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ORIGINAL ARTICLE

Multi-trip multi-compartment vehicle routing problem with backhauls

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Abstract The Multi-Trip Multi-Compartment Vehicle Routing Problem with Backhauls (MTMCVRPB) is a complex optimization problem that involves finding the most efficient routes for a heterogeneous fleet of multi-compartment vehicles to transport heterogeneous commodities simultaneously. The problem involves making deliveries and pickups while taking into account capacity constraints on compartments and other specific requirements such as limitation on route length. The MTMCVRPB aims to minimize the total cost of the routes, including the cost of vehicles, fuel and other expenses, while maximizing efficiency of vehicles and fulfilling the demand of customers. In order to address the MTMCVRPB, this article recommends a mixed integer linear programming formulation. The proposed model takes into consideration the demands of both linehaul and backhaul customers, ensuring that on each route linehaul deliveries are made before visiting any backhaul customer. The model was tested on randomly generated data sets and numerical experiments were performed to demonstrate effectiveness of the model. The proposed model can be beneficial for the industries such as grocery and food delivery, fuel distribution and garbage collection etc., as these represent some application areas of the model. This article also presents a saving based heuristic for the proposed model.

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1 Introduction

The vehicle routing problem (VRP) is a combinatorial optimization that entails determining the best route for a fleet of vehicles to traverse in order to serve a number of customers. The researchers G.B. Dantzig and J.H. Ramser firstly generalized the traveling salesman problem as the truck dispatching problem in their article (Dantzig and Ramser 1959). They provided the first mathematical programming model and algorithmic approach as well as a description of a practical application involving the distribution of fuel to service stations. Numerous models and techniques for the exact and ideal solution of the different VRP variants have been proposed after the publication of this landmark study and new VRP variants have been researched in order to better align with the real-world applications. The method developed by Dantzig and Ramser as an iterative process to solve the above mentioned truck dispatching problem was modified in Clarke and Wright (1964) and popularly known as Clarke and Wright's saving algorithm. This saving algorithm is used to solve both the basic and more complex versions of vehicle routing problems. The VRP is an NP-hard problem. So, there may be a limit to the size of problems that can be solved optimally using exact solution methods. Since, real-world VRPs are typically large scale problems, heuristic and meta-heuristic algorithms are therefore better choices for solving such problems. Some articles that discuss the application of heuristics and metaheuristics are: Coelho and Laporte (2015); Koch et al. (2016); Kurnia et al. (2018);

Reil et al. (2018); Sana et al. (2019); Ospina-Mateus et al. (2021); Hasani et al. (2022); Hosseini et al. (2022).

In logistics, distribution and transportation, the vehicle routing problem is the most researched combinatorial optimization problem. There is humongous literature available on vehicle routing problems e.g. Toth and Vigo (2014, 1997); Mor and Speranza (2022) analyzed more than thousands of the VRPs in their literature review articles. This article is based on combining three different variants of VRP, viz. Multi-Compartment Vehicle Routing Problems (MCVRP) (Ostermeier et al. 2021), Multi-Trip Vehicle Routing Problem (MTVRP) (Şen and Bülbül 2008) and one sub-category of pickup and delivery vehicle routing problem i.e. Vehicle Routing Problem with Backhauls (VRPB) (Santos et al. 2020). To the best of our knowledge, this study is a first of its kind that concurrently combines together the characteristics of these three types of vehicle routing problems. These VRPs are briefly discussed ahead.

1.1 Multi-trip vehicle routing problem

In multi-trip vehicle routing problem (MTVRP), numerous trips of the same vehicle are allowed in a predetermined time period. In transportation sector and many other industries, the demand of a large number of customers is satisfied by using vehicles having limited capacity that are employed repeatedly over time to maximise the profitability. Initially this problem was investigated in Fleischmann (1990), following that other articles such as Brandão and Mercer (1998); Mingozzi et al. (2013); Cattaruzza et al. (2016) made valuable contributions. In prior literature on MTVRPs, a variety of MTVRPs with various properties have been explored (Şen and Bülbül 2008). Recently, a MTVRP with time windows for waste collection was studied in Wong and Jamaian (2022).

1.2 Multi-compartment vehicle routing problem

In multi-compartment vehicle routing problems, vehicles with loaded space or containers divided into multiple compartments are used and these compartments are utilized to collect goods with different characteristics or to distribute several commodities being carried together. The use of multi-compartment vehicles enable the joint delivery of various types of products from a depot to the customers using a single vehicle. For instance, Chajakis and Guignard (2003) discussed that trucks that transport liquid fuels have separated tanks to carry fuels of various varieties, such as gasoline of different grades and maritime boats that transport petroleum products from refineries to clients have numerous compartments that can carry more than five distinct goods simultaneously in a single trip. Over the years, various studies have already been published on MCVRP which are discussed in section 2 (Literature review).

1.3 Vehicle routing problem with backhauls

Vehicle routing problem with backhauls (VRPB) involves two different set of customers, first set is known as linehauls, each customer in this set requires a given quantity to be delivered from depot to customers and second set is called backhauls, each backhaul customer requires a given quantity to be picked up by vehicles and delivered to depot. This is a variant of pickup and delivery vehicle routing problem (PDVRP) in which all delivery customers (linehauls) should be satisfied prior to picking up from any pickup point (or backhaul customer) by the vehicle on its route. The significance of the linehaul-backhaul problem is related to the ongoing effort to reduce distribution costs by utilizing the unused capacity of an empty vehicle returning to the depot. The initial studies (Golden et al. 1985; Goetschalckx and Jacobs-Blecha 1989) discussed the vehicle routing problem with backhauls and recently, VRPB for two-echelon with capacitated satellites is presented in Dumez et al. (2023).

The problem addressed in this article is multi-trip multi-compartment vehicle routing problem with backhauls (MTMCVRPB) as the three aforementioned categories are combined. In this vehicle routing problem, a multi-compartment vehicle performs numerous trips to fulfill the demands of pickup and delivery customers. In each trip, vehicle first delivers goods to all linehauls (delivery points), if the multi-commodity requirements of every linehaul customer on the vehicle's route have been met, it then picks up goods from backhauls (pickup locations) before returning to the depot.

The remainder of this work is structured as follows: The relevant literature is covered in Sect. 2. In Sect. 3, MTMCVRPB and its underlying assumptions are discussed. The MILP formulation of the given problem is provided in Sect. 4. The solution approach based on Clarke and Wright's saving algorithm is explained in Sect. 5. A validation of the proposed model is provided in Sect. 6. Section 7 discusses implications, limitations and future research scope. Section 8 presents conclusion.

2 Literature review

The Multi-Compartment Vehicle Routing Problem (MCVRP) is a variant of the classical Vehicle Routing Problem (VRP) that considers vehicles with multiple compartments for carrying different products. The MCVRP adds an extra layer of complexity to the VRP by requiring the consideration of capacity constraints for each compartment of the vehicle. These multi-compartment vehicle routing problems first time addressed in Chajakis and Guignard (2003) for the transportation of foods to convenience stores. This study aims to reduce cooling and delivery costs. The 0-1 programming and mixed integer programming models were developed and solved using the Lagrangean Relaxations. On the subject of applications, models and heuristics for vehicle routing with compartments, Derigs et al. (2011) gave a good overview and summary. An integer programming model of the MCVRP was provided in this paper.

The study (Asawarungsaengkul et al. 2013) describes that the reason for using vehicles with multiple compartments is to transport products of varying temperatures or compositions in the same vehicle. They addressed the concept in two applications that make use of vehicles with multiple compartments, first deals with transportation of petroleum products and second deals with the delivery of grocery and food items to convenience stores. In the case of a fleet of vehicles operating from a single depot, food and groceries are delivered to a set of customers. Customer orders are divided into three categories: frozen, refrigerated and dry.

The MCVRP can be used in many real-world applications where the goods are different in characteristics, such as milk (cows and goats) and fluid product (gasoline and diesel); different in quality, e.g. different types of crude oil; different in treatment conditions, such as level of storage temperatures and so on. Vehicles with multiple compartments are used to save transportation costs by transporting heterogeneous goods in different compartments on the same vehicle.

