Espectro de um sinal

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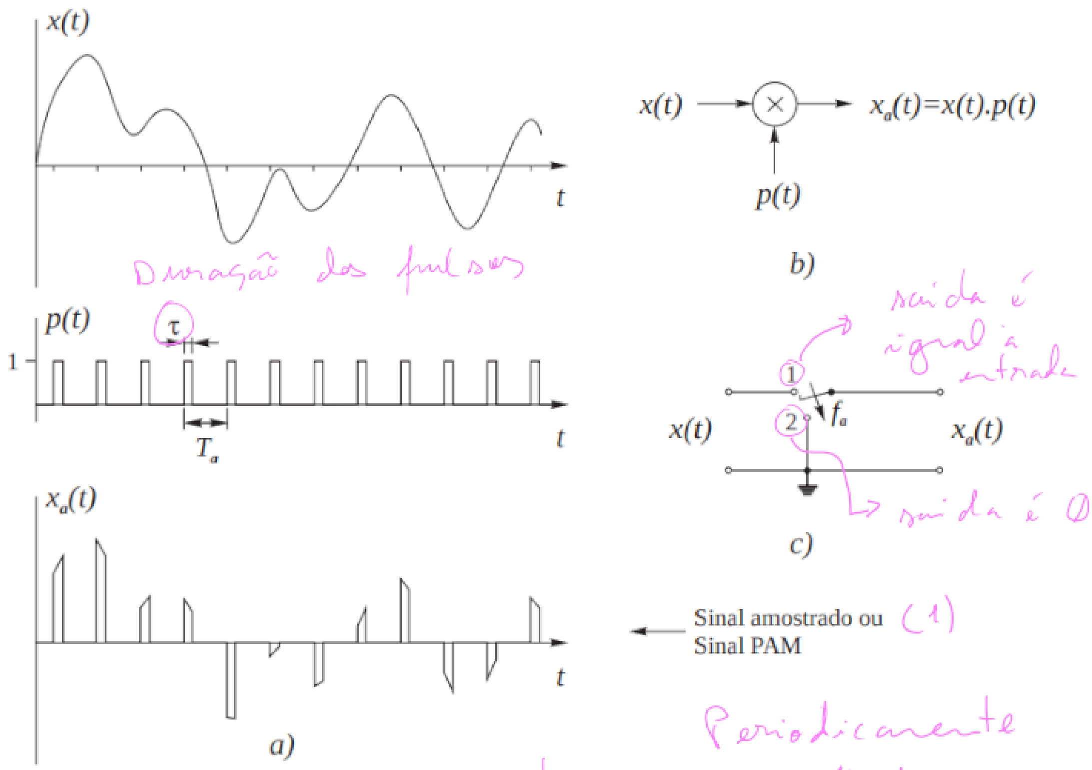
Tags: #FCD #SoftwareEngineering #uni

AMOSTRAGEM

A operação de amostragem do sinal $x(t)$ consiste em tomar os valores do sinal em instantes regularmente espaçados no tempo e considerar que nos intervalos de tempo entre esses instantes o sinal tem amplitude nula.

o *período de amostragem* que designaremos por T_a e cujo inverso é a *frequência de amostragem* em Hertz, $f_a = 1/T_a$ Hz.



Periodicamente toca na posição 1

(1) Pulse Amplitude Modulation

ESPECTRO

→ Define representation of general signal as a sum of sinusoids

Any $x(t)$ can be described on $0 \le t < T$ using a sum of sinusoids

$$x(t) = A_0 + A_1\cos(2\pi f_1 t + \phi_1) + A_2\cos(2\pi f_2 t + \phi_2) + \cdots + A_N\cos(2\pi f_N t + \phi_N)$$

