

Interpolation of Voltage Values Across State of Charge and Temperature Ranges

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Abstract

This study aims to develop a comprehensive model for predicting voltage values across varying states of charge (SoC) and temperature ranges. The primary objectives include analyzing the behavior of experimental data to propose a representative function, verifying the parameters whose coefficients can be expressed as functions of temperature, and determining a general function that accurately represents voltage in terms of SoC and temperature. The proposed model will be validated by comparing it to experimental data, ensuring it fits at least 90% of the observed values. Additionally, a detailed table will be created to organize the data in an Excel sheet, facilitating further analysis and application.

Key words: **State of Charge, Fitting Data, Temperature, Voltage, Interpolation, OCV**

1 Introduction

The performance and longevity of batteries are heavily influenced by their State of Charge (SoC) and operating temperature. Voltage, a key indicator of battery health and performance, varies significantly based on these two factors. Therefore, understanding the relationship between voltage, SoC, and temperature is essential for accurate battery monitoring and simulation. Developing a model that can predict voltage based on these parameters provides valuable insights for Battery Management Systems (BMS), improving the efficiency of energy storage systems, particularly in electric vehicles and renewable energy applications.

In this study, the objective is to propose a function that accurately represents voltage as a function of SoC and temperature. By analyzing the behavior of experimental data, the model will determine the coefficients that depend on temperature and validate them against real-world data. The ultimate goal is to achieve a model that fits at least 90% of the observed data, ensuring its reliability for practical use.

This task is not only crucial for monitoring the battery's electrical performance but also for simulating the thermal behavior of the battery. Efficient thermal management, achieved through an optimized cooling system, helps maintain battery temperature within an optimal range, ensuring enhanced performance, safety, and longevity. By integrating this voltage model with a cooling system simulation, we can better control the battery's operating conditions, improving the overall energy efficiency of the system and mitigating potential thermal-related issues such as overheating. Additionally, organizing the experimental data into a structured format, such as an Excel sheet, will facilitate further analysis and enhance the practical application of the model in different scenarios.

2 Methodology

This section outlines the approach taken to develop and validate a predictive model for voltage based on State of Charge (SoC) and temperature. The methodology is structured into four main stages: analyzing the voltage-SoC rela-

tionship, validating the variation of temperature-dependent coefficients, determining the general voltage function, and verifying the model's performance against experimental data. These steps ensure the model's accuracy and reliability, making it suitable for practical applications in battery management and thermal regulation systems.

2.1 Voltage and SoC relationship

The relationship between voltage and SoC is analyzed using the experimental data to identify key trends and behaviors. This analysis forms the foundation for deriving the mathematical function that will represent voltage as a function of SoC and temperature

The data illustrated in Figure 2b displays a distinctive "N" shape. This characteristic pattern can be effectively modeled using an arc-tangent hyperbolic function, as depicted in Figure 3 and mathematically represented in Equation 1.

$$F(S) = A \cdot \tanh^{-1}(B \cdot S - C) + D \quad (1)$$

The coefficients A , B , C , and D are fitted parameters, each varying with temperature as shown in Figure 5. Some of these parameters may exhibit significant variability, indicating a potential dependence on temperature. Therefore, it is essential to determine which coefficients are temperature-dependent.

2.2 Coefficient Variation Validation

The coefficients obtained from the voltage-SoC analysis are validated by examining their coefficient of variation. This step ensures which coefficients can be represented as temperature-dependent functions, enhancing the flexibility and applicability of the model.

The coefficient of variation (CV) is a statistical measure that indicates the relative variability of data points in a data series around the mean. It is calculated by dividing the standard deviation by the mean and is often expressed as a percentage. A low CV indicates low variability relative to the mean, suggesting more consistency in the data, and vice versa, high CV values indicates high variability relative to the mean, suggesting less consistency in the data, therefore the coefficient

is dependant from an external factor, in this case the temperature [1].

$$CV = \frac{\sigma}{\bar{x}} \quad (2)$$

The coefficient of variation for A , B , C , and D resulted as follows:

- A : 30.27%
- B : 4.22%
- C : 0.97
- D : 0.40

Values exceeding 5% indicate a statistically significant level of dispersion. Consequently, the parameter can be considered a function of an external variable [2]. In this context, parameter A is the only one that satisfies this criterion. Therefore, the fixed values are as follows:

- B : 1.6
- C : 1
- D : 3.8

To develop a comprehensive model, it is essential to understand how parameter A varies with respect to temperature.

