Energy Efficient Resource Allocation for Mixed RF/VLC Heterogeneous Wireless Networks

Mohamed Kashef, *Member, IEEE*, Muhammad Ismail, *Member, IEEE*, Mohamed Abdallah, *Senior Member, IEEE*, Khalid A. Qaraqe, *Senior Member, IEEE*, and Erchin Serpedin, *Fellow, IEEE*

***Abstract*—Developing energy efficient wireless communication networks has become crucial due to the associated environmental and financial benefits. Visible light communication (VLC) has emerged as a promising candidate for achieving energy efficient wireless communications. Integrating VLC with radio frequency (RF)-based wireless networks has improved the achievable data rates of mobile users. In this paper, we investigate the energy efficiency benefits of integrating VLC with RF-based networks in a heterogeneous wireless environment. We formulate and solve the problem of power and bandwidth allocation for energy efficiency maximization of a heterogeneous network composed of a VLC system and an RF communication system. Then, we investigate the impact of the system parameters on the energy efficiency of the mixed RF/VLC heterogeneous network. Numerical results are conducted to corroborate the superiority in performance of the proposed hybrid system. The impact of hybrid system parameters on the overall energy efficiency is also quantified.**

***Index Terms*—Energy efficiency, resource allocation, visible light communications, wireless networks.**

I. Introduction

ENERGY efficient communication solutions are motivated by the increasing energy consumption of wireless networks caused by the high demand for wireless communication services, including video streaming and data applications. The annual energy consumption of a mobile service operator is around 50–100 GWh [1]. From an environmental impact viewpoint, the CO_2 emissions of the telecommunications industry represent 2% of the total CO_2 emissions worldwide and are expected to reach 4% by 2020 [2].

Recently, visible light communication (VLC) has been proposed as an energy efficient solution that exploits the illumination energy, which is already consumed for lighting, in data transmission with high achievable data rates. This energy saving has been realized by the introduction of light emitting diodes (LEDs) that require approximately twenty times less power than the conventional light sources and five times less power than the fluorescent light sources [3]. Hence, replacing the current inefficient illumination sources by LEDs reduces significantly the CO_2 emissions due to lighting. The energy saving benefits of using LEDs as a more efficient replacement for the conventional lighting sources are described in [4]. Additionally, LEDs can be readily used as transmitters for VLC communication systems whereby communication takes place by modulating the intensity of light in such a way that it is unnoticeable to the human eyes. The receiver is a photo sensitive detector that demodulates the light signal into an electrical signal. Thus, data transfer is performed using intensity modulation and direct detection (IM/DD) [5], which can be practically obtained by a number of pulsed modulation schemes [6].

The energy efficiency of data transmission has been improved as a result of using white LEDs for both illumination and communication due to their low-power consumption and high achievable data rates [3]. Hence, various potential applications of VLC techniques have been proposed including broadband indoor communication, civilian applications in sensitive environments such as hospitals and aircraft, and military applications that demand anti-jamming. The adoption of LEDs in communications using VLC techniques is reviewed in [7] where brief comparisons to other communication techniques are studied and the VLC presented many advantages including cheap transmitters and receivers, low power consumption, and good safety features. These comparisons assume different performance criteria including energy efficiency. A more detailed comparison of the performance of VLC systems against radio frequency (RF) communication systems can be found in [8] where better area spectral efciency is achieved for VLC systems. Moreover, in [9], the authors have proposed an improved VLC modulation schemes to enhance energy efficiency in a stand-alone VLC system.

On the other hand, heterogeneous wireless networks have been proposed to enable wireless networks with different technologies to work together to enhance the overall system capacity. Such enhancements are achieved because of the diversity in fading channels, propagation losses, and the available resources at different networks. However, such gains can be only achieved by tackling the major challenge of developing resource allocation algorithms that assign the power and bandwidth among the heterogeneous networks to achieve different service requirements.

Employing multiple radio interfaces in communication systems has proven to enhance energy efficiency in RF

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M. Kashef, M. Ismail, and K. Qaraqe are with the Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar (e-mail: mohamed.kashef@qatar.tamu.edu; m.ismail@qatar.tamu.edu; Khalid.qaraqe@qatar.tamu.edu).

M. Abdallah is with the Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar and also with the Department of Electronics and Communications Engineering, Cairo University, Giza, Egypt (e-mail: mohamed.abdallah@qatar.tamu.edu).

E. Serpedin is with the Electrical and Computer Engineering, Texas A&M University, College Station, TX 77840 USA (e-mail: serpedin@ece.tamu.edu).

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communication systems. In [11], the cooperation between mobile terminals (MTs) is exploited by allowing the MTs to transmit their data efficiently to base stations using space-time coding over multiple radio interfaces. Also, MTs are used to relay source data using multiple radio interfaces in [12]. Moreover in [13], the authors have discussed the enhancement in the energy efficiency of MTs equipped with multi-homing capabilities where the MTs are allowed to aggregate the available resources from different networks in a downlink communication scenario.

Although VLC can attain high data rates in the presence of a line of sight (LOS) between the transmitter and the receiver, its performance degrades significantly in the absence of the LOS. Thus, reliability is a major concern for VLC systems. Also, ambient light interference is another challenge that can be mitigated by using optical filters and robust signaling schemes. On the contrary, RF wireless communication networks exhibit a higher transmission reliability even in the absence of a LOS between the transmitter and the receiver. The main drawback of RF wireless systems compared with the VLC networks is that RF networks consume higher amounts of energy.

Cooperation in VLC networks between the nodes has not been investigated comprehensively yet. Cooperation in VLC systems has been considered in [10] where the total rate of a VLC network contains multiple interfering transmitters is improved by employing cooperative transmission power control. Few papers have discussed the complementary use of VLC and RF communication systems to achieve throughput and reliability gains. In [14] and [15], the authors have discussed the potential benefits of the RF/VLC combination and the optimal handover techniques and have shown that lower data transfer delay and higher data rates can be achieved because of the nodes ability to switch their access between the VLC and RF networks. The hybrid simultaneous use of VLC and RF systems has been discussed in [16]–[22] where the authors investigate the feasibility and potential benefits of RF/VLC hybrid systems in enhancing the throughput and increasing the coverage. The energy efficiency optimization of this system has not been investigated before and hence we consider the energy efficiency of hybrid RF/VLC systems via radio resource allocation which is considered for first time to the best of our knowledge.

Fifth generation (5G) wireless access aims to achieve higher system data rates, network capacity, and reliability of communications [23]. Moreover, it aims to achieve lower latency, and energy consumption. In [23]–[25], the suitability of VLC technology to serve indoor communication in future 5G networks was demonstrated. The main factors that help this idea are the availability of the huge visible light spectrum bandwidth, the non-existence of the interference towards the existing RF transmissions, the spatial reuse capability the visible light characteristics, and the low cost for data transmission. These VLC benefits are gained beside the currently reported high data rate of 3.5 Gb/s [26]. Due to the requirements of 5G networks to have ubiquitous connectivity and hence achieving high data rates for very dense networks, they need to be energy efficient to reduce the total cost per transmitted bit [27]. A promising proposal to achieve this goal is employing small communication cells that exploit low-cost base stations and have short-distance communication links [28]. VLC technology represents an excellent candidate to satisfy the requirements of having small cells with low energy cost per transmitted bit [29].

VLC and RF communication systems can work together to take advantage of the benefits yielded by both systems to enhance the communication energy efficiency while maintaining good reliability. However, up to the best of authors' knowledge, no research work has studied the problem of radio resource allocation in a heterogeneous RF/VLC communication system from an energy efficiency viewpoint. In this paper, we investigate the energy efficiency of an indoor heterogeneous network composed of a single RF access point (AP) and a single VLC AP transmitting to a number of MTs located in the coverage region of both APs. The VLC system employs its illumination power for data transmission while consuming additional power for data processing. On the other hand, the RF communication system consumes both data processing and transmission powers. We formulate and analyze the problem of maximizing the heterogeneous network energy efficiency constrained by the required data rates for the MTs and the maximum allowable transmission powers for the APs. MTs are equipped with multi-homing capability and can receive data from both VLC and RF communication systems. We compare the performance of the heterogeneous network consisting of VLC and RF communication systems to the benchmarks represented by an RF only network and a heterogeneous network composed of two RF communication systems. We compare the energy efficiency of these systems to quantify the impact of using mixed RF/VLC systems on the network energy efficiency.

