Problemas de momento - energia

25 . 10 . 2022

1 $E = 6.25$ kg $\quad p_x = 1.25$ kg $\quad p_y = p_z = 2.50$ kg

$\Rightarrow$ Quadri-vetor: (E, p_x, p_y, p_z)

$= (6.25, 1.25, 2.50, 2.50)$

$\hookrightarrow$ O seu módulo é um invariante e chama-se massa.

Portanto, $\boxed{m} = \sqrt{(6.25)^2 - (1.25)^2 - (2.50)^2 \times 2}$

$= 5$ kg

(1) $\hookrightarrow m^2 = E^2 - p^2$

$(\vec{p} = 0 \Rightarrow E = m)$

2 a) 3 kg de massa

$\Delta x = 8$ metros

$\Delta t = 10$ metros

0 $\quad$ 8 $\quad x$

$v = \frac{\Delta x}{\Delta t} = \frac{8}{10} = 0.8$

$(E, 8, 0, 0)$

$\boxed{E = m \times \gamma}$

$E = m \times (1 - v^2)^{-\frac{1}{2}}$

$= 3 \times (1 - 0.8^2)^{-\frac{1}{2}} = 5$ kg

Fator de Lorentz

$\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}$

(1)

$E^2 = m^2 + p^2$

$\Leftrightarrow p = \sqrt{E^2 - m^2}$

$\Leftrightarrow p = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$ kg

$\hookrightarrow$ momento linear

b) $E_0 = m = 5$ kg ; Energia em repouso $\Leftrightarrow$ Massa

c) $E = E_0 + E_c$

$\Leftrightarrow E - m = E_c$

$\Leftrightarrow E_c = 5 - 3 = 2$ kg

d) $E_c = \frac{m \times v^2}{2} \Rightarrow E_c = \frac{3 \times (0.8)^2}{2} = 0.96$ N

3 a) $E_c = 3E_0$; $E = m + 3m = 4m$

$m^2 = (4m)^2 - p^2$

$\Leftrightarrow m^2 - 16m^2 = -p^2 \Leftrightarrow \sqrt{15}\, m = p$

x

$(4m, m\sqrt{15}, 0, 0)$

b) $E_c = m$; $E = 2m$ (os mesmos cálculos)

$(2m, m\sqrt{3}, 0, 0)$

c) yy $\quad p = 2m$; $m^2 = E^2 - (2m)^2$

$\Leftrightarrow m^2 + 4m^2 = E^2$

$\Leftrightarrow \sqrt{5}\, m = E$

$(\sqrt{5} m, 0, 2m, 0)$

d) $xx\ (-)$ $\quad E = 4m$

$m^2 = (4m)^2 - p^2$

$\Leftrightarrow m^2 - 16m^2 = -p^2 \Leftrightarrow p = \sqrt{15}\, m$

$(4m, -\sqrt{15} m, 0, 0)$

e) $p_x = p_y = p_z$; $E_c = 4 \times E_0$; $E = 4m + m = 5m$

$m^2 = (5m^2) - p^2 \Leftrightarrow m^2 - 25m^2 = p^2$

$\Leftrightarrow \sqrt{24}\, m = p$

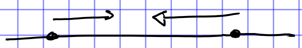
$p = \sqrt{x^2 + x^2 + x^2}$

$= \sqrt{3x^2}$

$\Rightarrow x\sqrt{3} = \sqrt{24}\, m \Leftrightarrow x = \frac{\sqrt{24}\, m}{\sqrt{3}} = \sqrt{8}\, m$

$(5m, \sqrt{8} m, \sqrt{8} m, \sqrt{8} m)$

④ 2 comboios : $m = 5 \times 10^6$ Kg
(direções opostas) ↳ $v = 42$ m/s



a) $E_c = \dfrac{m \times v^2}{2}$; $v = \dfrac{42}{3 \times 10^8} = 1{,}4 \times 10^{-7}$

$\Rightarrow E_c = \frac{1}{2} \times \left(5 \times 10^6 \times (1{,}4 \times 10^{-7})^2\right) = 4{,}9 \times 10^{-8}$ Kg $= 0{,}049$ mg ≈ 0.05 mg

b)
$$\begin{array}{l} (m + E_c \,,\, p) \\ +\,(m + E_c \,,\, -p) \\ \hline (2(m+E_c) \,,\, 0) \end{array}$$

