Preparação para o teste

24 de outubro de 2023 18:09

$undistr : A\times B + A\times C \rightarrow A\times(B+C)$

$undistr = [id\times i_1, id\times i_2]$

$distr : A\times(B+C) \rightarrow A\times B + A\times C$

$distr \cdot (id \times coswap)\cdot undistr$

$= coswap = [i_2, i_1]$

$\alpha \cdot h = K \equiv h = \alpha^{\circ}\cdot K$

$h \cdot \alpha = K \equiv h = K \cdot \alpha^{\circ}$

$\overset{\alpha}{distr} \overset{\circ}{\cdot} \underbrace{(id\times coswap)\cdot undistr}_{}^{h} = \overset{K}{coswap}$

$\Rightarrow h = \alpha^{\circ}\cdot K \quad (34)$

$(id\times coswap)\cdot undistr = \underline{undistr \cdot coswap}$

$[id\times i_1, id\times i_2]\cdot[i_2, i_1]$ Ref de undistr e coswap

Fusão-+ (20)

$[[id\times i_1, id\times i_2]\cdot i_2, [id\times i_1, id\times i_2]\cdot i_1]$

(18) $\{ [f,g]\cdot i_1 = f$

$[id\times i_2, id\times i_1]$

$$\begin{tikzcd}
(A \cdot \\
id\times[i_1,i_2]) \arrow[u] \\
A\times B \arrow[u, dash] \\
(id, A+B) \\
{[id\times i_1, id\times i_2]} \arrow[u] \\
A \arrow[u, dash]
\end{tikzcd}$$

(18) Ao contrário Descancelamento

$[id\times([i_2,i_1]\cdot i_1), id\times([i_2,i_1]\cdot i_2)]$

$[(id\cdot id)\times([i_2,i_1]\cdot i_1), (id\times id)\times([i_2,i_1]\cdot i_1)]$

(14) $[(id\times[i_2,i_1])\cdot(id\times i_1), (id\times[i_2,i_1])\cdot(id\cdot i_1)]$

(20) $(id\times[i_2,i_1])\cdot[id\times i_1, id\times i_2]$

def. $(id\times coswap)\cdot undistr$,

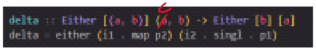
4. $\delta = [map\ \pi_2, singl\cdot\pi_1]$

$singl\ x = [x]$

$$\begin{tikzcd}
A \arrow[r, "singl"] \arrow[d, "f"'] & {[A]} \arrow[d, "h"] \\
A' \arrow[r, "singl"] & {[A']}
\end{tikzcd}$$

$singl\cdot f = h\cdot singl$

mais genérico



```
delta :: Either [(a, b)] (a, b) -> Either [b] [a]
delta = either (i1 . map p2) (i2 . singl . p1)
```

Etiqueta 1 : $map\ \pi_2$

Etiquet 2 : $singl\cdot\pi_1$; output do mesmo tipo pois não se usa i_1 ou i_2

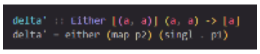
$[(a,b)] \rightarrow [b]$

$(b,c) \rightarrow [b]$

ou

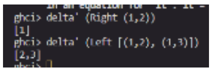
$[(a,a)] \rightarrow [a]$

$(a,a) \rightarrow [a]$



```
delta' :: Either [(a, a)] (a, a) -> [a]
delta' = either (map p2) (singl . p1)
```

Exemplo de utilização:



```
    In an equation for `it': it =
ghci> delta' (Right (1,2))
[1]
ghci> delta' (Left [(1,2), (1,3)])
[2,3]
ghci> 
```

$$\begin{tikzcd}
(A\times A)^* + (A\times A) \arrow[r, "\delta"] \arrow[d, "(f\times f)^* + (f\times f)"'] & A^* \arrow[d, "f"] \\
(A'\times A')^* + (A'\times A') \arrow[r, "\delta"] & A'^*
\end{tikzcd}$$

$$\delta \circ (f\times f)^* + (f\times f) = f \cdot \delta$$

$$\begin{tikzcd}
(A\times B)^* + (B,C) \arrow[r, "\delta"] \arrow[d, "(f\times g)^* + (g\times h)"'] & B^* \arrow[d, "g^*"] \\
(A'\times B')^* + (B',C') \arrow[r, "\delta"] & B'^*
\end{tikzcd}$$

$$\boxed{\delta \cdot (f\times g)^* + (g\times h) = g^* \cdot \delta}$$

Propriedade grátis

Afinal é
melhor fazer isto
com tipos diferentes
já que é possível
ao contrário de
outras situações

⟹ MAIS
GENÉRICO!

