Lista de exercícios 01 - PCC116

RODRIGO SENNA DA SILVA

Maio 2025

1 Item 1.5.7 - Prove os seguintes sequentes usando dedução natural. Tente demonstrá-los sem utilizar a regra RAA.

(a)

$$\{(P \land Q) \land R,\ S \land T\} \vdash Q \land S$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(P \land Q) \land R}\,\{_{ID}\}}{P \land Q}\,\{\land_{EE}\}}{Q}\,\{\land_{ED}\} \qquad \dfrac{\dfrac{}{S \land T}\,\{_{ID}\}}{S}\,\{\land_{EE}\}}{Q \land S}\,\{\land_{I}\}$$

(b)

$$\{(P \land Q) \land R\} \vdash (P \land R) \lor Z$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(P \land Q) \land R}\,\{_{ID}\}}{P \land Q}\,\{\land_{EE}\}}{P}\,\{\land_{EE}\} \qquad \dfrac{\dfrac{}{(P \land Q) \land R}\,\{_{ID}\}}{R}\,\{\land_{ED}\}}{P \land R^1}\,\{\land_{I}\}}{(P \land R) \lor Z}\,\{\lor_{IE}\}^1$$

(c)

$$\{P,\ P \to (P \to Q)\} \vdash Q$$

$$\dfrac{\dfrac{\dfrac{}{P \to (P \to Q)}\,\{_{ID}\} \qquad \dfrac{}{P}\,\{_{ID}\}}{P \to Q}\,\{\to_{E}\} \qquad \dfrac{}{P}\,\{_{ID}\}}{Q}\,\{\to_{E}\}$$

(d)

$$\vdash (P \land Q) \to P$$

$$\dfrac{\dfrac{\dfrac{}{(P \land Q)^1}\,\{_{ID}\}}{P}\,\{\land_{EE}\}}{(P \land Q) \to P}\,\{\to_{I}\}^1$$

(e)

$$\{P\} \vdash Q \to (P \wedge Q)$$

$$\dfrac{\dfrac{\dfrac{}{P}\{_{ID}\} \qquad \dfrac{}{Q^1}\{_{ID}\}}{P \wedge Q}\{\wedge_I\}}{Q \to (P \wedge Q)}\{\to_I\}^1$$

(f)

$$\{P\} \vdash (P \to Q) \to Q$$

$$\dfrac{\dfrac{\dfrac{}{P \to Q^1}\{_{ID}\} \qquad \dfrac{}{P}\{_{ID}\}}{Q}\{\to_E\}}{(P \to Q) \to Q}\{\to_I\}^1$$

(g)

$$\vdash (P \wedge Q) \to (P \vee Q)$$

$$\dfrac{\dfrac{\dfrac{}{P \wedge Q^1}\{_{ID}\}}{P \vee Q}\{\vee_{IE}\} \qquad \dfrac{\dfrac{}{P \wedge Q^1}\{_{ID}\}}{P \vee Q}\{\vee_{ID}\}}{(P \wedge Q) \to (P \vee Q)}\{\to_I\}^1$$

(h)

$$\{Q \to (P \to R),\ \neg R,\ Q\} \vdash \neg P$$

$$\dfrac{\dfrac{\dfrac{}{Q \to (P \to R)}\{_{ID}\} \qquad \dfrac{}{Q}\{_{ID}\}}{P \to R}\{\to_E\} \qquad \dfrac{}{\neg R}\{_{ID}\}}{\neg P}\{\to_E\}$$

(i)

$$\{P\} \vdash Q \to (P \wedge Q)$$

$$\dfrac{\dfrac{P \qquad Q^1}{P \wedge Q}\{\wedge_1\}}{Q \to (P \wedge Q)}\{\to_I\}^1$$