$E = E_0 + E_c \rightarrow = 0$

$2m + 2E_c = m'$

$2 \times 5 \times 10^6 + 2 \times 0.05 = m'$

Portanto $m - m' = 2 \times E_c = 2 \times 0.05 = 0.1$ mg

⑥ SLAC (Standford Linear Accelerator)
↳ acelera eletrões até uma Ec final de 47 GeV.
↳ 47×10^9 eV

a) $d = 3000$ m

$E_0 = m \approx 0.5$ MeV $= 0.5 \times 10^6$ eV

$1\,eV = 1{,}6 \times 10^{-19}$ J

$E \xrightarrow{+1} E + \delta \xrightarrow{+1} E + 2\delta$ (…) ↳ $E + K\delta = 47$ GeV

$0.5 \times 10^6 + K = 47 \times 10^9$

$\Leftrightarrow K = 4{,}69 \times 10^{10}$

$\dfrac{4.69 \times 10^{10}}{3000} = 1{,}57 \times 10^7$

$\Rightarrow 1{,}57 \times 10^7 / 10^6 = 15{,}7$ MeV/m

Newton: $E = \dfrac{0.5 \text{ MeV} \times 1^2}{2} = 0.25$ MeV/m

1 m — 15,7 MeV
x — 0.25 MeV

$x = \dfrac{1 \times 0.25}{15.7} = 0{,}016$ m

b) No referencial do Laboratório ; $\gamma = \dfrac{1}{\sqrt{1 - v^2}}$

$E = m \times \gamma = \dfrac{m}{\sqrt{1 - v^2}}$

Sugestão: para v muito próximo de um, $1 - v^2 = (1+v)(1-v) \approx 2(1-v)$

$47 \times 10^9 = (0.5 \times 10^6) / \sqrt{1 - v^2}$

$\Leftrightarrow 47 \times 10^9 = (0.5 \times 10^6) / \sqrt{2(1-v)}$

$\Leftrightarrow (1 - v) = \left(\dfrac{0.5 \times 10^6}{47 \times 10^9}\right)^2 / 2$

$\Leftrightarrow (1 - v) = 5{,}587 \times 10^{-11}$ (Basicamente, $v \approx c$)

c) $d = 12740$ Km $= 12740 \times 10^3$ m

$v = d/t \Rightarrow t = d/v = 12740 \times 10^3 / 1 = 12740 \times 10^3$ m

$v' = d/t \Rightarrow t = (12740 \times 10^3)/(1 - 5.66 \times 10^{-11})$

$= 12740000{,}00072$

Portanto, $v' - v = 0.72$ mm

d) 3000 m

$x = \begin{cases} t = 0 \\ x = 0 \end{cases}$ $\quad x = \begin{cases} t = \\ x = 3000 \end{cases}$

$x' = \begin{cases} t = 0 \\ x = 0 \end{cases}$ $\quad x' = \begin{cases} t = \\ x' = ? \end{cases}$

Proper Length

$L = \dfrac{L_0}{\gamma}$ $\quad \dfrac{L_0}{\frac{1}{\frac{1}{\sqrt{\ldots}}}}$

$\gamma = \dfrac{1}{\sqrt{1 - (1 - 5{,}66 \times 10^{-11})^2}}$

$L = 0.03192$ m

≈ 0.032 m

7 a) 2500 raios cósmicos ; $E > 4 \times 10^{17}$ eV
5 com $E \approx 10^{20}$ eV
(a energia de repouso do protão é $\approx 10^9$ eV)

$E = 10^{20}$ eV
$d = 10^5$ anos-luz

$$10^{20} = \frac{10^9}{\sqrt{1-v^2}} \iff v = \sqrt{1 - \frac{10^9}{10^{20}}} \approx 1$$

$X_1 = \begin{cases} x = 0 \\ t = 0 \end{cases}$ $\qquad X_2 = \begin{cases} x = 10^5 \text{ anos luz} \\ t = 10^5 \text{ anos} \end{cases}$

$X_1' = \begin{cases} x' = 0 \\ t' = 0 \end{cases}$ $\qquad X_2' = \begin{cases} x' = ? \\ t' = 0.32 \text{ anos luz} \end{cases}$

$v = 1 \Rightarrow v = d/t \iff t = d/v$
$\iff t = \frac{10^5}{v} = 10^5$ anos $= 1000 \times 100 = 1000$ séculos

