

# 3Blue1Brown - Anotações

27 de junho de 2024 17:53

$$\frac{s(t + dt) - s(t)}{dt}$$

Chapter 2:

$dt = 0.01$ , for example  
 ↪ na realidade.

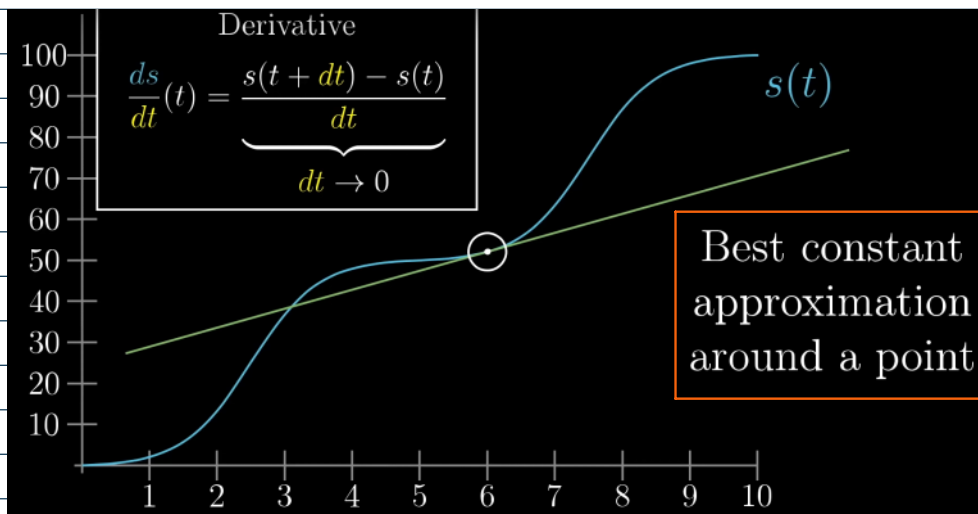
Em "pure math":

$$\frac{s(t + 0.00...0001) - s(t)}{0.00...0001}$$

↪ approaches 0  
 $dt \rightarrow 0$

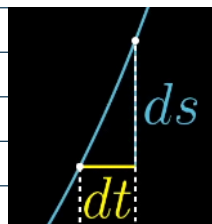
⇒ declive da tangente num  
 ponto de  $s(t)$

$dt$  is not "infinitely small"  
 $dt$  is not 0  
 it just approaches 0



$$s(t) = t^3$$

$$\begin{aligned} \frac{ds}{dt}(2) &= \frac{s(2 + dt) - s(2)}{dt} \\ &= \frac{(2 + dt)^3 - 2^3}{dt} \end{aligned}$$



