Design and Analysis of a Hybrid Radio Frequency and Visible Light Communication System

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***Abstract*— In this paper, a hybrid radio frequency (RF) and visible light communication (VLC) system is considered. A hybrid system with multiple VLC access points (APs) and RF APs is designed and analyzed. In indoor environments, VLC APs provide very high data rates whilst proving illumination, and RF APs offer ubiquitous coverage with moderate data rates. Since VLC networks piggyback on existing lighting infrastructures, they may not always be able to provide full coverage despite supporting very high data rates in some areas. Hence in practical deployments, the standalone VLC networks should be augmented in order to improve the per user data rate coverage. In this context, RF APs can be used to improve the per user rate coverage of VLC networks as well as to provide the ubiquitous control functionalities. In this paper, a simple RF deployment is proposed in order to improve the per user outage data rate performance of standalone VLC networks. It is assumed that the VLC system resources are fixed, and this paper quantifies the minimum spectrum and power requirements for a RF system, which after introduction to the VLC system, the hybrid RF/VLC system achieves certain per user rate coverage performances.**

***Index Terms*— Visible light, indoor wireless communication, channel models, outage probability, radio frequency.**

I. INTRODUCTION

THE global demand for wireless data has been exponentially increasing. The visual networking index (VNI) published by Cisco systems reports that global mobile data traffic reached 2.5 exabytes (2.5 billions GB) per month in 2014, and is predicted to be increased nearly 9-fold by 2019 [1]. Heterogeneous networks (HetNets) have been proposed to meet this demand, and it is widely regarded as a pragmatic approach to this exponential growth [2], [3]. One of the key technologies among many HetNet technologies is visible light communication (VLC) [4]–[6]. VLC is a technology which enables the transformation of conventional lighting infrastructures into high speed APs. It uses the unregulated and unused optical spectrum for wireless communication. In addition to its ability to combine illumination and communication, VLC technology can achieve high data rates. Modern VLC systems which use intensity modulation (IM) and direct detection (DD) with optical orthogonal frequency division multiplexing (OFDM) have been shown to achieve data rates in the Gbps range [10]–[13]. The advantages of VLC include: no spectrum license required, high physical link security, and no radio frequency (RF) interference. Also, VLC networks can be used for indoor localization in addition to high speed communication [7], and various technologies can be used for backhaul communication of VLC networks [8], [9].

In indoor environments, it is envisioned to deploy hybrid VLC/RF systems in order to provide enhanced coverage [14], [15]. Since light originating from an LED source is naturally confined to a small area, and light beams are susceptible to blockages, the achievable throughput could spatially fluctuate. The hybrid integration of VLC and RF systems are expected to significantly improve the user experience, because VLC systems can support very high data rates in specific areas, while RF systems can support moderate data rates ubiquitously. The synergistic effect of VLC and RF systems should provide better data rate performance and user experience. Therefore, in this study, a hybrid VLC and RF system to cover a large indoor area is designed and analyzed. It is important to note here that from technological stand point RF only systems can also be used to provide wireless coverage to indoor areas [16], [17], but in this study VLC system is supposed to play the dominant role, and a simple RF system is used to fill the coverage holes. This is deemed acceptable since the VLC system is capable of carrying a large portion of the data traffic. This approach may be attractive because VLC provides both illumination and communication, and causes no interference to other RF systems. Furthermore, this approach may be a choice for places such as hospitals because it may significantly reduce the electromagnetic radiation which can in some cases be harmful [19]. In fact in 2011, the World Health Organization (WHO) International Agency for Research on Cancer (IARC) categorized the non-ionizing electromagnetic radiation in the frequency range 30 kHz to 300 GHz, as a Group 2B, i.e. a possible, human carcinogen [20].

Firstly, the spatial distribution of the per user data rate of a standalone VLC deployment is evaluated. The simulation results show that even though the average per user data rate is higher, per user outage performance can still be significantly low. Herein the outage performance is denoted as the probability that the per user data rate falls below a certain threshold. In order to improve the per user rate outage performance, the standalone VLC system is augmented by a RF communication system. Secondly, an important question is the minimum resource requirements for the RF system to achieve a certain outage performance of the hybrid system.

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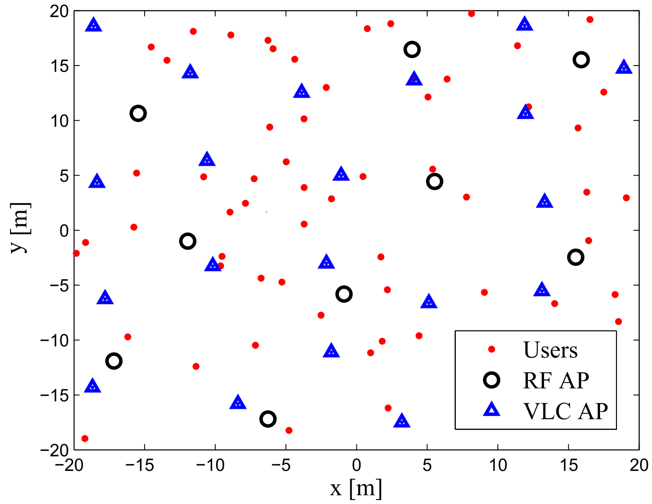

Fig. 1. Illustration of a hybrid VLC/RF system which covers a large indoor area. VLC APs piggyback on existing lighting fixtures and also provide illumination. Here only VLC enabled APs are shown. More lighting fixtures may be needed in order to fully satisfy illumination requirements.

In particular, this study analytically quantifies the minimum RF resource requirements, i.e., spectrum and power for the RF system which in combination with the VLC system achieve a certain per user rate outage performance. The resource requirements are dependent on a number of system parameters and settings, especially the deployment scenario. This study is based on a typical deployment scenario as shown in Fig. 1. The main contributions of the current paper are as follows.

- The per user data rate outage performance of standalone VLC networks is comprehensively analyzed, and limitations and challenges that arise in standalone VLC networks are identified.
- A simple blockage resilient indoor hybrid VLC/RF system model is proposed in order to improve the coverage of standalone VLC networks. In particular, the RF system avoids inter-cell interference (ICI) and the necessity of complex RF transmission schemes such as multiple-input-multiple-output (MIMO), and give precedence to the VLC network through an SINR based tier association scheme.
- Key parameters of the proposed hybrid VLC/RF networks are studied through computer simulation. Furthermore, an analytical framework is presented using tools from stochastic geometry, and methods to evaluate key parameters for the proposed VLC/RF network are studied.

The rest of the paper is organized as follows: The hybrid system model is introduced in Sec. II along with a detailed description of the channel models and deployment scenarios considered in the paper. The problem description with supporting results is presented in Sec. III. The performance evaluation and analysis are given in Sec. IV, and Sec. V respectively. How the analytical framework is extended to practical hybrid systems is discussed in Sec. VI followed by conclusions in Sec. VII.

TABLE I
VLC SIMULATION PARAMETERS

Name of Parameters	Value
Transmitted optical power per VLC AP, P_V	9 W
Modulation bandwidth for LED lamp, B_V	40 MHz
The physical area of a PD, A_p	1 cm^2
Half-intensity radiation angle, $\theta_{1/2}$	60 deg.
Gain of optical filter, $T_s(\theta)$	1.0
Receiver FOV semiangle, Θ_F	90 deg.
Refractive index, n	1.5
Optical to electric conversion efficiency, κ	0.53 A/W
Noise power spectral density, N_0	10^{-21} A^2/Hz

II. SYSTEM MODEL

A hybrid VLC/RF communication system for the downlink which covers a large indoor area as shown in Fig. 1 is designed.[1] The system consists of n_V VLC access points (AP) and n_R RF APs. Active users are uniformly distributed over the coverage area, and are served by VLC APs. The number of users is Poisson distributed with parameter $\lambda_u = c/A_0$, where c is the average number of active users in the coverage area, and A_0 is the area of the coverage area. Hence, users form a spatial Poisson point process (PPP) with intensity, λ_u. Each VLC AP consists of an LED lamp which contains several LEDs. The VLC system specifications are summarized in Table I. A VLC AP covers a small area which is termed in this study as an optical attocell. Every user is served by a single VLC AP from which the user has the strongest channel gain. All the optical APs reuse the same bandwidth so that there is inter-cell interference (ICI). In this study, optical ICI is treated as noise. If a VLC AP serves more than one user, which is likely if the system is exposed to a large number of users, it employs time division multiple access (TDMA) with round robin (RR) scheduling.

In this study, the original VLC system is augmented by a millimeter-wave RF communication system, and this introduction creates a two tier/layer network. RF APs with omni-directional transmit antennas are introduced to the system as shown in Fig. 1, and RF APs are assumed to cover the entire indoor environment. As suggested in [15], and experimentally confirmed in [17], beamforming using multiple antennas is not required for short range indoor communication. Even though a single RF AP could cover the whole coverage area in this deployment, n_R RF APs are considered for two main reasons. Firstly, it significantly reduces the minimum access distance, and secondly reduces the link outage probability, because all n_R links in outage is less likely than a single link in outage. Furthermore, It also significantly increases the line-of-sight (LoS) probability that is very beneficial in mmWave communication [18]. Even though, there are multiple RF APs, they function as a single cell with distributed radio heads. This

[1]This paper is a system level study, and hence many link level intricacies of both VLC and RF links have been omitted. The readers are directed to [31] and [25] for link level details of RF and VLC links respectively.