Several studies in the literature have utilized various real-world cases to address different variations of the Multi-Compartment Vehicle Routing Problem (MCVRP). The studies (Ostermeier and Hübner 2018; Hübner and Ostermeier 2019; Martins et al. 2019; Frank et al. 2021) utilized grocery product-based cases, the studies (Chen et al. 2019; Chen and Shi 2019; Kim and Park 2020; Chen et al. 2022) used temperature-based multi-compartment vehicles to address other real-cases in MCVRP. Manufacturing-based applications can be found in Pasha et al. (2020); Mahapatra et al. (2020); Zou et al. (2021). Additionally, liquid or fuel containers based cases are addressed in Cornillier et al. (2008); Asawarungsaengkul et al. (2013); Lahyani et al. (2015); Febriandini et al. (2020); Wang et al. (2020); Chowmali and Sukto (2021); Yindong et al. (2021); Ramadhani et al. (2021); Guo et al. (2022). The work by Henke et al. (2015); Gajpal et al. (2017); Mofid-Nakhaee and Barzinpour (2019); Paksaz et al. (2020) focused on garbage collection-related issues.

The MCVRP is a challenging optimization problem that has gained significant attention in the past decade (Ostermeier et al. 2021). Despite numerous studies and advancements made in recent years, the MCVRP continues to be an active area of research, with ongoing efforts to develop efficient and effective solution methods. A comparison of proposed study and prior studies on MCVRP is provided in Table 1. Authors reviewed a number of research papers in which multi-compartment vehicles perform multiple trips, use heterogeneous fleet of vehicles and involve backhauls as well. Additionally, studies based on multi-period MCVRP are: Cornillier et al. (2008); Lahyani et al. (2015); Martins et al. (2019); Kim and Park (2020); Frank et al. (2021), discussion on MCVRP with multi-depot can be found in Alinaghian and Shokouhi (2018); Paksaz et al. (2020) and application of time windows in MCVRP is studied in Chen and Shi (2019); Chen et al. (2019); Martins et al. (2019); Pasha et al. (2020); Febriandini et al. (2020); Eshtehadi et al. (2020); Paksaz et al. (2020); Zou et al. (2021); Yindong et al. (2021); Chang (2022). Mainly, four kinds of solution methods have been utilized to solve these types of vehicle routing problems. First category is exact solution methods for solving VRPs that provide optimal solutions, e.g. branch and bound (B & B) (Henke et al. 2015; Ostermeier and Hübner 2018), branch and cut (B & C) (Lahyani et al. 2015; Ostermeier et al. 2018) and branch and price (B & P) (Mirzaei and Wøhlk 2019). Second category is heuristics, e.g. genetic algorithm (GA) (Yahyaoui et al. 2020), simulated annealing (SA) (Pasha et al. 2020), variable neighborhood search (VNS) (Henke et al. 2015; Pasha et al. 2020) and large neighborhood search (LNS) (Ostermeier and Hübner 2018; Ostermeier et al. 2018; Hübner and Ostermeier 2019). Third one is metaheuristics, e.g. adaptive large neighborhood search (ALNS) (Alinaghian and Shokouhi 2018; Yahyaoui et al. 2020; Mofid-Nakhaee and Barzinpour 2019; Chen et al. 2019; Wang et al. 2020; Frank et al. 2021; Chang 2022) and evolutionary algorithms (EA) (Pasha et al. 2020). Fourth category is of hybrid algorithms such as hybrid genetic algorithms (HGA) (Chen et al. 2022) and hybrid particle swarm optimization (Chen and Shi 2019).

Salhi (1987) pioneered MTVRP by conducting multiple trips in the context of vehicle fleet mix. Within a refinement process, a matching algorithm is proposed to assign routes to vehicles that are limited to two trips. Taillard et al. (1996) proposed a three-phase heuristic algorithm for the MTVRP, which is based on the classical VRP. In the first phase, tabu search is used to generate a population of routes that satisfy the capacity constraint; In phase two, a set of different VRP solutions is obtained. In the final phase, routes are assigned to vehicles by solving the bin-packing problem (BPP). Furthermore, In their study, a set of classical MTVRP instances were generated, which have been widely used as benchmarks in the literature. Brandao and Mercer (1997) investigated a real-world application of MTVRP with time windows and a heterogeneous fleet and solved the problem using a tabu search algorithm. The methodology developed in this study was adapted by Brandão and Mercer (1998), who solved and compared classical MTVRP instances. Use of multi-trip in

Table 1 Literature reviewed for MCVRP

Study	MT	MC	HF	Backhauls	Algorithms
This article	√	√	√	√	Saving algorithm
(Wang et al. 2022)		√	√		MOVND
(Chang 2022)		√	√		ALNS
(Yousra and Ahmed 2022)		√	√		
(Chen et al. 2022)		√	√		HGA
(Guo et al. 2022)		√	√		ACO with VND
(Frank et al. 2021)		√	√		ALNS
(Ramadhani et al. 2021)	√	√	√		
(Yindong et al. 2021)		√			EDA-LF
(Zou et al. 2021)		√			IG
(Chowmali and Sukto 2021)		√	√		FJA-ALNS
(Paksaz et al. 2020)		√	√		
(Wang et al. 2020)	√	√			ALNS
(Eshtehadi et al. 2020)		√			EALNS
(Febriandini et al. 2020)		√	√		
(Kim and Park 2020)		√	√		
(Pasha et al. 2020)		√	√		EA,VNS,TS,SA
(Martins et al. 2019)		√			ALNS
(Chen and Shi 2019)		√			HPSO
(Hübner and Ostermeier 2019)		√	√		LNS
(Chen et al. 2019)		√	√		ALNS
(Mofid-Nakhaee and Barzinpour 2019)		√	√		ALNS & WO
(Mirzaei and Wøhlk 2019)		√	√		B & P
(Ostermeier et al. 2018)		√	√		LNS
(Yahyaoui et al. 2020)		√	√		ALNS, GA
(Alinaghian and Shokouhi 2018)		√			ALNS
(Göçmen and Rızvan 2018)		√	√		
(Ostermeier and Hübner 2018)		√	√		B & B, LNS
(Gajpal et al. 2017)		√			ACO
(Henke et al. 2015)		√			VNS
(Lahyani et al. 2015)		√	√		B & C
(Asawarungsaengkul et al. 2013)		√	√		Saving Algorithm
(Cornillier et al. 2008)	√	√	√		MPH
(Chajakis and Guignard 2003)		√	√		LR

MC: Multi-Compartment; MT: Multi-Trip; HF: Heterogeneous Fleet of Vehicles; For abbreviations given in “Algorithms” readers can refer the corresponding “Study”

MCVRP was studied in Alinaghian and Shokouhi (2018); Wang et al. (2020); Ramadhani et al. (2021). In Janinhoff et al. (2023), a multi-trip vehicle routing problem was studied with a data-driven application to the parcel industry.

The VRPB deals with the efficient routing of vehicles from a depot to a set of customers, with the constraint of collecting and delivering goods in the process. The VRPB differs from the classical Vehicle Routing Problem as it requires vehicles to return to the depot with goods collected from certain customers (Goetschalckx and Jacobs-Blecha 1989). Though numerous studies are available on VRPB and various advancements have been made in recent years, the VRPB still continues to be an active area of research, with ongoing efforts to develop efficient and effective solution methods. Previous research in the area of VRPB also focused on developing efficient and effective solution algorithms. For example, in a study (WA et al. 2012), a genetic algorithm was proposed to solve the VRPB, while Toth and Vigo (1997) developed a branch-and-bound algorithm to solve the VRPB to optimality. The model proposed in this article adds to the existing body of research by considering the use of multi-compartment vehicles in the VRPB, making it a valuable contribution to the field. Some review articles on VRPB are: Koç and Laporte (2018); Santos et al. (2020).