Amplit

$$= A_0 + \sum_{k=0}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

phases

(may require N large)

$$x(t) \longleftrightarrow A_0, (f_1, A_1, \phi_1), \ldots, (f_N, A_N, \phi_N)$$

varies w. time t varies w. frequency f_k

Spectrum $X(f)$ uses A_k, ϕ_k to represent $x(t)$ as a function of frequency

EXAMPLE :

$$x(t) = A\cos(2\pi f_0 t + \phi)$$

→ Graph Spectrum $X(f)$

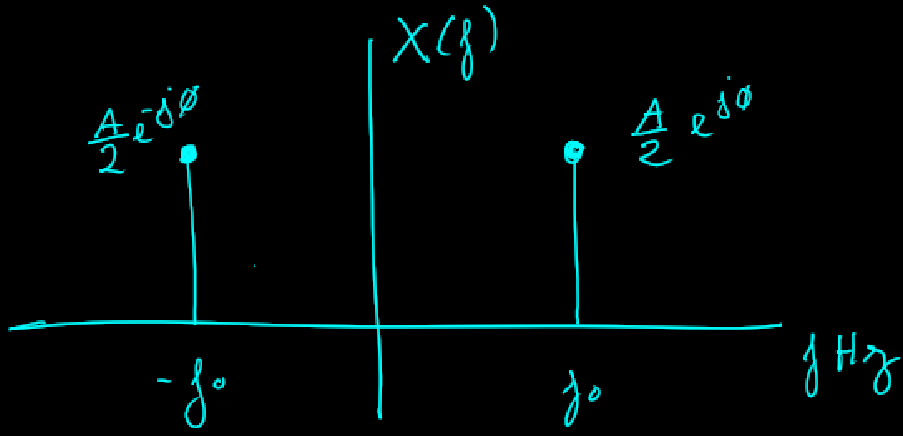
→ Decompose $x(t)$ (EULER EXPANSION)

Euler expansion:

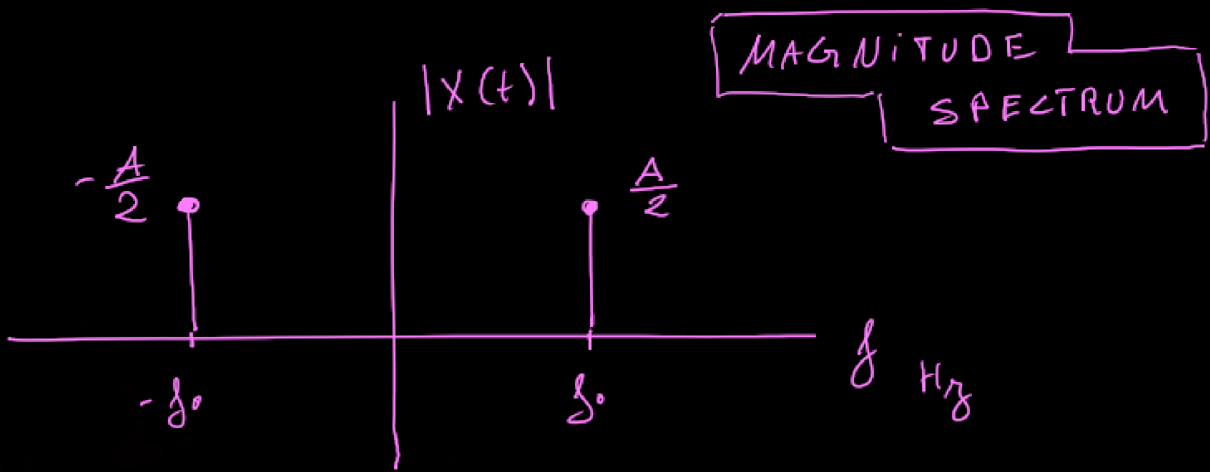
$$x(t) = \frac{A}{2} e^{j(2\pi f_0 t + \phi)} + \frac{A}{2} e^{-j(2\pi f_0 t + \phi)}$$

$$= \frac{A}{2} e^{j\phi} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j\phi} e^{-j2\pi f_0 t}$$

$$X(f) = \begin{cases} A/2\, e^{j\phi}, & f = f_0 \\ A/2\, e^{-j\phi}, & f = -f_0 \end{cases}$$



Now, without Ø (phase):



