Homework 6

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Question 1

a) $P(A, B, B), P(x, y, z)$

$\theta = \{x/A, y/B, z/B\}$

b) $Q(y, G(A, B)), Q(G(x, x), y)$

$\perp$

c) $R(x, A, z), R(B, y, z)$

$\theta = \{x/B, y/A\}$

d) $\text{Older}(\text{Father}(y), y), \text{Older}(\text{Father}(x), \text{John})$

$\theta = \{y/\text{John}, x/\text{John}\}$

e) $\text{Knows}(\text{Father}(y), y), \text{Knows}(x, x)$

$\perp$

Question 2

1. John likes all kinds of food
2. Apples are food
3. Chicken is food
4. Anything someone eats and isn't killed by is food
5. If you are killed by something, you are not alive
6. Bill eats peanuts and is still alive *
7. Sue eats everything Bill eats

a) First order logic:

1. $\forall x$ Food$(x) \Rightarrow$ Likes(John, x)
2. Food(Apples)
3. Food(Chicken)
4. $\forall x \exists y$ Eats$(y, x) \wedge \neg$Killed$(x, y) \Rightarrow$ Food(x)
5. $\forall x \forall y$ Killed$(x, y) \Rightarrow \neg$Alive(y)
6. Eats(Bill, Peanuts) $\wedge$ Alive(Bill)
7. $\forall x$ Eats(Bill, x) $\Rightarrow$ Eats(Sue, x)

b) To CNF:

1. $\neg$Food$(x) \vee$ Likes(John, x)
2. Food(Apples)
3. Food(Chicken)
4. $\neg$Eats$(y, x) \vee$ Killed$(x, y) \vee$ Food(x)
5. $\neg$Killed$(x, y) \vee \neg$Alive(y)
6. Eats(Bill, Peanuts)
7. Alive(Bill)
8. $\neg$Eats(Bill, x) $\vee$ Eats(Sue, x)

c) Prove John likes Peanuts using resolution:

Number	Rule	Composition
9	$\neg$Likes(John, Peanuts)	negate conclusion
10	$\neg$Food(Peanuts)	rules 1, 9
11	$\neg$Eats$(y,$ Peanuts$) \vee$ Killed(Peanuts, y)	rules 4, 10
12	Killed(Peanuts, Bill)	rules 6, 11
13	$\neg$Alive(Bill)	rules 5, 12
14	$\perp$	rules 7, 14

d) Use resolution to answer the question "What does Sue eat?"

Number	Rule	Composition
9	$\neg$Eats(Sue, x)	negate conclusion
10	$\neg$Eats(Bill, x)	rules 8, 9

Number	Rule	Composition
11	$\perp, \theta = \{x/\text{Peanuts}\}$	rules 6, 10

Sue eats peanuts.

e) Use resolution to answer the question "What does Sue eat?" with a modified set of rules.

1. John likes all kinds of food
2. Apples are food
3. Chicken is food
4. Anything someone eats and isn't killed by is food
5. If you are killed by something, you are not alive
6. If you don't eat, you die
7. If you die, you are not alive
8. Bill is alive
9. Sue eats everything Bill eats

To first order logic:

1. $\forall x$ Food$(x) \Rightarrow$ Likes(John, x)
2. Food(Apples)
3. Food(Chicken)
4. $\forall x \exists y$ Eats$(y, x) \wedge \neg$Killed$(x, y) \Rightarrow$ Food(x)
5. $\forall x \forall y$ Killed$(x, y) \Rightarrow \neg$Alive(y)
6. $\forall x$ $\neg[\exists y$ Eats$(x, y)] \Rightarrow$ Dead(x)
7. $\forall x$ Dead$(x) \Rightarrow \neg$Alive(x)
8. Alive(Bill)
9. $\forall x$ Eats(Bill, x) $\Rightarrow$ Eats(Sue, x)

To CNF:

1. $\neg$Food$(x) \vee$ Likes(John, x)
2. Food(Apples)
3. Food(Chicken)
4. $\neg$Eats$(y, x) \vee$ Killed$(x, y) \vee$ Food(x)
5. $\neg$Killed$(x, y) \vee \neg$Alive(y)
6. Eats$(x, FoodEatenBy(x)) \vee$ Dead(x)
7. $\neg$Dead$(x) \vee \neg$Alive(x)
8. Alive(Bill)
9. $\neg$Eats(Bill, x) $\vee$ Eats(Sue, x)

Proof:

Number	Rule	Composition
10	$\neg$Eats(Sue, x)	negate conclusion
11	$\neg$Eats(Bill, x)	rules 10, 9

Number	Rule	Composition
12	¬Dead(Bill)	rules 7, 8
13	Eats(Bill, FoodEatenBy(Bill))	rules 12, 6

Now we're stuck, we can prove that Bill eats food because he's alive, but we can't continue because there's no rules that allow us to determine what food Bill eats.

Question 3

a) Graph 1 with 3 colors is not satisfiable.

b) Graph 1 with 4 colors is satisfiable.

c) The two SAT instances tell us that graph 1 is 4 colorable but not 3 colorable. We can use the result of the SAT solver to obtain a coloring for graph 1:

Node	Color 1	Color 2	Color 3	Color 4
1	-1	-2	-3	4
2	-5	-6	7	-8
3	-9	10	-11	-12
4	-13	-14	15	-16
5	17	-18	-19	-20
6	-21	22	-23	-24
7	25	-26	-27	-28

This table was created directly from the results of RSAT, where each literal represents giving a node a specific coloring.

More concisely, the coloring here is:

- Node 1 $\leftarrow$ Color 4
- Node 2 $\leftarrow$ Color 3
- Node 3 $\leftarrow$ Color 2
- Node 4 $\leftarrow$ Color 3
- Node 5 $\leftarrow$ Color 1
- Node 6 $\leftarrow$ Color 2
- Node 7 $\leftarrow$ Color 1

d) The minimum number of colors required to properly color graph 2 is 8.