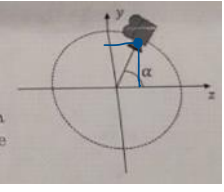
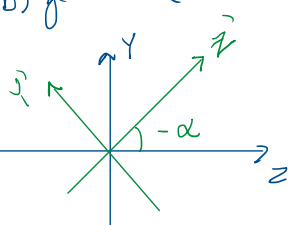


$$\textcircled{1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 1 \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

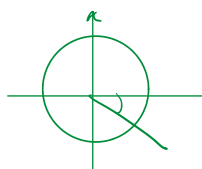
R.: a)

$\textcircled{2}$   
 a) 
 $P = (0, \sin(\alpha), \cos(\alpha))$   
 $gluLookAt(0, \sin(\alpha), \cos(\alpha), 0, 0, 0, 0, 1, 0)$   
 UP  
 Ponto para onde a câmara aponta

b)  $glRotate(-\alpha, 1, 0, 0)$   

 Regra da mão direita

$glTranslate(0, 0, 1)$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

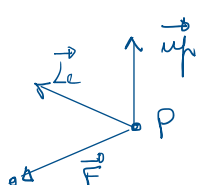


$\cos(-\alpha) = \cos(\alpha)$   
 $\sin(-\alpha) = -\sin(\alpha)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\textcircled{3} gluLookAt(p_1, p_2, p_3, l_1, l_2, l_3, u_1, u_2, u_3)$

$P(p_1, p_2, p_3) ; L(l_1, l_2, l_3) ; \vec{UP}(u_1, u_2, u_3)$



$\vec{F} = L - P$   
 $= (l_1 - p_1, l_2 - p_2, l_3 - p_3)$

$\vec{Le} = \frac{\vec{F}}{|\vec{F}|} \cdot \vec{UP}$



$$L_e = F \cdot up$$

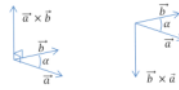


Figure 3.10 - Cross product

(se o F estiver a apontar para ti)  $F \times up = Left$   
e  $up \times F = Right$

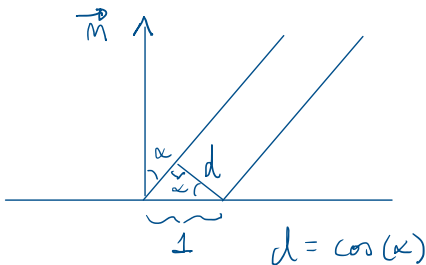
$$\vec{upReal} = \frac{\vec{F} \times \vec{L_e}}{\|\vec{F} \times \vec{L_e}\|} \rightarrow \text{Normalizar}$$

$$P' = P + \vec{upReal} \quad ; \quad L' = L + \vec{upReal}$$

$$glLookAt(P', L', U)$$

mesmo  $u$   
inicial

#### ④ Iluminação Difusa



Lambert

$$I_d = L_d \cdot K_d \cdot \cos(\alpha)$$

Atenuação  $\rightarrow I_{difund} = \underbrace{I_{at}}_{\rightarrow \frac{1}{d^2}} \times I_d$



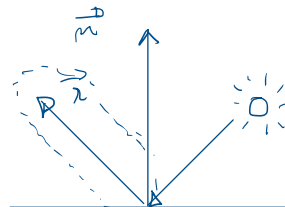
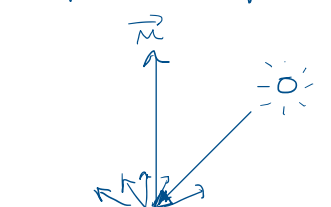
$A_1 \neq A_2$   
Área da esfera  $4\pi r^2$

#### Iluminação Ambiente

$$I_a = L_a K_a$$

$$\Rightarrow I = I_a + I_d$$

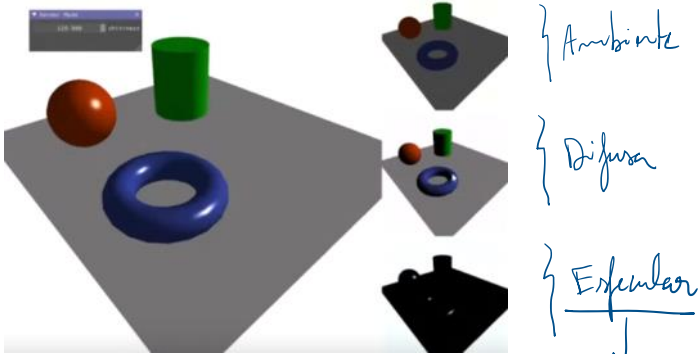
#### Componente especular $\rightarrow$ "espelho"



