Genetic Algorithms for the Nesting Problem in the Packing Industry

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Abstract— **The problem of parts nesting in plane areas is found in several industries, with restrictions and different objectives. Among these industries having this process they are packings and clothes industries, which use irregulars parts as concave and convex ones. A peculiarity of the packing industry is the frequent use of an only part type for each nesting process. This article uses genetic algorithms (GA) and heuristical rules to solve the nesting problem. The results are also presented and discussed.**

Index Terms— **nesting problem, genetic algorithms, packing industry.**

I. Introduction

The nesting problem in the packing industry can be stated as the attainment of the maximum number of packages in an arrangement in a paper sheet of known size, with the least loss possible of material. The objective of this process is the optimization of production by means of a minimization of production costs. Fig. 1 shows an example of six boxes that will be drawn from a standard paper sheet.

An important factor in the search of an optimal solution for this problem is the number of parts that will be manipulated in the mounting settle. There is a combinatorial explosion as the number of parts increases, leading to unfeasible computational costs. For actual problems, the number of parts is usually not larger than 20.

Genetic Algorithms [10] have been used successfully in the last decades for several complex combinatorial problems and also for problems similar to the above-mentioned one [5], [12]. Therefore, the objective of this work is to propose a new method using genetic algorithms (GA) and heuristical rules to solve the problem in actual situations.

II. Previous Work

The nesting/cutting problem of parts in plane stock has been widely studied in recent literature, considering it is a common problem faced in several industries, like packing, plating, clothing, furniture, and other ones. According to [7] such kind of problem can be divided into two main groups, according to how the parts of material are shaped: the first considers only rectangular-like parts and the second, irregular-shaped parts.

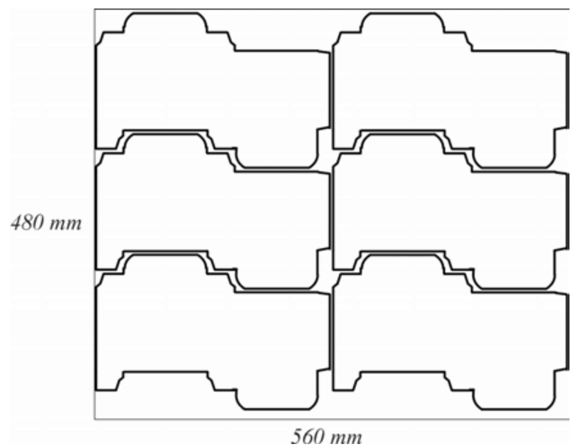

FIGURE 1 - Arrangement of six packages in a paper sheet.

Probably, [8], [9] have been the precursors of this area. They used rectangular parts and tackled the problem using linear programming techniques. They also have succeeded in working with problems in 1, 2 or 3 dimensions.

The problem was studied for [11] without the restriction of the number of parts to be cut in a sheet of a given material. The method they have developed consists in obtaining a rectangle that encloses one or more irregular parts using the smallest possible area. Such rectangle was called a module. Modules were then grouped in a sheet of material by means of dynamic programming. This algorithm requested that the rectangular module be positioned in one of the corners of the sheet. Later, [1] proposed an improvement in this algorithm, eliminating such limitation. This algorithm was used in the naval construction industry.

Concerning irregular parts that are a much more complex problem, [2] proposed a technique that use heuristic search methods. Also using heuristic methods, [14] has proposed an algorithm for the nesting problem in the steel plating industry.

The first use of Genetic Algorithms (GA) for this problem is the work of [13]. Since then, other authors have followed ahead, like [6] who presented a hybrid approach using both a GA and a local minimization algorithm. The method presented for [3] is based on GA where the combination of parts is represented in a tree.

More recently [18] have described a new approach for the attainment of optimized arrangements of packings. The solution is based on a hybrid system that uses parallel genetic algorithms and a heuristic process based on the combination of contouring characteristics. Some topologies for the communication between the subpopulations as well as several migration techniques are tested in the experiments. The simulations demonstrate that the proposed approach generates good results in this type of problem and in others with wide search domain.

In the work of [4] some approaches for optimized arrangements layouts have been applied. Irregular flat shapes (convex and concave) were used. Amongst the techniques that have been used, it was found the genetic algorithms technique. However, the good results obtained in the work can be improved with human aid.