(j)

$$\{(P \to R) \wedge (Q \to R),\ P \wedge Q\} \vdash Q \wedge R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(P \to R) \wedge (Q \to R)}\{_{ID}\}}{Q \to R}\{\wedge_{ED}\} \qquad \dfrac{\dfrac{}{P \wedge Q}\{_{ID}\}}{Q}\{\wedge_{ED}\}}{R}\{\to_E\} \qquad \dfrac{\dfrac{}{P \wedge Q}\{_{ID}\}}{Q}\{\wedge_{ED}\}}{Q \wedge R}\{\wedge_I\}$$

(k)

$$\{P \to Q \to R,\ P \to Q\} \vdash P \to R$$

$$\dfrac{\dfrac{\dfrac{}{P \to Q \to R}\{{}_{ID}\} \qquad \dfrac{}{P^1}\{{}_{ID}\}}{Q \to R}\{\to_E\}^1 \qquad \dfrac{\dfrac{}{P \to Q)}\{{}_{ID}\} \qquad \dfrac{}{P^1}\{{}_{ID}\}}{Q}\{\to_E\}}{P \to R}\{\to_E\}^1$$

(l)

$$\{P \to Q,\ R \to S\} \vdash (P \vee R) \to (Q \vee S)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \to Q}\{{}_{ID}\}}{Q \vee S}\{\vee_{IE}\} \qquad \dfrac{\dfrac{}{R \to S}\{{}_{ID}\}}{Q \vee S}\{\vee_{ID}\}}{Q \vee S}\{\vee_E\}}{(P \vee R) \to (Q \vee S)}\{\to_I\}$$

(m)

$$\{Q \to R\} \vdash (P \to Q) \to (P \to R)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{Q \to R}\{{}_{ID}\} \qquad \dfrac{\dfrac{}{P}\{{}_{ID}\} \qquad \dfrac{}{P \to Q^1}\{{}_{ID}\}}{Q}\{\to_E\}}{R}\{\to_E\}}{P \to R}\{\to_I\}}{(P \to Q) \to (P \to R)}\{\to_I\}^1$$

(n)

$$\{(P \wedge Q) \vee (P \wedge R)\} \vdash P \wedge (Q \vee R)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(P \wedge Q) \vee (P \wedge R)}\{{}_{ID}\}}{P \wedge Q}\{\wedge_{IE}\}}{P}\{\wedge_{EE}\} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{(P \wedge Q) \vee (P \wedge R)}\{{}_{ID}\}}{P \wedge Q}\{\wedge_{EE}\}}{Q}\{\wedge_{EE}\}}{Q \vee R}\{\vee_{IE}\}}{P \wedge (Q \vee R)}\{\wedge_I\}$$

(o)

$$\{P \to (Q \wedge R)\} \vdash (P \to Q) \wedge (P \to R)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P \to (Q \wedge R)}\{{}_{ID}\} \qquad \dfrac{}{P^1}\{{}_{ID}\}}{Q \wedge R}\{\to_E\}}{Q}\{\wedge_{EE}\}}{P \to Q}\{\to_I\} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{P \to (Q \wedge R)}\{{}_{ID}\} \qquad \dfrac{}{P^1}\{{}_{ID}\}}{Q \wedge R}\{\to_E\}^1}{R}\{\wedge_{ED}\}}{P \to R}\{\to_I\}}{(P \to Q) \wedge (P \to R)}\{\wedge_I\}$$

(p)

$$\{(P \to Q) \wedge (P \to R)\} \vdash P \to (Q \wedge R)$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{(P \to Q) \land (P \to R)}\{ID\}}{P \to Q}\{\land EE\} \qquad \dfrac{}{P_1}\{ID\}}{Q}\{\to E\} \qquad \dfrac{\dfrac{\dfrac{}{(P \to Q) \land (P \to R)}\{ID\}}{P \to R}\{\land ED\} \qquad \dfrac{}{P_1}\{ID\}}{R}\{\to E\}}{\dfrac{Q \land R}{P \to (Q \land R)}\{\to I\}^1}\{\land I\}
$$