General case

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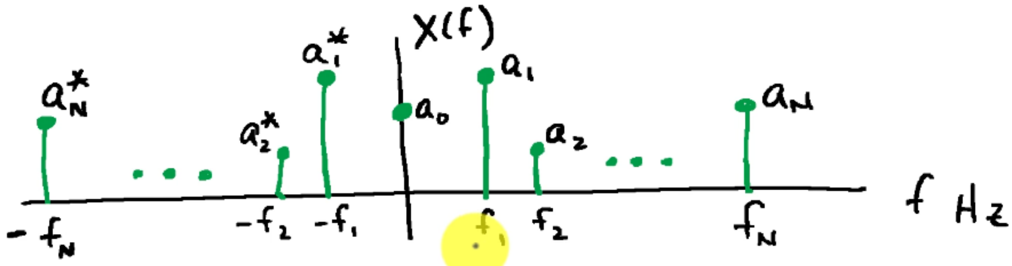
$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

$$= A_0 + \sum_{k=1}^{N} \left(\frac{A_k}{2} e^{j\phi_k} e^{j2\pi f_k t} + \frac{A_k}{2} e^{-j\phi_k} e^{-j2\pi f_k t} \right)$$

$$= a_0 + \sum_{k=1}^{N} \left(a_k e^{j2\pi f_k t} + a_k^* e^{-j2\pi f_k t} \right)$$

$$a_0 = A_0, \quad a_k = \frac{A_k}{2} e^{j\phi_k}$$



Example: Saxophone Concert A (440 Hz)

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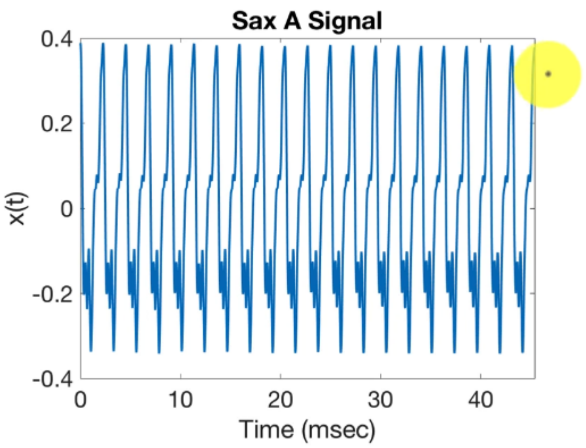

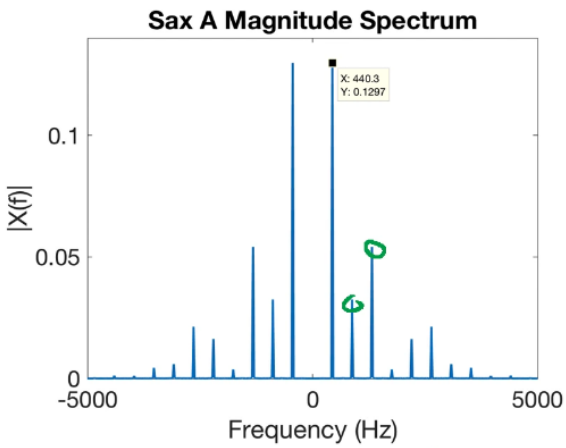

frequency components at 440, 880, 1320, etc

amplitude of 1320 > amplitude of 880

Example: $x(t) = 3 + 2\cos(20\pi t + \pi/3) + \cos(30\pi t - \pi/4)$

Graph $X(f)$.

$$x(t) = \underbrace{3}_{a_0} + \underbrace{\frac{2}{2}e^{j\pi/3}}_{a_1} e^{j2\pi \underbrace{10}_{f_1} t} + \underbrace{\frac{2}{2}e^{-j\pi/3}}_{a_1^*} e^{-j2\pi 10 t}$$

$$+ \underbrace{\frac{1}{2}e^{-j\pi/4}}_{a_2} e^{j2\pi \underbrace{15}_{f_2} t} + \underbrace{\frac{1}{2}e^{j\pi/4}}_{a_2^*} e^{-j2\pi 15 t}$$

Compare to

$$x(t) = a_0 + \sum_{k=1}^{N} a_k e^{j2\pi f_k t} + a_k^* e^{-j2\pi f_k t}$$

$N = 2$