$t' = t/\gamma$

PROPER TIME =

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~316227.76601683786639977871
(10^5)/(1/sqrt(1-(1-(10^9/10^20))))
0.316227766016838
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≈ 0.32

$$\frac{10^5}{\frac{1}{\sqrt{1-v^2}}} \iff 10^5 \times \sqrt{1 - \left(1 - \frac{10^9}{10^{20}}\right)}$$

b) $L = L_0/\gamma$

$10^5 = 10^{-24}\sqrt{1-v^2}$

10^5 anos luz
10^5 anos

$$\frac{10^5 \times 365 \times 24 \times 60 \times 60\, s}{3 \times 10^8 \text{ m/s}}$$

$\iff \left(\frac{10^5}{10^{24}}\right)^2 - 1 = -v^2$

$\iff \left(1 - \left(\frac{10^5}{10^{24}}\right)^2\right) = v^2 \iff v = \sqrt{1 - \left(\frac{10^5}{10^{24}}\right)^2} = 1$

$\begin{cases} L = L_0/\gamma \\ E = m \times \gamma \end{cases} \iff \begin{cases} 10^{-15} = 9{,}4 \times 10^{20}/\gamma \\ E = 10^9\gamma \end{cases}$

$\gamma = 9{,}4 \times 10^{20} / 10^{-15} = 9{,}4 \times 10^{35}$

$E = 10^9 \times 9{,}4 \times 10^{35} = 9{,}4 \times 10^{44}$ eV

1 eV — 1.60×10^{-19}
$9{,}4 \times 10^{44}$ — x

$x = \frac{9{,}4 \times 10^{44} \times 1{,}6 \times 10^{-19}}{1} = 1{,}504 \times 10^{26}$ J

1 J — 1.12×10^{-17}
$1{,}5 \times 10^{26}$ — x

$x = 1{,}68 \times 10^9$ kg
$= 1{,}68 \times 10^6$ toneladas

MASSA DE UM SISTEMA

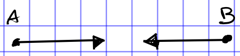
8 $p_{sis} = p_A + \cancel{p_B}^{\,0}$

a) $\Rightarrow (p_{sis})^2 = (p_A)^2 = (E_A)^2 - m^2 = (4m)^2 - m^2$
$\iff \sqrt{15}\, m$

$M_{sis} = \sqrt{E_{sis} - P_{sis}} = \sqrt{(5m)^2 - 15m^2}$
$= \sqrt{10}\, m$

resolução alternativa: $P_A = (4m, \sqrt{15}\, m) \Rightarrow P_A + P_B = (5m, \sqrt{15}\, m)$
$P_B = (m, 0)$

$m_{sistema} = \sqrt{(5m)^2 + (\sqrt{15}\, m)^2}$



b) $E_c = 5m$

$E^2 = p^2 - m^2 \iff p^2 = E^2 - m^2 = (6m)^2 - m^2 \iff p_A = \sqrt{35}\,m$

$\tilde{P}_A = (6m, \sqrt{35}\,m) \Rightarrow \tilde{P}_A + \tilde{P}_B = (12m, 0)$ ↳ momento do sistema é 0