RF system model is significantly different from contemporary assumptions for heterogeneous networks in [21] and [22]. As a result, severe interference between RF APs is avoided in this ultra dense indoor deployment. Furthermore, there is no interference between the RF and VLC systems, a key benefit of introducing VLC systems to future heterogeneous networks. The whole RF system has a fixed spectrum, B_R, and power budget, P_R. It is assumed that there is a central unit (CU) which controls both VLC and RF systems. The VLC system is assumed to be the primary system, and based on the strength of the channel including the interference, the CU dynamically selects an AP to serve a particular user. In order to give the precedence to the VLC network, in particular, CU firstly detects the SINR (or per user rate) achievable by a particular user via the VLC system, and if the SINR (or per user rate) falls below a certain threshold, the particular user is migrated to the RF system. This system association is significantly different and superior to the biasing based tier association in [22]. Unlike in the VLC system, RF APs do not interfere with each other because each RF AP is allocated a non-overlapping spectrum.

In this study, two deployment scenarios of a RF system denoted as D1 and D2 are considered. In D1, the RF spectrum and power are equally split among RF APs so that each RF AP has access to a bandwidth of B_R/n_R, and a power budget of P_R/n_R. Each RF AP meets its users' traffic demands using orthogonal frequency division multiplexing access (OFDMA), and it is further discussed in Sec. II-B. The spectrum and power are equally allocated because RF APs are placed approximately uniformly, and a uniform user distribution is assumed. If RF APs are not uniformly placed, the optimum average spectrum and power allocation would be different. In contrast, in D2, the CU dynamically splits the spectrum and power to RF APs depending on the instantaneous traffic demands. For instance, if the ith RF AP has a higher number of users than the jth RF AP at a particular time, the CU allocates more spectrum and power to the ith RF AP, and so on. D2 deployment requires some form of out-of-band communication between RF APs. This study focuses on the per user performance rather than system performance. In particular, the objective is to improve the outage per user data rate performance. This study quantifies the resources,[2] (i.e., B_R and P_R) requirements for the RF system in order to achieve a certain per user rate outage performance of the hybrid system.

A. Visible Light Communication (VLC) System

According to [23] and [24], the optical channel gain of a LoS channel is defined as

$$H = \begin{cases} \dfrac{(m+1)A_p}{2\pi d^2} T_s(\theta) g(\theta) \cos^m(\phi)\cos(\theta) & \theta \le \Theta_F, \\ 0 & \theta > \Theta_F, \end{cases} \tag{1}$$

where m is the Lambertian index which is a function of the half-intensity radiation angle $\theta_{1/2}$, expressed as $m = -1/\log_2(\cos(\theta_{1/2}))$; A_p is the physical area of the photo diode of the receiver; d is the distance from the VLC AP to the optical receiver; ϕ is the angle of irradiation and θ is the angle of incidence; Θ_F is the half angle of the receiver's field of view (FoV); $T_s(\theta)$ is the gain of the optical filter; and the concentrator gain $g(\theta)$ can be written as:

$$g(\theta) = \begin{cases} \dfrac{n^2}{\sin^2 \Theta_F} & 0 \le \theta \le \Theta_F \\ 0 & \theta > \Theta_F \end{cases} \tag{2}$$

where n is the refractive index. Each user is served by a single VLC AP which is physically closest, and have a LoS path to the user. VLC channels do not undergo time varying fading, and their strength is dependent on the distance between the transmitter and the receiver. Therefore, physically closest AP typically provides the strongest channel in VLC. Signal-to-interference-plus-noise ratio (SINR) of a typical user can then be written as:

$$\mathrm{SINR} = \frac{(\kappa H_0 P_V)^2}{\ell^2 N_0 B_V + \sum_k (\kappa H_k P_V)^2}, \tag{3}$$

where κ is the optical to electric conversion efficiency at the receiver; P_V and B_V are the transmitted optical power and the modulation bandwidth of VLC APs respectively; N_0 [A^2/Hz] is the noise power spectral density; H_0 is the channel gain between the typical user and the nearest LoS VLC AP; and H_k is the channel gain between the typical user and the kth interfering LoS VLC AP. It is assumed that the VLC system employs DC biased optical OFDM (DCO-OFDM). In DCO-OFDM, DC bias is added to the information signal in order to ensure that the output signal is positive. Furthermore, ℓ in (3) is denoted to the ratio between the average transmitted optical power, P_V which is also proportional to the DC bias and the electrical power of the information signal without the DC bias. Typically, as ℓ increases the probability of information signal being outside the LED linear operating region decreases. For instance, $\ell = 3$ means that approximately 0.3% of the signal is clipped. In this case, the clipping noise can be considered negligible. In DCO-OFDM, the achievable data rate by a typical user can be expressed as [25]:

$$\Gamma_V = \frac{B_V}{2}\log_2(1+\mathrm{SINR}) \quad \text{bps}, \tag{4}$$

where frequency flat channel gains are assumed. When each VLC AP is exposed to multiple users, it is assumed that the system uses TDMA with RR scheduling for multiple access. Even though, TDMA is considered in this paper, other multiple access schemes such as OFDMA are also viable solutions for VLC systems [26]. If the typical user is assigned to a VLC AP which has N active users, the achievable rate becomes:

$$\Gamma_V = \frac{B_V}{2N}\log_2(1+\mathrm{SINR}) \quad \text{bps}. \tag{5}$$

Note that the number of users assigned to each VLC AP varies with the user distribution and the user density of the system, hence N is a random number.

[2] Often there is a transmit power constraint for RF APs. In order to satisfy transmit power constraints, one can simply fix P_R according to the transmit power constraint. In that case, the main challenge is finding the optimum RF spectrum in order to achieve a certain per user rate outage probability of the hybrid system.

B. Radio Frequency System

In this study, a wideband RF communication system in the region of 60 GHz is considered [27]. Also, only the LoS operation is assumed. In a mmWave spectrum, the scattered components of the transmitted signal are extremely weak, and in most of the scenarios their power can be neglected. This assumption is further confirmed by recent measurements in the mmWave band [18], [28]. It is further assumed that OFDM is employed with a total bandwidth of B_R, and transmit power of P_R. The complex channel gain from the serving RF AP to the typical user in the nth sub-channel is given by

$$H^n = \sqrt{10^{\frac{-L(d)}{10}}}\, H_w^n, \tag{6}$$

where H_w^n is the normalized channel transfer function from the serving RF AP to the typical user in the nth sub-channel, and $L(d)$ is the corresponding large-scale fading loss in decibels at the separation distance d which is given by [29]

$$L(d) = L(d_0) + 10\gamma \log_{10}(d/d_0) + X, \tag{7}$$

where $L(d_0) = 68$ dB is the reference path loss at some reference distance $d_0 = 1$m; $\gamma = 1.6$ is the path loss exponent; and X represents the shadowing component which is assumed to be a zero mean Gaussian distributed random variable with standard deviation $\Omega = 1.8$ dB [29], [30]. Even though there are multiple RF APs in the system, each users is served by a single RF AP from which it has the strongest channel. Let H^n be denoted as the typical user's channel response in the nth sub-channel through which the communication takes place. From [31], the typical user's rate through the RF system can be given as:

$$\Gamma_R = \sum_{n\in\mathcal{N}} \Delta B_R \log_2\left(1 + \frac{|H^n|^2\,\Delta P_R}{N_0\,\Delta B_R}\right), \tag{8}$$

where $\mathcal{N}$ is the set of sub-channels allocated to the typical user; ΔB_R is sub-channel bandwidth; and ΔP_R is the power allocated to each sub-channel allocated to the typical user. The constant, N_0 is the power spectral density of white noise for the RF system. It is assumed here that $N_0 = -174$ dBm/Hz which is also equal to $N_0 = 4.002\text{x}10^{-21}$ W/Hz. Since no reflected paths are considered in the power-delay profile, the frequency response of the channel is assumed to be flat over the bandwidths considered in this study. Therefore, all sub-channel channel gains are equal, thus superscript, n of H^n may be removed. It is also assumed that the path loss is identical on all sub-channels.