The rest of the paper is organized as follows. The mixed RF/VLC heterogeneous network model is presented in Section II. The energy efficiency maximization resource allocation problem is formulated in Section III. In Section IV, a framework for solving the problem is presented. In Section V, numerical results are presented. Finally, in Section VI, conclusions are drawn.

II. System Model

We consider an indoor downlink scenario in which M MTs equipped with VLC receivers and RF receivers are communicating with a single RF AP and a single VLC AP as shown in Fig. 1. Examples of RF wireless communication networks include cellular networks (e.g., femto-cells). The set of MTs is denoted by $\mathcal{M} = \{1, 2, \cdots ., M\}$. All MTs are in the coverage areas of both APs. Each MT has multi-homing capabilities that allow simultaneous association with both networks and enable the MTs to aggregate the available resources from both networks to provide services with high performance requirements such as improving network capacity [13]. The VLC can offer benefits when light is turned on, however, the potential deployment for VLC is focused on the public areas where lights are always on such as airports, waiting areas, and offices. As a result, we focus our investigation to this case where light sources are always on. Moreover, in the cases where light can be switched on/off, the proposed power and bandwidth allocation optimization framework can be used during the times

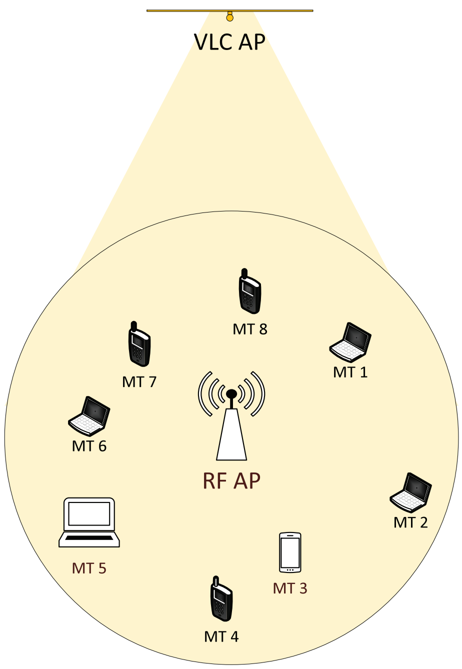

Fig. 1. Illustration of the APs Coverage Areas.

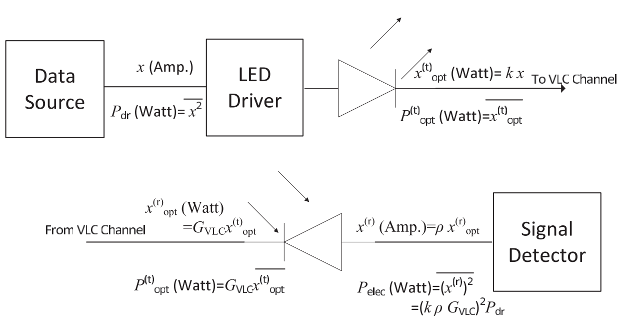

Fig. 2. VLC Single-User Transmitter and Receiver Models.

where VLC is on and within the coverage of VLC access point to optimize the system performance.

A. VLC System Model

The single-user VLC system transmitter and receiver models are shown in Fig. 2 as well as the signal flow through the system to calculate the electrical signal to noise power ratio at the receiver which controls the performance of the VLC system.

At the transmitter side, the driving current of the LED expressed in Amperes (Amp) is denoted by x and is proportional to the modulated data of the source. The driving power is denoted by P_{dr} and is calculated as the average of the squared driving current $\overline{x^2}$. In a multi-user system, the mth MT is allocated a proportion of P_{dr} that is denoted by $P_{\text{VLC},m}$. These proportions of the driving power $P_{\text{VLC},m}$ are to be controlled while optimizing the system performance. The output of the LED is an optical intensity signal which is proportional to the driving current, when operating in the LED dynamic range, and is denoted by $x_{\text{opt}}^{(\text{t})} = kx$ where k (Watt/Amp) is the proportionality factor of the electrical to optical conversion and is determined by the LED characteristics. The average transmission optical power, which is denoted by $P_{\text{opt}}^{(\text{t})}$, is the average of the transmitted optical intensity signal. It is calculated as follows

$$P_{\text{opt}}^{(\text{t})} = \overline{x_{\text{opt}}^{(\text{t})}}, \tag{1}$$

where $\overline{(.)}$ is the average of the signal. This average optical transmitted power determines the illumination level of the LED.

At the receiver of the mth MT, a proportion of the received optical signal, which carries the intended information to the mth MT, is extracted. This proportion is denoted by $x_{\text{opt},m}^{(r)}$ and it represents the light intensity at the photo-sensitive detector. It is expressed as follows

$$x_{\text{opt},m}^{(\text{r})} = G_{\text{VLC},m} x_{\text{opt},m}^{(\text{t})}, \tag{2}$$

where $G_{\text{VLC},m}$ is the VLC channel power gain between the VLC AP to the mth MT. The photo-detector has a responsivity ρ (Amp/Watt) and converts the received optical signal into an electrical current which is proportional to the optical intensity. At the mth MT, the received electrical signal, which is denoted by $x_m^{(\text{r})}$, is calculated as follows

$$x_m^{(\text{r})} = \rho x_{\text{opt},m}^{(\text{r})}. \tag{3}$$

The average electrical power of the received signal is calculated as follows

$$\begin{aligned} P_{\text{elec},m} &= \overline{\left(x_m^{(\text{r})}\right)^2}, \\ P_{\text{elec},m} &= \left(k\rho G_{\text{VLC},m}\right)^2 P_{\text{VLC},m}. \end{aligned} \tag{4}$$

This evaluated received electrical power represents the signal power while calculating the signal to noise ratio of the VLC system.

B. RF/VLC Heterogeneous System

The maximum bandwidths of the VLC and RF systems are denoted by $B_{\text{VLC,max}}$ and $B_{\text{RF,max}}$, respectively. The bandwidth of the VLC system is usually limited by the used LED and photo-detector bandwidths. The mth user is assigned a bandwidth of $B_{\text{VLC},m}$ and $B_{\text{RF},m}$ by the VLC and RF communication systems, respectively. The transmission powers allocated to the mth MT by VLC and RF APs are $P_{\text{VLC},m}$ and $P_{\text{RF},m}$, respectively. The total transmission power of an AP is constrained by the maximum allowed power, and it is denoted correspondingly by $P_{\text{VLC,max}}$ and $P_{\text{RF,max}}$. The fixed powers needed by the APs are denoted by Q_{VLC} and Q_{RF}. These fixed powers include the powers used for circuits' operation and data processing before transmission. The value of Q_{VLC} includes also any required power to compensate for the losses in the LED efficiency due to data transmission. Moreover, we have accounted for the modulation power consumption in the fixed powers Q_{VLC} where we show in our results the effect of the change of the fixed power on the system performance that corresponds to various modulation strategies. The values of this power can be found in few works in literature such as [30] and [31].

The data rates achieved by the mth MT are denoted by $R_{\mathrm{VLC},m}$ and $R_{\mathrm{RF},m}$, respectively, where the sum of the data rates of the mth MT is constrained to be larger than or equal to a required minimum data rate $R_{\min,m}$.

