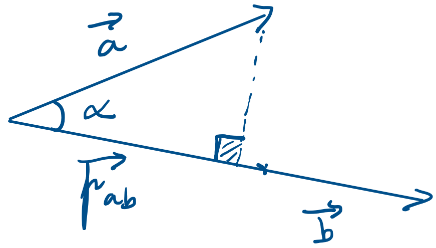
The Geometric Pipeline

17 de fevereiro de 2024 15:35

$\boxed{\text{Normal}}$ de um vetor

$$\vec{b}_N = \frac{\vec{b}}{|\vec{b}|}$$

↳ Unit length vector for the direction $\vec{b}$



PROJEÇÃO

$$|\vec{p}_{ab}| = |\vec{a}| \cos(\alpha)$$

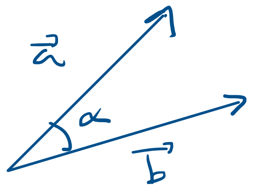
$$\vec{p}_{ab} = \underbrace{|\vec{a}| \cos(\alpha)}_{\text{AMPLITUDE}} \times \underbrace{\frac{b}{|\vec{b}|}}_{=\vec{b}_N}$$

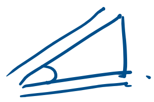
Normalizado

⇒ DIRECTION

DOT PRODUCT

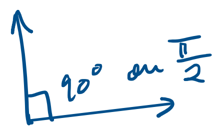
$$\vec{a} \cdot \vec{b} = \cos(\alpha)|\vec{a}||\vec{b}|$$





$$\cos(\alpha) = \frac{A}{H}$$

$$\begin{cases} \vec{a} \cdot \vec{b} < 0 \;\; if \;\; \frac{\Pi}{2} < \alpha \leq \Pi \\ \vec{a} \cdot \vec{b} = 0 \;\; if \;\; \alpha = \frac{\Pi}{2} \\ \vec{a} \cdot \vec{b} > 0 \;\; if \;\; 0 \leq \alpha < \frac{\Pi}{2} \end{cases}$$

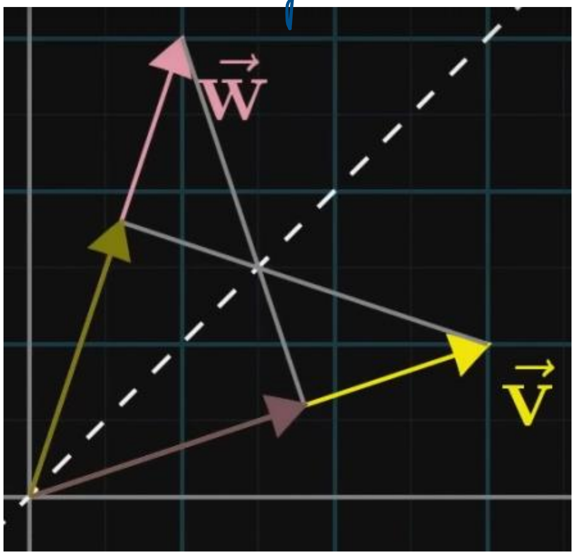


$\Rightarrow \vec{a} \cdot \vec{b} = 0$

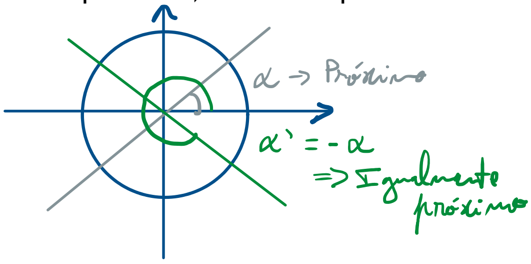
ORDEM não importa

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

ORDEM não importa



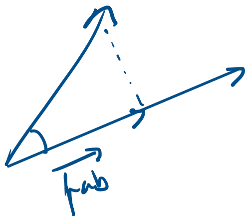
O valor da multiplicação das amplitudes dos dois vetores, geralmente dimuida através da relação entre as suas direções. Mais próximos, maior a amplitude.



$$\vec{a} \cdot \vec{b} = \cos(\alpha)\,|\vec{a}|\,|\vec{b}|$$
$$= |\vec{p_{ab}}|\,|\vec{b}|$$

Portanto, o produto dos vetores a . b é a amplitude da projeção de a em b multiplicada pela amplitude de b.