Users' achievable rates vary with the scenario. Let B_{Rk}, P_{Rk} be the fixed spectrum and power allocated to the kth AP in D1, respectively. It is assumed that sub-channel power allocation is uniform, and the RF APs equally split their available RF spectrum among all its users. The uniform power allocation across sub-channels ensures equal power allocation to users. Hence, the typical user's achievable rate in D1 becomes:

$$\Gamma_R = \frac{B_{R_k}}{U_{R_k}} \log_2\left(1 + \frac{|H|^2\,P_{R_k}}{N_0\,B_{R_k}}\right) \quad \text{bps}, \tag{9}$$

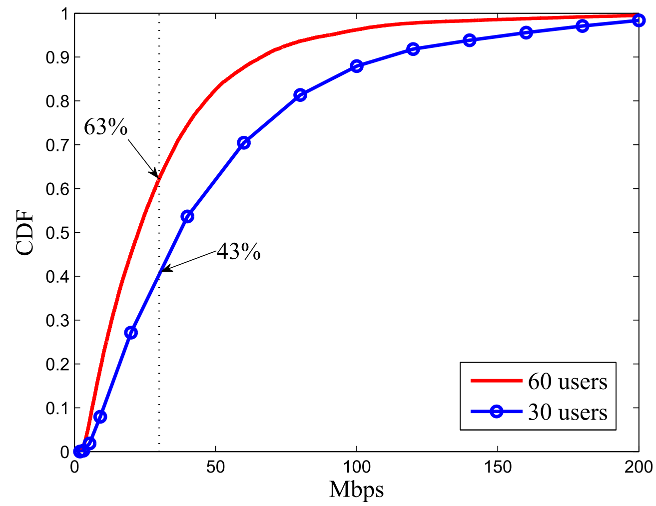

Fig. 2. Cumulative distribution function (CDF) of per user data rates of a VLC system illustrated in Fig. 1 with 20 VLC APs for system parameters given in Table I.

where U_{Rk} is the number of users assigned to the kth RF AP which the typical user is assigned to. In D2, the typical user's achievable rate is given by:

$$\Gamma_R = \frac{B_R}{U_R} \log_2\left(1 + \frac{|H|^2\,P_R}{N_0\,B_R}\right) \quad \text{bps}, \tag{10}$$

where U_R is the total number of users in the RF system, and i.e., $U_R = \sum_k U_{R_k}$. Note that the channel transfer function, H in LoS environment is modeled as:

$$H = \sqrt{10^{\frac{-L(d)}{10}}}\left(\sqrt{\frac{K}{1+K}}H_d + \sqrt{\frac{1}{1+K}}H_s\right) \tag{11}$$

where the direct path fading channel, $H_d = \sqrt{1/2}\,(1+j)$, and the fading channel of the scattered path is modeled as a zero mean unit variance complex Gaussian random variable, which is denoted as $H_s \sim \mathcal{CN}(0,1)$. The distance, d is the distance between the typical user and the RF AP which the typical user is associated to. Here, K is the Rician factor for indoor mmWave links. Recent measurements confirms that it is approximately 10 dB [33]. One must not confuse as to why shadow fading is introduced in (7) for LoS propagations. The reason is that shadowing can still occurs if one or more Fresnel zones are blocked by large objects or humans in indoor environments while the geometric LoS path is not blocked.

III. Problem Description

Fig. 2 shows the per user data rate outage performance of a VLC system which consists of $n_V = 20$ VLC enabled APs, and covers an area of 40 m × 40 m × 3.5 m with specifications as given in Table I. The VLC APs are positioned as illustrated in Fig. 1. Due to the propagation characteristics of light, the coverage area of a single VLC AP may be small. The inter-cell interference and the limited coverage normally degrade the spatial distribution of per user data rates in VLC networks. For instance, if the VLC system has 30 users on average, the per user data rate could be

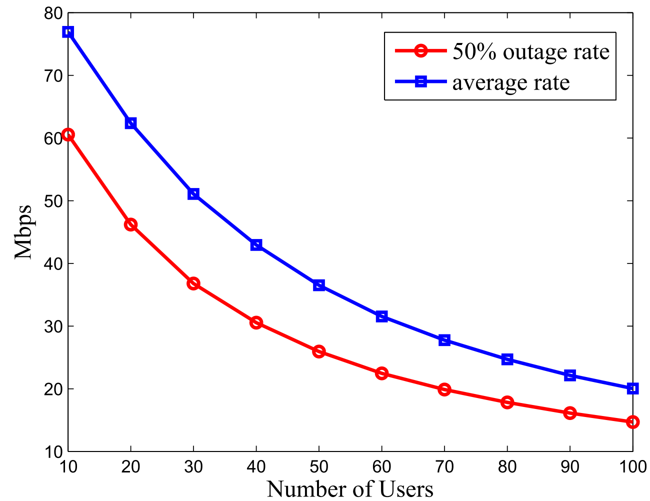

Fig. 3. Average and outage per user rate performance of a VLC system illustrated in Fig. 1 with 20 VLC APs for system parameters given in Table I.

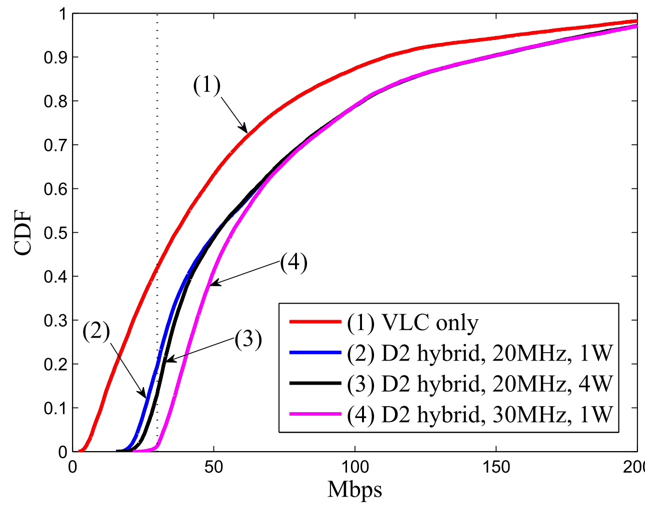

Fig. 4. CDF performance comparison of a hybrid VLC/RF system in Fig. 1 in D2. There are 30 users on average in this simulation.

less than 30 Mbps[3] at 43% of the locations. With 60 users, per user data rate could be less than 30 Mbps at 63% of the locations. As shown in Fig. 3, both average and 50% outage data rate steadily decrease with increasing user density. Here ϵ% outage data rate, z is defined as:

$$\Pr\left(\Gamma_V \leq z\right) = \epsilon\%, \tag{12}$$

where Γ_V is the per user data rate. On the good side however, as shown in Fig. 2, a certain number of users can achieve per user rate performance greater than 100 Mbps in a VLC system with 30 users. The results in this section clearly highlight the benefits of VLC networks along with some challenges. In order to address these challenges, specially the per user rate outage performance, the VLC system is augmented by a RF system. It is assumed that the system parameters of the VLC system are fixed. The objective is to understand how the RF communication system affects the overall user performance of the hybrid system; in particular, the effect of the RF bandwidth and power, i.e., B_R and P_R on the per user rate outage performance of the original VLC system.

IV. Performance Evaluation and Discussion

In this section, the per user rate performance of the VLC/RF hybrid system is evaluated using Monte Carlo simulation. The CU monitors the VLC system at constant intervals and users who are unable to achieve per user rates above Γ_T are migrated to the RF system. The rate threshold, Γ_T, is a design parameter which can be optimized in practical systems, but in this section, it is assumed to be known and set to $\Gamma_T = 30$ Mbps. Simulation results consist of several scenarios.

1) Scenario 1 (S1): Firstly, a moderately loaded system with 30 users on average is considered, and users are uniformly distributed on the coverage area. As shown in Fig. 4 for the VLC system, 43% of users experience data rates of 30 Mbps or less, and this is a significant proportion of

[3]This particular value is chosen arbitrarily, but as being discussed in the UK parliament for Digital Economy Bill 2017 [34], this is also a reasonable choice.

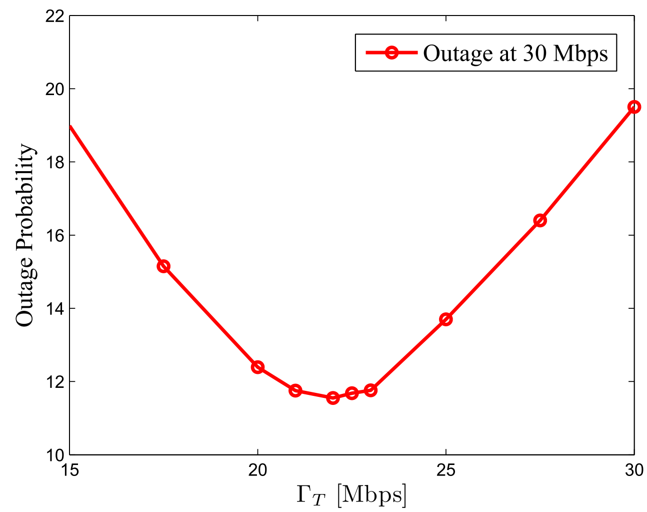

Fig. 5. Variation of the per user rate outage probability at 30 Mbps with the threshold, Γ_T of a VLC/RF system with $B_V = 40$ MHz, $B_R = 20$ MHz, and $P_R = 1$ W.

users. The VLC system is augmented with 10 RF access points (i.e. $n_R = 10$) with $B_{\mathrm{R}} = 20$ MHz and $P_{\mathrm{R}} = 1$ W, and D2 is considered. The results are shown in Fig. 4. In order to understand the trade-offs between spectrum and power, different combinations of resources are also considered. The results show that the per user outage probability of the hybrid VLC/RF system with $B_{\mathrm{R}} = 20$ MHz and $P_{\mathrm{R}} = 1$ W at 30 Mbps is now reduced to 19%. The outage probability can be further reduced or completely eliminated by increasing either the spectrum or transmit power further. For instance, if RF system can afford bandwidth of $B_{\mathrm{R}} = 30$ MHz, all the users will certainly achieve rates greater than 30 Mbps (see magenta curve). Herein the rate threshold, Γ_T is set to 30 Mbps. However, the research also shows that there is an optimum rate threshold which gives the minimum outage probability at a certain rate for a given hybrid system. For instance, results in Fig. 5 show that $\Gamma_T = 22$ Mbps is optimum for the outage probability at 30 Mbps for a VLC/RF system with $B_V = 40$ MHz, $B_R = 20$ MHz, and $P_R = 1$ W.

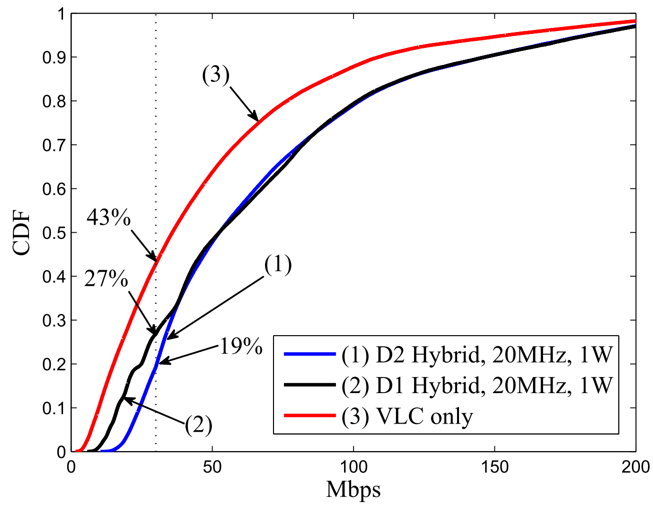

Fig. 6. CDF performance comparison of a hybrid VLC/RF system in Fig. 1 in D1 and D2. There are 30 users on average in this simulation.

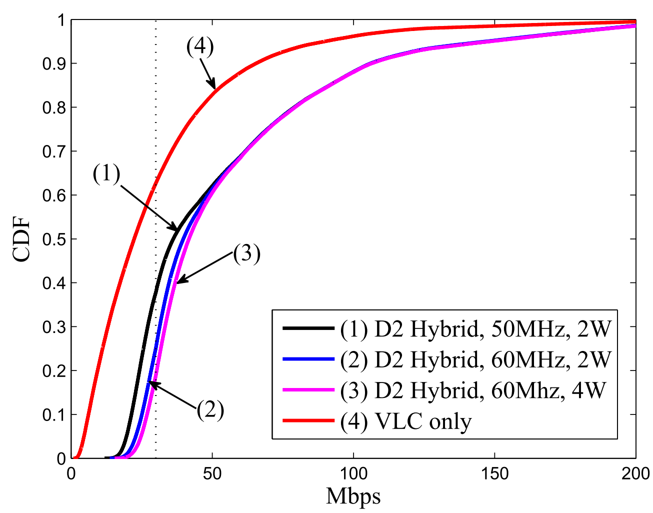

Fig. 7. CDF performance comparison of a hybrid VLC/RF system in Fig. 1 in D2. There are 60 users on average in this simulation.

2) Scenario 2 (S2): Secondly, the outage performance of D1 and D2 are compared. In this simulation scenario, the original VLC system is augmented with four RF access points with $B_{\mathrm{R}} = 20$ MHz and $P_{\mathrm{R}} = 1$ W. Each RF AP is allocated 5 MHz spectrum and 0.25 W power in D1. The results are shown in Fig. 6. In this case, average achievable user data rates are comparable, but in D2 the outage probability at 30 Mbps is 8% lower than that in D1. Therefore, it appears that dynamic resource partitioning among participating RF APs could significantly improve the user outage probability.

3) Scenario 3 (S3): In $S3$, a system loaded with 60 active users on average is considered. This scenario is an extension of $S1$ and $S2$. In this case, per user rate performance of the original VLC system with $n_V = 20$ is significantly decreased mainly due to the user density of the system. For instance, 63% of users could possibly experience data rates of 30 Mbps or less. The results in Fig. 7 show that outage probability at 30 Mbps could be reduced to about 15% with a hybrid VLC/RF system, but the RF system should have at least 60 MHz spectrum and 2 W power budget in D2.

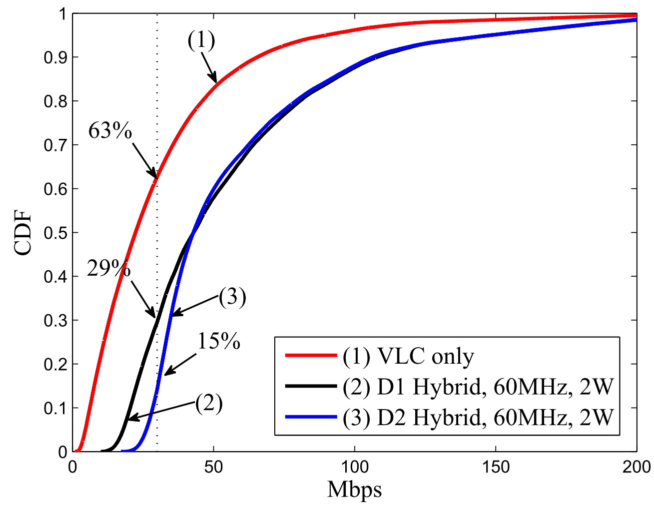

Fig. 8. CDF performance comparison of a hybrid VLC/RF system in Fig. 1 in D1 and D2. There are 60 users on average in this simulation.

It is considered here that $n_R = 10$. Similar to the trends in $S1$, with the same resources (60 MHz spectrum and 2 W power), per user outage probability in D1 at 30 Mbps is 14% higher than that in D2 deployment, but average data rates are comparable in both deployments. In summary, similar rate performance trends are observed in both loading scenarios, but resource requirements are notably higher in later case. For instance, the D2 RF system needs at least 60 MHz spectrum and 2 W power to reduce the outage probability from 63% to 15% (blue in Fig. 8).

4) Scenario 4 (S4): The proposed RF system for the hybrid VLC/RF system is very effective not only for per user rate outage performance but also against the link blocking as well. Fig. 9 shows the per user outage performance of a hybrid VLC/RF system with severe blocking (i.e. a blocking probability of 0.5) and without blocking. The system parameters are borrowed from $S1$, and hence $P_R = 1$ W, $B_R = 30$ MHz and $B_V = 40$ MHz. There are 30 users on average, $n_R = 10$ and $n_V = 20$ in this simulation. It is apparent that the performance degradation is negligible.

V. Analytical Calculation of Key Parameters

The Monte-Carlo simulation study in Sec. IV shows that hybrid VLC/RF systems could be designed in order to achieve certain rate and reliability requirements. It has also shown that there are a number of parameters to be carefully optimized. The goal of this section is analytically deriving the minimum resources, i.e., B_R and P_R for the RF system and the optimum rate threshold, Γ_T. In this section therefore, an analytical model is developed to analyze hybrid VLC/RF systems. The original hybrid system is described in Sec. II, but in this section a general random VLC and RF AP deployments are considered. How the analytical results obtained here can be used to study the performance of practical system deployments are considered in Sec. VI.

For the sake of analytical tractability in the sequel, a number of simplifications and assumptions are used. Specially,

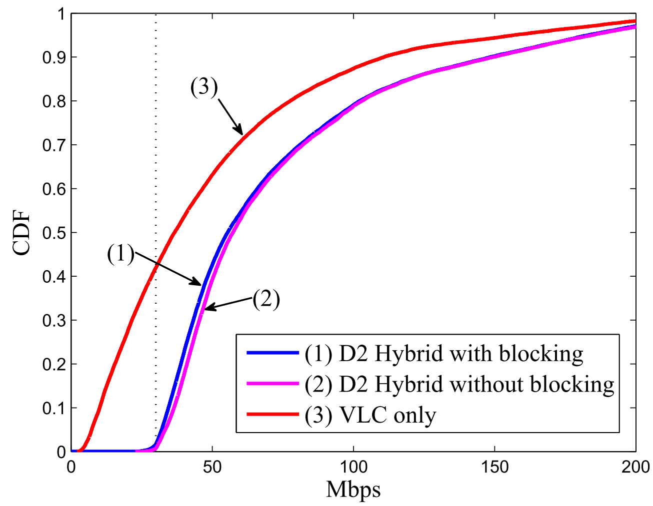

Fig. 9. Per user rate outage performance of a hybrid VLC/RF system for $S4$. "Without Blocking" case no RF link can ever be blocked, and "With Blocking", it is assumed that the probability of any RF link being blocked is 50%.

the locations of VLC and RF APs and users are modeled as random Poisson point processes (PPP) with uniform intensity of λ_V, λ_R and λ_u respectively. PPP networks are not typical for hybrid VLC/RF deployments, but their outage performance gives an upper bound for the outage performance of other practical networks [25]. Hence, studies on random hybrid network are an invaluable source of information. In [25], SINR coverage performance has been studied extensively, where in this work the focus is on per user rate coverage.

A. Random VLC Network

In this section, per user rate coverage of a standalone VLC network is studied, where it is assumed that all the users are served by the VLC system. If the user's PD faces directly upward, the angle of irradiance and the angle of incidence are equal which yields the following expression for the typical user's SINR [35]:

$$\mathrm{SINR} = \frac{(r_0^2 + h^2)^{-m-3}}{\sum_k (r_k^2 + h^2)^{-m-3} + \tilde{N}_0}, \tag{13}$$

where $\tilde{N}_0 = \left(\frac{2\pi\ell}{\kappa P_V (m+1) h^{m+1}}\right)^2 N_0 B_V$. Here r_0 is the horizontal distance from the typical user to the nearest VLC AP, r_k is the horizontal distance from the typical user to the kth interfering VLC AP, and h is the vertical distance from the users to the ceiling where VLC APs are located which is assumed to be equal to $h = 3$ m in this study. The SINR outage probability is studied firstly. It can be evaluated approximately, and the result is available in Theorem 1 in Appendix A.