The power gains of the channels between the mth MT and the RF and VLC APs are denoted by $G_{\mathrm{RF},m}$ and $G_{\mathrm{VLC},m}$, respectively. The channel power gain for the RF communication system captures both the channel fading and path loss. For the VLC system, the channel power gain captures LOS path loss for the optical wireless signal. The distances between the APs to the mth MT are denoted by $d_{\mathrm{RF},m}$ and $d_{\mathrm{VLC},m}$. The thermal noise power spectral density affecting the RF receivers is denoted by $N_{0,\mathrm{RF}}$, and is given by $N_{0,\mathrm{RF}} = k_{\mathrm{B}}T$, where k_{B} stands for Boltzmann's constant and T denotes the ambient temperature. The noise power spectral density affecting the VLC receivers is dominated by the light shot noise and is denoted by $N_{0,\mathrm{VLC}}$. It is shown in [32], Chapter 2, that the noise affecting VLC systems can be well approximated by Gaussian noise independent of the received signal. The characteristics of the channel models can be found in [8].

The RF communication path loss, denoted by PL, typically takes the form [33]

$$\mathrm{PL[dB]} = A\log_{10}(d_{\mathrm{RF},m}) + B + C\log_{10}\left(\frac{f_c}{5}\right) + X, \quad (5)$$

where f_c is the carrier frequency in GHz, A, B and C are constants depending on the propagation model, and X stands for an environment specific term. For the LOS scenario, $A = 18.7$, $B = 46.8$ and $C = 20$. For the non-LOS (NLOS) scenario, $A = 36.8$, $B = 43.8$, $C = 20$ and $X = 5(n_w - 1)$ in case of light walls or $X = 12(n_w - 1)$ in case of heavy walls, where n_w denotes the number of walls between the AP and the MT. The channel power gain is defined as

$$G_{\mathrm{RF},m} = 10^{-\mathrm{PL[dB]}/10}. \quad (6)$$

We denote the channel power gain in the presence of LOS by $G_{\mathrm{RF},m}^{\mathrm{LOS}}$ and for the NLOS scenario by $G_{\mathrm{RF},m}^{\mathrm{NLOS}}$. For VLC systems, the channel power gain is given in [8] by

$$G_{\mathrm{VLC},m} = \frac{(n+1)\cos^n(\phi_m)A_m\cos(\theta_m)}{2\pi d_{\mathrm{VLC},m}^2}, \quad (7)$$

where A_m stands for the physical area of the photo detector at the mth MT, ϕ_m denotes the angle of irradiance from the LED to the mth MT, n is the order of the Lambertian emission defined by the LED's semi-angle at half power Φ, which is $n = \ln(1/2)/\ln(\cos(\Phi))$, and θ_m represents the angle of incidence.

The LOS availability probabilities for RF and VLC systems are defined as the probability that there are no obstacles in the communication link between the MT and the corresponding AP, and are denoted by ρ_{RF} and ρ_{VLC}, respectively. In the case of RF transmissions, the channel path loss exponent increases with the LOS absence as discussed earlier. For the case of VLC, the signal is degraded significantly in the absence of LOS that may result in unsuccessful data transmissions. In this work, we assume that the NLOS VLC transmissions are unsuccessful that we focus only on the system performance with LOS transmissions.

III. Energy Efficiency Maximization Problem

In this section, we formulate the energy efficiency maximization problem, where energy efficiency is defined as the total achieved data rate per unit power consumption.

The average received electrical signal to noise power ratios for the mth MT corresponding to the RF and VLC systems are denoted by $\gamma_{\mathrm{RF},m}$ and $\gamma_{\mathrm{VLC},m}$, respectively, and are expressed as [8]

$$\gamma_{\mathrm{RF},m} = \frac{P_{\mathrm{RF},m}G_{\mathrm{RF},m}}{B_{\mathrm{RF},m}N_{0,\mathrm{RF}}}, \quad (8)$$

$$\gamma_{\mathrm{VLC},m} = \frac{P_{\mathrm{VLC},m}\left(k\rho G_{\mathrm{VLC},m}\right)^2}{B_{\mathrm{VLC},m}N_{0,\mathrm{VLC}}}, \quad (9)$$

where the value of $G_{\mathrm{RF},m}$ can be substituted by $G_{\mathrm{RF},m}^{\mathrm{LOS}}$ or $G_{\mathrm{RF},m}^{\mathrm{NLOS}}$ to obtain the corresponding $\gamma_{\mathrm{RF},m}^{\mathrm{LOS}}$ and $\gamma_{\mathrm{RF},m}^{\mathrm{NLOS}}$ for the LOS and NLOS channels. Also, the value of $\gamma_{\mathrm{VLC},m}$ is calculated by dividing the received electrical power in (4) by the noise power.

The achieved data rates by the mth MT exhibited by different networks are denoted by $R_{\mathrm{VLC},m}$ and $R_{\mathrm{RF},m}$, respectively. In this work, we employ Shannon's capacity equation to represent the achievable rate as a function in the channel signal to noise ratio. This achievable rate represents a limit for the performance without considering specific modulation and coding schemes. It is used for its mathematical tractability while the optimality of the achieved strategies does not deviate significantly while applying different modulation schemes [34]. Using the multi-homing capability, the sum of the achievable data rates of the mth MT via the VLC and RF APs should not be less than the required data rate $R_{\min,m}$. The expected values of the achievable data rates from each AP are calculated over the probability mass function of LOS availability and are calculated as follows

$$R_{\mathrm{RF},m} = B_{\mathrm{RF},m}\Big(\rho_{\mathrm{RF}}\log_2\left(1+\gamma_{\mathrm{RF},m}^{\mathrm{LOS}}\right) + (1-\rho_{\mathrm{RF}})\log_2\left(1+\gamma_{\mathrm{RF},m}^{\mathrm{NLOS}}\right)\Big), \quad (10)$$

$$R_{\mathrm{VLC},m} = B_{\mathrm{VLC},m}\rho_{\mathrm{VLC}}\log_2\left(1+\gamma_{\mathrm{VLC},m}\right). \quad (11)$$

The total achieved data rate in the heterogeneous RF/VLC network is denoted by R_{T} and its value is calculated as follows

$$R_{\mathrm{T}} = \sum_{m\in\mathcal{M}} R_{\mathrm{VLC},m} + \sum_{m\in\mathcal{M}} R_{\mathrm{RF},m}. \quad (12)$$

The total consumed power for communication is denoted by P_{T} and its value is calculated as follows

$$P_{\mathrm{T}} = Q_{\mathrm{VLC}} + Q_{\mathrm{RF}} + \sum_{m\in\mathcal{M}} P_{\mathrm{RF},m}. \quad (13)$$

where the first term in (13) represents the consumed power for the VLC AP and is calculated using the fact that the transmission power is the optical power used for illumination by design, and hence only the fixed power consumption Q_{VLC} is accounted for as a power cost. The second and third terms represents the RF power consumption which accounts for both the processing and transmission powers. The percentage of power to be used for transmission in VLC systems is still a part of the used illumination power so the transmission power is actually an optical portion of the illumination power. This feature leads to the fact that in order to maximize the energy efficiency, the full illumination power should be employed for communications and the RF system is used to achieve the rest of the required data rate.

