

$$\begin{aligned}
 (1) \quad f \cdot (g \times h) &= ap \cdot (id \times h) \cdot f \cdot g \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((ap \cdot (id \times h) \cdot f \cdot g) \times id) \quad \{35\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((ap \cdot (id \times h) \cdot f \cdot (g \times id)) \times id) \quad \{38\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((ap \cdot (id \times h) \cdot f \cdot (g \times id)) \times (id \cdot id)) \quad \{1\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((ap \cdot (id \times h) \times id) \cdot (f \cdot (g \times id) \times id)) \quad \{14\} \\
 \Rightarrow f \cdot (g \times h) &= (ap \cdot (ap \cdot (id \times h) \times id)) \cdot (f \cdot (g \times id) \times id) \quad \{Associative\} \\
 \Rightarrow f \cdot (g \times h) &= (ap \cdot (id \times h)) \cdot (f \cdot (g \times id) \times id) \quad \{36\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((id \times h) \cdot (f \cdot (g \times id) \times id)) \quad \{Associative\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot (f \cdot (g \times id) \times h) \quad \{12, 1(x2)\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((f \cdot g) \times h) \quad \{38\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((f \cdot g) \times (id \cdot h)) \quad \{1\} \\
 \Rightarrow f \cdot (g \times h) &= ap \cdot ((f \times id) \cdot (g \times h)) \quad \{14\} \\
 \Rightarrow f \cdot (g \times h) &= (ap \cdot (f \times id)) \cdot (g \times h) \quad \{Associative\} \\
 \Rightarrow f \cdot (g \times h) &= f \cdot (g \times h) \quad \{36\} \\
 &=
 \end{aligned}$$

Solução Alternativa

$$\begin{aligned}
 f \cdot (g \times h) &= ap \cdot (id \times h) \cdot f \cdot g \\
 &= ap \cdot (id \times h) \cdot (f \times id) \cdot g \quad \{38\} \\
 &= ap \cdot (f \times h) \cdot g \quad \{14, 1(x2)\} \\
 &= ap \cdot (f \times h) \cdot (g \times id) \quad \{38\} \\
 &= ap \cdot ((f \cdot g) \times (h \cdot id)) \quad \{14, 1\} \\
 &= ap \cdot (f \cdot (g \times id) \times h) \quad \{38\} \\
 &= ap \cdot ((f \cdot (g \times id) \times id) \cdot (id \times h)) \quad \{1(x2), 1\} \\
 &= f \cdot (g \times id) \cdot (id \times h) \quad \{Associative, 36\} \\
 &= f \cdot (g \times h) \quad \{12, 1(x2)\} \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \text{flip}(\text{flip } f) = f \\
 \Rightarrow & \overline{(\text{flip } f) \cdot \text{Swap}} = f \quad \{F2\} \\
 \Rightarrow & \overline{\overline{f \cdot \text{Swap}} \cdot \text{Swap}} = f \quad \{F2\} \\
 \Rightarrow & \overline{f \cdot \text{Swap} \cdot \text{Swap}} = f \quad \{\text{unerry} \cdot \text{erry} = \text{id}\} \\
 \Rightarrow & \overline{f} = f \quad \{\text{Swap} \cdot \text{Swap} = \text{id}\} \\
 \Rightarrow & f = f \quad \{\text{unerry} \cdot \text{erry} = \text{id}\}
 \end{aligned}$$

$$\begin{aligned}
 \text{flip } f \pi y &= \overline{f \cdot \text{Swap}} \pi y \quad \{F2\} \\
 &= (\overline{f \cdot \text{Swap}})(\pi, y) \quad \{85\} \\
 &= \overline{f}(\text{Swap}(\pi, y)) \quad \{73\} \\
 &= \overline{f}(y, \pi) \quad \{\text{Def Swap}\} \\
 &= f y \pi \quad \{86\}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & \text{junc} \cdot \text{unjunc} = \text{id} \\
 \Rightarrow & \forall K \mid (\text{junc} \cdot \text{unjunc}) K = \text{id } K \quad \{72\} \\
 \Rightarrow & \forall K \mid \text{junc}(\text{unjunc } K) = K \quad \{73, 74\} \\
 \Rightarrow & \forall K \mid \text{junc}(K \cdot i_1, K \cdot i_2) = K \quad \{F6 - \text{Def unjunc}\} \\
 \Rightarrow & \forall K \mid [K \cdot i_1, K \cdot i_2] = K \quad \{F6 - \text{Def junc}\} \\
 \Rightarrow & \forall K \mid K \cdot i_1 = K \cdot i_1 \wedge K \cdot i_2 = K \cdot i_2 \quad \{17\} \\
 \Rightarrow & \forall K \mid \text{True} \\
 & \text{unjunc} \cdot \text{junc} = \text{id} \\
 \Rightarrow & \forall f, g \mid (\text{unjunc} \cdot \text{junc})(f, g) = \text{id}(f, g) \quad \{72\} \\
 \Rightarrow & \forall f, g \mid ([f, g] \cdot i_1, [f, g] \cdot i_2) = (f, g) \quad \{F6(x2), 74\} \\
 \Rightarrow & \forall f, g \mid (f, g) = (f, g) \quad \{18(x2)\} \\
 \Rightarrow & \forall f, g \mid \text{True}
 \end{aligned}$$

(4)

$$\begin{cases} \text{for } b \ i \ 0 = i \\ \text{for } b \ i \ (n+1) = b \ (\text{for } b \ i \ n) \end{cases}$$

$$\Rightarrow \begin{cases} \text{for } b \ i \ (0 \ n) = \underline{i} \ n \\ \text{for } b \ i \ (\text{succ } n) = b \ (\text{for } b \ i \ n) \end{cases} \quad \{75(x2), \text{Def succ}\}$$

$$\Rightarrow \begin{cases} ((\text{for } b \ i) \cdot 0) \ n = \underline{i} \ n \\ ((\text{for } b \ i) \cdot \text{succ}) \ n = (b \cdot (\text{for } b \ i)) \ n \end{cases} \quad \{73(x3)\}$$

$$\Rightarrow \begin{cases} (\text{for } b \ i) \cdot 0 = \underline{i} \\ (\text{for } b \ i) \cdot \text{succ} = b \cdot (\text{for } b \ i) \end{cases} \quad \{72(x2)\}$$

$$\Rightarrow \begin{cases} (\text{for } b \ i) \cdot \text{in} \cdot i_1 = \underline{i} \\ (\text{for } b \ i) \cdot \text{in} \cdot i_2 = b \cdot (\text{for } b \ i) \end{cases} \quad \{24 \text{ in}, 18(x2)\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i}, b \cdot (\text{for } b \ i)] \quad \{17\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i} \cdot \text{id}, b \cdot (\text{for } b \ i)] \quad \{11\}$$

$$\Rightarrow (\text{for } b \ i) \cdot \text{in} = [\underline{i}, b] \cdot (\text{id} + (\text{for } b \ i)) \quad \{22\}$$

$$\Rightarrow \text{for } b \ i = ([\underline{i}, b]) \quad \{46\}$$

$$\text{logo, } g = [\underline{i}, b]$$

(5)

$$\begin{cases} a + 0 = a \\ a + (n+1) = 1 + (a + n) \end{cases}$$

$$\Rightarrow \begin{cases} (a+) \ (0 \ n) = \underline{a} \ n \\ (a+) \ (\text{succ } n) = (1+) \ ((a+) \ n) \end{cases} \quad \{75(x2), \text{Def succ}\}$$

$$\Rightarrow \begin{cases} ((a+) \cdot 0) \ n = \underline{a} \ n \\ ((a+) \cdot \text{succ}) \ n = (1+) \cdot ((a+) \ n) \end{cases} \quad \{73(x3)\}$$

$$\Rightarrow \begin{cases} (a+) \cdot 0 = \underline{a} \\ (a+) \cdot \text{succ} = \text{succ} \cdot (a+) \end{cases} \quad \{72(x2)\}$$

$$\Rightarrow (a+) \cdot \text{in} = [\underline{a}, \text{succ} \cdot (a+)] \quad \{18(x2), 17, \text{Def in}\}$$

$$\Rightarrow (a+) \cdot \text{in} = [\underline{a}, \text{succ}] \cdot (\text{id} + (a+)) \quad \{1, 22\}$$

$$\Rightarrow (a+) = ([\underline{a}, \text{succ}]) \quad \{46\} \quad \boxed{\text{logo, } (a+) = \text{for succ } a.}$$