(q)

$$\{P \to Q\} \vdash ((P \land Q) \to P) \land (P \to (P \land Q))$$

$$
\dfrac{\dfrac{\dfrac{}{P^1}\{ID\}}{(P \land Q) \to P}\{\to I\}^1 \qquad \dfrac{\dfrac{\dfrac{}{P^1}\{ID\} \qquad \dfrac{\dfrac{}{P \to Q}\{ID\}}{Q}\{\to E\}}{P \land Q}\{\land I\}}{P \to (P \land Q)}\{\to I\}}{((P \land Q) \to P) \land (P \to (P \land Q))}\{\land I\}
$$

(r)

$$\{P \to (Q \lor R),\ Q \to S,\ R \to S\} \vdash P \to S$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{P}\{ID\} \qquad \dfrac{}{P \to (Q \lor R)}\{ID\}}{Q \lor R}\{\to E\} \qquad Q \to S \qquad R \to S}{S}\{\lor E\}}{P \to S}\{\to I\}
$$

(s)

$$\vdash \neg P \to P \to P \to Q$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P^1}\{ID\} \qquad \dfrac{}{P^2}\{ID\}}{\bot}\{\to E\}}{Q}\{CTR\}}{P \to Q}\{\to I\}}{P \to (P \to Q)}\{\to I\}^2}{\neg P \to (P \to (P \to Q))}\{\to I\}^1
$$

(t)

$$\{P \land Q \to R,\ R \to S,\ Q \land \neg S\} \vdash \neg P$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{Q \land \neg S}\{ID\}}{\neg S}\{\land ED\} \qquad \dfrac{\dfrac{}{R \to S}\{ID\} \qquad \dfrac{\dfrac{}{P \land Q \to R}\{ID\} \qquad \dfrac{\dfrac{}{P}\{ID\} \qquad \dfrac{\dfrac{}{Q \land \neg S}\{ID\}}{Q}\{\land EE\}}{P \land Q}\{\land I\}}{R}\{\to E\}}{S}\{\to E\}}{\bot}\{\to E\}}{\neg P}\{CTR\}^1
$$

(u)

$$\{(P \to Q) \to R,\ S \to \neg P,\ T,\ \neg S \wedge T \to Q\} \vdash R$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$(P \to Q) \to R$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$\neg S \wedge T \to Q$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$\neg S^1$}
\RightLabel{$\{\to I\}$}
\UnaryInfC{$\neg P \to \neg S$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$S \to \neg P$}
\RightLabel{$\{\to E\}$}
\UnaryInfC{$\neg P$}
\RightLabel{$\{\to E\}$}
\BinaryInfC{$\neg S$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{T}
\RightLabel{$\{\wedge I\}^1$}
\BinaryInfC{$\neg S \wedge T$}
\RightLabel{$\{\to E\}$}
\BinaryInfC{Q}
\RightLabel{$\{\to I\}$}
\UnaryInfC{$P \to Q$}
\RightLabel{$\{\to E\}$}
\BinaryInfC{R}
\end{prooftree}
$$

(v)

$$\{(S \to P) \vee (T \to Q)\} \vdash (S \to Q) \vee (T \to P)$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$(S \to P) \vee (T \to Q)$}
\RightLabel{$\{\vee EE\}$}
\UnaryInfC{$S \to P$}
\RightLabel{$\{\to E\}$}
\UnaryInfC{P}
\RightLabel{$\{\to I\}$}
\UnaryInfC{$T \to P$}
\RightLabel{$\{\vee ID\}$}
\UnaryInfC{$(S \to Q) \vee (T \to P)$}
\end{prooftree}
$$