The per user data rate of a communication network is dependent on both the proximity effect which is captured by the SINR, and the network congestion which is captured by the number of users in the network. The per user data rate achieved by a typical user in a VLC network is given in (5),

where N is the total number of users associated with the VLC AP which the typical user is associated with. In the context of PPP VLC networks, the probability mass function (PMF) of N is given by the following Lemma.

Lemma 1: Let there be a PPP network with AP and user intensities of λ_V and λ_u respectively. Let N denote the total number of users associated with AP which a given user is associated with. Hence, N is always $N \geq 1$. The probability mass function (PMF) of N can be approximated by:

$$\Pr(N = n) \approx \frac{\zeta_0^{\zeta_1} \Gamma(n + \zeta_1) \left(\frac{\lambda_u}{\lambda_V}\right)^n}{n! \Gamma(\zeta_1) \left(\frac{\lambda_u}{\lambda_V} + \zeta_0\right)^{n+\zeta_1}}, \tag{14}$$

where ζ_0 and ζ_1 are two auxiliary parameters which can be evaluated empirically. For instance, the simulation studies show that $\zeta_0 = 5.2$ and $\zeta_1 = 8.4$ provide a very good approximation for $\lambda_u = 60$ and $\lambda_V = 20$.

Proof: The expression is motivated by the PMF of the number of users per PPP Voronoi cell [38], [39]. Both parameters, ζ_0 and ζ_1 are evaluated empirically. Similar expressions are available in [21] and [40] where $\zeta_0 = 3.5$ and $\zeta_1 = 4.5$. The expression in (14) is significantly more accurate than the expressions that exit in the literature, and can also be used to characterize N for practical VLC AP deployments. Henceforth, it is assumed that PMF of N can be accurately obtained either semi-analytically by (14) or wholly by empirical methods. ■

Lemma 1 in conjunction with Theorem 1, per user rate outage probability in PPP VLC attocell networks can be approximated, and the result is given in Corollary 1.

Corollary 1: The per user rate outage probability which is denoted as $O_V(x) = \Pr(\Gamma_V \leq x)$ in a PPP VLC network with λ_V and λ_u, AP and user densities respectively can be tightly approximated by:

$$O_V(x) = \sum_{n=1}^{\infty} F_V\left(2^{\frac{2nx}{B_V}} - 1\right) \Pr(N = n), \tag{15}$$

where $\Pr(N = n)$ is given in (14) and $F_V(.)$ is given in (34).

Proof: The per user rate outage probability can be written using the law of total probability as:

$$\Pr(\Gamma_V \leq x) = \sum_{n=1}^{\infty} \Pr(\Gamma_V \leq x | N = n) \Pr(N = n). \tag{16}$$

Here the conditional probability, $\Pr(\Gamma_V \leq x | N = n)$ can be given as $\Pr(\Gamma_V \leq x | N = n) = F_V\left(2^{\frac{2nx}{B_V}} - 1\right)$ as a result of Theorem 1. The summation in (15) is not needed to be evaluated until n is very large. It is evaluated only until $n = 10 - 20$ in the simulations of this study. ■

Per user rate performance of a random PPP network is shown in Fig. 10. Here the intensity of the VLC AP process, λ_V is set to $\lambda_V = 80/A_0$, and the intensity of the user process is set to $\lambda_u = 60/A_0$. As shown in Fig. 10, the analysis tightly approximates the simulated per user outage rate performance. However, there is a slight mismatch as rate increases which is mainly due to the approximation in Theorem 1.

B. Random RF Network

In this section, per user rate coverage of a standalone RF network is studied, where it is assumed that all the users

$$F_R(x) = \frac{1}{\Gamma(w)\Gamma(a)} \int_0^\infty \int_0^\infty \frac{2\pi \lambda_R y u^{a-1}}{e^{u+\lambda_R \pi y^2}} \gamma\left(w, \frac{wxbuB_R N_0 10^{6.8}}{P_R\left(y^2+h^2\right)^{-\frac{\gamma}{2}}}\right) dudy \tag{23}$$

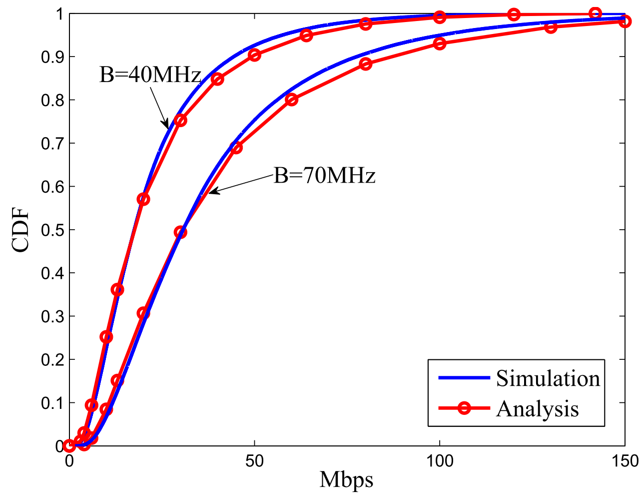

Fig. 10. Analytical and simulated per user rate outage performance comparison of a random VLC system with $\lambda_V = 80/A_0$ and $\lambda_u = 60/A_0$.

are served by the RF system. A random RF network with D2 deployment is considered due to its superior performance over D1 deployment. From (10), the rate achieved by a typical user in the RF system is given by:

$$\Gamma_R = \frac{B_R}{U_R} \log_2\left(1 + \frac{|H|^2 P_R}{N_0 B_R}\right), \quad U_R \neq 0 \tag{17}$$

where H denotes the random channel coefficient from the typical user to the closest LoS RF AP. The typicality herein means that the statistical performance of all the users in the RF system is as same as the performance of the typical user, and the per user rate outage performance is considered. There are four random variables caused by the number of RF users, U_R, multipath fading, shadow fading and distance dependent path loss. From (17), the relevant SNR can be defined as:

$$\mathrm{SNR} = \frac{|H|^2 P_R}{N_0 B_R}, \tag{18}$$

and can be rewritten using (7) and (11) as:

$$\mathrm{SNR} = 10^{-6.8} \frac{10^{-\frac{1.8X}{10}} |H_0|^2 P_R}{\left(r_0^2+h^2\right)^{\frac{\gamma}{2}} N_0 B_R}, \tag{19}$$

where $X \sim \mathcal{N}(0,1)$, r_0 is the horizontal distance to the serving RF AP, and

$$H_0 = \sqrt{\frac{K}{1+K}} H_d + \sqrt{\frac{1}{1+K}} H_s. \tag{20}$$

Due to the reasons explained in Sec. II, RF inter-cell interference is avoided. Hence the term, SNR is used in (19) in contrast to the SINR used in the VLC analysis.

Lemma 2: The cumulative distribution function of SNR in (19) *which is denoted as* $F_R(x) = \Pr(\mathrm{SNR} \leq x)$ *can be evaluated as in* (23)*, as shown at the top of this page, where*

$$w = \frac{K^2+2K+1}{2K+1}, \quad a = \frac{1}{e^{\sigma^2}-1}, \tag{21}$$

$$b = e^{\frac{3\sigma^2}{2}} - e^{\frac{\sigma^2}{2}}, \quad \sigma = \ln 10^{\frac{1.8}{10}}, \tag{22}$$

where $\gamma(.,.)$ *is the lower incomplete Gamma function.*

Proof: See Appendix B. ■

Corollary 2: Per user rate outage probability which is denoted as $O_R(x) = \Pr(\Gamma_R \leq x)$ *of a random PPP RF network with* λ_R *and* λ_u *AP and user intensities in D2 deployment is given by:*

$$O_R(x) = \sum_{n=1}^{\infty} F_R\left(2^{\frac{nx}{B_R}} - 1\right) \Pr(U_R = n), \tag{24}$$

where $F_R(.)$ *is given in* (23)*, and* $\Pr(U_R = n)$ *is given in:*

$$\Pr(U_R = n) = \frac{(\lambda_u A_0)^n e^{-\lambda_u A_0}}{n!}. \tag{25}$$

One can easily note the difference between (14) and (25). The reason is that even though there are multiple APs in the RF system, they function as a single cell with distributed radio heads. As also explained in Sec. II-B, the aforementioned deployment of the RF system significantly reduces the access distance. and avoid strong inter-cell interference.

C. Random VLC/RF Hybrid Network

In this section, the per user rate outage performance of standalone random VLC and RF systems are combined to quantify the per user rate outage performance of a random hybrid network. Furthermore, the following specific assumptions are also made.

- Simple System Association (SSA): In SSA, users whose SINR via VLC system fall below a certain threshold (i.e., SINR_T) served by the RF system. SSA is significantly different and superior to popular weighted path loss based association schemes commonly used for HetNets in the literature [21].
- VLC Cell Association: Each user served by the VLC system is associated with the physically closest VLC AP.
- RF Cell Association: Users served by the RF system are associated with the physically closest RF AP.

The following corollary provides the probability that a given user is associated with either the VLC or the RF system in SSA.

Corollary 3: Let $\Pr(\mathcal{A}_V)$ *and* $\Pr(\mathcal{A}_R)$ *be the probability that a given user is associated with the VLC and the RF systems respectively. If the users whose* SINR *achieved by the*

VLC network falls below SINR_T *are served by the RF system,* $\Pr(\mathcal{A}_R)$ *is given by:*

$$\Pr(\mathcal{A}_R) = F_V(\mathrm{SINR}_T), \tag{26}$$

where $F_V(.)$ *is given in Theorem 1. Hence,* $\Pr(\mathcal{A}_V) = 1 - F_V(\mathrm{SINR}_T)$.