PROVAR ANALITICAMENTE:

$$[\text{map } \pi_2, \text{singl} \cdot \pi_1] \circ (f\times g)^* + (g\times h) = g^* \circ [\text{map } \pi_2, \text{singl} \cdot \pi_1]$$

$$(22)\ [\text{map } \pi_2 \cdot (f\times g)^*, \text{singl} \cdot \pi_1 \circ (g\times h)] = [g^* \circ \text{map}\pi_2, g^* \cdot \text{singl} \cdot \pi_1]$$

$$\equiv \begin{cases} \text{map } \pi_2 \circ (f\times g)^* = g^* \circ \text{map } \pi_2 \\ \text{singl} \cdot \pi_1 \circ (g\times h) = g^* \cdot \text{singl} \circ \pi_1 \end{cases} = \begin{cases} \underline{(\pi_2)^* \circ (f\times g)^*} = g^* \cdot (\pi_2)^* \end{cases}$$

$$\equiv \begin{cases} \underline{(\pi_2)^* \circ (f\times g)^*} = (g \cdot \pi_2)^* \end{cases} = \begin{cases} (\underline{\pi_2 \circ (f\times g)})^* \overset{(13)}{=} (g \cdot \pi_2)^* \end{cases}$$

$$\equiv \begin{cases} (g \cdot \pi_2)^* = (g \cdot \pi_2)^* \\ \text{singl} \circ g \cdot \pi_1 = \underbrace{g^* \circ \text{singl}}_{\text{propriedade grátis de singl}} \circ \pi_1 \end{cases}$$

$$\begin{tikzcd}
A \arrow[r, "\text{singl}"] \arrow[d, "g"'] & {[A]} \arrow[d, "g^*"] \\
A' \arrow[r, "\text{singl}"] & {[A']}
\end{tikzcd}$$

$$\text{singl} \circ g = g^* \circ \text{singl}$$

$$\equiv \begin{cases} (g \cdot \pi_2)^* = (g \circ \pi_2)^* \\ \text{singl} \circ g \circ \pi_1 = \text{singl} \circ g \circ \pi_1 \end{cases}$$

$$\equiv \text{TRUE}$$

$$\begin{tikzcd}
D & & D \\
C\times D \arrow[u, "\pi_2"] & & B \arrow[u, "g"] \\
A\times B \arrow[u, "f\times g"] \arrow[uu, dashed, bend right, "g"'] & & A\times B \arrow[u, "\pi_2"]
\end{tikzcd}$$

Informação perdida devido a p2

A

~~⟨id, True → i1 (), i2⟩~~

$$\begin{tikzcd}
A\times B \\
A \arrow[u, "\langle f, g\rangle"]
\end{tikzcd}$$

$$\begin{tikzcd}
A+A & \overset{A\times1+A\times1}{3} \arrow[l, "\overset{\pi_1+\pi_1}{1+2}"'] & A\times(1+1) \arrow[l, "\overset{\text{distr}}{4}"'] & A\times 2 \arrow[l, "id\times"'] & A \arrow[l, "\langle id, p\rangle"'] \arrow[llll, bend left, "p?"]
\end{tikzcd}$$

$$\begin{tikzcd}
A\times(1+1) & (A\times 2)\times(1+1) \arrow[l, "\neq"'] \\
A\times 2 \arrow[u, "id\times[\underline{True}, \underline{False}]"] & A\times 2 \arrow[u, "\langle id, f\rangle"]
\end{tikzcd}$$

$$\text{hasDup} : A^* \to 2$$

$$\text{hasDup } [\,] = \text{False}$$

$$\text{hasDup } (x:xs) = \underline{\text{elem}}\ (x, xs) \lor \text{hasDup } x$$