2.3 General Function Determination

A general mathematical function that represents voltage as a function of SoC and temperature is determined. The function is developed by integrating the findings from the previous steps and is optimized to ensure accurate representation across various operating conditions.

To propose a function that represents the behavior of the parameter A , it is essential to first analyze its behavior, as illustrated in Figure 5a. The plot resembles the second half of a sigmoidal function, as depicted in Figure 4. The general form of the sigmoid function is given by Equation 3.

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$$A(T) = \frac{F}{1 + e^{-\frac{G \cdot T}{10} + H}} \quad (3)$$

The coefficients for Equation 3 are fixed, with their values specified as follows:

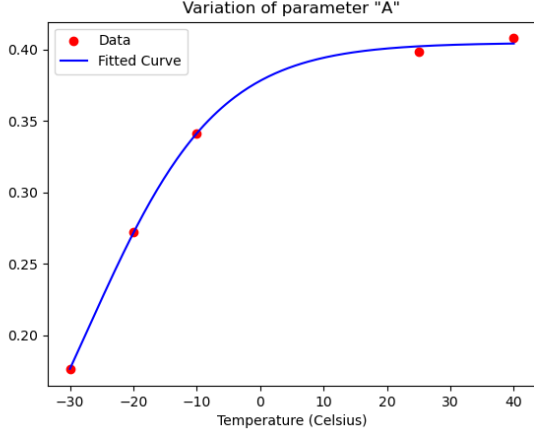


Figure 1: Coefficient A predicted by using Sigmoid Function

- F : 0.4046
- G : 0.9700
- H : -2.6520

By substituting Equation 3 into Equation 1, Equation 4 is obtained as the general model.

$$V(T, S) = \frac{F \cdot \tanh^{-1}(B \cdot S - C)}{1 + e^{-\frac{G \cdot T}{10} + H}} + D \quad (4)$$

2.4 Performance Verification

The accuracy of the proposed model is evaluated by comparing its predictions to experimental data. The model's performance is verified to ensure it fits at least 90% of the observed values, confirming its suitability for real-world battery monitoring and simulation.

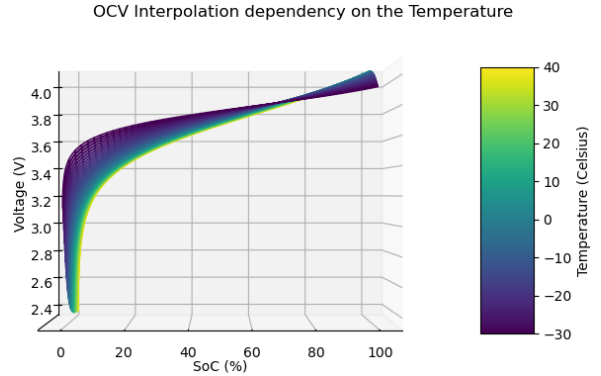
The analysis of the five plots of Figure 7 demonstrates that the model consistently achieves an accuracy rate exceeding 90%, indicating its high predictive power, robustness, and reliability. Furthermore, the low root mean squared error (RMSE) relative to the data scale suggests that the model's predictions closely align with the actual values. These results validate the model's efficacy as a simulation tool, making it suitable for estimating battery autonomy based on battery temperature and voltage.

To further illustrate the model's performance, a 3D plot can be created based on Equation 4,

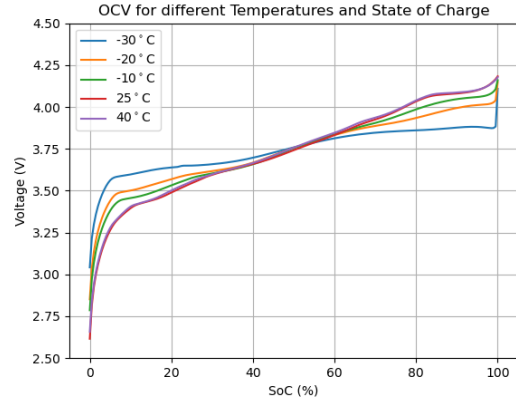
which is a function of two variables. This plot will display all the voltages across the State of Charge from 0% to 100% and the Temperature from -30°C to 40°C.

3 Gradient Plot Comparison

In this section, we will compare the 3D plot generated from the model against the experimental data. This comparison will help to visually assess the model's accuracy and reliability in predicting battery performance across different states of charge and temperature ranges.



(a) OCV Interpolation based on Equation 4



(b) OCV raw data

Figure 2: OCV General Model comparison against experimental OCV data

It can be confirmed from Figure 2 that the model effectively generalizes the raw data. Essentially, the general model interpolates the gaps in the plotted raw data, thereby providing information on the remaining battery capacity at any state of charge and temperature.

