1.

$$[i_1, i_2] = id$$

$$\Leftrightarrow \begin{cases} id \cdot i_1 = i_1 \\ id \cdot i_2 = i_2 \end{cases} \quad \{17\} \quad \Leftrightarrow \begin{cases} i_1 = i_1 \\ i_2 = i_2 \end{cases} \quad \{1\,(\times 2)\}$$

2.

$$[\underline{K}, \underline{K}] = \underline{K}$$

$$\Leftrightarrow \begin{cases} \underline{K} \cdot i_1 = \underline{K} \\ \underline{K} \cdot i_2 = \underline{K} \end{cases} \quad \{17\} \quad \Leftrightarrow \begin{cases} \underline{K} = \underline{K} \\ \underline{K} = \underline{K} \end{cases} \quad \{3\,(\times 2)\}$$

3.

$$coswap \cdot coswap = id$$

$$\Leftrightarrow [i_2, i_1] \cdot [i_2, i_1] = id \qquad \{\text{def coswap } (\times 2)\}$$

$$\Leftrightarrow [\,[i_2, i_1] \cdot i_2, [i_2, i_1] \cdot i_1\,] = id \qquad \{20\}$$

$$\Leftrightarrow [i_1, i_2] = id \qquad \{18\,(\times 2)\}$$

$$\Leftrightarrow True \qquad \{19\}$$

$$\begin{tikzcd} & B+A & \\ A \arrow[r, "i_1"'] \arrow[ur, "i_2"] & A+B \arrow[u, "coswap"] & B \arrow[l, "i_2"] \arrow[ul, "i_1"'] \end{tikzcd}$$

4.

<u>false</u>

$$\frac{A \longrightarrow \mathbb{B} \;,\; C \xrightarrow{id} C}{A \xrightarrow{\langle false, id \rangle} \mathbb{B} \times A} \quad \{C = A\}$$

* Num split, os conjuntos de ~~chegada~~ partida têm de ser iguais

O tipo de $\langle True, id \rangle$ é igual, vamos usar apenas letras diferentes.

$$\frac{A \xrightarrow{\langle false, id \rangle} \mathbb{B} \times A \;,\; C \xrightarrow{\langle True, id \rangle} \mathbb{B} \times C}{A + A \xrightarrow{\alpha} \mathbb{B} \times A} \quad \{A = C\}$$

* Num either, os conjuntos do chegada têm de ser iguais.

$\alpha = [\langle false, id\rangle, \langle True, id\rangle]$

$$\Leftrightarrow \begin{cases} \alpha \cdot i_1 = \langle false, id\rangle \\ \alpha \cdot i_2 = \langle True, id\rangle \end{cases} \qquad \{6\}$$

$$\Leftrightarrow \begin{cases} \alpha\,(Left\ a) = (false, a) \\ \alpha\,(Right\ a) = (True, a) \end{cases} \qquad \{72(\times 2), 73(\times 2), 77(\times 2), 74(\times 2), 75(\times 2)\}$$

5.

$id + id = id$ (functor-id+)

$\Leftrightarrow [i_1 \cdot id, i_2 \cdot id] = id$ $\{21\}$

$\Leftrightarrow [i_1, i_2] = id$ $\{1(\times 2)\}$

$\Leftrightarrow True$ $\{19\}$

$(f+g) \cdot i_1 = i_1 \cdot f$ (Natural-i_1)

$\Leftrightarrow [i_1 \cdot f, i_2 \cdot g] \cdot i_1 = i_1 \cdot f$ $\{21\}$

$\Leftrightarrow i_1 \cdot f = i_1 \cdot f$ $\{18\}$

$\Leftrightarrow True$

$(f+g) \cdot i_2 = i_2 \cdot g$ (Natural-i_2)

$\Leftrightarrow [i_1 \cdot f, i_2 \cdot g] \cdot i_2 = i_2 \cdot g$ $\{21\}$

$\Leftrightarrow i_2 \cdot g = i_2 \cdot g$ $\{18\}$

$\Leftrightarrow True$

6.

