# First order logic

## Resolution

Prove: $\forall x$ Eat$(x,$ Amanita$) \Rightarrow$ Dead$(x)$ Premise: 1. $\neg$Red$(x) \vee \neg$Mushroom$(x) \vee$ Poisonous$(x)$ 2. $\neg$Eat$(x, y) \vee \neg$Poisonous$(y) \vee$ Dead$(x)$ 3. Red(Amanita) 4. Mushroom(Amanita)

- $\forall x$ Eat$(x,$ Amanita$) \Rightarrow$ Dead$(x)$
- $\forall x$ $\neg$Eat$(x,$ Amanita$) \vee$ Dead$(x)$

set conclusion as false: - $\neg(\forall x\ \neg$Eat$(x,$ Amanita$)) \vee$Dead$(x)$ - $\exists x\ \neg\neg$Eat$(x,$ Amanita$) \wedge \neg$Dead$(x)$ - $\exists x\ Eat(x,$ Amanita$) \wedge \neg$Dead$(x)$ - Eat$(c,$ Amanita$) \wedge \neg$Dead$(c)$

5. Eat($c$, Amanita)
6. $\neg$Dead$(c)$
7. // rules 2 + 5, $\theta = \{x/c, y/\text{Amanita}\}$
   $\neg$Poisonous(Amanita) $\vee$ Dead$(c)$
8. // rules 6 + 7
   $\neg$Poisonous(Amanita)
9. // rules 1 + 8, $\theta = \{x/Amanita\}$
   $\neg$Read(Amanita) $\vee$ $\neg$Mushroom(Amanita)
10. // rules 3 + 9
    $\neg$Mushroom(Amanita)
11. // rules 4 + 10
    FALSE, contradiction

## Connection between $\forall$ and $\exists$

### Example 1

- $\forall x$ $\neg$Likes$(x,$ Brocolli$)$ // everyone dislikes brocolli
- $\neg\exists x$ Likes$(x,$ Brocolli$)$ // there does not exist someone who like brocolli

### Example 2

- $\forall x$ Likes$(x,$ icecream$)$ // everyone likes icecream
- $\neg\exists x$ $\neg$Likes$(x,$ icecream$)$ // there is no one who dislikes icecream

**Equivalence**

- $\forall x \; \neg P === \neg \exists x \; P$
- $\neg \forall x \; P === \exists x \; \neg P$
- $\forall x \; P === \neg \exists x \; \neg P$
- $\exists x \; P === \neg \forall x \; \neg P$

## Convert first order logic to CNF

- everyone who loves all animals is loved by someone:

  $\forall x \; [\forall y \; \text{Animal}(y) \Rightarrow \text{Loves}(x, y)] \Rightarrow [\exists y \; \text{Loves}(y, x)]$

1. eliminate implications

   $\forall x \; [\neg \forall y \; \neg \text{Animal}(y) \vee \text{Loves}(x, y)] \vee [\exists \; y \text{Loves}(y, x)]$

2. move $\neg$ inward (tricky)
   - $\forall x \; [\exists y \; \neg(\neg \text{Animal}(y) \vee \text{Loves}(x, y))] \vee [\exists y \; \text{Loves}(y, x)]$
   - $\forall x \; [\exists y \; \text{Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee [\exists y \; \text{Loves}(y, x)]$
3. standardize variables
   - $\forall x \; [\exists y \; \text{Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee [\exists z \; \text{Loves}(z, x)]$
   - if $(\exists x \; P(x)) \vee (\exists x \; Q(x))$ then change the name of one variable
   - in our case, we change the name of $y$ to $z$
4. skolemization
   - $\forall x \; \exists y \rightarrow$ use skolen function $F(x)$ to replace $y$
   - $\exists y \rightarrow$ use skolen constant c for $y$
   - $\forall x \; [\text{Animal}(F(x)) \wedge \neg \text{Loves}(x, F(x))] \vee [\text{Loves}(G(z), x)]$ // G(z) or G(x) ????
5. drop universal quantifiers
   - $[\text{Animal}(F(x)) \vee \neg \text{Loves}(x, F(x))] \vee \text{Loves}(G(z), x)$
   - $G(z)$ means 'someone'
6. distribute $\vee$ over $\wedge$
   - $[\text{Animal}(F(x)) \vee \text{Loves}(G(z), x)] \wedge [\neg \text{Loves}(x, F(x)) \vee \text{Loves}(G(z), x)]$