3 Problem description

This paper introduces a multi-trip multi-compartment vehicle routing problem with backhauls (MTMCVRPB) for the delivery of heterogeneous demands together to the linehaul customers and collection of a variety of goods from the backhaul customers (such as suppliers, returning of expired products etc.). Split deliveries are not considered in this article. So, each demand for a particular product should be lower than the corresponding compartment capacity. The orders of the linehaul customers are loaded from the depot and then transferred to the linehaul customers. Further, the collections from backhaul customers are transported to the depot itself, using same vehicle on a delivery route in such a way that all the linehauls are satisfied first before visiting any backhaul customer. The objective of this article is to find the delivery routes by minimizing two types of costs: fixed cost for minimizing the number of vehicles and routing cost for minimizing the distance (or time) of traveling. The primary activities of this problem are to meet the demand of linehaul and backhaul customers by reducing the number of vehicles, utilizing the capacity of each compartment and maximizing the number of trips of a vehicle to minimize the idle time of vehicle for a given time period.

Figure 1 illustrates an example of the multiple-trip multi-compartment vehicle routing problem with backhauls. It depicts a network consisting of 20 linehaul customers (C1, C2, ..., C20) represented by blue dots, 10 backhaul customers (C21, C22, ..., C30) denoted by green triangles and a central depot node namely Depot depicted by a red square. Within the example figure, three types of arcs can be observed. The blue arc represents the delivery route to linehaul customers, indicating the path taken by the vehicles to transport goods to these destinations. The green arcs signify the vehicle's pickups from the backhaul customers, indicating the locations from where the vehicle collects items. Lastly, the orange arc represents a period when the vehicle is empty, indicating a transitional phase in the route where the vehicle is not carrying any cargo. Multi-compartment vehicles given in Table 2 were used to solve this problem. As seen in Table 2, there are only three vehicles but routes in this example (Fig. 1) are six. Therefore, some of the vehicles have to perform multiple trips to meets the requirements of linehaul and backhaul customers.

3.1 Assumptions

The following assumptions and constraints govern the characteristics of the proposed MTMCVRPB:

- Every customer should be visited only once in a period.
- Weight or volume of the item to be loaded in a compartment of the vehicle should not exceed the capacity of the compartment.
- One compartment can transport one type of product at a time.
- The loading and unloading time of orders are assumed to be constant and negligible as time constraints are not considered in this study.
- Each compartment of vehicle should be empty before visiting any backhaul customer.
- There are two types of costs considered in this study, viz. fixed cost of using vehicles and routing cost of covering distance.
- Customer demands may consist of one or more than one types of items.

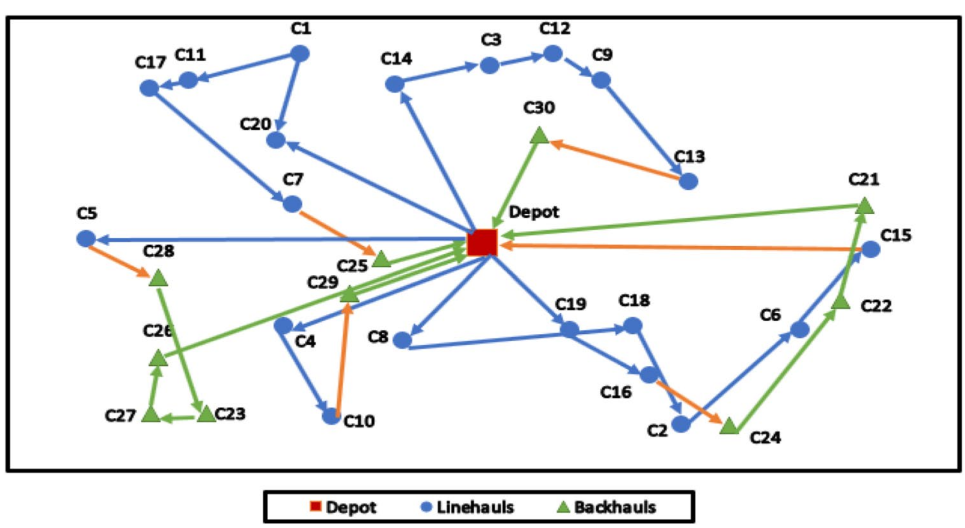


Fig. 1 Example of MTMCVRPB

- Order of one customer must be delivered by exactly one vehicle as no split delivery is allowed. The products in a customer's order must therefore be loaded in a proper compartment of the same vehicle.
- During a single trip from the depot, each vehicle can deliver to one or more customers.
- It is believed that each customer's order will be smaller compared to the weight and volume limitations of each truck. Additionally, it is expected that each item in a customer's order will be light in weight and tiny in volume compared to the weight and volume of the compartment on which it will be loaded.
- The products to be loaded on the vehicle should be chosen in such a way that they utilize full vehicle capacity or there is a negligible loss of space.
- Vehicle can not go directly to backhaul customers but it may return directly from any linehaul if the demand of all backhaul customers is already fulfilled.
- The quantities to be delivered and picked up are fixed and known in advance.
- Fixed cost for using a vehicle is independent of the number of trips performed by the vehicle. Therefore, if a single vehicle is used for multiple trips, fixed cost will only be taken into account once.

4 Mathematical model

Mixed integer linear programming (MILP) provides a powerful and flexible framework for modeling and solving the vehicle routing problems. By incorporating binary and integer variables, defining an objective function and considering constraints, MILP systematically explores the solution space using branch & bound or other similar methods and finds the optimal solution that minimizes the overall cost or distance traveled while satisfying all the defined constraints. So, MILP formulation for the proposed MTMCVRPB is developed in this article.

The MTMCVRPB is defined on a directed graph G = (V, A) where vertex $V = \{0\} \bigcup \{1, 2, ..., N\} \bigcup \{N + 1, N + 2, ..., N + M\}$ represents a union of a depot node $\{0\}$, the set of N linehaul customers and the set of M backhaul customers respectively. The set A is the set of arcs such that $A = \{(i, j) : i, j \in V \; \& \; i \neq j\}$ and a distance d_{ij} is associated with each arc (i, j) ∈ A. All distances are considered to be asymmetric. The model is applicable for symmetric distances as well.

4.1 Notations

The notations involved in the MILP formulation are given as follows:

Index sets

{0}	Depot Node
{1, 2, ..., N, N+1, ..., N+M}	Set of Customers
{1, 2, ..., K}	Set of Vehicles
{1, 2, ..., R}	Set of Trips
{1, 2, ..., L}	Set of Compartments at Vehicles

Parameters

q_j^l	Demand of product transported through compartment 'l' at customer 'j'
d_{ij}	Distance from customer 'i' to customer 'j'
F_k	Fixed cost of using vehicle 'k'
C_k	Routing cost per km for vehicle 'k'
Q_k^l	Capacity of compartment 'l' at vehicle 'k'
N	Number of linehaul customers
M	Number of backhaul customers
K	Number of Vehicles
R	Number of Trips
L	Number of Compartments
MDT	Maximum distance a vehicle travels in a single trip
MDP	Maximum distance a vehicle can travel in its all trips

Decision variables

X_{ijkr}	Binary variable having value 1 if vehicle 'k' travels from node 'i' to node 'j' in trip 'r', and 0 otherwise
Y_{ijkr}^l	Flow of delivery products transporting through compartment 'l' of vehicle 'k' during trip 'r' from node 'i' to node 'j'
Z_{ijkr}^l	Flow of pickup products transporting through compartment 'l' of vehicle 'k' during trip 'r' from node 'i' to node 'j'

4.2 Model formulation

Objective function

$$Min\ Total\ Cost = \sum_{j=1}^{N} \sum_{k=1}^{K} F_k * X_{0jk1} + \sum_{i=0}^{N+M} \sum_{j=0}^{N+M} \sum_{k=1}^{K} \sum_{r=1}^{R} d_{ij} * C_k * X_{ijkr}$$

