udp Instituto de Ciencias Básicas
FACULTAD DE INGENIERÍA

Cálculo II
Ayudantía N°3 - Pauta
Primer Semestre 2017

Use descomposición en fracciones parciales, o utilice la sustitución de la tangente del ángulo medio, para calcular las siguientes integrales.

1. $\displaystyle\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx$

Utilizando fracciones parciales:

$$\frac{5x^2+3x-2}{x^3+2x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+2)}$$
$$5x^2+3x-2 = Ax(x+2) + B(x+2) + Cx^2$$

. . . . **[1 punto]**

Si $x = 0$:

$$-2 = 2B$$
$$B = -1$$

Si $x = -2$:

$$20 - 6 - 2 = 4C$$
$$C = 3$$

Si $x = 1$:

$$\begin{aligned} 5+3-2 &= 3A + 3B + C \\ 6 &= 3A + 3(-1) + 3 \\ 6 &= 3A \\ A &= 2 \end{aligned}$$

. . . . **[1 punto]**

Obteniendo lo siguiente:

$$\frac{5x^2+3x-2}{x^3+2x^2} = \frac{2}{x} - \frac{1}{x^2} + \frac{3}{(x+2)}$$

Luego al aplicarlo en la integral:

$$\begin{aligned} \int \frac{5x^2+3x-2}{x^3+2x^2}\,dx &= \int \frac{2}{x}\,dx - \int \frac{1}{x^2}\,dx + \int \frac{3}{(x+2)}\,dx \\ &= 2\ln x + \frac{1}{x} + 3\ln(x+2) + C \end{aligned}$$

. . . . **[1 punto]**

2. $\displaystyle\int \frac{dx}{x^4 - x^2}$

Utilizando fracciones parciales:

$$\begin{aligned}\frac{1}{x^4 - x^2} &= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} + \frac{D}{(x+1)} \\ 1 &= Ax(x^2-1) + B(x^2-1) + Cx^2(x+1) + Dx^2(x-1)\end{aligned}$$

. . . . **[1 punto]**

Si $x = 0$:

$$\begin{aligned}1 &= -B \\ B &= -1\end{aligned}$$

Si $x = 1$:

$$\begin{aligned}1 &= 2C \\ C &= \frac{1}{2}\end{aligned}$$

Si $x = -1$:

$$\begin{aligned}1 &= -2D \\ D &= -\frac{1}{2}\end{aligned}$$

Si $x = 2$:

$$\begin{aligned}1 &= 6A + 3B + 12C + 4D \\ 1 &= 6A - 3 + 6 - 2 \\ A &= 0\end{aligned}$$

. . . . **[1 punto]**

Obteniendo lo siguiente:

$$\frac{1}{x^4 - x^2} = -\frac{1}{x^2} + \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$$

Luego al aplicarlo en la integral:

$$\begin{aligned}\int \frac{1}{x^4 - x^2}\,dx &= -\int \frac{1}{x^2}\,dx + \int \frac{1}{2(x-1)}\,dx - \int \frac{1}{2(x+1)}\,dx \\ &= \frac{1}{x} + \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2} + C\end{aligned}$$

. . . . **[1 punto]**

3. $\displaystyle\int \frac{x^4}{x^4-1}\,dx$

Cabe notar que no es posible realizar fracciones parciales al integrando directamente, ya que tanto el polinomio del numerador como el del denominador son de grado 4. En su lugar, considere:

$$\int \frac{x^4}{x^4-1}dx = \int \frac{x^4-1+1}{x^4-1}dx = x + \int \frac{dx}{x^4-1} = x + \int \frac{dx}{(x^2+1)(x-1)(x+1)}$$

Se aplica fracciones parciales a la integral resultante:

$$\frac{1}{(x^2+1)(x-1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+1}$$
$$1 = (Ax+B)(x^2-1) + C(x^2+1)(x+1) + D(x^2+1)(x-1)$$

.... **[1 punto]**

Si $x=1$:

$$1 = 4C$$
$$C = \frac{1}{4}$$

Si $x=-1$:

$$1 = -4D$$
$$D = -\frac{1}{4}$$

Si $x=0$:

$$0 = -B + C - D$$
$$0 = -B + \frac{1}{4} + \frac{1}{4}$$
$$B = -\frac{1}{2}$$

Si $x=2$:

$$1 = 6A - \frac{3}{2} + \frac{15}{4} - \frac{5}{4}$$
$$A = 0$$

.... **[1 punto]**

Luego al aplicarlo en la integral:

$$\int \frac{x^4}{x^4-1} = x + \int \frac{-1/2}{x^2+1}dx + \int \frac{1/4}{x-1}dx + \int \frac{-1/4}{x+1}dx$$
$$\int \frac{x^4}{x^4-1} = x + -\frac{1}{2}\arctan(x) + \frac{1}{4}\ln(x-1) - \frac{1}{4}\ln(x+1) + C$$

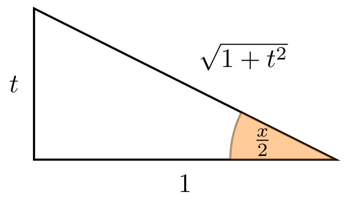
.... **[1 punto]**

4. $\displaystyle\int \frac{dx}{2\,\text{sen}\,x + \text{sen}(2x)}$

$$\int \frac{dx}{2\,\text{sen}\,x + \text{sen}(2x)} = \int \frac{dx}{2\,\text{sen}\,x + 2\,\text{sen}\,x\cos x} = \int \frac{dx}{2\,\text{sen}\,x(1+\cos x)}$$

Utilizamos la siguiente sustitución:

$$\begin{aligned} t &= \tan\left(\frac{x}{2}\right) \\ dx &= \frac{2}{1+t^2}\,dt \\ \text{sen}\,x &= \frac{2t}{1+t^2} \\ \cos x &= \frac{1-t^2}{1+t^2} \end{aligned}$$



. . . . **[1 punto]**

Obtenemos lo siguiente:

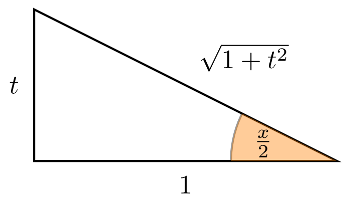
$$\begin{aligned} \int \frac{dx}{2\,\text{sen}\,x + \text{sen}(2x)} &= \int \frac{\frac{2\,dt}{1+t^2}}{2(\frac{2t}{1+t^2})(1+\frac{1-t^2}{1+t^2})} \\ I &= \int \frac{(1+t^2)^2\,dt}{4t(1+t^2)} \\ I &= \int \frac{(1+t^2)\,dt}{4t} \\ I &= \int \frac{dt}{4t} + \int \frac{t^2\,dt}{4t} \\ I &= \int \frac{dt}{4t} + \int \frac{t\,dt}{4} \\ I &= \frac{1}{4}\ln t + \frac{t^2}{8} + C \\ I &= \frac{1}{4}\ln\left(\tan\left(\frac{x}{2}\right)\right) + \frac{1}{8}\tan^2\left(\frac{x}{2}\right) + C \end{aligned}$$

. . . . **[1 punto]**

5. $\displaystyle\int \frac{dx}{3\operatorname{sen} x + 4\cos x}$

Utilizamos la siguiente sustitución:

$$\begin{aligned} t &= \tan\left(\frac{x}{2}\right) \\ dx &= \frac{2}{1+t^2}\,dt \\ \operatorname{sen} x &= \frac{2t}{1+t^2} \\ \cos x &= \frac{1-t^2}{1+t^2} \end{aligned}$$



. . . . **[1 punto]**

Obtenemos lo siguiente:

$$\begin{aligned} \int \frac{dx}{3\operatorname{sen} x + 4\cos x} &= \int \frac{\frac{2\,dt}{1+t^2}}{3(\frac{2t}{1+t^2}) + 4(\frac{1-t^2}{1+t^2}))} \\ I &= \int \frac{2(1+t^2)\,dt}{(1+t^2)(6t+4-4t^2)} \\ I &= \int \frac{dt}{(2-t)(2t+1)} \end{aligned}$$

. . . . **[1 punto]**

Utilizando fracciones parciales:

$$\begin{aligned} \frac{1}{(2-t)(2t+1)} &= \frac{A}{2-t} + \frac{B}{2t+1} \\ 1 &= A(2t+1) + B(2-t) \end{aligned}$$

Si $x = -\frac{1}{2}$:

$$\begin{aligned} 1 &= B\frac{5}{2} \\ B &= \frac{2}{5} \end{aligned}$$

Si $x = 2$:

$$\begin{aligned} 1 &= 5A \\ A &= \frac{1}{5} \end{aligned}$$

. . . . **[1 punto]**

Obteniendo lo siguiente:

$$\begin{aligned} \frac{1}{(2-t)(2t+1)} &= \frac{1}{5(2-t)} + \frac{2}{5(2t+1)} \\ \frac{1}{(2-t)(2t+1)} &= -\frac{1}{5}\frac{1}{t-2} + \frac{2}{5(2t+1)} \end{aligned}$$

Luego volviendo a la integral:

$$
\begin{aligned}
I &= \int \left(-\frac{1}{5}\frac{1}{t-2} + \frac{2}{5(2t+1)} \right) dt \\
I &= -\frac{1}{5}\ln(t-2) + \frac{1}{5}\ln(2t+1) + C \\
I &= -\frac{1}{5}\ln\left(\tan\left(\frac{x}{2}\right) - 2\right) + \frac{1}{5}\ln\left(2\tan\left(\frac{x}{2}\right) + 1\right) + C
\end{aligned}
$$

. . . . **[1 punto]**