

161 Homework 5

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November 9, 2017

Problem 1

Part A

- $P \Rightarrow \neg Q$
 - $(\neg P \vee \neg Q)$
- $Q \Rightarrow \neg P$
 - $(\neg Q \vee \neg P)$

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
F	F	T	T	T	T
F	T	T	F	T	T
T	F	F	T	T	T
T	T	F	F	F	F

Part B

- $P \iff \neg Q$
 - $(P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$
 - $(\neg P \vee \neg Q) \wedge (Q \vee P)$
- $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$\neg P$	$\neg Q$	$P \iff \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
F	F	T	T	F	F
F	T	T	F	T	T
T	F	F	T	T	T
T	T	F	F	F	F

Problem 2

Part A

Let F = Fire, S = Smoke, and H = Heat.

- $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
- $S \Rightarrow F$
 - $\neg S \vee F$
- $\neg S \Rightarrow \neg F$
 - $S \vee \neg F$

S	F	$\neg S$	$\neg F$	$\neg S \vee F$	$S \vee \neg F$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
F	F	T	T	T	T	T
F	T	T	F	T	F	F **
T	F	F	T	F	T	T
T	T	F	F	T	T	T

Neither valid or unsatisfiable, relationship does not hold under all possible inputs (but is not universally unsatisfiable). See double-starred (**) for a counterexample (if the left side of an implication is true, the right side must be true as well).

Part B

- $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
- $(S \Rightarrow F)$
 - $\neg S \vee F$
- $((S \vee H) \Rightarrow F)$
 - $\neg(S \vee H) \vee F$
 - $(\neg S \wedge \neg H) \vee F$

S	F	H	$\neg S$	$\neg F$	$\neg H$	$\neg S \vee F$	$(\neg S \wedge \neg H) \vee F$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
F	F	F	T	T	T	T	T	T
F	F	T	T	T	F	T	F	F **
F	T	F	T	F	T	T	T	T
F	T	T	T	F	F	T	T	T
T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	F	F	T
T	T	F	F	F	T	T	T	T
T	T	T	F	F	F	T	T	T

Neither valid or unsatisfiable, the relationship does not hold under all possible inputs, see the double-starred (**) row.

Part C

- $((S \wedge H) \Rightarrow F) \iff ((S \Rightarrow F) \vee (H \Rightarrow F))$
 - $(\neg(S \wedge H) \vee F) \iff ((\neg S \vee F) \vee (\neg H \vee F))$
 - $((\neg S \vee \neg H) \vee F) \iff ((\neg S \vee F) \vee (\neg H \vee F))$

S	F	H	$\neg S$	$\neg F$	$\neg H$	$(\neg S \vee \neg H) \vee F$	$(\neg S \vee F) \vee (\neg H \vee F)$	$((S \wedge H) \Rightarrow F) \iff ((S \Rightarrow F) \vee (H \Rightarrow F))$
F	F	F	T	T	T	T	T	T

S	F	H	$\neg S$	$\neg F$	$\neg H$	$(\neg S \vee \neg H) \vee F$	$(\neg S \vee F) \vee (\neg H \vee F)$	$((S \wedge H) \Rightarrow F) \iff ((S \Rightarrow F) \vee (H \Rightarrow F))$
F	F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	T	T
F	T	T	T	F	F	T	T	T
T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	F	F	T
T	T	F	F	F	T	T	T	T
T	T	T	F	F	F	T	T	T

Valid, both sides of the if and only if always result in the same value.

Problem 3

Let M = Mythical, I = Immortal, H = Horned, Y = Magical, and Z = Mammal

Part 1: Knowledge Base

English	Propositional sentence
<i>If the unicorn is mythical, then it is immortal</i>	$M \Rightarrow I$
<i>but if it is not mythical, then it is a mortal mammal</i>	$\neg M \Rightarrow (\neg I \wedge Z)$
<i>If the unicorn is either a immortal or a mammal, then it is horned</i>	$(I \vee Z) \Rightarrow H$
<i>The unicorn is magical if it is horned</i>	$H \Rightarrow Y$

Part 2: Convert to CNF

Propositional sentence	Sentence	CNF
$M \Rightarrow I$	$\neg M \vee I$	$\neg M \vee I$
$\neg M \Rightarrow (\neg I \wedge Z)$	$M \vee (\neg I \wedge Z)$	$(M \vee \neg I) \wedge (M \vee Z)$
$(I \vee Z) \Rightarrow H$	$(\neg I \wedge \neg Z) \vee H$	$(\neg I \vee H) \wedge (\neg Z \vee H)$
$H \Rightarrow Y$	$\neg H \vee Y$	$\neg H \vee Y$

These rules are shared between each part of Part 3:

Rule	CNF
1	$\neg M \vee I$
2	$M \vee \neg I$
3	$M \vee Z$
4	$\neg I \vee H$
5	$\neg Z \vee H$
6	$\neg H \vee Y$

Part 3

Is the unicorn mythical?

Rule	CNF	Combining Rules
7	$\neg M$	Assume the unicorn is not mythical
8	$I \vee Z$	1, 3
9	$Z \vee H$	8, 4
10	H	9, 5
11	Y	10, 6
12	Z	7, 3

It cannot be shown from the knowledge base that the unicorn is mythical.

Is the unicorn magical?

Rule	CNF	Combining Rules
7	$\neg Y$	Assume the unicorn is not magical
8	$I \vee Z$	1, 3

Rule	CNF	Combining Rules
9	$Z \vee H$	8, 4
10	H	9, 5
11	Y	10, 6
12	FALSE	11, 7

We conclude that the unicorn is magical.

Is the unicorn horned?

Rule	CNF	Combining Rules
7	$\neg H$	Assume the unicorn is not horned
8	$I \vee Z$	1, 3
9	$Z \vee H$	8, 4
10	H	9, 5
11	FALSE	10, 7

We conclude that the unicorn is horned.