Cálculo Numérico - IME/UERJ

Gabarito - Lista de Exercícios 3

Sistemas Lineares - Métodos diretos e iterativos

1.
$$X = (0.7681, -0.189, -0.6549)^t$$

2. A última linha do sistema triangular é nula, portanto é satisfeita para qualquer valor de z, o que significa que o conjunto de soluções do sistema linear é infinito e está definido por:

$$S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x = \frac{1}{16}(z + 13); y = \frac{1}{8}(11z - 17) \right\}$$

3. Há necessidade de troca da primeira e terceira linhas.

 $(x,y,z) = (10/7, -5/3, 9/5) \approx (1.423, -1.667, 1.800)$, usando 4 dígitos e arredondamento.

4.

$$L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; \ U = \begin{bmatrix} 1 & -4 \\ 0 & 5 \end{bmatrix}.$$

(a) (i)
$$X = (6/5, -1/5)^t$$

(ii)
$$X = (-3/5, -2/5)^t$$

(b) Primeira maneira:

Sabemos que

$$UU^{-1}=I$$

Como a inversa de uma matriz triangular superior também é triangular superior, logo,

$$U^{-1} = C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}$$

Assim,

$$UU^{-1} = \begin{bmatrix} 1 & -4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

De onde tiramos:

$$c_{11} = 1$$

 $c_{12} - 4c_{22} = 0$
 $5c_{22} = 1 \Rightarrow c_{22} = 1/5 \Rightarrow c_{12} = 4/5$
Logo,

$$U^{-1} = \left[\begin{array}{cc} 1 & 4/5 \\ 0 & 1/5 \end{array} \right]$$

Analogamente,

$$LL^{-1} = I$$

A inversa de uma matriz triangular inferior também é triangular inferior. Neste caso,

$$L^{-1} = D = \begin{bmatrix} 1 & 0 \\ d_{21} & 1 \end{bmatrix}$$

Assim,

$$LL^{-1} = \left[\begin{array}{cc} 1 & 0 \\ 1 & 1 \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ d_{21} & 1 \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

De onde tiramos:

$$d_{21} = -1$$

$$L^{-1} = \left[\begin{array}{cc} 1 & 0 \\ -1 & 1 \end{array} \right]$$

Sabemos que sem pivoteamento parcial:

$$A = LU \Rightarrow A^{-1} = (LU)^{-1} \Rightarrow A^{-1} = U^{-1}L^{-1}$$

Então,

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} 1 & 4/5 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \left[\begin{array}{cc} 1/5 & 4/5 \\ -1/5 & 1/5 \end{array} \right]$$

Segunda maneira: Usando forma escalonada por linhas para achar as inversas de L e U.

Assim,

$$A = LU \Rightarrow A^{-1} = U^{-1}L^{-1}$$

Cálculo de U^{-1} :

$$U \mid I = \begin{bmatrix} 1 & -4 & | & 1 & 0 \\ 0 & 5 & | & 0 & 1 \end{bmatrix} \begin{array}{c} L_1 \\ L_2 \end{array}$$

$$piv\hat{o} = a_{22} = 5;$$

$$m_{12} = \frac{a_{12}}{a_{22}} = -\frac{4}{5} \Rightarrow L_1 \leftarrow L_1 + \frac{4}{5}L_2$$

$$U^{(1)} \mid D^{(1)} = \begin{bmatrix} 1 & 0 & | & 1 & 4/5 \\ 0 & 5 & | & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

Dividindo L_2 por 5, obtemos:

$$I \mid U^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & | & 1 & 4/5 \\ 0 & 1 & | & 0 & 1/5 \end{array} \right] \begin{array}{c} L_1 \\ L_2 \end{array}$$

Cálculo de L^{-1} :

$$L \mid I = \begin{bmatrix} 1 & 0 & | & 1 & 0 \\ 1 & 1 & | & 0 & 1 \end{bmatrix} \begin{array}{c} L_1 \\ L_2 \end{array}$$

$$piv\hat{o} = a_{11} = 1;$$

$$m_{21} = \frac{a_{21}}{a_{11}} = 1 \Rightarrow L_2 \leftarrow L_2 - L_1$$

$$I \mid L^{-1} = \left[\begin{array}{ccccc} 1 & 0 & | & 1 & 0 \\ 0 & 1 & | & -1 & 1 \end{array} \right] \begin{array}{c} L_1 \\ L_2 \end{array}$$

