Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$
\begin{aligned}
&AND: \\
&\quad \Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \end{bmatrix} \\
&NOR: \\
&\quad \Theta^{(1)} = \begin{bmatrix} 10 & -20 & -20 \end{bmatrix} \\
&OR: \\
&\quad \Theta^{(1)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}
\end{aligned}
$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \end{bmatrix} \rightarrow \begin{bmatrix} a^{(3)} \end{bmatrix} \rightarrow h_\Theta(x)
$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$
\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 2010 & -20 & -20 \end{bmatrix}
$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$$
\Theta^{(2)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}
$$

Let's write out the values for all our nodes:

$$
\begin{aligned}
&a^{(2)} = g(\Theta^{(1)} \cdot x) \\
&a^{(3)} = g(\Theta^{(2)} \cdot a^{(2)}) \\
&h_\Theta(x) = a^{(3)}
\end{aligned}
$$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm: