

# Aula PL #04

10 de outubro de 2023 17:17

## Ficha 4

$$[\langle f, g \rangle, \langle h, k \rangle] = \langle [f, h], [g, k] \rangle$$

(1)

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]}^{k?} = [f, h] \\ \pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [g, k] \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \left\{ \begin{array}{l} (\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_1 = f \\ (\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_2 = h \end{array} \right. \\ \left\{ \begin{array}{l} (\pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_1 = g \\ (\pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_2 = k \end{array} \right. \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\left\{ \begin{array}{l} \pi_1 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_1) = f \\ \pi_2 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_1) = g \end{array} \right.}^k \\ \left\{ \begin{array}{l} \pi_1 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_2) = h \\ \pi_2 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_2) = k \end{array} \right. \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} [\langle f, g \rangle, \langle h, k \rangle] \cdot i_1 = \langle f, g \rangle \\ [\langle f, g \rangle, \langle h, k \rangle] \cdot i_2 = \langle h, k \rangle \end{array} \right.$$

$$\Rightarrow [\langle f, g \rangle, \langle h, k \rangle] = [\langle f, g \rangle, \langle h, k \rangle]$$

$\Rightarrow$  TRUE

(00)

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]}^{k?} = [f, h] \\ \pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [g, k] \end{array} \right.$$

$$\begin{array}{l} 20 \\ \Rightarrow \left\{ \begin{array}{l} [\pi_1 \cdot \langle f, g \rangle, \pi_1 \cdot \langle h, k \rangle] = [f, h] \\ [\pi_2 \cdot \langle f, g \rangle, \pi_2 \cdot \langle h, k \rangle] = [g, k] \end{array} \right. \end{array}$$

$$\Rightarrow \left\{ \begin{array}{l} [f, h] = [f, h] \\ [g, k] = [g, k] \end{array} \right.$$

$\Rightarrow$  TRUE

$$(2) \quad \kappa \cdot \text{undistl} = \text{id}$$

$$\Leftrightarrow \kappa \cdot [i_1 \times \text{id}, i_2 \times \text{id}] = \text{id}$$

$$\Leftrightarrow [\kappa \cdot i_1 \times \text{id}, \kappa \cdot i_2 \times \text{id}] = \text{id}$$

$$\Leftrightarrow \begin{cases} \text{id} \circ i_1 = \kappa \cdot (i_1 \times \text{id}) \\ \text{id} \circ i_2 = \kappa \cdot (i_2 \times \text{id}) \end{cases}$$

$$\Leftrightarrow \begin{cases} i_1 = \kappa \cdot \langle i_1 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle \\ i_2 = \kappa \cdot \langle i_2 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle \end{cases}$$

$$\Leftrightarrow \begin{cases} i_1 = \kappa \cdot \langle i_1 \cdot \pi_1, \pi_2 \rangle \\ i_2 = \kappa \cdot \langle i_2 \cdot \pi_1, \pi_2 \rangle \end{cases}$$

→ Penso que teria de introduzir variáveis para continuar

$$\begin{aligned} & \hookrightarrow [\langle (i_1 \times \text{id}) \cdot \pi_1, \langle (i_2 \times \text{id}) \cdot \pi_2 \rangle] \\ & = \langle [i_1 \cdot \pi_1, i_2 \cdot \pi_1], [i_2 \cdot \pi_2, \text{id} \cdot \pi_2] \rangle \quad \leftarrow \text{LEI DA TROCA} \\ & = \langle \pi_1 + \pi_1, [\pi_2, \pi_2] \rangle \end{aligned}$$

$$3. \quad f \rightarrow \kappa, \kappa = \kappa$$

$$(p? + p?) \cdot p? = (i_1 + i_2) \cdot p?$$

$$\langle (p \rightarrow f, h), (p \rightarrow g, i) \rangle = p \rightarrow \langle f, g \rangle, \langle h, i \rangle$$

$$\Leftrightarrow \begin{cases} \pi_1 \cdot (p \rightarrow \langle f, g \rangle, \langle h, i \rangle) = p \rightarrow f, h \\ \pi_2 \cdot (p \rightarrow \langle f, g \rangle, \langle h, i \rangle) = p \rightarrow g, i \end{cases}$$

$$\Leftrightarrow \begin{cases} p \rightarrow \pi_1 \cdot \langle f, g \rangle, \pi_1 \cdot \langle h, i \rangle = p \rightarrow f, h \\ p \rightarrow \pi_2 \cdot \langle f, g \rangle, \pi_2 \cdot \langle h, i \rangle = p \rightarrow g, i \end{cases}$$

$$\Leftrightarrow \begin{cases} p \rightarrow f, g = p \rightarrow f, g \\ p \rightarrow g, i = p \rightarrow g, i \end{cases} \quad \Leftrightarrow \text{TRUE}$$

$$\langle f, (p \rightarrow g, h) \rangle = p \rightarrow \langle f, g \rangle, \langle f, h \rangle$$

$$\Rightarrow \begin{cases} \pi_1 \cdot p \rightarrow \langle f, g \rangle, \langle f, h \rangle = f \\ \pi_2 \cdot p \rightarrow \langle f, g \rangle, \langle f, h \rangle = p \rightarrow g, h \end{cases}$$

$$\Rightarrow \begin{cases} p \rightarrow f, f = f \\ p \rightarrow g, h = p \rightarrow g, h \end{cases} \Rightarrow \text{TRUE}$$