$P_B = (6m, -\sqrt{35}\,m)$

$E_{sis}^2 = p_{sis}^2 - m^2$

$(12m)^2 - 0^2 = m^2 \iff m^2 = 12m$

c) $p = \sqrt{(10m)^2 - (3m)^2} = \sqrt{91}\,m$ 7+3 E_0 $p^2 = E^2 - m^2$

$Q_A = (10m, \sqrt{91}\,m) \Rightarrow Q_A + Q_B = (11m, \sqrt{91}\,m)$

$Q_B = (m, 0)$

$M_{sis}^2 = E_{sis}^2 - P_{sis}^2 = \sqrt{(11m)^2 - (\sqrt{91}\,m^2)}$

$= \sqrt{121m^2 - 91m^2} \iff M_{sis} = \sqrt{30}\,m$

d) $p^2 = (7m)^2 - m^2 \iff p = \sqrt{48}\,m$

$Q_A = (7m, \sqrt{48}m, 0)$ e $Q_B = (7m, 0, \sqrt{48}m)$

$Q_A + Q_B = (14m, \sqrt{48}m, \sqrt{48}m)$

$M_{sis} = \sqrt{(14m)^2 - (\sqrt{48}m)^2 - (\sqrt{48}m)^2} = 10m$

9. $E_{0\,fotão} = 0$

a) $E^2_{fotão} = P^2_{fotão} \Rightarrow Q_A = (3m, 3m)$, $Q_B = (m, 0)$

16 - 9 = 7

$Q_A + Q_B = (4m, 3m)$

$M_{sistema} = \sqrt{(4m)^2 - (3m)^2} = \sqrt{7}\,m$

b) $Q_A = (3E, 3E)$ $Q_B = (E, E)$

$Q_A + Q_B = (4E, 4E)$

$M_{sistema} = \sqrt{(4E)^2 - (4E^2)} = 0$

c) $Q_A = (3E, 3E)$ $Q_B = (E, -E)$

$M_{sistema} = \sqrt{(4E)^2 - (2E)^2} = \sqrt{12}\,E$

d) $Q_A = (E, E, 0)$ e $Q_B = (3E, 0, 3E)$

$Q_A + Q_B = (4E, E, 3E)$

$M_{sistema} = \sqrt{(4E)^2 - E^2 - (3E)^2} = \sqrt{6}\,E$

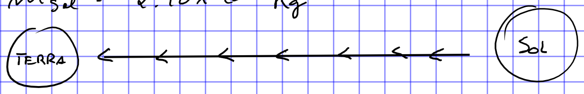
CONVERSÃO DE ENERGIA EM MASSA

10. 1372 W/m² → Constante Solar   Joule/seg

$r_{Terra} = 6,4 \times 10^6\,m$ e $d_{T-S} = 1.5 \times 10^{11}\,m$

$m_{Sol} = 2.10 \times 10^{30}$ Kg

a)



$A_0 = 4\pi \times r^2$ $1372 \times 2\pi \times (6,4 \times 10^6)^2 \approx 3,53 \times 10^7\,W$

$\Rightarrow 3,73 \times 10^7$ J/s   ?

(11) a) 100 W

100 J/s

$100 \times 365 \times 24 \times 60 \times 60$

$= 3,15 \times 10^{9}$ J

3.5×10^{-8} Kg $= 35\,\mu g$

1 J —— 1.12×10^{-17} Kg

x

b) $E_{Total} = [1, 2 \times 10^{13}$ KW-h$]$ → KW por hora

$10^3 \times (1,5 \times 10^{13}) \times 3600 \times 1.12 \times 10^{-17} = 600$ Kg

(2)

~~c)~~ 373 J/s

$0.25 \times 373 = 93,25$ J/s (útil)

$93,25 \times k \times 1.12 \times 10^{-17} = 1$

$\Leftrightarrow k = 9.57 \times 10^{14}$ s (tá errado)

$93,25 \times k = 1 \times (3 \times 10^8)^2$ ↳ é suposto dar 6×10^{13} segundos

$\Leftrightarrow k = 9,65 \times 10^{4}$

Welp

(12) 1Kg H_2 + 8 Kg O_2 → H_2O

↳ 10^8 J libertados

$10 \times 1000 + 80 \times 1000 = 90\,000$ Kg

9 Kg —— 10^8 J

90 000 —— x

$x = 10^{12}$ J

$\frac{10^{12}}{(3 \times 10^8)^2} = m \Leftrightarrow m = 1,11 \times 10^{-5}$ Kg

↳ 11 mg

(13) ~~feixe de luz: 1W = 1J/s~~

~~$\Delta p = F \times \Delta t \Leftrightarrow \Delta p = F \times \frac{\Delta E}{P} \Leftrightarrow F = \frac{\Delta p}{\frac{1}{\frac{\Delta E}{P}}}$~~ $= \frac{\Delta p P}{\Delta E}$

↳ 0? ↳ 1J/s

variação do momento linear ⟺ aceleração?

(2)

(11)

E inicial (J)	E útil (%)
373	0.25
x	1

$x = 1492$ W

$\frac{1492 \text{ kg}/m^2/s^2}{9 \times 16 \; m^2/s^2}$

1492 W $= 1492$ J/s $= 1492\,(kg/m^2)/s^3$

1 s —— 1492 J

$E = mc^2 \Rightarrow 1492 / 9 \times 10^{16} = 1,66 \times 10^{-14}$ Kg

$1,66 \times 10^{-14}$ —— 1 s

1 —— x

$x = \frac{1 \times 1}{1,66 \times 10^{-14}} = 6 \times 10^{13}$

(13) $N = P/(hf) \Rightarrow F = (1-r)\,P/c$

↳ fraction between 0 and 1 of reflection

$F = 1/(3 \times 10^8) = 3,3 \times 10^{-9}$ N

The momentum of a photon is hf/c, where f is the photon's frequency, h is Planck's constant, and c is the speed of light. This is a fact given by quantum physics. If you want things in wavelength λ instead, use $f\lambda = c$ to get the momentum h/λ.

Force is the rate of change of momentum, so if you can just calculate how much momentum you absorb from the photons each second, you know the force they exert on you.

Let's say you're struck by N photons each second, and that all photons have the same frequency/wavelength. If you're absorbing all photons, you're just sucking up their momentum, so the momentum per second — i.e. the force — is $F = hfN/c$. But if you're reflecting them instead, you have to both absorb the momentum and give them an equal push in the other direction, which doubles the force to $F = 2hfN/c$. If you absorb some and reflect some, the force is $F = (1+r)hfN/c$, where r is basically your albedo: the fraction (between 0 and 1, i.e. between 0% and 100%) of photons that you reflect.

But what if you don't know the number of photons per second, but only the light power P? Well, power is energy per unit time, so since we know the energy of each photon (hf), we get $N = P/hf$. Thus, we get the force $F = (1+r)P/c$. Voilà: the frequency has disappeared! As long as the albedo doesn't depend on frequency, we can therefore drop the assumption that all photons have the same frequency.

So there we have your final formula. Note that the force is typically tiny: to get a single newton of force, we need between 150 and 300 megawatts of power depending on the albedo.

b) $1.372 \times 10^3 \, W/m^2$

$F = 2 \times \frac{hf}{c} \times N$ → momentum

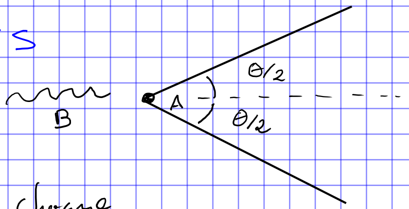
$N = P/hf$ → fotões / seg

$\Rightarrow F = \left(2 \times \frac{P}{hf} \times hf\right)/c$ → $(1-r)$

$= (1-r)/c$

Colisões

(14)



Antes do choque

$Q_A(m + E_c, E_c, 0)$ $\quad Q_B(m, 0, 0)$

$Q_A + Q_B = (2m + E_c, E_c, 0)$

Depois do choque

$Q_A'\left(\frac{E_c}{2} + m, \cos\left(\frac{\theta}{2}\right)\frac{E_c}{2}, \frac{E_c}{2}\operatorname{sen}\left(\frac{\theta}{2}\right)\right)$ $\quad Q_B'\left(\frac{E_c+m}{2}, \frac{E_c}{2}\cos\left(\frac{\theta}{2}\right), \operatorname{sen}\left(\frac{\theta}{2}\right)\frac{E_c}{2}\right)$

$Q_A' + Q_B' = \left(E_c + 2m, E_c\cos\left(\frac{\theta}{2}\right), E_c \operatorname{sen}\left(\frac{\theta}{2}\right)\right)$

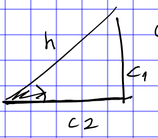
$m^2 = (E_c + 2m)^2 - \left(E_c \cos\left(\frac{\theta}{2}\right)\right)^2 - \left(E_c \cos\left(\frac{\theta}{2}\right)\right)^2$

$\Leftrightarrow m^2 = E_c^2 + 4E_c m + 4m^2 - E_c^2\cos^2\left(\frac{\theta}{2}\right) - E_c^2\cos^2\left(\frac{\theta}{2}\right)$

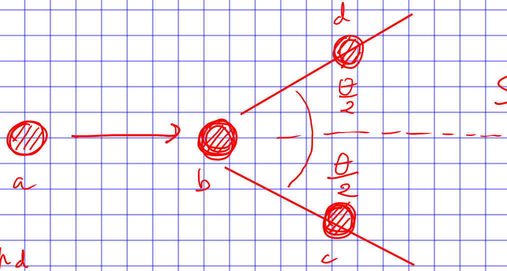
$\Leftrightarrow m^2 = E_c^2 + 4E_c m + 4m^2 - E_c^2\left(\cos^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right)\right)$

$\Leftrightarrow \frac{m^2 - 4m^2 - 4E_c m}{-2E_c} = \cos^2\left(\frac{\theta}{2}\right)$