III. PROPOSED METHODOLOGY

The proposed methodology for the attainment of packing arrangements layout in a sheet of paper will be presented in the next sections. The method consists of a sequence of steps: the packing representation, the packing operations, the heuristic search, the search space encoding schemes and the formulation of the cost function.

A. Packing representation

The implementation of the algorithm for layout optimization requires the creation of the packaging design, called E, based on an actual package (Fig. 2(a)). This model is composed for a set of rectangles with given positions (P) and dimensions (D). The *i* index represents each one of the rectangles (Fig. 2(b)).

$$
\begin{aligned}
E &= \{P,D\} \\
P &= (x_i, y_i) \\
D &= (l_i, h_i)
\end{aligned} \tag{1}
$$

Given that:

xi = Horizontal coordinate of the origin of the i rectangle referenced at its left corner.

y_i = Vertical coordinate of the origin of the *i* rectangle referenced at its lower corner.

l_i = Horizontal dimension of the *i* rectangle.

h_i = Vertical dimension of the *i* rectangle.

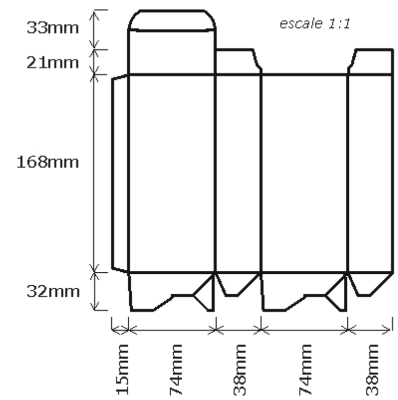

FIGURE 2(a) – Packing sample.

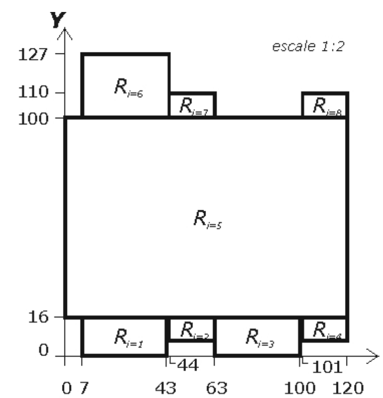

FIGURE 2(b) – Packing sample model represented by rectangles.

The codification of the packing sample model is represented in the Fig. 2(b) as follows:

$$
\begin{aligned}
&P = \{(x_1 = 7, y_1 = 0), (x_2 = 44, y_2 = 8), (x_3 = 63, y_3 = 0), (x_4 = 101, y_4 = 8), \\
&(x_5 = 0, y_5 = 16), (x_6 = 7, y_6 = 100), (x_7 = 44, y_7 = 100), (x_8 = 100, y_8 = 100\}\} \\
&D = \{(l_1 = 36, h_1 = 16), (l_2 = 19, h_2 = 8), (l_3 = 37, h_3 = 16), (l_4 = 19, h_4 = 8), \\
&(l_5 = 120, h_5 = 84), (l_6 = 36, h_6 = 27), (l_7 = 19, h_7 = 10), (l_8 = 19, h_8 = 10\}\})
\end{aligned}
$$

B. Basic packing operations

In this section the operations of rotation and displacement will be defined. The objective of the actions is to allow the movement of the packings inside the search space to determine a disposal with the least loss possible of material and without overlapping each other.

1) Rotation of the Packings

Practical cases of the packaging industry have demonstrated that, most of the time, the rotation of the packings has occurred in accordance with the angles of 0^0, 90^0, 180^0 and 270^0. In this work each one of these angles is represented by ϕ.

2) Displacement of the Packings

The attainment of the packings arrangement requires also the displacement. This occurs with the addition of a value x_a and y_a to all the x_i horizontal coordinates and the y_i vertical ones that define the rectangles disposal. The new set of coordinates, which will define the new disposal of the packing, will be called P_{new} and is presented as in (2).

$$Pnew = \{(x_a + x_i, y_a + y_i)\} \tag{2}$$

Given that :
xa = horizontal displacement.
ya = vertical displacement.

