udp Instituto de Ciencias Básicas
FACULTAD DE INGENIERÍA

Cálculo II

Ayudantía N°5 - Pauta

Primer Semestre 2017

1. Determine si las siguientes afirmaciones son verdaderas o falsas:

   a) $\int_0^1 x\,dx \leq \int_0^1 x^2\,dx$

$$\begin{aligned} 0 \leq x \leq 1 &\Rightarrow x^2 \leq x \\ &\Rightarrow \int_0^1 x^2 dx \leq \int_0^1 x dx \end{aligned}$$

   $\therefore$ La afirmación es Falsa .... **(1 punto)**

   b) Si f y g son funciones integrables en todo intervalo $[a,b] \subset R$ tales que

$$\int_1^2 f(x)\,dx = -1 \qquad \int_1^5 f(x)\,dx = 3 \qquad \int_1^5 g(x)\,dx = 1$$

   Entonces $\int_2^5 f(x)\,dx = 4$ y $\int_1^5 g(x)\left(\int_1^5 f(u)du\right)\,dx = 3$

$$\begin{aligned} \int_1^5 f(x)dx &= \int_1^2 f(x)dx + \int_2^5 f(x)dx \\ 3 &= -1 + \int_2^5 f(x)dx \\ \int_2^5 f(x)dx &= 4 \end{aligned}$$

$\therefore$ Esta afirmación es Verdadera .... **(1 punto)**

$$\begin{aligned} \int_1^5 g(x)(\int_1^5 f(u)du)dx &= \int_1^5 g(x)(3)dx \\ &= (3) * 1 \\ &= 3 \end{aligned}$$

$\therefore$ Esta afirmación es Verdadera .... **(1 punto)**

2. Calcular:

$$\lim_{x\to 0} \frac{\ln(1+x^2) + \displaystyle\int_1^{\cos x} e^{-t^2}\,dt}{xe^{x^3} + x^2 - \displaystyle\int_0^x e^{t^3}\,dt}$$

Aplicando L'Hopital

$$\begin{aligned} L &= \lim_{x\to 0} \frac{\frac{2x}{1+x^2} - \operatorname{sen}(x)e^{-\cos(x)}}{e^{x^3} + 3x^3e^{x^3} + 2x - e^{x^3}} \\ &= \lim_{x\to 0} \frac{\frac{2x}{1+x^2} - \operatorname{sen}(x)e^{-\cos(x)}}{3x^3e^{x^3} + 2x} = \frac{0}{0} \end{aligned}$$

.... **(2 puntos)**

Nuevamente aplicando L'Hopital

$$\begin{aligned} L &= \lim_{x\to 0} \frac{\frac{2-2x^2}{(1+x^2)^2} - [\cos(x)e^{-\cos(x)} - \operatorname{sen}(x)e^{-\cos(x)}2\cos(x\operatorname{sen}(x))]}{9x^2e^{x^3} + 9x^5e^{x^3} + 2} \\ &= \frac{2-\frac{1}{e}}{2} \\ &= 1 - \frac{1}{2e} \end{aligned}$$

.... **(1 punto)**

3. Sea F la función definida por $F(x) = \int_{\frac{1}{2}}^{x} f(u)du$, donde $f(u) = \int_{0}^{u} \frac{1}{1+t^2}dt$. Determine $F'(1)$.

Sabiedo que:

$$\begin{aligned} f(u) &= \int_{0}^{u} \frac{1}{1+t^2}dt \\ &= \arctan(t)|_0^u \\ &= \arctan(u) \end{aligned}$$

.... **(1 punto)**

Luego

$$\begin{aligned} F'(x) &= f(x) = \arctan(x) \\ F'(1) &= \arctan(1) = \frac{\pi}{4} \end{aligned}$$

.... **(1 punto)**

4. Sea $f(x) = \displaystyle\int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$, donde $g(x) = \displaystyle\int_0^{\cos(x)} \left(1+\operatorname{sen}(t^2)\right) dt$. Calcule $f'\left(\frac{\pi}{2}\right)$.

$$
\begin{aligned}
f'(x) &= \frac{1}{\sqrt{1+g^3(x)}} g'(x) \\
g'(x) &= 1 + \operatorname{sen}(\cos^2(x))(-\operatorname{sen}(x))
\end{aligned}
$$

.... **(2 puntos)**

Como estamos buscando $f'\left(\frac{\pi}{2}\right)$, al evaluar g(x) en el punto $\frac{\pi}{2}$ los limites de integración quedarían de la siguiente manera:

$$
\begin{aligned}
g(x) &= \int_0^{\cos(\frac{\pi}{2})} \left(1+\operatorname{sen}(t^2)\right) dt \\
&= \int_0^{0} \left(1+\operatorname{sen}(t^2)\right) dt \\
&= 0
\end{aligned}
$$

.... **(1 punto)**

Luego

$$
\begin{aligned}
f'(x) &= \frac{1}{\sqrt{1+g^3(x)}} g'(x) \\
f'(x) &= \frac{1}{\sqrt{1+g^3(x)}} (1 + \operatorname{sen}(\cos^2(x))(-\operatorname{sen}(x))) \\
f'(\frac{\pi}{2}) &= \frac{1}{\sqrt{1+g^3(\frac{\pi}{2})}} (1 + \operatorname{sen}(\cos^2(\frac{\pi}{2}))(-\operatorname{sen}(\frac{\pi}{2}))) \\
f'(\frac{\pi}{2}) &= \frac{1}{\sqrt{1}} (1 + \operatorname{sen}(0)(-1) = -1
\end{aligned}
$$

.... **(1 punto)**

5. *a)* Sean $\int_1^{11} f(x)\,dx = 15$ y $\int_1^{5} f(x)\,dx = 8$. Calcule $\int_2^{4} f(3x-1)\,dx$

Utilizando la sustitución $u = 3x - 1$ y $du = 3dx$

Si $x = 2 \Rightarrow u = 5$

Si $x = 4 \Rightarrow u = 11$

Luego:

$$\begin{aligned}
\int_2^4 f(3x-1)\,dx &= \frac{1}{3}\int_5^{11} f(u)\,du \\
&= \frac{1}{3}\int_5^{11} f(u)\,du \\
&= \frac{1}{3}(\int_1^{11} f(u)\,du - \int_1^{5} f(u)\,du) \\
&= \frac{1}{3}(15-8) = \frac{7}{3}
\end{aligned}$$

.... **(1 punto)**

b) Calcule $\int_{-1}^{1} \sqrt{|x| - x}\,dx$

$$\begin{aligned}
\int_{-1}^{1} \sqrt{|x|-x}\,dx &= \int_{-1}^{0} \sqrt{-x-x}\,dx + \int_0^1 \sqrt{x-x}\,dx \\
&= \int_{-1}^{0} \sqrt{-2x}\,dx + \int_0^1 \sqrt{0}\,dx \\
&= \int_{-1}^{0} \sqrt{-2x}\,dx
\end{aligned}$$

.... **(1 punto)**

Utilizando la sustitución $u = -2x$ y $du = -2dx$

$$\begin{aligned}
\int_{-1}^{0} \sqrt{-2x}\,dx &= -\frac{1}{2}\int_2^0 \sqrt{u}\,du \\
&= \frac{1}{2}\int_0^2 \sqrt{u}\,du \\
&= \frac{1}{2}\frac{2}{3}\,u^{\frac{3}{2}}\Big|_0^2 \\
&= \frac{1}{3}2^{\frac{3}{2}} = \frac{2\sqrt{2}}{3}
\end{aligned}$$

.... **(1 punto)**