Proof: The proof is straightforward from the definition of $\Pr(\mathcal{A}_R)$ and Theorem 1. ■

The analysis in Sections V-A and V-B in conjunction with Corollary 3 provides the per user rate outage probability of the random hybrid VLC/RF systems as follows.

Corollary 4: Let there be a random hybrid network as defined in Section V-C with VLC AP, RF AP and user densities of λ_V*,* λ_R *and* λ_u *respectively. Then the per user rate outage probability of the hybrid network which is defined here as* $O_H(x)$ *can be tightly approximated by:*

$$O_H(x) \approx O_R(x)\Pr(\mathcal{A}_R) + \left(\sum_{n=1}^{\infty} \Psi(x,n)\Pr(N=n)\right)\Pr(\mathcal{A}_V), \tag{27}$$

where $\Psi(x,n)$ *is defined as*

$$\Psi(x,n) = \begin{cases} 0 & xn \le \Gamma_T, \\ \dfrac{F_V\left(2^{\frac{2nx}{B_V}} - 1\right) - \Pr(\mathcal{A}_R)}{1 - \Pr(\mathcal{A}_R)} & xn > \Gamma_T, \end{cases} \tag{28}$$

where $\Gamma_T = \frac{B_V}{2}\log_2(1+\mathrm{SINR}_T)$*. Here* $\Pr(\mathcal{A}_R)$*,* $O_R(x)$ *and* $F_V(.)$ *are available in Corollary 3, Corollary 2, and Theorem 1 respectively. Furthermore,*

$$\Pr(N=n) \approx \frac{\zeta_0^{\zeta_1}\Gamma(n+\zeta_1)\left(\frac{\lambda_u \Pr(\mathcal{A}_V)}{\lambda_V}\right)^n}{n!\Gamma(\zeta_1)\left(\frac{\lambda_u \Pr(\mathcal{A}_V)}{\lambda_V} + \zeta_0\right)^{n+\zeta_1}}, \tag{29}$$

$$\Pr(U_R = n) \approx \frac{(\lambda_u \Pr(\mathcal{A}_R) A_0)^n e^{-\lambda_u \Pr(\mathcal{A}_R) A_0}}{n!}. \tag{30}$$

Proof: See Appendix C. ■

Fig. 11 shows the analytical and simulated per user rate performance of random hybrid VLC/RF systems with following parameters. System A has $B_V = 40$ MHz, $B_R = 20$ MHz, $P_R = 1$ W, $\Gamma_T = 30$ Mbps, System B has $B_V = 40$ MHz, $B_R = 100$ MHz, $P_R = 1$ W, $\Gamma_T = 30$ Mbps, and system C has $B_V = 60$ MHz, $B_R = 120$ MHz, $P_R = 1$ W, and $\Gamma_T = 50$ Mbps. For all systems, $\lambda_V = 80/A_0$, $\lambda_R = 10/A_0$, and $\lambda_u = 60/A_0$ which implies that on average there are 80 VLC APs, 10 RF APs, and 60 users within the coverage area of $A_0 = 40\text{x}40\ \mathrm{m}^2$. It is notable that per user outage rate performance curves are not smooth and have very complex behaviors, but analysis captures their behaviors successfully. There is a notable performance difference between System A, and Systems B and C. The reason is that the RF bandwidth of System A is significantly lower than System B even though both systems offload the same number of users (because B_V and Γ_T is the same in both systems) to the RF system. System C is superior to all other systems is because it has significantly higher resources for both VLC and RF system.

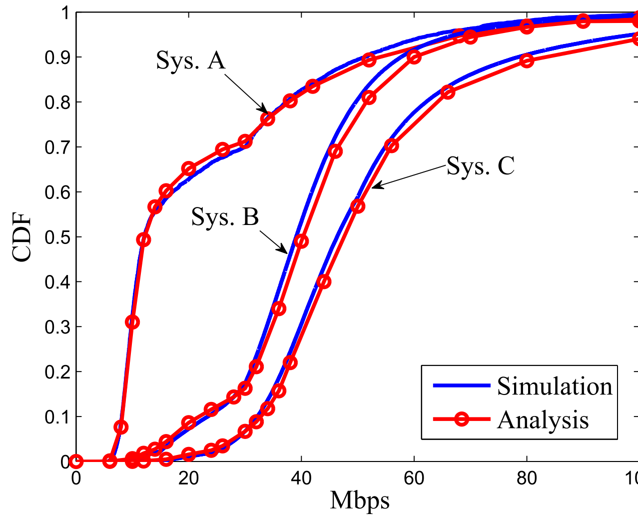

Fig. 11. Analytical and simulated per user rate outage performance comparison of random hybrid VLC/RF systems in D2. System A has $B_V = 40$ MHz, $B_R = 20$ MHz, $P_R = 1$ W, $\Gamma_T = 30$ Mbps, System B has $B_V = 40$ MHz, $B_R = 100$ MHz, $P_R = 1$ W, $\Gamma_T = 30$ Mbps, and system C has $B_V = 60$ MHz, $B_R = 120$ MHz, $P_R = 1$ W, and $\Gamma_T = 50$ Mbps. For all systems, $\lambda_V = 80/A_0$, $\lambda_R = 10/A_0$, and $\lambda_u = 60/A_0$

VI. Network Tuning and Design Guidelines

In this study, hybrid VLC/RF networks with AP deployments similar to the one shown in Fig. 1 is considered, and a random hybrid VLC/RF network (i.e. PPP) is comprehensively analyzed. While per user rate outage performance of random hybrid networks can be regarded as an upper bound to the outage performance of other fixed deployments, it is not clear how the outage statistics of random PPP networks derived in Sec. V can be used to study practical real world hybrid deployments such as the deployments in Fig. 1. In this section, a method to approximate the performance of practical VLC and hybrid VLC/RF deployments using the statistics of random VLC and hybrid VLC/RF systems is proposed.

The main motivation for the study in this section is, even though the number and the locations of VLC and RF APs are fixed in practical deployments, from a given user's point of view the number and the locations of VLC and RF APs can be regarded as dynamic. There are mainly two reasons for this assumption. Firstly, APs do not transmit if there are no users within their coverage area. Therefore, the number of active APs in the network changes frequently. Secondly, non line of sight (NLoS) communication is also not considered here, hence the number and the locations of desired and interfering APs to a given user changes frequently depending on the obstacles and human behavior in the coverage area. Therefore, extending performance results obtained for fully random networks to practical deployments is reasonable.

A. VLC Networks

Firstly, the per user rate outage performance of the stand-alone VLC system is considered. Let the SINR in (13) be defined concisely as:

$$\mathrm{SINR} = \frac{S}{I + \tilde{N}_0}, \tag{31}$$

where $S = (r_0^2 + h^2)^{-m-3}$, and $I = \sum_k (r_k^2 + h^2)^{-m-3}$. Based on (31) and (5), the following proposition for the performance of a standalone VLC system in fixed deployments such as in Fig. 1 can be made.

Proposition 1: The achievable data rate of practical/fixed standalone VLC network deployments can be approximated by:

$$\Gamma_V \approx \frac{\alpha_0 B_V}{2N} \log_2 \left(1 + \frac{\alpha_1 S}{I + \tilde{N}_0}\right) \quad \text{bps}, \tag{32}$$

where α_0 and α_1 are constants which are mainly dependent on the deployment, and as defined in (5) *and* (31)*, S, I and N are for random PPP networks.*

Proof: The proposition is an empirical expression with some key motivations. One of the main differences of fixed deployments and random deployments is that in fixed deployments, the aggregate interference is significantly lower than that in random deployments, thus better SINR. Hence factor, α_1, is introduced in (32). Furthermore, the number of users per cell also varies with the characteristics of the deployment, such as inter AP distance etc. Therefore, α_0 is introduced to capture the differences in the number of users per cell in PPP networks and practical networks. It is further assumed that a large number of data samples for N, I and S from PPP networks, and also for Γ_V from the original/fixed network are available at the CU. Then CU can find the optimum α_0 and α_1 in a minimum mean squared error (MMSE) sense. Note that squared error is evaluated on the sorted per user rate values of original/fixed network and the values generated from Proposition 1. Since, it is required to evaluate α_0 and α_1 once for the entire network, its computational complexity and memory requirements can be omitted. Specially memory requirements are omitted because data samples for N, I, S, and Γ_V can be stored in files on disk storages. This is deemed acceptable, since they are not accessed in real time. However, α_0 and α_1 should be reevaluated after major relocation of VLC APs. ■

As a result of Proposition 1, one can obtain the per user rate outage probability of a practical VLC deployment analytically (and of course approximately) using the SINR outage statistics of PPP networks, and α_0 and α_1 as:

$$O'_V(x) = \sum_{n=1}^{\infty} F_V \left(\frac{2^{\frac{2nx}{\alpha_0 B_V}} - 1}{\alpha_1} \right) \Pr(N = n), \tag{33}$$

where $F_V(.)$ is given in (34) and $\Pr(N = n)$ is given in Lemma 1 or obtained similarly as described therein for the corresponding PPP network. The corresponding PPP network herein means that if fixed VLC deployment has n_V VLC APs, the corresponding PPP network has $\lambda_V = n_V/A_0$ VLC AP density. Note that we differentiate $O'_V(.)$ in (33) from $O_V(.)$ in Corollary 1, because $O'_V(.)$ is based on (32) for real VLC deployments , but $O_V(.)$ is based on (5) for PPP VLC networks.