"or"

$$\lor :: \mathbb{B}\times\mathbb{B} \to \mathbb{B}$$

$$\hookrightarrow \text{elem} :: A\times A^* \to \mathbb{B}$$

$$\begin{tikzcd}
E & E^B \\
C\times D \arrow[u, "f"] & \\
A\times B \arrow[u, "g\times h"] & A \arrow[uu, "\overline{f\cdot(g\times h)}"']
\end{tikzcd}$$

$$\overline{f\cdot(g\times h)} = \overline{ap\cdot(id\times h)}\cdot\overline{f}\cdot g$$

$$= \overline{ap\circ(id\times h)}\circ\overline{f\cdot(g\times id)}$$

$$= \overline{ap\circ((id\circ id)\times(id\times h))}\cdot\overline{f\cdot(g\times id)}$$

$$= \overline{ap\circ(id\times id)\cdot(id\times h)}\circ\overline{f\cdot(g\times id)}$$

$$= \overline{\overline{id}\cdot(id\times h)}\circ\overline{f\cdot(g\times id)}$$

$$= \overline{id\times h}\cdot\overline{f\cdot(g\times id)}$$

$$\begin{tikzcd}
A\times C \arrow[r, "ap"] & D \\
(A\times B)^X \arrow[u, dash, "(id\times h)"] & D^B \\
Y \arrow[u, "\overline{f}"] & A^X \arrow[u, "\overline{ap\cdot(id\times h)}"'] \\
Z \arrow[u, "g"] & Y \arrow[u, "\overline{f}"] \\
 & Z \arrow[u, "g"]
\end{tikzcd}$$

?

$$\overline{ap\circ(id\times h)}\cdot\overline{f\circ(g\times id)}$$

$$\overline{ap\circ(id\times h)}\cdot f^A\cdot\overline{(g\times id)}$$

$$\overline{ap\cdot(id\times h)}\cdot\overline{f\cdot ap}\cdot\overline{(g\times id)}$$

$$\begin{tikzcd}
C^B \\
A \arrow[u]
\end{tikzcd}$$

$$\begin{tikzcd}
C^B & B \arrow[r, "ap"] & C \\
A\times B \arrow[u, "\overline{f}"] \arrow[ur, "id"'] \arrow[urr, "f"'] & &
\end{tikzcd}$$

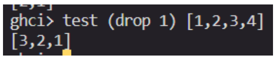
$$ap\cdot(id\times h)$$

$$\overline{ap\cdot(id\times h)}$$

$$= curry\ \$\ ap\circ(id\times h)$$

$$\begin{tikzcd}
A\times C \arrow[r, "ap"] & {} \\
A\times B \arrow[u, "id\times h"] &
\end{tikzcd}$$

Com h= reverse

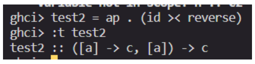


```
ghci> test (drop 1) [1,2,3,4]
[3,2,1]
```

$$test :: ([a]\to c)\to[a]\to c$$

mais genérico : $(A\to B)\to A\to B$;

Caso não fosse feito curry:



```
ghci> test2 = ap . (id >< reverse)
ghci> :t test2
test2 :: ([a] -> c, [a]) -> c
```

$$((A\to B), A)\to B$$

$$\begin{tikzcd}
B \\
B^A \arrow[u] \\
A \arrow[u]
\end{tikzcd}$$

$$\begin{tikzcd}
B & \text{vs} & B^A \\
(f, A) \arrow[u] & & f \arrow[u]
\end{tikzcd}$$

$$test\ f = f\cdot h$$

$$\begin{tikzcd}
(A\to B, A) \arrow[r, "ap"] & B \\
(A\to B)\times C \arrow[u, "id\times h"] \arrow[ur, "\alpha"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
B^A \\
(A\to B) \arrow[u, "\overline{ap\cdot(id\times h)}"']
\end{tikzcd}$$

$B\to A$

$$\begin{tikzcd}
D^C \\
D^C \arrow[u, "\overline{ap\cdot(id\times h)}"'] \\
B \arrow[u, "\overline{f}"'] \\
A \arrow[u, "g"']
\end{tikzcd}$$

$C\to D$

$$\overline{ap\cdot(id\times h)}\ f = f\cdot h$$

→ A função h dá um output que pode ser recebido pela função recebida como argumento

função recebida como
argumento

$\overline{g} \cdot f = \overline{g \cdot (f \times id)}$

$$\overbrace{\overline{ap \cdot (id \times h)}}^{\overline{g'}} \cdot \overbrace{\overline{f}}^{g'} \cdot g$$

$\underline{\underline{38}}$ $= \overline{ap \circ (id \times h) \cdot (\overline{f} \times id)} \cdot g$