(w)

$$\{\neg(P \to Q)\} \vdash Q \to P$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$\neg(P \to Q)$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{Q^1}
\RightLabel{$\{\to_I\}$}
\UnaryInfC{$(P \to Q)$}
\BinaryInfC{$\perp$}
\RightLabel{$\{CTR\}^1$}
\UnaryInfC{P}
\RightLabel{$\{\to_I\}^1$}
\UnaryInfC{$Q \to P$}
\end{prooftree}
$$

2 Item 2.9 - Prove os seguintes sequentes usando dedução natural:

(a)

$$\{\forall x.(P(x) \to Q(x))\} \vdash (\forall x.\neg Q(x)) \to (\forall x.\neg P(x))$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$\forall x.(P(x) \to Q(x))$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$\forall x.\neg Q(x)$}
\RightLabel{$\{\forall E\}$}
\UnaryInfC{$\neg Q(a)$}
\AxiomC{}
\RightLabel{$\{ID\}$}
\UnaryInfC{$P(a)$}
\RightLabel{$\{\to E\}$}
\BinaryInfC{$Q(a)$}
\RightLabel{$\{\neg E\}$}
\BinaryInfC{$\perp$}
\RightLabel{$\{\neg I\}$}
\UnaryInfC{$\neg P(a)$}
\RightLabel{$\{\forall I\}$}
\UnaryInfC{$\forall x.\neg P(x)$}
\RightLabel{$\{\to I\}$}
\UnaryInfC{$(\forall x.\neg Q(x)) \to (\forall x.\neg P(x))$}
\end{prooftree}
$$

(b)

$$\{\forall x.(P(x) \to \neg Q(x))\} \vdash \neg(\exists x.(P(x) \land Q(x)))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\exists x.(P(x) \land Q(x))}\{ID\}}{P(t) \land Q(t)}\text{[hipótese]}}{P(t)}\{\land E\} \qquad \dfrac{\dfrac{}{\forall x.(P(x) \to \neg Q(x))}\{ID\}}{P(t) \to \neg Q(t)}\{\forall E\}}{\neg Q(t)}\{\to E\} \qquad \dfrac{\dfrac{}{P(t) \land Q(t)}\{ID\}}{Q(t)}\{\land E\}}{\bot}}{\neg \exists x.(P(x) \land Q(x))}\{\neg I\}$$

(c)

$$\{\forall x.(A(x) \to (B(x) \lor C(x))),\ \forall x.\neg B(x)\} \vdash (\forall x.A(x)) \to (\forall x.C(x))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x.A(x)}\text{[hipótese]}}{A(t)}\forall E \qquad \dfrac{\dfrac{}{\forall x.(A(x) \to (B(x) \lor C(x)))}\{ID\}}{A(t) \to (B(t) \lor C(t))}\forall E}{B(t) \lor C(t)}\to E \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\forall x.\neg B(x)}\{ID\}}{\neg B(t)}\forall E \qquad \dfrac{}{B(t)}\text{[hipótese]}}{\bot}\to E}{C(t)}\bot E \qquad \dfrac{}{C(t)}\text{[hipótes}}{C(t)}\lor E}{\forall x.C(x)}\forall I}{(\forall x.A(x)) \to (\forall x.C(x))}\to I$$

(d)

$$\{\exists x.(P(x) \land Q(x)), \forall x.(P(x) \to R(x))\} \vdash \exists x.(R(x) \land Q(x))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x.(P(x) \to R(x))}\{ID\}}{P(t) \land Q(t)}\text{[hipótese]}}{P(t)}\{\land E\} \qquad \dfrac{\dfrac{}{\forall x.(P(x) \to R(x))}\{ID\}}{P(t) \to R(t)}\{\forall E\}}{R(t)}\{\to E\} \qquad \dfrac{\dfrac{}{P(t) \land Q(t)}\{ID\}}{Q(t)}\{\land E\}}{R(t) \land Q(t)}\{\land I\}}{\exists x.(R(x) \land Q(x))}\{\exists I\}$$