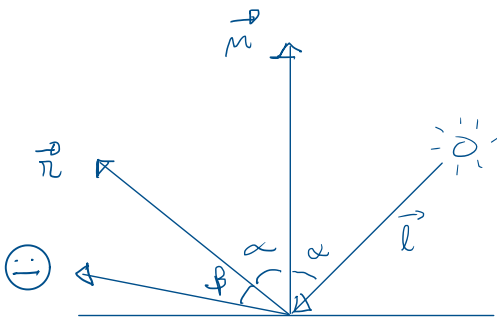
Superfície difusa  
perfeita

Superfície  
especular

Um objeto tem na  
combinação destas  
duas componentes

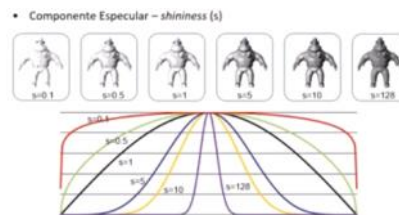
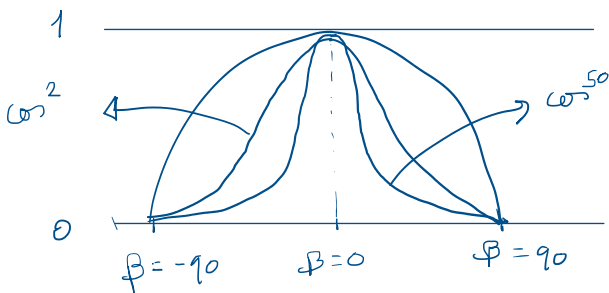


Dependem da  
posição do  
utilizador



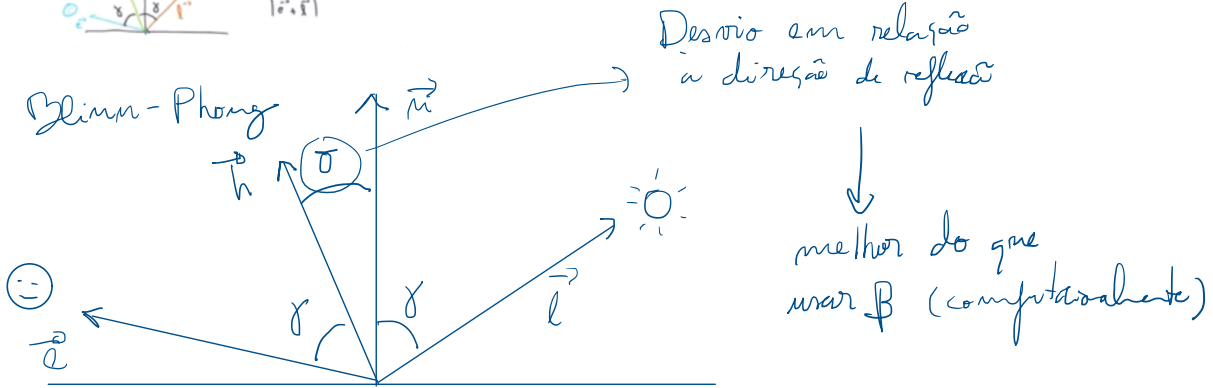
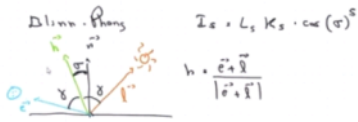
$$I_s = I_s \cdot k_s \cdot (\cos(\beta))^s$$

↓  
[0, 128]