We study the transmission power allocation problem to maximize the energy efficiency of the heterogeneous network over the assigned transmission powers and bandwidths to the MTs by the APs. The hybrid RF/VLC network energy efficiency is denoted by $\eta = R_{\mathrm{T}}/P_{\mathrm{T}}$. The user total achieved data rates, i.e., $R_{\mathrm{VLC},m} + R_{\mathrm{RF},m}$ are constrained by the minimum required data rates for the MTs. For each AP, the total transmission power consumption, i.e., $\sum_{m\in\mathcal{M}} P_{\mathrm{VLC},m}$ and $\sum_{m\in\mathcal{M}} P_{\mathrm{RF},m}$, is constrained by the maximum allowable transmission power. The total allocated bandwidth of each AP is constrained by the maximum allowable bandwidth as well. Given these constraints, the problem is formulated as follows

$$\begin{aligned}
\max_{P_{\mathrm{VLC},m},P_{\mathrm{RF},m},B_{\mathrm{VLC},m},B_{\mathrm{RF},m}} \quad & \eta \\
\text{s.t.} \quad & R_{\mathrm{VLC},m} + R_{\mathrm{RF},m} \geq R_{\min,m}, \forall m \in \mathcal{M}, \\
& \sum_{m\in\mathcal{M}} P_{\mathrm{VLC},m} \leq P_{\mathrm{VLC},\max}, \\
& \sum_{m\in\mathcal{M}} P_{\mathrm{RF},m} \leq P_{\mathrm{RF},\max}, \\
& \sum_{m\in\mathcal{M}} B_{\mathrm{VLC},m} \leq B_{\mathrm{VLC},\max}, \\
& \sum_{m\in\mathcal{M}} B_{\mathrm{RF},m} \leq B_{\mathrm{RF},\max}, \\
& P_{\mathrm{VLC},m}, P_{\mathrm{RF},m}, B_{\mathrm{VLC},m}, B_{\mathrm{RF},m} \geq 0, \forall m \in \mathcal{M}.
\end{aligned} \tag{14}$$

The problem (14) is referred to as concave-convex fractional program [35] since the numerator of the objective function is concave with respect to the decision variables and the denominator is affine. The proof for concavity of the numerator of the objective function is found in Appendix A.

IV. Energy Efficient Resource Allocation

In this section, we transform the energy efficiency maximization problem into a convex optimization problem. Then, an iterative algorithm is used to calculate the optimal allocated powers and bandwidths.

We use a parametric approach to transform the problem into a convex optimization problem for a parameter λ. The optimal solution of the problem in (14) can be obtained by finding the roots of the equation $F(\lambda) = 0$ [36]. By considering the same constraints as in (14), we solve the following optimization problem for a given λ

$$F(\lambda) = \max_{P_{\mathrm{VLC},m},P_{\mathrm{RF},m},B_{\mathrm{VLC},m},B_{\mathrm{RF},m}} R_{\mathrm{T}} - \lambda P_{\mathrm{T}}. \tag{15}$$

Then, we use the following Dinkelbach-type algorithm to update λ to find the roots for $F(\lambda) = 0$ and hence the optimal solution for the optimization problem in (14).

Algorithm 1. Dinkelbach-type Procedure

Initialization: $P_{\mathrm{VLC},m}(0)$, $P_{\mathrm{RF},m}(0)$, $B_{\mathrm{VLC},m}(0)$, $B_{\mathrm{RF},m}(0) > 0\ \forall m$, $\lambda(1) = \eta(0)$, $i = 1$;
while $F(\lambda(i)) \neq 0$ **do**
  Solve (15) for optimal $\{P_{\mathrm{VLC},m}(i), P_{\mathrm{RF},m}(i), B_{\mathrm{VLC},m}(i), B_{\mathrm{RF},m}(i)\}\forall m \in \mathcal{M}$;
  $\lambda(i+1) = \eta(i)$;
  $i \leftarrow i + 1$
end while
Output: $P_{\mathrm{VLC},m}$, $P_{\mathrm{RF},m}$, $B_{\mathrm{VLC},m}$, $B_{\mathrm{RF},m}\forall m \in \mathcal{M}$

Algorithm 1 is guaranteed to converge to the optimal solution of (15) in a finite number of iterations [36]. In the following, we derive the solution of (15) which is a crucial step in Algorithm 1.

Problem (15) is a convex optimization problem and the Lagrangian function for the problem can be expressed as

$$\begin{aligned}
L = {} & R_{\mathrm{T}} - \lambda P_{\mathrm{T}} + \sum_{m\in\mathcal{M}} \mu_m \{R_{\mathrm{VLC},m} + R_{\mathrm{RF},m} - R_{\min,m}\} \\
& + \nu_{\mathrm{VLC}}\{P_{\mathrm{VLC},\max} - \sum_{m\in\mathcal{M}} P_{\mathrm{VLC},m}\} + \nu_{\mathrm{RF}}\{P_{\mathrm{RF},\max} - \sum_{m\in\mathcal{M}} P_{\mathrm{RF},m}\} \\
& + \beta_{\mathrm{VLC}}\{B_{\mathrm{VLC},\max} - \sum_{m\in\mathcal{M}} B_{\mathrm{VLC},m}\} + \beta_{\mathrm{RF}}\{B_{\mathrm{RF},\max} - \sum_{m\in\mathcal{M}} B_{\mathrm{RF},m}\},
\end{aligned} \tag{16}$$

where μ_m is a Lagrangian multiplier for the data rate constraint of the mth MT, ν_{VLC} and ν_{RF} are the Lagrangian multipliers for the power constraints of the VLC and RF networks, respectively, and β_{VLC} and β_{RF} are the Lagrangian multipliers for the bandwidth constraints of the VLC and RF networks, respectively.

In the following subsections, we maximize the Lagrangian function in (16) for a given λ. For given values of μ_m and λ, the problems of allocating the power and bandwidth for each the RF and VLC networks are separated. Hence in algorithms 2–5, we solve these separated problems for given values of λ and μ_m. Then in Algorithm 6, we obtain the optimal values for μ_m to maximize the Lagrangian function in (16) for a given λ. Finally, Algorithm 6 is used to solve (15) within Algorithm 1 by taking into account the power and bandwidth allocation in VLC and RF networks.

A. Power Allocation in the VLC Network

In the following, we derive the optimal VLC power allocation for a given $\mu_m\ \forall m \in \mathcal{M}$. From the Karush-Kuhn-Tucker (KKT) conditions [37], we have

$$\frac{\partial L}{\partial P_{\mathrm{VLC},m}} = 0. \tag{17}$$

Thus, we have

$$P_{\mathrm{VLC},m} = B_{\mathrm{VLC},m}\left[\rho_{\mathrm{VLC}}\frac{1+\mu_m}{(\nu_{\mathrm{VLC}}+\lambda)\ln(2)} - \frac{N_{0,\mathrm{VLC}}}{\left(k\rho G_{\mathrm{VLC},m}\right)^2}\right]^+, \tag{18}$$

Algorithm 2. Power Allocation for VLC network

Input: $B_{\mathrm{VLC},m}$, μ_m, and λ;
Initialization: $\nu_{\mathrm{VLC}}(1) \geq 0$;
for $i = 1 : I$ **do**
  for $m \in \mathcal{M}$ **do**
    $P_{\mathrm{VLC},m}(i) = B_{\mathrm{VLC},m}\left[\rho_{\mathrm{VLC}}\frac{1+\mu_m}{(\nu_{\mathrm{VLC}}(i)+\lambda)\ln(2)} - \frac{N_{0,\mathrm{VLC}}}{(k\rho G_{\mathrm{VLC},m})^2}\right]^+$;
  end for
  $\nu_{\mathrm{VLC}}(i+1) = \left[\nu_{\mathrm{VLC}}(i) - \eta_1(P_{\mathrm{VLC,max}} - \sum_{m\in\mathcal{M}} P_{\mathrm{VLC},m})\right]^+$;
end for
Output: $P_{\mathrm{VLC},m} \forall m \in \mathcal{M}$

where $[.]^+$ is a projection on the positive quadrant that takes into account the constraint $P_{\mathrm{VLC},m} \geq 0$. The variable ν_{VLC} guarantees that the total VLC power is constrained by the maximum allowable power and can be found using the gradient descent method as follows

$$\nu_{\mathrm{VLC}}(i+1) = \left[\nu_{\mathrm{VLC}}(i) - \eta_1(P_{\mathrm{VLC,max}} - \sum_{m\in\mathcal{M}} P_{\mathrm{VLC},m})\right]^+, \tag{19}$$

where η_1 is a sufficiently small step size. Algorithm 2 describes the optimal power allocation for the VLC network, where I stands for the number of iterations required for convergence.

