Preparação - o outro documento estava muito grande

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$$\overline{f \cdot (g \times h)} = \overline{f \cdot ((id \cdot g) \times (h \cdot id))} = \overline{f \cdot \underbrace{(id \times h)}_{\text{"}h\text{"}} \cdot \underbrace{(g \times id)}_{\text{"}t \times id\text{"}}}$$

é possível formar um (função >< id) do lado direito

Fusão-Exp

$$= \overline{f \cdot (id \times h)} \cdot g$$

$$= \overline{ap \cdot (\overline{f} \times id) \cdot (id \times h)} \cdot g$$

$$= \overline{ap \cdot (\overline{f} \cdot id) \times (id \cdot h)} \cdot g$$

"Troca"

$$= \overline{ap \cdot (id \cdot \overline{f}) \times (h \cdot id)} \cdot g$$

$$= \overline{ap \cdot (id \times h) \cdot (\overline{f} \times id)} \cdot g \qquad \text{Fusão-Exp}$$

$$= \overline{ap \cdot (id \times h)} \cdot \overline{f} \cdot g$$

$$const\ (b, a) = \langle const\ b,\ const\ a \rangle$$

$$\Leftrightarrow \begin{cases} \pi_1 \cdot const\ (b, a) = const\ b \\ \pi_2 \cdot const\ (b, a) = const\ a \end{cases} \qquad \text{Universal-X}$$

$$\Leftrightarrow \begin{cases} const\ (\pi_1 \cdot (b, a)) = const\ b \\ const\ (\pi_2 \cdot (b, a)) = const\ a \end{cases} \qquad \text{Fusão-const}$$

$$\Leftrightarrow \begin{cases} const\ b = const\ b \\ const\ a = const\ a \end{cases} \qquad \text{Def-proj}$$

$$\Leftrightarrow True = True$$

$$\Leftrightarrow TRUE$$

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$$\alpha = swap \cdot (id \times swap)$$

$$\begin{array}{c} (B \times A) \\ \uparrow swap \\ (A \times B) \end{array}$$

$$\begin{array}{c} (C \times B) \times A \\ \uparrow swap \\ A \times (C \times B) \\ \uparrow id \times swap \\ A \times (B \times C) \end{array}$$

$$\begin{array}{ccc} A \times (B \times C) & \xrightarrow{\alpha} & (C \times B) \times A \\ \downarrow f \times (g \times h) & & \downarrow (h \times g) \times f \\ A' \times (B' \times C') & \xrightarrow{\alpha} & (C' \times B') \times A' \end{array}$$

$$\alpha \cdot f \times (g \times h) = (h \times g) \times f \cdot \alpha$$

Demonstração: Trivial, confio

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$$A \times (B + C) \cong A \times B + A \times C$$

$$(B+C) \times A \xrightarrow{swap} A \times (B \times C) \longrightarrow (A \times B) + (A \times C) \xrightarrow{swap + swap} (B \times A) + (C \times A)$$

ou

$[i_1 \cdot swap, i_2 \cdot swap]$

pois o tipo final não é o mesmo para as duas partes.

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$$(p \to g, h) \times f = p \cdot \pi_1 \to g \times f, h \times f$$

$$[\overline{i_1}, i_2] = \overline{distl}$$

$$\overline{(p \cdot \pi_1)?} = p \to \overline{i_1}, \overline{i_2}$$

$= \overline{(p \cdot \pi_1)?} = \overline{[\overline{i_1}, \overline{i_2}] \cdot p?} = \overline{distl} \cdot p?$ (def. $\overline{distl}$)

$= \overline{(p \cdot \pi_1)?} = distl \cdot (p? \times id)$ (Fusão - Exp)

$= (p \cdot \pi_1)? = distl \cdot (p? \times id)$ (uncurry nos dois lados)

$= [i_1 \times id, i_2 \times id] \cdot (p \cdot \pi_1)? = (p? \times id)$ undistl nos dois lados

$= p \cdot \pi_1 \to (i_1 \times id), (i_2 \times id) = (p? \times id)$ Shunt-Right (34)

$= (p \to i_1, i_2) \times id = (p? \times id)$ regra dada

$= ([i_1, i_2] \cdot p?) \times id = (p? \times id)$

$= (p? \times id) = (p? \times id)$

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$(p \to f, g) \cdot h$

$= [f, g] \cdot p? \cdot h$

$= [f, g] \cdot \alpha^{\circ} \cdot \langle p, id \rangle \cdot h$

$= [f, g] \cdot \alpha^{\circ} \cdot \langle p \cdot h, id \cdot h \rangle$ { Passar h para o lado esquerdo

$$
\begin{aligned}
&= [f,g] \cdot \alpha^\circ \cdot \langle p\cdot h,\ id\cdot h\rangle && \left\{ \text{Passar } h \text{ para o lado esquerdo} \right.\\
&= [f,g] \cdot \alpha^\circ \cdot \langle id\cdot p\cdot h,\ h\cdot id\rangle \\
&= [f,g] \cdot \alpha^\circ \cdot (id\times h)\cdot \langle p\cdot h,\ id\rangle && \left\{ \text{Propriedade grátis de } \alpha^\circ \right.\\
&= [f,g] \cdot (h+h)\cdot \alpha^\circ \cdot \langle p\cdot h,\ id \\
&= [f\cdot h,\ g\cdot h] \cdot \alpha^\circ \cdot \langle p\cdot h,\ id\rangle \\
&= [f\cdot h,\ g\cdot h] \cdot (p\cdot h)? \\
&= (p\cdot h) \to f\cdot h,\ g\cdot h
\end{aligned}
$$

$$
\begin{tikzcd}
2\times A \arrow[r, "\alpha^\circ"] \arrow[d, "id\times f"'] & A+A \arrow[d, "f+f"] \\
2\times A' \arrow[r, "\alpha^\circ"] & A'+A'
\end{tikzcd}
$$

$$(f+f)\cdot\alpha^\circ = \alpha^\circ\cdot(id\times f)$$

$$
\begin{tikzcd}
A \arrow[d, "h"] \\
B \arrow[d, "{\langle p, id\rangle}"] \arrow[dd, bend left, "p?"] \\
2\times B \arrow[d, shift left, "\alpha^\circ"] \\
B+B \arrow[u, shift left, bend left, "\alpha"] \arrow[d, "{[f,g]}"] \\
C
\end{tikzcd}
$$

$[\langle T, id\rangle, \langle F, id\rangle]$

$[f,g]$ (join)

$$
\begin{tikzcd}
& C+B & \\
C \arrow[ur, bend left, "id\cdot i_1"] \arrow[r] & C+B \arrow[u, "{[id\cdot i_1,\ f\cdot i_2]}"] & B \arrow[l] \arrow[ul, "f\cdot i_2"'] \\
& A+B \arrow[u, "f"] &
\end{tikzcd}
$$

$\to f\ B \to B$

$$
\begin{tikzcd}
(Point\ A)^{A^A} \\
A \arrow[u, "Point"]
\end{tikzcd}
\qquad
\begin{tikzcd}
(Point\ A)^A \\
A\times A \arrow[u, "\widehat{Point}"]
\end{tikzcd}
\qquad
\begin{tikzcd}
Point\ A \\
(A\times A)\times A \arrow[u, "\widehat{\widehat{Point}}"]
\end{tikzcd}
$$