Maior coeficiente  
⇒ Menor probabilidade  
de se ver a reflexão  
especular

$$\Rightarrow I = I_a k_a + f_{att} (L_d k_d \cos(\alpha)) + L_s k_s \cos^s(\beta)$$



$$h = \frac{\vec{e} + \vec{l}}{|\vec{e} + \vec{l}|}$$

⑤ Curva Bézier - grau 3

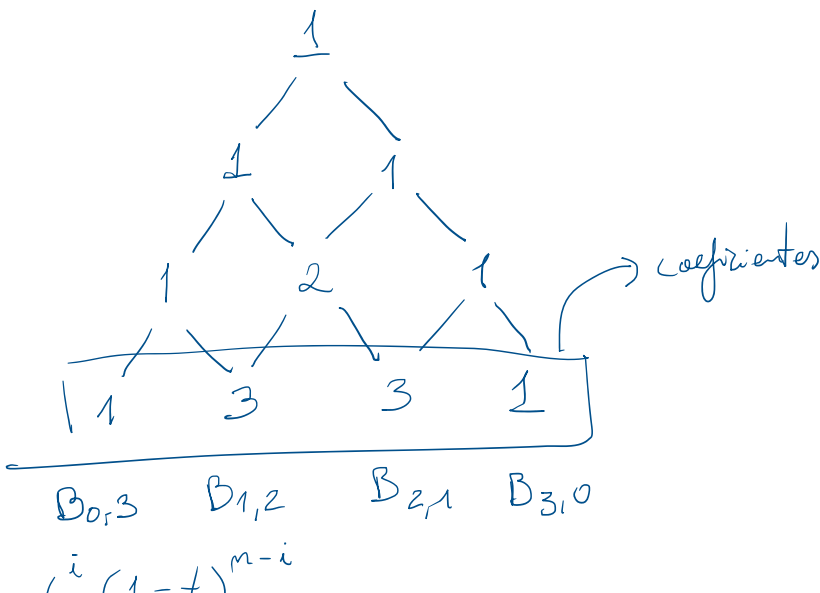
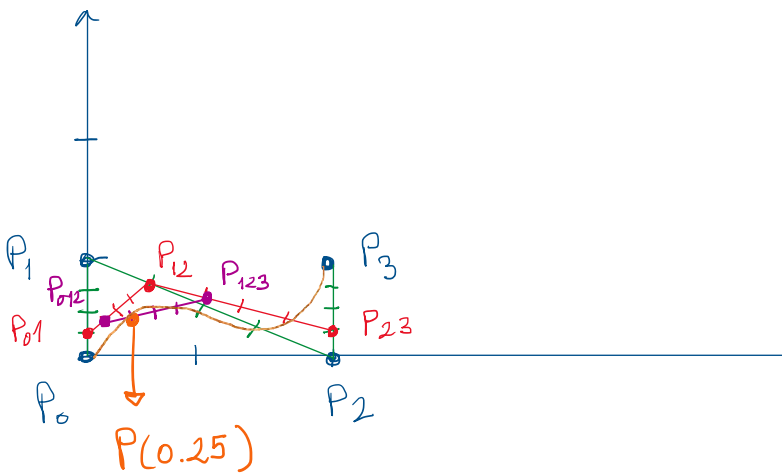
$$P_0 = (0,0)$$

$$P_2 = (2,0)$$

$$P_1 = (0,1)$$

$$P_3 = (2,1)$$

$$t = 0,25$$



00,3 11,2 10,1 01,0

$$C_i t^i (1-t)^{n-i}$$

$$1 \times 1 \times (1-t)^3_{P_0} + 3 \times t \times (1-t)^2_{P_1} + 3 \times t^2 \times (1-t)_{P_2} + 1 \times t^3 \times 1 \times P_3$$

