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Sovereign Debt Sustainability in Developed Countries : a General Concept or a Country-Specific Notion?

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Introduction

The increase in public debt across developed economies over recent decades has raised concerns among economists about whether such debt levels are sustainable, and whether a threshold exists beyond which sustainability breaks down. For instance, in the Organisation for Economic Co-operation and Development (OECD), the average gross public debt-to-gross domestic product (GDP) ratio rose from approximately 60% in the early 1990s to over 120% following the COVID-19 pandemic.

Although the issue has attracted renewed attention in recent years, concerns about debt sustainability are not new. As early as the 19th century, David Ricardo expressed concerns about the debt burden in post-Napoleonic Britain, and Evsey Domar, in the 1940s, laid out the first formal framework for analyzing long-term debt dynamics.

To respond to these concerns, modern economists have sought to establish criteria and conditions under which public debt can be considered sustainable. Yet, despite abundant literature and institutional tools, evaluating sovereign debt sustainability remains particularly complex for advanced economies. While some countries have maintained market access with debt ratios well above traditional thresholds, others have experienced abrupt crises at much lower levels of indebtedness. This heterogeneity raises a fundamental question: can a universal definition of debt sustainability apply across countries, or is the concept inherently country-dependent?

This dissertation aims to explore how sovereign debt sustainability can be defined and assessed in developed economies, and to examine the reasons why common criteria may fail to capture country-specific vulnerabilities. We first review the main theoretical and empirical approaches to sustainability, focusing on both academic and institutional approaches. In the second part, we analyze the concept of fiscal space as a complement to sustainability, highlighting how it allows for a more dynamic, forward-looking interpretation of a government's capacity to absorb shocks. This discussion is complemented by an empirical review of how ageing pressures could affect public finances in some euro area countries.

1. The Concept of Sovereign Debt Sustainability

1.1. Sovereign Debt in Advanced Economies : Specific Features

Certain features unique to developed-country governments are critical to understanding their debt sustainability.

First, unlike private agents such as households or firms, a government in a developed economy is considered **eternal**. This fundamental difference has important implications for debt management, as it allows the government to adopt a **longer time horizon** and, crucially, to **roll over debt indefinitely**.
Second, the government has institutional and legal mechanisms to determine its future income in a more direct and stable way than private agents, through its **sovereign power to raise future taxes**. However, this power is not unlimited: tax increases can provoke resistance, leading to constitutional or legislative constraints on taxation power [1](Brennan and Buchanan, 1980).
Third, most developed nations possess a **strong historical track record** in repaying their debt. This is especially true for the period after World War II as political stability enabled governments to repay their debt, except a few notable exceptions such as postwar Germany (1953), Greece (2012), or Iceland (2008).
Fourth, economically advanced countries benefit from **robust domestic financial markets** and **strong institutional frameworks,** all of which support stable and reliable access to funding.

These unique features imply that government debt of developed countries is often considered as **risk-free** by lenders. For instance, U.S. Treasuries are considered the benchmark for risk-free rates in dollar-denominated debt, while German Bunds (or more recently EU-Bonds issued by the European Commission) serve as the equivalent benchmark for euro-denominated debt. Other European countries with higher perceived risk than Germany face a sovereign yield **spread** over the Bund benchmark.

[1] A good example of such limitation is the "Tax Revolt" movement that began in the 1970s in the United States. Voters concerned about rising tax levels began imposing new statutory and constitutional limits on state and local government revenues and expenditures.

The perception of government debt as risk-free brings clear benefits. It reduces borrowing costs and enhances the liquidity premium attached to sovereign bonds, a key factor in their pricing, as discussed later.

In this paper, we assume that developed economies typically issue debt in their own currencies (e.g., Germany borrowing exclusively in euros, or the U.S. in dollars). While this assumption simplifies the analysis, it slightly departs from reality, as some advanced economies do issue a small share of their debt in foreign currencies (e.g., France issues a minor proportion of its debt in U.S. dollars). This deviation is minor and does not alter the analytical validity of our framework.

This capacity to borrow in domestic currency is a key advantage over emerging and developing markets that often resort to borrowing in foreign currencies, predominantly US dollars. This practice exposes them to **exchange rate risk** and makes their ability to service debt **dependent on the volume and value of their foreign-currency export earnings.**

1.2. Theoretical Foundations of Sovereign Debt Sustainability

We examine debt sustainability on **two distinct levels.**

First, we present what we refer to as the **traditional approach**, which defines sustainability from a theoretical standpoint, primarily through the lens of the government's intertemporal solvency.

Second, we explore two **practical frameworks** used to assess a country's debt sustainability. The first, developed by economist **Henning Bohn** evaluates sustainability based on the historical response of the primary balance to increases in the debt-to-GDP ratio.

The second, employed by international institutions such as the **International Monetary Fund (IMF)**, or the **European Commission**, focuses on the projected convergence of the debt-to-GDP ratio toward a stable level and compliance with key liquidity conditions; here, we concentrate on the IMF's Debt Sustainability Framework (DSF).

1.2.1. The Traditional Approach: Intertemporal Solvency and the Budget Constraint

The traditional approach is mainly based on the Intertemporal Government Budget Constraint (IGBC) which determines the level of primary surpluses a government must generate, given its initial stock of debt, to ensure fiscal sustainability.

This approach is constructed around a key premise that the economy is **dynamically efficient**, meaning that, on average (not necessarily at each period), the government borrowing interest rate *r* exceeds the GDP growth rate *g*. When the traditional debt sustainability literature took off in the 1980s, this condition $r > g$ prevailed in most countries, although not in all, making it the relevant case for analyzing debt sustainability. We will return to this assumption later, as **it carries important implications when applying the theory to real-world situations.**

For simplicity, we assume in the following derivations that the government cannot monetize its debt.

A central element of debt sustainability analysis is the government's flow budget constraint at time *t*:

$$T_t + D_t - D_{t-1} = G_t + r_t D_{t-1} + OT_t \tag{1}$$

where T_t denotes government revenues, G_t government primary spending, D_t the stock of debt, r_t the government borrowing rate, and OT_t other financial flows[2]. Assuming $OT_t = 0$, the equation simplifies to:

$$D_t = (1 + r_t)D_{t-1} - PB_t \tag{2}$$

where $PB_t = T_t - G_t$ defines the government primary balance.
One common interpretation is to consider D_t as the market value of government debt[3], and r_t as the holding return, which includes both interest payments (coupons) and any capital gains or losses due to price fluctuations.

In institutional settings, public debt is typically expressed as a share of GDP, which is considered a proxy for the government's fiscal capacity. In this context, we can rewrite equation (*2*) in terms of the debt-to-GDP ratio (d_t) and the primary balance-to-GDP ratio (pb_t)[4]:

$$d_t = \frac{(1 + r_t)}{(1 + g_t)} d_{t-1} - pb_t \tag{3}$$

---

[2] E.g., asset purchases and expenditures not included in G_t like bank recapitalization or guaranteed state enterprise debt.
[3] Alternatively, D_t may be interpreted as the face value of short-term debt maturing within one period, with r_t representing the associated contractual interest rate.
[4] For a full derivation, see Appendix 1.

By iterating equation (3) forward n periods and rearranging terms, the debt-to-GDP ratio at time t can be expressed as[5]:

$$d_t = f_{t,t+n}d_{t+n} + \sum_{j=1}^{n} f_{t,t+j}pb_{t+j} \tag{4}$$

Where the growth-adjusted discount factor ($f_{t,t+n}$) to be applied between period t and $t+n$ is defined as :

$$f_{t,t+n} = \prod_{b=t}^{t+n-1} \frac{1+g_b}{1+r_b}$$

From Equation (4), we find that today's debt must equal the discounted value of terminal debt (i.e., $f_{t,t+n}d_{t+n}$) plus the discounted sum of future primary surpluses between $t+1$ and $t+n$ (i.e., $\sum_{j=1}^{n} f_{t,t+j}pb_{t+j}$).

At this stage, new conditions are added to consider that a debt is sustainable and operationalize the equation above. However, there are two possible ways.

The first one, proposed by Hamilton and Flavin (1986), is based on the **transversality condition**, which requires that $\lim_{n\to\infty} f_{t,t+n}\, d_{t+n} = 0$. This condition is a more restrictive version of the **Non-Ponzi Game Condition** which stipulates that $\lim_{n\to\infty} f_{t,t+n}\, d_{t+n} \leq 0$ and prevents government from accumulating infinite debt by borrowing to pay off existing debt.

The problem is that, under $r > g$, the debt ratio d_t can diverge towards infinity, yet the transversality condition may still hold if the discount factor $f_{t,t+n}$ converges to zero more rapidly. However, this outcome could be considered unrealistic: if the debt ratio were to grow without bound, the government's ability to service its obligations would eventually collapse under the weight of excessive taxation demands. That is why Kremers (1988, 1989) defines fiscal sustainability not in terms of the transversality condition, but as the ability to prevent the debt-to-GDP ratio from exploding over time, that is $\lim_{n\to\infty} d_{t+n} = \bar{d}$, where $\bar{d}$ is a finite constant.

[5] For a full derivation, see Appendix 2

Regardless of which condition is adopted, under an environment $r > g$[6], the discounted value of terminal debt disappears in Equation (4), yielding Equation (5), which corresponds to the intertemporal solvency condition, also known as the Intertemporal Government Budget Constraint (IGBC) :

$$d_t = \sum_{j=1}^{n} f_{t,t+j} pb_{t+j} \tag{5}$$

Besides the above mentioned conditions, a sovereign debt is considered sustainable if the IGBC is respected that is the current debt-to-GDP ratio must equal the discounted sum of future primary surpluses relative to GDP. We deduce from that IGBC **that, starting from an initial positive debt-to-GDP ratio ($\boldsymbol{d_t > 0}$), it is impossible for a government to run a permanent deficit relative to GDP (i.e., $\boldsymbol{pb < 0}$ at each period) without being in a situation of unsustainability.**

1.2.2. Limits of the Traditional Approach

In the 1990s, concerns about U.S. debt sustainability intensified as the federal government ran several consecutive years of substantial primary deficits, a pattern that, under the IGBC framework, is incompatible with long-run debt stabilization unless it is offset by credible expectations of future fiscal adjustment. Although it was theoretically possible to argue that the government would eventually adjust its fiscal policy to generate surpluses, this adjustment path remained unverifiable ex ante. Moreover, this pattern of sustained primary deficits was not unprecedented: between 1916 and 1995, the United States recorded an average primary deficit of -1.2% of GDP (Bohn, 1998), suggesting that such fiscal behavior had characterized much of the twentieth century.

This pattern is further explored by Hakkio and Rush (1991), who examined whether U.S. fiscal behavior over 1950-1988 was consistent with the IGBC. While they did find evidence of cointegration between revenues and expenditures, the estimated elasticity was well below one: for each additional dollar of per capita government spending, revenues increased by only 68 cents. This implies that spending systematically outpaced revenues, violating the condition required for debt stabilization. The authors interpret this as a breach of the IGBC, whereby

[6] If we assume constant average r and g and $r > g$, then $\lim_{n\to\infty} f_{t,t+n} = 0$. Since $\lim_{n\to\infty} f_{t,t+n} = 0$ and $\lim_{n\to\infty} d_{t+n} = \bar{d}$ (finite), it follows that $\lim_{n\to\infty} f_{t,t+n}\, d_{t+n} = 0$. The discounted value of terminal debt disappears in the second approach and, by definition, in the first approach.

deficits were effectively financed through continual debt issuance, a dynamic akin to a "Ponzi-type" scheme.
Yet, despite this apparent violation, U.S. debt did not trigger any particular market distress during the period. This paradox invites a closer examination of the reasons why the IGBC may not serve as a reliable guide for assessing debt sustainability in practice.

First, from a practical standpoint, verifying whether the IGBC is satisfied **requires anticipating the entire future path of primary surpluses, as well as the future evolution of interest rates and economic growth.** Such information is, by nature, **unavailable and highly uncertain**, making empirical validation of the IGBC extremely difficult.

Second, when the economic growth rate exceeds the interest rate ($r < g$)[7], that is the economy is deemed dynamically inefficient, the IGBC in its classical form becomes irrelevant.
Indeed, since $r < g$ implies that the discount factor diverges, i.e., $\lim_{n\to\infty} f_{t,t+n} = +\infty$, the transversality condition ($\lim_{n\to\infty} f_{t,t+n}\, d_{t+n} = 0$) cannot hold, unless a **very restrictive assumption** is imposed: namely, that the debt-to-GDP ratio converges to zero at infinity (i.e., $\lim_{n\to\infty} d_{t+n} = \bar{d} = 0$) at a **speed faster than the divergence of the discount factor ($f_{t,t+n}$).**
Only under this additional very restrictive assumption can the IGBC be recovered in its classical form ($d_t = \sum_{j=1}^{n} f_{t,t+j} pb_{t+j}$)[8].