(e)

$$\{\forall x.P(a,x,x),\ \forall x.\forall y.\forall z.(P(x,y,z) \to P(f(x),y,f(z)))\} \vdash P(f(a),a,f(a))$$

$$\dfrac{\dfrac{\forall x.P(a,x,x)}{P(a,a,a)}\{\forall E\} \qquad \dfrac{\dfrac{\dfrac{\forall x.\forall y.\forall z.(P(x,y,z) \to P(f(x),y,f(z)))}{\forall y.\forall z.(P(a,y,z) \to P(f(a),y,f(z)))}\{\forall E\}}{\forall z.(P(a,a,z) \to P(f(a),a,f(z)))}\{\forall E\}}{P(a,a,a) \to P(f(a),a,f(a))}\{\forall E\}}{P(f(a),a,f(a))}\{\to E\}$$

(f)

$$\{\forall x.(P(x) \to Q(x))\} \vdash \forall x.P(x) \to \forall x.Q(x)$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{[hipótese]}
\UnaryInfC{$\forall x.P(x)$}
\RightLabel{$\{\forall E\}$}
\UnaryInfC{$P(t)$}
\AxiomC{}
\RightLabel{$\{_{ID}\}$}
\UnaryInfC{$\forall x.(P(x) \to Q(x))$}
\RightLabel{$\{\forall E\}$}
\UnaryInfC{$P(t) \to Q(t)$}
\RightLabel{$\{\to E\}$}
\BinaryInfC{$Q(t)$}
\RightLabel{$\{\forall I\}$}
\UnaryInfC{$\forall x.Q(x)$}
\RightLabel{$\{\to I\}$}
\UnaryInfC{$\forall x.P(x) \to \forall x.Q(x)$}
\end{prooftree}
$$

(g)

$$\{\exists x.\neg P(x)\} \vdash \neg\forall x.P(x)$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{\{ID\}}
\UnaryInfC{$\exists x.\neg P(x)$}
\AxiomC{}
\RightLabel{[hipótese]}
\UnaryInfC{$\neg P(t)$}
\AxiomC{}
\RightLabel{[hipótese]}
\UnaryInfC{$\forall x.P(x)$}
\RightLabel{$\forall E$}
\UnaryInfC{$P(t)$}
\RightLabel{$\perp$}
\BinaryInfC{$\perp$}
\RightLabel{$\neg I$}
\UnaryInfC{$\neg\forall x.P(x)$}
\RightLabel{$\exists E$}
\BinaryInfC{$\neg\forall x.P(x)$}
\end{prooftree}
$$

3 Item 5.2.1 - Prove os seguintes teoremas

(a). Para todo $n \in \mathbb{N}$,

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$$

Base da indução: Para $n = 0$, temos:

$$\sum_{i=0}^{0} 3^i = 3^0 = 1$$

e

$$\frac{3^{0+1} - 1}{2} = \frac{3 - 1}{2} = \frac{2}{2} = 1$$

Logo, a base da indução é válida.

Passo indutivo: Suponha que a fórmula seja válida para um certo $n \in \mathbb{N}$, ou seja,

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$$

Vamos provar que:

$$\sum_{i=0}^{n+1} 3^i = \frac{3^{n+2} - 1}{2}$$

De fato:

$$\begin{aligned}
\sum_{i=0}^{n+1} 3^i &= \left(\sum_{i=0}^{n} 3^i\right) + 3^{n+1} \\
&= \frac{3^{n+1}-1}{2} + 3^{n+1} \\
&= \frac{3^{n+1}-1+2\cdot 3^{n+1}}{2} \\
&= \frac{3\cdot 3^{n+1}-1}{2} \\
&= \frac{3^{n+2}-1}{2}
\end{aligned}$$

Portanto, a fórmula vale para $n+1$. Pelo princípio da indução matemática, a identidade é válida para todo $n \in \mathbb{N}$.

