilis, C.T. Kiranoudis. 2010. An adaptive memory methodology for the vehicle routing problem with simultaneous pick-ups and deliveries. *European Journal of Operational Research* **202**(2) 401–411.

Electronic Companion : Detailed experimental results

Tables 7 to 38 present the detailed results of UHGS and other state-of-the-art methods for a variety of vehicle routing variants and benchmarks. The first group of columns displays the instance identifier, number of customers n, vehicle fleet limit m when applicable, and also the number of customer clusters c in the GVRP case, the number of vehicle types w in fleet size and mix settings, and the number of periods t and depots d in the MDPVRP case. The next group of columns presents the results of actual state-of-the-art methods for each problem, as well as the results of UHGS. When available, both average and best results on several runs (number of runs specified in the headings of the Table) are provided.

We indicate in boldface the best average result among algorithms for each instance as well as the previous Best Known Solution (BKS) in the last column. New best known solutions are underlined. Finally, average measures over sets of instances are presented in the last lines: the computation time for each method, the average percentage of error (Gap) relative to the previous BKS, and the processor used. Some specific details for each problem and benchmark are listed in the following.

VRP with Backhauls (VRPB). "Double" precision values have been used for distance computations. Comparison is made with methods that rely on the same assumption. A fleet size value m is specified in the instances. As in the previous works, we consider a fixed fleet size without allowing less or more than m vehicles. To that extent, the distance matrix has been modified by setting $d_{00} = +\infty$. Other variants of the VRPB, such as the vehicle routing with mixed backhauls, the VRPB with time windows or with multiple depots, can be addressed by UHGS. For the sake of conciseness, results on these variants are not reported in this paper.

Cumulative VRP (CCVRP). Following the guidelines of Ngueveu et al. (2010), the duration constraint is not considered and the fleet size limit is fixed to the minimum feasible value. In the original paper of NPC10, only the best solution on the benchmark instances of Christofides et al. (1979) were reported. This algorithm was then provided to Ribeiro and Laporte (2012) who ran more extensive experiments on all instances. We rely on these latter values for our experimental comparison.

VRP with Simultaneous Deliveries and Pickups (VRPSDP). "Double" precision values have been used for distances and demands. We only compare to recent methods that rely on the same convention.

VRP with Mixed Deliveries and Pickups (VRPMDP). This problem is also sometimes called Mixed VRP with Backhauls (MVRPB). "Double" precision values have been used for distance computations. Comparison is made with methods that rely on the same assumption.

Load Dependent VRP (LDVRP). As in previous work, maximum trip duration constraints are taken into account (sum of driving length plus service time) and the fleet size is unconstrained.

Generalized VRP (GVRP). Distances were rounded to the closest integer to compare with the recent works of Bektas et al. (2011) and Moccia et al. (2012) using the same assumptions.

Open VRP (OVRP). The hierarchical objective of fleet minimization and then distance is used. As almost all recent methods succeeded in reaching the same best known fleet size for each problem, this fleet size "m_{BKS}" is presented in a single column. The same convention as Repoussis et al. (2010) is used: the route duration from the benchmark instances of Christofides et al. (1979) is multiplied by a factor of 0.9, whereas for the benchmark instances of Golden et al. (1998) the duration constraint is not considered.

Vehicle Fleet Mix Problem with Time Windows (VFMPTW). The benchmark instances of Liu and Shen (1999) are considered with three different fleet cost settings (type A,B,C instances). As in the former paper and several following works, we addressed the minimization of fixed fleet cost plus the *trip duration*, i.e., the time elapsed between departure from the depot and return minus the service time (Tables 28-30). Is should be noted that the departure time is not constrained to be time 0, and several best known solutions require a delayed departure from the depot. Additional experiments have also been conducted to address the more standard objective of fixed fleet cost plus total distance (Tables 31-33).

VRP with Soft Time Windows (VRPSTW). A classification and notation for soft time windows settings is introduced in Fu et al. (2007). Type 1 and type 2 soft time windows have been addressed in this paper. Several criteria have been considered by previous authors to assess on the quality of solutions, these criteria include the fleet size, the number of customers serviced outside of their time windows, the amount of lateness and earliness, and the route distance. To optimize on these objectives, we implemented a general formulation of service costs $c_i(t_i)$ to customer as a function of the service date t_i, given in Equation (34). In these equations, γ represents a fixed penalty for servicing a customer outside of its time window, and α and β are respectively the penalties for one unit of tardiness or earliness.

$$c_i(t_i) = \begin{cases} \gamma + \beta(e_i - t_i) & \text{if } t_i < e_i \\ 0 & \text{if } e_i \leq t_i \leq l_i \\ \gamma + \alpha(t_i - l_i) & \text{if } l_i < t_i \end{cases} \tag{34}$$

Two settings of type 1 soft time windows have been addressed. To address the hierarchical objective of minimizing first the fleet size, then the number of customers serviced outside of their time windows, then lateness, and finally distance, the parameters values have been to $\beta = +\infty$, $\gamma = 100000$ and $\alpha = 100$ and the fleet size has been minimized by iteratively reducing the fleet limit (Table 34). Another objective, involving the minimization of distance plus lateness with $\alpha = 1$ has been also addressed (Table 35). In this case, the other parameters have been set to $\beta = +\infty$ and $\gamma = 0$.

Furthermore, two settings of type 2 soft time windows have been addressed. The hierarchical objective has been addressed by setting $\beta = 100$, $\gamma = 100000$ and $\alpha = 100$ (Table 36). We also provide results for the objective seeking to minimize the sum of distance, earliness and lateness by setting $\beta = 1$, $\gamma = 0$ and $\alpha = 1$ (Table 37).

VRP with Backhauls and Time Windows (VRPBTW). "Double" precision values have been used for distance computations. Comparison is made with methods that rely on the same assumption. The objective is to minimize first the fleet size, and the the distance.

Comparison is made with methods that rely on the same assumption. A fleet size value m is specified in the instances. As in the previous works, we consider a fixed fleet size without allowing less or more than m vehicles. To that extent, the distance matrix has been modified by setting $d_{00} = +\infty$. Other variants of the VRPB, such as the vehicle routing with mixed backhauls, the VRPB with time windows or with multiple depots, can be addressed by UHGS. For the sake of conciseness, results on these variants are not reported in this paper.

Time-Dependent VRP with Time Windows (TDVRPTW). The same setting as Kritzinger et al. (2012) is considered, involving the minimization of the total time dependent travel time. The same convention as the authors is used, and thus all routes are constrained to start at time 0, waiting time being allowed only at a customer location upon an early arrival. We consider the same fleet size limits as in Kritzinger et al. (2012). The benchmark instances of Solomon and Desrosiers (1988) are used, along with the three travel time scenarios of Ichoua et al. (2003). The results are reported in Tables (25-27). The planning horizon is divided into three parts of 20%, 60% and 20%, respectively, as in Ichoua et al. (2003). The arc category matrix of Balseiro et al. (2011) is used. Some problem instances may be infeasible, and thus the computation of overall gaps to BKS was based on the subset of instances for which a feasible solution has been found by all methods.

Multi-Depot Periodic VRP with Time Windows (MDPVRPTW). The set of MDPVRP instances from Vidal et al. (2012a), originally issued from the combination of multi-depot and periodic instances of Cordeau et al. (1997, 2001), has been completed with the time windows values from Cordeau et al. (2001). Experiments have been conducted to set the fleet size value close to the minimum fleet size. The corresponding values of m are reported, along with the results, in Table 38.

Table 6: Results on the AVRP, instances of Fischetti et al. (1994)

Inst	**n**	**m**	**SUO13**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
A034-02f	33	2	**1406.00**	**1406.00**	**1406.00**	**1406.00**	0.19	1406.00
A034-04f	33	4	**1773.00**	**1773.00**	**1773.00**	**1773.00**	0.15	1773.00
A034-08f	33	8	**2672.00**	**2672.00**	**2672.00**	**2672.00**	0.11	2672.00
A036-03f	35	3	**1644.00**	**1644.00**	**1644.00**	**1644.00**	0.15	1644.00
A036-05f	35	5	**2110.00**	**2110.00**	**2110.00**	**2110.00**	0.16	2110.00
A036-10f	35	10	**3338.00**	**3338.00**	**3338.00**	**3338.00**	0.12	3338.00
A039-03f	38	3	**1654.00**	**1654.00**	**1654.00**	**1654.00**	0.17	1654.00
A039-06f	38	6	**2289.00**	**2289.00**	**2289.00**	**2289.00**	0.15	2289.00
A039-12f	38	12	**3705.00**	**3705.00**	**3705.00**	**3705.00**	0.13	3705.00
A045-03f	44	3	**1740.00**	**1740.00**	**1740.00**	**1740.00**	0.21	1740.00
A045-06f	44	6	**2303.00**	**2303.00**	**2303.00**	**2303.00**	0.23	2303.00
A045-11f	44	11	**3544.00**	**3544.00**	**3544.00**	**3544.00**	0.24	3544.00
A048-03f	47	3	**1891.00**	**1891.00**	**1891.00**	**1891.00**	0.27	1891.00
A048-05f	47	5	**2283.00**	**2283.00**	**2283.00**	**2283.00**	0.30	2283.00
A048-10f	47	10	**3325.00**	**3325.00**	**3325.00**	**3325.00**	0.26	3325.00
A056-03f	55	3	**1739.00**	**1739.00**	**1739.00**	**1739.00**	0.42	1739.00
A056-05f	55	5	**2165.00**	**2165.00**	**2165.00**	**2165.00**	0.34	2165.00
A056-10f	55	10	**3263.00**	**3263.00**	**3263.00**	**3263.00**	0.32	3263.00
A065-03f	64	3	**1974.00**	**1974.00**	**1974.00**	**1974.00**	0.42	1974.00
A065-06f	64	6	2571.70	**2567.00**	2567.40	**2567.00**	0.70	2567.00
A065-12f	64	12	3904.90	**3902.00**	**3902.00**	**3902.00**	0.47	3902.00
A071-03f	70	3	**2054.00**	**2054.00**	**2054.00**	**2054.00**	0.48	2054.00
A071-05f	70	5	2457.90	**2457.00**	**2457.00**	**2457.00**	0.57	2457.00
A071-10f	70	10	3492.90	**3486.00**	3488.40	**3486.00**	0.64	3486.00
Time			2.24 min		0.30 min			
Gap			+0.02%	+0.00%	+0.00%	+0.00%		
CPU			Xeon 2.93G		Opt 2.2G			

Table 7: Results on the VRPB, instances of Goetschalckx and Jacobs-Blecha (1989)

Inst	**n**	**m**	**GA09**		**ZK11**		**UHGS**			**BKS**
			Avg 8	Best X	Avg 10	Best 10	Avg 10	Best 10	T(min)	
A1	25	8	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	0.11	229.89
A2	25	5	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	0.12	180.12
A3	25	4	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	0.13	163.41
A4	25	3	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	0.16	155.80
B1	30	7	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	0.14	239.08
B2	30	5	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	0.15	198.05
B3	30	3	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	0.18	169.37
C1	40	7	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	0.22	250.56
C2	40	5	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	0.24	215.02
C3	40	5	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	0.23	199.35
C4	40	4	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	0.24	195.37
D1	38	12	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	0.18	322.53
D2	38	11	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	0.17	316.71
D3	38	7	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	0.19	239.48
D4	38	5	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	0.24	205.83
E1	45	7	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	0.27	238.88
E2	45	4	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	0.32	212.26
E3	45	4	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	0.36	206.66
F1	60	6	**263.17**	**263.17**	263.27	**263.17**	**263.17**	**263.17**	0.38	263.17
F2	60	7	**265.21**	**265.21**	265.66	**265.21**	**265.21**	**265.21**	0.39	265.21
F3	60	5	241.48	**241.12**	**241.12**	**241.12**	**241.12**	**241.12**	0.49	241.12
F4	60	4	**233.86**	**233.86**	234.60	**233.86**	**233.86**	**233.86**	0.54	233.86
G2	57	6	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	0.38	245.44
G3	57	5	**229.51**	**229.51**	229.97	**229.51**	**229.51**	**229.51**	0.43	229.51
G4	57	6	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	0.45	232.52
G5	57	5	**221.73**	**221.73**	**222.87**	**221.73**	**221.73**	**221.73**	0.46	221.73
G6	57	4	**213.46**	**213.46**	214.38	**213.46**	**213.46**	**213.46**	0.54	213.46
H1	68	6	269.00	**268.93**	270.06	**268.93**	**268.93**	**268.93**	0.62	268.93
H2	68	5	**253.37**	**253.37**	253.91	**253.37**	**253.37**	**253.37**	0.57	253.37
H3	68	4	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	0.64	247.45
H4	68	5	**250.22**	**250.22**	251.09	**250.22**	**250.22**	**250.22**	0.59	250.22
H5	68	4	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	0.62	246.12
H6	68	5	**249.14**	**249.14**	250.06	**249.14**	**249.14**	**249.14**	0.59	249.14
I1	90	10	350.40	**350.25**	351.08	**350.25**	350.37	**350.25**	0.89	350.25
I2	90	7	310.32	**309.94**	309.98	**309.94**	**309.94**	**309.94**	0.85	309.94
I3	90	5	294.84	**294.51**	294.79	**294.51**	**294.51**	**294.51**	0.99	294.51
I4	90	6	296.13	**295.99**	297.91	**295.99**	**295.99**	**295.99**	0.92	295.99
I5	90	7	301.83	**301.24**	303.49	**301.24**	**301.24**	**301.24**	0.82	301.24
J1	94	10	335.12	**335.01**	335.78	**335.01**	**335.01**	**335.01**	0.83	335.01
J2	94	8	**310.42**	**310.42**	312.51	**310.42**	**310.42**	**310.42**	0.84	310.42
J3	94	6	279.34	**279.22**	280.43	**279.22**	**279.22**	**279.22**	0.93	279.22
J4	94	7	296.58	**296.53**	298.32	**296.53**	**296.53**	**296.53**	1.12	296.53
K1	113	10	396.14	**394.07**	397.38	**394.07**	394.35	**394.07**	1.33	394.07
K2	113	8	362.56	**362.13**	365.46	**362.13**	**362.13**	**362.13**	1.40	362.13
K3	113	9	366.71	**365.69**	369.44	**365.69**	**365.69**	**365.69**	1.30	365.69
K4	113	7	350.32	**348.95**	349.72	**348.95**	**348.95**	**348.95**	1.29	348.95
L1	150	10	420.06	**417.90**	421.68	**417.90**	418.16	**417.90**	3.94	417.90
L2	150	8	401.36	**401.23**	405.20	**401.23**	**401.23**	**401.23**	2.96	401.23
L3	150	9	404.32	**402.68**	405.76	**402.68**	**402.68**	**402.68**	2.73	402.68
L4	150	7	384.83	**384.64**	388.14	**384.64**	**384.64**	**384.64**	2.35	384.64
L5	150	8	390.33	**387.56**	390.46	387.57	**387.56**	**387.56**	2.72	387.56
M1	125	11	399.12	**398.59**	400.50	**398.59**	398.66	**398.59**	1.58	398.59
M2	125	10	398.16	**396.92**	401.91	**396.92**	396.93	**396.92**	2.39	396.92
M3	125	9	377.81	**375.70**	378.07	**375.70**	375.93	**375.70**	2.70	375.70
M4	125	7	348.46	**348.14**	352.03	**348.14**	348.20	**348.14**	1.72	348.14
N1	150	11	408.17	**408.10**	411.72	**408.10**	**408.10**	**408.10**	2.72	408.10
N2	150	10	408.25	**408.07**	412.31	**408.07**	408.13	**408.07**	2.55	408.07
N3	150	9	394.70	**394.34**	398.76	**394.34**	394.94	**394.34**	2.46	394.34
N4	150	10	394.87	**394.79**	396.16	**394.79**	395.13	**394.79**	2.37	394.79
N5	150	7	374.12	**373.48**	376.90	**373.48**	373.55	**373.48**	3.12	373.48
N6	150	8	374.79	**373.76**	379.78	**373.76**	373.76	**373.76**	3.42	373.76
Time			1.13 min		1.09 min		0.99 min			
Gap			+0.09%	+0.00%	+0.38%	+0.00%	+0.01%	+0.00%		
CPU			T5500 1.67G		Xe 2.4G		Opt 2.4G			

Table 8: Results on the CCVRP, instances of Christofides et al. (1979)

Inst	**n**	**m**	**NPC10**		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
p01	50	5	**2230.35**	**2230.35**	2235.27	**2230.35**	**2230.35**	**2230.35**	0.44	2230.35
p02	75	10	2443.07	2421.90	2401.72	**2391.63**	2394.00	**2391.63**	0.70	2391.63
p03	100	8	4073.12	4073.12	4063.98	**4045.42**	**4045.42**	**4045.42**	0.93	4045.42
p04	150	12	5020.75	**4987.52**	4994.93	**4987.52**	**4987.52**	**4987.52**	1.84	4987.52
p05	199	17	5842.00	5810.20	5857.76	5838.32	5809.94	**5806.02**	4.02	5810.12
p11	120	7	7395.83	7317.98	7341.28	7315.87	**7314.55**	**7314.55**	1.35	7315.87
p12	100	10	3559.23	**3558.92**	3566.06	**3558.92**	**3558.93**	**3558.93**	0.64	3558.93
Time			5.20 min		2.69 min		1.42 min			
Gap			+0.74%	+0.28%	+0.37%	+0.07%	+0.01%	-0.01%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

Table 9: Results on the CCVRP, instances of Golden et al. (1998)