C. Heuristic search

To improve the GA performance it is suggested the creation of a heuristic approach composed by a sequence of rules, defined by experts, which will guide the movement of the packings. The quality of the obtained results are not limited by these rules, on the contrary, they allow the organization of the packings in the disposal and also reduce the algorithm's search space. The rules are presented below:

1. All the packings are lined up in columns.
2. Rotation is applied to all the packings from the same column.
3. First column has its horizontal origin coordinate lined up with the origin of the search space.
4. Each one of the columns can move to the left within the horizontal search region (R_x), except the first column that cannot move horizontally.
5. The horizontal origin coordinate of each column is based on the origin of the column that is at its left.
6. Each one of the columns can be displaced within the vertical search region (R_y) above the horizontal axis.
7. The packings of the columns can be displaced among themselves, within a 'between boxes' search region (R_{ec}).

The sizes of the R_x, R_y *e* R_{ec} regions should be defined to allow the application of the present sequence of rules. The values which will be composed by integer numbers, are obtained from the value of M_{me}, that is the larger packing measure, considering its height and width.

$$R_{xk} = [0..M_{me}] \tag{3}$$

$$R_{yk} = [0..M_{me}] \tag{4}$$

$$R_{eck} = [0..2.M_{me}] \tag{5}$$

$$\phi_k = [0..3] \tag{6}$$

Given that:
R_{xk} = Horizontal search region.
R_{yk} = Vertical search region.
R_{eck} = Between-boxes search regions.
ϕ_k = Rotation angle (represented by integer numbers).
k = Studied column rate.

Table I presents an example of R_{xk}, R_{yk} *e* R_{eck} values based on the packing sample model (Fig. 2(a), 2(b)) whose M_{me} value is 127. In this case, the layout has four columns, as presented in the next section.

D. Search space encoding schemes

As the packing form of representation and operations had been defined, it is required to encode the problem through the definition of the chromosome struture. First, the number of columns *(k)* that will compose the arrangement should be calculated. The genes that constitute the chromosome will be defined with these values. Each gene has a specific function in the codification of the arrangement and can present values according to the R_{xk}, R_{yk}, R_{eck} and ϕ_k previously defined limits.

It is important do not forget to estimate the number of packings in each column *(NEC)*. This number must be sufficient to fulfill the sheet in the vertical sense. The sheet's horizontal dimension is *TFX* and the vertical dimension is *TFY*, so the packing disposal must be contained in these dimensions. When the dimensions are defined, they must accord with the adopted scale, which is 1:2.

$$Chromosome = x_1\ y_1\ ec_1\ \phi_1 \quad \ldots \quad x_k\ y_k\ ec_k\ \phi_k \tag{7}$$

TABLE I – SEARCH REGIONS SPECIFICATION OF SIZE FOR THE MODEL IN THE FIG. 2(A) AND 2(B).

Horizontal Search Regions	Vertical Search Regions
$R_{x1} = 0$	$R_{y1} = [0..127]$
$R_{x2} = [0..127]$	$R_{y2} = [0..127]$
$R_{x3} = [0..127]$	$R_{y3} = [0..127]$
$R_{x4} = [0..127]$	$R_{y4} = [0..127]$
'Between Boxes' Search Regions	Possible Rotation Angles
$R_{ec1} = [0..254]$	$\phi_1 = [0..3]$
$R_{ec2} = [0..254]$	$\phi_2 = [0..3]$
$R_{ec3} = [0..254]$	$\phi_3 = [0..3]$
$R_{ec4} = [0..254]$	$\phi_4 = [0..3]$

Following, there is an example based on the model presented in Fig. 2(b). The packing layout is represented by the pieces of information contained in the chromosome as in (7), which are interpreted, and then a graphic model is generated (Fig. 3).