4 Conclusions

This study successfully developed a predictive model for voltage based on the State of Charge (SoC) and temperature. Through a detailed analysis of experimental data, a representative function was proposed, with temperature-dependent coefficients validated to enhance the model's adaptability. The model achieved a fit of over 90% accuracy compared to experimental data, confirming its reliability for practical use in battery management systems (BMS).

The results demonstrate that the arc-tangent hyperbolic function effectively captures the voltage-SoC relationship, while the sigmoid function provides a robust representation of temperature-dependent variations in the coefficients. The validated model allows for precise voltage predictions under varying SoC and temperature conditions, supporting more accurate battery performance simulations.

This model plays a crucial role in thermal management systems, as it integrates well with cooling system simulations, helping to maintain optimal battery temperatures. By providing accurate voltage predictions, this model can be instrumental in optimizing energy efficiency and ensuring battery longevity in applications such as electric vehicles and renewable energy storage systems.

Additionally, the creation of an organized Excel sheet allows for further analysis and facilitates broader application of the model across different scenarios. Overall, this study contributes to enhancing battery simulations, improving both electrical and thermal management strategies.

5 Future Tasks

- Develop an AI model to predict the values and conduct a comparative analysis with the model presented in this report.
- Simulate the system to forecast the expected battery duration across various temperatures and voltages.

References

- [1] Jim Frost. Coefficient of variation in statistics. 2024. [Coefficient of Variation](#).
- [2] Zach Bobbitt. What is considered a good coefficient of variation? *Statology*, 2021. [Coefficient of Variation Indicator](#).

6 Appendix: Data Visualizations

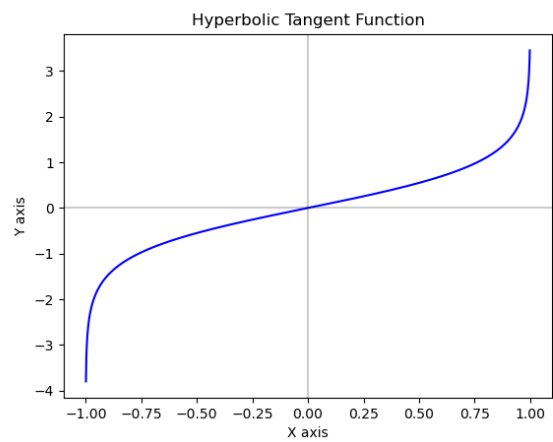


Figure 3: Arc-Tangent Hyperbolic Function

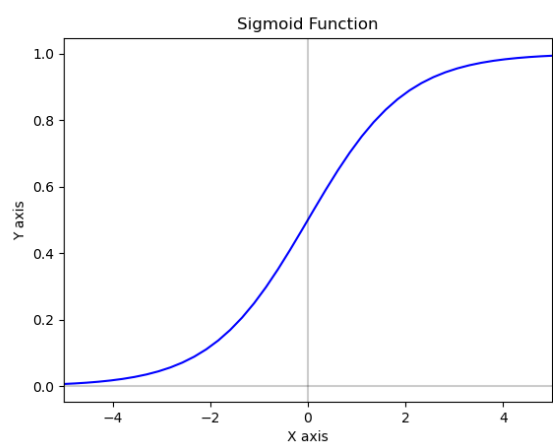
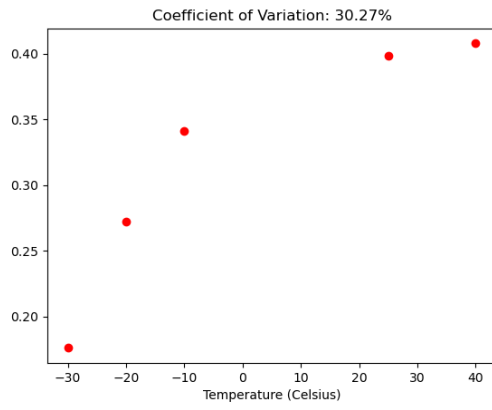
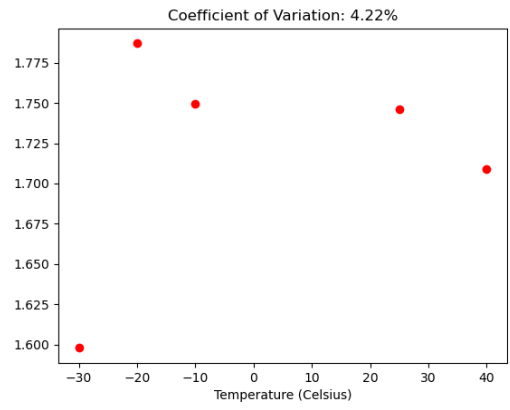