But two problems arise.
First, from a practical perspective, it is impossible to verify empirically the **very restrictive assumption.**
Second, inherent saving characteristics of developed economies raise reasonable reasons to think that $\lim_{n\to\infty} d_{t+n} = \bar{d} > 0$. Brender, Pisani, and Gagna (2021), showed that households, being the savers in developed economies, want to transfer their wealth through time. They do so often by saving significantly during their working years to prepare for retirement or future healthcare expenses. These savings are typically invested in debt instruments. Even in retirement, households tend to decumulate their savings more slowly than expected, often due

[7] We consider here that g and r are constant.
[8] Without that assumption, we cannot eliminate the term ($f_{t,t+n} d_{t+n}$.) that allowed us to find the IGBC.

to overestimating their life expectancy. As a result, the demand for savings remains high throughout all stages of life.

But in the economy, one agent's assets are necessarily another agent's liabilities. Thus, in the developed economies, households' ability to save depends on the willingness of another entity to borrow. Since it is observed that corporations issue limited amounts of debt, the only remaining borrower, or **the "borrower of last resort", is often the government.** As long as households continue to save rather than fully decumulate and their wealth remains positive, **there is no reason to assume that public debt has to decline**. In other words, if household accumulated net savings persist and corporate borrowing remains limited, public debt will naturally remain a counterpart to this wealth, making it possibly unnecessary that the transversality condition will hold in practice (i.e., $\lim_{n\to\infty} d_{t+n} > 0$ could possibly hold at infinity, and thus $\lim_{n\to\infty} f_{t,t+n}\, d_{t+n} = +\infty$ under $r < g$).

Furthermore, to assert that $\lim_{n\to\infty} d_{t+n} = \bar{d} = 0$ would imply that, at infinity, all households are deceased and the government has fully repaid its debt, an assumption that is questionable, especially given our premise that the government is eternal.

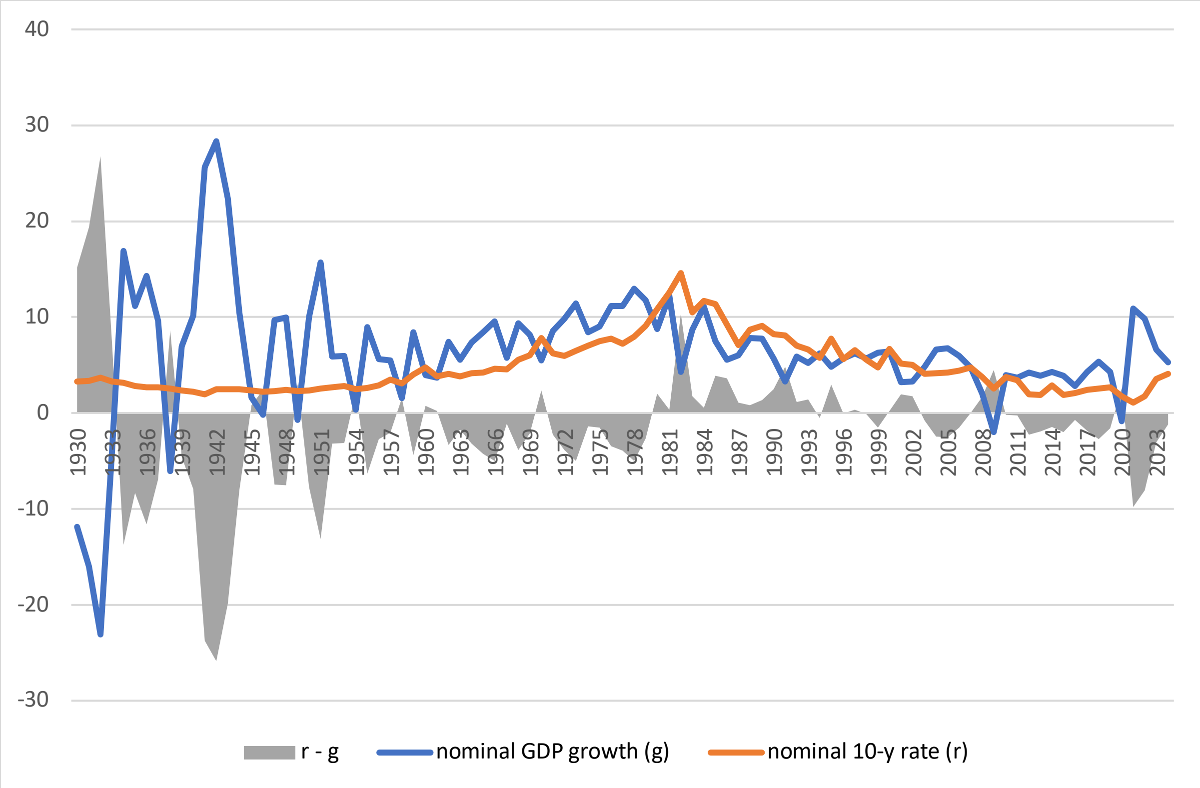
In light of these structural features of advanced economies, assuming that $\lim_{n\to\infty} d_{t+n} > 0$ appears both realistic and economically coherent. Yet under such an assumption, the IGBC in its standard form cannot be derived when $r < g$, since the transversality condition fails to hold.

Yet, one might question whether it is realistic to assume that $r < g$ (i.e., dynamic inefficiency) holds in developed economies. Bohn (1995)[9], and later Blanchard (2019) have both pointed out that the government borrowing rate has often been lower than the GDP growth rate on average, not only in the U.S., but also in many advanced economies, even though r may occasionally exceed g.

Figure 1 shows shows the evolution of the U.S. nominal GDP growth rate and the 10-year Treasury bond rate. On average, over the period 1930-2024, 10-year nominal interest rates have been lower than the nominal growth rate ; $r - g$ averaged -1.54%.

Figure 1 : U.S. Nominal GDP Growth Rate and 10-year Bond Rate, 1930–2024

[9] In the case of the U.S. only, not for other developed countries.



Source : *FRED (nominal GDP), R.Schiller (10-y rate)*

This suggests that in the context of advanced economies, where $r < g$ may prevail[10], the traditional interpretation of the IGBC becomes fragile, if not misleading, and should be applied with caution.

1.3. Beyond the IGBC: Bohn's Empirical Perspective on Debt Sustainability

Henning Bohn significantly reshaped the academic discussion on debt sustainability with two key papers in the mid-1990s. His 1995 article reinterprets the notion of the IGBC in a uncertain environment and offers valuable insights into how governments can satisfy intertemporal solvency even while running persistent primary deficits. His 1998 paper, the most widely cited empirical paper on debt sustainability, introduces a practical method for assessing fiscal sustainability.

[10] As Bohn notes, the observation that the safe interest rate is below the growth rate *(r < g)* does not, by itself, imply dynamic inefficiency. Building on Abel et al. (1989), he argues that the relevant criterion is whether the marginal product of capital (a risky return) exceeds *g*. Using this definition, Bohn finds that the U.S. economy in the 1990s was dynamically efficient, that is, the risky return exceeded the growth rate, even though the safe rate remained below it. See Bohn (1995).

1.3.1. Bohn (1995) : Rethinking the IGBC Under Uncertainty

Building on concerns about the empirical relevance of the dynamic efficiency condition (as formulated under the traditional approach), Bohn explained in his 1995 paper that the traditional approach overlooked critical factors like uncertainty and risk aversion, which are essential for understanding debt sustainability.

He notably challenged the relevance of the discount factor used in the IGBC. In a stochastic setting, agents do not discount future fiscal variables using the government borrowing rate. Instead, they use a discount factor aligned with their marginal rate of substitution between present and expected future consumption. This has important implications because if the discount factor used is irrelevant, the transversality condition as well as the IGBC are both irrelevant.

To address these inconsistencies, and **adapt or "bring closer" the traditional approach to reality, Bohn proposed a new framework.**

First, assuming a representative consumer/investor with a time-separable utility function and a constant discount factor $\beta \in (0,1)$, the government-borrowing rate-based discount factor $f_{t,t+n}$ can be replaced by a **stochastic discount factor** $\widetilde{f_{t,t+j}}$ **involving the growth-adjusted marginal rate of consumption substitution** $u_{t,t+j}$ **:**

$$\widetilde{f_{t,t+j}} = E_t\left[u_{t,t+j} \prod_{b=t}^{t+j-1} (1+g_b)\right] \tag{6}$$

$$u_{t,t+j} = \beta^n \cdot \frac{U'\left(PB_{t+j}\right)}{U'(PB_t)}$$

where:

- U represents the concave utility function of the consumer : the concavity helps to capture the notion of **risk aversion**.
- E_t is the expectation of a future value based on information available at time t.

Now, the debt's value in the secondary market, as well as the primary surpluses and utility that the consumer can expect to receive from holding the debt, will depend on the state of the economy.[11]

Bohn shows that in a model with complete markets, a version of Lucas's (1978) stochastic endowment economy, the correct transversality condition can now be written as

$$\lim_{n\to\infty} E_t\left[\widetilde{f_{t,t+n}} d_{t+n}\right] = 0 \tag{7}$$

Here, $E_t\left[\widetilde{f_{t,t+n}}\right]$ has the interpretation of the **price of a safe n-period bond.**

From this new transversality condition, a new IGBC can be deduced :

$$d_t = E_t\left[\sum_{j=1}^{n}\left(\widetilde{f_{t,t+j}}\, pb_{t+j}\right)\right] \tag{8}$$

At first sight, there is nothing that significantly changes compared to equation (5). The real change is the use of $E_t\left[\widetilde{f_{t,t+n}}\right]$ for discounting instead of $f_{t,t+n}$.

But, this small change is critical and allows Bohn to express the new IGBC with a term discounted by $f_{t,t+n}$ and a covariance term that was not considered before:

$$d_t = E_t\left[\sum_{j=1}^{n}\left(f_{t,t+j}\, pb_{t+j} + \operatorname{cov}\left(\widetilde{f_{t,t+j}}, pb_{t+j}\right)\right)\right] \tag{9}$$

A key implication of equation (9) is that fiscal sustainability depends not only on expected discounted primary surpluses but also on their covariance with the stochastic discount factor. The covariance term allows the new IGBC to be satisfied even when expected primary surpluses are negative, provided there is a sufficiently positive correlation between pb_{t+j} and $\widetilde{f_{t,t+j}}$.

[11] If the state is bad, consumers will expect low future consumption, so the marginal utility $U'\left(PB_{t+j}\right)$ will be high, and the discount factor can exceed 1. This is conceptually different from the traditional framework, where discounting depends mechanically on $\left(\frac{1+g}{1+r}\right)$, and can only exceed 1 if $g > r$. In contrast, in a stochastic setting, even with $g \leq r$, risk and marginal utility dynamics can produce discount factors above 1.

If agents anticipate that, in bad states of the economy, the government will be able to generate fiscal surpluses — effectively providing a stream of "dividends" to debt holders — then public debt will be perceived as offering insurance-like properties. This implies that debt is valued more highly in downturns, **as it delivers resources precisely when marginal utility is high**. Formally, this translates into a **positive covariance between the primary balance and the stochastic discount factor**: in bad times, utility is low and marginal utility is high, so the discount factor rises. If fiscal surpluses also increase in such periods, the covariance is positive.

In practice, however, the covariance should be negative. This is because governments typically run countercyclical policies: in bad times, they are more likely to incur deficits rather than surpluses, meaning they provide negative "dividend-like" flows precisely when agents would value resources the most. As a result, the covariance between the primary balance and the stochastic discount factor is negative, which tightens the intertemporal budget constraint. This highlights a key limitation.

Yet, Bohn makes an important and more general point: **if public debt is perceived as an insurance-like asset, this perception can ease the government's intertemporal constraint**. In other words, a country that enjoys strong market confidence related to the safety of its debt may be able to sustain structural deficits without jeopardizing debt sustainability. This could provide one possible explanation of why even though the U.S. government showed on average primary deficits between 1916 and 1995, its debt did not become unsustainable. It is likely that investors granted a "safe premium" to this debt.

In Part 2, we will return to this interpretation, which helps explain why some advanced economies can carry persistently high debt levels without triggering adverse market reactions and benefit from $r < g$.