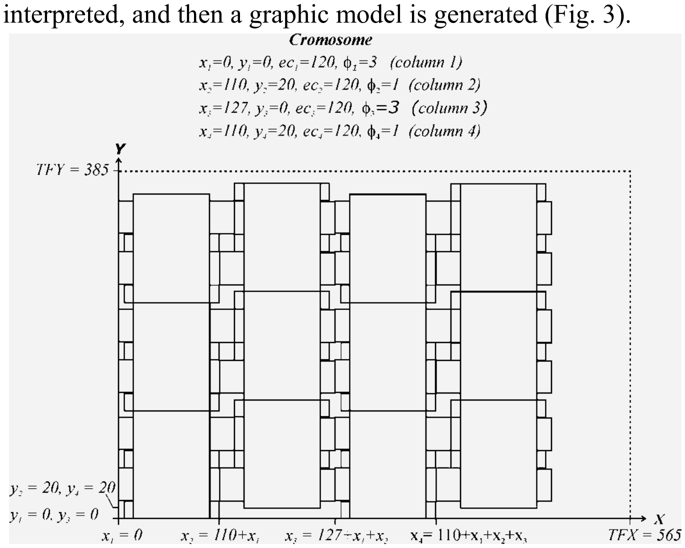

FIGURE 3 – Interpretation of the information contained in the chromosome example and its graphical representation.

E. Cost funtion formulation of the problem

To evaluate the quality of a solution it is necessary a pointer that measure this quality. For that is used the *fitness* $f(s^1)$ function. All the tested layouts have its *fitness* values compared and the solution with the best performance is chosen. According to [16] the *fitness* function is composed by two terms, the first being the total area index *IA(s)* that shows the total area taken up by the arrangement, and the second being an index of packing overlapping *IS(s)* which determine the overllaping between the packages. The objective is to determine an arrangement with the least total area index *IA(s),* and without overlapping between packings or *IS(s)=0*.

To improve the performance of the algorithm some studies apply techniques that modify dynamically the *fitness* function during the search [15], [17], [19]. The variation through the term *IVD(s),* which represents the index of dynamic variation, was chosen in this work. The attainment of the *IVD(s)* equation was accomplished in experimental form and is described below.

$$IVD(s) = k_1 + k_2.s \qquad (8)$$

Given that :
k_1 = Constant one.
k_2 = Constant two.
s = Solution that is being evaluated.

$$f(s) = 1 - \left(\frac{IA(s)^{IVD(s)} + IS(s)^{IVD(s)}}{2} \right) \qquad (9)$$

IV. RESULTS

The proposed method was validated through the comparison with the results gotten by the expert's solutions. As three types of packings frequently found in the industry had been used in the tests, the simulations represent real situations 120 evaluations with different populations and generations had been carried through for each one of the tested packings. The index used to compare the performance of each arrangement was the *paper area/packing*. Solution with a performance equal or superior to the best arrangements gotten by the experts had been called successful solution and the others failed solutions. Finally, it was measured the time of processing for each one of the tests.

A. Packing 1 results

Before the beginning of the simulations, the experts have gotten arrangements with **725** cm^2/pac for the packing 1. The result of one of the algorithm tests, with a population of 200 individuals, 600 generations and a random seed equal to 0.125, is shown in Fig. 4.

In Table II a summary of the results of the simulations is presented. For each one of the generations 10 tests with distinct random seeds had been carried through. Each one of the 10 processings had its time measured and the average time is presented.

[1] the variable "*s*" represents the solution that is being evaluated.

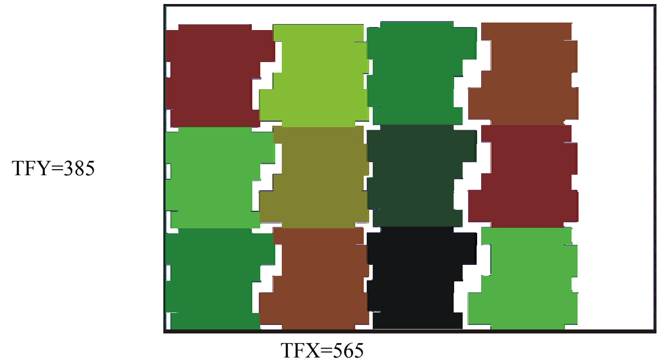

FIGURE 4 – Example of arrangement generated by genetic algorithm.

TABLE II – SUMMARY OF THE RESULTS OF SIMULATIONS WITH PACKING 1.

Populations	Generations	Successful Solutions (%)	Average Time (s)
200	200	20%	89
	400	20%	178
	600	40%	266
	800	70%	356
400	200	20%	177
	400	10%	353
	600	70%	530
	800	40%	710
800	200	30%	353
	400	70%	706
	600	50%	1070
	800	80%	1414

B. Packing 2 results

For packing 2 the lesser *paper area/packing* that experts have gotten was **544** cm^2/pac. The result of one of the algorithm tests, with a population of 200 individuals, 400 generations and a random seed equal to 0.925, is shown in Fig. 5.

