# Amplitude Modulation (AM) Derivation

**Level:** complete | **Date:** 2025-11-15

## Step 1: Definition of Amplitude Modulation

We start with the basic definition of modulation: multiplying a carrier signal by a message signal.

$$s_{AM}(t) = A_c [1 + m(t)] \cos(2\pi f_c t)$$

*Where $A_c$ is carrier amplitude, $m(t)$ is the normalized message signal, and $f_c$ is the carrier frequency.*

## Step 2: Expand the AM Signal

Distribute the carrier cosine term across the bracketed expression.

$$s_{AM}(t) = A_c \cos(2\pi f_c t) + A_c m(t) \cos(2\pi f_c t)$$

*This separates the carrier component from the modulated component.*

## Step 3: Identify Signal Components

The signal consists of two parts: carrier and sidebands.

$$s_{AM}(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{A_c m(t) \cos(2\pi f_c t)}_{\text{Sidebands}}$$

*The first term is the unmodulated carrier. The second term creates upper and lower sidebands.*

## Step 4: Fourier Transform to Frequency Domain

Taking the Fourier transform to analyze the spectrum.

$$S_{AM}(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] + \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]$$

*The carrier appears as impulses at $\pm f_c$, and the message spectrum $M(f)$ is translated to $\pm f_c$.*

## Step 5: Power Distribution

Calculate the power in carrier and sidebands.

$$P_{total} = P_c + P_{sidebands} = \frac{A_c^2}{2} + \frac{A_c^2 \langle m^2(t) \rangle}{2}$$

*The total power is the sum of carrier power and sideband power. Sideband power depends on message signal power.*

## Final Result

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s_{AM}(t) = A_c [1 + m(t)] \cos(2\pi f_c t)
```

## Key Results

- AM signal consists of carrier + two sidebands
- Bandwidth: BW = 2*f_m (twice the message bandwidth)
- Power efficiency: $\eta \leq 33.3\%$ (maximum with $\mu=1$)
- Simple demodulation with envelope detector

## Assumptions

- Message signal m(t) is normalized: $|m(t)| \leq 1$
- Carrier frequency f_c >> message bandwidth f_m
- Linear operation (no distortion)

## Related Topics

DSB-SC, SSB, VSB, Envelope detection