tiny changes in distance  
 tiny changes in time

$$3(2)^2 + 3(2)(dt) + (dt)^2$$

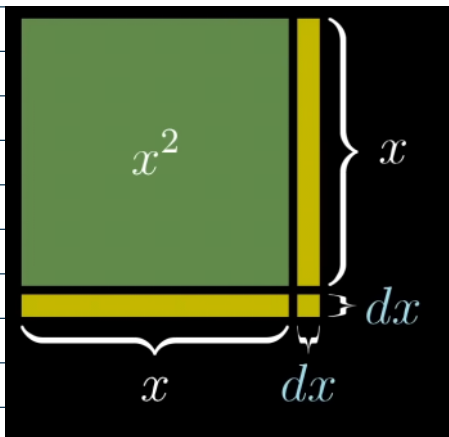
Contains  $dt$

$$\downarrow$$

$$3 \times t^2$$

$\hookrightarrow$  approaches 0

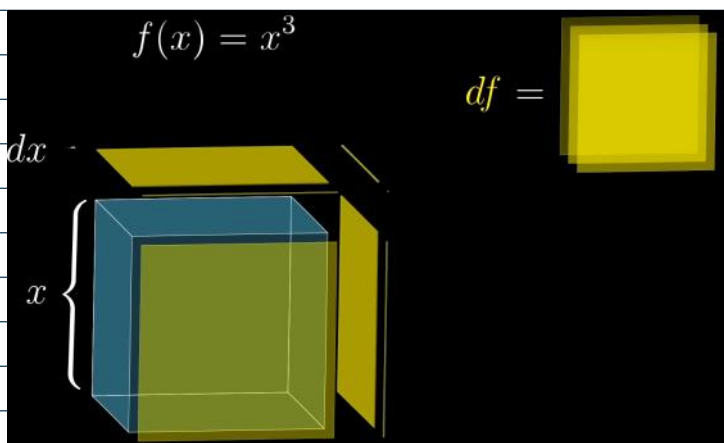
### Chapter 3



$$2 \times (x dx) + \underbrace{dx^2}$$

$\hookrightarrow$  negligibly tiny

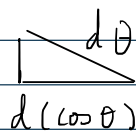
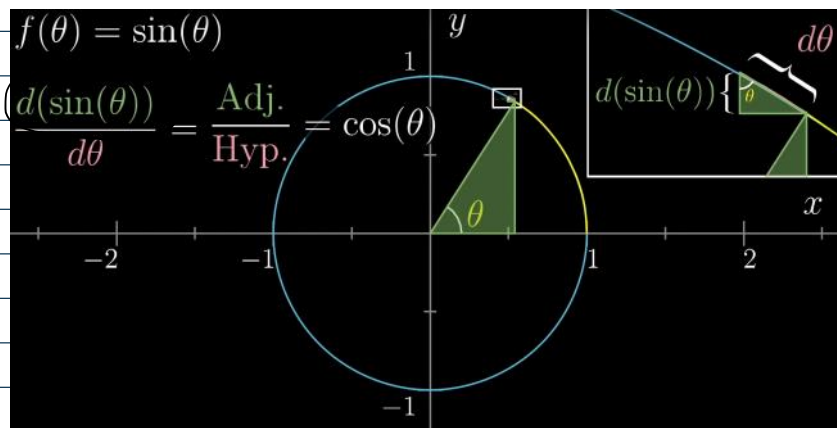
$$\Rightarrow \frac{df}{dx} = 2x$$



$$\rightarrow 3x dx$$

$$\begin{aligned}\frac{d(x^1)}{dx} &= 1x^0 \\ \frac{d(x^2)}{dx} &= 2x^1 \\ \frac{d(x^3)}{dx} &= 3x^2 \\ \frac{d(x^4)}{dx} &= 4x^3 \\ \frac{d(x^5)}{dx} &= 5x^4 \\ \frac{d(x^n)}{dx} &= nx^{n-1}\end{aligned}$$

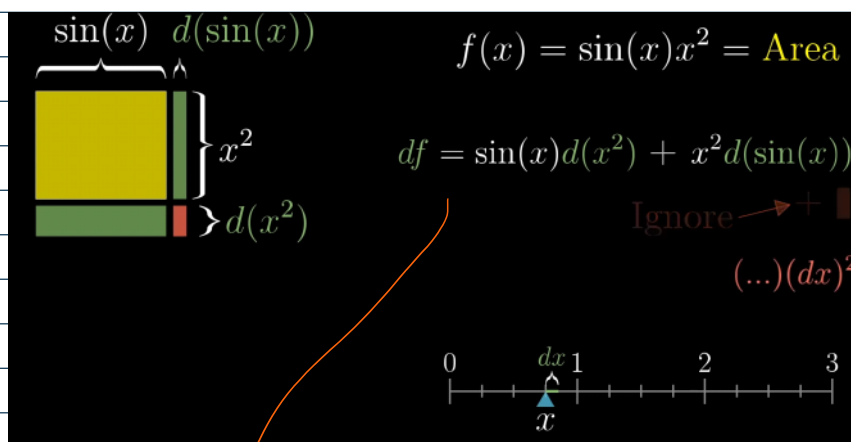
"Power rule"