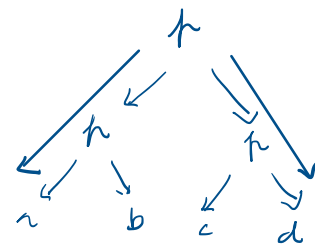
$$p \rightarrow (p \rightarrow a, b), (p \rightarrow c, d) = p \rightarrow a, d$$

$$\{ p?a = \begin{cases} p a \Rightarrow i_1 a \\ \neg(p a) \Rightarrow i_2 a \end{cases} \}$$

$$p \rightarrow [a, b] \cdot p?, [c, d] \cdot p?$$

$$\Rightarrow [[a, b] \cdot p?, [c, d] \cdot p?] \cdot p?$$

$$\Rightarrow \begin{cases} p \Rightarrow i_1 \cdot a \\ \neg p \Rightarrow i_2 \cdot d \end{cases} \Rightarrow p \rightarrow a, d$$



$$4. \text{ out} \cdot \text{in} = \text{id}$$

Universal-+

$$\text{out} \cdot [\underline{0}, \text{succ}] = \text{id}$$

$$\Rightarrow [\text{out} \cdot \underline{0}, \text{out} \cdot \text{succ}] = \text{id}$$

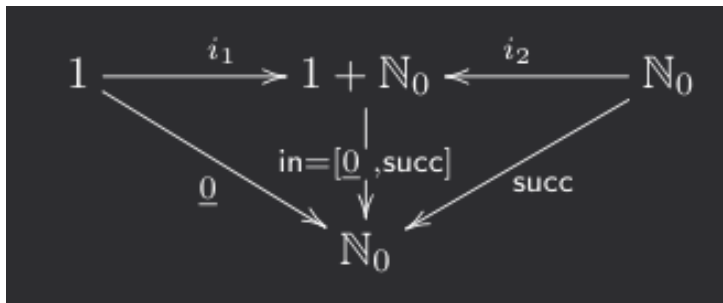
$$\textcircled{17} \Rightarrow \begin{cases} \text{out} \cdot \underline{0} = \text{id} \cdot i_1 \\ \text{out} \cdot \text{succ} = \text{id} \cdot i_2 \end{cases}$$

$$\Rightarrow \begin{cases} \forall x \text{ out} \cdot \underline{0} x = i_1 x \\ \forall x \text{ out} \cdot \text{succ} x = i_2 x \end{cases}$$

$$\Rightarrow \begin{cases} \forall n \text{ out } 0 = i_1 () \rightarrow x \text{ do} \\ \forall n \text{ out } x+1 = i_2 x \end{cases} \quad \begin{matrix} + i_1 \text{ do} \\ + i_2 \text{ do} \end{matrix}$$

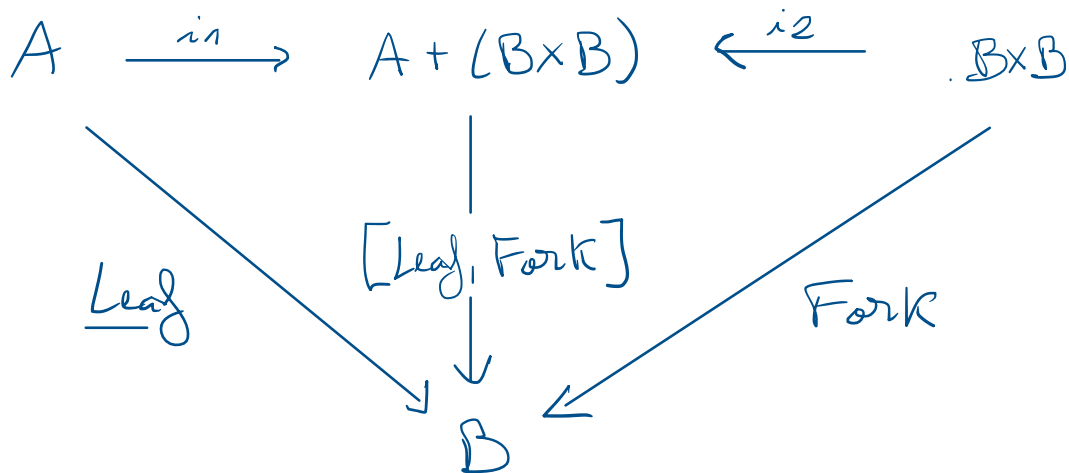
⑥

data LTree a = Leaf a | Fork (LTree a, LTree a)



$$\text{out}(\text{Leaf } a) = i_1 a$$

$$\text{out}(\text{Fork } (x, y)) = i_2 (x, y)$$



$$\text{in} = [\text{Leaf}, \text{Fork}]$$

$$\Rightarrow \text{out} \circ [\text{Leaf}, \text{Fork}] = \text{id}$$

$$\Rightarrow [\text{out} \circ \text{Leaf}, \text{out} \circ \text{Fork}] = \text{id}$$

$$\Rightarrow \begin{cases} \text{out} \circ \text{Leaf} = \text{id} \circ i_1 = i_1 \\ \text{out} \circ \text{Fork} = \text{id} \circ i_2 = i_2 \end{cases}$$

$$\begin{cases} \text{out} \circ \text{Leaf} = i_1 \circ i_1 = i_1 \\ \text{out} \circ \text{Fork} = i_2 \circ i_2 = i_2 \end{cases}$$

$$\Rightarrow \begin{cases} \forall a \quad \text{out} \circ \text{Leaf}(a) = i_1 a \\ \forall x, y \quad \text{out} \circ \text{Fork}(x, y) = i_2(x, y) \end{cases}$$

$$\Rightarrow \begin{aligned} \text{out}(\text{Leaf } a) &= i_1 a \\ \text{out}(\text{Fork}(x, y)) &= i_2(x, y) \end{aligned}$$

$$\textcircled{5} \quad [f \circ \underline{0}, f \circ \text{succ}] = [\underline{0} \circ \text{id}, \text{add} \circ \langle \text{odd}, f \rangle]$$

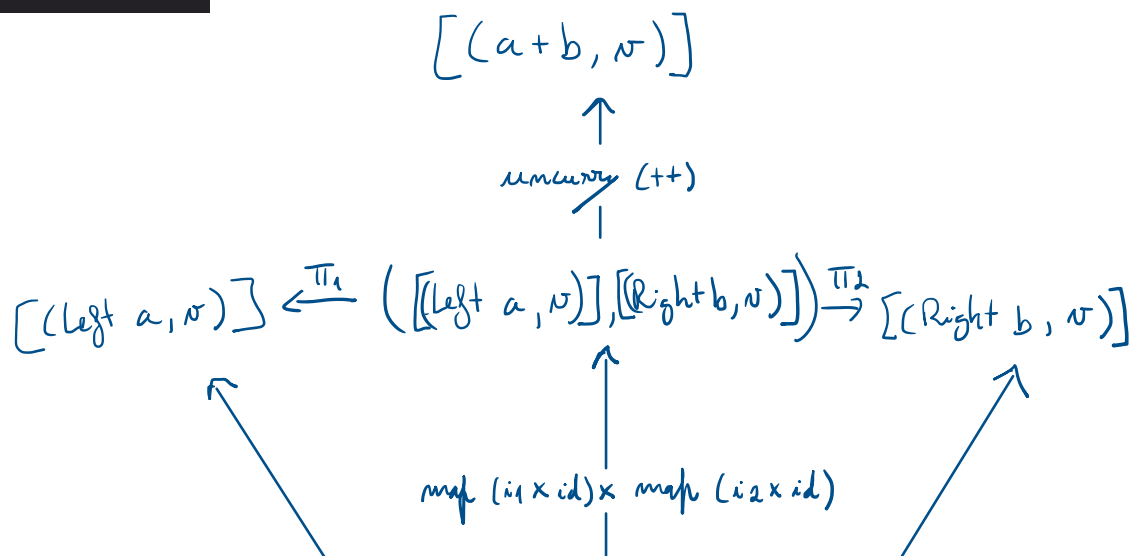
$$\Rightarrow \begin{cases} f \circ \underline{0} = \underline{0} \circ \text{id} \\ f \circ \text{succ} = \text{add} \circ \langle \text{odd}, f \rangle \end{cases}$$

$$\Rightarrow \begin{cases} f 0 = 0 \\ f(m+1) = \text{add}(\text{odd } m, f m) \end{cases}$$

$$\Rightarrow \begin{cases} f 0 = 0 \\ f(m+1) = (2m+1) + f m \end{cases}$$

```
-- ++ passa a receber um par de args,
glue :: ([a, v], [(b, v)]) -> [(Either a b, v)]
glue = uncurry (++) . (map (i1 >> id) >> map (i2 >> id))
```

```
ghci> uncurry (++) ([1,2], [3])
[1,2,3]
```



$$\begin{array}{c}
 \text{map } (i_1 \times \text{id}) \times \text{map } (i_2 \times \text{id}) \\
 | \\
 ([ (a, \sigma) ], [ (b, \sigma) ])
 \end{array}$$