Aula T5 - Anotações

16 de outubro de 2023 22:06

POLIMORFISMO

$\text{reverse} :: \underbrace{[a] \to [a]}_{\text{Paramétrica}}$

Um tipo qq "a"

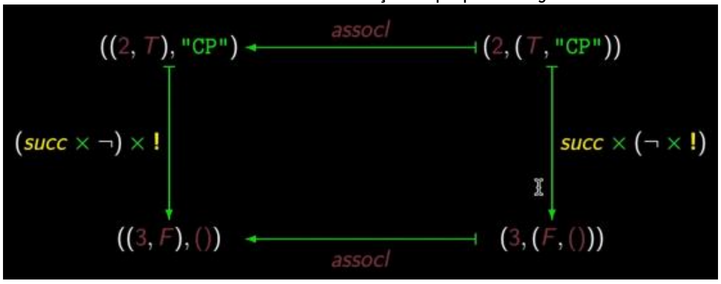
Caso contrário:

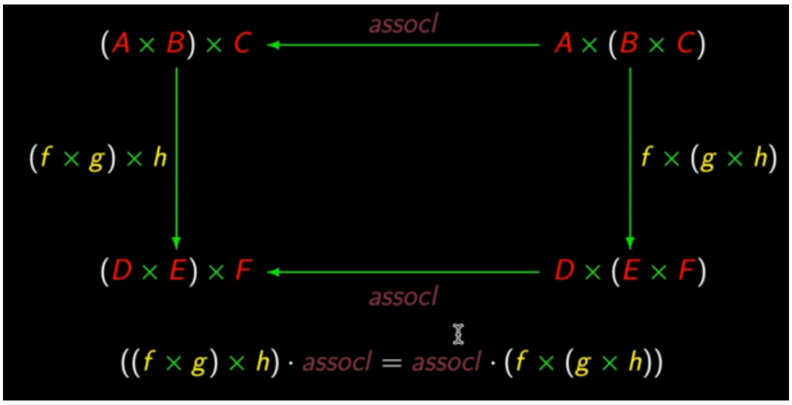
$reverse :: [Int] \to [Int]$
$reverse :: String \to String$
...

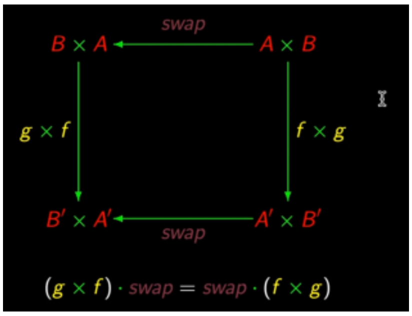
```
Prelude>
Prelude> maybe :: b -> (a->b) -> Maybe a -> b ; maybe = undefined
Prelude>
Prelude> :t maybe
maybe :: b -> (a -> b) -> Maybe a -> b
Prelude>
Prelude>
Prelude>
Prelude> -- maybe (s b) (s . f) = s . (maybe b f)
```