Inst	**n**	**m**	**NPC10**[1]		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
pr01	240	9	54878.25	54815.17	54853.76	54786.92	54742.20	**54739.85**	7.40	54786.92
pr02	320	10	100918.54	100836.90	100934.34	100662.53	100562.52	**100560.16**	11.00	100662.53
pr03	400	10	171400.35	171277.26	172231.14	171613.59	170964.42	**170923.53**	23.95	171277.26
pr04	480	10	262830.96	262584.23	265207.46	263433.03	262044.19	**261993.33**	26.14	262584.23
pr05	200	5	114237.00	114163.64	114846.27	114494.66	**114163.63**	**114163.63**	6.95	114163.64
pr06	280	7	140456.96	140430.09	140929.71	140804.64	**140430.08**	**140430.08**	12.65	140430.09
pr07	360	8	186702.15	183282.64	181610.82	180481.56	178976.20	**178880.44**	26.66	180481.56
pr08	440	10	194510.99	194312.60	195174.85	194988.74	193683.21	**193659.14**	26.07	194273.58
pr09	255	14	4740.42	4730.70	4728.05	4725.58	4724.01	**4722.06**	6.97	4725.58
pr10	323	16	6747.10	6732.36	6717.76	6713.92	6720.04	**6713.26**	10.30	6713.92
pr11	399	18	9259.66	9243.05	9216.60	**9214.07**	9222.92	9219.42	14.61	9214.07
pr12	483	19	12649.21	12629.37	12543.04	12526.17	12516.98	**12500.52**	28.66	12526.17
pr13	252	26	3660.93	3653.07	3638.50	3628.30	3632.63	**3627.45**	7.16	3628.30
pr14	320	29	6045.20	5770.02	5257.95	5216.80	5206.53	**5187.56**	16.25	5216.80
pr15	396	33	7140.11	7077.48	7023.12	7010.41	7015.51	**7005.47**	18.70	7010.41
pr16	480	37	9339.45	9300.74	9268.30	9250.98	9247.68	**9239.10**	27.28	9250.98
pr17	240	22	3103.99	3089.99	3068.29	3065.46	3061.28	**3060.14**	6.06	3065.46
pr18	300	27	4582.44	4528.16	4244.60	4221.14	4211.80	**4199.43**	11.64	4221.14
pr19	360	33	5589.12	5570.35	5531.78	5523.38	5502.59	**5496.39**	17.06	5523.38
pr20	420	38	7473.69	7413.58	7240.86	7223.08	7188.59	**7184.19**	21.82	7223.08
Time			94.13 min		21.11 min		17.16 min			
Gap			+2.03%	+1.38%	+0.34%	+0.07%	-0.14%	-0.23%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

Table 10: Results on the VRPSDP, instances of Salhi and Nagy (1999)

Inst	**n**	**ZTK10**	**SDBOF10**		**SUO13**		**UHGS**			**BKS**
		—	Avg 50	Best 50	Avg 10	Best 10	Avg 10	Best 10	T(min)	
CMT1X	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.72	466.77
CMT1Y	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.71	466.77
CMT2X	75	**684.21**	684.49	**684.21**	684.78	**684.21**	684.43	**684.21**	1.32	684.21
CMT2Y	75	**684.21**	684.43	**684.21**	684.59	**684.21**	684.36	**684.21**	1.35	684.21
CMT3X	100	**721.27**	**721.27**	**721.27**	721.46	**721.27**	**721.27**	**721.27**	1.69	721.27
CMT3Y	100	**721.27**	**721.27**	**721.27**	721.50	**721.27**	**721.27**	**721.27**	1.79	721.27
CMT12X	100	**662.22**	**662.22**	**662.22**	663.44	**662.22**	**662.22**	**662.22**	1.74	662.22
CMT12Y	100	**662.22**	662.25	**662.22**	663.12	**662.22**	**662.22**	**662.22**	1.69	662.22
CMT11X	120	**833.92**	842.78	**833.92**	848.65	846.23	**833.92**	**833.92**	2.75	833.92
CMT11Y	120	**833.92**	842.78	**833.92**	848.74	846.23	**833.92**	**833.92**	2.66	833.92
CMT4X	150	**852.46**	**852.46**	**852.46**	853.02	**852.46**	852.65	**852.46**	4.16	852.46
CMT4Y	150	**852.46**	**852.46**	**852.46**	852.73	**852.46**	852.72	**852.46**	3.95	852.46
CMT5X	199	1030.55	1029.66	**1029.25**	1029.52	**1029.25**	1029.60	**1029.25**	7.99	1029.25
CMT5Y	199	1030.55	1029.71	**1029.25**	**1029.25**	**1029.25**	1029.79	**1029.25**	6.60	1029.25
Time		—	256×0.36 min		—		2.79 min			
Gap		+0.11%	+0.16%	+0.00%	+0.30%	+0.21%	+0.01%	+0.00%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table 11: Results on the VRPSDP, instances of Montané and Galvão (2006)

Inst	**n**	**ZTK10**	**SDBOF10**		**SUO13**		**UHGS**			**BKS**
		—	Avg 50	Best 50	Avg 10	Best 10	Avg 10	Best 10	T(min)	
r101	100	**1009.95**	1010.54	**1009.95**	1010.08	**1009.95**	1011.60	**1009.95**	1.23	1009.95
r201	100	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	2.39	666.20
c101	100	1220.99	1220.64	**1220.18**	1220.43	**1220.18**	1220.99	1220.99	1.10	1220.18
c201	100	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	1.63	662.07
rc101	100	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	1.21	1059.32
rc201	100	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	1.99	672.92
R1_2_1	200	3376.30	3369.93	3360.02	3355.04	**3353.80**	3364.40	3355.37	6.77	3353.80
R2_2_1	200	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	8.95	1665.58
C1_2_1	200	3643.82	3635.87	3629.89	3636.53	**3628.51**	3639.00	3637.42	7.25	3628.51
C2_2_1	200	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	5.52	1726.59
RC1_2_1	200	3323.56	3317.51	3306.00	3306.73	**3303.70**	3315.35	3304.39	8.10	3303.70
RC2_2_1	200	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	7.35	1560.00
R1_4_1	400	9691.60	9647.24	9618.97	9539.56	**9519.45**	9594.08	9547.85	24.53	9519.45
R2_4_1	400	3572.38	3557.43	3551.38	3549.49	**3546.49**	3555.10	**3546.49**	24.57	3546.49
C1_4_1	400	11179.36	11118.98	11099.54	11075.60	**11047.19**	11113.63	11077.26	29.67	11047.19
C2_4_1	400	3549.27	3558.92	3546.10	3543.65	**3539.50**	3541.44	**3539.50**	29.76	3539.50
RC1_4_1	400	9645.27	9564.86	9536.77	9478.12	**9447.53**	9509.39	9469.44	29.84	9447.53
RC2_4_1	400	3423.62	3404.62	**3403.70**	**3403.70**	**3403.70**	3404.08	**3403.70**	24.10	3403.70
Time		—	256×3.11 min		7.23 min		12.00 min			
Gap		+0.47%	+0.30%	+0.17%	+0.08%	+0.00%	+0.20%	+0.07%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table 12: Results for the VRPMDP, instances of Salhi and Nagy (1999)

Inst	**n**	**m**	**GA09**		**SUO13**		**UHGS**			**BKS**
			Single	Best 10	Avg 10	Best 10	Avg 10	Best 10	T(min)	
CMT01H	50	4	**465.02**	**465.02**	465.03	**465.02**	**465.02**	**465.02**	0.69	465.02
CMT01Q	50	6	**489.74**	**489.74**	**489.74**	**489.74**	**489.74**	**489.74**	0.55	489.74
CMT01T	50	7	**520.06**	**520.06**	**520.06**	**520.06**	**520.06**	**520.06**	0.52	520.06
CMT02H	75	5	663.92	**662.63**	**662.63**	**662.63**	**662.63**	**662.63**	1.02	662.63
CMT02Q	75	7	733.15	732.76	731.40	**731.26**	731.45	**731.26**	1.46	731.26
CMT02T	75	9	785.00	**782.77**	**782.77**	**782.77**	**782.77**	**782.77**	0.83	782.77
CMT03H	100	3	702.94	701.31	**700.94**	**700.94**	**700.94**	**700.94**	1.94	700.94
CMT03Q	100	4	747.46	**747.15**	**747.15**	**747.15**	**747.15**	**747.15**	1.33	747.15
CMT03T	100	5	**798.07**	**798.07**	**798.07**	**798.07**	**798.07**	**798.07**	1.62	798.07
CMT04H	100	6	831.39	831.39	831.59	**828.12**	829.64	**828.12**	5.67	828.12
CMT04Q	100	8	918.57	**913.93**	915.27	915.27	915.46	915.27	4.18	913.93
CMT04T	100	9	**990.39**	**990.39**	**990.39**	**990.39**	**990.39**	**990.39**	2.50	990.39
CMT05H	120	4	1007.99	992.37	**978.74**	**978.74**	980.58	**978.74**	4.95	978.74
CMT05Q	120	6	1137.58	1134.72	1105.79	**1104.87**	1106.95	**1104.87**	5.79	1104.87
CMT05T	120	7	1232.53	1232.08	1220.24	**1218.77**	1220.53	**1218.77**	5.96	1218.77
CMT06H	150	6	**555.43**	**555.43**	557.35	**555.43**	**555.43**	**555.43**	0.45	555.43
CMT06Q	150	9	**555.43**	**555.43**	557.15	**555.43**	**555.43**	**555.43**	0.45	555.43
CMT06T	150	11	**555.43**	**555.43**	556.64	**555.43**	**555.43**	**555.43**	0.46	555.43
CMT07H	200	9	900.84	900.84	900.84	900.54	900.59	**900.12**	1.03	900.54
CMT07Q	200	12	902.95	**900.69**	902.62	**900.69**	901.09	**900.69**	1.19	900.69
CMT07T	200	15	**903.05**	**903.05**	**903.05**	**903.05**	**903.05**	**903.05**	0.73	903.05
CMT08H	50	7	**865.50**	**865.50**	**865.50**	**865.50**	**865.50**	**865.50**	1.31	865.50
CMT08Q	50	7	**865.50**	**865.50**	**865.50**	**865.50**	**865.50**	**865.50**	1.32	865.50
CMT08T	50	7	**865.54**	**865.54**	**865.54**	**865.54**	**865.54**	**865.54**	1.31	865.54
CMT09H	75	13	1163.85	1161.63	1162.17	**1160.68**	**1160.68**	**1160.68**	3.66	1160.68
CMT09Q	75	14	1161.97	1161.51	1161.69	**1161.24**	1161.32	**1161.24**	3.46	1161.24
CMT09T	75	14	1162.80	1162.68	1164.37	**1162.55**	**1162.55**	**1162.55**	3.68	1162.55
CMT10H	100	10	1392.39	1383.78	1377.23	1372.52	1381.47	**1372.47**	6.56	1372.52
CMT10Q	100	10	1392.23	1386.54	1379.47	**1374.18**	1378.41	**1374.18**	7.91	1374.18
CMT10T	100	10	1401.56	1400.22	1388.17	**1381.04**	1390.82	**1381.04**	5.49	1381.04
CMT11H	100	11	823.06	820.35	818.06	**818.05**	**818.05**	**818.05**	4.53	818.05
CMT11Q	100	11	**939.36**	**939.36**	**939.36**	**939.36**	**939.36**	**939.36**	2.99	939.36
CMT11T	100	11	**998.80**	**998.80**	998.81	**998.80**	**998.80**	**998.80**	2.34	998.80
CMT12H	120	12	646.74	**629.37**	**629.37**	**629.37**	**629.37**	**629.37**	1.49	629.37
CMT12Q	120	12	729.55	729.46	**729.25**	**729.25**	**729.25**	**729.25**	1.53	729.25
CMT12T	120	12	**787.52**	**787.52**	**787.52**	**787.52**	**787.52**	**787.52**	0.95	787.52
CMT13H	150	16	1542.97	**1542.86**	1544.54	**1542.86**	**1542.86**	**1542.86**	2.81	1542.86
CMT13Q	150	16	1542.97	1542.97	1544.05	**1542.86**	**1542.86**	**1542.86**	2.85	1542.86
CMT13T	150	16	1542.97	1542.97	1544.11	**1542.86**	1542.95	**1542.86**	2.86	1542.86
CMT14H	199	20	**821.75**	**821.75**	**821.75**	**821.75**	**821.75**	**821.75**	0.96	821.75
CMT14Q	199	20	**821.75**	**821.75**	**821.75**	**821.75**	**821.75**	**821.75**	0.97	821.75
CMT14T	199	20	**826.77**	**826.77**	**826.77**	**826.77**	**826.77**	**826.77**	1.15	826.77
Time			0.85 min		2.97 min		2.46 min			
Gap			+0.41%	+0.22%	+0.09%	+0.00%	+0.06%	+0.00%		
CPU			Xeon 2.4G		I7 2.93G		Opt 2.2G			

Table 13: Results on the VFMP-F, only fixed vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
F3	20	5	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	0.20	961.03
F4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
F5	20	5	**1007.05**	**1007.05**	1008.76	**1007.05**	**1007.05**	**1007.05**	0.23	1007.05
F6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
F13	50	6	**2406.36**	**2406.36**	2411.31	**2406.36**	2406.57	**2406.36**	1.02	2406.36
F14	50	3	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	0.88	9119.03
F15	50	3	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	0.73	2586.37
F16	50	3	**2720.43**	2729.08	2724.55	**2720.43**	**2720.43**	**2720.43**	0.66	2720.43
F17	75	4	1741.95	1746.09	1744.23	**1734.53**	1735.37	**1734.53**	1.75	1734.53
F18	75	6	**2369.65**	**2369.65**	2373.79	**2369.65**	2374.16	**2369.65**	1.73	2369.65
F19	100	3	8665.05	8665.12	8662.54	**8661.81**	8663.97	8662.86	3.70	8661.81
F20	100	3	4044.68	4044.78	4038.63	4032.81	4037.77	4034.42	2.26	4029.74
Time			8.34 min	0.71 min	0.15 min		1.13 min			
Gap			+0.07%	+0.12%	+0.13%	+0.01%	+0.04%	+0.01%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 14: Results on the VFMP-V, only variable vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
V3	20	5	NC	NC	**623.22**	**623.22**	**623.22**	**623.22**	0.17	623.22
V4	20	3	NC	NC	387.34	**387.18**	**387.18**	**387.18**	0.19	387.18
V5	20	5	NC	NC	**742.87**	**742.87**	**742.87**	**742.87**	0.20	742.87
V6	20	3	NC	NC	**415.03**	**415.03**	**415.03**	**415.03**	0.22	415.03
V13	50	6	**1491.86**	**1491.86**	1492.01	**1491.86**	**1491.86**	**1491.86**	0.72	1491.86
V14	50	3	**603.21**	**603.21**	605.00	**603.21**	**603.21**	**603.21**	0.56	603.21
V15	50	3	**999.82**	**999.82**	1001.03	**999.82**	**999.82**	**999.82**	0.61	999.82
V16	50	3	**1131.00**	**1131.00**	1131.85	**1131.00**	**1131.00**	**1131.00**	0.57	1131.00
V17	75	4	**1038.60**	**1038.60**	1042.48	**1038.60**	**1038.60**	**1038.60**	1.14	1038.60
V18	75	6	**1800.80**	**1800.80**	1802.89	**1800.80**	1801.40	1801.40	1.34	1800.80
V19	100	3	**1105.44**	**1105.44**	1106.71	**1105.44**	1106.93	**1105.44**	1.71	1105.44
V20	100	3	1533.24	1535.12	1534.23	**1530.43**	1531.82	**1530.43**	2.80	1530.43
Time			8.85 min	0.41 min	0.06 min		0.85 min			
Gap			+0.02%	+0.03%	+0.17%	+0.00%	+0.03%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 15: Results on the VFMP-FV, fixed and variable vehicle costs, instances of Golden (1984)

Inst	n	w	ISW09	P09	SPUO12		UHGS			BKS
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
FV3	20	5	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	0.17	1144.22
FV4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
FV5	20	5	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	0.17	1322.26
FV6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
FV13	50	6	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	0.51	2964.65
FV14	50	3	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	0.79	9126.90
FV15	50	3	**2634.96**	2635.21	**2634.96**	**2634.96**	2635.06	**2634.96**	0.71	2634.96
FV16	50	3	3169.10	3169.14	**3168.92**	**3168.92**	**3168.92**	**3168.92**	0.80	3168.92
FV17	75	4	2008.14	**2004.48**	2007.12	**2004.48**	2007.04	**2004.48**	1.33	2004.48
FV18	75	6	3157.20	3153.16	3148.91	**3147.99**	3148.99	3148.99	1.28	3147.99
FV19	100	3	8665.88	8664.67	8662.89	**8661.81**	8663.04	**8661.81**	3.91	8661.81
FV20	100	3	4154.87	4154.49	4153.12	**4153.02**	**4153.02**	**4153.02**	1.73	4153.02
Time			8.42 min	0.39 min	0.13 min		0.99 min			
Gap			+0.05%	+0.02%	+0.01%	+0.00%	+0.01%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table 16: Results on the LDVRP, instances of Christofides et al. (1979)