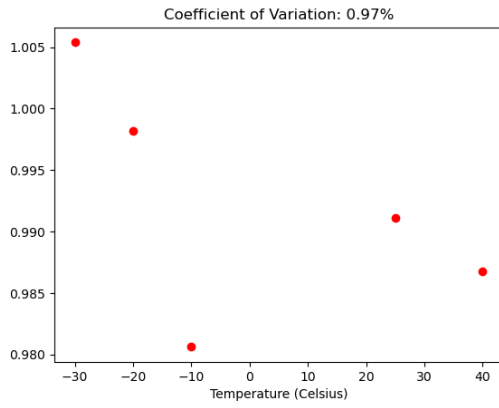
Figure 4: Sigmoid Function



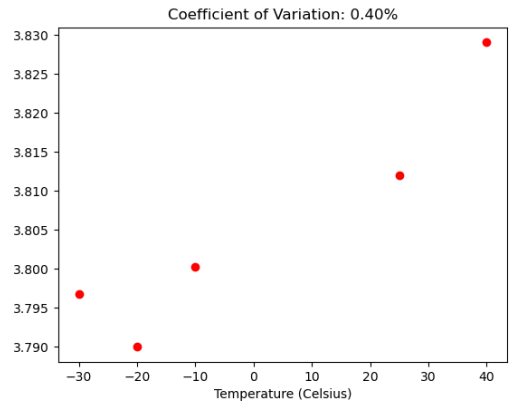
(a) Coefficient A



(b) Coefficient B

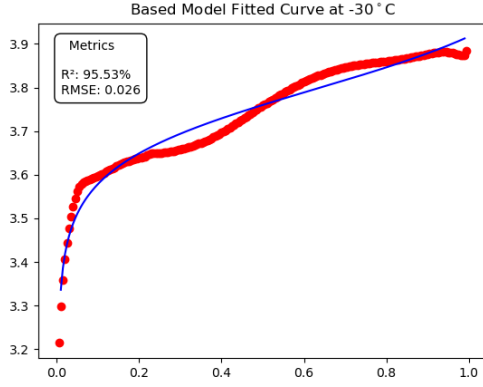


(c) Coefficient C

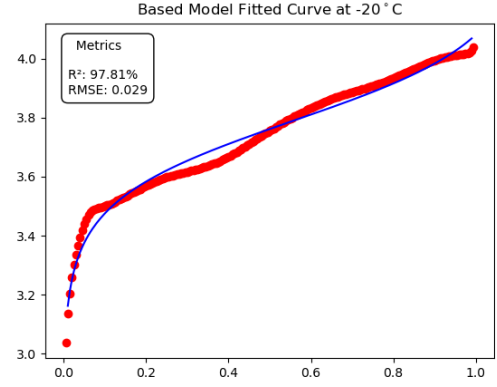


(d) Coefficient D

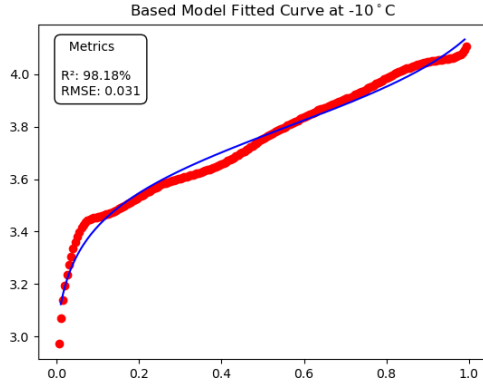
Figure 5: Variation of Coefficients in Equation 1 in terms of the Temperature



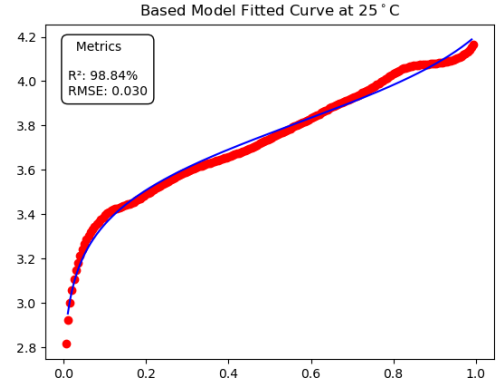
(a) Arc-Tangent Hyperbolic Base Model Performance for -30°C data