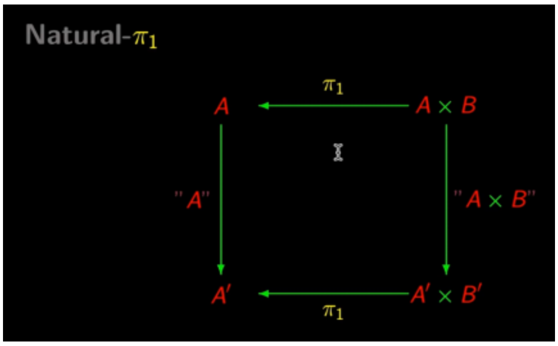
PROPRIEDADES GRÁTIS

Ilustração da propriedade grátis do assocl

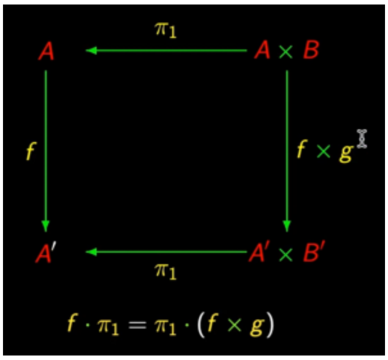


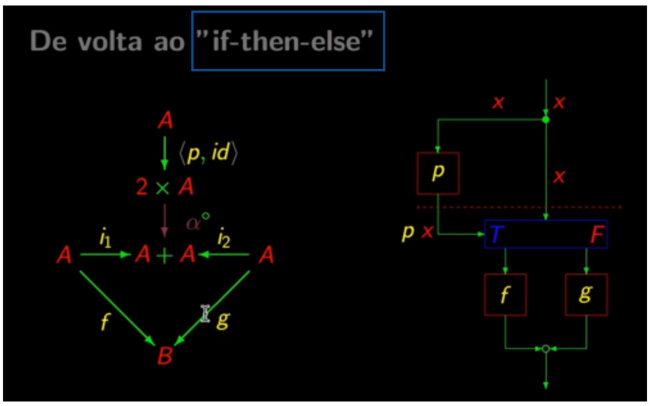




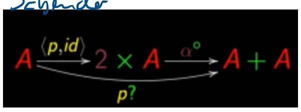


$\Rightarrow$





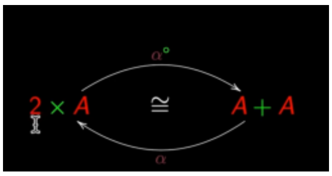
Schröder

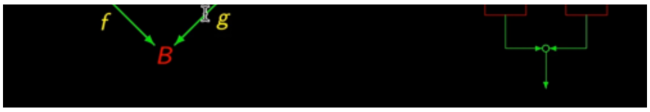


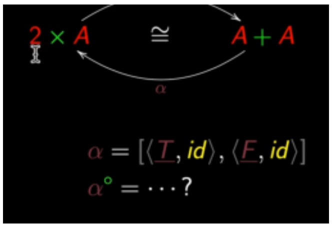
$p? : A \to A+A$

Se o predicado por Verdadeiro para A, dá A com etiqueta no lado esquerdo, se der Falso para A, dá A com etiqueta no lado direito.

$p? \cdot f = (f+f) \cdot (p \cdot f)?$



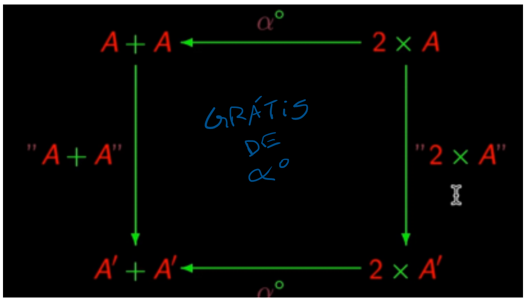




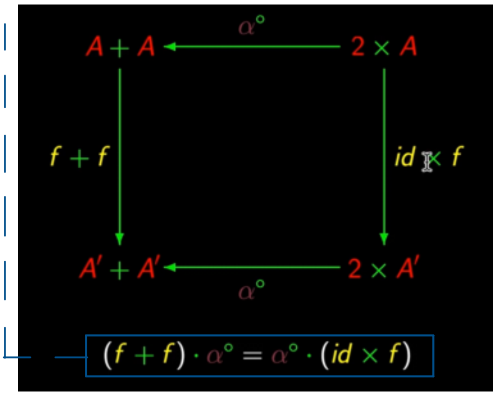
$$
\begin{array}{ll}
 & p?\cdot f \\
= & \{\ p? = \alpha^\circ\cdot\langle p, id\rangle\ \} \\
 & \alpha^\circ\cdot\langle p, id\rangle\cdot f \\
= & \{\ \text{fusão-}\times\ \} \\
 & \alpha^\circ\cdot\langle p\cdot f, id\cdot f\rangle \\
= & \{\ \text{natural-}id\text{ duas vezes}\ \} \\
 & \alpha^\circ\cdot\langle id\cdot p\cdot f, f\cdot id\rangle
\end{array}
$$

$= \alpha^\circ \cdot (id \times f) \cdot \langle p \cdot f, id\rangle$

$$
\begin{array}{ll}
= & \{\ \text{grátis de } \alpha^\circ\ \} \\
 & (f + f)\cdot\alpha^\circ\cdot\langle p\cdot f, id\rangle \\
= & \{\ p? = \alpha^\circ\cdot\langle p, id\rangle\ \} \\
 & (f + f)\cdot(p\cdot f)?
\end{array}
$$



$\Rightarrow$



PARTE B:

RECURSION POINT-FREE STYLE

- Como nasce um programa?
  - ↳ O que é um programa?
- Como nasceram os algoritmos célebres?

$$
\begin{cases}
a \times 0 = 0 \\
\boxed{a \times 1 = a} \\
a \times (b + c) = a \times b + a \times c
\end{cases}
$$

Consequência dos dois

↳ $a \times (b + 1) = a \times b + a$

Se $c = 1$, $b = 0$

$$
\Rightarrow \begin{cases}
a \times 0 = 0 \\
a \times (b + c) = a \times b + a \times c
\end{cases}
$$

Em HASKELL:
$$
\begin{cases}
a \ .\!*\ 0 = 0 \\
a \ .\!*\ (b + 1) = (a \ .\!*\ b) + a
\end{cases}
$$

CURRIED
$$
\begin{cases}
(a\times)\,(0\ x) = \underline{0}\ x \\
(a\times)\,(\mathsf{succ}\ b) = (a+)\ ((a\times)\ b)
\end{cases}
$$

$$
\Leftrightarrow \begin{cases}
((a\times)\cdot\underline{0})\ \cancel{x} = \underline{0}\ \cancel{x} \\
((a\times)\cdot\mathsf{succ})\ \cancel{b} = ((a+)\cdot(a\times))\ \cancel{b}
\end{cases}
$$

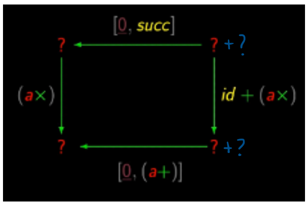
$$\Leftrightarrow \begin{cases} (a\times)\cdot\underline{0} = \underline{0} \\ (a\times)\cdot succ = (a+)\cdot(a\times) \end{cases}$$

$$[f\,,g] = [h\,,k] \Leftrightarrow \begin{cases} f = h \\ g = k \end{cases}$$

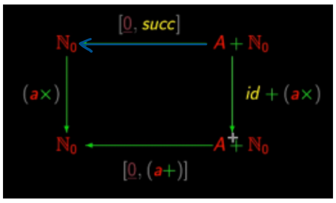
$$\Leftrightarrow [(a\times)\cdot\underline{0},\ (a\times)\cdot succ] = [\underline{0},\ (a+)\cdot(a\times)]$$

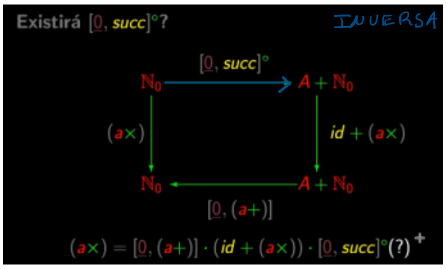
$$\Leftrightarrow (a\times)\cdot[\underline{0},\ succ] = [\underline{0}\cdot id,\ (a+)\cdot(a\times)]$$

$$\Leftrightarrow (a\times)\cdot[\underline{0},\ succ] = [\underline{0},\ (a+)]\cdot(id+(a\times))$$



$\Rightarrow$





$$out : \mathbb{N}_0 \longrightarrow A + \mathbb{N}_0$$

$$out\cdot[\underline{0}, succ] = id$$

$$\Leftrightarrow [out\cdot\underline{0},\ out\cdot succ] = id$$

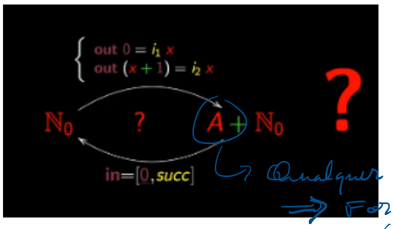
Introduzir Variáveis

$$\Leftrightarrow \begin{cases} out\cdot\underline{0} = id\cdot i_1 \\ out\cdot succ = id\cdot i_2 \end{cases} \Leftrightarrow \begin{cases} out\,(\underline{0}\,x) = i_1\,x \\ out\,(succ\ x) = i_2\,x \end{cases}$$

$$\Leftrightarrow \begin{cases} out\ 0 = i_1\ x \\ out\ (x+1) = i_2\ x \end{cases} \longleftarrow$$

0 tem diversas imagens (?)

