udp Instituto de Ciencias Básicas
FACULTAD DE INGENIERÍA

Cálculo II
Ayudantía N°1 - Ejercicio N°5
Primer Semestre 2017

5. Escriba $\operatorname{sen}^n x \cos^m x = \operatorname{sen}^{n-1} x \cos^m x \operatorname{sen} x$, y recordando que $\cos^2 x = 1 - \operatorname{sen}^2 x$ demuestre la fórmula de reducción

$$\int \cos^m x \operatorname{sen}^n x dx = -\frac{\cos^{m+1} x \operatorname{sen}^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \operatorname{sen}^{n-2} x dx$$

Luego, úsela para calcular $\int \operatorname{sen}^4 x \cos^2 x dx$

Solución:

$$I = \int \operatorname{sen}^n(x) \cos^m(x)\, dx$$
$$I = \int \operatorname{sen}^{(n-1)}(x) \cos^m(x) \operatorname{sen}(x)\, dx$$

Usando integración por partes:

$$u = (\operatorname{sen}(x))^{(n-1)} \qquad v = -\frac{(\cos(x))^{(m+1)}}{m+1}$$
$$du = (n-1)(\operatorname{sen}(x))^{(n-2)} \cos(x)\, dx \qquad dv = (\cos(x))^m \operatorname{sen}(x) dx$$

Obtenemos lo siguiente:

$$I = \operatorname{sen}^{(n-1)}(x) \cos^m(x) \operatorname{sen}(x)\, dx$$
$$I = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^{(m+2)} (\operatorname{sen}(x))^{(n-2)}\, dx$$
$$I = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)} (\cos(x))^2\, dx$$

Utilizando $\cos^2 x = 1 - \operatorname{sen}^2 x$ obtenemos lo siguiente:

$$\int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)} (\cos(x))^2\, dx$$
$$\int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)} (1 - (\operatorname{sen}(x))^2)\, dx$$
$$\int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \left(\int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)}\, dx - \int \operatorname{sen}^n(x) \cos^m(x)\, dx \right)$$
$$\int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)}\, dx - \frac{n-1}{m+1} \int \operatorname{sen}^n(x) \cos^m(x)\, dx$$

$$\int \operatorname{sen}^n(x) \cos^m(x)\, dx + \frac{n-1}{m+1} \int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)}\, dx$$
$$\left(1 + \frac{n-1}{m+1}\right) \int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)}\, dx$$
$$\left(\frac{m+n}{m+1}\right) \int \operatorname{sen}^n(x) \cos^m(x)\, dx = -\frac{(\cos(x))^{(m+1)}}{m+1} (\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1} \int (\cos(x))^m (\operatorname{sen}(x))^{(n-2)}\, dx$$

$$\int \operatorname{sen}^n(x)\cos^m(x)\,dx = (\frac{m+1}{m+n})(-\frac{(\cos(x))^{(m+1)}}{m+1}(\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+1}\int (\cos(x))^m(\operatorname{sen}(x))^{(n-2)}\,dx)$$
$$\int \operatorname{sen}^n(x)\cos^m(x)\,dx = -\frac{(\cos(x))^{(m+1)}}{m+n}(\operatorname{sen}(x))^{(n-1)} + \frac{n-1}{m+n}\int (\cos(x))^m(\operatorname{sen}(x))^{(n-2)}\,dx$$

Quedando demostrada la formula de reducción.

Luego calculamos $\int \operatorname{sen}^4 x\cos^2 x dx$

$$\int \operatorname{sen}^4 x\cos^2 x dx = -\frac{\cos^3(x)\operatorname{sen}^3(x)}{6} + \frac{3}{6}\int \cos^3(x)\operatorname{sen}^3(x)\,dx$$
$$\int \operatorname{sen}^4 x\cos^2 x dx = -\frac{\cos^3(x)\operatorname{sen}^3(x)}{6} + \frac{1}{2}(-\frac{\cos^3(x)\operatorname{sen}(x)}{4} + \frac{1}{4}\int \cos^2(x)\,dx)$$
$$\int \operatorname{sen}^4 x\cos^2 x dx = -\frac{\cos^3(x)\operatorname{sen}^3(x)}{6} - \frac{1}{2}\frac{\cos^3(x)\operatorname{sen}(x)}{4} + \frac{1}{8}(\frac{x}{2} + \frac{1}{4}\operatorname{sen}(2x) + C)$$
$$\int \operatorname{sen}^4 x\cos^2 x dx = -\frac{\cos^3(x)\operatorname{sen}^3(x)}{6} - \frac{\cos^3(x)\operatorname{sen}(x)}{8} + \frac{x}{16} + \frac{1}{32}\operatorname{sen}(2x) + C$$