Aula PI #07

31 de outubro de 2023 18:02

$$\begin{tikzcd}
T \arrow[d, "k"'] \arrow[r, shift right, "out"'] & FT \arrow[l, shift right, "in"'] \arrow[d, "Fk"] \\
B & FB \arrow[l, "g"]
\end{tikzcd}$$

} Tipo reconstruído

$T := \mathbb{N}_0$

$\Rightarrow FT = 1 + \mathbb{N}_0$

$id + k \rightarrow \mathbb{N}_0$

ou

$1 + A \times A^* \rightarrow L$

Catamorfismo com gene g

$$k = (\!| g |\!) \Leftrightarrow k \cdot in = g \cdot Fk$$

$$\begin{tikzcd}
\mathbb{N}_0 \arrow[d, "Rep\ a"'] \arrow[r, shift right, "out"'] & 1 + \mathbb{N}_0 \arrow[l, shift right, "in"'] \arrow[d, "F(rep\ A) = id + rep\ a"] \\
A^* & 1 + A^* \arrow[l]
\end{tikzcd}$$

$[nil, (a:)]$

rep True 0 = []

rep True 1 = [True]

3) a) $k\ [1,2,3] = 6$

↳ catamorfismo de listas

$$\begin{tikzcd}
\mathbb{N}_0^* \arrow[d, "k"'] \arrow[r, shift right, "out"'] & F\ 1 + \mathbb{N}_0 \times \mathbb{N}_0^* \arrow[l, shift right, "in"'] \arrow[d, "Fk = id + id \times k"] \\
\mathbb{N}_0 & 1 + \mathbb{N}_0 \times \mathbb{N}_0 \arrow[l]
\end{tikzcd}$$

Null, head, Tail

$g = ?$

Qual é o gene?

$$
\begin{tikzcd}
 & \mathbb{N}_0 & \\
1 \arrow[ur, "\underline{1}"] \arrow[r] & 1 + \mathbb{N}_0 \times \mathbb{N}_0 \arrow[u] & \mathbb{N}_0 \times \mathbb{N}_0 \arrow[l] \arrow[ul, "mul"']
\end{tikzcd}
$$

π da cauda

```
ghci> mycata g = g . (id -|- (mycata g)) . outNat
ghci> rep a = mycata (either nil (a:))
ghci> :t rep
rep :: Integral c => a -> c -> [a]
ghci> rep 5 2
[5,5]
ghci> rep 5 5
[5,5,5,5,5]
ghci> :l List.hs
[1 of 3] Compiling Cp               ( Cp.hs, interpreted )
[2 of 3] Compiling Nat              ( Nat.hs, interpreted )
[3 of 3] Compiling List             ( List.hs, interpreted )
Ok, three modules loaded.
ghci> mycataL g = g . (id -|- (id >< (mycataL g))) . outList
ghci> k = mycataL (either (const 1) mul)
ghci> k [1,2,3,4]
24
```

Naturais

Listas

b) Reverse

$$
\begin{tikzcd}
A^* \arrow[r, shift left, "out"] \arrow[d] & 1 + A \times A^* \arrow[l, shift left, "in"] \arrow[d] \\
A^* & 1 + A \times A^* \arrow[l]
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & A^* & \\
1 \arrow[ur, "nil"] \arrow[r, "i_1"] & 1 + A \times A^* & A \times A^* \arrow[l, "i_2"'] \arrow[ul]
\end{tikzcd}
$$

A^*

$\uparrow$

usamos (++) ou $(++)^{A:}$

$$
\begin{tikzcd}
{} \\
A^* \times A^* \arrow[u, "uncurry\ (++)\ ou\ (++)^{A}"'] \\
A^* \times A^* \arrow[u, "swap"'] \\
A \times A^* \arrow[u, "singl \times id"']
\end{tikzcd}
$$

```
ghci> myReverse = mycataL (either nil (uncurry (++) . swap . (singl >< id)))
ghci> myReverse [1,2,3,4]
[4,3,2,1]
ghci>
```

c) concat [[1,2]] = [1,2]

$$
\begin{tikzcd}
(A^*)^* \arrow[r, shift left, "out"] \arrow[d, "K"'] & 1 + A^* \times (A^*)^* \arrow[l, shift left, "in"] \arrow[d, "id + id \times K"] \\
A^* & 1 + A^* \times A^* \arrow[l, "g"]
\end{tikzcd}
$$

```
ghci> nil 1
[]
ghci> nil 123
[]
ghci> nil [1,2]
[]
```

```
ghci> uncurry (++) ([1,2], [3,4])
[1,2,3,4]
```

$$
\begin{tikzcd}
 & A^* & \\
1 \arrow[ur, "nil"] \arrow[r] & 1 + A^* \times A^* \arrow[u, "{[nil, (++)^A]}"] & A^* \times A^* \arrow[l] \arrow[ul, "(++)^A"']
\end{tikzcd}
$$

d) $map\ f :: A^* \to B^*$

$f : A \to B$

$$
\begin{tikzcd}
A^* \arrow[r, shift right, "out"'] \arrow[d, "map\ f"'] & 1 + A \times A^* \arrow[l, shift right, "in"'] \arrow[d, "id + id \times (map\ f)"] \\
B^* & 1 + A \times B^* \arrow[l, "g"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
B^* \\
B \times B^* \arrow[u, "(:)"', "cons"] \\
A \times B^* \arrow[u, "f \times id"']
\end{tikzcd}
$$

Vai até aos casos mais atómicos e esses resultados servirão como input para as chamadas anteriores

"colapsar até ao início depois de atingir o fim"

↳ CATAMORFISMO!

(pode não estar correto, é a minha interpretação)

$$
\begin{tikzcd}
 & B^* & \\
1 \arrow[ur, "nil"] \arrow[r] & 1 + A \times B^* \arrow[u, "{[\ldots]}"] & A \times B^* \arrow[l] \arrow[ul, "(:) \cdot (f \times id)"']
\end{tikzcd}
$$

```
ghci> mymap f = mycataL (either nil (uncurry (:) . (f >< id)))
ghci> f x = x + 1
ghci> mymap f [1,2,3]
[2,3,4]
ghci> 
```

i) maxLista

$$
\begin{tikzcd}
\mathbb{N}_0^* \arrow[d, "maxLista"'] \arrow[r, shift left] & 1 + \mathbb{N}_0 \times \mathbb{N}_0^* \arrow[l, shift left] \arrow[d, "id + id \times maxLista"] \\
\mathbb{N}_0 & 1 + \mathbb{N}_0 \times \mathbb{N}_0 \arrow[l, "{[\underline{0}, max]}"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
& \mathbb{N}_0 & \\
1 \arrow[ur, "\underline{0}"] \arrow[r] & 1 + \mathbb{N}_0 \times \mathbb{N}_0 & \mathbb{N}_0 \times \mathbb{N}_0 \arrow[l] \arrow[ul, "max"']
\end{tikzcd}
$$

j) filter p

$$
\begin{tikzcd}
A^* \arrow[r, shift left, "out"] \arrow[d, "filter\ p"'] & 1 + A \times A^* \arrow[l, shift left, "in"] \arrow[d, "id + id \times (filter\ p)"] \\
A^* & 1 + A \times A^* \arrow[l]
\end{tikzcd}
$$

$$A^* \xleftarrow{\;[nil,\ [\pi_2,\ (:)^A]\cdot distl \cdot (p? \times id)]\;} 1 + A \times A^*$$

$$\begin{tikzcd}
A^* \\
(A \times A^*) + (A \times A^*) \arrow[u, "{[\pi_2,\ (:)^A]}"'] \\
(A + A) \times A^* \arrow[u, "distl"'] \\
A \times A^* \arrow[u, "p? \times id"']
\end{tikzcd}$$

```
filter = mycata (either nil ((either p2 (uncurry (:))) . distl . (p? >< id>))
```

4

$\overbrace{f \cdot (\!| g |\!)}^{\text{função}} = \overbrace{(\!| h |\!)}^{\text{cata}}$

$\equiv$ { 46 (Universal - cata) }

$f \cdot (\!| g |\!) \cdot \mathsf{in} = h \cdot \mathsf{F}\,(f \cdot (\!| g |\!))$

$\equiv$ { 47, 50 }

$f \cdot g \cdot (id + (\!| g |\!)) = h \cdot (id + f \cdot (\!| g |\!))$

$\equiv$ { 1 }

$f \cdot g \cdot (id + (\!| g |\!)) = h \cdot (id \cdot id + f \cdot (\!| g |\!))$

$\equiv$ { 25 }

$f \cdot g \cdot (id + (\!| g |\!)) = h \cdot (id + f) \cdot (id + (\!| g |\!))$

$\Leftarrow$ { 5 }

$f \cdot g = h \cdot (id + f)$

$f \cdot (\!| g |\!) \cdot in = h \cdot F\,(f \cdot (\!| g |\!))$

5

$$\text{sumprod } a = (a*) \cdot \text{sum}$$

$$\Leftrightarrow (\!| [\text{Zero}, \text{add} \cdot ((a*) \times id)] |\!) = (a*) \cdot (\!| [\text{Zero}, \text{add}] |\!)$$

49 $$\Leftrightarrow (a*) \cdot [\text{Zero}, \text{add}] = [\text{Zero}, \text{add}((a*) \times id)] \cdot F (\text{add}$$

$$\Leftrightarrow (a*) \cdot [\text{Zero}, \text{add}] = [\text{Zero}, \text{add} \cdot ((a*) \times id] \cdot (id + id \times g)$$

20, 21, 1 $$\Leftrightarrow [(a*) \cdot \text{Zero}, (a*) \cdot \text{add}] = [\text{Zero}, \text{add} \cdot ((a*) \times id) \cdot ((id) \times (a*))]$$

27 $$\Leftrightarrow \begin{cases} (a*) \cdot \text{Zero} = \text{Zero} \\ (a*) \cdot \text{add} = \text{add} \cdot ((a*) \times id) \cdot (id \times (a*)) \end{cases}$$

$$\Leftrightarrow \begin{cases} \forall x \mid ((a*) \cdot \text{add})(x,y) = (\text{add} \cdot ((a*)))(x,y) \\ \forall x,y \mid ((a*) \cdot \text{add})(x,y) = (\text{add} \cdot ((a*) \times (a*)))(x,y) \end{cases}$$

$$\Leftrightarrow \begin{cases} \forall x \mid a \times 0 = 0 \\ \forall x,y \mid a \times (x+y) = \text{add}(a \times x, a \times y) \end{cases}$$

$$\Leftrightarrow \text{TRUE}$$