1.3.2. Bohn (1998) : Assessing Sustainability through Primary Balance Responsiveness

In his 1998 paper, Bohn devised a practical method to assess sustainability based on the positive response of primary surpluses to changes in the debt-to-GDP ratio. That method avoided the use of the IGBC.

For example, if historical data shows that overall, when the debt-GDP ratio increase, the primary surplus also increases, then the debt would be considered sustainable, **because the**

government is adjusting the primary surplus to the debt level relative to the GDP. It would prove that any increase in the debt-GDP ratio due to negative shocks, such as low growth, wars, or high interest rates, is eventually counteracted by primary surpluses.

The model is built as multivariate regressions where (ρ) represents the coefficient of the primary surplus's response to debt:

$$pb_t = \rho d_t + \alpha Z_t + \epsilon_t \ = \rho d_t + \mu_t \tag{10}$$

Where:

$$\mu_t = Z_t + \epsilon_t$$

If $r < g$, Bohn asserts that $\rho > 0$ is a sufficient condition[12] for sustainability.
Here, (Z_t) accounts for other determinants of the primary surplus (temporary government spendings and a business cycle indicator like the output gap), which is particularly relevant in scenarios like wars or crises that involve substantial expenses. Without accounting for these factors, a univariate regression $(pb_t = \rho d_t + \epsilon_t)$ might incorrectly suggest that a high debt-to-GDP ratio leads to increased deficits, implying no proof of sustainability $(\rho < 0)$, when it is merely a cyclical phenomenon due to a war[13] or a crisis.

For the 1916-1995, the fact that $r < g$ was empirically verified in the U.S.[14], and Bohn found a significantly positive fiscal response coefficient (ρ > 0). This was sufficient to demonstrate debt sustainability over the sample, despite the average primary balance being in deficit. These findings challenged the concerns raised by traditional IGBC-based assessments[15] at the time.
As $r < g$ still prevails in most developed countries, **this condition $(\rho > 0)$ is still adapted to assess sustainability in developed countries.**
It is important to note that if the interest rate were to permanently exceed the growth rate, the condition $(\rho > 0)$ may no longer be sufficient to guarantee debt sustainability.

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[12] For proof, see Appendix 3.
[13] For instance, during World War II in 1942, the deficit of the US was already high but continued to rise in 1943 due to ongoing war expenses. In such contexts, the debt-to-GDP ratio may increase alongside larger deficits, not because of unsustainable fiscal behaviour, but due to exceptional, temporary shocks. If not properly accounted for, this can bias the estimated reaction coefficient ρ downward, suggesting a negative response of the primary balance to debt. Including controls such as a war dummy or a temporary spending indicator helps correct for this, ensuring that the regression captures long-term fiscal behaviour rather than cyclical or crisis-related effects.
[14] Bohn computed average rate $r = 0.1\%$ and average growth rate $g = 3.3\%$).
[15] Hakkio and Rush (1991) for example.

One interesting feature is that this test is not based on the stability of the debt-to-GDP ratio, as, according to Bohn:

*"A rising debt-GDP ratio does not necessarily indicate unsustainable policies, **nor does a stable ratio imply sustainability**"* (Bohn, 1998).

In fact, a stable debt-to-GDP ratio does not guarantee sustainability, as even policies that appear stable in expectation can be unsustainable under certain risk scenarios. For instance, a government that continuously sets the primary balance to zero while rolling over its debt might experience a declining debt-to-GDP ratio on average if r < g, but the policy could still be unsustainable if there **is a non-zero probability that interest rates rise above the growth rate in the future (**i.e. we arrive to r > g). This highlights the need for a more dynamic and risk-aware understanding of sustainability, as developed in his earlier 1995 paper.
Bohn emphasizes that governments must not only stabilize the debt ratio, but also **actively adjust it in response to macroeconomic conditions. For example, as discussed in 1.2.2**., **if excess savings to absorb decline, the government must reduce its debt ratio; otherwise, excess demand may fuel inflation, prompt rate hikes that raise debt servicing costs and further erode fiscal sustainability.**
By analogy, **even if the debt-to-GDP ratio appears to follow an upward-trending path** — as has often been observed over the past decades in several developed economies such as the US, France, Japan, Spain, and Italy — **the observation of a positive fiscal reaction coefficient** (i.e., $\rho > 0$) under the condition $r < g$ can be taken, in Bohn's approach, as **sufficient evidence that the upward-trending path of the debt-to-GDP ratio will eventually stabilize**, thereby ensuring sustainability.

In this sense, Bohn's view stands in contrast to the European approach institutionalized in the Maastricht criteria, which define fiscal sustainability through rigid numerical thresholds, namely, a nominal (overall) deficit below 3% of GDP and a debt-to-GDP ratio below 60%, regardless of country-specific dynamics.

Empirical Application for France and Greece

While insightful, Bohn's approach remains incomplete: it abstracts from many factors, such as market confidence, political economy constraints, and nonlinear fiscal behavior, that can critically influence sustainability outcomes.

Empirically, relying solely on fiscal reaction functions can also lead to contradictory interpretations. For instance, Lamé, Lequien, and Pionnier (2010) apply adapted versions of Bohn's tests to French and Greek national accounts over the 1978–2007 period. Their results suggest that Greece fulfilled the sufficient condition for sustainability over the period, while France did not, outcomes that are clearly at odds with subsequent developments after 2007, as Greece entered a deep sovereign debt crisis while France retained stable market access.

This highlights the need to complement traditional fiscal response analyses with broader frameworks that better account for latent vulnerabilities and shifts in market sentiment, a theme that will be explored in Part 2.

1.4. The IMF's DSA Approach to Sovereign Debt Sustainability

International institutions such as the IMF define a government debt as sustainable if the government has the capacity to stabilize the debt-to-GDP ratio in the future. As Blanchard (2022) notes, maintaining sustainability under this framework implies keeping the debt ratio stable with a high degree of probability. The IMF definition, the one we will focus on, also integrates a liquidity components by considering notably the rollover risk.

1.4.1. The Debt Dynamics Equation and the Stabilizing Role of the Primary Balance

This approach is mainly constructed around the **debt dynamics equation**, which is obtained by subtracting d_{t-1} from equation (3) on each side :

$$\Delta d_t = d_t - d_{t-1} = \frac{r_t - g_t}{1 + g_t} d_{t-1} - pb_t$$

For *r* & *g* sufficiently small, we can approximate $\frac{(1+r_t)}{(1+g_t)} \approx 1 + r_t - g_t$; with this approximation, we obtain:

$$\Delta d_t \approx (r_t - g_t) d_{t-1} - pb_t \qquad (11)$$

From equation (11), we observe that if the interest rate paid by the government exceeds the growth of the economy $r_t > g_t$ the **dynamics of debt is unfavorable** as $r_t - g_t > 0$. In this case, starting from an initial debt strictly positive, even if the primary deficit is indefinitely equal to 0, the term $(r_t - g_t)d_{t-1} > 0$ makes the debt-to-GDP ratio increasing indefinitely. The term $(r_t - g_t)d_{t-1}$ is often referred to in the literature as the **snowball effect,** as it captures the mechanical component of debt accumulation resulting from the interest-growth differential.

But in the reverse situation $r_t > g_t$ **the snowball effect is favorable**. The government can even sustain a primary deficit in *t* (i.e., $pb_t < 0$) while keeping its debt-to-GDP ratio unchanged ($\Leftrightarrow$ $d_t - d_{t-1} = 0$). This is due to the negative contribution of the snowball effect as $r_t - g_t < 0$. Surprisingly, the higher the debt-to-GDP ratio, the larger the primary deficit relative to GDP that can be sustained while still stabilizing the ratio.

Box 1 : The Stabilizing Power of Negative *r – g*: a U.S. Debt Simulation

A favorable snowball effect may help explain how the United States saw its debt-to-GDP ratio increase only moderately, from around 2.7% in 1916 to approximately 54% in 1995[16], despite running an average primary balance of -1.2% of GDP over the period.

The stabilizing role of average negative $r - g$ can be quantified through a counterfactual simulation. Maintaining the observed average primary deficit (-1.2% of GDP) but assuming $r - g = 0$ yields :

$$d_t = d_{t-1} + 1.2\%$$

Iterating this over the 80-year period yields a counterfactual 1995 debt ratio of:

$$d_{1995} = 2.7\% + 80 \times 1.2\% = 98.7\% \text{ of GDP}$$

compared to the observed 54%. This implies that the historical $r - g < 0$ averted approximately 45 percentage points of GDP in additional debt accumulation[17].

[16] Debt figures are sourced from the Global Debt Database, compiled by the IMF (see Mbaye et al. 2018 for methodology).

[17] This illustrative simulation assumes parameter constancy, whereas actual dynamics were modulated by: (i) time-varying *r - g* (e.g., positive during the Volcker era), (ii) structural breaks (e.g., post-WWII growth surges and financial repression), and (iii) discretionary fiscal adjustments. While the magnitude aligns with theoretical debt-stabilizing thresholds (Blanchard 2019), a complete decomposition requires regime-specific analysis using annual data.

Another way to observe that dynamic is to establish a rule on primary surplus-to-GDP ratio that must be followed for a stable debt-to-GDP ratio (ie. $\Delta d_t = 0$):

$$pb_t = \frac{r_t - g_t}{1 + g_t} d_{t-1} \approx (r_t - g_t) d_{t-1} \tag{12}$$

In the quest for sustainability, the **primary surplus is a key adjustment tool for the government**, as it is variable of the debt dynamics equation the most directly controlled by the government in the short to medium term. Though it is not a flawless policy tool, the primary surplus can be adjusted through decisions on primary spending and revenue. In this way, tax and expenditure policy can influence solvency even in the short run. By extension, this is an important variable for the IMF, as it enables the institution to advise a country on the adjustment it needs to make to stabilize its debt. This emphasis on the primary surplus aligns with Bohn's approach, which also places the government's capacity to adjust the primary balance in response to rising debt at the core of fiscal sustainability.

Other variables of the debt dynamics equation are beyond the government's control.
In fact, as outlined by Alogoskoufis (2012), in an open economy, in the short and the medium run, the growth rate of GDP depends on the state of the economic cycle, on the determinants of domestic investment, and in particular the expectations of domestic firms and households about the future profitability of investment in physical and human capital, as well as on the determinants of domestic savings and the real interest rate. All these factors can be affected by government policy indirectly. One limitation to this view is that fiscal authorities can directly smooth economic fluctuations, particularly through countercyclical policies, which affect GDP growth. However, empirical studies show that these effects are typically short-lived. Blanchard and Perotti (2002), for instance, find that government spending shocks have a positive but transitory effect on output. Likewise, the Congressional Budget Office (2012) notes that fiscal expansions can support short-term growth but do not significantly influence long-run growth dynamics. These findings support the idea that fiscal policy may influence the economic cycle, but not the structural determinants of long-term growth. In the long run, the growth rate may thus well be exogenous, driven primarily by productivity, demographics, and technological innovation.

Interest rates are largely beyond the direct control of national governments, particularly in advanced economies where short-term rates are set by independent central banks, and long-term rates reflect market expectations of the future path of those central bank rates.

The historical sovereign debt-to-GDP is predetermined and cannot be changed by the government, unless the government chooses to default.

1.4.2. The Debt Sustainability Analysis: an Operational Tool to Assess Sustainability

The IMF translated the framework outlined before into a practical tool: the Debt Sustainability Analysis (DSA). The DSA aims to determine whether a country's debt trajectory is sustainable by analyzing the evolution of the debt-to-GDP ratio, using key variables that influence debt dynamics, such as the interest rate, the growth rate, and the primary balance, to assess whether the ratio is likely to stabilize in the future. This forward-looking analysis incorporates assumptions about economic conditions, fiscal policies, and potential government adjustments.

Compared to the IGBC framework, which relies on long-run solvency conditions and unobservable assumptions about future fiscal policy, the DSA is more operational: it provides a concrete, model-based projection of the debt path under various scenarios, without requiring the transversality condition or a strict comparison of r and g.

The DSA, like Bohn's model, departs from the IGBC by focusing on actual fiscal behavior rather than theoretical solvency constraints. Both approaches share the idea that sustainability is not defined by a debt level per se, but by whether the debt trajectory can be stabilized through credible fiscal behavior, either projected (IMF) or econometrically estimated (Bohn). For instance, under both approaches, a rising debt ratio may still be considered sustainable if fiscal policy is expected to adjust sufficiently in the future.

The IMF employs two distinct DSA frameworks: one for low-income countries and another for market access countries (SRDSF / MAC-DSF). We will study the latter that corresponds more to the situation of developed economies which have access to capital on markets.