B. Power Allocation in the RF Network

Similar to the previous subsection, we derive the optimal RF power allocation for a given μ_m. Using the KKT conditions, we have

$$\frac{\partial L}{\partial P_{\mathrm{RF},m}} = 0. \tag{20}$$

Thus, it follows that

$$\frac{B_{\mathrm{RF},m}}{\ln 2}\left(\rho_{\mathrm{RF}}\frac{G_{\mathrm{RF},m}^{\mathrm{LOS}}}{P_{\mathrm{RF},m}G_{\mathrm{RF},m}^{\mathrm{LOS}} + B_{\mathrm{RF},m}N_{0,\mathrm{RF}}} + (1-\rho_{\mathrm{RF}})\frac{G_{\mathrm{RF},m}^{\mathrm{NLOS}}}{P_{\mathrm{RF},m}G_{\mathrm{RF},m}^{\mathrm{NLOS}} + B_{\mathrm{RF},m}N_{0,\mathrm{RF}}}\right) = \frac{\lambda + \nu_{\mathrm{RF}}}{1+\mu_m}. \tag{21}$$

The allocated power $P_{\mathrm{RF},m}$ is obtained as the positive real root of (21) using Newton's method.

The optimal variable ν_{RF} can be obtained using the gradient descent method to satisfy the total RF power constraint as follows

$$\nu_{\mathrm{RF}}(i+1) = \left[\nu_{\mathrm{RF}}(i) - \eta_2(P_{\mathrm{RF,max}} - \sum_{m\in\mathcal{M}} P_{\mathrm{RF},m})\right]^+, \tag{22}$$

where η_2 is a sufficiently small step size. Algorithm 3 describes the optimal power allocation for the RF network.

Algorithm 3. Power Allocation for RF Network

Input: $B_{\mathrm{RF},m}$, μ_m, and λ;
Initialization: $\beta(1) \geq 0$;
for $i = 1 : I$ **do**
  for $m \in \mathcal{M}$ **do**
    Find $P_{\mathrm{RF},m}(i)$ as the positive real root of (21);
  end for
  $\nu_{\mathrm{RF}}(i+1) = \left[\nu_{\mathrm{RF}}(i) - \eta_2(P_{\mathrm{RF,max}} - \sum_{m\in\mathcal{M}} P_{\mathrm{RF},m})\right]^+$;
end for
Output: $P_{\mathrm{RF},m} \forall m \in \mathcal{M}$

C. Bandwidth Allocation in the VLC Network

In the following, we derive the optimal VLC bandwidth allocation for a given μ_m $\forall m \in \mathcal{M}$. From the KKT conditions [37], we have

$$\frac{\partial L}{\partial B_{\mathrm{VLC},m}} = 0. \tag{23}$$

Thus, we have

$$\frac{\partial R_{\mathrm{VLC},m}}{\partial B_{\mathrm{VLC},m}} = \frac{\beta_{\mathrm{VLC}}}{1+\mu_m}, \tag{24}$$

which can be rewritten as follows

$$\rho_{\mathrm{VLC}}\left(\log_2\left(1 + \frac{(k\rho G_{\mathrm{VLC},m})^2 P_{\mathrm{VLC},m}}{B_{\mathrm{VLC},m}N_{0,\mathrm{VLC}}}\right) - \frac{P_{\mathrm{VLC},m}(k\rho G_{\mathrm{VLC},m})^2}{\ln(2)\left(B_{\mathrm{VLC},m}N_{0,\mathrm{VLC}} + P_{\mathrm{VLC},m}(k\rho G_{\mathrm{VLC},m})^2\right)}\right) = \frac{\beta_{\mathrm{VLC}}}{1+\mu_m}. \tag{25}$$

The allocated bandwidth $B_{\mathrm{VLC},m}$ is obtained as the positive real root of (25) using Newton's method.

The variable β_{VLC} guarantees that the total VLC bandwidth is constrained by the maximum allowable bandwidth and can be found using the gradient descent method as follows

$$\beta_{\mathrm{VLC}}(i+1) = \left[\beta_{\mathrm{VLC}}(i) - \eta_3(B_{\mathrm{VLC,max}} - \sum_{m\in\mathcal{M}} B_{\mathrm{VLC},m})\right]^+, \tag{26}$$

where η_3 is a sufficiently small step size. Algorithm 4 describes the optimal bandwidth allocation for the VLC network.

D. Bandwidth Allocation in the RF Network

In the following, we derive the optimal VLC bandwidth allocation for a given μ_m $\forall m \in \mathcal{M}$. From the KKT conditions [37], we have

$$\frac{\partial L}{\partial B_{\mathrm{RF},m}} = 0. \tag{27}$$

Thus, we have

$$\frac{\partial R_{\mathrm{RF},m}}{\partial B_{\mathrm{RF},m}} = \frac{\beta_{\mathrm{RF}}}{1+\mu_m}, \tag{28}$$

Algorithm 4. Bandwidth Allocation for VLC network

Input: $P_{\mathrm{VLC},m}$, μ_m, and λ;
Initialization: $\beta_{\mathrm{VLC}}(1) \geq 0$;
for $i = 1 : I$ **do**
  for $m \in \mathcal{M}$ **do**
    Find $B_{\mathrm{VLC},m}(i)$ as the positive real root of (25);
  end for
  $\beta_{\mathrm{VLC}}(i+1) = \left[\beta_{\mathrm{VLC}}(i) - \eta_3(B_{\mathrm{VLC,max}} - \sum_{m\in\mathcal{M}} B_{\mathrm{VLC},m})\right]^+$;
end for
Output: $B_{\mathrm{VLC},m} \forall m \in \mathcal{M}$

Algorithm 5. Bandwidth Allocation for RF network

Input: $P_{\mathrm{RF},m}$, μ_m, and λ;
Initialization: $\beta_{\mathrm{RF}}(1) \geq 0$;
for $i = 1 : I$ **do**
  for $m \in \mathcal{M}$ **do**
    Find $B_{\mathrm{RF},m}(i)$ as the positive real root of (29);
  end for
  $\beta_{\mathrm{RF}}(i+1) = \left[\beta_{\mathrm{RF}}(i) - \eta_4(B_{\mathrm{RF,max}} - \sum_{m\in\mathcal{M}} B_{\mathrm{RF},m})\right]^+$;
end for
Output: $B_{\mathrm{RF},m} \forall m \in \mathcal{M}$

which can be rewritten as follows

$$\rho_{\mathrm{RF}}\left(\log_2\left(1+\frac{G^{\mathrm{LOS}}_{\mathrm{RF},m}P_{\mathrm{RF},m}}{B_{\mathrm{RF},m}N_{0,\mathrm{RF}}}\right) - \frac{P_{\mathrm{RF},m}G^{\mathrm{LOS}}_{\mathrm{RF},m}}{\ln(2)\left(B_{\mathrm{RF},m}N_{0,\mathrm{RF}}+P_{\mathrm{RF},m}G^{\mathrm{LOS}}_{\mathrm{RF},m}\right)}\right) + (1-\rho_{\mathrm{RF}})\left(\log_2\left(1+\frac{G^{\mathrm{NLOS}}_{\mathrm{RF},m}P_{\mathrm{RF},m}}{B_{\mathrm{RF},m}N_{0,\mathrm{RF}}}\right) - \frac{P_{\mathrm{RF},m}G^{\mathrm{NLOS}}_{\mathrm{RF},m}}{\ln(2)\left(B_{\mathrm{RF},m}N_{0,\mathrm{RF}}+P_{\mathrm{RF},m}G^{\mathrm{NLOS}}_{\mathrm{RF},m}\right)}\right) = \frac{\beta_{\mathrm{RF}}}{1+\mu_m}. \tag{29}$$

The allocated bandwidth $B_{\mathrm{RF},m}$ is obtained as the positive real root of (29) using Newton's method.

The variable β_{RF} guarantees that the total RF bandwidth is constrained by the maximum allowable bandwidth and can be found using the gradient descent method as follows

$$\beta_{\mathrm{RF}}(i+1) = \left[\beta_{\mathrm{RF}}(i) - \eta_4(B_{\mathrm{RF,max}} - \sum_{m\in\mathcal{M}} B_{\mathrm{RF},m})\right]^+, \tag{30}$$

where η_4 is a sufficiently small step size. Algorithm 5 describes the optimal bandwidth allocation for the RF network.