$\Leftrightarrow \frac{3m^2 - 4E_c m}{2E_c} = \cos\left(\frac{\theta}{2}\right)$



$\cos\frac{\theta}{2} = \frac{c_2}{h}$



Symmetry $\Rightarrow$ equal Energy and momentum (in magnitude)

$p_c = p_d$

$E_{c_C} = E_{c_D} = E_{c_A}/2 \quad \Rightarrow$ Conservation of energy

$p_{total} = p_a = p_c \cos(\theta/2) + p_d \cos(\theta/2) = 2 p_d \cos(\theta/2)$

$\Rightarrow$ Conservation of momentum

$p_a = \sqrt{E^2 - m^2} = \left((E_c + m)^2 - m^2\right)^{\frac{1}{2}} = \sqrt{E_c^2 + 2mE_c}$

$p_d = \sqrt{\left(\frac{E_c}{2}\right)^2 + 2m\left(\frac{E_c}{2}\right)^2}$

Portanto, $p_a = 2p_d \cos(\theta/2)$

$\Leftrightarrow \sqrt{E_c^2 + 2mE_c} = 2\sqrt{\left(\frac{E_c}{2}\right)^2 + 2m\frac{E_c}{2}} \times \cos(\theta/2)$

$\Leftrightarrow E_c^2 + 2mE_c = 4\left(\frac{E_c^2}{4} + E_c m\right) \times \cos^2(\theta/2)$

$\Leftrightarrow \frac{E_c^2 + 2mE_c}{E_c^2 + 2mE_c^2} = \cos^2(\theta/2)$

$\Leftrightarrow \sqrt{\frac{E_c + 2m}{E_c + 4m}} = \cos(\theta/2)$

$Q_A \left(E_c/2 + m,\ \frac{E_c}{2} \times \cos\left(\frac{\theta}{2}\right),\ \frac{E_c}{2} \operatorname{sen}\left(\frac{\theta}{2}\right)\right)$

$Q_B \left(E_c/2 + m,\ \frac{E_c}{2} \times \cos\left(\frac{\theta}{2}\right),\ \frac{E_c}{2} \operatorname{sen}\left(\frac{\theta}{2}\right)\right)$?

$Q_A + Q_B = \left(E_c + 2m,\ E_c \cos\left(\frac{\theta}{2}\right),\ E_c \operatorname{sen}\left(\frac{\theta}{2}\right)\right)$

$Q'_A + Q'_B = (E_c + 2m,\ E_c,\ 0)$

$(E_c + 2m)^2 - (E_c)^2 = (E_c + 2m)^2 - \left(E_c \cos\left(\frac{\theta}{2}\right)^2\right) - \left(E_c \operatorname{sen}\left(\frac{\theta}{2}\right)\right)^2$

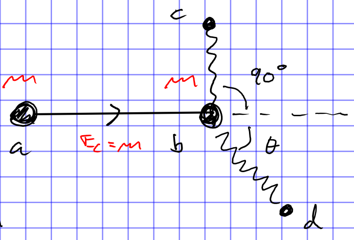
$-E_c^2 = -E_c^2 \cos^2\left(\frac{\theta}{2}\right) - E_c^2 \operatorname{sen}^2\left(\frac{\theta}{2}\right)$

$\Leftrightarrow -E_c^2 = -E_c^2\left(\cos^2\left(\frac{\theta}{2}\right) + \operatorname{sen}^2\left(\frac{\theta}{2}\right)\right) \Leftrightarrow -E_c^2 = -E_c^2$

Pah, true

15

$m = E_c$



$p_a = \sqrt{(2m)^2 - m^2} = \sqrt{3m^2} = \sqrt{3}\,m$

$p_a = p_c + p_d$ $\quad (2m,\ E_d \cos\theta,\ E_c + E_d \operatorname{sen}(\theta))$

$E_{total} = \underbrace{E_a}_{2m} + m = E_c + E_d$ $\quad$ ↳ $3m$

direções opostas

$p_{x\,total} = p_a = p_c \times \cos\theta$ (horizontal)

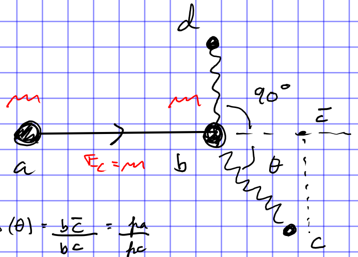
$p_{y\,total} = 0 = p_c \times \operatorname{sen}\theta - p_d$ (vertical)