Step-by-Step Breakdown of the DSA Process

To start with, the IMF typically defines a **"baseline scenario"** where it establishes projections for key economic variables over a five-year period. These variables serve **as inputs in the debt dynamics equation** and include factors such as GDP growth, interest rates, primary balance, and exchange rate depreciation/appreciation. The projected values are based **on historical data and anticipated future developments**. By entering these projections into the debt dynamics equation, the IMF can forecast the evolution of the debt-to-GDP ratio.

Next, the IMF evaluates the realism of its baseline scenario by comparing the projected fiscal path to the country's historical performance, especially regarding the primary balance, and to the experience of peer economies (e.g., from the same region or income group). If, for instance, the IMF finds that it's necessary for a country to sustain primary surplus of 5% of GDP over three years to stabilize the debt, while the country has never achieved more than 1% over the past three decades, the assumption is deemed implausible and subject to revision. This calibration step[18] ensures that the projected fiscal adjustments are not only theoretically stabilizing, but also feasible in practice.

After the baseline scenario is set and calibrated, near-term and medium-term risk assessments are conducted.

The Near-Term Risk Assessment uses a logit regression model to predict sovereign stress events within 1-2 years by evaluating structural, cyclical, debt-related, and global factors. This assessment yields a stress probability that serves as an early warning system, not a sustainability indicator, and users must apply judgment for unintuitive results or unique country situations.

The Medium-Term Risk Assessment produces **a mechanical assessment of debt sustainability**.

[18] A well-known case outside advanced economies is Argentina's 2018 Stand-By Arrangement, in which the IMF projected a sharp fiscal consolidation, from a primary deficit of 2.6% of GDP in 2017 to a surplus of 1.3% by 2019 (IMF 2018). The adjustment proved unrealistic and the program eventually broke down. At the time, the DSA did not yet include the realism tool introduced in the 2021 update to the SRDSF, which now flags implausible fiscal paths more systematically.

The IMF first assesses **solvency** by applying randomly drawn shocks to key macroeconomic variables of the debt dynamics equation — such as GDP growth, interest rates, exchange rates, and primary balances — and projecting multiple debt trajectories over a five-year horizon. These stochastic simulations reflect the uncertainty surrounding macroeconomic forecasts and generate a distribution of possible debt paths. The IMF then assesses whether the simulated trajectories significantly deviate from the baseline projection. The tool also estimates the probability of debt non-stabilization, based on the share of simulations in which the debt-to-GDP ratio fails to stabilize, given the projected primary surpluses. It thus enables the IMF to assess whether the projected fiscal path is likely to ensure debt stabilization with a sufficient likelihood.

Beyond this solvency analysis, which focuses on the medium-term sustainability of the debt ratio, **the IMF also conducts a liquidity assessment by examining a country's gross financing needs (GFNs)**, which include the primary deficit and debt service obligations (interests and debt amortization). The goal is to determine whether these short-term funding needs can be met under a range of economic conditions, particularly in the event of adverse shocks. For instance, the model simulates the impact of market stress or a tightening of domestic financial conditions on the government's ability to refinance its debt[19].

In addition to the near- and medium-term assessments, the IMF also conducts a long-term debt sustainability analysis, which extends the projection horizon beyond ten years. This long-run evaluation focuses on structural trends that may affect debt dynamics, such as demographic changes, long-term growth prospects, and climate-related fiscal pressures. While more uncertain by nature, this component helps assess whether current fiscal trajectories remain viable in the absence of further policy adjustments, and informs the design of long-term fiscal reform strategies.

Over time, the DSA framework has evolved in response to criticisms and lessons learned from past crises. The IMF has progressively expanded the tool's capacity to incorporate uncertainty and realism. For example, stochastic simulations, once absent, are now a central part of the medium-term risk analysis. Moreover, the new SRDSF framework (launched in 2021 for

[19] Egypt's 2016 program provides another illustrative example outside developed economies. The DSA flagged acute liquidity risks linked to large short-term gross financing needs and limited market access. To mitigate these, the IMF recommended front-loaded disbursements from multilateral institutions (IMF 2016), enabling Egypt to roll over its obligations and avoid a funding crisis during the initial phase of the adjustment program.

market-access countries) puts greater emphasis on identifying fiscal risks and incorporates broader indicators, such as contingent liabilities, financial sector vulnerabilities, and political risk. These updates aim to improve the forward-looking nature of the DSA and to avoid the overly deterministic projections that failed to capture the depth of crises like that of Greece.

Critical Challenges Facing the DSA Tool

While the DSA framework has become more sophisticated, it still faces two persistent challenges. The first relates to how fiscal adjustments interact with growth and debt dynamics; the second concerns the model's strong sensitivity to baseline assumptions. These issues remain difficult to fully address, even with recent methodological improvements.

One issue is that creating these projections is challenging due to the **interconnected and endogenous nature of macroeconomic variables.** For instance, if a country implements austerity measures by cutting public spending and increasing taxes, it may improve its primary fiscal balance, an adjustment that typically supports a declining debt-to-GDP ratio. However, such measures can simultaneously weaken economic activity and reduce GDP growth, which may counteract the intended improvement in debt dynamics by increasing the snowball effect. This tension **does not stem from a flaw in the DSA itself, but rather from the difficulty of assessing whether a proposed adjustment path is both economically and politically sustainable**. The DSA provides a framework for projections, but the success of any consolidation effort depends on how economic agents react, whether households maintain consumption, whether investment holds, and whether confidence is preserved.

This issue was starkly illustrated by Greece's experience. In 2010, Greece was advised to implement stringent austerity measures to stabilize its public finances. While these measures initially improved the primary balance, the sharp cuts in public spending and steep tax increases significantly hampered economic activity. The resulting slowdown was far more severe than anticipated by the IMF, triggering a contraction in GDP that worsened the debt-to-GDP ratio. Ultimately, the adjustment proved too harsh for the economy to absorb, and Greece defaulted again on its debt in 2015.

This episode highlighted the critical risks embedded in DSA when the complex interplay between fiscal consolidation and economic growth is not properly taken into account, particularly the risk of applying overly optimistic growth assumptions in a context of deep fiscal retrenchment.

Another problem lies in **the weight of initial assumptions on the evolution of economic variables** such as GDP growth or interest rates. A very slight difference in projected values can lead to very different conclusions on debt sustainability.

A striking illustration of the sensitivity of debt sustainability assessments to initial assumptions occurred in 2015, during the Greek debt crisis. At that time, both the IMF and the European Commission conducted DSAs on Greece, but reached sharply contrasting conclusions.

The IMF concluded that Greek public debt was **unsustainable**[20], and would require substantial debt relief to be stabilized over the long term. This judgment was based on a downward revision of Greece's potential growth rate: the IMF estimated long-run real GDP growth at just **1.5%**, citing low productivity growth and demographic challenges.

In contrast, the European Commission considered Greek debt to be **sustainable**[21], largely because it adopted significantly more optimistic growth assumptions. The Commission assumed rates of **2.1% to 2.3%**, reflecting confidence in the impact of structural reforms.

The divergence between the two assessments stemmed from a difference of only **about 0.6 to 0.8 percentage points** in assumed long-term growth. Yet, this seemingly small discrepancy led to **opposite policy implications**, debt relief being deemed essential by the IMF but not by the Commission.

[20] IMF. 2015. *Greece: Preliminary Draft Debt Sustainability Analysis*. IMF Country Report No. 15/165, June 26, p. 4.

[21] European Commission. 2015. *Greece – Compliance Report: Fourth Review*. June, p. 9.

2. Fiscal Space Beyond Rules: A Contry-Specific View of Debt Sustainability

While Part 1 reviewed the main approaches used to assess debt sustainability, their application to real-world cases reveals significant divergences. In particular, debt trajectories showing comparable increases have resulted in very different sustainability outcomes and market reactions, even though, under IMF analysis, a failure to stabilize the debt ratio typically raises concerns about sustainability. As seen in Part 1, we also identified inconsistencies between model predictions and observed behavior, such as the pre-2008 assessment of Greece and France under Bohn's framework.

Moreover, some institutions have attempted to define common debt thresholds, most notably the Maastricht Treaty's 60 percent of GDP target, and some academic contributions were interpreted as pointing to common benchmarks beyond which sustainability concerns would arise[22]. Yet these benchmarks were never empirically validated. Some euro area countries, such as France, exceeded the Maastricht limit for extended periods without facing market pressures. Japan went even further, reaching debt levels far beyond what had previously been considered compatible with sustainability, and still experienced no fiscal crisis.

This observation raises a key question: what factors explain such discrepancies or misleading model signals?

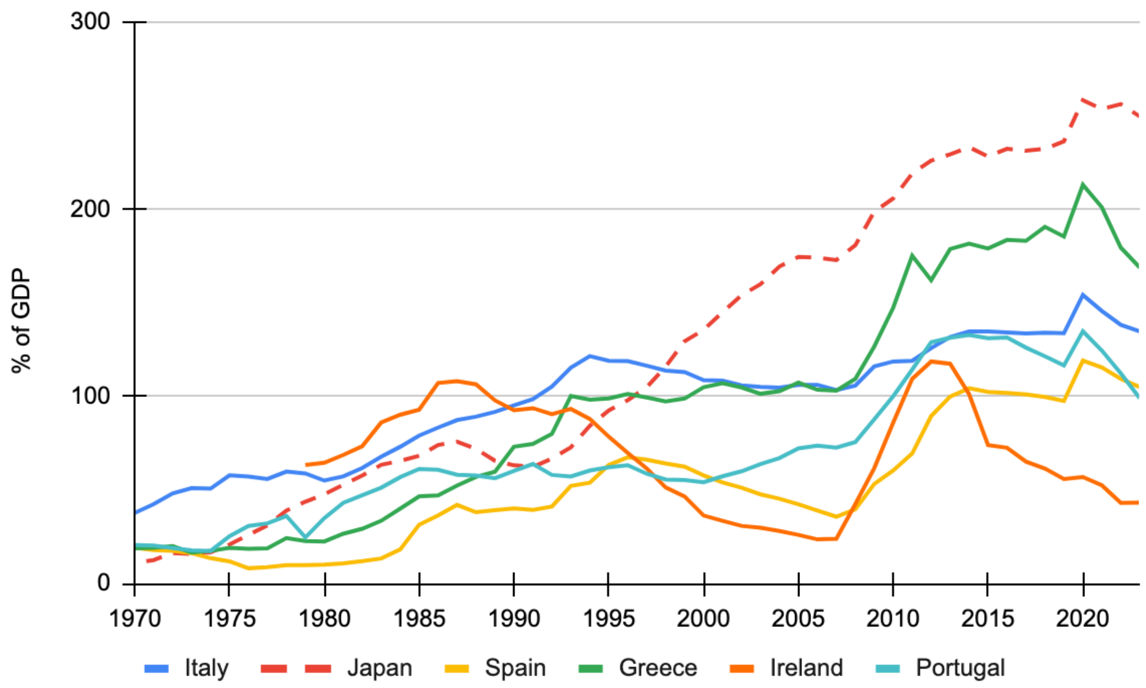
We begin by examining one striking empirical contrast, where upward debt trends led to sharply divergent outcomes, before turning to the concept of fiscal space to help answer this question.

2.1. When Debt Paths Diverge, so Do Outcomes

As show in Figure ((2) Japan has experienced a sustained rise in its debt-to-GDP ratio, increasing from about 50% in 1990 to nearly 260% by 2020, an average annual growth exceeding 5%.

[22] The idea of a universal debt threshold drew support from Reinhart and Rogoff (2010), who found that growth tends to slow when public debt exceeds 90% of GDP. Their findings, however, were later challenged for data omissions and methodological flaws (Herndon, Ash, and Pollin, 2013), and the 90% figure, often interpreted as a debt limit, was not presented as such by the authors.

Figure 2 : Gross Public Debt, Percent of GDP, 1970–2023

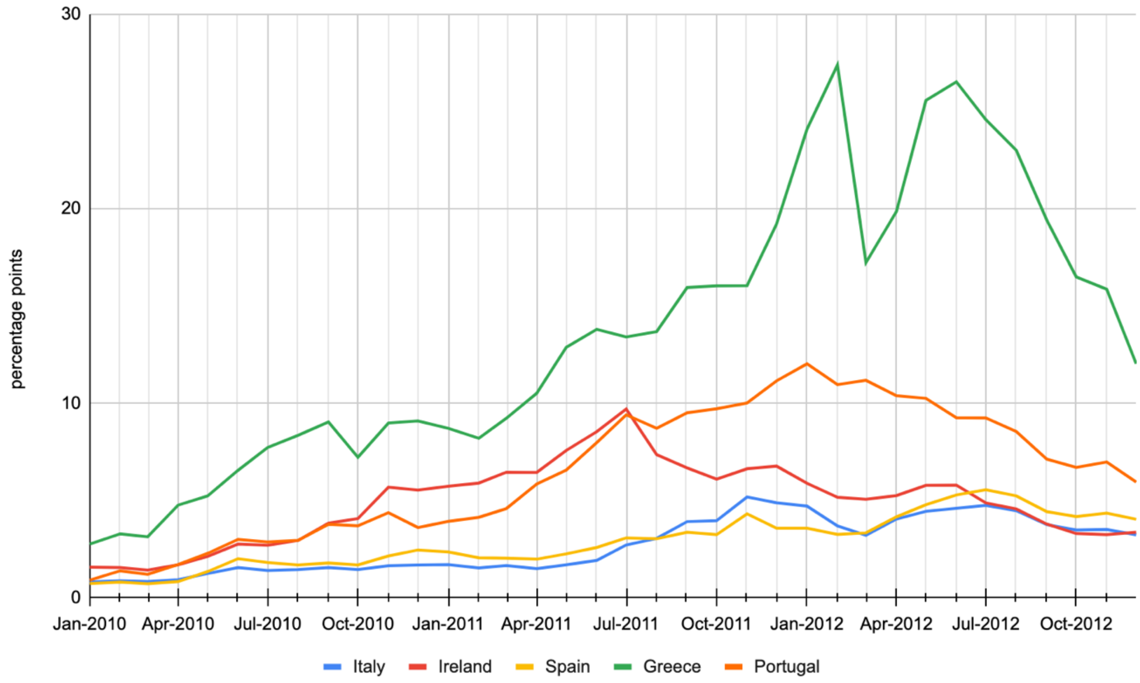


Source : *IMF, Global Debt Database (2025)*

Yet, despite this steep trajectory, Japan never experienced the kind of market turmoil that engulfed several euro area countries in 2011.

That year, fears of fiscal unsustainability, exacerbated by the absence of a central fiscal authority and doubts over the ECB's backstop, triggered a sharp spike in sovereign spreads across the periphery. As shown in Figure 3, over the period 2010-2012, yields peaked at 27 percentage points above the German bund in Greece, and to around 10 in Portugal and Ireland. Even Italy and Spain saw their spreads double. Crucially, these countries had significantly lower debt ratios in 2011: below 120% for Italy and Portugal, under 70% for Spain, and around 110% for Ireland. Only Greece approached Japan's but still at a lower level of 165%.

Figure 3 : 10-year Government Bond Spreads over German Bunds, 2010–2012



Source : *Author's calculations based on data from FRED (2025).*

By contrast, Japan's 10-year bond yields continued their steady decline. This resilience persisted even in the face of external warnings. In November 2011, the IMF cautioned: "Should JGB yields rise from current levels, Japanese debt could quickly become unsustainable. […] Recent events in other advanced economies have underscored how quickly market sentiment toward sovereigns with unsustainable fiscal imbalances can shift" (Wall Street Journal, 2011).

More than a decade later, in 2024, the IMF reiterated its concern, describing fiscal risks in Japan as "considerable" and urging fiscal consolidation as a "very important priority" (Japan Times). Yet, once again, markets remained unshaken.

These repeated warnings, met with persistent investor confidence, suggest that Japan operates under a higher implicit debt limit. This calls into question the notion that a uniform debt-to-GDP threshold[23] can meaningfully capture sovereign risk, a view that had influenced earlier

[23] On that point, an influential contribution came from Cecchetti, Mohanty, and Zampolli (2011), who estimated that public debt levels above 85% of GDP are associated with slower long-run economic growth in advanced economies. Their analysis, however, was not concerned with solvency or fiscal limits per se, but with the macroeconomic consequences of high indebtedness, in particular, the way in which debt may impair growth prospects over time. Interestingly, they point out that while economies have historically continued to grow amid

frameworks such as the Maastricht Treaty's 60% rule. Instead, it points to the need for a more nuanced approach grounded in the concept of fiscal space, which allows for country-specific assessments of debt tolerance based on market expectations, institutional credibility, and macro-financial resilience.

2.2. Understanding Fiscal Space: a Central Concept in Sustainability Analysis

Fiscal space refers to the room a government has to increase spending or reduce taxes without compromising debt sustainability. In other words, it represents the buffer between a country's current debt level and the critical threshold beyond which market confidence could collapse.

This concept can be used to determine a country's exposure to sudden shifts in investor sentiment. The literature refers to this as debt market panics, pure liquidity crises, rollover crises, or self-fulfilling debt crises. Rollover risk represents the potential difficulties the government might face when renewing or refinancing an existing debt obligation.
It can also be used to assess the time available for adjustment when a country's debt-to-GDP ratio is on an upward trajectory, for example because of a structural change, but has not yet reached unsustainable levels. In this case, fiscal space indicates how long the government has to correct the path before sustainability is compromised.

This notion of fiscal space and its implications for sustainability are poorly taken into account in Bohn's initial model.
First, Bohn's test assumes a continuous and linear adjustment of the primary balance to rising debt. In practice, however, this relationship may become non-linear beyond a certain point. Political resistance, social fatigue, or macroeconomic rigidities may prevent governments from increasing their primary surplus in response to rising debt once a critical threshold is passed; this concept is sometimes referred as **fiscal fatigue**. As a result, a country may appear to have

rising debt levels, the sustainability of high debt ultimately hinges on continued GDP growth: *"Without rising GDP, there will be no way to raise the revenues governments need to reduce their exploding debts."* This perspective, though not a fiscal threshold in a narrow sense, establishes a link between growth trajectories, fiscal fatigue and sustainability.

behaved "sustainably" in the past, yet find itself unable to maintain that response at very high debt levels[24].

Second, while the model can be extended to a stochastic setting, most applications ignore shocks that may suddenly push debt onto an unsustainable path, especially for countries near their fiscal limits.

By contrast, the IMF's approach gives more weight to these dimensions, notably through the GFNs module, which aims to capture rollover risk. Its long-term assessments also incorporate structural shocks and contingent liabilities, such as demographic or climate-related risks. However, this was not always the case: DSA models were less refined a decade ago, which may partly explain why early warnings, such as those issued for Japan, did not translate into actual market stress.

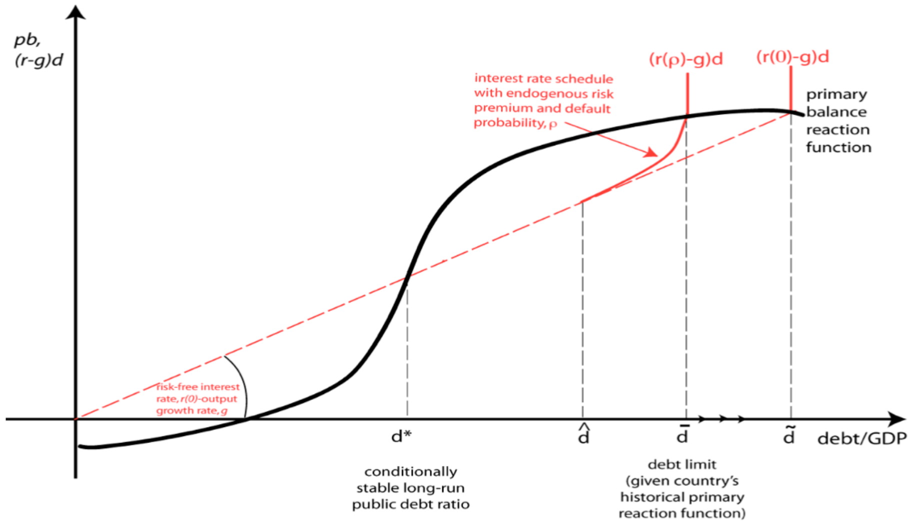
2.2.1. A Stylized Model of Fiscal Space: Thresholds and Debt Limits

To provide a more intuitive grasp of the concept of fiscal space, we now turn to a **stylized model** devised by the IMF (Ostry, Ghosh, Kim, Qureshi, 2010)**.** This model, illustrated in the Figure 4 below, formalizes the interaction between the government's fiscal response and the rising cost of debt, and helps visualize the conditions under which a debt trajectory can become unsustainable.

Authors use a more formal definition of fiscal space in this model. It is viewed as the distance between the current debt-to-GDP ratio and the debt limit implied by a country's historical capacity to adjust its fiscal balance (i.e., fiscal space = debt-to-GDP ratio limit – current debt-to-GDP ratio). When a country retains significant fiscal space (i.e., in the model, its current debt-to-GDP ratio is far from the limit) it can afford temporary deviations from fiscal discipline without triggering a crisis. Conversely, once this margin has been exhausted, a country may face immediate market pressures, even if its debt remains stable or increases only gradually. The purpose of this model is to show which factors influence the building of this debt-to-GDP limit.

[24] Abiad and Ostry (2005) and Mendoza and Ostry (2008) find (looking at the broad cross-country evidence) that the marginal response of the primary balance to debt is significantly weaker at high levels of debt than at more moderate levels.

Figure 4 : Determination of Debt Limit



Source : *IMF (2010)*

Authors first define a standard deterministic case, based on two simplifying assumptions. First, **governments are assumed to behave responsibly by increasing their primary surplus as debt rises ($\rho > 0$ in Bohn's method),** this is reflected in the upward-sloping primary balance reaction function. However, this relationship weakens at high debt levels, where further fiscal adjustment may become politically or economically unfeasible. This is reflected graphically by **a flattening of the primary balance reaction function at higher debt ratios.**

Second, the interest-growth differential is assumed to be positive and constant, so that the snowball effect ($(r(0) - g) * d$ curve) increases linearly with the debt level[25].

These two curves generate two key intersection points. The first, at d^*, corresponds to the stable long-run debt level the economy converges to, provided debt remains below a critical threshold. The second, at $\tilde{d}$, **marks the point beyond which the primary surplus is insufficient to cover the snowball effect**, triggering an explosive debt dynamic. This threshold defines the debt limit, and the gap between current debt and $\tilde{d}$ reflects the available fiscal space.

[25] As said in the Part 1, this not the case of many developed countries, but this is just for illustrative purpose. Actually, the reasoning still holds when $r - g$ is negative; authors still manage to compute a fiscal space for countries whose historical $r - g$ is negative.

To better reflect real-world dynamics, authors extend the framework to incorporate **uncertainty and investor risk aversion**. In the stochastic version, adverse shocks may affect growth, the primary balance, or interest rates. Even before debt reaches $\tilde{d}$, markets may anticipate rising default risk and demand a risk premium due to rising default risk ρ, causing the effective interest rate to increase with debt. As a result, the slope of the snowball effect curve rises endogenously (it corresponds graphically to$(r(\rho) - g) * d\ curve$), making it more convex as debt approaches its perceived limit. Graphically, the debt limit shifts from $\tilde{d}$ to $\bar{d}$, with an inflection point $\widehat{d}$ marking where markets begin to reprice risk. In this sense $\bar{d},$ marks a forward-looking limit shaped by market psychology, while $\tilde{d}$, remains the structural maximum beyond which debt becomes unsustainable without exceptional measures. How markets perceive and price that uncertainty thus compresses fiscal space even before the fundamental limit is reached. **Under uncertainty and markets risk aversion,** the remaining fiscal space can be expressed as the gap between the estimated debt limit and the current debt level :