Portanto,

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} 1 & 4/5 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & 4/5 \\ -1/5 & 1/5 \end{bmatrix}$$

Terceira maneira:

$$LU \cdot \begin{bmatrix} | & | \\ d_1 & d_2 \\ | & | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ I_1 & I_2 \\ | & | \end{bmatrix}}_{I = A^{-1}}$$

onde

$$d_1 = \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix}; \ d_2 = \begin{bmatrix} d_{12} \\ d_{22} \end{bmatrix};$$

$$I_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \ I_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Assim, resolvemos 4 sistemas:

$$LUd_1 = I_1 \underset{Ud_1 = y_1}{\Longrightarrow} \left\{ \begin{array}{rcl} Ly_1 & = & I_1 & (1) \Rightarrow \text{Acho } y_1 \\ Ud_1 & = & y_1 & (2) \Rightarrow \text{Acho } d_1 \end{array} \right..$$

$$d_1 = \begin{bmatrix} 1/5 \\ -1/5 \end{bmatrix}.$$

$$LUd_2 = I_2 \underset{Ud_2 = y_2}{\Longrightarrow} \left\{ \begin{array}{lcl} Ly_2 & = & I_2 & (3) \Rightarrow \text{Acho } y_2 \\ Ud_2 & = & y_2 & (4) \Rightarrow \text{Acho } d_2 \end{array} \right..$$

$$d_2 = \begin{bmatrix} 4/5 \\ 1/5 \end{bmatrix}.$$

Logo,

$$A^{-1} = \begin{bmatrix} 1/5 & 4/5 \\ -1/5 & 1/5 \end{bmatrix}.$$

5. (a) Primeiro, achar as matrizes $L \in U$ usando eliminação de Gauss, as quais serão:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix}; \qquad U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1/2 \end{bmatrix}$$

Primeira maneira:

Sabemos que

$$UU^{-1}=I$$

Como a inversa de uma matriz triangular superior também é triangular superior, logo,

$$U^{-1} = C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}$$

Assim,

$$UU^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

De onde tiramos:

$$c_{11} = 1/2$$

$$2c_{12} + c_{22} = 0$$

$$2c_{13} + c_{23} = 0$$

$$c_{22} = 1 \Rightarrow 2c_{12} + 1 = 0 \Rightarrow c_{12} = -1/2$$

$$c_{23} + c_{33} = 0$$

$$(1/2)c_{33} = 1 \Rightarrow c_{33} = 2 \Rightarrow c_{23} = -2 \Rightarrow c_{13} = 1$$
Logo,

$$U^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

Analogamente,

$$LL^{-1} = I$$

A inversa de uma matriz triangular inferior também é triangular inferior. Neste caso,

$$L^{-1} = D = \begin{bmatrix} 1 & 0 & 0 \\ d_{21} & 1 & 0 \\ d_{31} & d_{32} & 1 \end{bmatrix}$$

Assim,

$$LL^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ d_{21} & 1 & 0 \\ d_{31} & d_{32} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

De onde tiramos:

$$d_{21} = -2$$

$$(1/2) + (1/2)d_{21} + d_{31} = 0 \Rightarrow d_{31} = 1/2$$

$$(1/2) + d_{32} = 0 \Rightarrow d_{32} = -1/2$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1/2 & -1/2 & 1 \end{bmatrix}$$

Sabemos que sem pivoteamento parcial:

$$A=LU\Rightarrow A^{-1}=(LU)^{-1}\Rightarrow A^{-1}=U^{-1}L^{-1}$$
 Então,

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1/2 & -1/2 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

Segunda maneira:

Temos que:

$$LU \cdot \underbrace{\begin{bmatrix} | & | & | \\ d_1 & d_2 & d_3 \\ | & | & | \end{bmatrix}}_{D=A^{-1}} = \underbrace{\begin{bmatrix} | & | & | \\ I_1 & I_2 & I_3 \\ | & | & | \end{bmatrix}}_{I}$$

onde

$$d_{1} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \end{bmatrix}; d_{2} = \begin{bmatrix} d_{12} \\ d_{22} \\ d_{32} \end{bmatrix}; d_{3} = \begin{bmatrix} d_{13} \\ d_{23} \\ d_{33} \end{bmatrix};$$

$$I_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; I_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; I_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Assim, resolvemos 6 sistemas:

$$LUd_1 = I_1 \underset{Ud_1 = y_1}{\Longrightarrow} \left\{ \begin{array}{l} Ly_1 = I_1 & (1) \Rightarrow \operatorname{Acho} y_1 \\ Ud_1 = y_1 & (2) \Rightarrow \operatorname{Acho} d_1 \end{array} \right.$$

$$LUd_2 = I_2 \underset{Ud_2 = y_2}{\Longrightarrow} \left\{ \begin{array}{l} Ly_2 = I_2 & (3) \Rightarrow \operatorname{Acho} y_2 \\ Ud_2 = y_2 & (4) \Rightarrow \operatorname{Acho} d_2 \end{array} \right.$$

$$LUd_3 = I_3 \underset{Ud_3 = y_3}{\Longrightarrow} \left\{ \begin{array}{l} Ly_3 = I_3 & (5) \Rightarrow \operatorname{Acho} y_3 \\ Ud_3 = y_3 & (6) \Rightarrow \operatorname{Acho} d_3 \end{array} \right.$$

Assim,

$$A^{-1} = \begin{bmatrix} & | & | & | \\ d_1 & d_2 & d_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

Terceira maneira: Usando forma escalonada por linhas para achar as inversas de $L \in U$.

Assim,

$$A = LU \Rightarrow A^{-1} = U^{-1}L^{-1}$$

Cálculo de U^{-1} :

$$U \mid I = \begin{bmatrix} 2 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 1/2 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

pivô =
$$a_{33} = 1/2$$
;
 $m_{23} = \frac{a_{23}}{a_{33}} = 2 \Rightarrow L_2 \leftarrow L_2 - 2L_3$

$$U^{(1)} \mid D^{(1)} = \begin{bmatrix} 2 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & -2 \\ 0 & 0 & 1/2 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

pivô =
$$a_{22} = 1$$
;
 $m_{12} = \frac{a_{12}}{a_{22}} = 1 \Rightarrow L_1 \leftarrow L_1 - L_2$

$$U^{(2)} \mid D^{(2)} = \begin{bmatrix} 2 & 0 & 0 & | & 1 & -1 & 2 \\ 0 & 1 & 0 & | & 0 & 1 & -2 \\ 0 & 0 & 1/2 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

Fazendo as operações:

$$L_1 \leftarrow \frac{1}{2}L_1;$$

$$L_3 \leftarrow 2L_3$$

obtemos:

$$I \mid U^{-1} = \begin{bmatrix} 1 & 0 & 0 & | & 1/2 & -1/2 & 1 \\ 0 & 1 & 0 & | & 0 & 1 & -2 \\ 0 & 0 & 1 & | & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

Cálculo de L^{-1} :

$$L \mid I = \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 2 & 1 & 0 & | & 0 & 1 & 0 \\ 1/2 & 1/2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

$$\begin{aligned} &\text{piv\^o} = a_{11} = 1; \\ &m_{21} = \frac{a_{21}}{a_{11}} = 2 \Rightarrow L_2 \leftarrow L_2 - 2L_1; \\ &m_{31} = \frac{a_{31}}{a_{11}} = \frac{1}{2} \Rightarrow L_3 \leftarrow L_3 - \frac{1}{2}L_1 \end{aligned}$$

$$L^{(1)} \mid D^{(1)} = \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 1/2 & 1 & | & -1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

pivô =
$$a_{22} = 1$$
;
 $m_{32} = \frac{a_{32}}{a_{22}} = \frac{1}{2} \Rightarrow L_3 \leftarrow L_3 - \frac{1}{2}L_2$

$$I \mid L^{-1} = \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1/2 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

Portanto,

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1/2 & -1/2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Primeiro, achar as matrizes L, U, P usando eliminação de Gauss **com pivoteamento parcial**, as quais serão:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{bmatrix}; \qquad U = \begin{bmatrix} 4 & 3 & 1 \\ 0 & -1/2 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix}; \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Primeira maneira:

Sabemos que:

$$PA = LU \Rightarrow (PA)^{-1} = (LU)^{-1} \Rightarrow A^{-1}P^{-1} = U^{-1}L^{-1}$$

$$\Rightarrow A^{-1}\underbrace{P^{-1}P}_{I} = U^{-1}L^{-1}P \Rightarrow A^{-1} = U^{-1}L^{-1}P.$$

