

Kieha 4 | 23/24

$$\textcircled{1} \quad [\langle f, g \rangle, \langle h, k \rangle] = \langle [f, h], [g, k] \rangle$$

$$\Rightarrow \begin{cases} \pi_1 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [f, h] \\ \pi_2 \cdot [\langle f, g \rangle, \langle h, k \rangle] = [g, k] \end{cases} \quad \{6\}$$

$$\Rightarrow \begin{cases} [f, h] = [f, h] \\ [g, k] = [g, k] \end{cases} \quad \{20(x2), 7(x2)\}$$

$\Rightarrow \text{True}$

$$\textcircled{2} \quad \text{undist} = [i_1 \times \text{id}, i_2 \times \text{id}]$$

$$= [\langle i_1 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle, \langle i_2 \cdot \pi_1, \text{id} \cdot \pi_2 \rangle] \quad \{10(x2)\}$$

$$= \langle [i_1 \cdot \pi_1, i_2 \cdot \pi_1], [\pi_2, \pi_2] \rangle \quad \{28, 1(x2)\}$$

$$= \langle \pi_1 + \pi_1, [\pi_2, \pi_2] \rangle \quad \{21\}$$

③

$$\begin{aligned}
 \text{Ex)} & \langle (p \rightarrow f, h), (p \rightarrow g, i) \rangle = \\
 & = \langle [f, h] \cdot p?, [g, i] \cdot p? \rangle \quad \{30(x2)\} \\
 & = \langle [f, h], [g, i] \rangle \cdot p? \quad \{9\} \\
 & = \langle \langle f, g \rangle, \langle h, i \rangle \rangle \cdot p? \quad \{28\} \\
 & = p \rightarrow \langle f, g \rangle, \langle h, i \rangle \quad \{30\}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex)} & \langle f, p \rightarrow g, h \rangle \\
 & = \langle p \rightarrow f, p \rightarrow g, h \rangle \quad \{F2\} \\
 & = \langle [f, f] \cdot p?, [g, h] \cdot p? \rangle \quad \{30(x2)\} \\
 & = \langle [f, f], [g, h] \rangle \cdot p? \quad \{9\} \\
 & = \langle \langle f, g \rangle, \langle f, h \rangle \rangle \cdot p? \quad \{27\} \\
 & = p \rightarrow \langle f, g \rangle, \langle f, h \rangle \quad \{30\}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex)} & p \rightarrow (p \rightarrow a, b), (p \rightarrow c, d) = \\
 & = [(p \rightarrow a, b), (p \rightarrow c, d)] \cdot p? \quad \{30\} \\
 & = [[a, b] \cdot p?, [c, d] \cdot p?] \cdot p? \quad \{30(x2)\} \\
 & = ([[a, b], [c, d]] \cdot (p? + p?)) \cdot p? \quad \{22\} \\
 & = [[a, b], [c, d]] \cdot (i_1 + i_1) \cdot p? \quad \{Associative comp. F3\} \\
 & = [a, d] \cdot p? \quad \{22, 18(x2)\} \\
 & = p \rightarrow a, d
 \end{aligned}$$

④

$$out \cdot in = id$$

$$\Rightarrow \begin{cases} out \cdot 0 = i_1 \\ out \cdot succ = i_2 \end{cases} \quad \{20, 17, 1(x2)\}$$

$$\Rightarrow \begin{cases} out \ 0 = i_1() \\ out(n+1) = i_2 \ n \end{cases} \quad \{72(x2), 73(x2), 75\}$$

③) $f \cdot [0, \text{succ}] = [0, \text{add}] \cdot (\text{id} + \langle \text{odd}, f \rangle)$

$\Rightarrow \begin{cases} f \cdot 0 = 0 \\ f \cdot \text{succ} = \text{add} \cdot \langle \text{odd}, f \rangle \end{cases}$

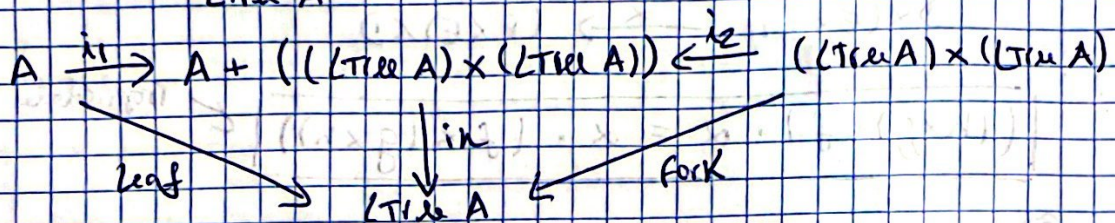
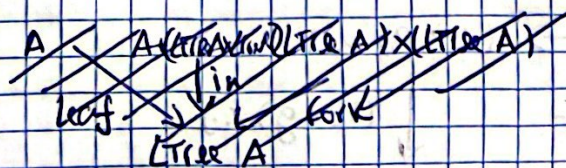
$\{20, 22, 1, 27\}$

$\Rightarrow \begin{cases} f \cdot 0 = 0 \\ f(n+1) = (2n+1) + f n \end{cases}$

$\{72(x_2), 73(x_3), 75(x_2), 77\}$

$\begin{aligned} f \cdot 0 &= 0 \\ f \cdot 1 &= 1 \\ f \cdot 2 &= 4 \\ f \cdot 3 &= 9 \\ f \cdot 4 &= 16 \\ &\vdots \\ f \cdot n &= n^2 \end{aligned}$

⑥)



$\text{out} \cdot \text{in} = \text{id}$

$\Rightarrow \begin{cases} \text{out} \cdot \text{leaf} = i_1 \\ \text{out} \cdot \text{fork} = i_2 \end{cases}$

$\{20, 17, 1(x_2)\}$

$\Rightarrow \begin{cases} \text{out}(\text{leaf } a) = i_1 \cdot a \\ \text{out}(\text{fork}(a, b)) = i_2(a, b) \end{cases}$

$\{72(x_2), 73(x_2)\}$