Serie de Fourier

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Obtengo la serie de Fourier asociada a ala siguiente función:

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < 0 \\ 1 & \text{for } 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} < x \leq \pi \end{cases}$$

Calculo a_0:

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx \\ a_0 &= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \ dx + \int_{0}^{\frac{\pi}{2}} 1 \ dx + \int_{\frac{\pi}{2}}^{\pi} 0 \ dx \right] \\ a_0 &= \frac{1}{\pi} \left[\int_{0}^{\frac{\pi}{2}} 1 \ dx \right] \\ a_0 &= \frac{1}{\pi} \left[x \Big|_{0}^{\frac{\pi}{2}} \right] \\ a_0 &= \frac{1}{\pi} \cdot \left[\frac{\pi}{2} - 0 \right] \\ a_0 &= \frac{1}{2} \end{aligned}$$

Calculo a_n:

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx) dx \\ a_n &= \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} 1 \cdot \cos(nx) dx \\ a_n &= \frac{1}{\pi} \left[\frac{1}{n} \sin(nx) \Big|_{0}^{\frac{\pi}{2}} \right] \\ a_n &= \frac{1}{n\pi} \sin\left(n\frac{\pi}{2} \right) \end{aligned}$$

Los separo en pares e impares:
$n = 2n$:

$$a_{2n} = \frac{1}{2n\pi} \sin(\cancel{2}n\frac{\pi}{\cancel{2}})$$
$$a_{2n} = \frac{1}{2n\pi} \underbrace{\sin(n\pi)}_{=0}$$
$$a_{2n} = 0$$

$n = 2n - 1$:

$$a_{2n-1} = \frac{1}{\pi(2n-1)} \sin\left((2n-1)\frac{\pi}{2}\right)$$
$$a_{2n-1} = \frac{1}{\pi(2n-1)} \underbrace{\sin\left(n\pi - \frac{\pi}{2}\right)}_{*}$$
$$a_{2n-1} = \frac{1}{\pi(2n-1)}(-1)^{n+1}$$

Calculo b_n:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot \sin(nx)dx$$
$$b_n = \frac{1}{\pi}\int_{0}^{\frac{\pi}{2}} 1 \cdot \sin(nx)dx$$
$$b_n = \frac{1}{\pi}\left[-\frac{1}{n}\cos(nx)\Big|_0^{\frac{\pi}{2}}\right]$$
$$b_n = \frac{1}{\pi}\left[-\frac{1}{n}\left[\cos\left(n\frac{\pi}{2}\right) - \cos(n\cdot 0)\right]\right]$$
$$b_n = -\frac{1}{n\pi}\left[\cos\left(n\frac{\pi}{2}\right) - 1\right]$$

Separo en pares:
$n = 2n$:

$$b_{2n} = -\frac{1}{\pi 2n}\left[\cos\left(\cancel{2}n\frac{\pi}{\cancel{2}}\right) - 1\right]$$
$$b_{2n} = -\frac{1}{\pi 2n}\left[(-1)^n - 1\right]$$

Dentro de los pares separo en múltiplos y no múltiplos de 4.

$n = 4n$:

$$b_{4n} = -\frac{1}{\pi 4n}\left[(-1)^{2n} - 1\right]$$
$$b_{4n} = -\frac{1}{\pi 4n}\left[1 - 1\right]$$
$$b_{4n} = 0$$

$n = 4n - 2$:

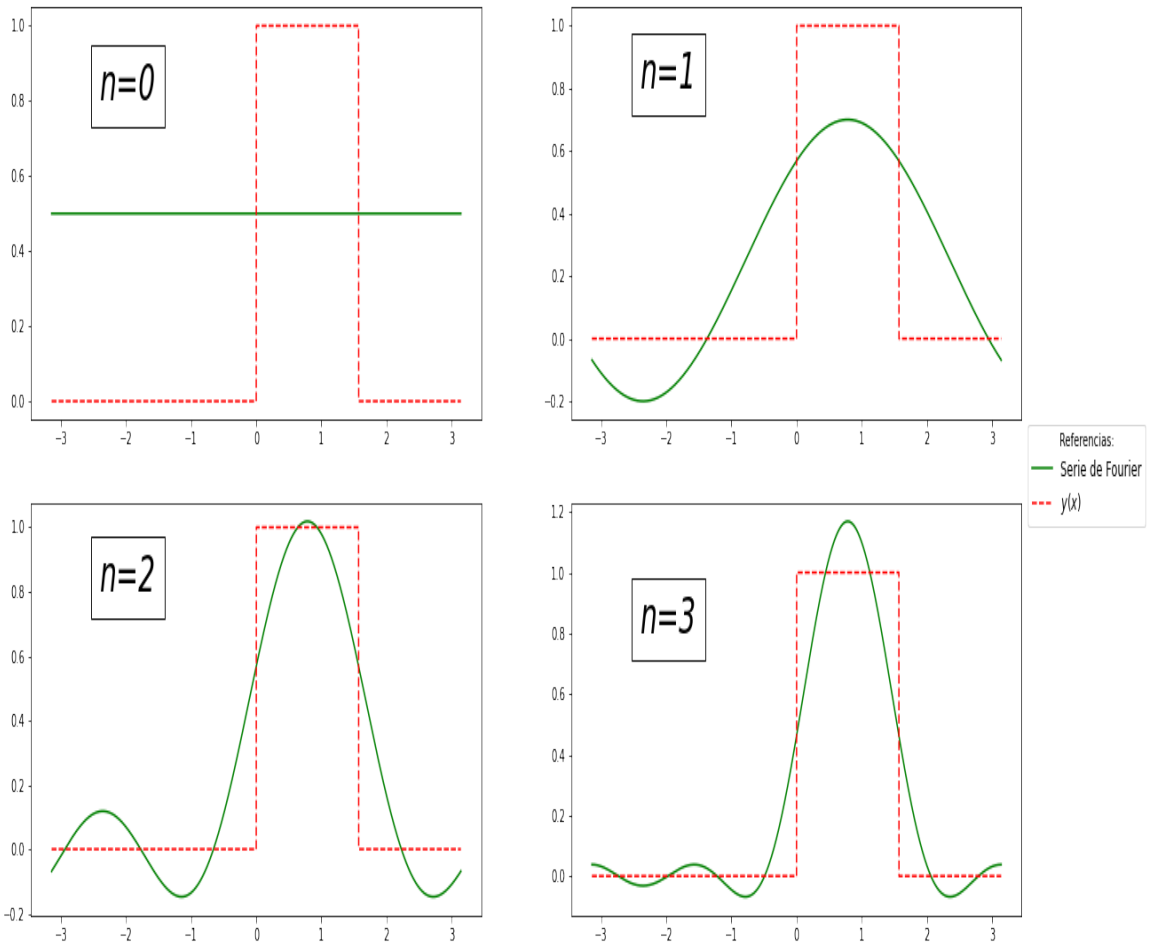
$$b_{4n-2} = -\frac{1}{\pi 2(2n-1)}\left[(-1)^{2n-1} - 1\right]$$
$$b_{4n-2} = -\frac{1}{\pi 2(2n-1)}[-1 - 1]$$
$$b_{4n-2} = \frac{1}{\pi(2n-1)}$$

E impares:
$n = 2n - 1$:

$$b_{2n-1} = -\frac{1}{(2n-1)\pi}\left[\underbrace{\cos\left((2n-1)\frac{\pi}{2}\right)}_{*} - 1\right]$$
$$b_{2n-1} = \frac{1}{\pi(2n-1)}$$

$\therefore$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{+\infty}\left[\frac{(-1)^{n+1}}{\pi(2n-1)}\cos\big((2n-1)x\big) + \frac{1}{\pi(2n-1)}\sin\big((4n-2)x\big) + \frac{1}{\pi(2n-1)}\sin\big((2n-1)x\big)\right]$$


n=0
n=1
n=2
n=3
Referencias:
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y(x)