

# Aula PL #04

10 de outubro de 2023 17:17

## Ficha 3

$$[\langle f, g \rangle, \langle h, k \rangle] = \langle [f, h], [g, k] \rangle$$

(1)

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]}^{k?} = [f, h] \\ \pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [g, k] \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \left\{ \begin{array}{l} (\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_1 = f \\ (\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_2 = h \end{array} \right. \\ \left\{ \begin{array}{l} (\pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_1 = g \\ (\pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle]) \cdot i_2 = k \end{array} \right. \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\pi_1 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_1)}^k = f \\ \overbrace{\pi_2 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_1)}^k = g \\ \pi_1 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_2) = h \\ \pi_2 \cdot ([\langle f, g \rangle, \langle h, k \rangle] \cdot i_2) = k \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} [\langle f, g \rangle, \langle h, k \rangle] \cdot i_1 = \langle f, g \rangle \\ [\langle f, g \rangle, \langle h, k \rangle] \cdot i_2 = \langle h, k \rangle \end{array} \right.$$

$$\Rightarrow [\langle f, g \rangle, \langle h, k \rangle] = [\langle f, g \rangle, \langle h, k \rangle]$$

$\Rightarrow$  TRUE

(00)

$$\Rightarrow \left\{ \begin{array}{l} \overbrace{\pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle]}^{k?} = [f, h] \\ \pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [g, k] \end{array} \right.$$

$$\Rightarrow \left\{ [\pi_1 \cdot \langle f, g \rangle, \pi_1 \cdot \langle h, k \rangle] = [f, h] \right.$$

$$\stackrel{20}{\Rightarrow} \begin{cases} [\pi_1 \cdot \langle f, g \rangle, \pi_1 \cdot \langle h, k \rangle] = [f, h] \\ [\pi_2 \cdot \langle f, g \rangle, \pi_2 \cdot \langle h, k \rangle] = [g, k] \end{cases}$$

$$\Rightarrow \begin{cases} [f, h] = [f, h] \\ [g, k] = [g, k] \end{cases}$$

$$\Rightarrow \text{TRUE}$$

$$(2) \quad \kappa \cdot \text{undistl} = \text{id}$$

$$\Rightarrow \kappa \cdot [i_1 \times \text{id}, i_2 \times \text{id}] = \text{id}$$

$$\Rightarrow [\kappa \cdot i_1 \times \text{id}, \kappa \cdot i_2 \times \text{id}] = \text{id}$$

$$\Rightarrow \begin{cases} \text{id} \cdot i_1 = \kappa \cdot (i_1 \times \text{id}) \\ \text{id} \cdot i_2 = \kappa \cdot (i_2 \times \text{id}) \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = \kappa \cdot \langle i_1 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle \\ i_2 = \kappa \cdot \langle i_2 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle \end{cases}$$

$$\Rightarrow \begin{cases} i_1 = \kappa \cdot \langle i_1 \cdot \pi_1, \pi_2 \rangle \\ i_2 = \kappa \cdot \langle i_2 \cdot \pi_1, \pi_2 \rangle \end{cases}$$

→ Penso que teria de introduzir variáveis para continuar

$$\begin{aligned} & \checkmark [\langle (i_1 \times \text{id}) \cdot \pi_1 \rangle, \langle (i_2 \times \text{id}) \cdot \pi_2 \rangle] \\ &= \langle [i_1 \cdot \pi_1, i_2 \cdot \pi_1], [i_2 \cdot \pi_2, \text{id} \cdot \pi_2] \rangle \quad \leftarrow \text{LEI DA TROCA} \\ &= \langle \pi_1 + \pi_1, [\pi_2, \pi_2] \rangle \end{aligned}$$

$$3. \quad f \rightarrow \kappa, \kappa = \kappa$$

$$(h? + h?) \cdot h? = (i_1 + i_2) \cdot h?$$

$$(p^? + p^?) \cdot p^? = (i_1 + i_2) \cdot p^?$$

$$\langle (p \rightarrow f, h), (p \rightarrow g, i) \rangle = p \rightarrow \langle f, g \rangle, \langle h, i \rangle$$

$$\hookrightarrow \begin{cases} \pi_1 \circ (p \rightarrow \langle f, g \rangle, \langle h, i \rangle) = p \rightarrow f, h \\ \pi_2 \circ (p \rightarrow \langle f, g \rangle, \langle h, i \rangle) = p \rightarrow g, i \end{cases}$$