Inst	n	XZKX12		UHGS			BKS
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
p01	50	751.43	751.11	<u>**746.39**</u>	<u>**746.39**</u>	0.50	751.11
p02	75	1188.62	1179.53	<u>**1172.62**</u>	<u>**1172.62**</u>	0.99	1179.53
p03	100	1153.56	**1147.83**	**1147.83**	**1147.83**	1.49	1147.83
p04	150	1461.69	1452.88	<u>**1446.64**</u>	<u>**1446.64**</u>	4.49	1452.88
p05	199	1865.30	1844.87	1840.54	<u>**1834.31**</u>	6.26	1844.87
p11	120	1516.42	1513.48	<u>**1511.99**</u>	<u>**1511.99**</u>	1.74	1513.48
p12	100	1175.59	**1174.02**	**1174.02**	**1174.02**	0.92	1174.02
Time		1.3 min		2.34 min			
Gap		+0.48%	+0.00%	-0.28%	-0.33%		
CPU		—		Opt 2.2G			

Table 17: Results for the LDVRP, instances of Golden et al. (1998)

Inst	**n**	**XZKX12**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
pr01	240	7714.29	7683.952	7661.10	**7660.64**	12.24	7683.95
pr02	320	11195.02	11172.71	11178.93	**11148.74**	25.71	11172.71
pr03	400	14566.73	14497.64	14525.36	**14480.67**	25.90	14497.64
pr04	480	18605.37	18327.03	18225.56	**18206.84**	30.43	18327.03
pr05	200	8576.91	8561.53	8457.61	**8457.60**	5.71	8561.53
pr06	280	11121.04	11102.22	11056.72	**11056.47**	12.51	11102.22
pr07	360	13477.07	13422.16	13408.06	**13392.93**	27.56	13422.16
pr08	440	16098.60	15928.26	15538.15	**15491.34**	29.76	15928.26
pr09	255	858.34	850.80	835.55	**834.73**	23.17	850.80
pr10	323	1090.85	1083.00	1062.66	**1061.36**	27.86	1083.00
pr11	399	1360.20	1352.32	1319.47	**1316.59**	30.34	1352.32
pr12	483	1661.07	1630.81	1599.59	**1596.68**	30.02	1630.81
pr13	252	1269.37	1261.93	1235.32	**1232.99**	13.84	1261.93
pr14	320	1604.83	1595.48	1564.18	**1562.73**	22.87	1595.48
pr15	396	1987.76	1970.43	1934.13	**1930.84**	28.80	1970.43
pr16	480	2408.72	2391.12	2340.21	**2337.60**	30.00	2391.12
pr17	240	1033.88	1027.21	1018.17	**1018.02**	11.22	1027.21
pr18	300	1469.97	1462.31	1440.00	**1435.34**	20.15	1462.31
pr19	360	2014.26	2007.62	1967.85	**1966.77**	28.32	2007.62
pr20	420	2699.29	2687.85	2626.61	**2621.48**	30.18	2687.85
Time		3.3 min		23.81 min			
Gap		+0.66%	+0.00%	-1.38%	-1.52%		
CPU		—		Opt 2.2G			

Table 18: Results on the GVRP, instances of Bektas et al. (2011)

Inst	n	c	m	BER11	MCL11	UHGS			BKS
				Single	Single	Avg 10	Best 10	T(min)	
A-n32-k5-C16-V2	32	16	2	**519.00**	**519.00**	**519.00**	**519.00**	0.64	519.00
A-n33-k5-C17-V3	33	17	3	**451.00**	**451.00**	**451.00**	**451.00**	0.69	451.00
A-n33-k6-C17-V3	33	17	3	**465.00**	**465.00**	**465.00**	**465.00**	0.69	465.00
A-n34-k5-C17-V3	34	17	3	**489.00**	**489.00**	**489.00**	**489.00**	0.73	489.00
A-n36-k5-C18-V2	36	18	2	**505.00**	**505.00**	**505.00**	**505.00**	0.83	505.00
A-n37-k5-C19-V3	37	19	3	**432.00**	**432.00**	**432.00**	**432.00**	0.76	432.00
A-n37-k6-C19-V3	37	19	3	**584.00**	**584.00**	**584.00**	**584.00**	0.83	584.00
A-n38-k5-C19-V3	38	19	3	**476.00**	**476.00**	**476.00**	**476.00**	0.89	476.00
A-n39-k5-C20-V3	39	20	3	**557.00**	**557.00**	**557.00**	**557.00**	0.99	557.00
A-n39-k6-C20-V3	39	20	3	**544.00**	**544.00**	**544.00**	**544.00**	1.05	544.00
A-n44-k6-C22-V3	44	22	3	**608.00**	**608.00**	**608.00**	**608.00**	1.36	608.00
A-n45-k6-C23-V4	45	23	4	**613.00**	**613.00**	**613.00**	**613.00**	1.10	613.00
A-n45-k7-C23-V4	45	23	4	**674.00**	**674.00**	**674.00**	**674.00**	1.21	674.00
A-n46-k7-C23-V4	46	23	4	**593.00**	**593.00**	**593.00**	**593.00**	1.00	593.00
A-n48-k7-C24-V4	48	24	4	**667.00**	**667.00**	**667.00**	**667.00**	1.26	667.00
A-n53-k7-C27-V4	53	27	4	**603.00**	**603.00**	**603.00**	**603.00**	1.33	603.00
A-n54-k7-C27-V4	54	27	4	**690.00**	**690.00**	**690.00**	**690.00**	1.39	690.00
A-n55-k9-C28-V5	55	28	5	**699.00**	**699.00**	**699.00**	**699.00**	1.32	699.00
A-n60-k9-C30-V5	60	30	5	**769.00**	**769.00**	**769.00**	**769.00**	1.46	769.00
A-n61-k9-C31-V5	61	31	5	**638.00**	**638.00**	**638.00**	**638.00**	1.59	638.00
A-n62-k8-C31-V4	62	31	4	**740.00**	**740.00**	**740.00**	**740.00**	1.89	740.00
A-n63-k10-C32-V5	63	32	5	**801.00**	**801.00**	**801.00**	**801.00**	1.63	801.00
A-n63-k9-C32-V5	63	32	5	**912.00**	**912.00**	**912.00**	**912.00**	1.71	912.00
A-n64-k9-C32-V5	64	32	5	**763.00**	**763.00**	**763.00**	**763.00**	1.84	763.00
A-n65-k9-C33-V5	65	33	5	**682.00**	**682.00**	**682.00**	**682.00**	1.62	682.00
A-n69-k9-C35-V5	69	35	5	**680.00**	**680.00**	**680.00**	**680.00**	1.83	680.00
A-n80-k10-C40-V5	80	40	5	**997.00**	**997.00**	**997.00**	**997.00**	2.65	997.00
B-n31-k5-C16-V3	31	16	3	**441.00**	**441.00**	**441.00**	**441.00**	0.63	441.00
B-n34-k5-C17-V3	34	17	3	**472.00**	**472.00**	**472.00**	**472.00**	0.70	472.00
B-n35-k5-C18-V3	35	18	3	**626.00**	**626.00**	**626.00**	**626.00**	0.65	626.00
B-n38-k6-C19-V3	38	19	3	**451.00**	**451.00**	**451.00**	**451.00**	0.86	451.00
B-n39-k5-C20-V3	39	20	3	**357.00**	**357.00**	**357.00**	**357.00**	0.79	357.00
B-n41-k6-C21-V3	41	21	3	**481.00**	**481.00**	**481.00**	**481.00**	1.08	481.00
B-n43-k6-C22-V3	43	22	3	**483.00**	**483.00**	**483.00**	**483.00**	1.15	483.00
B-n44-k7-C22-V4	44	22	4	**540.00**	**540.00**	**540.00**	**540.00**	1.13	540.00
B-n45-k5-C23-V3	45	23	3	**497.00**	**497.00**	**497.00**	**497.00**	1.28	497.00
B-n45-k6-C23-V4	45	23	4	**478.00**	**478.00**	**478.00**	**478.00**	1.23	478.00
B-n50-k7-C25-V4	50	25	4	**449.00**	**449.00**	**449.00**	**449.00**	1.08	449.00
B-n50-k8-C25-V5	50	25	5	**916.00**	**916.00**	**916.00**	**916.00**	1.36	916.00
B-n51-k7-C26-V4	51	26	4	**651.00**	**651.00**	**651.00**	**651.00**	1.44	651.00
B-n52-k7-C26-V4	52	26	4	**450.00**	**450.00**	**450.00**	**450.00**	1.18	450.00
B-n56-k7-C28-V4	56	28	4	**486.00**	492.00	**486.00**	**486.00**	1.35	486.00
B-n57-k7-C29-V4	57	29	4	**751.00**	**751.00**	**751.00**	**751.00**	1.43	751.00
B-n57-k9-C29-V5	57	29	5	**942.00**	**942.00**	**942.00**	**942.00**	1.60	942.00
B-n63-k10-C32-V5	63	32	5	**816.00**	**816.00**	**816.00**	**816.00**	1.74	816.00
B-n64-k9-C32-V5	64	32	5	**509.00**	**509.00**	**509.00**	**509.00**	1.37	509.00
B-n66-k9-C33-V5	66	33	5	**808.00**	**808.00**	**808.00**	**808.00**	1.88	808.00
B-n67-k10-C34-V5	67	34	5	**673.00**	**673.00**	**673.00**	**673.00**	1.91	673.00
B-n68-k9-C34-V5	68	34	5	**704.00**	**704.00**	**704.00**	**704.00**	1.87	704.00
B-n78-k10-C39-V5	78	39	5	**803.00**	804.00	**803.00**	**803.00**	2.38	803.00
G-n262-k25-C131-V12	262	131	12	3249.00	3319.00	3241.80	<u>**3229.00**</u>	22.98	3249.00
M-n101-k10-C51-V5	101	51	5	**542.00**	**542.00**	**542.00**	**542.00**	2.85	542.00
M-n121-k7-C61-V4	121	61	4	**719.00**	720.00	**719.00**	**719.00**	5.45	719.00

Table 19: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	n	c	m	BER11	MCL11	UHGS			BKS
				Single	Single	Avg 10	Best 10	T(min)	
M-n151-k12-C76-V6	151	76	6	**659.00**	**659.00**	**659.00**	**659.00**	4.94	659.00
M-n200-k16-C100-V8	200	100	8	**791.00**	805.00	<u>786.00</u>	<u>786.00</u>	11.47	791.00
P-n101-k4-C51-V2	101	51	2	**455.00**	**455.00**	**455.00**	**455.00**	4.83	455.00
P-n16-k8-C8-V5	16	8	5	**239.00**	**239.00**	**239.00**	**239.00**	0.09	239.00
P-n19-k2-C10-V2	19	10	2	**147.00**	**147.00**	**147.00**	**147.00**	0.15	147.00
P-n20-k2-C10-V2	20	10	2	**154.00**	**154.00**	**154.00**	**154.00**	0.15	154.00
P-n21-k2-C11-V2	21	11	2	**160.00**	162.00	**160.00**	**160.00**	0.19	160.00
P-n22-k2-C11-V2	22	11	2	**162.00**	163.00	**162.00**	**162.00**	0.24	162.00
P-n22-k8-C11-V5	22	11	5	**314.00**	**314.00**	**314.00**	**314.00**	0.18	314.00
P-n23-k8-C12-V5	23	12	5	**312.00**	**312.00**	**312.00**	**312.00**	0.23	312.00
P-n40-k5-C20-V3	40	20	3	**294.00**	**294.00**	**294.00**	**294.00**	1.00	294.00
P-n45-k5-C23-V3	45	23	3	**337.00**	**337.00**	**337.00**	**337.00**	1.13	337.00
P-n50-k10-C25-V5	50	25	5	**410.00**	**410.00**	**410.00**	**410.00**	1.08	410.00
P-n50-k7-C25-V4	50	25	4	**353.00**	**353.00**	**353.00**	**353.00**	1.23	353.00
P-n50-k8-C25-V4	50	25	4	**392.00**	421.00	**392.00**	**392.00**	1.39	392.00
P-n51-k10-C26-V6	51	26	6	**427.00**	**427.00**	**427.00**	**427.00**	1.04	427.00
P-n55-k10-C28-V5	55	28	5	**415.00**	**415.00**	**415.00**	**415.00**	1.22	415.00
P-n55-k15-C28-V8	55	28	8	**555.00**	565.00	**555.00**	**555.00**	1.07	555.00
P-n55-k7-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.41	361.00
P-n55-k8-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.36	361.00
P-n60-k10-C30-V5	60	30	5	**443.00**	**443.00**	**443.00**	**443.00**	1.52	443.00
P-n60-k15-C30-V8	60	30	8	**565.00**	**565.00**	**565.00**	**565.00**	1.24	565.00
P-n65-k10-C33-V5	65	33	5	**487.00**	**487.00**	**487.00**	**487.00**	1.55	487.00
P-n70-k10-C35-V5	70	35	5	**485.00**	**485.00**	**485.00**	**485.00**	1.74	485.00
P-n76-k4-C38-V2	76	38	2	**383.00**	**383.00**	**383.00**	**383.00**	2.80	383.00
P-n76-k5-C38-V3	76	38	3	**405.00**	**405.00**	**405.00**	**405.00**	2.44	405.00
A-n32-k5-C11-V2	32	11	2	**386.00**	**386.00**	**386.00**	**386.00**	0.33	386.00
A-n33-k5-C11-V2	33	11	2	318.00	**315.00**	**315.00**	**315.00**	0.30	315.00
A-n33-k6-C11-V2	33	11	2	**370.00**	**370.00**	**370.00**	**370.00**	0.28	370.00
A-n34-k5-C12-V2	34	12	2	**419.00**	**419.00**	**419.00**	**419.00**	0.39	419.00
A-n36-k5-C12-V2	36	12	2	**396.00**	**396.00**	**396.00**	**396.00**	0.39	396.00
A-n37-k5-C13-V2	37	13	2	**347.00**	**347.00**	**347.00**	**347.00**	0.42	347.00
A-n37-k6-C13-V2	37	13	2	**431.00**	**431.00**	**431.00**	**431.00**	0.47	431.00
A-n38-k5-C13-V2	38	13	2	**367.00**	**367.00**	**367.00**	**367.00**	0.46	367.00
A-n39-k5-C13-V2	39	13	2	**364.00**	**364.00**	**364.00**	**364.00**	0.43	364.00
A-n39-k6-C13-V2	39	13	2	**403.00**	**403.00**	**403.00**	**403.00**	0.53	403.00
A-n44-k6-C15-V2	44	15	2	**503.00**	**503.00**	**503.00**	**503.00**	0.74	503.00
A-n45-k6-C15-V3	45	15	3	**474.00**	**474.00**	**474.00**	**474.00**	0.65	474.00
A-n45-k7-C15-V3	45	15	3	**475.00**	**475.00**	**475.00**	**475.00**	0.62	475.00
A-n46-k7-C16-V3	46	16	3	**462.00**	**462.00**	**462.00**	**462.00**	0.87	462.00
A-n48-k7-C16-V3	48	16	3	**451.00**	**451.00**	**451.00**	**451.00**	0.90	451.00
A-n53-k7-C18-V3	53	18	3	**440.00**	**440.00**	**440.00**	**440.00**	1.10	440.00
A-n54-k7-C18-V3	54	18	3	**482.00**	**482.00**	**482.00**	**482.00**	1.04	482.00
A-n55-k9-C19-V3	55	19	3	**473.00**	**473.00**	**473.00**	**473.00**	1.10	473.00
A-n60-k9-C20-V3	60	20	3	**595.00**	**595.00**	**595.00**	**595.00**	1.33	595.00
A-n61-k9-C21-V4	61	21	4	**473.00**	**473.00**	**473.00**	**473.00**	1.18	473.00
A-n62-k8-C21-V3	62	21	3	**596.00**	**596.00**	**596.00**	**596.00**	1.61	596.00
A-n63-k10-C21-V4	63	21	4	**593.00**	**593.00**	**593.00**	**593.00**	1.33	593.00
A-n63-k9-C21-V3	63	21	3	**642.00**	643.00	**642.00**	**642.00**	1.51	642.00
A-n64-k9-C22-V3	64	22	3	**536.00**	**536.00**	**536.00**	**536.00**	1.64	536.00
A-n65-k9-C22-V3	65	22	3	**500.00**	**500.00**	**500.00**	**500.00**	1.53	500.00
A-n69-k9-C23-V3	69	23	3	**520.00**	**520.00**	**520.00**	**520.00**	1.60	520.00
A-n80-k10-C27-V4	80	27	4	**710.00**	**710.00**	**710.00**	**710.00**	2.14	710.00
B-n31-k5-C11-V2	31	11	2	**356.00**	**356.00**	**356.00**	**356.00**	0.35	356.00