Para calcular U^{-1} , sabemos que:

$$UU^{-1} = I$$

Como a inversa de uma matriz triangular superior também é triangular superior, logo,

$$U^{-1} = C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}$$

Assim,

$$UU^{-1} = \begin{bmatrix} 4 & 3 & 1 \\ 0 & -1/2 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

De onde tiramos:

$$c_{11} = 1/4$$

$$4c_{12} + 3c_{22} = 0$$

$$4c_{13} + 3c_{23} + c_{33} = 0$$

$$(-1/2)c_{22} = 1 \Rightarrow c_{22} = -2 \Rightarrow c_{12} = 3/2$$

$$(-1/2)c_{23} - (1/2)c_{33} = 0 \Rightarrow c_{23} + c_{33} = 0$$

$$(1/2)c_{33} = 1 \Rightarrow c_{33} = 2 \Rightarrow c_{23} = -2 \Rightarrow c_{13} = 1$$
Logo,

$$U^{-1} = \begin{bmatrix} 1/4 & 3/2 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

Analogamente, para calcular L^{-1} , sabemos que:

$$LL^{-1} = I$$

A inversa de uma matriz triangular inferior também é triangular inferior. Neste caso,

$$L^{-1} = D = \begin{bmatrix} 1 & 0 & 0 \\ d_{21} & 1 & 0 \\ d_{31} & d_{32} & 1 \end{bmatrix}$$

Assim,

$$LL^{-1} = \left[egin{array}{cccc} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{array}
ight] \left[egin{array}{cccc} 1 & 0 & 0 \\ d_{21} & 1 & 0 \\ d_{31} & d_{32} & 1 \end{array}
ight] \left[egin{array}{cccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}
ight]$$

De onde tiramos:

$$(1/2) + d_{21} = 0 \Rightarrow d_{21} = -1/2$$

 $(1/4) - (1/2)d_{21} + d_{31} = 0 \Rightarrow d_{31} = -1/2$
 $-(1/2) + d_{32} = 0 \Rightarrow d_{32} = 1/2$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/2 & 1 \end{bmatrix}$$

Então,

$$A^{-1} = U^{-1}L^{-1}P = \begin{bmatrix} 1/4 & 3/2 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

Segunda maneira:

Temos que:

$$LU \cdot \begin{bmatrix} | & | & | \\ d_1 & d_2 & d_3 \\ | & | & | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | & | \\ P_1 & P_2 & P_3 \\ | & | & | \end{bmatrix}}_{P}$$

onde

$$d_{1} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \end{bmatrix}; d_{2} = \begin{bmatrix} d_{12} \\ d_{22} \\ d_{32} \end{bmatrix}; d_{3} = \begin{bmatrix} d_{13} \\ d_{23} \\ d_{33} \end{bmatrix};$$

$$P_{1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; P_{2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; P_{3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Assim, resolvemos 6 sistemas:

$$LUd_1 = P_1 \underset{Ud_1 = y_1}{\Longrightarrow} \left\{ \begin{array}{l} Ly_1 & = & P_1 & (1) \Rightarrow \operatorname{Acho} y_1 \\ Ud_1 & = & y_1 & (2) \Rightarrow \operatorname{Acho} d_1 \end{array} \right.$$

$$LUd_2 = P_2 \underset{Ud_2 = y_2}{\Longrightarrow} \left\{ \begin{array}{l} Ly_2 & = & P_2 & (3) \Rightarrow \operatorname{Acho} y_2 \\ Ud_2 & = & y_2 & (4) \Rightarrow \operatorname{Acho} d_2 \end{array} \right.$$

$$LUd_3 = P_3 \underset{Ud_3 = y_3}{\Longrightarrow} \left\{ \begin{array}{l} Ly_3 & = & P_3 & (5) \Rightarrow \operatorname{Acho} y_3 \\ Ud_3 & = & y_3 & (6) \Rightarrow \operatorname{Acho} d_3 \end{array} \right.$$

Assim,

$$A^{-1} = \begin{bmatrix} & | & & | & & | \\ & d_1 & d_2 & d_3 & & \\ & | & & | & & | \end{bmatrix} = \begin{bmatrix} & 2 & -1 & & 1 \\ & -3 & & 2 & -2 \\ & 1 & -1 & & 2 \end{bmatrix}$$

6.
$$X = (4/7, -1/14, -5/14)^t$$

7. (a)
$$X = (-0.8780, -1.4390, -2.2195, 2.2195)$$