14 $= \overline{ap \circ (id \circ \overline{f}) \times (h \circ id)} \cdot g$

(Troca) 1 $= \overline{ap \circ (\overline{f} \circ id) \times (id \circ h)} \circ g$

14 $= \overline{ap \circ (\overline{f} \times id) \cdot (id \times h)} \circ g$

36 $= \overline{f \circ (id \times h)} \circ g$

38 $= \overline{f \circ (id \times h) \cdot (g \times id)}$

$= \overline{f \circ (id \circ g) \times (h \times id)}$

$= \overline{f \circ (g \times h)}$

$\Leftrightarrow$

$$\begin{tikzcd} C \times D & C \times D \\ C \times B \arrow[u, "id \times h"] & A \times D \arrow[u, "f \times id"] \\ A \times B \arrow[u, "f \times id"] & A \times B \arrow[u, "id \times h"] \end{tikzcd}$$

$$\begin{tikzcd} D^C \\ B \arrow[u, "\overline{g}"] \\ A \arrow[u, "f"] \end{tikzcd} \qquad \begin{tikzcd} D \\ B \times C \arrow[u, "g"] \\ A \times C \arrow[u, "f \times id"] \end{tikzcd} \qquad \begin{tikzcd} D^C \\ A \arrow[u, "\overline{g \cdot (f \times id)}"] \end{tikzcd}$$

$undistr = [id \times i_1, id \times i_2]$

$$\begin{tikzcd} & A \times (B+C) & \\ (A \times B) \arrow[ur, "id \times i_1"] \arrow[r, "i_1"] & (A \times B) + (A \times C) \arrow[u, "undistr"] & (A \times C) \arrow[l, "i_2"'] \arrow[ul, "id \times i_2"'] \end{tikzcd}$$

$$\begin{tikzcd} (A \times B) + (A \times C) \arrow[r, "undistr"] \arrow[d, "(f \times g) + (f \times h)"] & A \times (B+C) \arrow[d, "f \times (g+h)"] \\ (A' \times B') + (A' \times C') \arrow[r, "undistr"] & A' \times (B' + C') \end{tikzcd}$$

$undistr \cdot (f \times g) + (f \times h) = f \times (g+h) \cdot undistr$!

$\left(f \times (g+h)\right) \circ [id \times i_1, id \times i_2]$

$= [(f \times (g+h)) \cdot (id \times i_1), (f \times (g+h)) \cdot (id \times i_2))$

$= [f \cdot id \times (g+h) \cdot i_1, f \cdot id \times (g+h) \cdot i_2]$

$= [(id \cdot f) \times (i_1 \cdot g), (id \circ f) \times (i_2 \circ h)]$

$= [(id \times i_1) \cdot (f \times g), (id \times i_2) \cdot (f \times h)]$

(22) $= [(id \times i_1), (id \times i_2)] \circ (f \times g + f \times h)$

$= undistr\ ((f+g) \times (f+h))$

$[g, h] \cdot (i + j) = [g \cdot i, h \cdot j]$

$(g+h) \cdot i_1 \neq g$

$$\begin{tikzcd} C+D & \cdots \\ A+B \arrow[u, "g+h"] & \cdots \arrow[u, "i_1"] \\ A \arrow[u, "i_1"] & A \arrow[u, "g"] \end{tikzcd}$$

⊜ $g \cdot i_1$

$$\begin{tikzcd} E \\ C+D \arrow[u, "{[f, g]}"] \\ A+B \arrow[u, "i+j"] \arrow[uu, bend left] \end{tikzcd}$$

$\Leftrightarrow [f \cdot i, g \cdot j]$

$f = \langle id \times \pi_1, \pi_2 \cdot \pi_2 \rangle$
$g = \langle \pi_1 \cdot \pi_1, \pi_2 \times id \rangle$

Um split é
execução em paralelo,
dá um produto/ tuplo

$$\begin{tikzcd} A \arrow[d, "\pi_1"'] & A \times (B \times C) \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & (B \times C) \arrow[d, "\pi_2"] \\ {} & (A \times B) \times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] \arrow[u, "g"] \arrow[ul, "\pi_1 \cdot \pi_1"] \arrow[ur, "\pi_2 \times id"'] & {} \end{tikzcd}$$