$$\vec{p_{ab}} = \vec{a} \cdot \vec{b} \; \frac{\bar{b}}{|\vec{b}|}$$

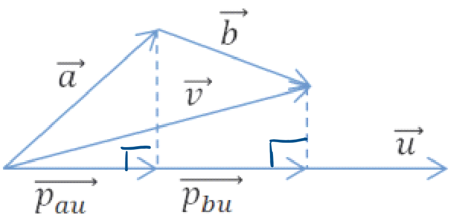


ASSUMING that $\vec{b}$ is a UNIT VECTOR

Assuming that $\vec{b}$ is a unit vector, we get that the dot product is the length of the projected vector. The projected vector can be defined as:

$$\overrightarrow{p_{ab}} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\vec{b} \tag{11}$$

$$\vec{v} = \vec{a} + \vec{b}$$



$$\vec{p_{au}} + \vec{p_{bu}} = p_{vu}$$

$$\overrightarrow{p_{vu}} = \overrightarrow{p_{au}} + \overrightarrow{p_{bu}} \Leftrightarrow \frac{(\vec{a}+\vec{b})\cdot\vec{u}}{|\vec{u}|}\vec{u} = \frac{\vec{a}\cdot\vec{u}}{|\vec{u}|}\vec{u} + \frac{\vec{b}\cdot\vec{u}}{|\vec{u}|}\vec{u} \Leftrightarrow (\vec{a}+\vec{b})\cdot\vec{u} = \vec{a}\cdot\vec{u} + \vec{b}\cdot\vec{u}$$

↳ unit vector $\vec{u}_N \circ \vec{u}$

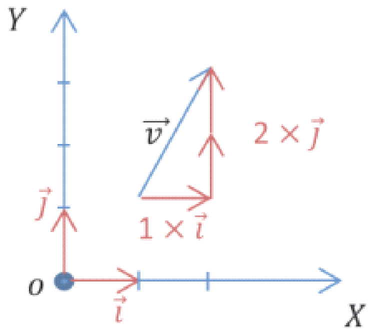
Hence, dot product satisfies the distributive law

Considering two vectors $\vec{u} = u_x\vec{\imath} + u_y\vec{\jmath} + u_z\vec{k}$ and $\vec{v} = v_x\vec{\imath} + v_y\vec{\jmath} + v_z\vec{k}$, we can write:

$$\vec{u}\cdot\vec{v} = (u_x\vec{\imath} + u_y\vec{\jmath} + u_z\vec{k})\cdot(v_x\vec{\imath} + v_y\vec{\jmath} + v_z\vec{k})$$

Applying the distributive law we get:

$$\vec{u}\cdot\vec{v} = \begin{matrix} u_xv_x(\vec{\imath}\cdot\vec{\imath}) + u_xv_y(\vec{\imath}\cdot\vec{\jmath}) + u_xv_z(\vec{\imath}\cdot\vec{k}) + \\ u_yv_x(\vec{\jmath}\cdot\vec{\imath}) + u_yv_y(\vec{\jmath}\cdot\vec{\jmath}) + u_yv_z(\vec{\jmath}\cdot\vec{k}) + \\ u_zv_x(\vec{k}\cdot\vec{\imath}) + u_zv_y(\vec{k}\cdot\vec{\jmath}) + u_zv_z(\vec{k}\cdot\vec{k}) \end{matrix}$$



$\vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$.

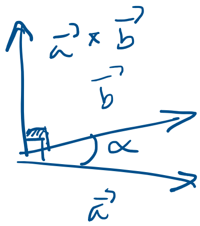
$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z$

cosine of two unit vectors:

$\cos(\alpha) = u_xv_x + u_yv_y + u_zv_z$

CROSS PRODUCT



Provides a vector
perpendicular to the plane
defined by the 2
input vectors

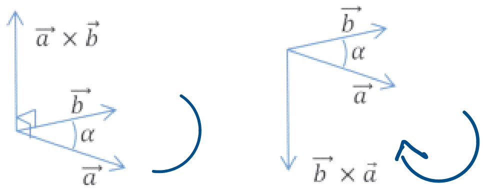


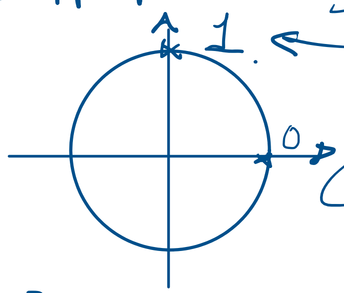
Figure 3.10 – Cross product

Right Hand Rule

Geometric Definition

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \, sen(\alpha) \, \boxed{\vec{n}}$$

Unit Vector

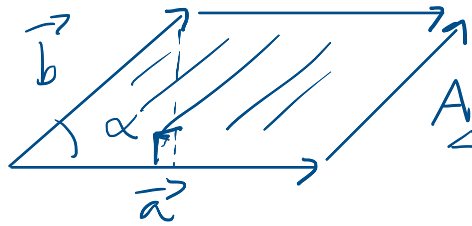


maior amplitude

não forma um plano

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \, sen(\alpha)$$

=



Base

$$A_{\square} = \underbrace{|\vec{a}|}_{} \, \underbrace{|\vec{b}| \, sen(\alpha)}_{Altura}$$