Table 20: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	n	c	m	BER11	MCL11	UHGS			BKS
				Single	Single	Avg 10	Best 10	T(min)	
B-n34-k5-C12-V2	34	12	2	**369.00**	**369.00**	**369.00**	**369.00**	0.35	369.00
B-n35-k5-C12-V2	35	12	2	**501.00**	**501.00**	**501.00**	**501.00**	0.36	501.00
B-n38-k6-C13-V2	38	13	2	**370.00**	**370.00**	**370.00**	**370.00**	0.53	370.00
B-n39-k5-C13-V2	39	13	2	**280.00**	**280.00**	**280.00**	**280.00**	0.42	280.00
B-n41-k6-C14-V2	41	14	2	**407.00**	**407.00**	**407.00**	**407.00**	0.57	407.00
B-n43-k6-C15-V2	43	15	2	**343.00**	**343.00**	**343.00**	**343.00**	0.73	343.00
B-n44-k7-C15-V3	44	15	3	**395.00**	**395.00**	**395.00**	**395.00**	0.54	395.00
B-n45-k5-C15-V2	45	15	2	422.00	**410.00**	**410.00**	**410.00**	0.73	410.00
B-n45-k6-C15-V2	45	15	2	**336.00**	**336.00**	**336.00**	**336.00**	0.76	336.00
B-n50-k7-C17-V3	50	17	3	**393.00**	**393.00**	**393.00**	**393.00**	1.00	393.00
B-n50-k8-C17-V3	50	17	3	**598.00**	**598.00**	**598.00**	**598.00**	0.98	598.00
B-n51-k7-C17-V3	51	17	3	**511.00**	**511.00**	**511.00**	**511.00**	0.72	511.00
B-n52-k7-C18-V3	52	18	3	**359.00**	**359.00**	**359.00**	**359.00**	0.99	359.00
B-n56-k7-C19-V3	56	19	3	**356.00**	**356.00**	**356.00**	**356.00**	1.10	356.00
B-n57-k7-C19-V3	57	19	3	**558.00**	**558.00**	**558.00**	**558.00**	1.26	558.00
B-n57-k9-C19-V3	57	19	3	**681.00**	**681.00**	**681.00**	**681.00**	1.18	681.00
B-n63-k10-C21-V3	63	21	3	**599.00**	**599.00**	**599.00**	**599.00**	1.61	599.00
B-n64-k9-C22-V4	64	22	4	**452.00**	**452.00**	**452.00**	**452.00**	1.34	452.00
B-n66-k9-C22-V3	66	22	3	**609.00**	**609.00**	**609.00**	**609.00**	1.43	609.00
B-n67-k10-C23-V4	67	23	4	**558.00**	**558.00**	**558.00**	**558.00**	1.37	558.00
B-n68-k9-C23-V3	68	23	3	**523.00**	**523.00**	**523.00**	**523.00**	1.71	523.00
B-n78-k10-C26-V4	78	26	4	**606.00**	**606.00**	**606.00**	**606.00**	1.83	606.00
G-n262-k25-C88-V9	262	88	9	2476.00	2463.00	2469.00	**2460.00**	13.80	2463.00
M-n101-k10-C34-V4	101	34	4	**458.00**	**458.00**	**458.00**	**458.00**	2.62	458.00
M-n121-k7-C41-V3	121	41	3	**527.00**	**527.00**	**527.00**	**527.00**	4.18	527.00
M-n151-k12-C51-V4	151	51	4	**483.00**	**483.00**	**483.00**	**483.00**	5.19	483.00
M-n200-k16-C67-V6	200	67	6	**605.00**	**605.00**	**605.00**	**605.00**	6.06	605.00
P-n101-k4-C34-V2	101	34	2	374.00	**370.00**	**370.00**	**370.00**	3.26	370.00
P-n16-k8-C6-V4	16	6	4	**170.00**	**170.00**	**170.00**	**170.00**	0.05	170.00
P-n19-k2-C7-V1	19	7	1	**111.00**	**111.00**	**111.00**	**111.00**	0.07	111.00
P-n20-k2-C7-V1	20	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n21-k2-C7-V1	21	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n22-k2-C8-V1	22	8	1	**111.00**	**111.00**	**111.00**	**111.00**	0.10	111.00
P-n22-k8-C8-V4	22	8	4	**249.00**	**249.00**	**249.00**	**249.00**	0.12	249.00
P-n23-k8-C8-V3	23	8	3	**174.00**	**174.00**	**174.00**	**174.00**	0.13	174.00
P-n40-k5-C14-V2	40	14	2	**213.00**	**213.00**	**213.00**	**213.00**	0.55	213.00
P-n45-k5-C15-V2	45	15	2	**238.00**	**238.00**	**238.00**	**238.00**	0.71	238.00
P-n50-k10-C17-V4	50	17	4	**292.00**	**292.00**	**292.00**	**292.00**	0.69	292.00
P-n50-k7-C17-V3	50	17	3	**261.00**	**261.00**	**261.00**	**261.00**	0.84	261.00
P-n50-k8-C17-V3	50	17	3	**262.00**	**262.00**	**262.00**	**262.00**	0.84	262.00
P-n51-k10-C17-V4	51	17	4	**309.00**	**309.00**	**309.00**	**309.00**	0.73	309.00
P-n55-k10-C19-V4	55	19	4	**301.00**	**301.00**	**301.00**	**301.00**	1.01	301.00
P-n55-k15-C19-V6	55	19	6	**378.00**	**378.00**	**378.00**	**378.00**	0.76	378.00
P-n55-k7-C19-V3	55	19	3	**271.00**	**271.00**	**271.00**	**271.00**	1.15	271.00
P-n55-k8-C19-V3	55	19	3	**274.00**	**274.00**	**274.00**	**274.00**	1.15	274.00
P-n60-k10-C20-V4	60	20	4	**325.00**	**325.00**	**325.00**	**325.00**	1.31	325.00
P-n60-k15-C20-V5	60	20	5	**382.00**	**382.00**	**382.00**	**382.00**	1.10	382.00
P-n65-k10-C22-V4	65	22	4	**372.00**	**372.00**	**372.00**	**372.00**	1.25	372.00
P-n70-k10-C24-V4	70	24	4	**385.00**	**385.00**	**385.00**	**385.00**	1.55	385.00
P-n76-k4-C26-V2	76	26	2	320.00	**309.00**	**309.00**	**309.00**	2.02	309.00
P-n76-k5-C26-V2	76	26	2	**309.00**	**309.00**	**309.00**	**309.00**	2.31	309.00
Time				0.01 min	0.34 min		1.53 min		
Gap				+0.06%	+0.11%	+0.00%	-0.01%		
CPU				Opt 2.4G	Duo 1.83G		Opt 2.4G		

Table 21: Results on the OVRP, instances of Christofides et al. (1979) and Fisher (1994)

Inst	n	m_{BKS}	ZK10		RTBI10	SUO13		UHGS			BKS
			Avg 10	Best 10	Single	Avg 10	Best 10	Avg 10	Best 10	T(min)	
p01	50	5	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	0.41	416.06
p02	75	10	568.38	**567.14**	**567.14**	**567.14**	**567.14**	568.15	**567.14**	0.51	567.14
p03	100	8	639.98	**639.74**	**639.74**	639.81	**639.74**	**639.74**	**639.74**	0.85	639.74
p04	150	12	733.93	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	1.73	733.13
p05	199	16	895.62	893.39	894.11	895.55	**883.50**	890.15	884.08	4.13	883.50
p06	50	6	—	—	**412.96**	**412.96**	**412.96**	**412.96**	**412.96**	0.55	412.96
p07	75	10	—	—	584.15	*582.07**	**583.19**	584.59	**583.19**	0.77	583.19
p08	100	9	—	—	**644.63**	644.95	**644.63**	644.79	**644.63**	1.79	644.63
p09	150	13	—	—	764.56	759.38	757.91	760.75	<u>**757.07**</u>	5.18	757.84
p10	199	17	—	—	888.46	877.68	**874.71**	875.49	**874.71**	6.10	874.71
p11	120	7	682.34	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	1.51	682.12
p12	100	10	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	0.61	534.24
p13	120	11	—	—	910.26	904.02	**899.16**	900.22	**899.16**	3.39	899.16
p14	100	11	—	—	**591.87**	**591.87**	**591.87**	**591.87**	**591.87**	1.70	591.87
f11	71	4	**177.00**	**177.00**	**177.00**	177.21	**177.00**	**177.00**	**177.00**	0.65	177.00
f12	134	7	770.57	**769.55**	**769.55**	770.00	**769.55**	769.68	**769.55**	1.71	769.55
Time			—		9.54 min	2.39 min		1.97 min			
Gap			—	—	+0.32%	*+0.16%**	+0.00%	+0.11%	+0.00%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

∗ The minimum fleet size was not attained by SUO13 on all runs.

Table 22: Results on the OVRP, instances of Golden et al. (1998)

Inst	n	m^*	ZK10		RTBI10	SUO13		UHGS			BKS
			Avg 10	Best 10	Single	Avg 10	Best 10	Avg 10	Best 10	T(min)	
pr01	240	9	4562.88	4557.38	4583.70	4551.74	4544.46	4546.35	<u>**4543.00**</u>	8.20	4544.46
pr02	320	10	7264.32	7253.20	7271.24	7229.56	7215.48	7218.41	<u>**7213.56**</u>	14.94	7215.48
pr03	400	9	9824.44	9793.72	9821.09	9784.52	9773.83	9763.39	<u>**9750.63**</u>	22.66	9773.83
pr04	480	10	12430.06	12415.36	12428.20	12393.40	12389.43	12387.82	<u>**12380.66**</u>	26.41	12389.43
pr05	200	5	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	5.71	6018.52
pr06	280	7	7735.10	7731.00	7733.77	7728.77	7721.16	7704.91	<u>**7704.59**</u>	11.23	7721.16
pr07	360	8	9243.69	9193.15	9254.15	9205.01	9180.93	9132.27	<u>**9127.70**</u>	19.01	9180.93
pr08	440	10	10363.28	10347.70	10363.40	10342.10	10326.57	10316.60	<u>**10289.70**</u>	26.43	10326.57
Time			14.79 min		17.53 min	64.07 min		16.82 min			
Gap			+0.39%	+0.21%	+0.47%	+0.13%	+0.00%	-0.11%	-0.19%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

Table 23: Results on the OVRPTW, instances of Solomon and Desrosiers (1988)

Inst	**n**	**RTI09**		KTDHS12		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	**19.00/1192.85**	**19/1192.85**	**19.00/1192.95**	**19/1192.85**	**19.00/1192.85**	**19/1192.85**	2.86	19/1192.85
R102	100	17.00/1081.65	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	2.81	17/1079.39
R103	100	13.00/1017.28	**13/1016.78**	13.00/1016.83	**13/1016.78**	**13.00/1016.78**	**13/1016.78**	3.57	13/1016.78
R104	100	9.53/844.32	9/869.63	9.00/837.28	9/834.44	9.00/833.52	**9/832.50**	4.86	9/834.44
R105	100	14.00/1055.98	**14/1055.04**	14.00/1055.34	**14/1055.04**	**14.00/1055.04**	**14/1055.04**	3.92	14/1055.04
R106	100	12.00/1001.04	12/1000.95	12.00/1003.15	12/1001.41	12.00/1000.93	**12/1000.36**	4.88	12/1000.95
R107	100	10.00/915.82	10/912.99	10.00/918.47	**10/910.75**	10.00/914.75	10/912.08	6.14	10/910.75
R108	100	9.00/760.30	9/760.30	9.00/765.63	9/760.30	9.00/760.32	**9/759.86**	4.80	9/760.30
R109	100	11.00/934.77	11/934.53	11.00/937.86	**11/934.15**	11.00/934.52	**11/934.15**	4.36	11/934.15
R110	100	10.00/851.01	**10/846.49**	10.00/881.91	10/874.64	10.00/885.18	**10/873.75**	5.08	10/846.49
R111	100	10.00/902.45	**10/895.21**	10.00/904.25	10/895.56	**10.00/895.21**	**10/895.21**	5.34	10/895.21
R112	100	9.47/814.33	9/811.73	9.00/815.43	9/805.17	9.00/811.76	**9/801.43**	5.81	9/805.17
C101	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.67	10/556.18
C102	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.00	10/556.18
C103	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.10	10/556.18
C104	100	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	1.06	10/555.41
C105	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.83	10/556.18
C106	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.85	10/556.18
C107	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.84	10/556.18
C108	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.95	10/555.80
C109	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.97	10/555.80
RC101	100	14.00/1228.76	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	3.37	14/1227.37
RC102	100	12.00/1205.65	12/1203.05	12.00/1197.16	**12/1185.43**	12.50/1125.04	12/1195.20	4.12	12/1185.43
RC103	100	11.00/927.62	11/923.15	11.00/919.32	**11/918.65**	**11.00/918.65**	**11/918.65**	3.92	11/918.65
RC104	100	10.00/789.21	**10/787.02**	10.00/790.58	**10/787.02**	**10.00/787.02**	**10/787.02**	4.51	10/787.02
RC105	100	13.00/1220.96	13/1195.20	13.00/1201.73	13/1195.20	13.10/1186.34	**13/1185.43**	4.45	13/1195.20
RC106	100	11.00/1113.20	11/1095.65	11.00/1073.78	**11/1071.83**	11.00/1074.52	**11/1071.83**	4.24	11/1071.83
RC107	100	11.00/862.44	11/861.28	11.00/862.88	11/861.28	11.00/860.62	**11/860.62**	4.03	11/861.28
RC108	100	10.00/832.05	**10/831.09**	10.00/837.38	10/833.03	10.00/832.27	**10/831.09**	4.31	10/831.09
R201	100	**4.00/1182.43**	**4/1182.43**	4.00/1187.99	**4/1182.43**	**4.00/1182.43**	**4/1182.43**	5.80	4/1182.43
R202	100	3.00/1150.07	**3/1149.59**	3.00/1152.70	3/1151.14	**3.00/1149.59**	**3/1149.59**	9.73	3/1149.59
R203	100	3.00/894.34	**3/889.12**	3.00/900.51	3/894.40	3.00/890.02	**3/889.12**	11.72	3/889.12
R204	100	2.00/803.54	2/801.46	2.00/821.67	2/803.50	2.00/798.13	**2/797.83**	7.67	2/801.46
R205	100	3.00/950.21	**3/943.33**	3.00/966.18	3/952.83	**3.00/943.33**	**3/943.33**	11.21	3/943.33
R206	100	3.00/870.96	3/869.27	3.00/883.18	3/874.78	**3.00/865.32**	**3/865.32**	13.01	3/869.27
R207	100	2.77/865.93	2/857.08	2.00/880.56	2/857.08	**2.00/854.40**	**2/854.40**	6.65	2/857.08
R208	100	2.00/700.67	2/700.53	2.00/710.74	2/700.63	2.00/694.67	**2/694.24**	8.00	2/700.53
R209	100	3.00/853.86	**3/851.69**	3.00/864.91	**3/851.69**	3.00/852.42	**3/851.69**	11.57	3/851.69
R210	100	3.00/894.38	3/892.45	3.00/908.77	3/901.87	3.00/891.23	**3/890.02**	11.00	3/892.45
R211	100	2.00/887.41	2/886.90	2.00/894.55	2/874.49	2.00/852.15	**2/846.92**	8.14	2/874.49
C201	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	1.33	3/548.51
C202	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	2.09	3/548.51
C203	100	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	2.73	3/548.13
C204	100	**3.00/547.55**	**3/547.55**	3.00/549.02	**3/547.55**	**3.00/547.55**	**3/547.55**	3.21	3/547.55
C205	100	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	1.76	3/545.83
C206	100	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	1.87	3/545.45
C207	100	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	1.92	3/545.24
C208	100	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	2.16	3/545.28
RC201	100	4.00/1309.06	**4/1303.73**	4.00/1321.87	4/1304.50	**4.00/1303.73**	**4/1303.73**	6.50	4/1303.73
RC202	100	3.00/1329.52	3/1321.43	3.00/1335.13	3/1292.35	3.00/1289.86	**3/1289.04**	10.92	3/1292.35
RC203	100	3.00/995.02	3/993.29	3.00/1004.88	3/993.22	3.00/987.28	**3/977.56**	10.81	3/993.22
RC204	100	3.00/719.92	**3/718.97**	3.00/736.97	3/722.20	**3.00/718.97**	**3/718.97**	9.52	3/718.97
RC205	100	4.00/1190.67	**4/1189.84**	4.00/1193.05	**4/1189.84**	**4.00/1189.84**	**4/1189.84**	7.92	4/1189.84
RC206	100	3.00/1092.09	3/1091.79	3.00/1102.53	3/1092.66	**3.00/1087.97**	**3/1087.97**	10.72	3/1091.79
RC207	100	3.00/1005.32	**3/998.70**	3.00/1015.46	3/1006.06	3.00/999.29	**3/998.70**	9.43	3/998.70
RC208	100	3.00/772.76	3/769.40	3.00/786.41	3/778.32	3.00/769.12	**3/768.75**	11.94	3/769.40
Time		10.0 min		10.0 min		5.27 min			
Gap		+0.89%/+0.42%	0%/+0.24%	0%/+0.79%	0%/+0.18%	+0.09%/-0.10%	0%/-0.10%		
CPU		P-IV 3G		Xe 2.67G		Opt 2.2G			

Table 24: Results on the VRPBTW, instances of Gélinas et al. (1995)

Inst	**n**	**RP06**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
BHR101A	100	22.0	**22/1818.86**	**22.0/1818.86**	**22/1818.86**	3.31	22.00/1818.86
BHR101B	100	23.0	23/1959.56	**23.0/1959.52**	**23/1959.52**	4.40	23.00/1959.56
BHR101C	100	24.0	**24/1939.10**	**24.0/1939.10**	**24/1939.10**	3.26	24.00/1939.10
BHR102A	100	19.0	19/1653.19	**19.0/1653.18**	**19/1653.18**	3.38	19.00/1653.19
BHR102B	100	22.0	**22/1750.70**	**22.0/1750.70**	**22/1750.70**	2.98	22.00/1750.70
BHR102C	100	22.0	**22/1775.76**	**22.0/1775.76**	**22/1775.76**	3.02	22.00/1775.76
BHR103A	100	15.0	15/1387.57	**15.0/1385.38**	**15/1385.38**	3.59	15.00/1387.57
BHR103B	100	15.0	15/1390.33	**15.0/1390.32**	**15/1390.32**	3.67	15.00/1390.33
BHR103C	100	17.0	**17/1456.48**	**17.0/1456.48**	**17/1456.48**	3.37	17.00/1456.48
BHR104A	100	11.0	11/1084.17	10.4/1157.92	**10/1204.57**	4.69	11.00/1084.17
BHR104B	100	11.0	**11/1154.84**	11.0/1155.71	**11/1154.84**	4.45	11.00/1154.84
BHR104C	100	11.0	11/1191.38	11.0/1190.93	**11/1190.20**	6.13	11.00/1191.38
BHR105A	100	15.4	15/1561.28	15.0/1561.61	**15/1560.15**	5.24	15.00/1561.28
BHR105B	100	16.0	**16/1583.30**	**16.0/1583.30**	**16/1583.30**	4.45	16.00/1583.30
BHR105C	100	16.5	16/1710.19	16.0/1709.88	**16/1709.66**	5.48	16.00/1710.19
Time		1.90 min		4.10 min			
Gap		+0.39%	+0%/+0%	-0.36%/+0.45%	-0.61%/+0.72%		
CPU		P-IV 1.5G		Opt 2.2G			