$\Rightarrow$ Não é uma função



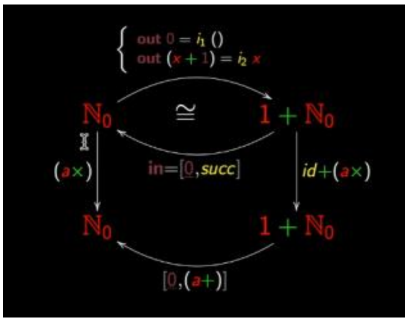
↳ Qualquer

$\Rightarrow$ Forçar um único x

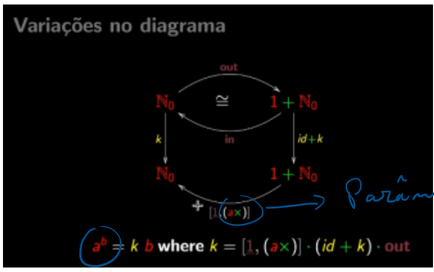
apenas uma $\Rightarrow$ Função

$$\begin{tikzcd} \mathbb{N}_0 \arrow[r, bend left, "out"] & 1 + \mathbb{N}_0 \arrow[l, bend left, "in"] \end{tikzcd}$$

Solução:



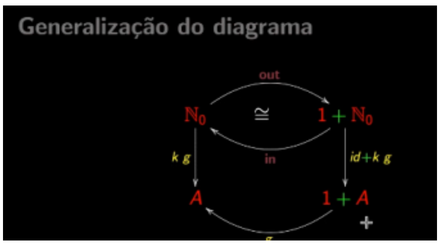
$$(a\times) = k,\ k = [\underline{0}, (a+)]\cdot(id + k)\cdot out$$



Parâmetro do problema?

Função de ordem superior

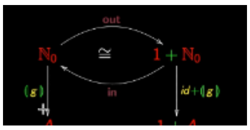
$\Rightarrow$



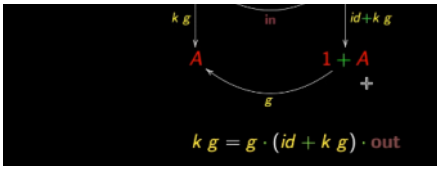
$$k\,g = (|\,g\,|)$$

"cata de g"

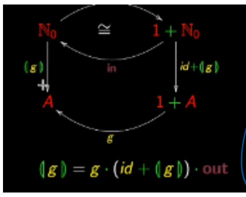
↳ catamorfismo de g



Estende uma função a uma função



⇝



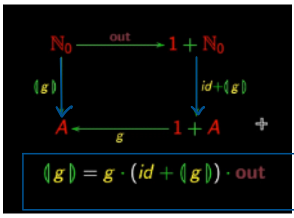
de g

Estende uma função a uma função recursiva sobre os números naturais.

$\underbrace{\text{cata}}_{\text{para baixo}}$ $(\kappa\alpha\tau\alpha)$ + $\underbrace{\text{morfismo}}_{\text{transformação}}$ $(\mu o \rho \phi \iota \sigma \mu o \varsigma)$

"Transformação descendente"

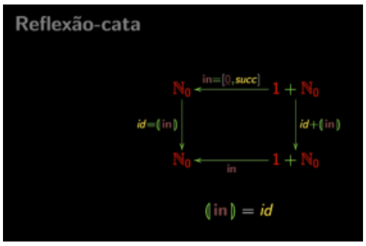
↳ Perde informação

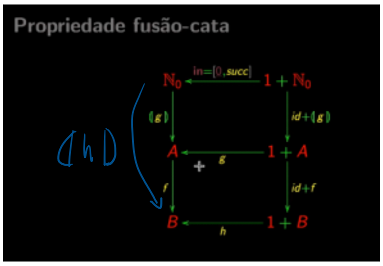


$$\boxed{(\!|g|\!) \cdot \mathsf{in} = g \cdot (id + (\!|g|\!))}$$

Universal-cata:

$$k = (\!|g|\!) \Leftrightarrow k \cdot \mathsf{in} = g \cdot (id + k)$$





$$f \cdot (\!|g|\!) = (\!|h|\!)$$

$$\Leftrightarrow f \cdot (\!|g|\!) \cdot in = h \cdot (id + f \cdot (\!|g|\!))$$

$$\Leftrightarrow f \cdot g \cdot (id + (\!|g|\!)) = h \cdot (id + f) \cdot (id + (\!|g|\!))$$

$$\Leftrightarrow f \cdot g = (id + f)$$

Em suma:

$$f \cdot (\!|g|\!) = (\!|h|\!) \Leftarrow f \cdot g = h \cdot (id + f)$$