(b). Para todo $n \in \mathbb{N}$,

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Base: Para $n = 0$,

$$\sum_{i=0}^{0} i^2 = 0, \quad \text{e} \quad \frac{0(0+1)(2\cdot 0+1)}{6} = 0$$

Passo indutivo: Suponha a fórmula válida para n,

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Prove para $n+1$:

$$\sum_{i=0}^{n+1} i^2 = \sum_{i=0}^{n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)+6(n+1)^2}{6} = \frac{(n+1)\left[n(2n+1)+6(n+1)\right]}{6}$$

$$= \frac{(n+1)(2n^2+7n+6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\Rightarrow \sum_{i=0}^{n+1} i^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

Logo, a fórmula vale para todo $n \in \mathbb{N}$.

(c). Para todo $n \in \mathbb{N}$,

$$\sum_{i=1}^{n}(8i-5) = 4n^2 - n$$

Base: Para $n = 1$,

$$8(1) - 5 = 3, \quad 4(1)^2 - 1 = 3$$

Passo indutivo: Suponha que

$$\sum_{i=1}^{n}(8i-5) = 4n^2 - n$$

então

$$\sum_{i=1}^{n+1}(8i-5) = \sum_{i=1}^{n}(8i-5) + (8(n+1)-5)$$

$$= 4n^2 - n + 8n + 3 = 4n^2 + 7n + 3 = 4(n+1)^2 - (n+1)$$

(d). Para todo $n \geq 1$,

$$5 \mid (n^5 - n)$$

Base: Para $n = 1$, temos $1^5 - 1 = 0$, e $5 \mid 0$

Passo indutivo: Suponha $5 \mid n^5 - n$, ou seja, $n^5 - n = 5k$

Verificar para $n + 1$:

$$(n+1)^5 - (n+1) = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1 - n - 1 = (n^5 - n) + 5(n^4 + 2n^3 + 2n^2 + n)$$

Como $5 \mid n^5 - n$ e o restante é múltiplo de 5, segue que $5 \mid (n+1)^5 - (n+1)$

(e). Para todo $n \geq 1$,

$$6 \mid (7^n - 1)$$

Provar que $6 \mid (7^n - 1)$ para todo $n \geq 1$

"'latex **Base:** Para $n = 1$, $7^1 - 1 = 6 \Rightarrow 6 \mid 6$

Passo indutivo: Suponha $6 \mid (7^n - 1) \Rightarrow 7^n \equiv 1 \mod 6$

Multiplicando ambos os lados por 7:

$$7^{n+1} = 7 \cdot 7^n \equiv 7 \cdot 1 = 7 \equiv 1 \mod 6 \Rightarrow 6 \mid (7^{n+1} - 1)$$

(f). Para todo $n \geq 1$,

$$6 \mid (n^3 + 5n)$$

Provar que $6 \mid (n^3 + 5n)$

"'latex **Base:** Para $n = 1$, $1 + 5 = 6 \Rightarrow 6 \mid 6$

Passo indutivo: Suponha $6 \mid (n^3 + 5n)$

$$(n+1)^3 + 5(n+1) = n^3 + 3n^2 + 3n + 1 + 5n + 5 = (n^3 + 5n) + 3n^2 + 3n + 6$$

Como $6 \mid (n^3 + 5n)$ e os demais termos somam múltiplo de 6, temos $6 \mid (n+1)^3 + 5(n+1)$

(g). Para todo $n \in \mathbb{N}$,

$$2 \mid (n^2 + n)$$

Provar que $2 \mid (n^2 + n)$

"'latex **Base:** Para $n = 0$, $0^2 + 0 = 0 \Rightarrow 2 \mid 0$

Passo indutivo: Suponha $2 \mid n^2 + n$

$$(n+1)^2 + (n+1) = n^2 + 2n + 1 + n + 1 = (n^2 + n) + 2n + 2$$

Como $2 \mid (n^2 + n)$ e $2n + 2$ é par, então $2 \mid (n+1)^2 + (n+1)$