Route construction constraints

$$X_{iikr} = 0 \; \forall \; i = 0, 1, 2, ..., N + M; \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{1}$$

$$\sum_{j=1}^{N} X_{0jkr} \le 1 \; \forall \; k = 1, 2, ..., K; \; r = 1, 2, ..., R; \tag{2}$$

$$\sum_{r=1}^{R} X_{0jkr} \le 1 \; \forall \; j = 1, 2, ..., N; \; k = 1, 2, ..., K; \tag{3}$$

$$\sum_{i=1}^{N+M} X_{i0kr} \le 1 \; \forall \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{4}$$

$$\sum_{r=1}^{R} X_{i0kr} \le 1 \; \forall \; i = 1, 2, ..., N + M; \; k = 1, 2, ..., K; \tag{5}$$

$$\sum_{k=1}^{K} \sum_{r=1}^{R} X_{ijkr} \le \; 1 \; \forall \; i, j = 0, 1, 2, ..., N + M; \tag{6}$$

$$\sum_{i=0}^{N+M} X_{ijkr} - \sum_{i=0}^{N+M} X_{jikr} = 0 \; \forall \; j = 1, 2, ..., N + M; \; k = 1, 2, ..., K; \; r = 1, 2, ..., R; \tag{7}$$

$$\sum_{j=0}^{N+M} \sum_{k=1}^{K} \sum_{r=1}^{R} X_{ijkr} = 1 \; \forall \; i = 1, 2, ..., N + M; \tag{8}$$

$$\sum_{i=0}^{N+M} \sum_{j=0}^{N+M} X_{ijkr} - \sum_{i=0}^{N+M} \sum_{j=0}^{N+M} X_{ijk(r-1)} \le 0 \; \forall \; k = 1, 2, ..., K; \; r = 2, 3, ..., R \tag{9}$$

$$\sum_{i=0}^{N} \sum_{r=1}^{R} X_{ijkr} \ge \sum_{r=1}^{R} X_{1jkr} \; \forall \; j = 0, 1, 2, ..., N + M; \; k = 1, 2, ..., K \tag{10}$$

Balancing flow constraints: A. Flow control of linehaul customers

$$\sum_{i=0}^{N} \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{l=1}^{L} Y^{l}_{ijkr} - \sum_{i=0}^{N} \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{l=1}^{L} Y^{l}_{jikr} = q^{l}_{j} \; \forall \; j = 1, 2, ..., N; \tag{11}$$

$$q^{l}_{j} * X_{ijkr} \le Y^{l}_{ijkr} \; \forall \; i, j = 0, 1, 2, ..., N; \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{12}$$

$$Y^{l}_{ijkr} \le (Q^{l}_{k} - q^{l}_{i}) * X_{ijkr} \; \forall \; i, j = 0, 1, 2, ..., N; \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{13}$$

B. Flow control of backhaul customers

$$\sum_{i=0,N+1}^{N+M} \sum_{k=1}^{K} \sum_{r=1}^{R} Z^{l}_{jikr} - \sum_{i=0}^{N+M} \sum_{k=1}^{K} \sum_{r=1}^{R} Z^{l}_{ijkr} = \sum_{i=0,N+1}^{N+M} \sum_{k=1}^{K} \sum_{r=1}^{R} q^{l}_{i} * X_{jikr} \; \forall \; j = N + 1, N + 2, ..., N + M; l = 1, 2, ..., L \tag{14}$$

$$q^{l}_{j} * X_{ijkr} \le Z^{l}_{ijkr} \; \forall \; i = 0, 1, 2, ..., N + M; \; j = 0, N + 1, N + 2, ..., N + M; \; k = 1, 2, ..., K; r = 1, 2, ..., R \tag{15}$$

$$Z^{l}_{ijkr} \le Q^{l}_{k} * X_{ijkr} \; \forall \; i = 0, 1, 2, ..., N + M; \; j = 0, N + 1, N + 2, ..., N + M; \; k = 1, 2, ..., K; r = 1, 2, ..., R \tag{16}$$

Maximum distance allowed per trip for a vehicle

$$\sum_{i=0}^{N+M} \sum_{j=0}^{N+M} d_{ij} * X_{ijkr} \le MDT \; \forall \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{17}$$

Maximum distance allowed in all trips for a vehicle

$$\sum_{i=0}^{N+M} \sum_{j=0}^{N+M} \sum_{r=1}^{R} d_{ij} * X_{ijkr} \le MDP \; \forall \; k = 1, 2, ..., K \tag{18}$$

Restrictions on decision variables

$$X_{ijkr} \in \{0, 1\} \; \forall \; i, j = 0, 1, 2, ..., N + M; \; k = 1, 2, ..., K; \; r = 1, 2, ..., R \tag{19}$$

$$Y^{l}_{ijkr}, Z^{l}_{ijkr} \ge 0 \; \forall \; i, j = 0, 1, 2, ..., N + M; \; k = 1, 2, ..., K; r = 1, 2, ..., R; \; l = 1, 2, ... L \tag{20}$$

The objective function of the given formulation minimizes total cost which is a combination of two types of costs: the fixed cost of using vehicles and the traveling cost of routing them. Constraint (1) ensures that an arc cannot be used to travel from a node to itself, while (2) ensures that a vehicle can travel to at the most one linehaul customer starting from depot node in one trip and (3) ensures that at the most one trip of a vehicle can be used to travel from depot node to any linehaul customer. Similarly, (4) ensures that a vehicle in a trip may return to the depot from at the most one customer and (5) requires the vehicle to return to the depot from any customer using a single trip. The constraint (6) ensures that at the most one trip of a vehicle can be used to travel from a node to another. Constraint (7) requires that if a vehicle enters a node during its r^{th} trip, it must also depart from that node in that trip. The constraint (8) ensures that a vehicle starting from any customer must visit exactly one node in a trip. The constraints (9) and (10) dictate that next trip by any vehicle can only start after the completion of the previous trip. Constraints (11), (12) and (13) balance the flow of linehaul customers, while (14), (15) and (16) regulate the flow of backhaul customers. Constraint (17) places a limit on the maximum distance that a vehicle can travel in a single trip, while (18) places restriction on the maximum distance travelled by a vehicle in all its trips. Finally, constraints (19) and (20) represent the restrictions on the decision variables.

4.3 Lemma

The proposed model always gives an optimal solution.

Proof From the given MILP formulation presented in section 4.2 defined by the objective function and constraints (1) to (20), a linear programming relaxation can be obtained by removing binary constraints, i.e. constraint (19). The modified problem is given below:

Linear programming relaxation
Objective function
Subject to:
Constraints (1) to (18)
Constraint (20)

The optimal solution of above linear programming relaxation can be found using extensions of simplex method such as Big-M method or Two-Phase method as the constraints (7, 8, 10, 11 and 14) are not ≤ type constraints. The optimal objective function value (say Z_{LB}) of this linear programming relaxation problem will be the lower bound to the MTMCVRPB formulation provided in subsection 4.2. Due to constraint (8), values of decision variables $X_{ijkr} \in [0, 1]$ as this constraint restricts summation of some of X_{ijkr} to be exactly equal to one, therefore this optimal solution may not satisfy constraint (19) as this constraint restricts $X_{ijkr} \in \{0, 1\}$. So, optimal solution of this linear programming relaxation may not be optimal to the MTMCVRPB given in 4.2. Hence, an approach is needed to converge decision variables X_{ijkr} to binary values as given ahead.

4.3.1 Optimal solution to MTMCVRPB

To identify the optimal solution, the branch and bound algorithm is employed, systematically exploring the LP relaxation through a series of iterations. This algorithm effectively utilizes branching, bounding and pruning techniques to identify the feasible region and gradually converging towards the optimal solution. The best solution found using this process is indeed an optimal solution. This guarantee is possible as all other nodes with bounds lower than Z_{LB} are infeasible and will be pruned by comparing respective objective function value with upper bound (Z_{UB}) (obtained using any heuristic such as saving based heuristic given in section 5) and lower bound Z_{LB} (obtained using linear programming relaxation as mentioned above).