Table 25: Results on the TDVRPTW, scenario 1. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
C101	100	10	**755.86**	**755.86**	**755.86**	**755.86**	2.87	755.86
C102	100	10	**738.87**	**738.87**	**738.87**	**738.87**	3.33	738.87
C103	100	10	713.82	**713.50**	**713.50**	**713.50**	3.32	713.50
C104	100	10	683.96	**683.27**	**683.27**	**683.27**	3.22	683.27
C105	100	10	**747.41**	**747.41**	**747.41**	**747.41**	3.35	747.41
C106	100	10	**741.08**	**741.08**	**741.08**	**741.08**	3.27	741.08
C107	100	10	**735.84**	**735.84**	**735.84**	**735.84**	3.16	735.84
C108	100	10	**705.61**	**705.61**	**705.61**	**705.61**	2.99	705.61
C109	100	10	691.57	691.26	**691.16**	**691.16**	3.61	691.26
R101	100	17	1320.22	1311.57	**1310.20**	**1310.20**	3.68	1311.57
R102	100	16	1132.68	1130.36	**1128.23**	**1128.23**	3.16	1130.36
R103	100	13	926.72	918.03	912.23	**911.74**	4.85	918.03
R104	100	10	761.08	754.67	753.04	**752.73**	6.25	754.67
R105	100	14	1033.27	1020.20	1020.43	**1020.16**	4.90	1020.20
R106	100	12	949.94	940.86	939.38	**939.19**	5.16	940.86
R107	100	10	843.05	837.10	831.69	**831.50**	6.86	837.10
R108	100	9	741.89	734.76	726.33	**725.08**	8.71	734.76
R109	100	11	842.03	833.93	**829.39**	**829.39**	5.14	833.93
R110	100	10	817.95	806.34	796.76	**795.00**	7.21	806.34
R111	100	10	808.21	798.35	797.70	**797.50**	7.80	798.35
R112	100	9	739.17	723.25	717.74	**714.75**	7.28	723.25
RC101	100	15	1241.72	**1236.04**	**1236.04**	**1236.04**	4.14	1236.04
RC102	100	13	1085.29	**1072.60**	1073.20	**1072.60**	4.79	1072.60
RC103	100	11	937.77	931.49	**928.22**	**928.22**	4.23	931.49
RC104	100	10	864.99	858.35	844.62	**844.57**	5.42	858.35
RC105	100	14	1161.43	1151.13	1148.75	**1147.51**	5.03	1151.13
RC106	100	12	997.20	994.23	989.83	**988.73**	6.14	994.23
RC107	100	11	915.44	907.27	899.97	**898.27**	6.12	907.27
RC108	100	10	853.37	843.04	**838.10**	**838.10**	5.52	843.04
C201	100	3	**620.77**	**620.77**	**620.77**	**620.77**	9.19	620.77
C202	100	3	601.93	**601.19**	**601.19**	**601.19**	10.32	601.19
C203	100	3	589.01	585.76	**585.75**	**585.75**	10.70	585.76
C204	100	3	574.05	568.48	**566.07**	**566.07**	14.23	568.48
C205	100	3	**595.79**	**595.79**	**595.79**	**595.79**	10.27	595.79
C206	100	3	**571.07**	**571.07**	**571.07**	**571.07**	8.21	571.07
C207	100	3	577.03	**577.02**	**577.02**	**577.02**	8.47	577.02
C208	100	3	565.75	**565.74**	**565.74**	**565.74**	8.14	565.74
R201	100	4	996.46	984.55	981.00	**980.95**	15.01	984.55
R202	100	3	947.05	933.24	920.36	**910.46**	28.28	933.24
R203	100	3	762.06	752.14	739.98	**738.55**	27.16	752.14
R204	100	2	644.02	636.21	616.33	**615.83**	29.64	636.21
R205	100	3	789.42	767.56	**758.00**	**758.00**	18.40	767.56
R206	100	3	735.59	719.59	703.65	**703.14**	24.68	719.59
R207	100	2	719.74	691.36	669.18	**669.10**	30.00	691.36
R208	100	2	580.47	569.78	557.38	**556.37**	27.14	569.78
R209	100	3	684.13	666.75	657.91	**656.62**	20.26	666.75
R210	100	3	748.70	735.30	718.03	**711.94**	22.17	735.30
R211	100	2	706.45	691.10	651.23	**649.69**	29.96	691.10
RC201	100	4	1171.56	1164.73	**1146.39**	**1146.39**	11.96	1164.73
RC202	100	3	1056.94	1037.16	1025.99	**1025.07**	22.89	1037.16
RC203	100	3	826.91	814.80	803.39	**803.18**	21.21	814.80
RC204	100	3	649.31	642.18	**623.96**	**623.96**	19.83	642.18
RC205	100	4	1034.77	1024.01	991.45	**989.01**	21.33	1024.01
RC206	100	3	922.09	910.09	894.04	**893.92**	21.02	910.09
RC207	100	3	809.39	777.23	764.24	**762.47**	20.89	777.23
RC208	100	3	643.09	628.81	603.76	**600.85**	16.24	628.81
Time			10.00 min		11.59 min			
Gap			+1.03%	+0.00%	-0.93%	-1.03%		
CPU			Xe 2.67G		Opt 2.2G			

Table 26: Results on the TDVRPTW, scenario 2. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
C101	100	10	**764.03**	**764.03**	**764.03**	**764.03**	3,18	764.03
C102	100	10	718.92	**715.31**	715.48	**715.31**	3,99	715.31
C103	100	10	672.71	663.36	659.41	**658.72**	4,17	663.36
C104	100	10	612.49	603.71	599.83	**599.13**	5,52	603.71
C105	100	10	746.40	**746.32**	**746.32**	**746.32**	3,89	746.32
C106	100	10	732.78	**731.99**	**731.99**	**731.99**	4,04	731.99
C107	100	10	715.35	**714.99**	**714.99**	**714.99**	3,39	714.99
C108	100	10	670.74	**669.33**	**669.33**	**669.33**	4,24	669.33
C109	100	10	629.12	620.83	**619.51**	**619.51**	3,95	620.83
R101	100	19	NF	NF	NF	NF	NF	NF
R102	100	19	NF	NF	NF	NF	NF	NF
R103	100	14	751.81	746.01	735.75	**732.36**	6,80	746.01
R104	100	10	635.86	625.36	607.71	**605.50**	8,44	625.36
R105	100	14	821.49	811.42	802.98	**800.78**	6,72	811.42
R106	100	12	755.56	742.81	734.75	**730.65**	7,74	742.81
R107	100	10	659.81	652.02	631.40	**630.14**	9,56	652.02
R108	100	9	597.09	587.72	568.98	**566.40**	9,54	587.72
R109	100	11	654.32	650.01	634.95	**634.57**	6,42	650.01
R110	100	10	632.51	625.43	599.93	**597.80**	8,77	625.43
R111	100	10	627.52	621.01	595.14	**593.28**	8,50	621.01
R112	100	9	575.88	564.50	547.51	**545.16**	8,84	564.50
RC101	100	15	1026.17	1018.11	1005.31	**1005.30**	5,02	1018.11
RC102	100	13	876.38	867.08	856.86	**855.51**	6,00	867.08
RC103	100	11	759.96	747.33	733.28	**732.82**	5,82	747.33
RC104	100	10	689.16	684.81	660.84	**654.81**	10,68	684.81
RC105	100	14	922.95	916.50	901.57	**897.46**	6,23	916.50
RC106	100	12	783.52	770.60	761.02	**758.36**	6,76	770.60
RC107	100	11	720.60	701.89	679.38	**678.92**	6,85	701.89
RC108	100	10	653.09	632.62	623.50	**623.36**	8,43	632.62
C201	100	3	**697.20**	**697.20**	**697.20**	**697.20**	9,50	697.20
C202	100	3	627.01	620.35	**617.86**	**617.86**	14,04	620.35
C203	100	3	583.87	572.65	**566.61**	**566.61**	14,89	572.65
C204	100	3	545.26	531.48	510.99	**510.59**	20,87	531.48
C205	100	3	**596.17**	**596.17**	**596.17**	**596.17**	10,74	596.17
C206	100	3	545.21	**545.08**	**545.08**	**545.08**	10,58	545.08
C207	100	3	554.77	554.32	**553.27**	**553.27**	12,77	554.32
C208	100	3	533.80	**532.02**	**532.02**	**532.02**	9,70	532.02
R201	100	4	852.81	844.66	**823.87**	**823.87**	16,56	844.66
R202	100	3	793.88	781.88	759.13	**755.61**	29,72	781.88
R203	100	3	643.09	630.52	615.90	**613.07**	27,64	630.52
R204	100	2	517.27	507.12	482.10	**481.46**	30,00	507.12
R205	100	3	643.97	632.85	605.37	**603.15**	28,55	632.85
R206	100	3	598.40	586.96	555.28	**547.38**	29,13	586.96
R207	100	2	571.91	538.92	523.27	**521.76**	30,03	538.92
R208	100	2	461.51	454.63	427.68	**427.63**	29,06	454.63
R209	100	3	536.58	525.55	492.47	**490.36**	25,09	525.55
R210	100	3	584.04	569.43	544.01	**540.99**	27,76	569.43
R211	100	2	543.88	526.89	488.20	**484.73**	30,02	526.89
RC201	100	4	1033.70	1018.62	999.00	**998.43**	25,58	1018.62
RC202	100	3	889.10	871.72	847.05	**843.30**	28,33	871.72
RC203	100	3	698.44	664.98	**653.55**	**653.55**	19,65	664.98
RC204	100	3	524.64	513.94	502.12	**500.32**	23,06	513.94
RC205	100	4	877.67	856.88	816.33	**815.50**	26,04	856.88
RC206	100	3	774.43	743.50	720.53	**717.99**	27,33	743.50
RC207	100	3	647.10	624.43	587.93	**582.32**	25,36	624.43
RC208	100	3	503.48	492.95	453.62	**450.33**	27,83	492.95
Time			10.00 min		14.51 min			
Gap			+1.58%	+0.00%	-2.37%	-2.62%		
CPU			Xe 2.67G		Opt 2.2G			

Table 27: Results on the TDVRPTW, scenario 3. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
C101	100	12	NF	NF	NF	NF	NF	NF
C102	100	10	909.53	900.45	896.39	**894.47**	8.48	900.45
C103	100	10	755.38	742.76	710.18	**710.02**	10.01	742.76
C104	100	10	662.57	647.76	617.49	**617.00**	8.09	647.76
C105	100	10	895.23	**895.00**	**895.00**	**895.00**	4.04	895.00
C106	100	10	932.83	**915.17**	**915.17**	**915.17**	9.68	915.17
C107	100	10	877.13	871.64	**870.94**	**870.94**	5.50	871.64
C108	100	10	766.17	**734.56**	736.55	**734.56**	6.90	734.56
C109	100	10	665.18	648.99	**632.93**	**632.93**	5.36	648.99
R101	100	19	NF	NF	NF	NF	NF	NF
R102	100	19	NF	NF	NF	NF	NF	NF
R103	100	14	713.96	704.73	699.58	**692.73**	7.20	704.73
R104	100	10	592.24	578.29	550.56	**549.02**	8.54	578.29
R105	100	14	678.92	671.42	654.26	**653.72**	5.87	671.42
R106	100	12	636.94	627.92	612.98	**610.20**	6.26	627.92
R107	100	10	560.92	547.57	520.70	**519.85**	9.41	547.57
R108	100	9	515.95	507.93	482.40	**480.52**	14.30	507.93
R109	100	11	577.02	565.35	540.73	**538.17**	10.25	565.35
R110	100	10	530.49	518.05	498.94	**497.33**	13.86	518.05
R111	100	10	525.84	508.77	482.45	**482.18**	9.11	508.77
R112	100	9	474.68	464.66	439.45	**438.07**	7.53	464.66
RC101	100	15	896.67	871.50	850.99	**846.58**	4.48	871.50
RC102	100	13	795.23	779.95	766.33	**762.66**	5.08	779.95
RC103	100	11	683.87	667.22	646.99	**645.27**	10.32	667.22
RC104	100	10	621.89	615.30	568.14	**564.99**	5.60	615.30
RC105	100	14	803.68	790.01	771.59	**769.48**	5.60	790.01
RC106	100	12	672.92	662.02	639.56	**634.64**	6.44	662.02
RC107	100	11	623.29	615.01	577.75	**569.49**	11.07	615.01
RC108	100	10	559.53	548.45	521.27	**521.11**	7.73	548.45
C201	100	4	834.21	**833.41**	**833.41**	**833.41**	9.65	833.41
C202	100	3	802.68	780.32	771.39	**766.15**	24.02	780.32
C203	100	3	670.12	638.98	**621.15**	**621.15**	25.44	638.98
C204	100	3	579.54	553.86	515.93	**515.48**	19.61	553.86
C205	100	3	**688.00**	**688.00**	**688.00**	**688.00**	14.56	688.00
C206	100	3	626.80	619.51	618.35	**618.17**	24.32	619.51
C207	100	3	646.75	630.40	**630.15**	**630.15**	20.85	630.40
C208	100	3	589.41	583.48	**578.45**	**578.45**	18.85	583.48
R201	100	4	815.22	803.84	794.53	**794.52**	21.92	803.84
R202	100	3	759.19	744.39	724.60	**706.64**	30.01	744.39
R203	100	3	629.25	608.87	593.41	**588.95**	30.00	608.87
R204	100	2	436.63	434.01	415.63	**414.71**	30.01	434.01
R205	100	3	518.43	507.67	477.15	**475.00**	30.00	507.67
R206	100	3	471.43	455.64	437.02	**435.85**	23.51	455.64
R207	100	2	460.94	444.56	412.53	**404.39**	30.00	444.56
R208	100	2	355.43	343.58	330.52	**326.62**	30.00	343.58
R209	100	3	408.84	394.44	369.08	**366.55**	30.13	394.44
R210	100	3	477.47	466.98	448.51	**447.80**	30.00	466.98
R211	100	2	412.42	394.92	364.25	**359.12**	30.01	394.92
RC201	100	4	1088.85	1082.22	1061.11	**1059.61**	30.00	1082.22
RC202	100	3	860.34	848.82	823.13	**822.73**	30.00	848.82
RC203	100	3	703.90	693.24	670.62	**661.74**	30.00	693.24
RC204	100	3	470.95	468.12	446.66	**440.64**	30.00	468.12
RC205	100	4	850.53	837.65	817.76	**815.48**	28.73	837.65
RC206	100	3	617.77	602.13	561.78	**558.25**	30.00	602.13
RC207	100	3	501.67	488.64	443.57	**435.80**	25.12	488.64
RC208	100	3	368.92	359.30	337.83	**333.68**	30.00	359.30
Time			10.00 min		17.24 min			
Gap			+2.10%	+0.00%	-3.42%	-3.88%		
CPU			Xe 2.67G		Opt 2.2G			