It is found that $\alpha_0 = 1.4$ and $\alpha_1 = 2$ give the best possible performance for the fixed VLC deployment in Fig. 1. It is important to note that α_0 and α_1 may have different values for different deployments. As shown in Fig. 12, the per user rate outage performance of the modified PPP VLC system is significantly closer to the corresponding performance of the original VLC system.

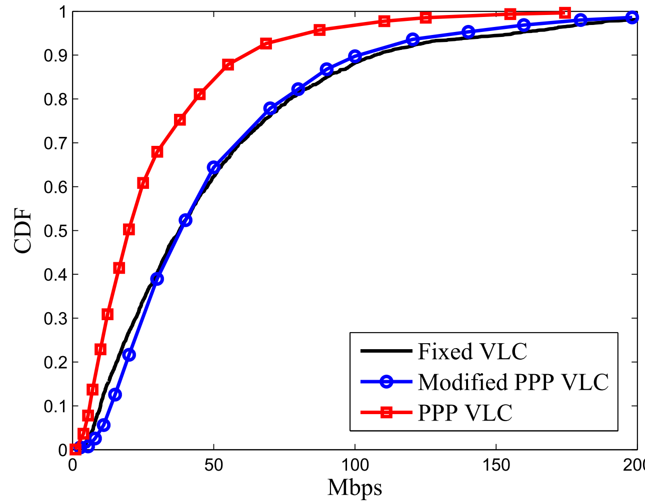

Fig. 12. Per user rate outage performance comparison of a random PPP VLC system and a fixed VLC deployment in Fig. 1. Here $B_V = 40$ MHz, $n_u = 30$, and $n_V = 20$, and for PPP VLC network, $\lambda_u = 30/A_0$, $\lambda_V = 20/A_0$.

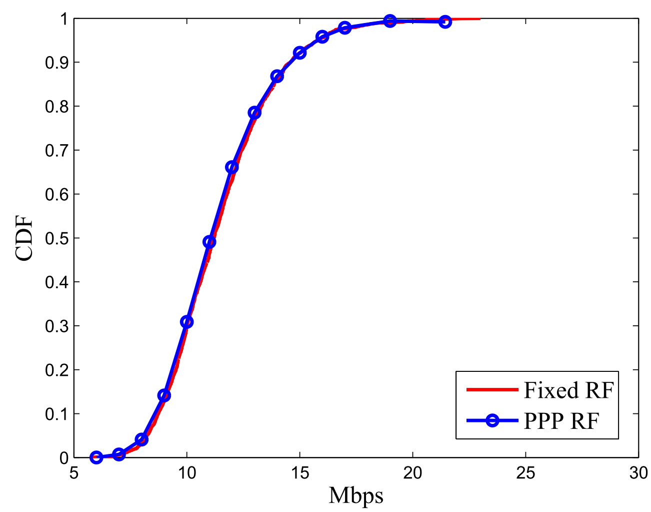

Fig. 13. Per user rate outage performance comparison of a standalone random RF system and the fixed RF deployment in Fig. 1. Here $n_R = 10$, $\lambda_u = 30/A_0$, $P_R = 1$ W and $B_R = 20$ MHz and for PPP RF network, $\lambda_R = 10/A_0$, $\lambda_u = 30/A_0$, $P_R = 1$ W and $B_R = 20$ MHz.

B. Hybrid VLC/RF Networks

Simulation studies also show that, it is possible to approximate the per user rate outage probability of fixed hybrid VLC/RF deployments using the results in Sec. V and Proposition 1. As shown in Figs. 12 and 13, the main performance deference is in the outage performance gap between the original VLC deployment in Fig. 1 and the PPP random VLC deployments. Since, there are several random processes stemming from fast fading and shadowing in the RF system, as shown in Fig. 13, the rate coverage performance of the original RF deployment in Fig. 1 and the PPP random RF deployment are almost the same. Therefore, Proposition 1 is used to approximate the per user outage performance of the VLC system, and the complete procedure is as follows.

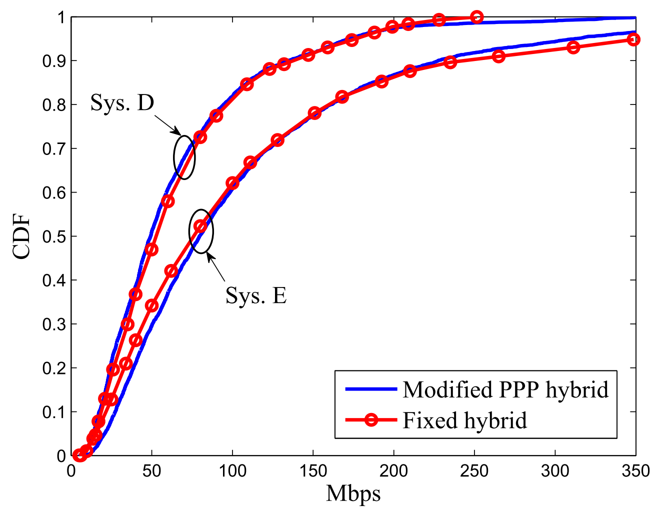

Fig. 14. Per user rate outage performance comparison of the fixed hybrid VLC/RF system in Fig. 1 and a modified random hybrid VLC/RF system. Sys. D has $B_V = 40$ MHz, $B_R = 20$ MHz, and $P_R = 1$ W, and Sys. E has $B_V = 80$ MHz, $B_R = 80$ MHz, and $P_R = 2$ W. Here it is assumed that $\Gamma_T = 30$ Mbps and $\Gamma'_T = 16.93$ Mbps and $\Gamma'_T = 18.04$ Mbps respectively for Sys. D and Sys. E.

Lets assume that there is a fixed hybrid VLC/RF deployment as shown in Fig. 1 with n_V, and n_R VLC and RF AP respectively. It is also assumed that the number of users are Poisson distributed with intensity, λ_u, and the per user rate outage performance of the hybrid network with Γ_T is considered. The following steps can be used to achieve this goal.

- Calculate the per user rate outage probability at Γ_T of the fixed standalone VLC system using Corollary 1 and Proposition 1. Let it holds that $\epsilon = O'_V(\Gamma_T)$.
- Define a random PPP VLC system with VLC AP intensity of $\lambda_V = n_V/A0$. Then find the rate at which the random PPP VLC achieves per user rate outage probability of ϵ. Let it be $\Gamma'_T = O_V^{-1}(\epsilon)$.
- Define a random PPP VLC/RF system with VLC AP density as in Step 2, and RF AP density of $\lambda_R = n_R/A_0$. Step 2 also implies that $\Pr(\mathcal{A}_R) = \epsilon$.
- A modified version of Corollary 4 incorporating Proposition 1 then provides the per user rate outage probability of the fixed hybrid VLC/RF deployments.

The results of two hybrid VLC/RF systems are shown in Fig. 14. System D has $B_V = 40$ MHz, $B_R = 20$ MHz, and $P_R = 1$ W, and System E has $B_V = 80$ MHz, $B_R = 80$ MHz, and $P_R = 2$ W. It is herein assumed that $\Gamma_T = 30$ Mbps, and as a result, $\Gamma'_T = 16.93$ Mbps and $\Gamma'_T = 18.04$ Mbps respectively for System D and System E.

Note that extensive simulation studies also reveal that using outage statistics of completely random network such as PPP for analytically quantifying the outage performance of practical/fixed hybrid VLC/RF networks as described in Sec. VI-B could be less accurate than the method described in Sec. VI-A for VLC only networks. Therefore, further studies may be required to understand the depth and breadth of the method described in Sec. VI-B.

C. Additional Remarks

The main objective of this study is designing hybrid RF/VLC networks that guarantees certain per user rate coverage specifications. The specifications often states that the hybrid network should be able to support certain per user data rate to certain percentage of the users when the network has certain total number of active users. Firstly, the designers should evaluate the per user rate coverage of the standalone VLC deployment with maximum possible resources, i.e., B_V and P_V. If the outage performance at the desired rate is below the specifications, the VLC system should be augmented with a RF system. Then the designer should quantify the minimum resource requirements, i.e., B_R, P_R for the RF system to achieve a specified per user rate outage probability for the hybrid system. Furthermore, it is also important to find the optimum rate threshold, Γ_T to achieve the minimum outage probability. There are two ways to calculate these critical parameters. The first method is to use the analytical results for random hybrid network in Sec. V. Since, outage performance of random hybrid networks forms an upper bound to the outage performance of other practical deployments, the optimum values derived for random networks certainly satisfy outage performance of other networks. On the other hand, the network tuning described in Sec. VI can be used to approximate the outage performance of practical hybrid VLC/RF networks and derive optimum values, B_R, P_R, and Γ_T directly for these networks.