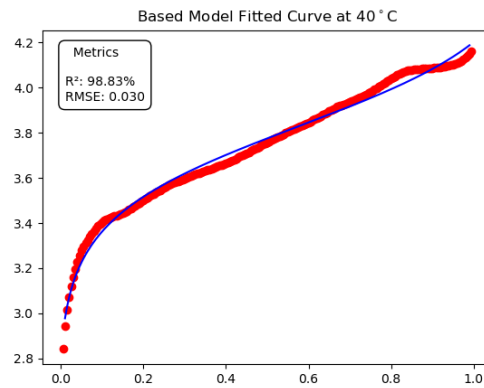
(b) Arc-Tangent Hyperbolic Base Model Performance for -20°C data



(c) Arc-Tangent Hyperbolic Model Performance for -10°C data

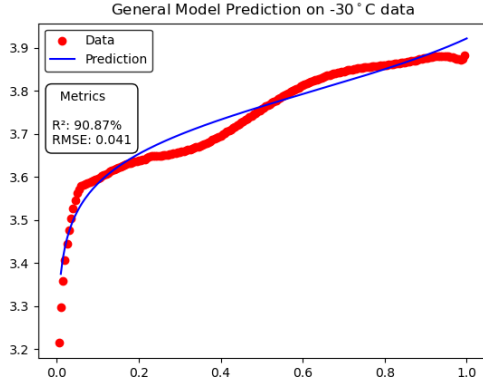


(d) Arc-Tangent Hyperbolic Base Model Performance for 25°C data

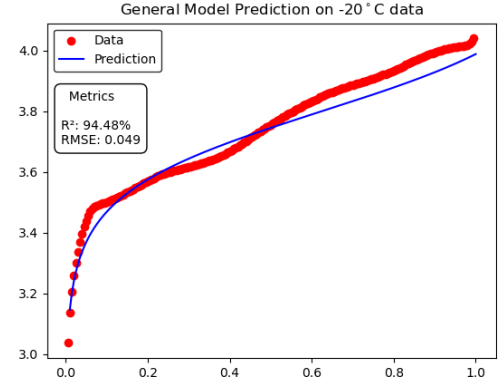


(e) Arc-Tangent Hyperbolic Base Model Performance for 40°C data

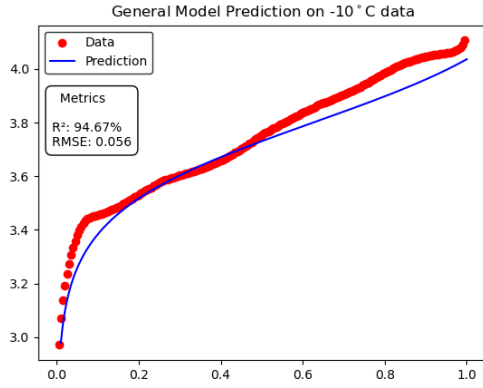
Figure 6: Base Model Performance against experimental data



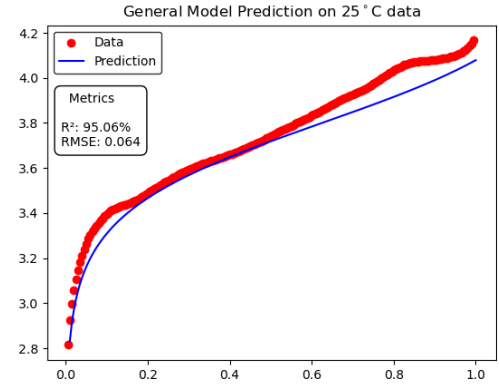
(a) General Model Performance for -30°C data



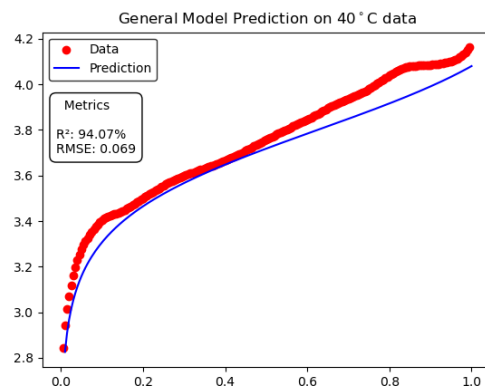
(b) General Model Performance for -20°C data



(c) General Model Performance for -10°C data



(d) General Model Performance for 25°C data



(e) General Model Performance for 40°C data

Figure 7: General Model Performance against experimental data