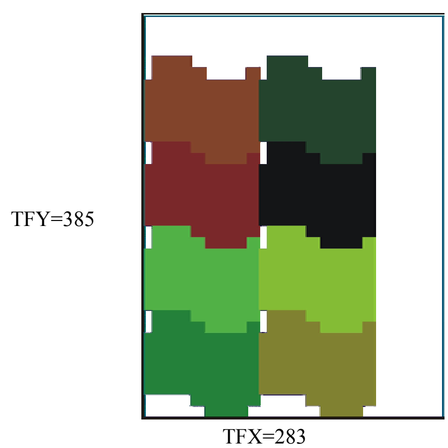

FIGURE 5 – Example of arrangement generated by genetic algorithm.

In Table III a summary of the results of the simulations is presented. For each one of the generations 10 tests with distinct random seeds had been carried through. Each one of the 10 processings had its time measured and the average time is presented.

TABLE III – SUMMARY OF THE RESULTS OF SIMULATIONS WITH PACKING 2.

Populations	Generations	Successful Solutions (%)	Average Time (s)
200	200	30%	26
	400	50%	52
	600	50%	78
	800	70%	103
400	200	30%	51
	400	80%	103
	600	60%	155
	800	100%	207
800	200	80%	103
	400	100%	205
	600	100%	309
	800	100%	412

C. *Packing 3 results*

For packing 3 the lesser *paper area/packing* that experts have gotten was **1452** cm^2/pac. The result of one of the algorithm tests, with a population of 400 individuals, 400 generations and a random seed equal to 0.025, is shown in Fig. 6.

TFY=385

TFX=565

FIGURE 6 – Example of arrangement generated by genetic algorithm.

In Table IV a summary of the results of the simulations is presented. For each one of the generations 10 tests with distinct random seeds had been carried through. Each one of the 10 processings had its time measured and the average time is presented.

V. FINAL CONSIDERATIONS

The methodology presented in this paper proved to be feasible by the good results obtained. Preliminary tests had demonstrated the possibility of development of an aid program to the attainment of packing arrangements using genetic algorithms (GA).

TABLE IV – SUMMARY OF THE RESULTS OF SIMULATIONS WITH PACKING 3.

Populations	Generations	Successful Solutions (%)	Average Time (s)
200	200	10%	17
	400	10%	32
	600	60%	48
	800	50%	64
400	200	20%	33
	400	20%	65
	600	40%	97
	800	40%	129
800	200	0%	65
	400	40%	130
	600	20%	194
	800	20%	259

The GA associated with the knowledge of the experts, by using heuristical rules, was essencial in the development of this new method. When the generated results were evaluated they revealed to be better than those obtained by the experts, most of the time (Table V). Another advantage of the proposed method was the considerable reduction of time for the attainment of arrangements (Table V).

TABLE V – COMPARISON BETWEEN THE EXPERTS AND THE GA RESULTS.

		Packing 1	Packing 2	Packing 3
GA	Number of Generated Arrangements	120	120	120
	Number os Successful Solutions	52	85	33
	Successful Solutions	43 %	71 %	28 %
Experts	Number of Generated Arrangements	12	9	9
	Number os Successful Solutions	4	5	3
	Successful Solutions	33 %	56 %	33 %
Percentage advantage of successful solutions through GA over the experts		**10 %**	**15 %**	**-5 %**

Some alterations in the methodology are proposed as future studies, in order to permit more flexibility to the layouts and to allow the use of packings with different shapes in the same arrangement. This option is not used frequently in the industry, although it represents a group that should not be disregarded and deserves special attention.

TABLE VI – COMPARISON OF TIME BETWEEN THE EXPERTS AND THE GA RESULTS.

		Packing 1	Packing 2	Packing 3
GA	Average time per arrangement	517s	150s	94s
Expert s	Average time per arrangement	690s	410s	993s
Percentage advantage of time for arrangements generation through GA over the experts		**33%**	**173%**	**956%**

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