$$\frac{A \xrightarrow{id} A \quad , \quad B \times C \xrightarrow{\pi_1} B}{A + (B \times C) \xrightarrow{id + \pi_1} A + B} \qquad \{\}$$

$$\frac{D \xrightarrow{i_2} E + D \quad , \quad F \times G \xrightarrow{\pi_2} G}{F \times G \xrightarrow{i_2 \cdot \pi_2} E + G} \qquad \{D = G\}$$

$$\frac{A + (B \times C) \xrightarrow{id + \pi_1} A + B \quad , \quad F \times G \xrightarrow{i_2 \cdot \pi_2} E + G}{F \times G \xrightarrow{\alpha} A + B} \qquad \{E + G = A + (B \times C)\}$$

$F \times (B \times C) \xrightarrow{\alpha} A + B$

Logo, $\boxed{\alpha : A \times (B \times C) \longrightarrow D + B}$

[illegible]

$\cancel{B\times C \xleftarrow{\pi_2} A\times(B\times C)}$

$$\begin{tikzcd}
 & A\times(B\times C) \arrow[d, "\pi_2"] & \\
 & B\times C \arrow[d, "i_2"] & \\
D \arrow[r, "i_1"] \arrow[d, "id"'] & D+(B\times C) \arrow[d, "id+\pi_1"] & B\times C \arrow[l, "i_2"'] \arrow[d, "\pi_1"] \\
D \arrow[r, "i_1"'] & D+B & B \arrow[l, "i_2"]
\end{tikzcd}$$

7.

$coassocl \cdot [id+i_1, i_2\cdot i_2] = id$

$\Leftrightarrow \begin{cases} coassocl \cdot (id+i_1) = i_1 \\ coassocl \cdot i_2 \cdot i_2 = i_2 \end{cases}$ {20, 17; 1(a2)}

$\Leftrightarrow \begin{cases} coassocl \cdot i_1 = i_1 \cdot i_1 \\ coassocl \cdot i_2 \cdot i_1 = i_1 \cdot i_2 \\ coassocl \cdot i_2 \cdot i_2 = i_2 \end{cases}$ {21, 1, 20, 17}

$\Leftrightarrow \begin{cases} coassocl \cdot i_1 = i_1 \cdot i_1 \\ coassocl \cdot i_2 = [i_1\cdot i_2, i_2] \end{cases}$ {Associative [illegible] 17}

$\Leftrightarrow coassocl = [i_1\cdot i_1, [i_1\cdot i_2, i_2]]$ {17}

$\Leftrightarrow coassocl = [i_1\cdot i_1, i_2+id]$ {1, 21}

coassocl (Left a) = Left (Left a)
coassocl (Right (Left b)) = Left (Right b)
coassocl (Right (Right c)) = Right c

(8.)

$fac \cdot [\underline{0}, succ] = [\underline{1}, mul \cdot \langle succ, fac \rangle]$

$\Leftrightarrow \begin{cases} fac \cdot \underline{0} = \underline{1} \\ fac \cdot succ = mul \cdot \langle succ, fac \rangle \end{cases}$ {20, 27}

$\Leftrightarrow \begin{cases} fac\ 0 = 1 \\ fac\ (n+1) = (n+1) \times fac\ n \end{cases}$

{72, 73, 75 (x2)}

{72, 73 (x2), 77, Def succ, Def mul}

(9.)

$(p \to f, g) \cdot h = ([f, g] \cdot p?) \cdot h$ {30}

$= [f, g] \cdot (p? \cdot h)$ {Associativa Comp}

$= [f, g] \cdot ((h+h) \cdot (p \cdot h)?)$ {29}

$= [f \cdot h, g \cdot h] \cdot (p \cdot h)?$ {Associativa Comp, 22}

$= p \cdot h \to f \cdot h, g \cdot h$ {30}