O split tem as
setas invertidas
em relação ao

$$\begin{tikzcd}
{} \arrow[d, "\pi_1"'] \arrow[dr, dash, "\pi_1\cdot\pi_1"] & {} \arrow[d, dash, "g"] & {} \arrow[d, "\pi_2"] \arrow[dl, dash, "\pi_2\times id"'] \\
A\times B & (A\times B)\times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & C \\
& A\times(B\times C) \arrow[ul, "id\times\pi_1"] \arrow[u, "f"] \arrow[ur, "\pi_2\cdot\pi_2"'] &
\end{tikzcd}$$

setas invertidos
em relação ao
either [ ]

$$\begin{tikzcd}
A & A\times(B\times C) \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & (B\times C) \\
& (A\times B)\times C \arrow[ul, "\pi_1\cdot\pi_1"] \arrow[u, "g"] \arrow[ur, "\pi_2\times id"'] &
\end{tikzcd}$$

$$\langle id\times\pi_1 , \pi_2\cdot\pi_2\rangle \circ \langle \pi_1\cdot\pi_1 , \pi_2\times id\rangle = id$$

$$\Leftrightarrow \langle id\times\pi_1 \circ \langle\pi_1\cdot\pi_1, \pi_2\times id\rangle, \pi_2\cdot\pi_2\circ\langle\pi_1\cdot\pi_1, \pi_2\times id\rangle\rangle = id$$

$$\Leftrightarrow \begin{cases} \pi_1\circ id = (id\times\pi_1)\circ\langle\pi_1\cdot\pi_1 , \pi_2\times id\rangle & (12) \\ \pi_2\cdot id = \pi_2\cdot\pi_2 \circ \langle\pi_1\cdot\pi_1 , \pi_2\times id\rangle \end{cases}$$

$$\Leftrightarrow \begin{cases} \pi_1 = \langle id\circ\pi_1\cdot\pi_1 , \underset{=\pi_2\circ\pi_1}{\pi_1\cdot\pi_2\times id}\rangle \\ \pi_2 = \underline{\qquad\qquad} \end{cases}$$

$$\Leftrightarrow \begin{cases} \pi_1\circ\pi_1 = \pi_1\cdot\pi_1 \\ \pi_2\cdot\pi_1 = \pi_2\circ\pi_1 \\ \pi_2 = \pi_2\cdot\pi_2\cdot\langle\pi_1\cdot\pi_1, \pi_2\times id\rangle \end{cases}$$

$$\Leftrightarrow \begin{cases} \pi_1\circ\pi_1 = \pi_1\cdot\pi_1 \\ \pi_2\cdot\pi_1 = \pi_2\cdot\pi_1 \\ \pi_2 = \pi_2\circ(\pi_2\times id) = id\cdot\pi_2 = \pi_2 \end{cases}$$

$\Leftrightarrow$ TRUE //

$$\pi_1 = g\circ\pi_2\cdot\pi_2$$
$$\pi_2 = g\cdot\pi_2\cdot\pi_1$$

$$(id\times\pi_2)\circ\langle id\times\pi_2 , id\times\pi_1\rangle\cdot g = id$$

$$= \langle (id\times\pi_2)\circ(id\times\pi_2)\circ g , (id\times\pi_2)\cdot(id\times\pi_1)\cdot g\rangle = id$$

$$= \langle (id\circ id)\times(\pi_2\circ\pi_2)\circ g , (id\cdot id)\times(\pi_2\circ\pi_1)\cdot g\rangle = id$$

$$= \langle \pi_2\circ\pi_2 , \pi_2\circ\pi_1\rangle\circ g = id$$

$$= \langle \pi_2\circ\pi_2\circ g, \pi_2\cdot\pi_1\circ g\rangle = id$$

$$\begin{tikzcd}
C\times B \\
A\times(B\times C) \arrow[u, "{\langle\pi_1\cdot\pi_2 , \pi_2\cdot\pi_1\rangle}"]
\end{tikzcd}$$

$$(id\times\pi_2)\circ\langle id\times\pi_2, id\times\pi_1\rangle\circ g = id$$

⑪

$$= \langle id\circ(id\times\pi_2), \pi_2\circ(id\times\pi_1)\rangle\circ g = id$$

$$= \langle id\times\pi_2\circ g, \pi_1\circ\pi_2\cdot g\rangle = id$$

$$= \begin{cases} \pi_1 = (id\times\pi_2)\circ g \\ \pi_2 = \pi_1\circ\pi_2\circ g \end{cases}$$

$$\begin{tikzcd}
(A\times C)\times B \\
(A\times C)\times(A\times B) \arrow[u, "(id\times\pi_2)"'] \\
A\times(B\times C) \arrow[u, "A\times(C\times B)"] \\
(A\times C)\times B \arrow[u, "g"']
\end{tikzcd}$$

$$\langle id\times\pi_2, \pi_1\cdot\pi_2\rangle$$

???