For decision variables where $0 < X_{ijkr} < 1$ make two sub problems (or two branches) of LP relaxation as follows:

Sub problem 1	Sub problem 2
Objective function	Objective function
Subject to:	Subject to:
Constraints (1) to (18)	Constraints (1) to (18)
Constraint (20)	Constraint (20)
$X_{ijkr} \leq 0$	$X_{ijkr} \geq 1$

Solve both the sub-problems (sub problem 1 and 2) as LP relaxation problems. If any of the sub-problems (branches) is infeasible, prune that branch by comparing the corresponding objective function value with lower bound, Z_{LB}. If the objective function value comes out to be greater than Z_{LB}, then we need to update Z_{LB} and this objective function value will be new Z_{LB}. If optimal objective function value of any of the sub problems exceeds Z_{UB} then prune that branch as well, otherwise make sub problems (next level branches) and solve them too with same solution approach as discussed before. Repeat this process of branching, bounding and pruning until all the branches are pruned or the solutions converge to feasible solutions satisfying binary constraint (19) as well. Select the best solution from these feasible solutions (which have binary values to all the decision variables X_{ijkr}) to MTMCVRPB formulation given in subsection 4.2 by evaluating all the branches.

4.4 Managerial insights

Some managerial insights of the proposed model are as follows:

1. The given optimization model tries to merge linehaul and backhaul customers for reducing overall cost.
2. The vehicles are allowed to perform multiple trips that helps in reducing the number of vehicles to be deployed.
3. If fixed cost is high, number of trips can be increased by decreasing MDT value. This will help in reducing overall cost.
4. If demand increases in a given period, then MDT can be increased to enable the vehicles to travel a larger distance for satisfying the increased demand.
5. We can control fixed cost by deploying vehicles of different sizes (heavy, medium and light vehicles in the proposed model).

5 Solution approach

This section introduces a working of saving based heuristic as shown below. This heuristic can be used to find a quick solution to MTMCVRPB and that solution will also be used as an upper bound (say objective function value = Z_{UB}) that helps in pruning the branches of branch and bound method.

5.1 Quick and feasible solution for MTMCVRPB:

The central idea of this algorithm is to optimize the savings achieved by consolidating two routes and having them serviced by a single vehicle, instead of using two different vehicles. To achieve this, the algorithm considers two distinct routes, represented by first route and second route and identifies the last customer on first route and the first customer on second route, denoted as i and j respectively. By combining these routes and having them serviced by a single vehicle, the total distance that the vehicle needs to travel decreases by a value equal to S_{ij}, which can be computed using the following function:

$$S_{ij} = d_{i0} + d_{0j} - d_{ij} \tag{5.1}$$

Toth and Vigo (2014) provides a detailed description of the algorithm as well as an analysis of its performance on a variety of test problems. The authors also discuss several extensions and variations of the algorithm, including a modified version that incorporates a time window constraint.

Step-wise description of proposed saving-based heuristic for this MTMCVRPB as given in flowchart Fig. 2 is given as follows:

Step 1 Calculate the savings for each pair of locations in the problem using the equation 5.1.

Step 2 Arrange the savings in descending order.

Step 3 Make a permutation set for all the vehicles with maximum number of trips allowed e.g. if there are the two vehicles namely v_1 and v_2 and maximum two trips each vehicle can perform in a given period of time then permutation set of vehicles will be P = {(v_{11}, v_{12}, v_{21}, v_{22}), (v_{21}, v_{22}, v_{11}, v_{12})} where v_{kr} represents the route of vehicle k during its r^{th} trip. Cardinality of this permutation set will be $K!$ where K is the number of vehicles and each element of P has cardinality of $K * R$ where R is the maximum number of trips a vehicle is allowed to perform.

Step 4 Select first tuple from P and start with these set of routes, where each route begins and ends at the depot means assign Depot-Depot to each v_{kr}

Step 5 Select the pair of locations with the highest savings and attempt to merge them into a single route v_{kr}. Identify the routes that include these locations and determine the optimal way to combine them, considering different merging possibilities. This can involve inserting one route into another, reversing a route and joining them or creating a new route that connects the two locations. Ensure that the basic assumptions of Vehicle Routing Problems with Backhauls (VRPBs) are accounted for. Choose the merging option that minimizes the total distance traveled.

Step 6 Update the set of routes by replacing the original routes that were combined with the merged route.

Step 7 Repeat steps 5 and 6 until all locations have been visited.

Step 8 Calculate total cost for each route v_{kr} and check feasibility of the solution and if so, store the solution.

Step 9 Repeat step 4 to step 8 till all the permutations in P are exhausted.

Step 10 Now select the best solution from stored solutions in step 8. This will be the near-optimal solution (may be optimal but not guaranteed).

This saving-based heuristic can provide near-optimal solutions for large-scale problems in a relatively short amount of time.

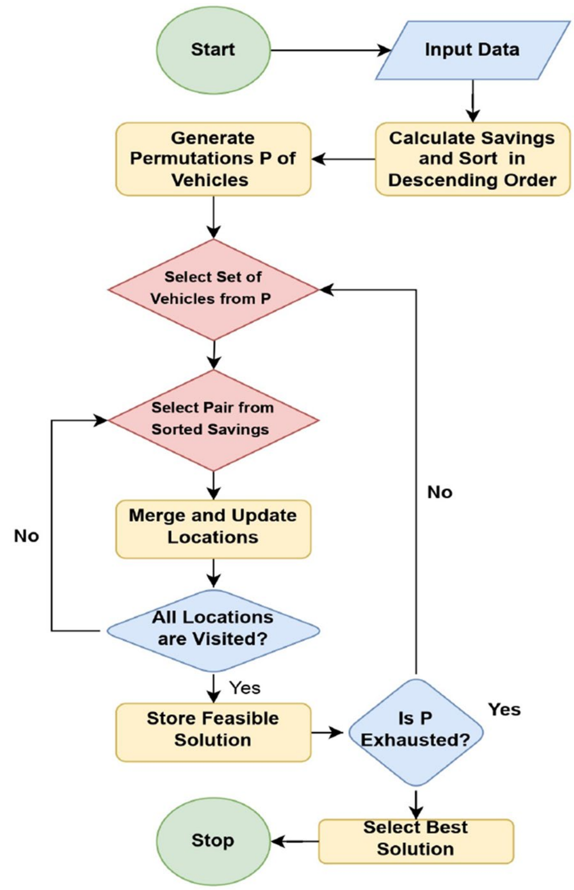


Fig. 2 Flow Chart for Saving Based Heuristic for MTMCVRPB

6 Computational experimentation

The results of the numerical experiments conducted on a simple example described in subsection 6.1 with randomly generated data sets are described below. The solutions obtained using LINGO 19.0 and saving based heuristic (subsection 5.1) are demonstrated here.

6.1 Example used for experiments

Let us assume that there is a single depot and 10 customers (six are linehauls C1 to C6 and four are backhauls C7 to C10 but we varied them for numerical experimentation purpose) with two types of products (product 1 and product 2) and three types of vehicles (heavy, medium and light) with different capacities, each having two equal sized compartments (say compartment 1 and compartment 2). The vehicles have fixed costs and routing cost i.e. per km costs associated with the vehicles. All the attributes of vehicles are given in Table 2.

The demand for both items from each customer is given in Table 3 and a randomly generated symmetric distance matrix is presented in Table 4.

Each vehicle may make a maximum of two trips and maximum distance per trip (MDT) is 250 kilometers. So, throughout the period vehicle can travel at most 500 kilometers.

The proposed MILP model is tested on the above example using LINGO software and saving based hueristic (explained in Sect. 5).

6.2 Various instances and their solutions

By varying the linehaul customers (LC) and backhaul customers (BC), the following Table 5 of solutions is created in which total cost (TC) is the summation of fixed cost (FC) and routing cost (RC). Note that after any route, values in brackets represent the vehicle name and then its trip number e.g. optimal route for instance 1 in Table 5 is Depot-C8-C10-C3-C1-C4-C2-Depot (Medium,1); Depot-C7-C9-C5-C6-Depot (Medium,2), where (Medium,1) represents that medium type vehicle travels on route Depot-C8-C10-C3-C1-C4-C2-Depot during its 1^{st} trip and (Medium,2) represents that medium type vehicle travels on route Depot-C7-C9-C5-C6-Depot in its 2^{nd} trip. For the instances in which LC = 1 and LC = 0, the solution was found to be infeasible. So, these instances are not included in Table 5.

