

$$\begin{cases} f\ x = 2 * x \\ g\ x = x + 1 \end{cases} \quad \begin{cases} f = \text{succ} \\ g\ x = 2 * x \end{cases}$$

```
ghci> f x = 2 * x
ghci> g x = x + 1
ghci> (f.g) 5
```

```
ghci> f = succ
ghci> g x = 2 * x
ghci> f.g 5
```

(f.g).h = f.(g.h)
 $\Leftrightarrow f(g(h\ x)) = f(g(h\ x))$
 \Leftrightarrow para todo o x, ((f.g).h) x = (f.(g.h)) x
 \Leftrightarrow para todo o x,

(f.g).h = f.(g.h)

A <- f - B <- g <- C
 A <- f.g - C

Ex.3

f . id = f
 (lei 72)
 \Leftrightarrow para todo o x, (f.id) x = f x
 (lei 73)
 \Leftrightarrow para todo o x f(id(x)) = f x
 (lei 74)
 \Leftrightarrow para todo o x f(x) = f(x)

Store c = take 10 . Nub . (x;
 Store aceita outros tipos ==> polimorfismo

Regras

$$(f \times g) \cdot (h \times k) = f \cdot h \times g \cdot h$$

$$\Leftrightarrow \forall x, y \mid (f \times g)(h(x), k(y)) = (f(h(x)), g(k(y)))$$

$$\Leftrightarrow \forall x, y \mid (f(h(x)), g(k(y))) = (f(h(x)), g(k(y)))$$

$$\Leftrightarrow \text{TRUE}$$

Igualdade extensional

Def-comp

Def-id

Def-const

Notação- λ

Def-split

Def-x

$$f = g \Leftrightarrow (\forall x :: f\ x = g\ x) \quad (72)$$

$$(f \cdot g)\ x = f(g\ x) \quad (73)$$

$$\text{id}\ x = x \quad (74)$$

$$\underline{k}\ x = k \quad (75)$$

$$f\ a = b \equiv f = \lambda a \rightarrow b \quad (76)$$

$$\langle f, g \rangle x = (f\ x, g\ x) \quad (77)$$

$$(f \times g)(a, b) = (f\ a, g\ b) \quad (78)$$

↳ Do formalismo

$$\pi_1 \cdot (f \times g) = f \cdot \pi_1$$

$$(72) \Leftrightarrow \forall x, y \mid (\pi_1 \cdot (f \times g))(x, y) = (f \cdot \pi_1)(x, y)$$

$$(73) \Leftrightarrow \forall x, y \mid \pi_1((f \times g)(x, y)) = f(\pi_1(x, y))$$

$$(78) \Leftrightarrow \forall x, y \mid \pi_1(f(x), g(y)) = f(x)$$

$$\Leftrightarrow \forall x, y \mid f(x) = f(x)$$

$$\Leftrightarrow \text{TRUE}$$

$$\text{length} :: [a] \rightarrow \mathbb{Z}$$

$$\text{length}\ [] = 0$$

$$\text{length}\ (x:xs) = 1 + \text{length}\ xs$$

$$\text{reverse} :: [a] \rightarrow [a]$$

$$\text{reverse}\ [] = []$$

$$\text{reverse}\ (x:xs) = \text{reverse}\ xs$$

$$(++) :: [a] \rightarrow [a] \rightarrow [a]$$

$$(++)\ []\ l = l$$

$$(++)\ (h:t)\ l = h : \underline{(++)\ t\ l}$$

$$(++)\ [1, 2, 3]\ [5, 6] \Downarrow$$

$$[1] : ([2] : ([3] : [5, 6]))$$

• CURRY

$$\text{add} :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$$

$$\text{add}\ x\ y = x + y$$

$$\text{add}\ 2 :: \text{Int} \rightarrow \text{Int}$$

$$\text{add}\ 2\ 3 :: \text{Int}$$

Series of
curried
functions

→ Curry is a process of converting
function that takes multiple arguments

→ Curry is a process of converting a function that takes multiple arguments into a series of functions, each taking a single argument

$\text{curry} :: (a, b) \rightarrow c \rightarrow a \rightarrow b \rightarrow c$

$\text{curry } f \ x \ y = f \ (x, y)$

curry the add function explicitly

$\text{addTwo} :: \text{Int} \rightarrow \text{Int}$

$\text{addTwo} = \text{curry } (+) \ 2$

• UNCURRY

$\text{uncurry} :: (a \rightarrow b \rightarrow c) \rightarrow (a, b) \rightarrow c$

$\text{uncurry } f \ (x, y) = f \ x \ y$

$\mu A :: (\text{Int}, \text{Int}) \rightarrow \text{Int}$

$\mu A = \mu A \ \text{add}$

$\Rightarrow \mu A \ (2, 3)$

```
-- Original function taking a tuple
addTuple :: (Int, Int) -> Int
addTuple (x, y) = x + y

-- Using uncurry to adapt it for curried arguments
addCurried :: Int -> Int -> Int
addCurried = uncurry addTuple
```

The uncurry function in Haskell is used to transform a curried function (a function that takes multiple arguments as a sequence of single-argument functions) into a function that takes a tuple as its argument.

• Flip

$\text{flip} :: (a \rightarrow b \rightarrow c) \rightarrow b \rightarrow a \rightarrow c$

$\text{flip } f \ x \ y = f \ y \ x$