$$Fiscal\ space_t = \bar{d} - d_t.$$

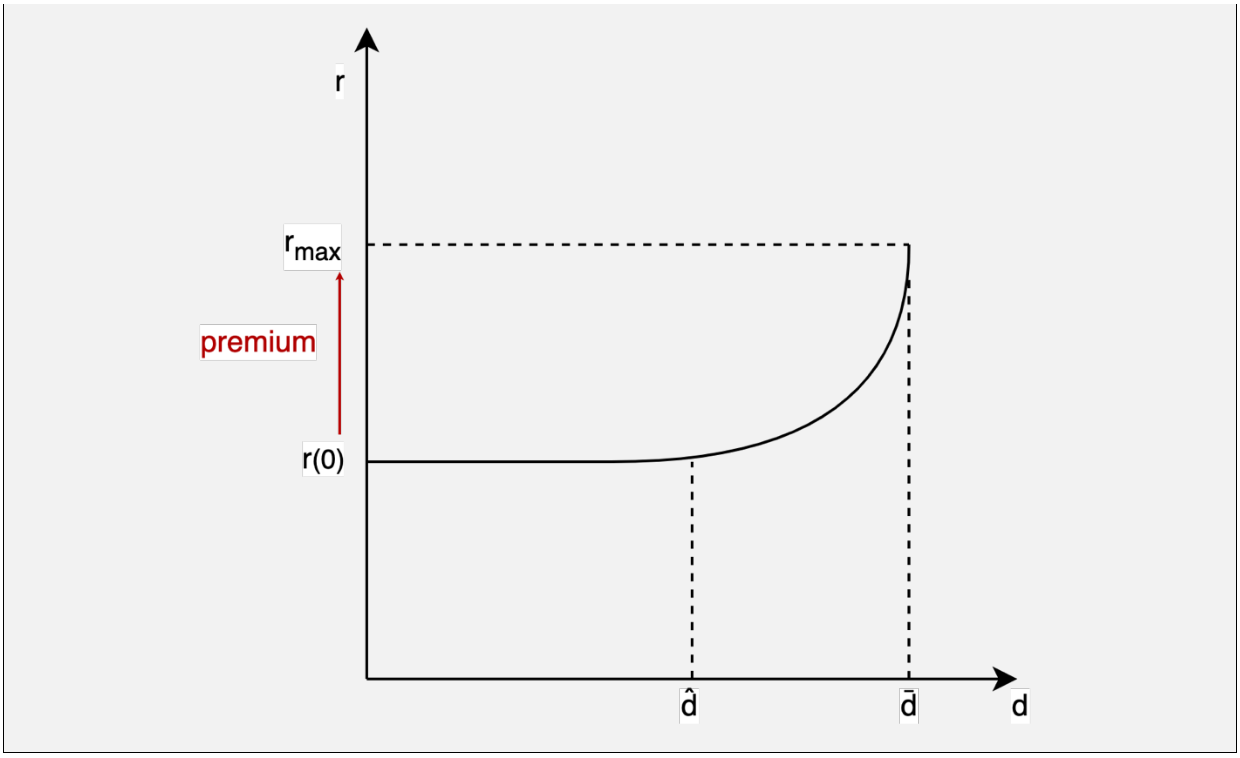
Box 2 : Why Does the Premium Increase after $\hat{d}$?

Investors look at the expected path of debt, but also at what could happen in a bad year. Suppose debt is close to its limit, and the expected primary balance is just enough to keep it stable. On average, the situation seems under control.

However, if a negative shock hits, for example, a much weaker-than-expected primary balance, this single event could push debt **above the limit**, even if it was previously below. That's the key point: from $\hat{d}$ onward, a bad realization in just one year can be enough to place the country on an explosive debt path.

Even if the probability of such a shock is low, its consequences are severe. This is why markets begin to demand a risk premium after $\hat{d}$: not because default is likely, but because it becomes a plausible outcome in an adverse scenario.

Diagram : The supply of funds to a risky sovereign



This stylized model sheds light on **the factors that determine a country's long term debt limit.**

The position of the primary balance curve depends on the government's historical ability and willingness to adjust: a more responsive or credible fiscal stance shifts the curve upward and pushes the limit outward.

Conversely, an increase in the interest-growth differential **steepens** the debt service line and reduces fiscal space.

Higher investor risk aversion or greater perceived probability of adverse shocks can have a similar effect, by raising the required risk premium and narrowing the space available before debt becomes unsustainable.

When extending Bohn's model with the concepts of fiscal fatigue and fiscal space, as in Ghosh et al. (2013) using 1985–2007 data, the analysis reveals that for five countries — Greece, Iceland, Italy, Japan, and Portugal — no debt limit $\bar{d}$ exists[26], meaning there is no evidence of any fiscal space being left. Notably, during 2008–2012, all of these countries except Japan experienced debt crises. **This highlights how this refined framework including fiscal space better captures real-world vulnerabilities.**

[26] Using market interest rate projected by the authors.

In the same paper, the authors notably estimate the debt limit $\bar{d}$ to vary significantly across countries, around 230% for Norway, 170% for Germany and the US, and approximately 160% for France. While, at the time, projected debt-to-GDP ratios for 2015 stood at roughly 50%, 80%, 110%, and 90% respectively, this implies theoretical fiscal space of about 180% for Norway, 90% for Germany, 60% for the US, and 70% for France, suggesting that fiscal space is highly country-dependent.

2.2.2. Fragilities Identified with the Model

The stylized framework helps identify several **vulnerabilities that can erode fiscal space** well before a debt trajectory becomes formally unsustainable. These fragilities are especially important under **uncertainty**, when small changes in expectations or external shocks can have outsized effects on debt dynamics.

The first risk stems from contingent liabilities, defined as obligations that may become due depending on future events but are not reflected in the current fiscal balance. These liabilities can materialize either gradually or suddenly.
Gradual materialization is illustrated by demographic pressures, where aging populations could incrementally affect the burden on pension and healthcare systems. This point will be further explained in 2.4.3.. Climate change follows a similar pattern, progressively degrading fiscal balances by increasing the frequency and severity of climate-related disasters and adaptation costs.
Conversely, contingent liabilities can also emerge abruptly, such as public guarantees, financial sector support, or the fiscal impact of sudden natural disasters, which can rapidly deplete fiscal space and expose hidden vulnerabilities.

A **second vulnerability arises from the risk of market repricing**, where investors revise their perception of sovereign risk even without already observed change in the financial fundamentals. Such shifts can occur due to deteriorating expectations regarding future growth or doubts about the government’s fiscal commitment. This repricing is often non-linear: borrowing costs may remain low until confidence suddenly erodes, causing a sharp rise in risk premiums. Critically, this can occur while debt dynamics appear stable, highlighting how fiscal space can disappear unexpectedly.

Contagion can amplify such episodes, as repricing in one country often spills over to seemingly comparable economies through rising spreads and capital flight. During the euro area crisis, for instance, Greece's fiscal turmoil quickly transmitted to Italy and Spain, despite differences in fundamental.

As highlighted in Box 2, a **third weakness is that debt may appear stable on average, but when it nears debt limit, a single adverse shock** (in investor sentiment or primary balance) **can flip the trajectory**, from convergence toward a sustainable level to an explosive path beyond $\bar{d}$. Everything may look under control until it suddenly isn't.

Even when a country breaches its estimated debt limit due to the materialization of one of these fragilities, the situation is not necessarily irreversible. If the loss of market confidence is not yet complete, decisive policy action can still restore sustainability. The following instruments are typically available :

First, **extraordinary fiscal adjustments**, going beyond historical norms, can be undertaken, often in coordination with international institutions such as the IMF. Second, **structural reforms** may help raise the economy's long-term debt tolerance by improving tax efficiency, optimizing public expenditure, or supporting higher potential growth through competitiveness gains.

Another option is the **debt restructuring** to reduce debt service and regain fiscal breathing space. However, if it entails a reduction in the nominal value of outstanding obligations, it amounts to a partial default, effectively confirming that the original debt path was not sustainable and necessitated resolution through involuntary adjustment[27].

Box 3 : Real-World Examples of Fiscal Fragility and Rollover Risk

Real-world episodes show how quickly sovereigns can shift from apparent stability to refinancing stress when the fragilities above mentioned surface.

Ireland (2008–2010): The Sudden Emergence of Rollover Risk

[27] Not all restructurings involve a reduction in the nominal value of debt. Some operations may only consist in extending maturities, lowering interest rates, or offering voluntary exchanges, which may still provide temporary relief without constituting a formal default event.

Before the global financial crisis, Ireland's public debt stood below 25% of GDP, suggesting ample fiscal space and little immediate rollover risk. However, when the government guaranteed domestic bank liabilities after the housing bubble burst, hidden contingent liabilities surfaced, causing debt to surge. Markets anticipated the future fiscal burden before it fully materialized. This anticipation shock caused the markets to think that Ireland is actually much closer to its debt limit that they thought before. As a consequence, markets demanded higher spreads, eventually forcing Ireland to seek IMF–EU support. This episode illustrates how contingent liabilities can abruptly narrow fiscal space and trigger rollover pressures even in seemingly sound economies.

United Kingdom (2022): Feedback Loops and Shrinking Fiscal Space

In September 2022, Prime Minister Liz Truss and Chancellor Kwasi Kwarteng announced a “mini-budget” that featured significant, unfunded tax cuts and expansive energy subsidies, introduced without endorsement from the Office for Budget Responsibility (OBR). This absence of a credible fiscal anchor led markets to perceive a shift toward fiscal irresponsibility. In response, yields on 10-year UK gilts surged by over 100 basis points within days, and the pound depreciated sharply. The Bank of England was ultimately forced to intervene with emergency bond purchases to prevent a full-blown financial crisis with massive selloffs.

This episode demonstrates how fiscal space can erode abruptly, **even without any change in the actual debt level.**

In the stylized framework, the UK’s debt-to-GDP ratio remained unchanged, but market perceptions of risk shifted dramatically, causing the perceived debt-GDP ratio to be lowered (graphically $\overline{\mathrm{d}}$ has shifted leftward). A sequential market repricing mechanism modified the components affecting that limit. (i) Rising risk premiums caused the effective **interest rate curve to steepen**, reflecting higher borrowing costs. (ii) If the shock is perceived as persistent (long-term interest rates persistently higher and GDP growth permanently lower), the **entire trajectory of $(r(\rho) - g)d$ shifts upward**, further narrowing fiscal space. (iii) **Markets reduced their expectations for future primary surpluses**, effectively shifting the primary balance reaction function downward.

The UK’s transition from a relatively flat segment of the curve (low risk premium) to a steeper one (higher premium) reflects its perceived proximity to a newly reduced debt limit.

2.3. Fiscal Space between Adjustment Margin and Crisis Trigger

This section advances the question of whether debt sustainability is a universal or context-specific concept by exploring the notion of fiscal space as both a time buffer and a vulnerability indicator.

First, assessing fiscal space helps determine how much time a country has to adjust an unsustainable fiscal trajectory before market pressures force abrupt correction. Two countries may face similar long-term challenges, such as ageing or declining growth potential, yet differ significantly in their sustainability outlook depending on the fiscal room they have to respond. Second, limited fiscal space implies heightened exposure to rollover risk, where adverse market and government behaviors can amplify perceived solvency concerns. Understanding the mechanisms behind this risk (notably as conceptualized by Sachs and Calvo) and the tools available to mitigate it is essential. These mitigation strategies are highly country-specific, reinforcing the idea that sustainability cannot be assessed through uniform thresholds alone.

2.3.1. Fiscal Space as a Margin for Policy Response

Fiscal space provides a critical measure of how much time a government has to adjust a rising debt trajectory, especially when facing long-term structural changes.

In the United States, public institutions such as the Congressional Budget Office (CBO) and research-based platforms like the Penn Wharton Budget Model rely on Overlapping Generations (OLG) models to simulate the long-run macroeconomic consequences of fiscal policy. These models, rooted in the foundational work of Diamond (1965) and Auerbach and Kotlikoff (1987), analyze how government debt interacts with household decisions, taxes, and spending over time and across generations.

To ensure coherent long-term projections, these models introduce a **closure rule**, that is, a future fiscal correction not embedded in current legislation but assumed to take place to stabilize debt dynamics. This rule often takes the form of a broad-based value-added tax, a proportional income or payroll tax, a reduction in spending, or a combination thereof. Its role is to cap the debt-to-GDP ratio at a sustainable level, avoiding explosive paths inconsistent with market behavior.

A key strength of this approach is that it enables economists **to estimate a maximum sustainable debt-to-GDP ratio**, beyond which the economy cannot remain in equilibrium. In the case of the United States, in 2023, PWBM estimates this limit to be around to 200 percent of GDP ($\tilde{d}$ in stylized model), depending on interest rate assumptions and the extent to which markets trust that the fiscal correction will eventually occur. Actually, integrating the investor risk behavior, this limit could in fact be lower, around 175% of GDP ($\bar{d}$ in stylized model).
More importantly, these models allow us **to calculate the time governments have left before such a correction becomes unavoidable.** Under current policy trajectories, **the U.S. would likely reach this critical threshold within 20 years, meaning an adjustment is necessary within that period to prevent the debt entering an explosive dynamic.**

2.3.2. From Fragile Fiscal Space to Full-Blown Sovereign Crisis: Dynamics of the Rollover Risk

Beyond indicating the time available for fiscal adjustment, the remaining fiscal space also provides a useful gauge of a sovereign's exposure to rollover risk. When this space has been eroded by the fragilities discussed in 2.2.2, two main mechanisms can trigger a transition from fragility to full-blown crisis.