E. Optimal Power and Bandwidth Allocation for the RF/VLC Network

The Lagrangian multiplier μ_m is used for the mth MT to coordinate the power and bandwidth allocated by both VLC and

Algorithm 6. Power and Bandwidth Allocation for the RF/VLC Network

Input: λ;
Initialization: $\mu_m(1) \geq 0$;
for $i = 1 : I$ **do**
  Allocate power of the VLC network using Algorithm 2;
  Allocate power of the RF network using Algorithm 3;
  Allocate bandwidth of the VLC network using Algorithm 4;
  Allocate bandwidth of the RF network using Algorithm 5;
  for $m \in \mathcal{M}$ **do**
    $\mu_m(i+1) = \left[\mu_m(i) - \eta_5(R_{\mathrm{VLC},m} + R_{\mathrm{RF},m} - R_{\mathrm{min},m})\right]^+$;
  end for
end for
Output: $P_{\mathrm{VLC},m}$, $P_{\mathrm{RF},m}$, $B_{\mathrm{VLC},m}$, $B_{\mathrm{RF},m} \forall m \in \mathcal{M}$

RF networks such that the minimum required data rate is satisfied. The optimal value of μ_m can be obtained by the gradient descent method as follows

$$\mu_m(i+1) = \left[\mu_m(i) - \eta_5(R_{\mathrm{VLC},m} + R_{\mathrm{RF},m} - R_{\mathrm{min},m})\right]^+, \tag{31}$$

where η_5 is a sufficiently small step size. Algorithm 6 describes the hybrid system joint transmission power and bandwidth allocation for a given λ.

By using Algorithms 2–6 in Algorithm 1, the optimal power allocation to maximize the energy efficiency of the heterogeneous RF/VLC system can be obtained. The optimal solution accounts for the RF and VLC characteristics (reliability and energy efficiency metrics). Algorithm 1 obtains the optimal solution of (14) in finite number of iterations while it is required to get the optimal solution of the convex problem in (15) for fixed λ. Algorithm 6 exploits gradient descent method to solve the convex problem (15) which converges to the optimal solution. Each of algorithms 2–5 obtains an optimal solution for one of the optimization variables at a fixed μ and a fixed set of parameters. Hence, Algorithm 6 converges to the optimal solution and the order of implementing these algorithms does not affect this convergence to optimality of this technique since (15) is a convex optimization problem.

F. Computational Complexity

In this subsection, we discuss the computational complexity of the proposed algorithm. Algorithm 1 has a complexity of the order of the finite number of iterations to achieve convergence of the Dinkelbach procedure [36]. We denote the number of the required iterations for the Algorithm 1 to converge by I_{R}. In [38], it is shown that the complexity of the gradient method is polynomial in the number of the used dual variables which is the number of the MTs. Hence, Algorithm 6 has a complexity of order M to solve a gradient descent method for the convex problem. Similarly the Algorithms 2, 3, 4, and 5 which are evaluated in parallel within Algorithm 6 have a complexity order of M. Therefore, the proposed strategy computational complexity is found to be $O\left(I_{\mathrm{R}}M^2\right)$ which is polynomial in the number of the

MTs. The number of MTs that could be accessed using a single VLC AP is commonly small and hence the computational complexity of the proposed strategy is reasonably low.

V. Numerical Results

In the following, we assess the performance of the proposed resource allocation strategy. We compare the energy efficiency of the proposed mixed RF/VLC heterogeneous networks to two benchmark systems. We refer to the proposed system in the following results by ‘RF-VLC’. We compare it to a system consisting of a single RF wireless network, which is denoted by ‘RF-Only’, and hence no multi-homing is performed, and we also compare it to a system comprising two RF APs over different frequency bands and which will be denoted by ‘RF-RF’, and hence multi-homing is achieved only over RF links. In the system with two RF APs, one of the RF systems is assigned a bandwidth equal to that of the VLC system to ensure a fair comparison.

The values of the VLC and RF systems are obtained from [8] and [39], respectively. In the following results, we set $R_{\min,m} = 2$ Mbps, $P_{\text{RF,max}} = 1$ watt, $Q_{\text{RF,max}} = 6.7$ watts, $Q_{\text{VLC,max}} = 4$ watts, $N_{0,\text{RF}} = 3.89\text{x}10^{-21}$ watts/Hz, $N_{0,\text{VLC}} = 10^{-21}$ watts/Hz, $B_{\text{RF}} = 10$ MHz, $B_{\text{VLC}} = 20$ MHz, $k = 10$ watts/Amp., $\rho = 0.8$ Amp./watt, and $M = 4$. The RF AP is located such that the MTs are randomly and uniformly-distributed located such the distance to the AP is in the range between 1 and 1.5 meters. The VLC AP is located such that the MTs are uniformly-distributed and randomly located such that the distance to the AP is in the range between 1.5 and 2 meters. The region of interest in this manuscript is the region of overlapping between the two APs. Moreover, considering a coverage area near the RF AP allows for low RF transmission powers and high RF energy efficiency of a stand-alone RF system (i.e., without VLC integration). We show in our manuscript that even under such circumstances the VLC integration enables more energy efficient network operation, which means that VLC is even more promising in regions which are far away from the RF AP. The VLC system maximum power is the product of the number of LEDs used at the VLC source by the maximum power driving each LED, and we set the number of LEDs to 38 with the maximum power to drive a LED is 300 milli-watts. The value of the maximum driving power is set to generate around 900 lumens from the VLC source which is practically a suitable value for lighting. These LEDs are concentrated in a small area to form a LED array. The LED array VLC systems can be modeled as point VLC sources for link characterization as shown in [40].

In Fig. 3, we show the energy efficiency of the different systems against the number of MTs. The performance of the RF-VLC system is significantly better than the performance of the RF-only system because of the multi-homing capability of the MTs and the energy efficient nature of the VLC systems. Using multi-homing, MTs can have links with better channel conditions with at least one AP, leading to high achieved data rates and low power consumption. Also, the performance of the RF-VLC system is better than that of the RF-RF system because of the low cost of VLC AP power consumption compared to

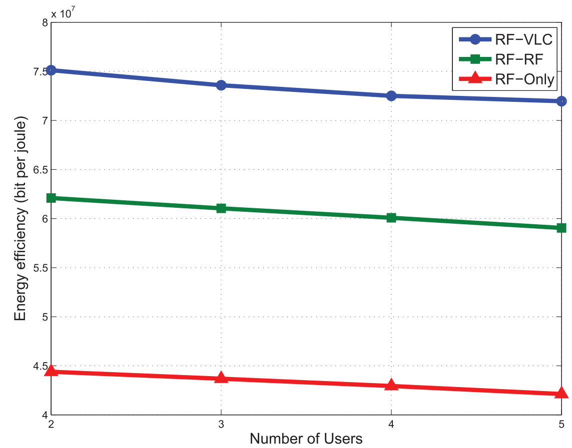

Fig. 3. Energy efficiency against the number of MTs.

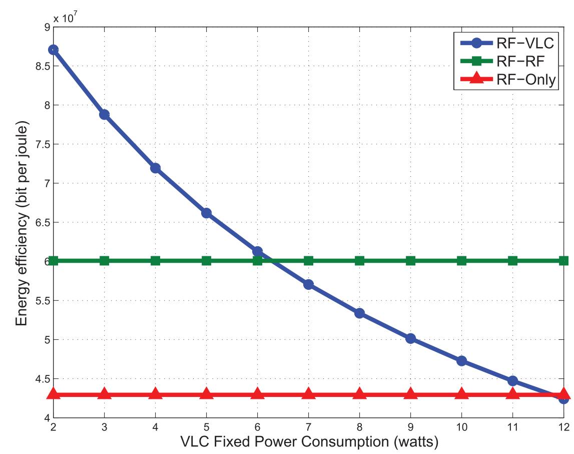

Fig. 4. Energy efficiency against the fixed power of the VLC system.

the RF communication systems. The power consumption in RF systems is the sum of the fixed and transmission powers, while in VLC systems, the power is due only to the fixed power component since no power is dedicated for transmission as its transmission power is already used for illumination.