Table 2 Vehicles used in the experiments

Vehicles		Heavy	Medium	Light
Capacity	Compartment 1	300	200	100
	Compartment 2	300	200	100
Fixed cost (Rs)		1000	800	600
Routing cost (Rs/Km)		35	30	25

6.3 Sensitivity analysis

Sensitivity analysis is an important aspect of optimization problems to examine the behavior of the proposed model under different scenarios. In this regard, the sensitivity analysis is conducted on the maximum distance per trip (MDT), maximum distance per period for vehicle (MDP), fixed costs of each vehicle (F_k), cost per km for each vehicle (C_k), maximum number of trips (R) that each vehicle can perform and variation in the number of products or number of compartments (L).

6.3.1 Scenario 1: Changes in MDT

For the instance of 6 linehauls and 4 backhauls, if distance parameter MDT is changed from 350 to ∞ then the solution will remain unchanged and if MDT is varied between 0 and 350 then the total cost changes as shown in Fig. 3.

The following Tables 6 and7 shows the impact of MDT on number of trips of vehicles as you can see here MDT has reverse impact on total cost means if MDT decreases total cost will increase.

6.3.2 Scenario 2: Changes in MDP

Table 7 shows that if MDP decreases, FC may decrease but RC increases as MDP controls the number of trips a vehicle can perform during whole period.

6.3.3 Scenario 3: Changes in routing cost C_k

Next, we study the impact of varying the values of routing cost on objective function values (total cost). Instance 5 of Table 5 has been considered for this experiment. Results of the experiment are presented in Fig. 4. As seen in Fig. 4, variations in the routing cost of medium sized vehicles have a greater influence on total cost in comparison to heavy sized and light sized vehicles. Total cost is affected least by the variations in the routing cost of heavy sized vehicles.

6.3.4 Scenario 4: Changes in fixed cost F_k

In order to examine the effect of changes in the fixed cost parameter F_k on the solution of instance 5 presented in

Table 3 Demand of Linehaul and Backhaul Customers

Demand	Depot	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Product 1	0	9	26	95	56	77	20	82	25	13	29
Product 2	0	17	32	0	2	46	84	30	59	24	30

Table 4 Distance matrix

Distance (Km)	Depot	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Depot	0	49.5	19.31	20.81	55.04	34.23	37.7	60.96	32.65	41.11	24
C1	49.5	0	61.33	38.33	62.65	83.05	68.94	105.99	72.28	90.55	51.87
C2	19.31	61.33	0	39.82	43.86	34.83	53.46	44.78	45.22	37.16	42.58
C3	20.81	38.33	39.82	0	67.9	47.8	31.62	81.32	33.96	56.65	13.89
C4	55.04	62.65	43.86	67.9	0	78.52	92.7	69.72	86.95	79.4	77.49
C5	34.23	83.05	34.83	47.8	78.52	0	36.89	45.61	24.7	9.9	39.45
C6	37.7	68.94	53.46	31.62	92.7	36.89	0	81.54	12.21	46.62	17.8
C7	60.96	105.99	44.78	81.32	69.72	45.61	81.54	0	69.64	37.34	78.82
C8	32.65	72.28	45.22	33.96	86.95	24.7	12.21	69.64	0	34.41	21.02
C9	41.11	90.55	37.16	56.65	79.4	9.9	46.62	37.34	34.41	0	49.09
C10	24	51.87	42.58	13.89	77.49	39.45	17.8	78.82	21.02	49.09	0

Table 5 Different instances and solution routes

Instance	LC	BC	FC	RC	TC	Optimal Routes
1	10	0	800	12435	13235	Depot-C8-C10-C3-C1-C4-C2-Depot (Medium,1); Depot-C7-C9-C5-C6-Depot (Medium,2)
2	9	1	800	12093.6	12893.6	Depot-C2-C4-C1-C3-C6-C10-Depot (Medium,1); Depot-C8-C5-C9-C7-Depot (Medium,2)
3	8	2	800	12569.4	13369.4	Depot-C2-C4-C1-C3-C6-C10-Depot (Medium,1); Depot-C8-C5-C7-C9-Depot (Medium,2)
4	7	3	800	12167.4	12967.4	Depot-C4-C1-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C7-C5-C9-Depot (Medium,2)
5	6	4	1400	11233.5	12633.5	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
6	5	5	1400	10781.75	12181.75	Depot-C2-C4-C1-C3-C10-C6-C8-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1)
7	4	6	800	11343.9	12143.9	Depot-C4-C1-C3-C10-C6-C8-Depot (Medium,1); Depot-C2-C7-C9-C5-Depot (Medium,2)
8	3	7	1600	11072.2	12672.2	Depot-C2-C7-C9-C5-C8-C6-C10-Depot (Heavy,1); Depot-C3-C1-C4-Depot (Light,1)
9	2	8	800	12543.6	13343.6	Depot-C2-C7-C9-C5-C8-Depot (Medium,1); Depot-C1-C4-C3-C10-C6-Depot (Medium,2)

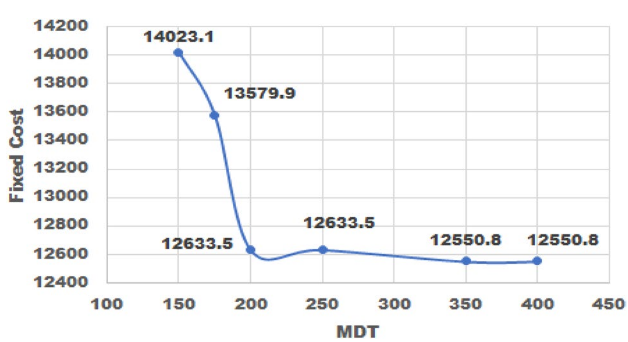


Fig. 3 Impact of maximum distance per trip on total cost

table 5, a sensitivity analysis was performed by varying the values of F_k in the objective function . The results of this analysis are displayed in Fig. 5. It was observed that modifications made to F_k of heavy vehicles had the least impact on the solution as compared to changes made to the F_k of medium and light vehicles. The model in this example exhibits high sensitivity to medium vehicle, as the total cost consistently increases with the associated fixed cost.

6.3.5 *Scenario 5: Changes in number of trips R*

The variation in total cost by varying the number of trips for the instances given in Table 5, is shown in Fig. 6. If number of trips increases, total cost either will remain same for some instances otherwise it decreases.

Table 6 Impact of MDT on trips of vehicles

MDT	FC	RC	TC	Optimal routes
150	1400	12623.1	14023.1	Depot-C6-C8-C10-Depot (Medium,1); Depot-C3-C1-Depot (Medium,2); Depot-C5-C9-C7-Depot (Light,1); Depot-C2-C4-Depot (Light,2) 14023.1
175	1400	12179.9	13579.9	Depot-C2-C3-C6-C8-C10-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1); Depot-C1-C4-Depot (Light,2)
200	1400	11233.5	12633.5	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
250	1400	11233.5	12633.5	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
350	1400	11150.8	12550.8	Depot-C2-C4-C1-C3-C6-C8-C10-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1)
∞	1400	11150.8	12550.8	Depot-C2-C4-C1-C3-C6-C8-C10-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1)