Coordination Failure and Sovereign Liquidity Crises (Sachs, 1994)

Sachs emphasizes a critical vulnerability that sovereigns face, even when their fundamentals appear broadly sound: the risk of a self-fulfilling liquidity crisis driven by a **coordination failure** among investors. This situation arises when a country's GFN exceed its available cash reserves. In such cases, the government **must roll over at least part of its maturing debt to avoid default.**

In this context, even a fundamentally sustainable sovereign can become vulnerable if investors lose confidence. Each investor faces a strategic dilemma: if they expect other investors to continue refinancing the sovereign's debt, they are likely to do the same, as refinancing appears safe. However, if they suspect others may withdraw, they may choose not to refinance to avoid potential losses.

This **coordination problem can result in a self-fulfilling crisis**: if all investors fear that others will refuse to refinance, they collectively withhold funds. The **government then loses access**

to market financing, leading to a liquidity crisis and potential default, not because it is fundamentally insolvent, but because it cannot access the liquidity needed to meet its obligations.

Sachs' analysis underscores a critical insight: while underlying fragilities create the conditions for financial stress, it is the coordination failure among investors that can abruptly amplify and accelerate a crisis. **A sovereign that appears to have exhausted its fiscal space can be pushed into default if investor panic triggers a sudden loss of market access**.

Strategic Government Inaction (Calvo, 1988)

Once fiscal space is eroded and market stress emerges, the government's response becomes crucial in determining the trajectory of the crisis. Even if investors are hypothetically still willing to refinance at high rates, the government itself may choose to default rather than continue making painful adjustments. This strategic decision is at the heart of Calvo's (1988) model, where the government faces a critical choice between two divergent scenarios.

In the first scenario, adjustment and stabilization (the good equilibrium), the government judges that default would be more costly than fiscal tightening. It enacts credible measures to restore investor confidence, risk premiums fall, and debt remains sustainable.

In the second scenario, strategic default (the bad equilibrium), the government judges that the cost of adjustment is politically or economically intolerable. Faced with social unrest, political backlash, or severe economic contraction, it refuses to impose further fiscal discipline and strategically abandons adjustment. Investor confidence collapses entirely, spreads widen further, and the country ultimately defaults.

Investor expectations and coordination matter, but **government choices are equally decisive**. In Calvo's original model, debt is assumed to be composed entirely of one-period bonds, meaning the entire stock of debt matures within one year. Consequently, a loss of confidence leads to an immediate spike in borrowing costs, leaving little time to adjust.

Alogoskoufis (2012) outlines **the Greek crisis of 2009–2010 as an appropriate example of a Calvo-style crisis**, where strategic government inaction and investor panic transformed a manageable situation into a full-blown crisis. Before 2008, Greece's debt-to-GDP ratio was high but appeared stable. Yet this apparent stability masked a fragile fiscal position with minimal fiscal space.

The tipping point came in late 2009 when Prime Minister George Papandreou revealed that Greece's budget deficit was far larger than previously reported, rising from 6–8% to 12.5% of GDP. Although this was merely a correction of past data, it triggered a sharp loss of market confidence. Greece essentially faced two options. The government could have reassured investors by implementing credible reforms, restoring confidence and maintaining access to market financing. Alternatively, it could fail to reassure, allowing panic to spread. Unfortunately, the government's response was slow and marked by mixed signals, which only fueled investor doubts. Borrowing costs surged, and Greece lost access to markets, unable to roll over its debt.
This episode illustrates how a Calvo-style crisis can emerge after a perceived erosion of fiscal space: initial fragilities lead to panic, and the government's inability to credibly reassure investors accelerates the collapse.

More recent research by Lorenzoni and Werning (2019) extend Calvo's logic to a more realistic setting with long-term debt. In their model, rising marginal rates feed into the average cost of debt more gradually, giving governments time to adjust policy. The risk of a bad equilibrium remains, but it unfolds more slowly, giving governments a chance to act before default becomes unavoidable.
The dilemma is unchanged: either adjust or risk drifting into strategic default, only now, the process is slower.

Solutions to Mitigate that Rollover Risk
Recognizing that fiscal fragilities can rapidly trigger rollover crises raises an essential question: how can countries protect themselves against such risks? Three key defenses emerge.

First, as argued by Willems and Zettelmeyer (2022), maintaining a low enough debt level can help eliminate rollover risk. In Calvo's framework, lower debt raises the interest rate at which a crisis becomes inevitable, making a bad equilibrium less likely. Building on this framework, **we propose that what ultimately matters is not the level of debt per se, but the perceived fiscal space and capacity for adjustment.** Even if low absolute debt can help, a country's resilience depends on its ability to maintain a credible margin for adjustment.

Second, the **structure of public debt is critical.** As Cole and Kehoe (1996, 2000) emphasize, short-term borrowing exposes countries to frequent refinancing needs, increasing their

vulnerability to sudden stops. In contrast, **a longer average maturity provides a buffer**. It slows the transmission of rising spreads to the average borrowing cost and gives the government time to implement corrective measures before default becomes unavoidable. This is consistent with Lorenzoni and Werning (2019).
As Willems and Zettelmeyer (2022) highlight, **countries can also limit their exposure to private, non-resident investors**, who are most likely to withdraw during a crisis. Even when borrowing in domestic currency, relying too heavily on foreign investors can amplify risks.

A third and perhaps most **powerful defense against rollover crises is the central bank.**
As Sachs illustrates, the central bank can serve as a lender of last resort, especially as developed economies borrow in their own currency, providing liquidity to the government when market conditions deteriorate, ensuring that temporary liquidity shortages do not escalate into a full-blown crisis. In Calvo's framework, the central bank can also cap borrowing costs by directly intervening in the bond market, preventing interest rates from rising to levels that would make default a rational choice for the government.
Beyond these direct interventions, central banks can also use asset purchase programs, such as quantitative easing, which replace privately held bonds of finite maturity with central bank reserves that do not need to be rolled over, further reducing refinancing risks.

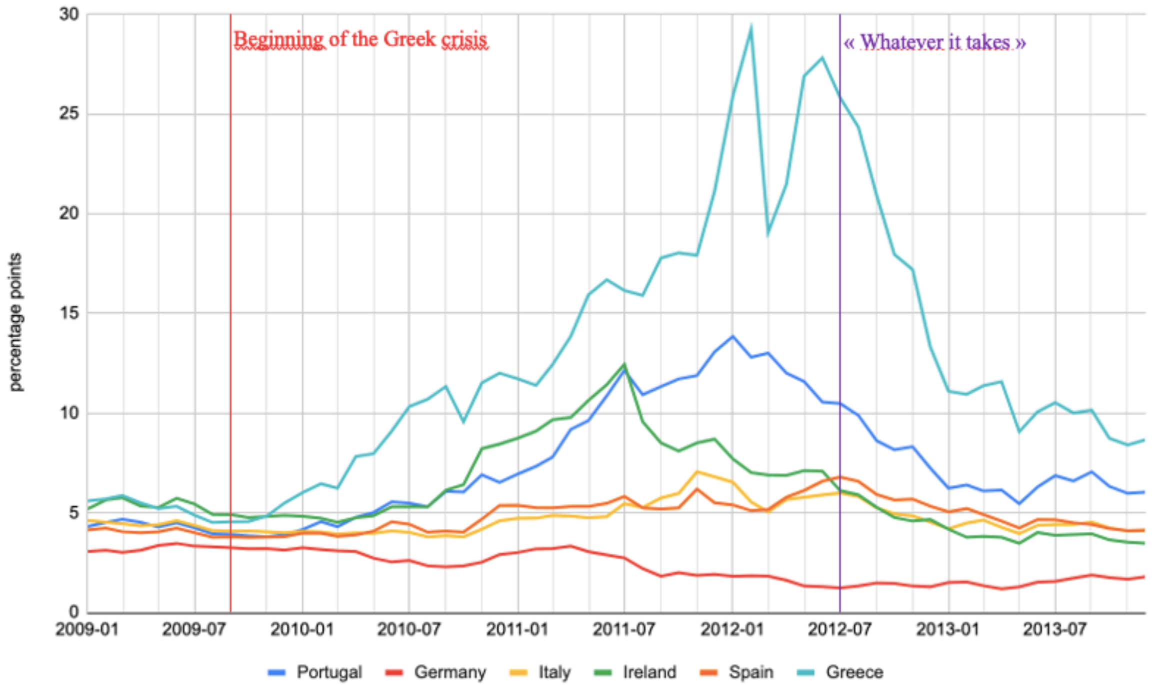
However, **these interventions must not undermine central bank's credibility (**Willems and Zettelmeyer, 2022), that is the central bank must retain control over inflation and some control over real interest rates. If market participants believe that central bank support will lead to uncontrollable inflation, they may still withdraw from sovereign debt, fearing that the value of their holdings will be eroded.

The central bank's power, as discussed above, helps explain why stress only materialized in the euro area countries in 2010, and not in Japan[28], even though the framework developed by Ghosh et al. (2013) identified no remaining fiscal space in Italy, Portugal, Spain, Greece, and Japan (see 2.2.1).

[28] It should be noted that this is not the only reason that could explain that discrepancy between Japan and euro area countries. As Yoshino and Vollmer (2014) note, Japanese investors face higher transaction costs and exchange rate risks when shifting out of domestic bonds. This limited substitutability has likely played a role in preventing sudden exits from JGBs during market stress. In contrast, euro area investors can reallocate funds across member states without currency risk, notably into triple-A-rated sovereigns such as Germany, or the Netherlands.

In Japan, central bank support was already anticipated by investors. In contrast, the ECB had not yet established a mechanism to mitigate liquidity risk, which contributed to the spread escalation in Europe. This changed after Draghi's "whatever it takes" statement, which led to a sharp decline in spreads, as can be observed on Figure 5.

Figure 5 : 10-year Government Bond Spreads over German Bunds, 2009–2013



Source : *FRED*

This comparison shows that by mitigating rollover risk, the central bank not only makes a crisis less likely but also **extends governments' fiscal space**, **raising the debt level at which they can still borrow.**

2.4. Long-Term Determinants and Constraints of Fiscal Space

The following section investigates long-term structural drivers that shape fiscal space. By examining how factors such as the interest-growth differential, the liquidity and safety of government debt, central bank credibility, private saving behavior, and demographic change influence governments' capacity to sustain debt, this analysis highlights how fiscal space is deeply rooted in each country's macro-financial environment and thus far from uniform across advanced economies.

2.4.1. When $r < g$: Free Lunch and Central Bank Support for Fiscal Space

The empirical reality of $r < g$ in advanced economies has led to a rethinking of public debt sustainability, as discussed in Part 1. Blanchard (2019) emphasizes that when the interest rate on government bonds is durably below the growth rate, debt becomes less costly to service, and the primary surplus adjustment required to maintain solvency is correspondingly reduced. This **mechanically expands fiscal space.**

However, **why governments can borrow at such low interest rates remains a central question.**

A growing literature offers a structural explanation: government bonds are not valued solely for their cash flows, but also because they provide **liquidity** and **safety** services to investors. This perspective, rooted in the work of Woodford (1990) and Holmström & Tirole (1998), has been further developed by Berentsen & Waller (2018), Brunnermeier, Merkel & Sannikov (2021), and Reis (2021).

Government bonds are **liquid** because they can be easily traded and converted into cash without large losses. They are considered **safe** because they are backed by the government's ability to tax future income**.** Countries carrying the most these characteristics on their government debt are often characterized as **safe haven**. **By definition, a safe-haven asset must maintain its value (i.e. its purchasing power in a given currency) in bad states of the world (**during an economic crisis for example). Traditional safe havens are the U.S., Germany, Japan, Switzerland and the U.K..

In advanced economies, Venditti, Habib, and Stracca (2020) outlines that **pollical risk rating**, a proxy for the quality of institutions of the issuing country, and the **size of the government debt market** can foster that safe haven status.

This interpretation implies that government bonds carry a **convenience yield, a premium investors are willing to pay for holding a liquid and safe asset.** As Willems and Zettelmeyer (2022) argue, this allows the government to issue debt at rates below the social discount rate, which could be interpretated as a **"free lunch"**.

Crucially, this modifies the traditional understanding of the IGBC. In the standard framework, debt must be backed by the present value of future primary surpluses; Equation (8) can be expressed as:

$$Value\ of\ debt\ stock = E[PV(future\ primary\ surpluses)] \tag{13}$$

But **once we account for the liquidity and safety services** provided by public debt, **its market value reflects not only fiscal backing, but also the value of these service flows**. Equation (8) becomes:

$$Value\ of\ debt\ stock = E[PV(future\ primary\ surpluses)] + E[PV(future\ service\ flow)] \quad (14)$$

This extension has important implications. As we interpret it, obtaining Equation (*14*) requires that $\boldsymbol{E[PV(terminaldebt)] = E[PV(futureserviceflow)]}$, which implies that the transversality condition (i.e., $E[PV(terminaldebt)] = 0$) no longer needs to hold[29] to respect intertemporal solvency.