We also know that both cross products point in the same direction (see Figure 3.13), therefore,

$$\vec{a} \times \vec{c} = \vec{a'} \times \vec{c} \quad (25)$$

$a = |\vec{a'}|$
$= sen(\alpha)|\vec{a}|$

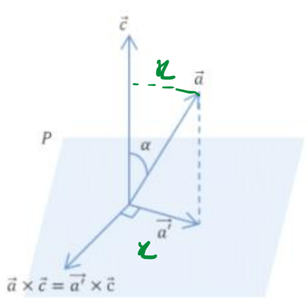


Figure 3.13 - $\vec{a} \times \vec{c} = \vec{a'} \times \vec{c}$

$$\vec{a} \times \vec{c} = \vec{a'} \times \vec{c}$$

$$(\vec{a} + \vec{b})' = \vec{a}' + \vec{b}'$$

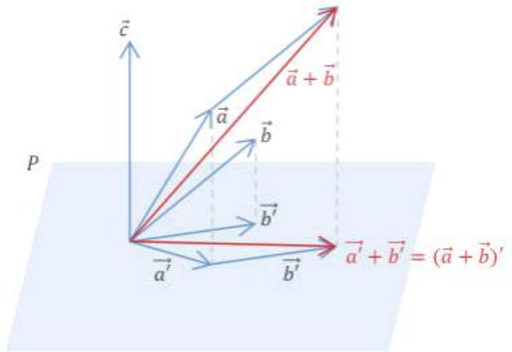


Figure 3.14 - $(\vec{a} + \vec{b})' = \vec{a'} + \vec{b'}$

Using the results in eq. 22) we get:

$$a \times b = +a_x b_y \vec{k} - a_x b_z \vec{j} - a_y b_x \vec{k} + a_y b_z \vec{\imath} + a_z b_x \vec{j} - a_z b_y \vec{\imath} \tag{33}$$

Which can be written as

$$a \times b = (a_y b_z - a_z b_y)\vec{\imath} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\vec{k} \tag{34}$$

To compute the components of the cross product vector $\vec{v} = \vec{a} \times \vec{b}$ we can use eq. 35)

$$\begin{aligned} v_x &= a_y b_z - a_z b_y \\ v_y &= a_z b_x - a_x b_z \\ v_z &= a_x b_y - a_y b_x \end{aligned} \tag{35}$$

The rule of Sarrus for computing determinants can be helpful to memorize this formula:

$$\begin{array}{cccccc} i & j & k & i & j & k \\ a_x & a_y & a_z & a_x & a_y & a_z \\ b_x & b_y & b_z & b_x & b_y & b_z \end{array}$$

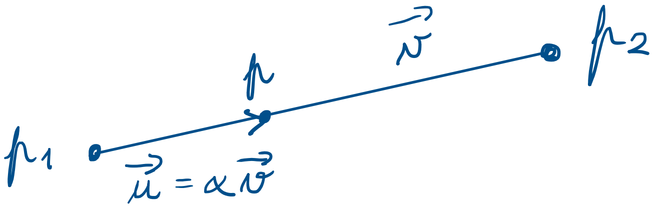
POINTS AND VECTORS

$$p' = p + \vec{v}$$

$$p' = p + k\vec{v}$$

$$\boxed{v = p' - p}$$

$$p = p_1 + \alpha \overbrace{(p_2 - p_1)}$$



$$p = p_1 + \alpha p_2 - \alpha p_1$$
$$= (1-\alpha) p_1 + \alpha p_2$$

Homogeneous Coordinates

$$P_1 = (P_x + V_x, P_y + V_y, 1)$$

$$P_1 = \overbrace{P_x \vec{i} + P_y \vec{j} + \underbrace{O}_{\text{Origem}}}^{P} + \underbrace{v_x \vec{i} + v_y \vec{j}}_{V}$$

$$\to O'$$

$$P_1 = P_x \vec{i} + P_y \vec{j} + O'$$

$$P_1 = [\vec{i}, \vec{j}, o'] \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix}$$

$$o' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + V_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + V_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} V_x \\ V_y \\ 1 \end{bmatrix}$$

$$P_1 = \begin{array}{c} \begin{array}{ccc} \vec{i} & \vec{j} & \vec{o} \end{array} \\ \begin{bmatrix} 1 & 0 & V_x \\ 0 & 1 & V_y \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} P_x + V_x \\ P_y + V_y \\ 1 \end{bmatrix}$$

$$P_1 = T_V \cdot P \iff P_1 = P + \vec{V}$$