ONLINE AUCTIONS: THEORY AND EXAMPLES

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Abstract. Online auctions have gathered a significant user base across the internet and, as a result, have become an active business model. This research paper aims to provide a comprehensive overview of auctions, focusing on online auctions. It starts by explaining their purpose, followed by an exploration of the advantages of online auctions and the key factors contributing to their success. Additionally, this study introduces foundational auction theory, covering various auction types, basic models, and a brief understanding of the Revenue Equivalence Theorem. To conclude, this paper lists some examples of contemporary auction websites.

Contents

1. Why auctions?

Most consumers are familiar with the standard posted prices, but a significant portion of business transactions takes place through negotiations or auctions. The larger the scale of the transaction in terms of cost and complexity, the more likely the transaction will be conducted via a negotiation process between the buyer and the seller or auction. One key factor influencing the choice between these mechanisms is the uncertainty about the correct price for the good or service.

It is crucial to recognize that products featuring posted prices, such as those found in supermarkets, typically belong to the world of mass-market commodities. Their prices have been carefully determined and their cost is low enough that it is inefficient to spend time on negotiation. On the other hand, when corporations engage in transactions with one another, the price is often determined through negotiation.

An auction serves either as an initial step leading to negotiation or as a standalone method for price discovery. For example, in real estate, potential buyers bid for the opportunity to negotiate with the seller, and for unique and highly desired items, an auction may be the sole practical means of sale.

Furthermore, auctions have an important benefit over negotiations. By following a strict protocol, auctions have a transparency that gives participants confidence that they are being treated fairly. [2]

2. Advantages of online auctions

The rise of online auctions has yielded numerous advantages. Transaction costs for both buyers and sellers have seen significant reductions, and the process of showing and describing products has become more straightforward. The digital platform facilitates the execution of intricate auctions and broadens participation. This accessibility not only diversifies the auction marketplace, theoretically a positive outcome, but also facilitates collaboration among geographically separated individuals — a departure from the constraints of traditional auctions.

3. Online auctions success and prices

In the world of online auctions, success and prices are influenced by an interplay of factors that can be categorized into three distinct categories: trust enhancement, transaction facilitation, and product attributes. [3]

3.1. Trust-Enhancing Factors.

One pivotal category revolves around elements designed to bolster trust and alleviate information asymmetry concerns. Notably, sellers' feedback scores serve as a critical indicator of reputation, playing a substantial role in cultivating trust among potential buyers and subsequently impacting auction success and prices. Additionally, comprehensive descriptions, including details about sellers, items, and transaction conditions, contribute significantly to the overall success and pricing dynamics.

3.2. **Transaction-Enhancing Factors.**

Precise adjustments to auction parameters play a crucial role. This includes establishing the initial bid and reserve price, providing various payment methods, determining the auction's duration, and setting shipping costs — each element contributing intricately to shaping the overall experience.

3.3. **Product Attributes.**

Product models, conditions, and the inclusion of bundled items all play a role in shaping the perceived value of the offerings. It's crucial to understand how these product attributes align with buyer preferences and market demands for achieving optimal results in online auctions.

4. Auction theory [1]

4.1. **The standard auction types.**

4.1.1. *Ascending-bid.*

In the ascending auction, the price is successively raised until one bidder remains, and that bidder wins the object at the final price. This auction can be run by having the seller announce prices, or by having the bidders call out prices themselves. In the model most commonly used by auction theorists (often called the Japanese auction), the price rises continuously while bidders gradually quit the auction. Bidders observe when their competitors quit, and once someone quits, they are not let back in. This way, there is no possibility for one bidder to preempt the process by making a large "jump bid".

4.1.2. *Descending-bid.*

The descending auction works in exactly the opposite way: the auctioneer starts at a very high price and then lowers the price continuously. The first bidder who calls out that they will accept the current price wins the object at that price.

4.1.3. *First-price sealed-bid auction.*

In the first-price sealed-bid auction, each bidder independently submits a single bid, without seeing others' bids, and the object is sold to the bidder who makes the highest bid. The winner pays their bid, i.e., the price is the highest or "first" price bid.

4.1.4. *Second-price sealed-bid auction.*

In the second-price sealed-bid auction, each bidder independently submits a single bid, without seeing others' bids, and the object is sold to the bidder who makes the highest bid. However, the price they pay is the second-highest bidder's bid, or "second price". This auction is sometimes called Vickrey auction.

4.2. **Basic models of auctions.**

A key feature of auctions is the presence of asymmetric information. (With perfect information most auction models are relatively easy to solve.)

4.2.1. *Private-value model.*

Within the private-value model, individual bidders ascribe a personal valuation to the item, which is uniquely personal and remains undisclosed to other participants.

4.2.2. *Pure common-value model.*

The value is equal to all, but each bidder has different private information about what that value actually is. For example, the value of an oil lease depends on how much oil is under the ground, and bidders may have access to different geological “signals” about that amount (different private information). In this case, a bidder would change their estimate of the value if they learnt another bidder’s signal, in contrast to the private-value case in which their value would be unaffected by learning any other bidder’s preferences or information.