$$\hookrightarrow \begin{cases} p \rightarrow \pi_1 \circ \langle f, g \rangle, \pi_1 \circ \langle h, i \rangle = p \rightarrow f, h \\ p \rightarrow \pi_2 \circ \langle f, g \rangle, \pi_2 \circ \langle h, i \rangle = p \rightarrow g, i \end{cases}$$

$$\hookrightarrow \begin{cases} p \rightarrow f, g = p \rightarrow f, g \\ p \rightarrow g, i = p \rightarrow g, i \end{cases} \quad \hookrightarrow \text{TRUE}$$

$$\langle f, (p \rightarrow g, h) \rangle = p \rightarrow \langle f, g \rangle, \langle f, h \rangle$$

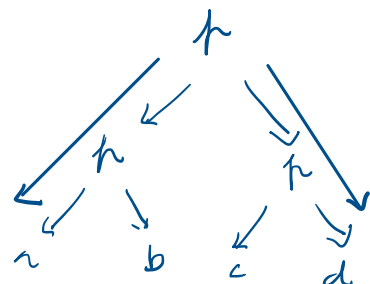
$$\hookrightarrow \begin{cases} \pi_1 \circ p \rightarrow \langle f, g \rangle, \langle f, h \rangle = f \\ \pi_2 \circ p \rightarrow \langle f, g \rangle, \langle f, h \rangle = p \rightarrow g, h \end{cases}$$

$$\hookrightarrow \begin{cases} p \rightarrow f, g = f \\ p \rightarrow g, h = p \rightarrow g, h \end{cases} \quad \hookrightarrow \text{TRUE}$$

$$p \rightarrow (p \rightarrow a, b), (p \rightarrow c, d) = p \rightarrow a, d$$

$$\{ p^? a = \begin{cases} p a \Rightarrow i_1 a \\ \neg(p a) \Rightarrow i_2 a \end{cases} \}$$

$$p \rightarrow [a, b] \cdot p^?, [c, d] \cdot p^?$$



$$p \rightarrow [a, b] \cdot p?, [c, d] \cdot p?$$

$$\Rightarrow [[a, b] \cdot p?, [c, d] \cdot p?] \cdot p?$$

$$\Rightarrow \begin{cases} p \Rightarrow i_1 \cdot a \\ \neg p \Rightarrow i_2 \cdot d \end{cases} \Rightarrow p \rightarrow a, d$$

$$4. \text{ out} \cdot \text{in} = \text{id}$$

Universal-+

$$\text{out} \cdot [\underline{0}, \text{succ}] = \text{id}$$

$$\Rightarrow [\text{out} \cdot \underline{0}, \text{out} \cdot \text{succ}] = \text{id}$$

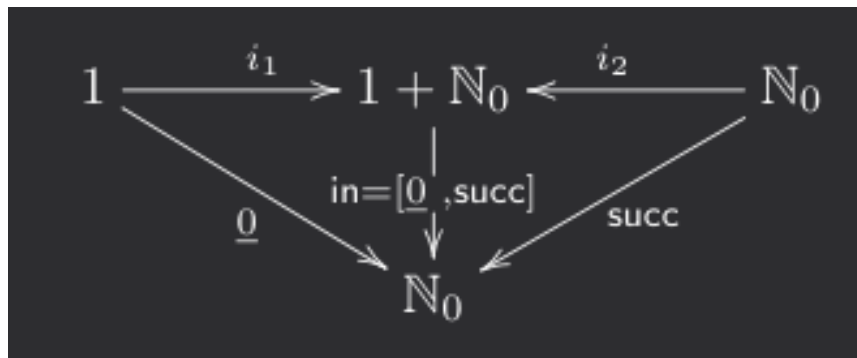
$$\textcircled{17} \quad \begin{cases} \text{out} \cdot \underline{0} = \text{id} \cdot i_1 \\ \text{out} \cdot \text{succ} = \text{id} \cdot i_2 \end{cases}$$

$$\Rightarrow \begin{cases} \forall x \text{ out} \cdot \underline{0} \ x = i_1 \ x \\ \forall x \text{ out} \cdot \text{succ} \ x = i_2 \ x \end{cases}$$

$$\Rightarrow \begin{cases} \forall x \text{ out} \ 0 = i_1 \ () \rightarrow x \text{ é do tipo } 1 \\ \forall x \text{ out} \ x+1 = i_2 \ x \end{cases}$$