Table 28: Results on the VFMPTW, minimization of duration, type A fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**		**UHGS**		**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	4631.31	**4536.40**	4617.95	4608.62	6.03	4536.40
R102	100	5	4401.30	**4348.92**	4376.11	4369.74	6.38	4348.92
R103	100	5	4182.16	**4119.04**	4149.67	4145.68	4.65	4119.04
R104	100	5	3981.28	3986.35	3965.21	**3961.39**	5.23	3981.28
R105	100	5	4236.84	4229.67	4215.84	**4209.84**	5.11	4229.67
R106	100	5	4118.48	4130.82	4112.20	**4109.08**	6.32	4118.48
R107	100	5	4035.96	4031.16	4012.58	**4007.87**	5.45	4031.16
R108	100	5	3970.26	3962.20	3936.47	**3934.48**	5.12	3962.20
R109	100	5	4060.17	4052.21	4037.40	**4020.75**	5.06	4052.21
R110	100	5	3995.18	3999.09	3971.53	**3965.88**	5.30	3995.18
R111	100	5	4017.81	4016.19	3992.07	**3985.68**	6.05	4016.19
R112	100	5	3947.30	3954.65	3923.21	**3918.88**	6.77	3947.30
C101	100	3	**7226.51**	**7226.51**	**7226.51**	**7226.51**	3.19	7226.51
C102	100	3	**7119.35**	7137.79	**7119.35**	**7119.35**	2.82	7119.35
C103	100	3	7107.01	7141.03	7104.46	**7102.86**	2.46	7107.01
C104	100	3	**7081.50**	7086.70	7081.51	7081.51	2.22	7081.50
C105	100	3	7199.36	**7169.08**	7196.06	7196.06	3.40	7169.08
C106	100	3	7180.03	**7157.13**	7177.41	7176.68	3.67	7157.13
C107	100	3	7149.17	**7135.38**	7144.73	7144.49	3.19	7135.38
C108	100	3	7115.81	7113.57	**7111.23**	**7111.23**	2.78	7113.57
C109	100	3	7094.65	7092.49	**7091.66**	**7091.66**	2.39	7092.49
RC101	100	4	5253.97	5237.19	5225.17	**5217.90**	5.03	5237.19
RC102	100	4	5059.58	5053.62	5044.63	**5018.47**	5.71	5053.48
RC103	100	4	4868.94	4885.58	4830.08	**4822.21**	6.03	4868.94
RC104	100	4	4762.85	4761.28	4741.69	**4737.00**	4.10	4761.28
RC105	100	4	5119.80	5110.86	5110.51	**5097.35**	5.61	5110.86
RC106	100	4	4960.78	4966.27	4947.46	**4935.91**	6.60	4960.78
RC107	100	4	4828.17	4819.91	4791.19	**4783.08**	5.32	4819.91
RC108	100	4	4734.15	4749.44	4710.71	**4708.85**	5.17	4734.15
R201	100	4	3922.00	**3753.42**	3791.54	3782.88	7.66	3753.42
R202	100	4	3610.38	3551.12	3540.39	**3540.03**	13.37	3551.12
R203	100	4	3350.18	3336.60	3314.09	**3311.35**	9.07	3334.08
R204	100	4	3390.14	3103.84	3076.13	**3075.95**	8.87	3103.84
R205	100	4	3465.81	3367.90	3334.35	**3334.27**	9.25	3367.90
R206	100	4	3268.36	3264.70	3246.09	**3242.40**	9.01	3264.70
R207	100	4	3231.26	3158.69	3145.79	**3145.08**	9.40	3158.69
R208	100	4	3063.10	3056.45	3020.52	**3017.12**	8.07	3056.45
R209	100	4	3192.95	3194.74	3186.18	**3183.36**	9.49	3191.63
R210	100	4	3375.38	3325.28	3288.82	**3287.66**	10.21	3325.28
R211	100	4	3042.48	3053.08	3021.67	**3019.93**	9.08	3042.48
C201	100	4	5891.45	**5820.78**	5878.54	5878.54	5.17	5820.78
C202	100	4	5850.26	5783.76	**5776.88**	**5776.88**	5.15	5779.59
C203	100	4	5741.90	**5736.94**	5741.82	5741.12	5.72	5736.94
C204	100	4	5691.51	5718.49	**5680.46**	**5680.46**	4.31	5691.51
C205	100	4	5786.71	**5747.67**	5782.53	5781.15	6.56	5747.67
C206	100	4	5795.15	**5738.09**	5767.70	5767.70	4.74	5738.09
C207	100	4	5743.52	**5721.16**	5731.54	5731.44	5.14	5721.16
C208	100	4	5884.20	5732.95	**5725.03**	**5725.03**	4.52	5732.95
RC201	100	6	4740.21	**4701.88**	4740.49	4737.59	5.28	4701.88
RC202	100	6	4522.36	4509.11	**4487.48**	**4487.48**	4.48	4509.11
RC203	100	6	4312.52	4313.42	4305.63	**4305.49**	5.88	4312.52
RC204	100	6	4141.04	4157.32	4140.16	**4137.93**	6.68	4141.04
RC205	100	6	4652.57	**4585.20**	4625.21	4615.04	6.40	4585.20
RC206	100	6	4431.64	4427.73	4408.63	**4405.16**	5.14	4416.95
RC207	100	6	4310.11	4313.07	4295.07	**4290.14**	6.52	4310.11
RC208	100	6	4091.92	4103.31	4076.12	**4075.04**	5.74	4091.92
Time			13.14 min	16.67 min		5.86 min		
Gap			+0.72%	+0.08%	-0.13%	-0.21%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table 29: Results on the VFMPTW, minimization of duration, type B fleet, instances of Liu and Shen (1999)

Inst	n	w	BDHMG08	RT10	UHGS			BKS
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2486.76	**2421.19**	2487.11	2486.77	3.89	2421.19
R102	100	5	2227.48	**2209.50**	2223.80	2222.15	4.31	2209.50
R103	100	5	1938.93	1953.50	1931.17	**1930.21**	4.18	1938.93
R104	100	5	1714.73	1713.36	1694.06	**1688.12**	4.34	1713.36
R105	100	5	2027.98	2030.83	**2017.56**	**2017.56**	3.83	2027.98
R106	100	5	1919.03	1919.02	1916.36	**1913.84**	5.10	1919.02
R107	100	5	1789.58	1780.52	1775.34	**1774.50**	4.27	1780.52
R108	100	5	**1649.24**	1665.78	1657.01	1654.68	5.83	1649.24
R109	100	5	1828.63	1840.54	1818.15	**1818.15**	5.09	1828.63
R110	100	5	1774.46	1788.18	1765.50	**1761.53**	5.77	1774.46
R111	100	5	1769.71	1772.51	1757.34	**1751.10**	5.57	1769.71
R112	100	5	1669.78	1667.00	1664.36	**1663.09**	6.33	1667.00
C101	100	3	**2417.52**	**2417.52**	**2417.52**	**2417.52**	2.06	2417.52
C102	100	3	2350.55	**2350.54**	2350.55	2350.55	2.98	2350.54
C103	100	3	2353.64	2347.99	2345.31	**2345.31**	4.02	2347.99
C104	100	3	2328.62	**2325.78**	2327.84	2327.84	2.43	2325.78
C105	100	3	**2373.53**	2375.04	**2373.53**	**2373.53**	3.35	2373.53
C106	100	3	2404.56	**2381.14**	2386.03	2386.03	3.17	2381.14
C107	100	3	2370.01	2357.67	2364.21	2364.21	3.10	2357.52
C108	100	3	**2346.38**	**2346.38**	**2346.38**	**2346.38**	3.28	2346.38
C109	100	3	2339.89	**2336.29**	**2336.29**	**2336.29**	2.60	2336.29
RC101	100	4	2462.60	2464.66	2461.29	**2456.10**	4.69	2462.60
RC102	100	4	2263.45	2272.68	2261.83	**2259.25**	4.24	2263.45
RC103	100	4	2035.62	2041.24	2028.38	**2025.30**	4.74	2035.62
RC104	100	4	1905.06	1916.85	1901.04	**1901.04**	4.37	1905.06
RC105	100	4	**2308.59**	2325.99	2329.30	2329.14	4.76	2308.59
RC106	100	4	2149.56	2160.45	2152.58	**2146.00**	3.68	2149.56
RC107	100	4	2000.77	2003.26	1990.20	**1989.34**	4.09	2000.77
RC108	100	4	1910.83	1908.72	1900.80	**1898.96**	3.26	1906.69
R201	100	4	2002.53	**1953.42**	1975.28	1973.43	6.39	1953.42
R202	100	4	1790.38	1751.12	1747.39	**1740.03**	8.05	1751.12
R203	100	4	1541.19	1536.60	1513.38	**1511.35**	6.44	1535.08
R204	100	4	1284.33	1303.84	1276.31	**1275.95**	7.58	1284.33
R205	100	4	1563.62	1560.07	**1534.27**	**1534.27**	6.45	1560.07
R206	100	4	1464.53	1464.70	1443.43	**1441.35**	5.92	1464.53
R207	100	4	1380.41	1358.69	1345.42	**1345.08**	6.97	1358.69
R208	100	4	1244.74	1256.45	1219.25	**1217.12**	6.00	1244.74
R209	100	4	1431.37	1394.74	1382.44	**1380.79**	7.75	1394.74
R210	100	4	1516.66	1525.28	1486.85	**1485.65**	7.72	1516.66
R211	100	4	1255.06	1253.08	1220.46	**1219.93**	7.36	1253.08
C201	100	4	1820.64	**1816.14**	1820.64	1820.64	2.90	1816.14
C202	100	4	1795.40	**1768.51**	1775.21	**1768.51**	5.22	1768.51
C203	100	4	**1733.63**	1734.82	**1733.63**	**1733.63**	3.29	1733.63
C204	100	4	1708.69	1716.18	**1680.46**	**1680.46**	3.30	1708.69
C205	100	4	1782.74	**1747.68**	1778.30	1778.30	5.48	1747.68
C206	100	4	1772.87	**1756.01**	1767.70	1767.70	3.84	1756.01
C207	100	4	1729.49	**1729.39**	1729.49	1729.49	3.48	1729.39
C208	100	4	1724.20	**1723.20**	1724.20	1724.20	3.40	1723.20
RC201	100	6	2343.79	**2230.54**	2331.33	2329.59	4.34	2230.54
RC202	100	6	2091.53	2022.15	2059.81	2057.66	6.69	2002.62
RC203	100	6	1852.74	1841.26	1825.14	**1824.54**	5.33	1841.26
RC204	100	6	1565.31	1575.18	1557.77	**1555.75**	5.50	1565.31
RC205	100	6	2195.75	**2166.62**	2179.31	2174.74	5.43	2166.62
RC206	100	6	1923.56	1893.13	**1883.08**	**1883.08**	4.33	1887.23
RC207	100	6	1745.85	1743.23	1719.07	**1714.14**	5.65	1743.23
RC208	100	6	1488.19	1526.78	**1483.20**	**1483.20**	4.80	1488.19
Time			9.12 min	16.67 min		4.80 min		
Gap			+0.59%	+0.23%	-0.16%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table 30: Results on the VFMPTW, minimization of duration, type C fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2199.78	**2134.90**	2199.79	2199.79	3.38	2134.90
R102	100	5	1925.55	**1913.37**	1926.50	1925.56	5.06	1913.37
R103	100	5	**1609.94**	1631.47	1616.42	1615.38	3.62	1609.94
R104	100	5	1370.84	1377.81	1365.32	**1363.26**	4.58	1370.84
R105	100	5	**1722.05**	1729.57	**1722.05**	**1722.05**	3.60	1722.05
R106	100	5	1602.87	1607.96	1603.06	**1599.04**	4.77	1602.87
R107	100	5	1456.02	1452.52	1447.86	**1442.97**	3.72	1452.52
R108	100	5	1336.28	1330.28	1321.96	**1321.68**	5.44	1330.28
R109	100	5	1507.77	1519.37	1508.36	**1506.59**	5.02	1507.77
R110	100	5	1446.41	1457.43	1446.96	**1443.92**	5.73	1446.41
R111	100	5	1447.88	1443.34	1427.82	**1423.47**	6.99	1443.34
R112	100	5	1335.41	1339.44	1329.24	**1329.07**	4.77	1335.41
C101	100	3	**1628.31**	1628.94	1628.94	1628.94	1.83	1628.31
C102	100	3	**1610.96**	**1610.96**	**1610.96**	**1610.96**	2.43	1610.96
C103	100	3	1619.68	**1607.14**	**1607.14**	**1607.14**	2.77	1607.14
C104	100	3	1613.96	**1598.50**	1599.90	1599.90	2.70	1598.50
C105	100	3	**1628.38**	1628.94	1628.94	1628.94	1.88	1628.38
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.88	1628.94
C107	100	3	**1628.38**	1628.94	1628.94	1628.94	1.96	1628.38
C108	100	3	**1622.89**	**1622.89**	**1622.89**	**1622.89**	3.00	1622.89
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.62	1614.99
RC101	100	4	2084.48	2089.37	2084.16	**2082.95**	4.91	2084.48
RC102	100	4	1895.92	1906.68	1898.52	**1895.05**	4.28	1895.92
RC103	100	4	1660.62	1666.24	1661.76	**1650.30**	3.98	1660.62
RC104	100	4	1537.09	1540.13	**1526.04**	**1526.04**	3.57	1537.09
RC105	100	4	1957.52	**1953.99**	1962.82	1957.14	4.71	1953.99
RC106	100	4	1776.08	1787.69	1775.84	**1774.94**	3.80	1776.08
RC107	100	4	1614.04	1622.90	1611.28	**1607.11**	3.83	1614.04
RC108	100	4	1535.14	1531.69	1524.10	**1523.96**	3.38	1531.69
R201	100	4	1729.92	1728.42	**1716.02**	**1716.02**	4.54	1728.42
R202	100	4	1537.35	1527.92	1524.96	**1515.03**	8.84	1527.92
R203	100	4	1308.70	1311.60	1287.36	**1286.35**	6.24	1308.70
R204	100	4	1062.46	1085.71	1051.19	**1050.95**	7.62	1062.46
R205	100	4	1311.84	1335.07	1309.29	**1309.27**	6.44	1311.84
R206	100	4	1251.51	1239.70	1216.87	**1216.35**	5.34	1239.70
R207	100	4	1149.23	1139.61	**1120.08**	**1120.08**	7.23	1139.61
R208	100	4	1009.26	1022.11	992.66	**992.12**	6.01	1009.26
R209	100	4	1178.45	1171.41	1156.97	**1155.79**	7.50	1171.41
R210	100	4	1289.35	1281.08	1259.42	**1257.89**	6.54	1281.08
R211	100	4	1013.84	1028.08	995.54	**994.93**	6.59	1013.84
C201	100	4	**1269.41**	**1269.41**	**1269.41**	**1269.41**	2.86	1269.41
C202	100	4	1242.66	1244.54	**1239.54**	**1239.54**	3.85	1242.66
C203	100	4	**1193.63**	1203.42	**1193.63**	**1193.63**	3.03	1193.63
C204	100	4	**1176.52**	1188.18	**1176.52**	**1176.52**	3.90	1176.52
C205	100	4	1245.62	1239.60	**1238.30**	**1238.30**	4.36	1239.60
C206	100	4	1245.05	**1229.23**	1238.30	1238.30	4.87	1229.23
C207	100	4	1215.42	1213.07	**1209.49**	**1209.49**	3.00	1213.07
C208	100	4	**1204.20**	1205.18	**1204.20**	**1204.20**	3.03	1204.20
RC201	100	6	2004.53	**1915.42**	1996.79	1996.79	3.67	1915.42
RC202	100	6	1766.52	**1677.62**	1733.23	1732.66	6.53	1677.62
RC203	100	6	1517.98	1504.35	1496.48	**1496.11**	6.15	1504.35
RC204	100	6	1238.66	1241.45	**1220.75**	**1220.75**	5.45	1238.66
RC205	100	6	1854.22	**1822.07**	1844.74	1844.74	5.07	1822.07
RC206	100	6	1590.22	1586.61	1557.19	**1553.65**	4.45	1586.61
RC207	100	6	1396.16	1406.26	1382.17	**1377.52**	6.06	1396.16
RC208	100	6	1145.84	1175.23	1141.47	**1140.10**	6.32	1145.84
Time			8.18 min	16.67 min		4.58 min		
Gap			+0.45%	+0.35%	-0.17%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table 31: Results on the VFMPTW, minimization of distance, type A fleet, instances of Liu and Shen (1999)