4.2.3. *General model.*

A general model encompassing both these as special cases assumes each bidder receives a private information signal, but allows each bidder’s value to be a general function of all the signals. For example, your value for a painting may depend mostly on your own private information (how much you like it) but also somewhat on other’s private information (how much they like it) because this affects the resale value and/or the prestige of owning it.

4.3. **Similarities in standard auction types.**

Consider first the descending auction. Each bidder must choose a price at which they will call out, conditional on no other bidder having yet called out; and the bidder who chooses the highest price wins the object at the price they call out. Thus this game is strategically equivalent to the first-price sealed-bid auction. This is why the descending auction is sometimes referred to as an open first-price auction.

With private values, now in the ascending auction, it is clearly a dominant strategy to stay in the bidding until the price reaches your value.

In other words, in an ascending auction with private values, the idea is that each bidder has a personal valuation for the item up for auction, known only to themselves. The strategy for bidders is to stay in the bidding until the price reaches their own valuation, the point at which they become indifferent between winning and not winning.

The second-to-last person drops out when their value is reached. The winner is the person with the highest valuation, and they end up paying a price equal to the value of the second-highest bidder. To clarify, since the second-to-last person drops out when their value is reached, that leaves the highest bidder as the winner at the price dictated by the second-highest bidder’s valuation.

Consider a second-price sealed-bid private-values auction. In this auction format, each bidder submits a sealed bid (a private bid) equal to their personal valuation. The highest bidder wins, but they pay the second-highest bid’s value. In this auction, it is optimal for each player to bid their true value because the highest bidder still wins at the price of the second-highest bidder.

Now comes the connection: the ascending auction is sometimes referred to as an open second-price auction. This is because in both auctions, the winner pays the value of the second-highest bidder. In the ascending auction, this happens as a result of the bidding process, while in the second-price sealed-bid auction, it’s explicit in the rules.

However, this equivalence only holds for private values or when there are just two bidders. If there are common components to valuations or more than two bidders, the dynamics change as players learn about their values based on when others drop out in an ascending auction, and they adjust their strategies accordingly.

4.4. Revenue Equivalence Theorem.

Theorem 4.4.1. *The revenue equivalence theorem states that,if all bidders are risk-neutral bidder and have independent private value for the auctioned items, then all four of the standard single unit auctions have the same expected sales price.*

[4]

In other words, revenue equivalence is a concept that suggests that, under certain conditions, different auction mechanisms can yield the same revenue for the auctioneer.

Revenue equivalence can be used to prove that many types of auctions are revenue equivalent. For example, the first price auction and second price auction are revenue equivalent when the bidders are symmetric.

Consider the second price single item auction. It is optimal for each player i to bid its own value $b_i = v_i$. Suppose i wins the auction, and pays $\max_{j\neq i} b_j$. The revenue from this auction is simply $\max_{j\neq i} b_j$.

In the first price auction, if all players bid using a bidding function $b(v) = E\Big(\max_{j\neq i} v(j) \mid v_j \leq v\forall_j\Big)$, this is a Nash equilibrium.

In other words, if each player bids such that they bid the expected value of second highest bid, assuming that theirs was the highest, then no player has any incentive to deviate. If this were true, then it is easy to see that the expected revenue from this auction is also $\max_{j\neq i} b_j$ if i wins the auction.

Proof. □

X is a set of possible outcomes.
There are n agents with different valuations for each outcome: $v_i : X \to \mathbb{R}_{\geq 0}$
(Expresses the value it has for each alternative, in monetary terms.)

Player 1 bids $b(z)$ where, $z < v$ (less than their actual valuation)
We want to find a value of z such that the player's expected payoff is maximized.
Probability of winning is $P(\max_{i>1} v_i < z)$ (highest valuation among opponents)

Expected cost is $E(\max_{i>1} v(i) \mid v_i \leq v\forall_i)$
(The expected value of the maximum value among opponents (excluding player 1), given that the value of any opponent is less than z.)

Expected payoff is $P \cdot (v - \text{cost})$
$\Leftrightarrow P(\max_{i>1} v_i < z) \cdot (v - E(\max_{i>1} v(i) \mid v_i \leq v\forall_i))$

(Using the general fact that:
$E(X \mid X \leq z) \cdot P(X < z) = \int_0^z P(X < z) - P(X < y)\ \mathrm{dy})$

$\Leftrightarrow P(X < z) \cdot v - \int_0^z P(X < z) - P(X < Y)\ \mathrm{dy}$
$\Leftrightarrow P(X < z) \cdot v - P(X < z) \cdot z - \int_0^z P(X < y)\ \mathrm{dy}$

Taking derivatives with respect to z, we obtain
$P(X < z)'(v - z) = 0 \Rightarrow v = z$

Thus bidding with your value v maximizes the player's expected payoff. Since $P(X < z)$ is monotone increasing, we verify that this is indeed a maximum point. [5]

5. Modern-day auction websites

List of modern-day auction websites

- https://www.liveauctioneers.com
- https://www.auctionzip.com/online-auctions
- https://www.catawiki.com/en
- https://www.invaluable.com/auctions
- https://www.dealdash.com
- https://www.christies.com
- https://www.artsy.net/auctions
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