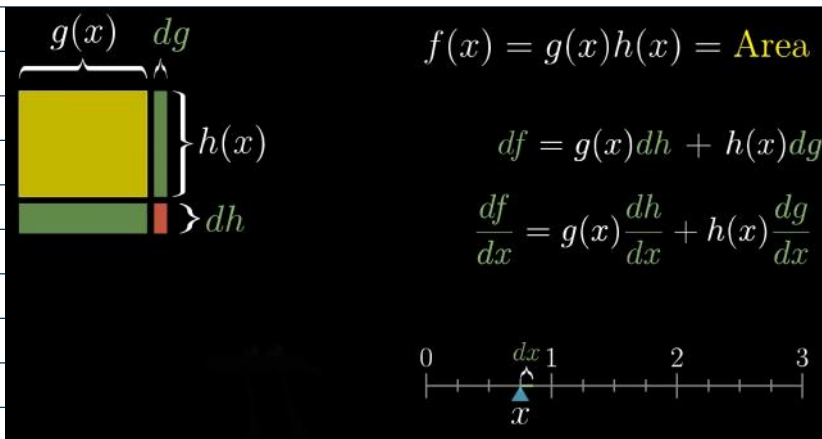
$$\frac{d(\cos(\theta))}{d\theta} = - \frac{\text{Opp}}{\text{Hyp}} = -\sin(\theta) ?$$

## Chapter 4

↳ Regra da cadeia e Regra do produto



$$\frac{df}{dx} = \sin(x) 2x + x^2 \cos(x)$$

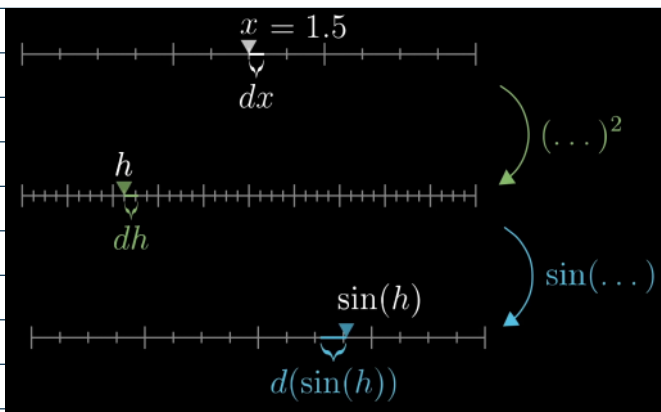


## FUNCTION COMPOSITION

$g(x) = \sin(x) \quad h(x) = x^2$

$g(h(x)) = \sin(x^2)$

Derivative?



