Ficha 1

22 de junho de 2024 13:44

1. $x^2 - 5x + 3 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4}}{2 \times 1}$$

$$x_1 = \frac{5 + \sqrt{13}}{2} \wedge x_2 = \frac{5 - \sqrt{13}}{2}$$

Calculadora:
3. Álgebra
↳ Ferramentas polinomiais
↳ Calcular raízes

2. $t(x) = (m-2)x^2 - 2mx + 3m$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a = \sqrt{m-2}\,x$$
$$b = \sqrt{3m}$$

$$-2mx = 2\left(\sqrt{m-2}\,x\right)\left(\sqrt{3m}\right)$$
$$\Leftrightarrow mx = \sqrt{m-2}\sqrt{3m}\,x$$
$$\Leftrightarrow m = \sqrt{m-2}\sqrt{3m}$$
$$\Leftrightarrow m^2 = 3m(m-2)$$
$$\Leftrightarrow m^2 = 3m^2 - 6m$$
$$\Leftrightarrow -2m^2 + 6m = 0$$
$$\Leftrightarrow m(-2m+6) = 0$$
$$\Leftrightarrow m = 0 \vee -2m + 6 = 0$$
$$\Leftrightarrow m = 0 \vee m = 3$$

$\Rightarrow (x-3)^2 = x^2 - 6x + 9$

3. Fatorizar:

a) $f(x) = 2x^2 - 4x - 6$

GROUPING:

$ax^2 + bx + c$

$u \cdot v = a \times c$ $\quad\Longrightarrow\quad (ax^2 + ux) + (vx + c)$

$u + v = b$

$2(x^2 - 2x - 3)$

$= 2(x^2 + x)(-3x - 3)$

$= 2x(x+1) - 3(x+1)$

$= 2(x-3)(x+1)$

C.A.

$\begin{cases} u \cdot v = 1 \times -3 = -3 \\ u + v = -2 \end{cases}$

fatores de -3: -1 e -3

$\ldots \Rightarrow u = 1$

$v = -3$

b) $x - 2\sqrt{x} - 8$

$x\left(1 - \frac{2}{\sqrt{x}}\right) - 8$?

4. $(a+b)^3 = (a+b)(a+b)(a+b)$

$= (a^2 + ab + ba + b^2)(a+b)$

$= (a^2 + 2ab + b^2)(a+b)$

$= a^3 + a^2b + 2a^2b + 2ab^2 + b^2a + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

→ Binómio de Newton

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^{n-k} y^k$$

(número de elementos agrupados k a k) $\rightarrow \frac{3!}{1!(3-1)!} = \frac{6}{2} = 3$

5. $a + (b - c) > b - 2a$, em ordem a a

5. $a + (b - c) > b - 2a$, em ordem a a

$\Leftrightarrow a - c > -2a$

$\Leftrightarrow a > -2a + c$

$\Leftrightarrow 3a > c$

$\Leftrightarrow a > c/3$

6. $x, y \in \mathbb{R}$: $x < y$

a) $|x| < |y|$

$x \geq 0$ e $y \geq 0 \Rightarrow |x| < |y|$

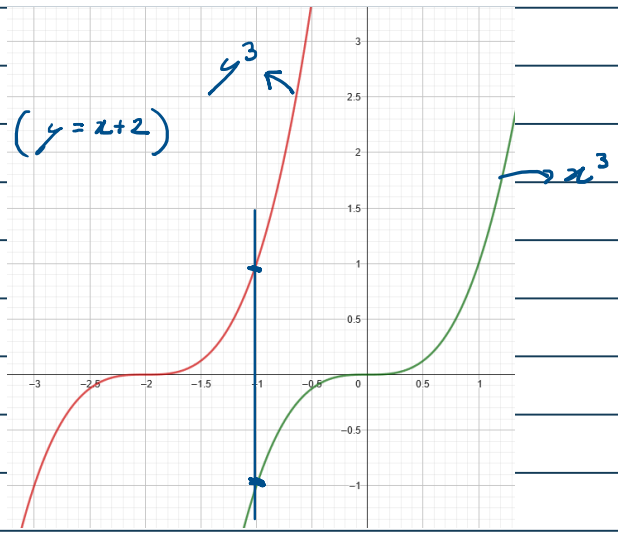
$x \leq 0$ e $y \geq 0 \not\Rightarrow |x| < |y|$

