Algorithm 1: Principal Component Analysis (PCA)

Input: Data matrix X of dimensions $N \times M$ (rows are observations, columns are features); number of principal components q

Output: Matrix Z of dimensions $N \times q$ containing the q principal components

1 Center the data by subtracting the mean of each column (feature) from the corresponding column in X

2 Compute the covariance matrix $\Sigma = \frac{1}{N} X^T X$

3 Perform eigenvalue decomposition on Σ to obtain eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_M$ and eigenvectors $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \ldots, \boldsymbol{\gamma}_M$

4 Sort the eigenvectors by decreasing eigenvalue magnitudes

5 Select the first q eigenvectors $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \ldots, \boldsymbol{\gamma}_q$

6 Form the projection matrix $\boldsymbol{\Gamma}_q$ using the selected eigenvectors as columns

7 Compute the principal components $Z = X\boldsymbol{\Gamma}_q$

8 **return** Z