$$0.42 P_0 + 0.42 P_1 + 0.14 P_2 + 0.02 P_3$$

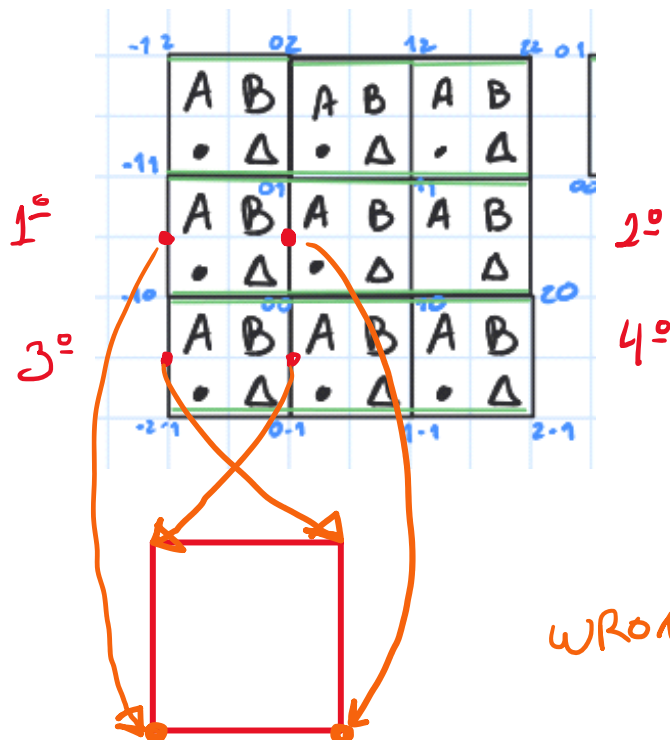
$$(0,0) + (0,0.42) + (0.28,0) + (0.04,0.02)$$

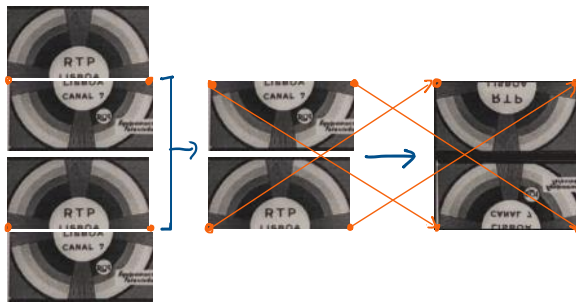
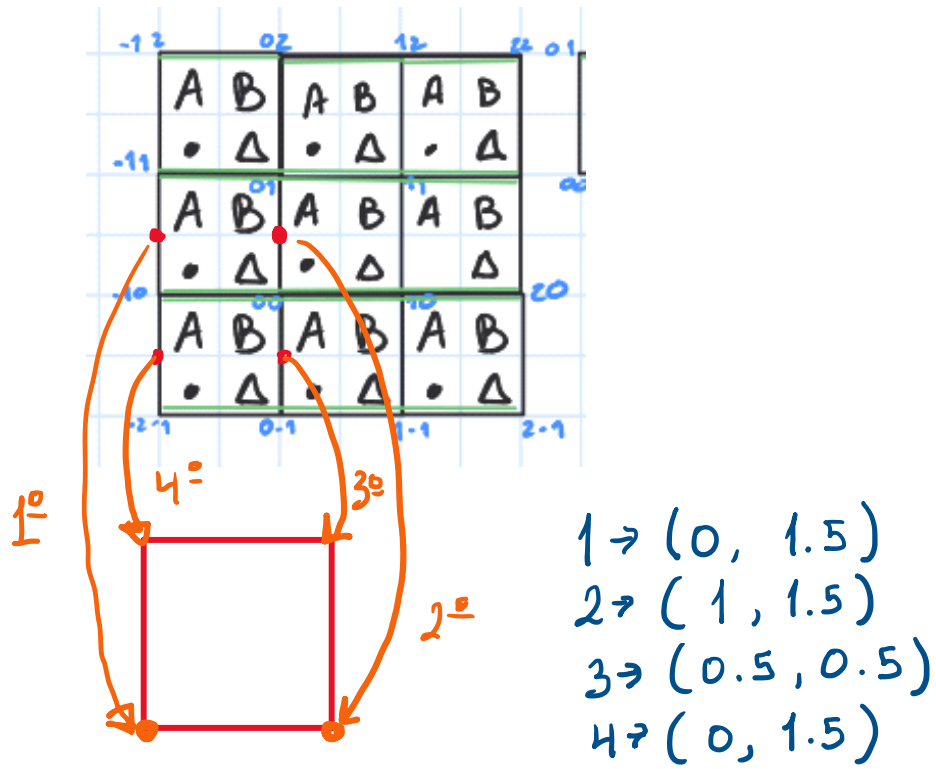
$$= (0.32, 0.44) \approx (0.3, 0.4)$$

6

01 4°	A	B	11 3°
00 1°	▽	△	10 2°

▽	△
△	▽





7

Como escolher os planos?