Inst	n	w	BDHMG08	BPDRT09	UHGS			BKS
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	4349.80	4342.72	4322.04	**4314.36**	4.61	4342.72
R102	100	5	4196.46	4189.21	4175.05	**4166.28**	6.03	4182.47
R103	100	5	4052.85	4051.62	4034.88	**4027.36**	5.35	4051.62
R104	100	5	3973.48	3972.65	3938.92	**3936.40**	4.81	3972.65
R105	100	5	4161.72	4152.50	4130.71	**4122.50**	6.49	4152.50
R106	100	5	4095.20	4085.30	4058.95	**4048.59**	5.57	4085.07
R107	100	5	4006.61	3996.74	3979.18	**3970.51**	5.56	3996.74
R108	100	5	3961.38	3949.50	3932.46	**3928.12**	4.68	3949.50
R109	100	5	4048.29	4035.89	4020.93	**4015.71**	4.80	4035.89
R110	100	5	3997.88	3991.63	3966.47	**3961.68**	6.49	3991.63
R111	100	5	4011.63	4009.61	3973.49	**3964.99**	5.28	4008.88
R112	100	5	3962.73	3954.19	3926.32	**3918.88**	4.92	3954.19
C101	100	3	7098.04	7097.93	**7093.45**	**7093.45**	2.96	7097.13
C102	100	3	7086.11	7085.47	**7080.17**	**7080.17**	2.14	7085.47
C103	100	3	7080.35	7080.41	**7079.21**	**7079.21**	2.09	7080.35
C104	100	3	7076.90	**7075.06**	**7075.06**	**7075.06**	2.19	7075.06
C105	100	3	7096.19	7096.22	**7093.45**	**7093.45**	3.33	7095.13
C106	100	3	7086.91	7088.35	**7083.87**	**7083.87**	2.28	7086.91
C107	100	3	7084.92	7090.91	**7084.61**	**7084.61**	2.23	7084.92
C108	100	3	7082.49	7081.18	**7079.66**	**7079.66**	2.19	7081.18
C109	100	3	7078.13	7077.68	**7077.30**	**7077.30**	2.04	7077.68
RC101	100	4	5180.74	5168.23	5154.95	**5150.86**	5.21	5168.23
RC102	100	4	5029.59	5025.22	5000.28	**4987.24**	4.81	5025.22
RC103	100	4	4895.57	4888.53	4821.61	**4804.61**	7.08	4888.53
RC104	100	4	4760.56	4747.38	4724.10	**4717.63**	5.30	4747.38
RC105	100	4	5060.37	5068.54	5035.76	**5035.35**	5.57	5060.37
RC106	100	4	4997.86	4972.11	4944.74	**4936.74**	5.63	4972.11
RC107	100	4	4865.76	4861.04	4795.35	**4788.69**	5.08	4861.04
RC108	100	4	4765.37	4753.12	4709.09	**4708.85**	4.78	4753.12
R201	100	4	3484.95	3530.24	**3446.78**	**3446.78**	6.51	3484.95
R202	100	4	3335.74	3335.61	**3308.16**	**3308.16**	7.68	3335.61
R203	100	4	3173.95	3164.03	**3141.09**	**3141.09**	5.65	3162.84
R204	100	4	3065.15	3029.83	3018.83	**3018.14**	6.96	3029.83
R205	100	4	3277.69	3261.19	3220.56	**3218.97**	6.40	3252.43
R206	100	4	3173.30	3165.85	3150.61	**3146.34**	10.30	3165.85
R207	100	4	3136.47	3102.79	3080.64	**3077.58**	8.70	3100.64
R208	100	4	3050.00	3009.13	2999.35	**2997.24**	5.37	3009.13
R209	100	4	3155.73	3155.60	3123.30	**3122.42**	6.37	3141.17
R210	100	4	3219.23	3206.09	3178.57	**3174.85**	6.93	3206.09
R211	100	4	3055.04	3026.02	3021.67	**3019.93**	9.10	3026.02
C201	100	4	5701.45	5700.87	**5695.02**	**5695.02**	3.71	5695.02
C202	100	4	5689.70	5689.70	**5685.24**	**5685.24**	3.78	5687.07
C203	100	4	5685.82	**5681.55**	**5681.55**	**5681.55**	4.21	5681.55
C204	100	4	5690.30	5677.69	**5677.66**	**5677.66**	4.27	5677.66
C205	100	4	5691.70	5691.70	**5691.36**	**5691.36**	3.98	5691.70
C206	100	4	5691.70	5691.70	**5689.32**	**5689.32**	3.82	5691.70
C207	100	4	5689.82	5692.36	**5687.35**	**5687.35**	4.24	5689.82
C208	100	4	5686.50	5689.59	**5686.50**	**5686.50**	3.86	5686.50
RC201	100	6	4407.68	4404.07	4378.21	**4374.09**	5.92	4398.21
RC202	100	6	4277.67	4266.96	4244.65	**4244.63**	4.63	4266.96
RC203	100	6	4204.85	4189.94	4171.47	**4170.17**	7.73	4185.70
RC204	100	6	4109.86	4098.34	**4087.11**	**4087.11**	5.79	4098.34
RC205	100	6	4329.96	4304.52	4295.41	**4291.93**	5.46	4304.52
RC206	100	6	4272.08	4272.82	4253.57	**4251.88**	5.12	4272.08
RC207	100	6	4232.81	4219.52	4186.43	**4185.98**	4.82	4213.66
RC208	100	6	4095.71	4093.83	4076.27	**4075.04**	4.08	4082.58
Time			4.13 min	—		5.09 min		
Gap			+0.25%	+0.06%	-0.41%	-0.48%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table 32: Results on the VFMPTW, minimization of distance, type B fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**2226.94**	—	2229.24	2228.67	5.05	2226.94
R102	100	5	**2071.90**	—	2073.91	2073.63	3.59	2071.90
R103	100	5	1857.22	—	1855.83	**1853.66**	4.57	1857.22
R104	100	5	1707.31	—	1686.09	**1683.33**	5.37	1707.31
R105	100	5	1995.07	—	**1988.86**	**1988.86**	3.30	1995.07
R106	100	5	1903.95	—	1889.58	**1888.31**	4.64	1903.95
R107	100	5	1766.18	—	1754.75	**1753.35**	4.15	1766.18
R108	100	5	1666.89	—	1651.75	**1647.88**	4.54	1666.89
R109	100	5	1833.54	—	1819.14	**1818.15**	3.66	1833.54
R110	100	5	1781.15	—	1765.34	**1758.64**	5.11	1781.15
R111	100	5	1768.74	—	1747.08	**1740.86**	5.32	1768.74
R112	100	5	1675.76	—	1664.41	**1661.85**	5.36	1675.76
C101	100	3	2340.98	—	**2340.15**	**2340.15**	3.12	2340.98
C102	100	3	2326.53	—	**2325.70**	**2325.70**	2.61	2326.53
C103	100	3	2325.61	—	**2324.60**	**2324.60**	3.03	2325.61
C104	100	3	**2318.04**	—	**2318.04**	**2318.04**	2.44	2318.04
C105	100	3	2344.64	—	**2340.15**	**2340.15**	3.00	2344.64
C106	100	3	2345.85	—	**2340.15**	**2340.15**	3.46	2345.85
C107	100	3	2345.60	—	**2340.15**	**2340.15**	3.20	2345.60
C108	100	3	2340.17	—	**2338.58**	**2338.58**	3.18	2340.17
C109	100	3	**2328.55**	—	**2328.55**	**2328.55**	2.72	2328.55
RC101	100	4	2417.16	—	2416.23	**2412.71**	4.16	2417.16
RC102	100	4	2234.47	—	2213.93	**2213.92**	5.86	2234.47
RC103	100	4	2025.74	—	2016.28	**2016.28**	3.50	2025.74
RC104	100	4	1912.65	—	1908.66	**1897.04**	4.07	1912.65
RC105	100	4	2296.16	—	2293.21	**2287.51**	4.54	2296.16
RC106	100	4	2157.84	—	2141.32	**2140.86**	3.95	2157.84
RC107	100	4	2008.02	—	1990.13	**1989.34**	2.99	2008.02
RC108	100	4	1920.91	—	1900.72	**1898.96**	3.91	1920.91
R201	100	4	1687.44	—	1721.54	**1646.78**	5.96	1687.44
R202	100	4	1527.74	—	1509.06	**1508.16**	9.15	1527.74
R203	100	4	1379.15	—	**1341.09**	**1341.09**	3.75	1379.15
R204	100	4	1243.56	—	1218.47	**1218.14**	5.11	1243.56
R205	100	4	1471.97	—	1419.52	**1418.97**	7.28	1471.97
R206	100	4	1400.84	—	1350.32	**1346.34**	7.23	1400.84
R207	100	4	1333.53	—	1279.63	**1277.58**	6.28	1333.53
R208	100	4	1225.37	—	1198.41	**1197.24**	4.33	1225.37
R209	100	4	1370.30	—	1323.60	**1322.42**	6.15	1370.30
R210	100	4	1418.54	—	1376.24	**1374.31**	6.97	1418.54
R211	100	4	1263.72	—	1219.99	**1219.93**	6.79	1263.72
C201	100	4	1700.87	—	**1695.02**	**1695.02**	2.97	1700.87
C202	100	4	1687.84	—	**1685.24**	**1685.24**	2.83	1687.84
C203	100	4	1696.25	—	**1681.55**	**1681.55**	3.36	1696.25
C204	100	4	1705.94	—	**1677.66**	**1677.66**	3.47	1705.94
C205	100	4	1711.00	—	**1691.36**	**1691.36**	3.21	1711.00
C206	100	4	1691.70	—	**1689.32**	**1689.32**	2.99	1691.70
C207	100	4	1704.88	—	**1687.35**	**1687.35**	3.44	1704.88
C208	100	4	1689.59	—	**1686.50**	**1686.50**	3.24	1689.59
RC201	100	6	1965.31	—	1942.19	**1938.36**	5.94	1965.31
RC202	100	6	**1771.87**	—	1773.04	1772.81	5.92	1771.87
RC203	100	6	1619.55	—	1606.56	**1604.04**	5.99	1619.55
RC204	100	6	1501.10	—	**1490.25**	**1490.25**	4.28	1501.10
RC205	100	6	1853.58	—	1835.74	**1832.53**	5.11	1853.58
RC206	100	6	1761.49	—	**1725.44**	**1725.44**	5.79	1761.49
RC207	100	6	1666.03	—	1651.09	**1646.37**	5.65	1666.03
RC208	100	6	1494.11	—	**1483.20**	**1483.20**	4.39	1494.11
Time			3.45 min	—	4.50 min			
Gap			+0.00%	—	-0.94%	-1.10%		
CPU			Ath 2.6G	Duo 2.4G	Opt 2.2G			

Table 33: Results on the VFMPTW, minimization of distance, type C fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**1951.20**	1951.89	**1951.20**	**1951.20**	4.60	1951.20
R102	100	5	**1770.40**	1778.29	1785.35	1785.35	2.92	1770.40
R103	100	5	1558.17	1555.26	1552.64	**1552.34**	3.55	1555.26
R104	100	5	1367.82	1372.08	1356.70	**1355.15**	5.37	1361.46
R105	100	5	1696.67	1698.26	**1694.56**	**1694.56**	3.25	1696.67
R106	100	5	1589.25	1590.11	1590.25	**1583.17**	4.12	1589.25
R107	100	5	1435.21	1439.81	1433.44	**1428.08**	5.36	1435.21
R108	100	5	1334.75	1334.68	1315.47	**1314.88**	5.32	1334.68
R109	100	5	1515.22	1514.13	1507.97	**1506.59**	4.68	1507.10
R110	100	5	1457.42	1461.85	1448.83	**1443.92**	5.06	1457.42
R111	100	5	1439.43	1439.14	1423.43	**1420.15**	5.58	1435.93
R112	100	5	1358.17	1343.26	1329.70	**1327.58**	4.97	1337.68
C101	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.12	1628.94
C102	100	3	**1597.66**	**1597.66**	**1597.66**	**1597.66**	2.35	1597.66
C103	100	3	**1596.56**	**1596.56**	**1596.56**	**1596.56**	2.88	1596.56
C104	100	3	1594.06	1590.86	**1590.76**	**1590.76**	2.32	1590.86
C105	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.96	1628.94
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.05	1628.94
C107	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.17	1628.94
C108	100	3	**1622.75**	**1622.75**	**1622.75**	**1622.75**	3.20	1622.75
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.88	1614.99
RC101	100	4	2048.44	2053.55	2047.33	**2043.48**	4.39	2048.44
RC102	100	4	1860.48	1872.49	1849.38	**1847.92**	4.10	1860.48
RC103	100	4	1660.81	1663.08	1654.30	**1646.35**	4.19	1660.81
RC104	100	4	1536.24	1540.61	1523.42	**1522.04**	5.64	1536.24
RC105	100	4	1913.09	1929.89	1925.66	**1913.06**	4.01	1913.09
RC106	100	4	1772.05	1776.52	1770.95	1770.95	3.77	1761.63
RC107	100	4	1615.74	1633.29	1609.88	**1607.11**	4.08	1615.74
RC108	100	4	1527.35	1527.87	1524.10	**1523.96**	3.35	1527.35
R201	100	4	1441.46	1466.13	1462.03	1443.41	4.90	1439.76
R202	100	4	1298.10	1296.78	1297.43	**1283.16**	7.91	1288.70
R203	100	4	1145.38	1127.28	**1116.09**	**1116.09**	3.95	1127.28
R204	100	4	1019.77	1000.89	993.16	**993.14**	6.63	1000.89
R205	100	4	1222.03	1240.74	1196.73	**1193.97**	7.33	1222.03
R206	100	4	1138.26	1141.13	1123.21	**1121.34**	6.00	1138.26
R207	100	4	1086.42	1067.97	1055.39	**1052.58**	6.97	1067.97
R208	100	4	976.11	979.50	971.36	**969.90**	5.78	976.11
R209	100	4	1140.96	1140.38	1098.89	**1097.42**	5.94	1123.19
R210	100	4	1161.87	1170.29	1153.34	**1149.85**	6.80	1161.87
R211	100	4	1015.84	1008.54	**994.93**	**994.93**	6.51	1008.54
C201	100	4	**1194.33**	**1194.33**	**1194.33**	**1194.33**	4.82	1194.33
C202	100	4	1189.35	**1185.24**	**1185.24**	**1185.24**	2.57	1185.24
C203	100	4	**1176.25**	**1176.25**	**1176.25**	**1176.25**	3.60	1176.25
C204	100	4	1176.55	1176.55	**1175.37**	**1175.37**	4.32	1176.55
C205	100	4	**1190.36**	**1190.36**	**1190.36**	**1190.36**	4.46	1190.36
C206	100	4	**1188.62**	**1188.62**	**1188.62**	**1188.62**	4.01	1188.62
C207	100	4	**1184.88**	1187.71	**1184.88**	**1184.88**	3.66	1184.88
C208	100	4	1187.86	**1186.50**	**1186.50**	**1186.50**	2.99	1186.50
RC201	100	6	1632.41	1630.53	1623.78	**1623.36**	6.81	1630.53
RC202	100	6	1459.84	1461.44	1450.70	**1447.27**	5.29	1459.84
RC203	100	6	1295.07	1292.92	**1274.04**	**1274.04**	4.49	1292.92
RC204	100	6	1171.26	1162.91	1159.37	**1159.00**	6.01	1162.91
RC205	100	6	1525.28	1532.67	1517.09	**1512.53**	5.24	1525.28
RC206	100	6	1425.15	1420.89	1400.62	**1395.18**	3.92	1420.89
RC207	100	6	1332.40	1328.29	1319.56	**1314.44**	6.24	1328.29
RC208	100	6	1155.02	1152.92	**1140.10**	**1140.10**	6.81	1152.92
Time			3.08 min	—	4.56 min			
Gap			+0.26%	+0.27%	-0.36%	-0.51%		
CPU			Ath 2.6G	Duo 2.4G	Opt 2.2G			

Table 34: Results on the type 1 VRPSTW (only lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall lateness "L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	**n**	**F10**				**UHGS**								
		Single				Avg 10				Best 10				
		Fleet	TW	L	Dist	Fleet	TW	L	Dist	Fleet	TW	L	Dist	T(min)
R101	100	12	56	—	1128.70	11.00	27.70	1644.70	1329.70	11	27	1720.97	1331.85	11.78
R102	100	11	46	—	1058.70	10.00	22.20	1069.17	1226.84	10	21	1206.72	1252.78	18.64
R103	100	10	34	—	1027.40	9.80	7.50	258.62	1171.84	9	5	90.68	1208.63	11.05
R104	100	9	18	—	947.30	9.00	0.50	17.15	1009.30	9	0	0.00	1007.31	7.79
R105	100	11	42	—	1073.50	10.20	19.50	1106.53	1239.15	10	9	680.04	1381.88	13.74
R106	100	10	33	—	1047.40	9.90	8.10	433.32	1236.89	9	6	358.86	1259.11	12.59
R107	100	10	24	—	987.60	9.00	7.00	354.81	1029.30	9	7	343.45	1042.96	11.24
R108	100	9	14	—	947.20	9.00	0.00	0.00	963.50	9	0	0.00	960.88	8.10
R109	100	10	28	—	1001.40	10.00	5.00	208.62	1194.64	10	5	200.72	1183.42	8.80
R110	100	9	29	—	1013.40	9.20	9.50	461.42	1043.08	9	0	0.00	1118.84	12.57
R111	100	10	26	—	983.30	9.00	7.80	385.46	1038.01	9	6	443.34	1047.09	12.72
R112	100	9	17	—	940.90	9.00	0.00	0.00	988.59	9	0	0.00	982.14	9.96
R201	100	3	56	—	984.00	3.00	2.00	197.62	1502.80	3	2	174.47	1497.06	27.05
R202	100	3	40	—	943.50	2.00	21.30	4703.07	991.60	2	20	4757.71	991.27	30.61
R203	100	2	30	—	901.80	2.00	6.10	1124.60	991.09	2	6	1076.65	995.76	30.04
R204	100	2	19	—	836.30	2.00	0.00	0.00	830.79	2	0	0.00	825.52	26.74
R205	100	3	36	—	911.90	2.00	14.40	2998.08	983.30	2	14	2767.13	973.84	26.49
R206	100	2	25	—	956.90	2.00	3.50	652.23	994.39	2	3	465.02	993.13	29.92
R207	100	2	18	—	876.60	2.00	0.00	0.00	893.85	2	0	0.00	893.33	29.54
R208	100	2	11	—	833.40	2.00	0.00	0.00	727.17	2	0	0.00	726.82	14.16
R209	100	2	26	—	950.50	2.00	7.30	1814.72	990.71	2	7	1386.37	994.41	29.28
R210	100	2	29	—	963.80	2.00	6.00	1339.34	995.19	2	6	1264.94	997.75	29.75
R211	100	2	14	—	906.80	2.00	0.00	0.00	900.56	2	0	0.00	892.72	27.23
C101	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.32
C102	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.28
C103	100	—	—	—	—	10.00	0.00	0.00	828.06	10	0	0.00	828.06	2.12
C104	100	—	—	—	—	10.00	0.00	0.00	824.78	10	0	0.00	824.78	1.96
C105	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.73
C106	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.56
C107	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.48
C108	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.19
C109	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.05
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	5.82
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	8.35
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	9.79
C204	100	—	—	—	—	3.00	0.00	0.00	590.60	3	0	0.00	590.60	8.08
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	6.40
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	6.53
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	6.54
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	6.51
RC101	100	11	44	—	1255.30	11.00	12.10	789.34	1486.11	11	12	808.44	1486.99	7.01
RC102	100	10	32	—	1230.10	10.30	10.80	476.57	1399.34	10	4	41.41	1539.29	10.51
RC103	100	10	25	—	1154.60	10.00	1.20	18.41	1320.22	10	1	4.18	1333.71	8.03
RC104	100	10	12	—	1083.90	10.00	0.00	0.00	1135.48	10	0	0.00	1135.48	5.57
RC105	100	11	38	—	1219.70	10.90	7.20	345.49	1516.88	10	6	278.42	1538.72	6.89
RC106	100	10	27	—	1150.30	10.00	7.60	308.59	1310.32	10	7	344.38	1307.76	8.85
RC107	100	10	28	—	1123.00	10.00	1.50	65.46	1300.57	10	1	64.05	1325.19	9.70
RC108	100	10	10	—	1071.60	10.00	0.00	0.00	1139.82	10	0	0.00	1139.82	6.97
RC201	100	3	48	—	1147.40	3.00	3.00	482.47	1663.69	3	3	482.47	1663.69	32.65
RC202	100	3	35	—	1073.50	3.00	0.00	0.00	1377.25	3	0	0.00	1365.65	26.22
RC203	100	3	29	—	906.30	2.10	13.80	3013.28	929.95	2	0	0.00	1064.14	30.49
RC204	100	2	14	—	850.70	2.00	2.00	261.17	915.94	2	2	257.81	916.79	29.27
RC205	100	3	40	—	1158.40	3.00	1.00	87.83	1623.09	3	1	9.38	1605.20	32.78
RC206	100	3	40	—	978.40	3.00	0.00	0.00	1149.37	3	0	0.00	1146.32	24.48
RC207	100	3	33	—	986.40	3.00	0.00	0.00	1061.49	3	0	0.00	1061.14	24.51
RC208	100	2	21	—	885.50	2.00	6.40	946.83	910.34	2	6	949.50	917.64	22.51
Time		9.69 min				18.62 min								
CPU		P-M 1.6G				Opt 2.2G								

Table 35: Results on the type 1 VRPSTW (only lateness) with $\alpha = 1$. Minimization of the sum of distance and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988).