$\hookrightarrow d(\sin(x^2)) = \cos(x^2) d(x^2)$

$= \cos(x^2) 2x dx$

$\Rightarrow \frac{d(\sin(x^2))}{dx} = \cos(x^2) 2x$

$\hookrightarrow$  CHAIN RULE

$\frac{d}{dx} \sin(x^2) = \cos(x^2) 2x$

Inner

Outer

$\frac{d}{dx} g(h(x)) = \frac{dg}{dh}(h(x)) \frac{dh}{dx}(x)$

$= \frac{dg}{dx}$

## Chapter 5

All exponential functions are proportional to their own derivative

But e has a proportionality constant = 1

$$M(t) = e^t \quad \frac{e^{0.00000001} - 1}{0.00000001} = 1.0000000 \dots$$

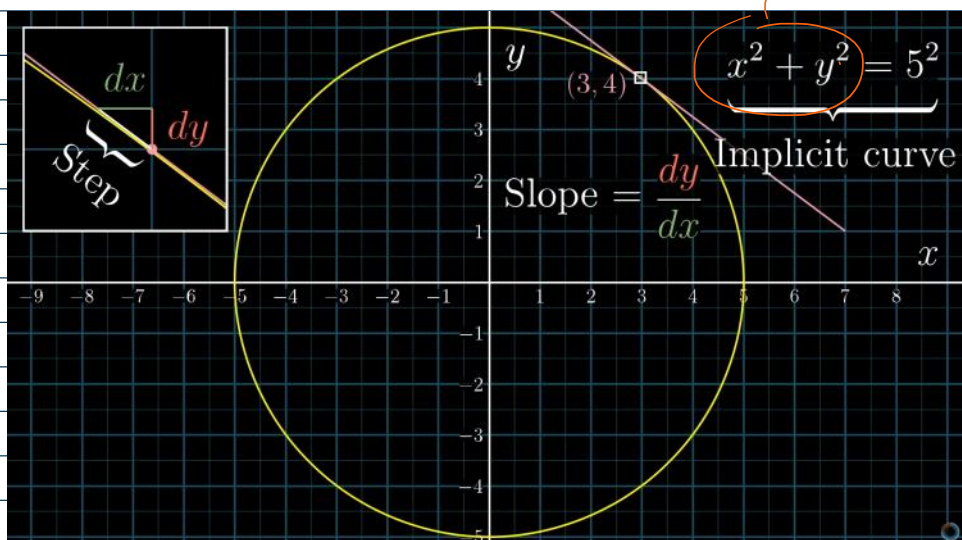
$$\frac{dM}{dt}(t) = e^t \underbrace{\left( \frac{e^{dt} - 1}{dt} \right)}_{dt \rightarrow 0}$$