Table 7 Impact of MDP on Fixed Cost and Routing Cost

MDP	MDT	FC	RC	TC	Optimal routes
≥ 300	150	1400	12623.1	14023.1	Depot-C6-C8-C10-Depot (Medium,1); Depot-C3-C1-Depot (Medium,2); Depot-C5-C9-C7-Depot (Light,1); Depot-C2-C4-Depot (Light,2)
250	150	1400	12739.5	14139.5	Depot-C4-C2-Depot (Medium,1); Depot-C3-C1-Depot (Medium,2); Depot-C6-C8-C10-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
225	150	2400	13403.8	15803.8	Depot-C1-C3-Depot (Heavy,1); Depot-C5-C9-C7-Depot (Medium,1); Depot-C6-C8-C10-Depot (Light,1); Depot-C4-C2-Depot (Light,2)
200	150	2400	14507.1	16907.1	Depot-C4-C2-Depot (Heavy,1); Depot-C5-C9-C7-Depot (Medium,1); Depot-C3-Depot (Medium,2); Depot-C6-C8-C10-Depot (Light,1); Depot-C1-Depot (Light,2)
190	150	2400	14831.4	17231.4	Depot-C5-C9-C7-Depot (Heavy,1); Depot-C3-C10-Depot (Medium,1); Depot-C2-C5-Depot (Medium,2); Depot-C6-C8-Depot (Light,1); Depot-C1-Depot (Light,2)
175	175	2400	12888.1	15288.1	Depot-C3-C6-C8-C10-Depot (Heavy,1); Depot-C2-C5-C9-C7-Depot (Medium,1); Depot-C1-C4-Depot (Light,1)
250	175	1400	12739.5	14139.5	Depot-C2-C4-Depot (Medium,1); Depot-C1-C3-Depot (Medium,2); Depot-C6-C8-C10-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
300	175	1400	12339.5	13739.5	Depot-C2-C5-C9-C7-Depot (Medium,1); Depot-C3-C6-C8-C10-Depot (Medium,2); Depot-C4-C1-Depot (Light,1)
≥ 350	175	1400	12179.9	13579.9	Depot-C2-C3-C6-C8-C10-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1); Depot-C1-C4-Depot (Light,2)
≥ 350	200	1400	11233.5	12633.5	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
300	200	1400	11482.8	12882.8	Depot-C3-C1-C4-C2-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1); Depot-C6-C8-C10-Depot (Light,2)
200	200	2400	12432.1	14832.1	Depot-C6-C8-C10-Depot (Heavy,1); Depot-C2-C4-C1-C3-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1)
250	250	1400	11482.8	12882.8	Depot-C3-C1-C4-C2-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1); Depot-C6-C8-C10-Depot (Light,2)
≥ 300	250	1400	11233.5	12633.5	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
≥ 350	≥ 350	1400	11150.8	12550.8	Depot-C2-C4-C1-C3-C6-C8-C10-Depot (Medium,1); Depot-C5-C9-C7-Depot (Light,1)

6.3.6 Scenario 6: Increasing the number of products

The sensitivity of the proposed mathematical model is also tested against the number of products or number of compartments involved in the experiments. In this experiment also, the instance in which LC = 6 and BC = 4 i.e. instance 5 in Table 5 is used. As you can see in the Table 8, number of products vary from 1 to 5. With increase in number of products, fixed cost first increases and later starts to decrease, therefore the fixed cost curve seems to be concave but routing cost gradually increases as number of

Fig. 4 Impact of variations in routing cost on total cost

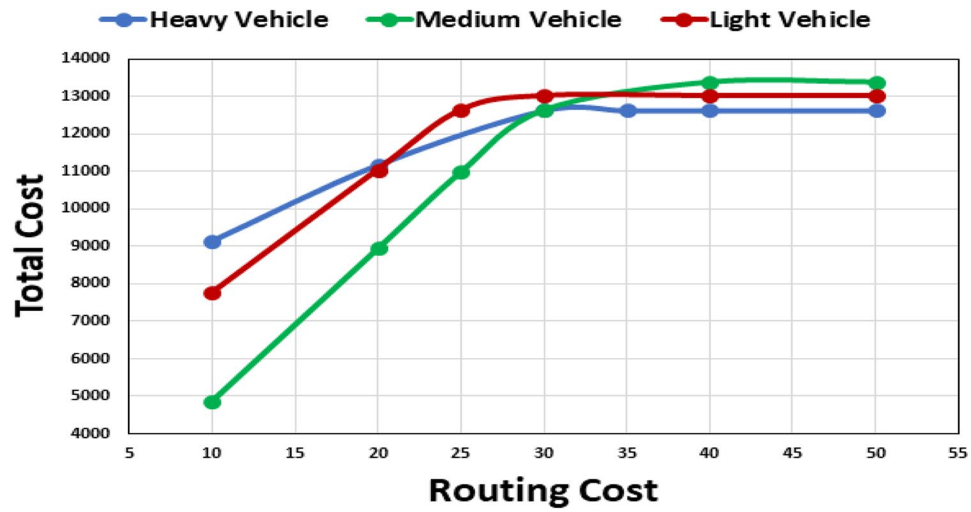
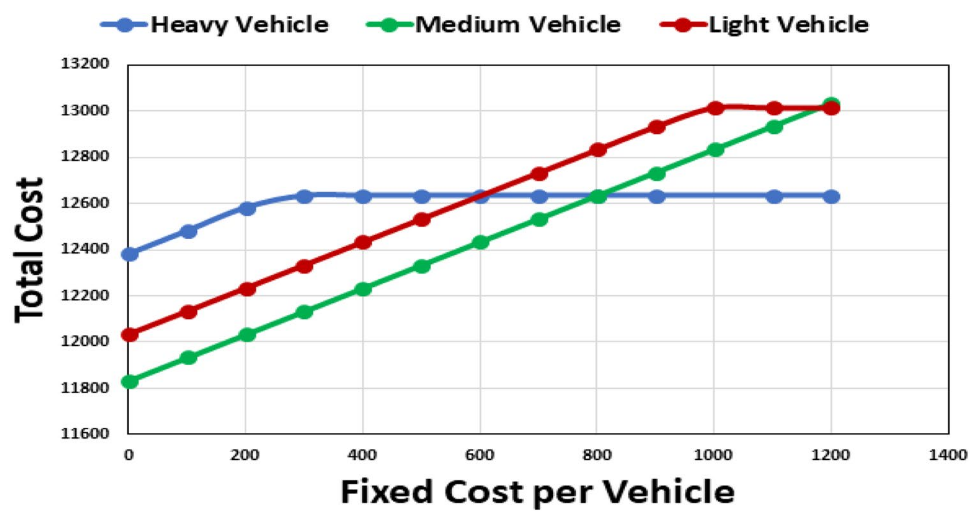


Fig. 5 Impact on Total Cost of Variations in Fixed Cost per Vehicle



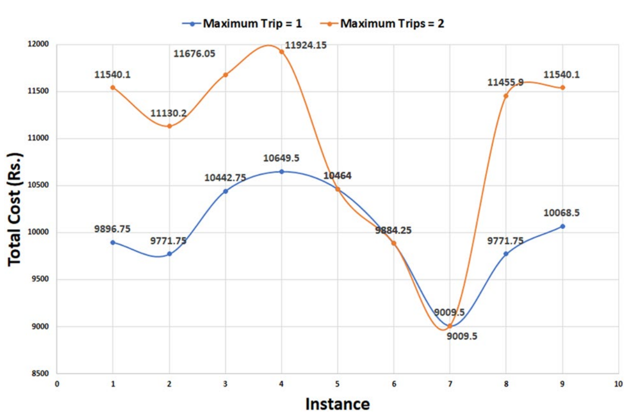


Fig. 6 Variations in total cost based on variations in the maximum number of trips

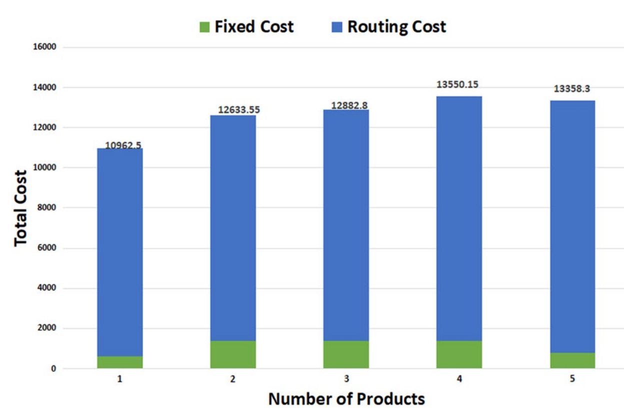


Fig. 7 Impact of variations in number of products on total cost

Table 8 Impact of variations in number of products on total cost

Products	Fixed cost	Routing cost	Total cost	Number of routes	Routes
1	600	10362.5	10962.5	2	Depot-C2-C4-C1-C3-C10-C8-Depot (Light,1); Depot-C6-C5-C9-C7-Depot (Light,2)
2	1400	11233.55	12633.55	3	Depot-C3-C6-C8-C10-Depot (Medium, 1); Depot-C2-C4-C1-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
3	1400	11482.8	12882.8	3	Depot-C2-C4-C1-C3-Depot (Medium,1); Depot-C6-C8-C10-Depot (Light,1); Depot-C5-C9-C7-Depot (Light,2)
4	1400	12150.15	13550.15	3	Depot-C3-C6-C8-C10-Depot (Medium,1); Depot-C1-C4-C2-Depot (Medium,2); Depot-C5-C9-C7-Depot (Light,1)
5	800	12558.3	13358.3	2	Depot-C4-C2-C5-C9-C7-Depot (Medium,1); Depot-C1-C3-C6-C8-C10-Depot (Medium,2)

Table 9 Computational time and total cost using LINGO 19 and saving algorithm

LC	BC	Optimal cost using LINGO (in Rs.)	Cost Using Saving Algorithm (in Rs.)	Time Using LINGO (in seconds)	Time Using Saving Algorithm (in seconds)
10	0	13235	17550.25	41.26	0.003995
9	1	12893.6	17621.15	4.83	0.003001
8	2	13369.4	18564.30	2.17	0.003994
7	3	12967.4	16646.75	1.65	0.003998
6	4	12633.5	16947.85	1.69	0.002998
5	5	12181.75	16129.2	2.37	0.002998

products increase and this impact can be understood by Fig. 7.

6.3.7 *Comparison between solutions of Lingo19 and saving algorithm*

As seen in Table 9, it is evident that there is significant deviation in the optimal solution and solution provided by the proposed algorithm i.e. saving based heuristic but the algorithm finds the near optimal solution very quickly.

7 Discussion

7.1 Practical implications

Based on the experiments, some recommendations for enhancing the delivery operations are given below:

1. Lesser fixed cost of using vehicles may allow vehicles to travel a longer distance.
2. Higher fixed cost of heavy vehicles, restricts them to travel smaller distance.
3. Variations in the routing cost of medium sized vehicles have a greater influence on total cost.
4. To solve the model quickly, saving based heuristic can be used.

7.2 Limitations

1. The study assumes fixed and known customer demand, which may not accurately reflect real-world scenarios where demand fluctuates over time. Future research can explore techniques for handling dynamic demand by incorporating predictive models or adapting the proposed approach to real-time data.
2. The absence of time constraints in the proposed model is another limitation. Time constraints play a crucial role in practical vehicle routing problems, as they affect delivery schedules and customer satisfaction. The incorporation of time-based constraints, including soft and hard time windows, would enhance the realism of the model and its applicability to real-world settings. Future research should aim to incorporate such time-related considerations to provide a more comprehensive solution.

By addressing these limitations, future studies can further refine and extend the proposed model, making it more applicable to real-world scenarios and enhancing its practical utility in optimizing multi-trip multi-compartment vehicle routing problems with backhauls.

7.3 Future research directions

Authors chose MTMCVRPB with the crisp environment to build a foundation before delving into more complex formulations. By studying the crisp setting first, they can understand the fundamental principles and properties of the vehicle routing problem. Once this understanding is established, they can then extend their research to fuzzy or intuitionistic fuzzy settings to explore and address the nuances and uncertainties that arise in real-world scenarios. Multiple-trip multi-compartment vehicle routing problem can be converted into a fuzzy vehicle routing problem by considering uncertainty or fuzziness in certain parameters or elements of the problem (such as uncertainty in demands of linehauls and backhauls). Here are a few approaches:

1. *Fuzzy demands:* Instead of having crisp demand values for each customer, you can represent the demands as fuzzy numbers (triangular fuzzy number, trapezoidal fuzzy number and etc.) as Kumar (2016) converted transportation problem into type 1 fuzzy transportation problem (fuzzy demands and fuzzy availabilities but crisp transportation cost). Fuzzy demands capture the uncertainty or imprecision associated with customer demands. For example, instead of a customer having a demand of exactly 50 units, you can express their demand as a fuzzy number with a range or membership function, such as "around 50 units" or "between 40 and 60 units."
2. *Fuzzy travel times:* In a traditional VRP, travel times between locations are usually assumed to be fixed or deterministic. However, in a fuzzy VRP, you can represent travel times as fuzzy numbers or fuzzy sets to account for uncertainty or variability. This can capture factors like traffic congestion, road conditions or unpredictable delays. For example, instead of assuming a fixed travel time of 30 minutes between two locations, you can represent it as a fuzzy number or membership function indicating a range of possible travel times.
3. *Fuzzy capacities:* Instead of assuming crisp capacities for vehicles, you can introduce fuzziness to represent the imprecision or variability in vehicle capacities. Fuzzy capacities can be used to account for factors like weight restrictions, varying vehicle conditions or uncertainties in load sizes. For example, instead of a vehicle having a fixed capacity of 1000 units, you can represent its capacity as a fuzzy number or fuzzy set indicating a range of possible load capacities.
4. *Fuzzy constraints:* In addition to demands and capacities, other constraints in the MTMCVRPB can also be fuzzified. For instance, time windows for customer visits can be represented as fuzzy intervals to account for uncertainties in arrival or service times. Other constraints, such as vehicle working hours or route durations, can also be fuzzified to handle uncertainties or imprecisions in these parameters.

By introducing fuzziness to various elements of the MTMCVRPB, the given VRP can be transformed into a fuzzy VRP, which allows for the optimization of routes under uncertain or imprecise conditions. Solving such a fuzzy VRP typically involves using fuzzy logic-based techniques, fuzzy mathematical programming or evolutionary algorithms designed for handling uncertainty. Some research papers depicting the implementation of fuzzy mathematical programming include: (Cheng et al. 1995; Kumar and Hussain 2016; Kumar 2018a, b, c, 2019, 2023, 2020a, b).

8 Conclusion

This research paper introduces a novel approach to address the Multi-Trip Multi-Compartment Vehicle Routing Problem with backhauls (MTMCVRPB). The objective of the study is to enhance the utilization of multi-compartment vehicles while minimizing the overall cost of transportation, considering both fixed and routing costs. The proposed model incorporates maximum distance constraints for each trip of the vehicle as well as throughout the entire period. The model is solved using LINGO 19.0 software and a saving-based heuristic algorithm is proposed for obtaining quick solutions.

The numerical experiments conducted on randomly generated data demonstrate the effectiveness of the proposed approach in diverse scenarios. The outcomes indicate that the proposed Mixed-Integer Linear Programming (MILP) model effectively solves the MTMCVRPB and enables the optimization of routes for a heterogeneous fleet of multi-compartment vehicles transporting different commodities concurrently. Comparative results between the optimal solution and the solution produced using the saving-based heuristic demonstrate that the suggested algorithm produces near-optimal solutions in a quick time but incurs some additional costs. The saving-based heuristic offers an effective way for obtaining rapid solutions to the problem.

The practical implications of this study are substantial, particularly for logistics companies seeking to optimize their delivery operations. By employing the proposed model, these companies can streamline their route planning by combining delivery and pickup operations and allowing vehicles with multiple compartments to perform multiple trips, thereby resulting in a reduction of cost and enhanced delivery operations.

In real life problems, time constraints play a crucial role. Time constraints are not available in the proposed model. The incorporation of time-based constraints, including soft and hard time windows, would enhance the realism of the model and its applicability to real-world settings. This study assumes the customer demand to be fixed but in real life, demand fluctuates over time. A stochastic version of the given model may be proposed to incorporate irregular demand.

Future research can also explore the techniques for simultaneously handling dynamic customer demand and time-based constraints. Furthermore, the MTMCVRPB can be converted into a fuzzy VRP by introducing uncertainty or fuzziness to certain parameters or elements of the problem. This fuzzy VRP can be solved using fuzzy logic-based techniques, fuzzy mathematical programming, or evolutionary algorithms designed for handling uncertainty.

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Declarations

Conflict of interest The authors declare that there is no potential conflict of interest.

Human participants and/or animals Not Applicable.

Informed consent Not Applicable.

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