exemplo: $-300 < 7$

$|-300| \not< |7|$

b) $\sqrt{x} < \sqrt{y}$, Verdadeiro $x, y \geq 0$

c) $x^3 < y^3$, Verdadeira



y^3 é sempre maior do que x^3

d) $\frac{1}{x} < \frac{1}{y}$, $(x, y \neq 0)$

Falso, quanto menor o denominador ($\neq 1$) maior o resultado, como x é menor do que y $\frac{1}{x}$ pode ser maior do que $\frac{1}{y}$

$\frac{1}{x} < \frac{1}{y}$ quando $x < 0$ e $y < 0$

$\frac{1}{x} > \frac{1}{y}$ quando $x > 0$ e $y > 0$

$\frac{1}{x} < \frac{1}{y}$ quando $x < 0$ e $y > 0$

} 3 CASOS POSSÍVEIS

7. Simplificar as expressões

a) $\frac{(4 \cdot 3)^{10} + 4^9}{8^4}$

$= \frac{(2^2)^{10} \times 3^{10} + (2^2)^9}{(2^3)^4} = \frac{2^{20} \times 3^{10} + 2^{18}}{2^{12}}$

$= 2^8 \times 3^{10} + 2^6$

b) $\frac{3^{-\frac{8}{11}}}{-9} - \frac{4}{11} = \frac{1}{\frac{3^{\frac{8}{11}}}{-9}} - \frac{4}{11}$

$\frac{22}{11} + \frac{8}{11} = \frac{30}{11}$

$= \frac{1}{3^2 \times 3^{\frac{8}{11}}} - \frac{4}{11} = \frac{1}{3^{\frac{30}{11}}} - \frac{4}{11}$

c) $x^{\frac{5}{2}}\left(x^{-\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{\frac{7}{2}}\right)$

$= x + 2x^3 + 3x^6$

$a^2 - b^2 = (a-b)(a+b)$

d) $\frac{4x^2 - 3y + 6x - y^2}{4x^2 - y^2 - 6y - 9}$

$4x^2 - y^2$

$= (2x - y)(2x + y) - 3(2x - y)/\ldots$

$= ((2x + y) - 3)(2x - y)$

$= \frac{(2x + y - 3)(2x - y)}{(2x)^2 - (y - 3)^2}$

$= \frac{(2x + y - 3)(2x - y)}{(2x - (y - 3))(2x + (y - 3))}$

$= \frac{(2x + y - 3)(2x - y)}{(2x - y + 3)(2x + y - 3)}$

$$= \frac{(2x+y-3)(2x-y)}{(2x-y+3)(2x+y-3)}$$

$$= \frac{2x-y}{2x-y+3}$$

$$(2a+1)^2 - (m+k)^2 = (2a+1-m-k)(2a+1+m+k)$$

e) $$\frac{4a^2+4a+1-m^2-2mk-k^2}{8a^2m+4am-4am^2-4amk}$$

$$= 4a^2+2a+2am+2ak + 2a+1+m+k + 2am+m+m^2+mk + 2ak+k+mk+k^2 = 4a^2+4a+4am+4ak+2m+1+\times$$

$$\frac{(2a+1-m-k)(2a+1+m+k)}{4ma(2a+1-m-k)}$$

$$= \frac{2a+1+m+k}{4ma}$$

f) $$\frac{1+5a}{6a^2-6} + \frac{1}{3a+3} + \frac{1}{2a-2}$$

$$\frac{1+5a}{6(a^2-1)} + \frac{2(a-1)}{3(a+1)2(a-1)}$$

$$\hookrightarrow \frac{2(a-1)}{6(a^2-1)} + \frac{3(a+1)}{6(a^2-1)}$$

$$= \frac{1+5a+2(a-1)+3(a+1)}{6(a^2-1)}$$

$$= (1+5a+2a-2+3a+3)/\ldots$$

$$= \frac{2+10a}{6(a^2-1)} = \frac{1+5a}{3a^2-3}$$

8. $x^3 - 11x^2 + 37x - 30$ | $x^2 - 5x + 6$

$$-x^3 + 5x^2 - 6x \qquad x-6$$

$$0 - 6x^2 + 31x - 30$$

$$+6x^2 - 30x + 36$$

$$0 \qquad x + 6$$

$$x^3 - 11x^2 + 37x - 30$$

$$= (x^2 - 5x + 6)(x-6) + (x+6)$$

9. a) $\{x \in \mathbb{R} : |x+4| = 3\}$

$$\begin{cases} x+4 = 3 \\ -(x+4) = 3 \end{cases} \Leftrightarrow \begin{cases} x = -1 \\ x = -7 \end{cases}$$

$\{x \in \mathbb{R} : x = -1 \vee x = -7\}$

b) $\{x \in \mathbb{R} : \sqrt{(x+1)^2} = 3\}$

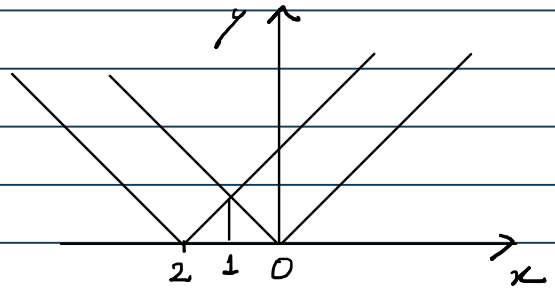
$= \{x \in \mathbb{R} : |x+1| = 3\}$

$= \{x \in \mathbb{R} : x+1 = 3 \vee -(x+1) = 3\}$

$= \{x \in \mathbb{R} : x = 2 \vee x = -4\}$

c) $\{x \in \mathbb{R} : |x| = |x+2|\}$

$= \{x \in \mathbb{R} : x = 1\}$



d) $\{x \in \mathbb{R} : (x^2-7)^2 = 0\}$

$(x^2-7)(x^2-7) = 0$

$\Leftrightarrow (x^2-7) = 0$

$\Leftrightarrow x^2 = 7 \Leftrightarrow x = \sqrt{7} \vee x = -\sqrt{7}$

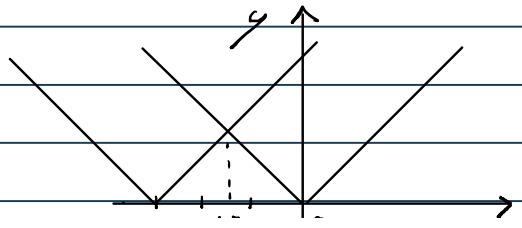
e) $\{x \in \mathbb{R} : \sqrt{3x+1} = 2x\}$

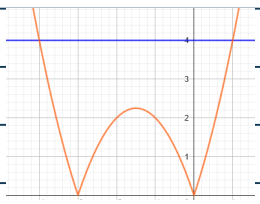
$3x+1 = 4x^2$

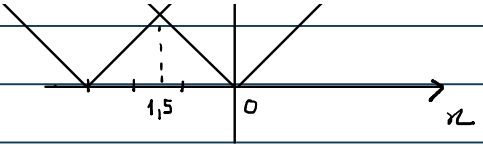
$\Leftrightarrow -4x^2+3x+1 = 0$

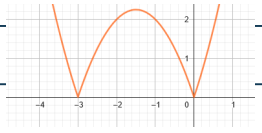
$\{x \in \mathbb{R} : x = 1 \vee x = -1/4\}$

f) $\{x \in \mathbb{R} : |x||x-3| = 4\}$

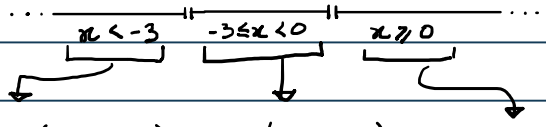








$|x \cdot y| = |x||y|$



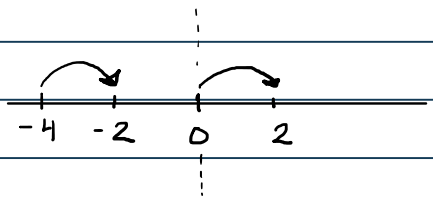
$(-x)(-(x+3)) = 4$ $\quad x(-(x+3)) = 4$ $\quad x(x+3) = 4$

$\Leftrightarrow x = -4$ $\quad \Leftrightarrow \{\}$ $\quad \Leftrightarrow x = 1$

10. a) $]-2, 2[$

$= -2 < x < 2 = |x| < 2$

b) $]-4, 0[$



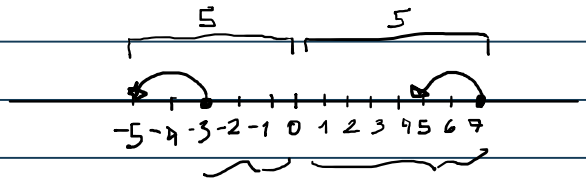
$-4 < x < 0$

$= -4+2 < x+2 < 0+2$

$= |x+2| < 2$

c) $]0, 4[\quad = \quad |x-2| < 2$

d) $]-3, 7[$



$= -3 < x < 7$

$= -5 < x-2 < 5$

$= |x-2| < 5$

11.

a) $\{x \in \mathbb{R} : 1 - x \leq 2\}$

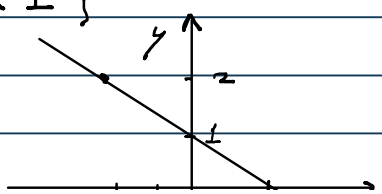
$1 - x \leq 2 \Leftrightarrow -x \leq 1 \Leftrightarrow x \geq -1$

b) $\{x \in \mathbb{R} : 0 \leq 1 - 2x < 1\}$

$y = -2x + 1$

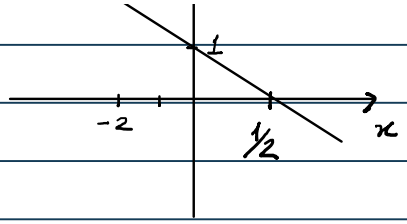
$0 = -2x + 1$

$\Leftrightarrow x = 1/2$



$0 = -2x + 1$

$\Leftrightarrow x = 1/2$

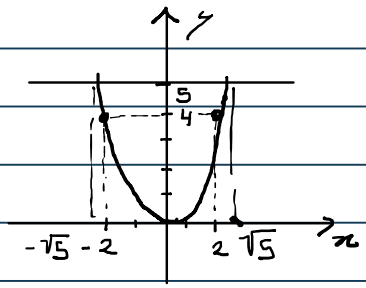


$]0, 1/2]$

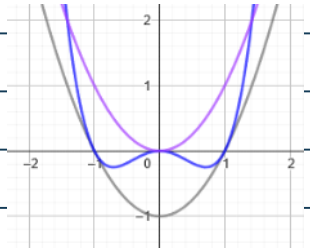
c) $\{ x \in \mathbb{R} : x^2 > 5 \}$

$x < -\sqrt{5}$ e $x > \sqrt{5}$

$]-\infty, -\sqrt{5}[\cup]\sqrt{5}, +\infty[$



d) $\{ x \in \mathbb{R} : x^2(x^2 - 1) \geq 0 \}$



$x^2 (x-1)(x+1) \geq 0$

$0 \quad 1 \quad -1$

	$x < -1$	$x = -1$		$x = 0$		$x = 1$
x^2	+	+	+	0	+	+
$x+1$	−	0	+	+	+	0
$x-1$	−	−	−	−	−	0
$x^2(x+1)(x-1)$	+	0	−	0	−	0

$x \leq -1$ ou $x = 0$ ou $x \geq 1$

e) $\{ x \in \mathbb{R} : |5 - \frac{1}{x}| < 1 \}$

$-1 < 5 - \frac{1}{x} < 1 \quad , x \neq 0$

$-1 < 5 - \frac{1}{x} \Leftrightarrow -6 < -\frac{1}{x}$

$\Leftrightarrow 6 > \frac{1}{x} \Leftrightarrow x > \frac{1}{6}$

$5 - \frac{1}{x} < 1 \Leftrightarrow -\frac{1}{x} < -4$

$\Leftrightarrow \frac{1}{x} > 4 \Leftrightarrow \frac{1}{4} > x$

$\rightarrow \quad]\frac{1}{6}, \frac{1}{4}[$

f) $\{x \in \mathbb{R} : |3-x| \geq 2\}$

$-2 \geq 3-x \geq 2$

$3-x \leq -2 \iff -x \leq -5 \iff x \geq 5$

$3-x \geq 2 \iff -x \geq -1 \iff x \leq 1$

$x \in]-\infty, 1] \cup [5, +\infty[$

g) $\{x \in \mathbb{R} : |5x+2| \leq 1\}$

$-1 \leq 5x+2 \leq 1$

$x \in [-\frac{3}{5}, -\frac{1}{5}]$

h) $\{x \in \mathbb{R} : x^3 \geq 4x\}$

$x^3 = 4x \iff x^2 = 4 \iff x = 2 \vee x = -2 \vee x = 0$

	$x < -2$	$x = -2$		$x = 0$		$x = 2$	$x > 2$
x^3	$-$	$-$	$-$	0	$+$	$+$	$+$
$4x$	$-$	$-$	$-$	0	$+$	$+$	$+$

↳ quem é menor ↳ quem é maior

$\Rightarrow$ a exponencial

$x \in [-2, 0] \cup [2, +\infty[$

"inverte" no lado negativo

MELHOR FORMA:

$x^3 \geq 4x$

$\iff x^3 - 4x \geq 0$

$\iff x(x^2-4) \geq 0$

$\iff x(x-2)(x+2) \geq 0$

$\{x=0, x=2, x=-2\}$ zeros

$\Leftrightarrow x(x-2)(x+2) \geq 0$

$\{ x=0 \ , \ x=2 \ , \ x=-2 \}$ zeros

	$x<-2$	$x=-2$		$x=0$		$x=2$	$x>2$
x	−	−	−	0	+	+	+
$x-2$	−	−	−	−	−	0	+
$x+2$	−	0	+	+	+	+	+
$x(x+2)(x-2)$	−	0	+	0	−	0	+

≥ 0

$x \in [-2, 0] \cup [2, +\infty[$

$a^2x + bx + c$

$\begin{cases} uv = ac = -6 \\ u+v = b = -5 \end{cases}$

$\begin{cases} u = -6 \\ v = 1 \end{cases}$

$(ax^2 + ux) + (vx + c)$

i) $\{ x \in \mathbb{R} : 6x^2 - 5x \leq 1 \}$

$\Leftrightarrow 6x^2 - 5x - 1 \leq 0$

$(6x^2 - 6x) + (x - 1)$

$6x(x-1) + (x-1)$

$(6x+1)(x-1)$

$x = -\frac{1}{6}$ $\quad x = 1$

	$x<-\frac{1}{6}$	$x=\frac{1}{6}$		$x=1$	$x>1$
$6x+1$	−	0	+	+	+
$x-1$	−	−	−	0	+
$(6x-1)(x+1)$	+	0	−	0	+

$[-1/6, 1]$

j) $\{ x \in \mathbb{R} : |3x - 2| \leq 1 \}$

$-1 \leq 3x - 2 \leq 1$

$\cdots \quad x \in [1/3, 1]$

k) $\{ x \in \mathbb{R} : 2 < |x| < 3 \}$

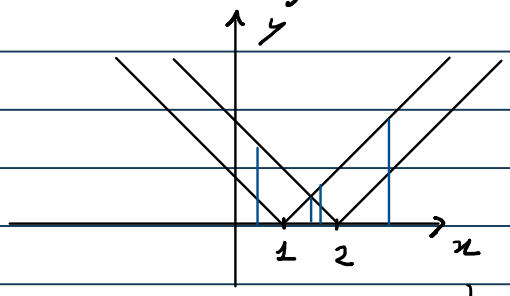
$x \in]-3, -2[\cup]2, 3[$

l) $\{ x \in \mathbb{R} : |x - 1| < |x - 2| \}$

l) $\{x \in \mathbb{R} : |x-1| < |x-2|\}$

Com o gráfico:

$]-\infty, \frac{3}{2}[$



Com cálculos:

$\left\{ \begin{array}{l} x-1 \geq 0 : x \geq 1 \,,\ |x-1| = x-1 \\ x-1 < 0 : x < 1 \,,\ |x-1| = -(x-1) \end{array} \right\}$

$\left\{ \begin{array}{l} x-2 \geq 0 : x \geq 2 \,,\ |x-2| = x-2 \\ x-2 < 0 : x < 2 \,,\ |x-2| = -(x-2) \end{array} \right\}$

$\Rightarrow \boxed{x < 1 \,,\ 1 \leq x < 2 \,,\ x \geq 2}$ Intervalos

	$x<1$		$x \geq 2$
$\lvert x-1 \rvert$	−	+	+
$\lvert x-2 \rvert$	−	−	+

$-(x-1) < -(x-2)$ Verdadeiro $\forall x$ $\wedge\ x < 1$ $\Rightarrow x < 1$

$x-1 < -(x-2) \iff x < 3/2$ $\wedge\ 1 \leq x < 2$ $\Rightarrow 1 \leq x \leq \frac{3}{2}$

Sem solução

União : $x \leq \frac{3}{2}$

m) $\{x \in \mathbb{R} : |x+2| + |x+2| < 10\}$

$2|x+2| < 10$

$\iff |x+2| < 5$

$\iff -5 < x+2 < 5$

$\iff x > -7 \ \wedge \ x < 3$

n) $\left\{ x \in \mathbb{R} : \frac{1-x}{2x+3} > 0 \right\}$ $\quad 2x+3 = 0 \iff x = -\frac{3}{2}$

	$x < -\frac{3}{2}$	$x = -\frac{3}{2}$		$x = 1$	$x > 1$
$1 - x$	+	+	+	0	−
$2x + 3$	−	0	+	+	+
$(1-x)/(2x+3)$	−	Ind.	+	0	−

$\hookrightarrow x \in]-\frac{3}{2}, 1[$

o) $\{x \in \mathbb{R} : |x^2 - 1| \leq 1\}$

$\sqrt{x^2} \geq \sqrt{0}$
$x \geq 0 \ \vee \ -x \geq 0$

$-1 \leq x^2 - 1 \leq 1$

$x^2 \geqslant 0 \quad \vee \quad x^2 \leq 2$

Verdadeiro $\forall x$ $\quad (x \leq \sqrt{2} \ \vee \ x \leq -\sqrt{2})$

$x \in [-\sqrt{2}, \sqrt{2}]$

$\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

n) $\{x \in \mathbb{R} : 2x^2 \leq 4\}$

$2x^2 - 4 \leq 0$

$(\sqrt{2}x - 2)(\sqrt{2}x + 2) \leq 0$

$x = \sqrt{2} \qquad x = -\sqrt{2}$

	$x < -\sqrt{2}$	$x = -\sqrt{2}$		$x = \sqrt{2}$	$x > \sqrt{2}$
$(\sqrt{2}x - 2)$	−	−	−	0	+
$(\sqrt{2}x + 2)$	−	0	+	+	+
	+	0	−	0	+

$x \in [-\sqrt{2}, \sqrt{2}]$

ou $x^2 \leq 2$

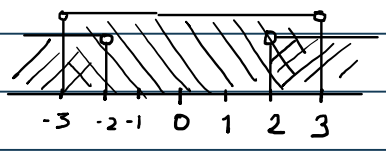
For $u^n \leq a$, n is even

$\Rightarrow -\sqrt[n]{a} \leq u \leq \sqrt[n]{a}$

q) $\{x \in \mathbb{R} : 4 < x^2 < 9\}$

$-2 > x > 2$

$-3 < x < 3$



$]-3, -2[\cup]2, 3[$

r) $\{x \in \mathbb{R} : \frac{x}{\quad} \leq 0\}$

r) $\left\{ x \in \mathbb{R} : \dfrac{x}{x-2} \leq 0 \right\}$

	$x<0$	$x=0$		$x=2$	$x>2$
x	−	0	+	+	+
$x-2$	−	−	−	0	+
$x/x-2$	+	0	−	Ind.	+

$\longrightarrow [0, 2[$

s) $\{ x \in \mathbb{R} : |x-3| < 2|x| \}$

$\left\{ \begin{array}{l} x-3 \geq 0 : \; x \geq 3, \; |x-3| = x-3 \\ x-3 < 0 : \; x < 3, \; |x-3| = -(x-3) \end{array} \right\}$

$\left\{ \begin{array}{l} x \geq 0 : \; x \geq 0, \; 2|x| = 2x \\ x < 0 : \; x < 0, \; 2|x| = -2x \end{array} \right\}$

$x < 0, \; 0 \leq x < 3, \; x \geq 3$

$x-3$	−	−	+
$2x$	−	+	+
	$-(x-3) < -2x$	$-(x-3) < 2x$	$x-3 < 2x$
	$(x-3) > 2x$	$3 < 3x$	$-3 < x$
	$-3 > x$	$1 < x$	$\wedge \; x \geq 3$
	$\wedge \; x < 0$	$\wedge \; 0 \leq x < 3$	
	$[-\infty, -3[$	$]1, 3[$	$[3, +\infty[$

$$]-\infty, -3[\cup]1, +\infty[$$

t) $\{ x \in \mathbb{R} : |x+1| > |x-3| \}$

$\left\{ \begin{array}{l} x+1 \geq 0 : \; x \geq -1, \; |x+1| = x+1 \\ x+1 < 0 : \; x < -1, \; |x+1| = -(x+1) \end{array} \right\}$

$\left\{ \begin{array}{l} x-3 \geq 0 : \; x \geq 3, \; |x-3| = x-3 \\ x-3 < 0 : \; x < 3, \; |x-3| = -(x-3) \end{array} \right\}$

$x < -1, \; -1 \leq x < 3, \; x \geq 3$

$x+1$	−	+	+

$x-3$ — | — | +

$-(x+1) > -(x-3)$	$x+1 > -x+3$	$x+1 > x-3$
$-x-1 > -x+3$	$x > 1$	$1 > -3$ ✓
$-1 > 3$ ✗	$\Rightarrow 1 < x < 3$	$x \geq 3$
Sem solução	$x \in]1, +\infty[$	

12. ...

$(a-b)(a+b) = b(a-b)$

$\Rightarrow a+b = b$ ✗

como $a=b$, $a-b=0$ e $\frac{0}{0}$ é indefinido

13.

a) F

b) V

c) F

d) V

e) F

f) F