$E_a + m = 2m + m = 3m = E_c + E_d$ (conservação de energia)

$p_a = E_c \times \cos\theta$ (conservação do momento horizontal)

$E_d = E_c \times \operatorname{sen}\theta$ (conservação do momento vertical)

$p_c = E_d$ e $p_d = E_d$



$\cos(\theta) = \frac{b\bar{c}}{bc} = \frac{p_a}{p_c}$

Conservação da Energia

$E = E_a + m = E_c + E_d$

Conservação do momento linear

$p_x = p_a = p_c \times \cos\theta$

$p_y = 0 = p_c \times \operatorname{sen}\theta - p_d$

$Q_{Tot_1} = (3m,\ m,\ 0)$ $\quad Q_{Tot\,2} = (E_c + E_d,\ p_c \times \cos\theta,\ p_c \times \operatorname{sen}\theta - p_d)$

Como são fotões (não têm massa)

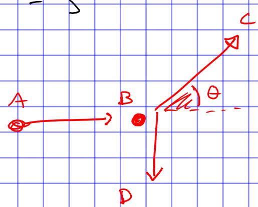
↳ $p_c = E_c$ e $p_d = E_d$ $\quad p_a = \sqrt{(2m)^2 - m^2} = \sqrt{3}\,m$

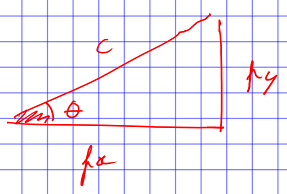
$\Rightarrow 3m = E_c + E_d$

$$\begin{cases} E_c + E_d = 3m \\ p_A = E_c \times \cos\theta \\ 0 = E_c \times \operatorname{sen}\theta - E_d \end{cases} \qquad \begin{cases} \frac{\sqrt{3}\,m}{\cos\theta} = E_c \\ E_d = \left(\frac{\sqrt{3}\,m}{\cos\theta}\right) \times \operatorname{sen}(\theta) \end{cases}$$

$$\Leftrightarrow \begin{cases} E_c = (\sqrt{3}\,m)/\cos\theta \\ E_d = \sqrt{3}\,m\,\operatorname{tg}\theta \end{cases} \Leftrightarrow \begin{cases} \sqrt{3}\,m/\cos\theta + \sqrt{3}\,m\,\operatorname{tg}\theta = 3m \\ \frac{m}{m}\left(\sqrt{3}/\cos\theta + \sqrt{3}\,\operatorname{tg}\theta\right) = 3 \end{cases}$$

$$\Leftrightarrow \left\{ \frac{\sqrt{3}}{\cos\theta} + \frac{\sqrt{3}\operatorname{tg}\theta\cos\theta}{\cos\theta} = 3 \right.$$





$$\begin{cases} E_c = 3m - E_d \\ E_A^2 - p_A^2 = (E_c + E_d)^2 - (E_c \cos\theta)^2 - (E_c \times \operatorname{sen}\theta + E_d)^2 \end{cases}$$

$$\Leftrightarrow \{ 9m^2 - 3m^2 = 9m^2 - E_c^2 \cos^2\theta - (E_c^2 \operatorname{sen}^2\theta + 2 E_d E_c \operatorname{sen}\theta + E_d^2)$$

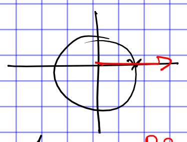
$$\Leftrightarrow \{ -3m^2 = -E_c^2 \cos^2\theta - E_c^2 \operatorname{sen}^2\theta + E_d E_c \operatorname{sen}\theta - E_d^2$$

$$\Leftrightarrow 3m^2 = E_c^2 + E_d^2 + E_c E_d \operatorname{sen}\theta$$

$$\Leftrightarrow 3m^2 = \left(\frac{\sqrt{3}m}{\cos\theta}\right)^2 + \left(\frac{\sqrt{3}m}{\cos\theta} \times \operatorname{sen}\theta\right)^2 + \left(\frac{\sqrt{3}m}{\cos\theta} \times \frac{\sqrt{3}}{\cos\theta} \times \operatorname{sen}^2\theta\right)$$

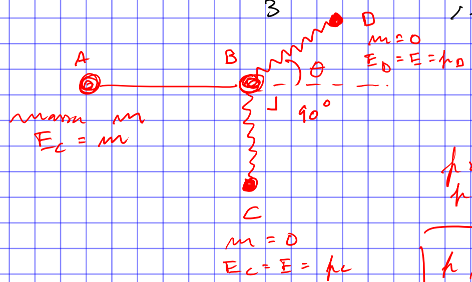
$$\Leftrightarrow 3m^2 = \frac{3m^2}{\cos^2\theta} + \frac{3m^2 \operatorname{sen}^2\theta}{\cos^2\theta} + \frac{3m^2 \times \operatorname{sen}^2\theta}{\cos^2\theta}$$

$$\Leftrightarrow 3m^2 = 3m^2 \frac{(1 + \operatorname{sen}^2\theta + \operatorname{sen}^2\theta)}{\cos^2\theta}$$



$$\Leftrightarrow \cos^2\theta = 1 + 2\operatorname{sen}^2\theta$$
$$\Leftrightarrow \cos^2\theta = 1 + 2(1 - \cos^2\theta)$$
$$\Leftrightarrow \cos^2\theta + 2\cos^2\theta = 1 + 2$$
$$\Leftrightarrow \cos^2\theta = \frac{3}{3} = 1 \quad \Leftrightarrow \cos\theta = \sqrt{1} = 1$$
$$\Leftrightarrow \theta = k\pi, \ k \in \mathbb{Z}$$

BRUH



$E = E_a + m = 3m$
$\quad = E_c + E_D$

$p_x = p_a = \boxed{p_c \times \cos\theta}$
$p_y = 0 = \boxed{p_c \times \operatorname{sen}\theta - p_D}$

$p_A = E_c \times \cos\theta$
$E_d = E_c \times \operatorname{sen}\theta$

$p_A = \sqrt{E_A^2 - m_A^2}$
$\quad = \sqrt{3}\, m$

$p_A^2 = p_c^2 + p_D^2$

~~$p_A = p_c + p_d$~~

$$\begin{cases} E_c = 3m - E_D \\ E_c^2 = P_A^2 - E_D^2 \end{cases}$$

$$\{ (\ldots) \Leftrightarrow \operatorname{sen}\theta = \frac{1}{2} \Leftrightarrow \theta = 30°$$

FÓRMULAS

$p^2 = E^2 - h^2$ $\quad \gamma = \frac{1}{\sqrt{1 - v^2}}$

$m^2 = E^2 - p^2$

$E = mc^2$; $E = m \times \gamma$

$L = L_0 / \gamma$

$\Delta t = \Delta t_0 \times \gamma$

$v = \frac{L_0}{\Delta t} = \frac{L}{\Delta t_0}$

$v = \frac{v' + v_2}{1 + v_2 v'}$

17

Antes $\quad$ Depois

A $\longrightarrow$ B $\quad$ D

$m_A = 2$ $\quad$ $m = 15$

$E_A = 6$ $\quad$ $E_c = 0$

a) $E_A + E_B = E_c$
$\quad 6 + E_B = 15 \Leftrightarrow E_B = 15 - 6$
$\quad = 9$

b) $m^2 = E_A - p_A^2$
$\Leftrightarrow p_A^2 = E_A^2 - m^2 \Leftrightarrow p_A = \sqrt{6^2 - 2^2}$
$\quad = \sqrt{36 - 4} = \sqrt{32} \approx 5{,}66$

$0 = p_A^2 + p_B^2 \Leftrightarrow -p_A = p_B$

$\Rightarrow p_B = -5{,}66$

c) $m^2 = E_B - p_B^2 \Leftrightarrow m = \sqrt{9^2 - (-\sqrt{32})^2} = 7$

d) A massa da partícula D é maior do que a soma das massas das partículas A e B

18 a)

$m_A = 0$ $m_B = m$ $m_c = 1.01\,m$

$E_c = pc$ $E_c = 0$ $E_c = ?$

$$(E, E) + (m, 0) \qquad\qquad (E', p)$$
$$= (E + m, E)$$

Conservação da energia

conservação do momento

$$\begin{cases} \boxed{E + m = E'} \\ \boxed{E = p} \\ m^2 = E'^2 - p^2 \end{cases} \Leftrightarrow \begin{cases} - \\ - \\ 1{,}021\, m^2 = (E + m)^2 - E^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} - \\ - \\ 1{,}02101\, m^2 = 2E \times m + m^2 \end{cases}$$

$$\Leftrightarrow E = 0{,}01005\, m$$