Inst	n	m	KTDHS12		UHGS			BKS
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	19	1562.98	**1562.58**	1562.89	**1562.58**	1.93	1562.58
R102	100	17	1379.62	**1379.11**	1379.21	**1379.11**	1.90	1379.11
R103	100	13	1160.64	1159.54	1159.51	**1159.28**	3.34	1159.54
R104	100	9	1009.02	1003.73	**999.77**	**999.77**	3.93	1003.73
R105	100	14	1348.89	**1347.75**	**1347.75**	**1347.75**	2.17	1347.75
R106	100	12	1237.29	**1236.58**	**1236.58**	**1236.58**	2.91	1236.58
R107	100	10	1089.84	1084.96	**1083.62**	**1083.62**	4.55	1084.96
R108	100	9	951.24	949.94	947.04	**946.60**	4.20	949.94
R109	100	11	1176.40	**1173.21**	**1173.21**	**1173.21**	3.54	1173.21
R110	100	10	1114.66	**1106.66**	1111.57	1107.26	3.55	1106.66
R111	100	10	1086.36	1080.25	1076.41	**1074.84**	4.97	1080.25
R112	100	9	981.82	972.11	975.78	**971.31**	5.13	972.11
C101	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.10	828.94
C102	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.15	828.94
C103	100	10	828.07	828.07	**828.06**	**828.06**	2.02	828.07
C104	100	10	**824.78**	**824.78**	**824.78**	**824.78**	1.96	824.78
C105	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.12	828.94
C106	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.91	828.94
C107	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.89	828.94
C108	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.94	828.94
C109	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.90	828.94
RC101	100	14	1591.59	**1590.22**	**1590.22**	**1590.22**	2.57	1590.22
RC102	100	12	1429.90	**1428.21**	**1428.21**	**1428.21**	2.83	1428.21
RC103	100	11	1242.33	**1239.54**	1239.73	**1239.54**	3.09	1239.54
RC104	100	10	1128.74	**1126.31**	**1126.31**	**1126.31**	3.25	1126.31
RC105	100	13	1451.38	**1450.84**	**1450.84**	**1450.84**	2.59	1450.84
RC106	100	11	1350.17	**1349.30**	1349.72	**1349.30**	3.55	1349.30
RC107	100	11	1208.96	**1208.81**	1208.98	**1208.81**	3.37	1208.81
RC108	100	10	1119.61	**1118.00**	1118.31	**1118.00**	3.72	1118.00
R201	100	4	1237.17	**1237.11**	**1237.11**	**1237.11**	4.13	1237.11
R202	100	3	1169.23	**1165.32**	**1165.32**	**1165.32**	6.19	1165.32
R203	100	3	942.96	937.35	934.01	**933.52**	10.55	937.35
R204	100	2	840.79	832.38	824.73	**824.02**	24.72	832.38
R205	100	3	1006.79	**994.43**	**994.43**	**994.43**	6.53	994.43
R206	100	3	920.13	912.81	**906.14**	**906.14**	7.39	912.81
R207	100	2	1044.87	908.70	888.44	**887.28**	24.63	908.70
R208	100	2	735.26	728.92	727.08	**726.82**	13.72	728.92
R209	100	3	917.21	909.30	**909.16**	**909.16**	6.98	909.30
R210	100	3	958.58	948.80	941.95	**938.34**	8.57	948.80
R211	100	2	923.85	901.18	892.50	**885.71**	19.66	901.18
C201	100	3	**591.56**	**591.56**	**591.56**	**591.56**	4.62	591.56
C202	100	3	**591.56**	**591.56**	**591.56**	**591.56**	6.90	591.56
C203	100	3	**591.17**	**591.17**	**591.17**	**591.17**	7.67	591.17
C204	100	3	**590.60**	**590.60**	**590.60**	**590.60**	6.84	590.60
C205	100	3	**588.88**	**588.88**	**588.88**	**588.88**	5.24	588.88
C206	100	3	**588.49**	**588.49**	**588.49**	**588.49**	5.84	588.49
C207	100	3	**588.29**	**588.29**	**588.29**	**588.29**	5.25	588.29
C208	100	3	**588.32**	**588.32**	**588.32**	**588.32**	5.89	588.32
RC201	100	4	1380.47	**1380.33**	**1380.33**	**1380.33**	4.34	1380.33
RC202	100	3	1322.17	**1317.28**	**1317.28**	**1317.28**	9.59	1317.28
RC203	100	3	1057.10	1046.05	1045.00	**1040.77**	10.73	1046.05
RC204	100	3	809.09	797.41	**797.04**	**797.04**	6.71	797.41
RC205	100	4	1305.97	1299.61	**1298.00**	**1297.65**	6.34	1299.61
RC206	100	3	1135.90	**1135.26**	**1135.26**	**1135.26**	6.56	1135.26
RC207	100	3	1073.58	1061.14	1058.16	**1056.88**	7.43	1061.14
RC208	100	3	834.82	829.00	827.90	**827.67**	7.98	829.00
Time			10.00 min		5.82 min			
Gap			+0.62%	+0.00%	-0.13%	-0.18%		
CPU			Xe 2.67G		Opt 2.2G			

Table 36: Results on type 2 VRPSTW (earliness and lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall earliness plus lateness "E+L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	**n**	**FEL07**				**UHGS**								
		Best X				Avg 10				Best 10				
		Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	T(min)
R101	100	14	44	—	1872.94	9.00	57.30	2998.08	1025.65	9	56	2742.45	1018.58	61.19
R102	100	13	29	—	1732.54	9.00	38.70	1825.32	1018.77	9	37	1934.47	1012.28	60.94
R103	100	12	9	—	1542.79	9.00	17.30	676.55	1020.72	9	16	621.25	1022.96	60.65
R104	100	10	0	—	1107.18	9.00	1.60	44.39	1014.39	9	1	19.05	1013.65	60.44
R105	100	—	—	—	—	9.00	39.10	2008.68	1030.78	9	37	2050.22	1037.70	61.04
R106	100	—	—	—	—	9.00	24.30	1107.09	1035.12	9	24	972.07	1021.48	60.73
R107	100	—	—	—	—	9.00	7.90	341.56	1031.17	9	7	294.55	1033.97	60.58
R108	100	10	0	—	968.34	9.00	0.00	0.00	980.60	9	0	0.00	970.15	60.37
R109	100	11	4	—	1379.87	9.00	22.40	961.34	1023.20	9	21	833.92	1028.18	60.66
R110	100	—	—	—	—	9.00	12.60	572.97	1025.82	9	11	552.12	1034.03	60.49
R111	100	—	—	—	—	9.00	8.10	334.60	1015.26	9	7	248.93	1013.65	60.65
R112	100	—	—	—	—	9.00	1.10	30.57	1018.88	9	1	1.25	1025.86	60.30
C101	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C102	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.25
C103	100	10	0	—	918.08	10.00	0.00	0.00	828.06	10	0	0.00	828.06	30.15
C104	100	10	0	—	899.00	10.00	0.00	0.00	824.78	10	0	0.00	824.78	30.32
C105	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.43
C106	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.78
C107	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C108	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.18
C109	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.15
RC101	100	13	26	—	1851.22	9.00	46.70	2631.68	1120.65	9	44	2787.38	1122.73	39.23
RC102	100	13	1	—	1772.42	9.00	31.30	1573.76	1123.74	9	29	1508.22	1119.92	38.13
RC103	100	11	0	—	1416.81	9.00	17.30	698.94	1125.40	9	16	640.61	1131.62	37.56
RC104	100	10	0	—	1262.55	9.00	5.80	155.00	1117.95	9	5	161.57	1126.69	33.99
RC105	100	12	1	—	1531.57	9.00	34.60	1648.93	1130.18	9	33	1580.23	1127.29	39.71
RC106	100	11	0	—	1224.72	9.00	28.63	1161.00	1123.24	9	28	1207.60	1111.51	38.08
RC107	100	—	—	—	—	9.00	18.67	720.80	1112.72	9	18	601.38	1106.99	35.73
RC108	100	—	—	—	—	9.00	11.20	352.37	1107.38	9	10	284.23	1123.11	35.30
R201	100	—	—	—	—	2.00	41.20	9159.21	985.32	2	40	8583.79	988.84	31.01
R202	100	—	—	—	—	2.00	25.90	4631.88	986.18	2	24	5062.88	986.21	31.00
R203	100	—	—	—	—	2.00	10.80	1924.46	979.73	2	10	1514.13	988.64	30.89
R204	100	—	—	—	—	2.00	0.00	0.00	873.07	2	0	0.00	851.66	30.96
R205	100	—	—	—	—	2.00	20.00	3979.80	985.48	2	19	3289.15	979.97	30.89
R206	100	—	—	—	—	2.00	9.00	1699.39	983.37	2	7	1624.34	988.52	30.89
R207	100	—	—	—	—	2.00	1.50	135.63	973.57	2	1	26.44	933.74	30.91
R208	100	—	—	—	—	2.00	0.00	0.00	741.26	2	0	0.00	730.54	30.69
R209	100	—	—	—	—	2.00	12.70	2383.53	980.17	2	10	1910.46	963.47	30.85
R210	100	—	—	—	—	2.00	12.30	2352.23	981.78	2	11	2015.68	983.07	30.88
R211	100	—	—	—	—	2.00	0.70	40.99	968.68	2	0	0.00	931.99	30.90
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.19
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.33
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	30.38
C204	100	—	—	—	—	3.00	0.00	0.00	590.93	3	0	0.00	590.60	30.37
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	30.17
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	30.22
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	30.18
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	30.24
RC201	100	—	—	—	—	2.00	52.70	12622.22	908.21	2	50	12420.98	912.51	31.01
RC202	100	—	—	—	—	2.00	33.80	7893.30	911.08	2	33	7335.92	907.59	30.98
RC203	100	—	—	—	—	2.00	16.30	3181.25	907.44	2	15	2418.65	910.63	30.89
RC204	100	—	—	—	—	2.00	3.40	601.79	902.05	2	2	460.90	894.01	30.72
RC205	100	—	—	—	—	2.00	39.30	8993.74	909.28	2	37	8706.52	911.66	30.96
RC206	100	—	—	—	—	2.00	35.30	8265.45	906.10	2	33	7424.35	913.25	30.82
RC207	100	—	—	—	—	2.00	25.00	5291.51	908.94	2	24	4805.29	908.80	30.85
RC208	100	—	—	—	—	2.00	12.70	1828.13	911.30	2	11	1694.30	912.64	30.74
Time		5.98 min				41.16 min								
CPU		P-II 0.6G				Opt 2.2G								

Table 37: Results on type 2 VRPSTW (earliness and lateness) with $\alpha = 1$. Minimization of the sum of distance, earliness and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988)

Inst	**n**	**m**	**UHGS**		
			Avg 10	Best 10	T(min)
R101	19	100	1546.91	1546.91	24.13
R102	17	100	1377.38	1377.38	26.93
R103	13	100	1158.83	1158.31	30.14
R104	9	100	1004.57	1000.33	30.10
R105	14	100	1342.57	1342.57	30.03
R106	12	100	1223.09	1223.09	30.12
R107	10	100	1080.90	1079.12	30.43
R108	9	100	948.23	945.64	30.08
R109	11	100	1164.68	1164.68	30.11
R110	10	100	1108.30	1104.59	30.16
R111	10	100	1065.76	1065.76	30.08
R112	9	100	991.50	969.91	30.10
C101	10	100	828.94	828.94	30.10
C102	10	100	828.94	828.94	30.08
C103	10	100	828.06	828.06	30.20
C104	10	100	824.78	824.78	29.34
C105	10	100	828.94	828.94	30.12
C106	10	100	828.94	828.94	30.07
C107	10	100	828.94	828.94	30.07
C108	10	100	828.94	828.94	30.33
C109	10	100	828.94	828.94	30.09
RC101	14	100	1584.20	1584.20	29.67
RC102	12	100	1409.36	1409.36	30.07
RC103	11	100	1231.67	1231.67	30.16
RC104	10	100	1123.25	1121.84	30.12
RC105	13	100	1433.37	1433.37	30.18
RC106	11	100	1334.89	1334.89	30.39
RC107	11	100	1203.06	1203.06	30.45
RC108	10	100	1115.44	1115.44	30.12
R201	4	100	1235.14	1235.14	30.28
R202	3	100	1159.76	1159.76	30.13
R203	3	100	937.04	934.10	30.15
R204	2	100	837.21	820.90	30.18
R205	3	100	996.24	994.43	30.12
R206	3	100	910.99	906.54	30.11
R207	2	100	937.79	906.81	30.19
R208	2	100	735.31	730.52	30.13
R209	3	100	911.61	909.16	30.11
R210	3	100	948.91	938.77	30.14
R211	2	100	921.81	912.39	30.17
C201	3	100	591.56	591.56	30.09
C202	3	100	591.56	591.56	30.11
C203	3	100	591.17	591.17	30.11
C204	3	100	590.60	590.60	30.09
C205	3	100	588.88	588.88	30.12
C206	3	100	588.49	588.49	30.13
C207	3	100	588.29	588.29	30.11
C208	3	100	588.32	588.32	30.11
RC201	4	100	1380.33	1380.33	30.12
RC202	3	100	1312.05	1312.05	30.10
RC203	3	100	1047.43	1044.74	30.13
RC204	3	100	796.91	796.68	30.13
RC205	4	100	1300.98	1297.86	30.11
RC206	3	100	1135.44	1135.26	30.12
RC207	3	100	1061.92	1056.88	30.12
RC208	3	100	832.30	827.67	30.13
Time				29.96 min	
Gap			+0.26%	+0.00%	
CPU				Opt 2.2G	

Table 38: Results on the new MDPVRPTW instances.

Inst	**n**	**m**	**t**	**d**	**UHGS**		
					Avg 10	Best 10	T(min)
pr01	48	1	4	4	2483.81	2482.78	0.87
pr02	96	2	4	4	4474.72	4468.60	2.93
pr03	144	3	4	4	5758.43	5735.59	6.93
pr04	192	4	4	4	6708.98	6680.76	18.96
pr05	240	4	4	4	7275.26	7202.79	29.02
pr06	288	5	4	4	8263.29	8207.18	29.68
pr07	72	1	6	6	5497.20	5496.76	2.06
pr08	144	2	6	6	7791.98	7716.08	11.21
pr09	216	3	6	6	10579.81	10504.77	29.49
pr10	288	4	6	6	13612.91	13343.55	30.00
pr11	48	1	4	4	2043.74	2043.74	0.85
pr12	96	2	4	4	3851.07	3825.34	3.47
pr13	144	3	4	4	4781.02	4755.10	9.48
pr14	192	4	4	4	5535.82	5471.17	21.36
pr15	240	4	4	4	5871.13	5830.71	30.00
pr16	288	5	4	4	6913.56	6832.53	30.01
pr17	72	1	6	6	4787.41	4782.74	2.47
pr18	144	2	6	6	6465.43	6402.79	21.78
pr19	216	3	6	6	8940.97	8785.80	30.01
pr20	288	3	6	6	10930.86	10662.62	30.00
pr01b	48	1	4	4	2423.29	2423.29	0.64
pr02b	96	2	4	4	4521.26	4486.88	2.84
pr03b	144	3	4	4	5664.54	5649.74	7.20
pr04b	192	4	4	4	6724.52	6694.32	16.69
pr05b	240	4	4	4	7345.82	7284.81	28.18
pr06b	288	5	4	4	8591.64	8551.01	30.00
pr07b	72	1	6	6	5255.06	5255.06	1.67
pr08b	144	2	6	6	7468.25	7444.70	10.10
pr09b	216	3	6	6	10930.40	10797.34	29.67
pr10b	288	4	6	6	12673.68	12494.65	30.00
pr11b	48	1	4	4	2100.23	2100.23	0.80
pr12b	96	2	4	4	3782.07	3748.45	2.85
pr13b	144	3	4	4	4891.31	4883.31	9.40
pr14b	192	4	4	4	5474.16	5442.94	21.13
pr15b	240	4	4	4	5902.56	5809.17	29.71
pr16b	288	5	4	4	6992.78	6941.25	29.90
pr17b	72	1	6	6	4794.89	4794.89	1.86
pr18b	144	2	6	6	6332.54	6288.46	22.25
pr19b	216	3	6	6	9003.87	8825.36	30.00
pr20b	288	3	6	6	10998.39	10774.34	30.00
Time					16.09 min		
Gap					+0.77%	+0.00%	
CPU					Opt 2.2G		