- 2 graus de liberdade
  - o orientação
  - o deslocamento

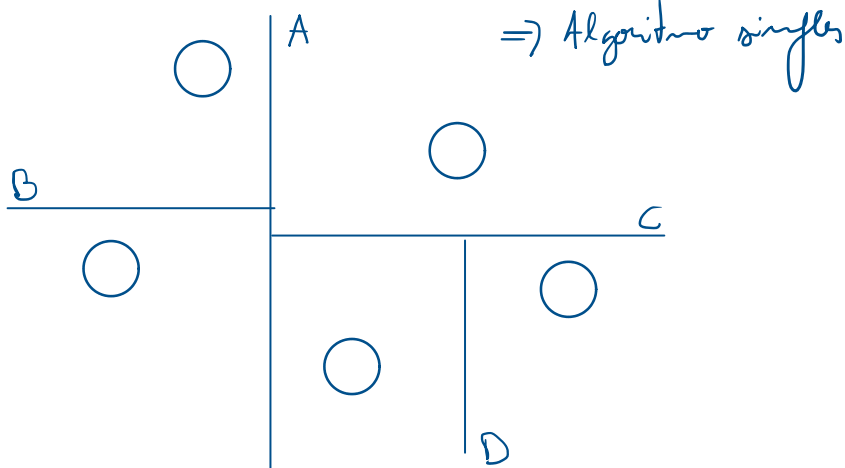
⇒ determinar qual é o melhor plano com base na cena

⇒ se a cena for dinamica, fica ainda mais complexo

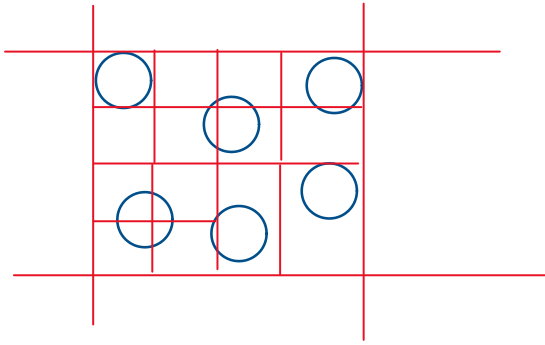
ou seja, há demasiados graus de liberdade

Uma das alternativas: k-D trees

- os planos são perpendiculares aos eixos
- raiz na origem (por exemplo)

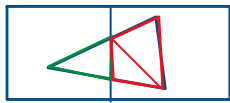


Ou Quadrees (2D) < = > Octrees (3D)



Quando parar a divisão?

- quando o número de triângulos for menor do que um determinado valor,  $t$
- quando o volume da célula for menor do que um determinado valor,  $c$
- quando a profundidade da árvore for maior do que um determinado valor,  $p$



A: Guardar as duas células como um nodo pai

- subutilização da placa gráfica

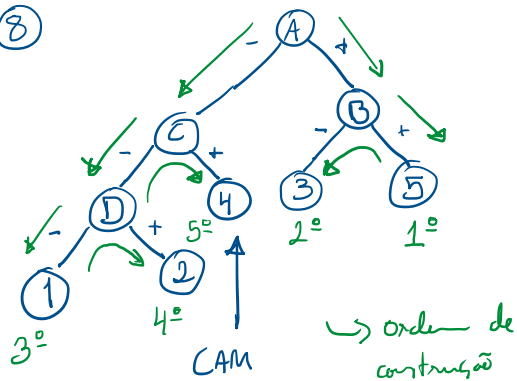
B: Partir o triângulo em 3 triângulos, e guardar cada um na célula correspondente

- processar mais vértices

C: Duplicar o triângulo  $\leftarrow$  mais benefícios em termos de CG

- a maior parte dos triângulos não atravessam fronteiras das células
- gasta-se mais memória (mas só em termos de índices)
- se e quando for desenhar o segundo, verifica que já foi desenhado outro nessa profundidade e os pixels não são processados

⑧



Ordem dos pixels a escrever

- 1° 4
- 2° 2
- 3° 1
- 4° 3
- 5° 5