It is important to note that, due to the analytical complexity of the results, it is not possible to directly evaluate the optimum values for B_R, P_R, and Γ_T, but simple algorithms can easily be developed to evaluate these values using the results presented in this paper. For instance, assume it is needed to evaluate the optimum Γ_T to achieve minimum outage probability at a certain rate, Γ_0 for a hybrid network with given P_R and B_R. Then one can set $\Gamma_T = \Gamma_0$, and evaluate $O_H(\Gamma_0)$ using Corollary 4. Then Γ_T is incremented slightly say $\Gamma_T = \Gamma_0 + \delta$, and $O_H(\Gamma_0 + \delta)$ is evaluated. If $O_H(\Gamma_0 + \delta) < O_H(\Gamma_0)$, Γ_0 can be incremented further to $\Gamma_T = \Gamma_0 + 2\delta$. Otherwise, Γ_T can be decremented. As such, simple line search algorithms can be developed to find the critical parameters. The main computational complexity of those algorithms arises in the evaluation of Corollary 4. The computational complexity of Corollary 4 is dependent on the threshold, Γ_T. Algorithms may requires Corollary 4 to be evaluated multiple times.

VII. Conclusion

In this study, a hybrid VLC/RF network is considered, where a simple RF system is introduced to a standalone VLC system in order to improve the per user rate outage performance. The system dynamically assigns users to either VLC or RF system depending on users' channel condition. A central unit denoted as CU monitors the VLC system at constant intervals, and users with lower achievable rates are migrated to the RF system. The resource requirements (spectrum and power) for the RF system is investigated in order to satisfy a certain per user data rate outage performance of the hybrid system. Two deployments of a RF system denoted as D1 and D2 are considered. In D1, a fixed spectrum and power is allocated to each RF AP, and in D2, CU dynamically allocates the power and spectrum to each AP depending on the traffic demands. The performance and trade-offs between spectrum and power are investigated. It is shown that from a per user rate outage probability perspective D2 performs better, but may require some form of out-of-band communication between RF APs. Exploiting the analytical results for random VLC/RF networks, an analytical framework is also developed to analyze the performance of practical hybrid VLC/RF networks. Furthermore, these analytical models can be used to study trends and trade-offs of hybrid network further, and to design algorithms for adaptive and self organizing hybrid networks.

Appendix A
SINR Outage Probability of the VLC System

Theorem 1 [25]: The cumulative distribution function (CDF) of SINR *in* (13) *which is denoted as* $F_V(x) = \Pr(\text{SINR} \leq x)$ *can be tightly approximated as:*

$$F_V(x) = \int_0^{\infty} \frac{2\pi\lambda_V u}{e^{\pi\lambda_V u^2}} \sum_{k=0}^{\infty} \frac{\Delta_k^{[1]}(u)\,\Delta_k^{[2]}(u)}{k!\Gamma(k+\alpha)} du, \tag{34}$$

where

$$\Delta_k^{[1]}(u) = \beta^{\alpha} \sum_{\ell_1=0}^{k} C_{\ell_1}^k \mu_{\ell_1}(I), \quad \text{and} \tag{35}$$

$$\Delta_k^{[2]}(u) = \sum_{\ell_2=0}^{k} \frac{C_{\ell_2}^k \Gamma\left(\ell_2+\alpha, \beta\left(\frac{X}{x} - \tilde{N}_0\right)\right)}{\beta^{\ell_2+\alpha}}. \tag{36}$$

X, α, β, C_ℓ^k, $\mu_\ell(I)$ are defined respectively as:

$$X = \left(u^2+h^2\right)^{-m-3}, \quad \alpha = \frac{\lambda_V\pi\,(2m+5)}{(m+2)^2\left(u^2+h^2\right)^{-1}} \tag{37}$$

$$\beta = \frac{(2m+5)}{(m+2)\left(u^2+h^2\right)^{-m-3}}, \tag{38}$$

$$C_\ell^k = (-1)^{k-\ell}\binom{k}{\ell}\beta^{\ell} S_\ell^k, \tag{39}$$

$$\mu_\ell(I) = \begin{cases} 1 & \ell = 0 \\ \kappa_1 & \ell = 1 \\ \kappa_\ell + \sum_{l=1}^{\ell-1}\binom{\ell-1}{l-1}\kappa_l \mu_{\ell-l}(I) & \ell \geq 2, \end{cases} \tag{40}$$

where κ_ℓ and S_ℓ^k are defined respectively as:

$$\kappa_\ell = \frac{\lambda_V\pi\left(u^2+h^2\right)^{1-\ell(m+3)}}{\ell(m+3)-1}, \tag{41}$$

$$S_\ell^k = \begin{cases} 1 & \ell > k-1 \\ \prod_{l=\ell}^{k-1}(\alpha+l) & \ell \leq k-1. \end{cases} \tag{42}$$

Proof: A more descriptive proof is available in [25]. For the sake of completeness, we outline a proof here. Firstly, the characteristic function (CF) of the aggregated interference, $I = \sum_k (r_k^2 + h^2)^{-m-3}$ is derived. The density function of I is obtained by inverting its CF. The exact inversion of CF appears to be complex for an arbitrary, m. Hence, the moments of I are extracted from its CF and the density of I is approximated using a method similar to the one described in [36]. Then $F_V(x)$ conditioned on r_0 is evaluated using the density function of I, and the conditional SINR outage probability is averaged over r_0 to obtain the final result as in (34). The probability density function of r_0 in random PPP networks is known, and is given by [37]:

$$f_{r_0}(u) = 2\pi\lambda_V u e^{-\lambda_V \pi u^2}, \quad u \geq 0. \tag{43}$$

■

Appendix B
Proof of Lemma 2

There are three independent random processes in (19), hence the analysis is straightforward. We use following density functions for the analysis [31].

$$f_{|H_0|^2}(t) = \frac{w^w}{\Gamma(w)} t^{w-1} e^{-wt}, \quad t \geq 0, \tag{44}$$

$$f_{\Phi}(u) = \frac{1}{b^a \Gamma(a)} u^{a-1} e^{-u/b}, \quad u \geq 0, \tag{45}$$

$$f_{r_0}(y) = 2\pi\lambda_R y e^{-\lambda_R \pi y^2}, \quad y \geq 0, \tag{46}$$

where $\Phi = 10^{-\frac{1.8X}{10}}$. Both $|H_0|^2$ and Φ are approximated as generalized Gamma variates [31], [32]. The analysis is briefly outlined here.

$$F_R(x) = \Pr(\text{SNR} \leq x), \tag{47}$$

$$= \mathcal{E}\left\{ \Pr\left(|H_0|^2 \leq \frac{x B_R N_0 10^{6.8}\Phi}{P_R\left(r_0^2+h^2\right)^{-\frac{\gamma}{2}}} \,\middle|\, \Phi, r_0 \right) \right\}, \tag{48}$$

$$= \mathcal{E}\left\{ \gamma\left(w, \frac{w x B_R N_0 10^{6.8}\Phi}{P_R\left(r_0^2+h^2\right)^{-\frac{\gamma}{2}}} \right) \right\}, \tag{49}$$

where the expectation in (48) and (49) is over Φ and r_0.

Appendix C
Proof of Corollary 4

Let Γ be the per user rate of the hybrid network, and from the law of total probability:

$$O_H(x) = \Pr(\Gamma_R \leq x)\Pr(\mathcal{A}_R) + \Pr(\Gamma_V \leq x)\Pr(\mathcal{A}_V), \tag{50}$$

$$= O_R(x)\Pr(\mathcal{A}_R) \tag{51}$$

$$+ \left(\sum_{n=1}^{\infty}\Pr(\Gamma_V \leq x | N=n)\Pr(N=n)\right)\Pr(\mathcal{A}_V) \tag{52}$$

From simple system association, it can be shown that $\Pr(\text{SINR} \leq \text{SINR}_T | \text{VLC}) = 0$. Hence:

$$\Pr\left(\frac{B_V}{2}\log_2(1+\text{SINR}) \leq \Gamma_T\right) = 0. \tag{53}$$

For exposition, we define $\tilde{\Gamma}_V = \frac{B_V}{2}\log_2(1+\text{SINR})$, and $\Psi(x,n) = \Pr(\Gamma_V \le x | N = n)$. $\Psi(x,n)$ can also be rewritten as $\Psi(x,n) = \Pr\left(\tilde{\Gamma}_V \le xn\right)$. From (53), it is clear that $\Psi(x,n) = 0$ if $xn \le \Gamma_T$. Otherwise, i.e., $xn > \Gamma_T$:

$$\Psi(x,n) = \Pr\left(\Gamma_V \le x \,|\, N = n, \tilde{\Gamma}_V > \Gamma_T\right), \tag{54}$$

$$= \Pr\left(\tilde{\Gamma}_V \le xn \middle| \tilde{\Gamma}_V > \Gamma_T\right), \tag{55}$$

$$= \frac{\Pr\left(\tilde{\Gamma}_V \le xn, \tilde{\Gamma}_V > \Gamma_T\right)}{\Pr\left(\tilde{\Gamma}_V > \Gamma_T\right)}, \tag{56}$$

$$= \frac{F_V\left(2^{\frac{2nx}{B_V}} - 1\right) - \Pr(\mathcal{A}_R)}{1 - \Pr(\mathcal{A}_R)} \tag{57}$$

Due to the simple system association scheme, a certain percentage of users fall in the VLC system and the rest of the users fall in the RF system. The users fall in the RF system can be approximated as a PPP with intensity, $\lambda_u \Pr(\mathcal{A}_R)$, and the users fall in the VLC system can also be approximated as a PPP with intensity, $\lambda_u \Pr(\mathcal{A}_V)$. These assumptions in conjunction with (14) and (25) give the results in (29) and (30) respectively. This completes the proof.

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