In Fig. 4, we show the energy efficiency performance versus the fixed power consumption of the VLC AP to investigate the impact of any increased power consumption in the VLC network. In [30], the total fixed power for a single LED transmitter cannot exceed 0.1 watts. In [31], a high power single LED can consume up to 1.6 watts. Hence, we study the effect of total VLC fixed power on the energy efficiency of the system in the range of 2–12 watts which lies within the practical ranges. The energy efficiency of RF-VLC system is equal to that of the RF-RF system when the fixed power is 6 watts which is nearly equal to the fixed power of an RF AP. As a result, the integration of a VLC system in a heterogeneous networking with RF communication will not be beneficial if the VLC AP fixed power is high compared to an equivalent RF AP.

We study in Fig. 5 the effect of the number of LEDs on the energy efficiency of the RF-VLC system. Increasing the number of LEDs allows higher transmission power for the VLC system which motivates the MTs to obtain most of their

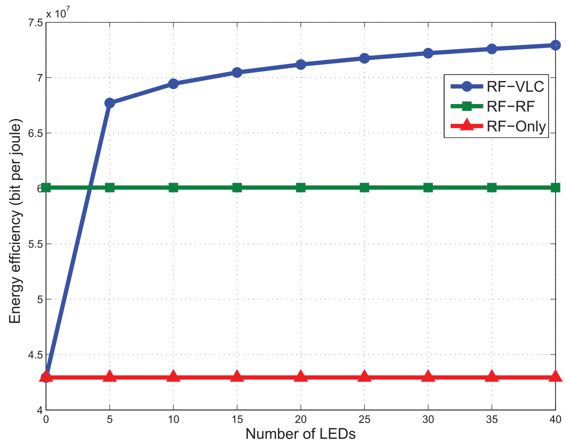

Fig. 5. Energy efficiency against the number of LEDs used by the VLC system.

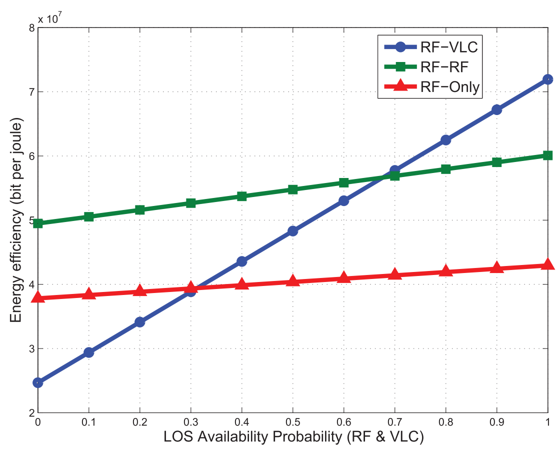

Fig. 6. Energy efficiency against the LOS availability probability in VLC and RF systems.

required data service from the VLC AP, reducing the transmission power consumption of RF AP, and hence improving the overall energy efficiency. Also, the figure shows that introducing the VLC network even with a small number of LEDs (only 5 LEDs) enhances the energy efficiency significantly.

In Fig. 6, we consider the case in which the LOS availability probabilities for RF and VLC systems are equal. We show the energy efficiency versus the LOS availability probability. The performance for the RF-VLC system is better than the benchmarks when the probability of LOS availability in the VLC system is higher than 0.7 because of the good energy efficiency properties of the proposed RF-VLC system. Also, the slope of the curve of the RF-VLC energy efficiency is higher than those of the benchmark systems because of the significance of the LOS availability to the VLC system compared with the RF systems.

Finally, in Fig. 7, we discuss the effect of LOS availability probability in the RF system on the energy efficiency of the RF-VLC heterogeneous system when $\rho_{\mathrm{VLC}} = 1$. In the RF-VLC system, the users exploit the less costly VLC energy for data transmission and exploit the RF transmission power when required. As a result, the enhancement of the performance with the increase of LOS availability probability presents a smaller slope than the RF benchmarks.

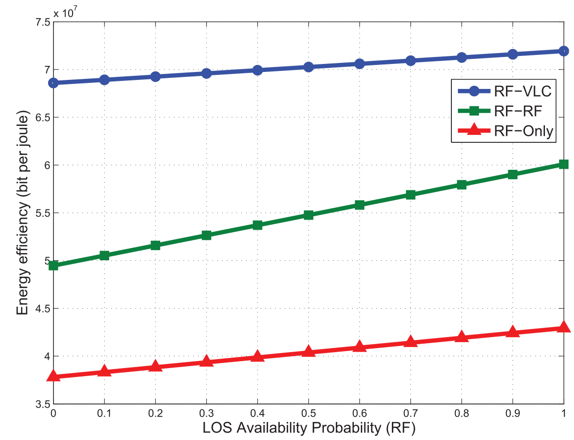

Fig. 7. Energy efficiency against the LOS availability probability in RF systems.

VI. Conclusion and Future Work

The research objectives in the area of energy efficient communications are highly influenced by a variety of environmental and financial considerations. Integrating VLC and RF APs in heterogeneous wireless networking environments has shown promising improvements in the achieved energy efficiency. Therefore, employing VLC in a heterogeneous networking environment is proposed for enhancing energy efficiency. The multi-homing capability of the MTs in a heterogeneous network with VLC and RF APs allows users to benefit from the huge unlicensed bandwidth of the visible light spectrum and the low cost of the transmission power. It also allows for improved reliability, as RF communications are employed in the absence of VLC LOS. We have shown the superior performance of the heterogeneous RF/VLC network compared to the benchmarks offered by an RF only network and a heterogeneous network consisting of two RF systems. The use of the hybrid RF/VLC system has been considered for throughput improvement while the energy efficiency optimization of this system has not been investigated before. As a result, this work investigates the promising hybrid RF/VLC system from a new viewpoint and paves the road for considering the energy efficiency in various RF/VLC systems architectures. This work serves as a building block for an indoor data network composed of multiple VLC and RF APs with different coverage regions. In this case, the problem of the coordination between the APs has to be discussed especially in VLC networks. Moreover, the MT association to an AP and interference impact are to be studied. Additionally, obtaining the optimal resource allocation algorithms for various modulation schemes is to be considered.

Appendix A
Proof of Concavity of the Total Achieved Rate

In order to prove that problem (14) is concave-convex, we need prove that R_{T} is concave with respect to the decision variables. We first prove the concavity of $R_{\mathrm{VLC},m}$ in the decision variables $B_{\mathrm{VLC},m}$ and $P_{\mathrm{VLC},m}$ as it is not a function of the remaining decision variables. The Hessian matrix of $R_{\mathrm{VLC},m}$

can be expressed as follows

$$H_{\mathrm{VLC},m} = \frac{\rho_{\mathrm{VLC}}}{\ln(2)\left(B_{\mathrm{VLC},m} + k_1 P_{\mathrm{VLC},m}\right)^2} \begin{bmatrix} A_1 & B_1 \\ B_1 & C_1 \end{bmatrix}, \tag{32}$$

where the constants k_1, A_1, B_1, and C_1 are calculated as follows

$$k_1 = \frac{\left(k\rho G_{\mathrm{VLC},m}\right)^2}{N_{0,\mathrm{VLC}}}, \tag{33}$$

$$A_1 = \frac{-\left(k_1 P_{\mathrm{VLC},m}\right)^2}{B_{\mathrm{VLC},m}}, \tag{34}$$

$$B_1 = k_1^2 P_{\mathrm{VLC},m}; \tag{35}$$

$$C_1 = -k_1^2 B_{\mathrm{VLC},m}. \tag{36}$$

Both the diagonal elements of the Hessian matrix are negative and the second principal minor is 0 and hence $H_{\mathrm{VLC},m}$ is negative semidefinite. Thus, $R_{\mathrm{VLC},m}$ is concave in both $B_{\mathrm{VLC},m}$ and $P_{\mathrm{VLC},m}$.

We then prove the concavity of $R_{\mathrm{RF},m}$ in $B_{\mathrm{RF},m}$ and $P_{\mathrm{RF},m}$. The Hessian matrix of $R_{\mathrm{RF},m}$ is obtained as follows

$$H_{\mathrm{RF},m} = \begin{bmatrix} A_2 & B_2 \\ B_2 & C_2 \end{bmatrix}, \tag{37}$$

where the constants A_2, B_2, and C_2 are expressed as follows

$$A_2 = \frac{-P_{\mathrm{RF},m}^2}{B_{\mathrm{RF},m}\ln(2)} D, \tag{38}$$

$$B_2 = \frac{P_{\mathrm{RF},m}}{\ln(2)} D, \tag{39}$$

$$C_2 = \frac{-B_{\mathrm{RF},m}}{\ln(2)} D, \tag{40}$$

where the values of D, k_2^{LOS}, and k_2^{NLOS} are obtained as follows

$$D = \left(\frac{\rho_{\mathrm{RF}}\left(k_2^{\mathrm{LOS}}\right)^2}{\left(B_{\mathrm{RF},m} + k_2^{\mathrm{LOS}} P_{\mathrm{RF},m}\right)^2} + \frac{\left(1-\rho_{\mathrm{RF}}\right)\left(k_2^{\mathrm{NLOS}}\right)^2}{\left(B_{\mathrm{RF},m} + k_2^{\mathrm{NLOS}} P_{\mathrm{RF},m}\right)^2} \right), \tag{41}$$

$$k_2^{\mathrm{LOS}} = \frac{G_{\mathrm{RF},m}^{\mathrm{LOS}}}{N_{0,\mathrm{RF}}}, \tag{42}$$

$$k_2^{\mathrm{NLOS}} = \frac{G_{\mathrm{RF},m}^{\mathrm{NLOS}}}{N_{0,\mathrm{RF}}}. \tag{43}$$

As a result, we found that both the diagonal elements of the Hessian matrix are negative and the second principal minor is 0. Thus, $R_{\mathrm{RF},m}$ is concave in both $B_{\mathrm{RF},m}$ and $P_{\mathrm{RF},m}$. The total achievable rate, R_{T}, is a sum of concave functions and hence it is concave as well.

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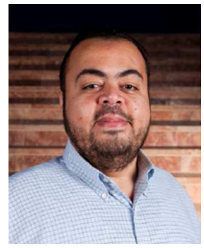
Mohamed Kashef (M'14) received the B.Sc. and M.S. degrees (with Hons.) in electronics and electrical communications engineering from the Cairo University, Cairo, Egypt, in June 2006 and August 2009, respectively, and the Ph.D. degree in electrical engineering from the University of Maryland at College Park, College Park, MD, USA, in 2013. He is a Postdoctoral Research Associate with Texas A&M University at Qatar, Doha, Qatar. His research interests include wireless communication systems and networks, visible light communication networks, and optimization of stochastic systems.

Muhammad Ismail (S'10–M'13) received the B.Sc. and M.Sc. degrees in electrical engineering (electronics and communications) from Ain Shams University, Cairo, Egypt, in 2007 and 2009, respectively, and the Ph.D. degree in electrical and computer engineering from the University of Waterloo, Waterloo, ON, Canada, in 2013. He is an Assistant Research Scientist with the Department of Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar. He research interests include distributed resource allocation, green wireless networks, cooperative networking, smart grid, and biomedical signal processing. He is a coauthor of two research monographs published by Springer and Wiley. He joined the IET Communications Editorial Board, as an Associate Editor, in December 2014. He was an Editorial Assistant for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY from January 2011 to July 2013. He served as a TPC member in the ICWMC from 2010 to 2014 and the IEEE ICC 2014 and 2015. He served on the IEEE INFOCOM 2014 Organizing Committee as a Web Chair. He has been a Technical Reviewer for several IEEE conferences and journals. He was the corecipient of the Best Paper Awards in the IEEE ICC 2014, the IEEE Globecom 2014, and the SGRE 2015.



Mohamed Abdallah (M'06–SM'13) was born in Giza, Egypt. He received the B.Sc degree (with Hons.) from Cairo University, Giza, Egypt, in 1996, and the M.Sc. and Ph.D. degrees in electrical engineering from University of Maryland at College Park, College Park, MD, USA, in 2001 and 2006, respectively. He joined Cairo University, in 2006, where he still holds the position of Associate Professor with the Department of Electronics and Electrical Communication. He is currently an Associate Research Scientist with Texas A&M University at Qatar, Doha, Qatar. His research interests include the design and performance of physical layer algorithms for cognitive networks, cellular heterogeneous networks, sensor networks, smart grids, visible light and free-space optical communication systems, and reconfigurable smart antenna systems.

Khalid A. Qaraqe (M'97–SM'00) was born in Bethlehem. He received the B.S. degree (with Hons.) in electrical engineering from the University of Technology, Bagdad, Iraq, in 1986, the M.S. degree in electrical engineering from the University of Jordan, Amman, Jordan, in 1989, and the Ph.D. degree in electrical engineering from Texas A&M University, College Station, TX, USA, in 1997. From 1989 to 2004, he has held a variety positions in many companies and has more than 12 years of experience in the telecommunication industry. He has worked on numerous GSM, CDMA, and WCDMA projects and has experience in product development, design, deployments, testing, and integration. He joined the Department of Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar, in July 2004, where he is now a Professor.

He has been awarded 15 research projects consisting of more than USD 9.0 M from local industries in Qatar and the Qatar National Research Foundation (QNRF). He has authored 90 journal papers in top IEEE journals, and published and presented 194 papers at prestigious international conferences. He has 13 book chapters published, 2 books slated to appear in 2015, 4 patents, and presented 5 tutorials and talks. His research interests include communication theory and its application to design and performance, analysis of cellular systems and indoor communication systems, mobile networks, broadband wireless access, cooperative networks, cognitive radio, diversity techniques, and beyond 4G systems.

Dr. Qaraqe was the recipient of the Itochu Professorship Award (2013–2015), the Best Researcher Award from QNRF 2013, the Best Paper Award from the IEEE First workshop On Smart Grid And Renewable Energy (March 2015), the Best Paper Award from the IEEE Globecom 2014, the Best Poster Award from IEEE Dyspan Conference (October 2012), the TAMUQ Research Excellence Award (April 2010), the Best Paper Award from ComNet 2010, the Best Paper Award from CROWNCOM 2009, and the Best Paper Award from ICSPC 2000 and 2007.

Erchin Serpedin (F'13) received the specialization degree in signal processing and transmission of information from Ecole Superieure DElectricite (SUPELEC), Paris, France, in 1992, the M.Sc. degree from the Georgia Institute of Technology, Atlanta, GA, USA, in 1992, and the Ph.D. degree in electrical engineering from the University of Virginia, Charlottesville, VA, USA, in January 1999. He is a Professor with the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX, USA. He is the author of 2 research monographs, 1 textbook, 9 book chapters, 110 journal papers, and 180 conference papers. His research interests include signal processing, biomedical engineering, bioinformatics, and machine learning. He is currently serving as an Associate Editor of the *IEEE Signal Processing Magazine* and as the Editor-in-Chief of the journal *EURASIP Journal on Bioinformatics and Systems Biology* (Springer). He served as an Associate Editor of a dozen journals, such as the IEEE TRANSACTIONS ON INFORMATION THEORY, the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE SIGNAL PROCESSING LETTERS, the IEEE COMMUNICATIONS LETTERS, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, *Signal Processing* (Elsevier), *Physical Communications* (Elsevier), *EURASIP Journal on Advances in Signal Processing*, and as a Technical Chair for five major conferences. He was the recipient of numerous awards and research grants.