$$\begin{tikzcd}
((A\times B)\times D)\times B \\
(A\times B)\times(C\times D) \arrow[u]
\end{tikzcd}$$

$$(id\times\pi_2)\circ\langle id\times\pi_2 , id\times\pi_1\rangle\circ g = id$$

$$\langle id \cdot (id \times \pi_2), \pi_2 \cdot (id \times \pi_1) \rangle$$

$$= \langle id \times \pi_2 , \pi_1 \cdot \pi_2 \rangle \cdot g = id$$ ISOMORFISMO

$$\begin{tikzcd}
(A \times B) \times D & ((A\times B)\times D)\times C \arrow[l, "\pi_1"'] \arrow[r, "\pi_2"] & C \\
 & (A \times B) \times (C \times D) \arrow[ul] \arrow[u, "\langle id \times \pi_2 , \pi_1 \cdot \pi_2 \rangle"] \arrow[ur, "\pi_1 \cdot \pi_2"'] &
\end{tikzcd}$$

$$g :: ((A \times B) \times D) \times B \longrightarrow (A \times B) \times (D \times D)$$

———— // ————

$$p \rightarrow (q \rightarrow a, b), b = (p \wedge q) \rightarrow a, b$$

$$(p \wedge q)? = p \rightarrow q?, i_2$$

$$[q \rightarrow a, b, b] \cdot p? = [[a,b] \cdot q?, b \cdot id] \cdot p?$$

$$= [[a,b], b] \cdot (q? + id) \cdot p?$$

$$= [[a,b], b] \cdot [i_1 \cdot q?, i_2] \cdot p?$$

$$= [[[a,b], b] \cdot i_1 \cdot q?, [[a,b],b] \cdot i_2] \cdot p?$$

$$= [b \cdot p?, [a,b]] \cdot p?$$

$$= p \rightarrow b \cdot q , [a,b] \quad \times \qquad p? =$$

$$p \rightarrow (q \rightarrow a, b), b$$

$$[q \rightarrow a, b, b] \cdot p?$$

$$[[a,b] \cdot q?, b] \cdot p?$$

$$[[a,b] \cdot q?, b \cdot id] \cdot p?$$

$$[[a,b], b] \cdot (q? + id) \cdot p?$$

$$[[a,b], b] \cdot [i_1 \cdot q?, id \times i_2] \cdot p?$$

$$[[a,b], b] \cdot [i_1, id] \cdot (q? + i_2) \cdot p?$$

$$[[[a,b],b] \cdot i_1, [[a,b],b]] \cdot (q? + i_2) \cdot p?$$

$$[b, [[a,b],b]] \cdot (q? + i_2) \cdot p?$$

$$[b \cdot q?, b] \cdot p?$$???? TENTEI

$$p \rightarrow b \cdot q?, b$$

$[q \to a, b \,;\, b] \cdot p?$

$[[a,b] \circ q?, b] \circ p?$

$[[a,b] \circ q, id \times b] \cdot p? = [[a,b], id] \circ (q? + b) \cdot p?$

$= \quad \times \qquad\qquad [i_1 \cdot q?, i_2 \circ b]\, p?$

$(p \cdot q)? = p \longrightarrow q?, i_2$

$(p \wedge q) \to a, b$ $\Leftarrow$ AO CONTRÁRIO

$[a,b] \cdot (p \wedge q)?$

$= [a,b] \cdot (p \to q?, i_2)$

$= [a,b] \cdot [q?, i_2] \cdot p?$

$= [[a,b] \circ q?, [a,b] \cdot i_2] \cdot p?$

$= [q \to a, b \,;\, b] \cdot p?$

$= p \to (q \to a, b), b$ //

$\overline{\overline{f \cdot (g \times h)}} = \overline{f \circ ((g \circ id) \times (h \circ i}$

$= \overline{f \cdot (id \times h)} \circ g$