$$2^t = 2^{\ln(2)t}$$

## Chapter 6 : Diferenciação Implícita

→ Variáveis interdependentes

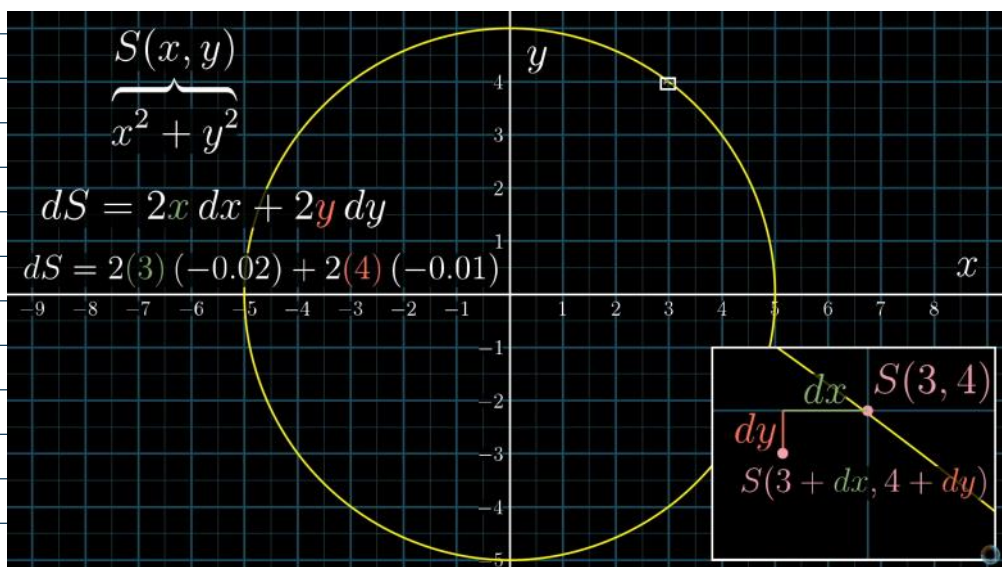
≠ Função

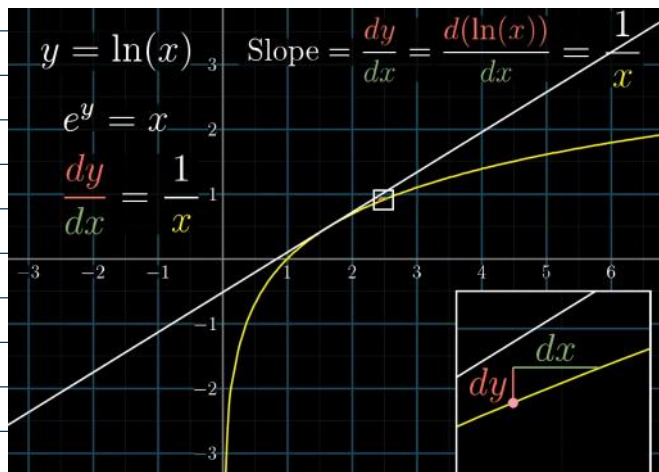


$$2x dx + 2y dy = 0$$

$$\implies \frac{dy}{dx} = -\frac{x}{y}$$

↓  
implicit differentiation

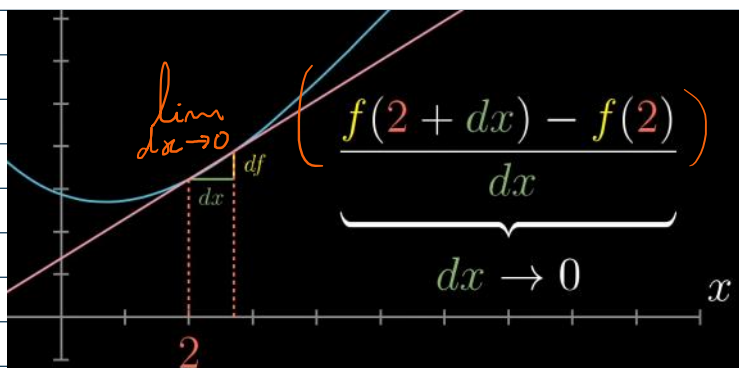




## Capítulo 7: Limites, regra de L'Hopital e definições de $\delta$ , $\epsilon$

"Calculus required continuity, and continuity was supposed to require the infinitely little; but nobody could discover what the infinitely little might be."

- Bertrand Russel



How to compute limits?

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{\tan x} \quad \sin^2 x = 1 - \cos^2 x$$

$$\frac{1 - (\cos^2 x - \sin^2 x)}{\frac{\sin x}{\cos x}}$$

$$= \frac{1 - \cos^2 x + \sin^2 x}{\frac{\sin x}{\cos x}} = \frac{2 \sin^2 x \cos x}{\sin x}$$

$$= 2 \sin x \cos x$$

$$= \sin(2x)$$

$$= \sin(0)$$

$$= 0$$

ou

usar a regra  
de L'Hopital

— // —

# TAYLOR SERIES

Non polinomial function ~~~~> Polinomial function

Near  $x = 0$

$$e^x \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

easier to deal with

$$\cos(x) \xrightarrow{x=0} 1$$

$$P(x) = 1 + 0x + \left(-\frac{1}{2}\right)x^2$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{dP}{dx}(x) = 0 + 2\left(-\frac{1}{2}\right)x$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^2P}{dx^2}(x) = 2\left(-\frac{1}{2}\right)$$

Encontrar m, em  $m \cdot x$ , para que o valor da segunda derivada seja igual à segunda derivada da função que se quer aproximar

Melhoria:

$$\cos(0) = 1$$

$$P(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$$

$$\frac{d(\cos)}{dx}(0) = -\sin(0) = 0$$

$$\frac{d^4P}{dx^4}(x) = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \frac{1}{24}$$

$$= 24 \cdot \frac{1}{24}$$

$$\frac{d^2(\cos)}{dx^2}(0) = -\cos(0) = -1$$

$$\frac{d^3(\cos)}{dx^3}(0) = \sin(0) = 0$$

$$\frac{d^4(\cos)}{dx^4}(0) = \cos(0) = 1$$

$$\cos(0) = 1$$

$$P(x) = 1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!} + \dots$$

"Taylor polynomial"

$$-\sin(0) = 0$$

$$-\cos(0) = -1$$

$$\sin(0) = 0$$

$$\cos(0) = 1$$

VERSÃO GENÉRICA:

