CS161: Homework 7

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Problem 1

Prove Generalized Product Rule

$$\Pr(A, B|K) = \Pr(A|B, K)\Pr(B|K)$$

To prove the generalized product rule, we use the axiom of conditional probability, which states:

$$\Pr(X|Y) = \frac{\Pr(X, Y)}{\Pr(Y)} \tag{1}$$

From this follows the less general product rule, for the intersection of two events:

$$\Pr(Y|X) = \frac{\Pr(X, Y)}{\Pr(X)}$$
$$\Pr(X, Y) = \Pr(Y|X)\Pr(X)$$

Substitute $X = A \cap B$ and $Y = K$ into (1)

$$\Pr(A, B|K) = \frac{\Pr(A, B, K)}{\Pr(K)}$$

Substitute $X = A$ and $Y = B \cap K$ into (1)

$$\Pr(A|B, K) = \frac{\Pr(A, B, K)}{\Pr(B, K)}$$

Now combine and simplify using the $\Pr(B|K) = \frac{\Pr(B,K)}{\Pr(K)}$:

$$\Pr(A, B|K) = \frac{\Pr(A|B, K)\Pr(B, K)}{\Pr(K)} = \Pr(A|B, K)\Pr(B|K)$$

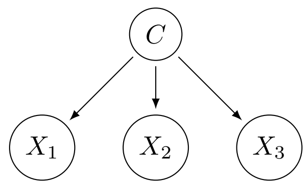
Prove Generalized Baye's Rule

$$\Pr(A|B,K) = \frac{\Pr(B|A,K)\Pr(A|K)}{\Pr(B|K)}$$

$$\begin{aligned}
\Pr(A,B) &= \Pr(A|B)\Pr(B) \\
&= \Pr(B|A)\Pr(A) \\
\Pr(B|A)\Pr(A) &= \Pr(A|B)\Pr(B) \\
\Pr(A|B) &= \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}
\end{aligned}$$

$$\begin{aligned}
\Pr(A|B,K) &= \Pr(A|B,K)\Pr(B|K) \\
\Pr(B|A,K) &= \Pr(B|A,K)\Pr(A|K) \\
\Pr(B|A,K)\Pr(A|K) &= \Pr(A|B,K)\Pr(B|K) \\
\Pr(A|B,K) &= \frac{\Pr(B|A,K)\Pr(A|K)}{\Pr(B|K)}
\end{aligned}$$

Problem 2



where $n \in \{1, 2, 3\}$, the CPTs for the coin tosses X_1, X_2 and X_3 are:

C	X_n	$\Pr(X_n \mid C)$
a	h	0.2
a	t	0.8
b	h	0.6
b	t	0.4
c	h	0.8
c	t	0.2

and the distribution for C is:

C	$\Pr(C)$
a	$0.\overline{3}$
b	$0.\overline{3}$
c	$0.\overline{3}$

Problem 3

world	N	S	C	$\Pr(N, S, C)$
1	1	square	black	2/13
2	1	square	white	1/13
3	1	circle	black	1/13
4	1	circle	white	1/13
5	2	square	black	4/13
6	2	square	white	1/13
7	2	circle	black	2/13
8	2	circle	white	1/13

- α_1: the object is black
- α_2: the object is square
- α_3: if the object is one or black, then it is also a square

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- α_1 holds in worlds 1, 3, 5 and 7 with a probability of

$$\Pr(\alpha_1) = \frac{2}{13} + \frac{1}{13} + \frac{4}{13} + \frac{2}{13} = \frac{9}{13}$$

- α_2 holds in worlds 1, 2, 5 and 6 with a probability of

$$\Pr(\alpha_2) = \frac{2}{13} + \frac{1}{13} + \frac{4}{13} + \frac{1}{13} = \frac{8}{13}$$

- α_3 holds in worlds 1, 2 and 5 with a probability of

$$\Pr(\alpha_3) = \frac{2}{13} + \frac{1}{13} + \frac{4}{13} = \frac{7}{13}$$

Problem 4

A

These should be read as: the variable (first parameter to I) is independent of its non-descendants (third parameter to I), when giving values for the variable's parents.

- $\mathrm{I}(A, \varnothing, \{B, E\})$
- $\mathrm{I}(B, \varnothing, \{A, C\})$
- $\mathrm{I}(C, A, \{D, B, E\})$
- $\mathrm{I}(D, \{A, B\}, \{C, E\})$

- $\mathrm{I}(E, B, \{A, C, D, F, G\})$
- $\mathrm{I}(F, \{C, D\}, \{A, B, E\})$
- $\mathrm{I}(G, F, \{A, B, C, D, E, H\})$
- $\mathrm{I}(H, \{F, E\}, \{A, B, C, D, G\})$

B

- d_separated(A, BH, E): false, there is a path

$$A \to C \to F \to H \leftarrow E$$

  where the collider H does not block the path because it is given.

- d_separated(G, D, E): false, there is a path

$$G \leftarrow F \leftarrow C \leftarrow A \to D \leftarrow B \to E$$

  where the collider D does not block the path because it is given.

- d_separated(AB, F, GH): false, there is a path

$$B \to E \to H$$

C

$$\Pr(a, b, c, d, e, f, g, h) = \Pr(a)\Pr(b)\Pr(c|a)\Pr(d|a, b)\Pr(e|b)\Pr(f|c, d)\Pr(g|f)\Pr(h|f, e)$$

D

1. $\Pr(A = 0, B = 0) = \Pr(A = 0)\Pr(B = 0) = (.8)(.3) = 0.24$
2.

- $\Pr(E = 1|A = 1) = \Pr(E = 1)$
- $\Pr(E = 1) = \Pr(E = 1|B = 0)\Pr(B = 0) + \Pr(E = 1|B = 1)\Pr(B = 1)$
- $\Pr(E = 1) = (0.9)(0.3) + (0.1)(0.7) = 0.27 + 0.07 = 0.34$