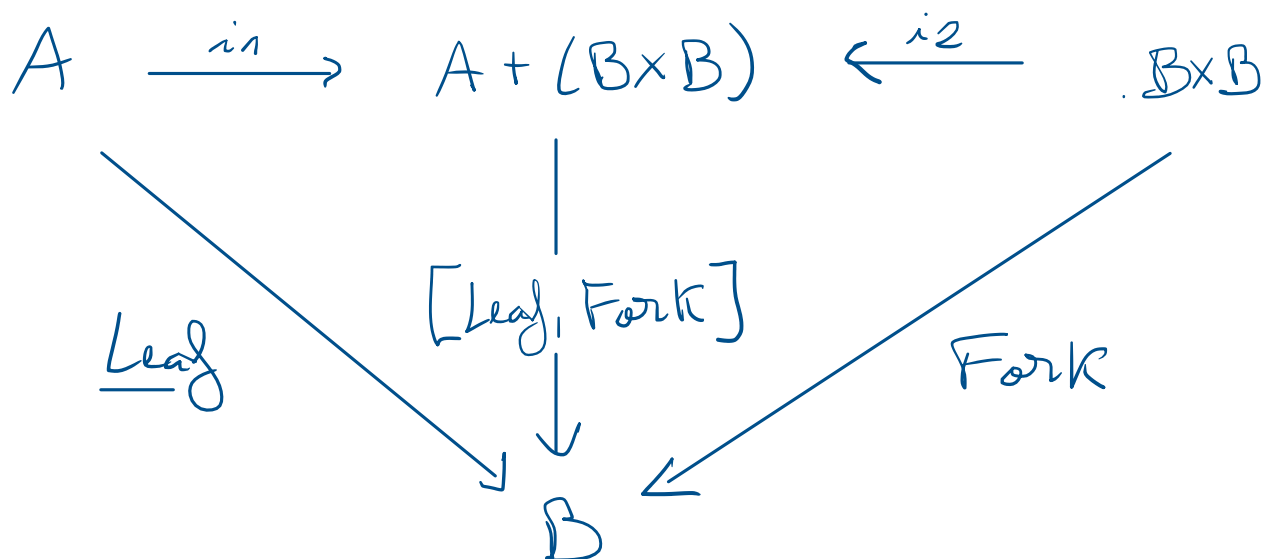
⑥

data LTree a = Leaf a | Fork (LTree a, LTree a)



$$\text{out}(\text{Leaf } a) = i_1 a$$

$$\text{out}(\text{Fork}(x, y)) = i_2(x, y)$$



$$\text{in} = [\text{Leaf}, \text{Fork}]$$

$$\Rightarrow \text{out} \circ [\text{Leaf}, \text{Fork}] = \text{id}$$

$$\Rightarrow [\text{out} \circ \text{Leaf}, \text{out} \circ \text{Fork}] = \text{id}$$

$$\Rightarrow \begin{cases} \text{out} \circ \text{Leaf} = \text{id} \circ i_1 = i_1 \\ \text{out} \circ \text{Fork} = \text{id} \circ i_2 = i_2 \end{cases}$$

$$\Rightarrow \begin{cases} \forall a & \text{out} \circ \text{Leaf}(a) = i_1 a \\ \forall x, y & \text{out} \circ \text{Fork}(x, y) = i_2(x, y) \end{cases}$$

$$\Rightarrow \begin{aligned} \text{out}(\text{Leaf } a) &= i_1 a \\ \text{out}(\text{Fork}(x, y)) &= i_2(x, y) \end{aligned}$$

$$\textcircled{5} \quad [f \circ \underline{0}, f \circ \text{succ}] = [\underline{0} \circ \text{id}, \text{add} \circ \langle \text{odd}, f \rangle]$$

$$\Rightarrow \begin{cases} f \circ \underline{0} = \underline{0} \circ \text{id} \\ f \circ \text{succ} = \text{add} \circ \langle \text{odd}, f \rangle \end{cases}$$

$$\Rightarrow \begin{cases} f 0 = 0 \\ f(m+1) = \text{add}(\text{odd } m, f m) \end{cases}$$

$$\Rightarrow \begin{cases} f 0 = 0 \\ f(m+1) = (2m+1) + f m \end{cases}$$