Crucially, Equation (*14*) implies that **public debt can respect the intertemporal solvency constraint even with a persistent primary deficit**, as long as the service flow component remains sufficiently valuable[30]. We have already mentioned the case of the U.S., but other countries showed an average primary deficit that has been fostered by their safe and liquid status. For example, over the period 1961–2023, the average primary deficit of the U.K., France, and Japan, were respectively of 0.44%, 0.70%, and 0.94%.

Yet, this interpretation does not imply that such deficits can be sustained without limit: they **remain constrained by the magnitude and credibility of the service flow component** attributed to public debt. Jiang et al. (2019) estimated that, for the U.S., the service flow was worth about 65% of GDP on average.

This offers a reinterpretation of Bohn's (1995) framework: **his "positive covariance" between surpluses and debt value can be seen as an early attempt to capture the role of debt as a safe and liquid asset**[31], here captured by the service flow component.

Box 4 : Are There Differences between Traditional Safe Haven Countries ?

U.S. Treasuries remain the strongest safe asset, reflecting the special role of the U.S. economy in global markets during times of financial stress. In crisis episodes, investors engage

[29] This conclusion aligns with our earlier discussion on the inconsistency of transversality condition in Section 1.2.2.; it should be noted that this transversality condition could hold only if $E[PV(futureserviceflow)]$ is equal to 0.

[30] While this perspective reintroduces the IGBC as a relevant identity, it does so under broader assumptions, highlighting that what ensures solvency is not just future fiscal effort, but also the sustained demand for government bonds as safe and liquid assets.

[31] This is aligned with Schoder (2013)'s interpretation of the covariance term as the value of safety attached to government bonds.

in flight-to-safety behavior and buy Treasuries massively. Venditti et al. (2020) explain that this special status is especially fostered by two reasons.

First, the Treasuries market is incomparable in size, depth, and liquidity. Treasuries represent more than 28.6 trillion dollars in circulation, with an average daily turnover exceeding 1.1 trillion dollars, which far surpasses any other sovereign bond market. In comparison, outstanding German Bunds amounted to 1.2 trillion euros at the end of 2023, with a daily volume of around 27 billion euros.

Second, this special status is reinforced by the dominance of the dollar in foreign currency reserves. Since the dollar is used to price goods and contract debt worldwide, economic agents, including consumers and firms, have a permanent need for dollars, even in times of crisis. To meet this need, they hold liquid USD assets, primarily Treasuries. This behavior mechanically reinforces the stability and global demand for Treasuries, which contributes to their status as a global safe asset.

Nenova (2025) also highlights that **the U.S. Treasuries provide a global safe asset**, whereas **German Bunds function more as a regional safe asset**. In times of stress, investors flee into German Bunds and sell government bonds from peripheral euro area countries such as Italy or Spain. These « peripheral » bonds lose their substitutability with Bunds, a mechanism that leads to the « fragmentation » of the euro area government bonds market. The same behavior is observed, although to a lesser extent, with better-rated « core » countries such as France or Belgium, which maintain higher substitutability with Bunds.

This is problematic, because during a crisis, the transmission of monetary policy, for example through a fall in interest rates on safe assets like Bunds, has much weaker effects on the borrowing costs of peripheral countries. This issue is even more relevant given that the gap between core and periphery persists, even after the ECB starts purchasing all euro area sovereign bonds as part of the Public Sector Purchase Programme (PSPP). Fragmentation therefore remains, even outside of immediate market turmoil, and further complicates single monetary policy transmission.

How Central Banks Can Enhance That Service Flow Component

In this context, the **role of the central bank becomes critical**. As Willems and Zettelmeyer (2022) emphasize, a **credible central bank enhances the service flow component of government debt by ensuring both its safety and its liquidity.**

Safety depends on the ability to conduct countercyclical monetary policy credibly: when a recession hits, a central bank that can lower rates without losing control of inflation expectations boosts the price of existing fixed-rate bonds, providing capital gains to holders. Investors value such assets because they serve as a hedge, generating gains when other investments may be losing value (Brunnermeier, Merkel & Sannikov, 2021; Cochrane, 2021a). If the central bank lacks credibility, it may hesitate to cut rates, fearing inflation, and thus fails to maintain this safety profile.

Beyond maintaining safety, **credible central banks also enhance liquidity**. This occurs in two ways.

First, they support the development of deep, liquid local-currency debt markets by maintaining stable inflation expectations, a factor that encourages investors to hold and trade government bonds. Studies such as Jeanne (2003), Du, Pflueger & Schreger (2020), and Engel & Park (2022) have shown that credible central banks are essential for the growth of these markets.

Second, central banks can act as market makers of last resort, stepping in during periods of stress to ensure that government bonds remain tradable.

This ability to guarantee liquidity is critical because it means that investors can convert their bonds to cash even during a crisis.

To formalize the central bank's contribution to this mechanism, Willems and Zettelmeyer (2022) introduce the idea that its credibility can be viewed as a **fiscal asset**. In their view, a credible central bank possesses **reputational capital**, accumulated over time through a consistent record of maintaining low and stable inflation. This credibility, much like a financial asset, can be used to support fiscal policy by extending fiscal space.

However, this asset is exhaustible: if governments rely too heavily on central bank support, they may erode the institution's credibility. Once that credibility is lost, the debt loses its safe asset status, and the convenience yield vanishes, **along with the fiscal space it enabled.**

2.4.2. Private Saving Patterns and Their Impact on Sovereign Fiscal Capacity

In Part 1, we highlighted, drawing on Brender, Pisani, and Gagna (2021), how structural saving patterns in advanced economies challenge the theoretical relevance of the transversality condition. Here, we extend that analysis by examining how these same behaviors affect the fiscal space and the service flow component, particularly when monetary policy becomes constrained.

Excess Savings: A Source of Deflationary Pressure

In developed economies, households consistently accumulate savings across the life cycle, often favoring low-risk financial assets such as government bonds. This persistent saving behavior creates a structural surplus of funds seeking safe investment outlets.

When these excess savings cannot be absorbed, demand remains chronically insufficient, creating persistent disinflationary or deflationary pressure. This phenomenon has been particularly evident in Japan since the 1990s and can be amplified through mechanisms such as debt deflation or a collapse in money velocity.

Monetary Policy Reaches Its Limits: Zero Lower Bound and Weak Transmission

To avoid deflationary spirals, central banks lower interest rates to stimulate demand and absorb excess savings. However, the transmission of lower rates is often limited and heterogeneous across agents.

Firms may remain reluctant to invest when expected demand is weak, regardless of borrowing costs, a pattern observed throughout the 2010s as many companies preferred to accumulate cash reserves, despite historically low borrowing costs.

Households, too, may respond only partially. Consumer credit, such as credit cards and personal loans, is often priced with higher interest rate premiums, making it less responsive to central bank policy. In contrast, **mortgage loans are highly sensitive to interest rates**. Lower rates can significantly increase debt capacity for homebuyers, but this effect can have **unintended consequences**. If low rates do not stimulate new housing construction, **they may instead drive-up property prices**, benefiting existing homeowners rather than first-time buyers. Although the rise in property values can increase the wealth of homeowners, this does not necessarily translate into higher consumption, as homeowners may choose to maintain their savings instead of spending more.

As a result, low interest rates may end up not having any effect on excess savings.

In open economies, lower interest rates may also weaken the currency, improving competitiveness through the uncovered risk interest parity mechanism. However, if other countries are simultaneously experiencing weak demand, the effectiveness of this external adjustment channel is also limited.

The Government as the Borrower of Last Resort: Absorbing Excess Savings

In such an environment, characterized by weak private demand, constrained monetary policy, and limited external adjustment, the economy risks falling into a prolonged state of low growth and low inflation. When interest rates approach the zero lower bound and savings remain unabsorbed, the economy enters a **liquidity trap**: monetary policy becomes ineffective, and the government must step in as the « borrower of last resort » to sustain aggregate demand.
This role is essential for maintaining full employment and preventing a deflationary spiral.
Such onfiguration supports high public debt levels as long as excess private saving remains strong.
Governments can achieve this by directing public investment (in infrastructure, education) or making direct transfers to households (ideally transfer to households with high consumption propensity), converting excess savings into actual spending.

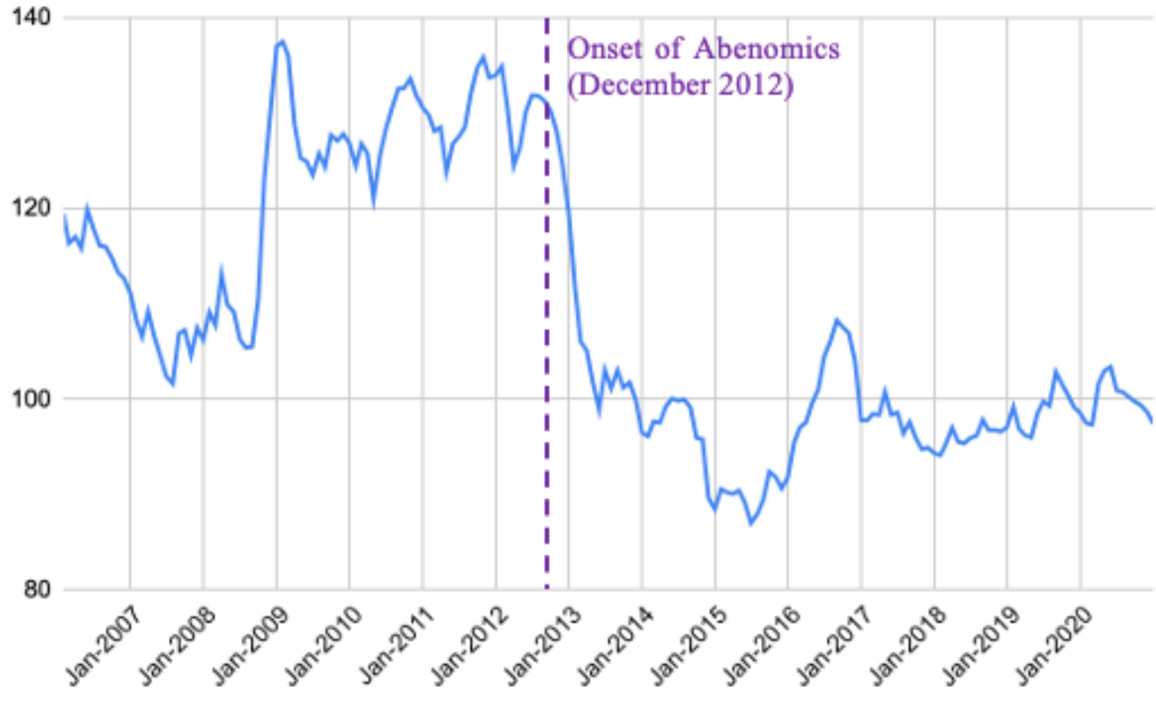
However, if households begin to save less and reduce their amount of wealth to transfer into the future, **the government must also reduce the relative size of its debt in order to prevent imbalances.** Otherwise, excess public borrowing in a context of diminished private savings may create an excess of aggregate demand, putting upward pressure on prices. In response, central banks may raise interest rates, thereby increasing debt servicing costs and ultimately undermining fiscal sustainability.
This lends support to the point made by Bohn and worth restating here (see 1.3.2): *"a rising debt-GDP ratio does not necessarily indicate unsustainable policies, nor does a stable ratio imply sustainability"* (Bohn, 1998). This distinction is especially relevant when the government is the only agent capable of absorbing excess private savings. A rising debt-to-GDP ratio may not be unstable if there is excess savings to be absorbed, and the government is the only agent able to do that. Conversely, a stable debt-to-GDP ratio may not be sustainable if excess savings are diminishing and the government maintains its borrowing level.

Box 5 : Japan's Excess Savings and High Public Debt

Since the 1990s, Japan has shown how government borrowing can maintain demand in an economy trapped by excess savings. Faced with weak private demand, the Japanese government consistently borrowed to offset the savings of households and businesses. Over two decades, public debt surged by over 110% of GDP. Yet this borrowing was not a sign of fiscal irresponsibility but a necessary tool to prevent a deeper deflationary spiral. Without this government intervention, Japan's economy would likely have suffered even more severe deflation and stagnation.

Between 2010 and 2012, however, the current account began to deteriorate, partly due to the appreciation of the yen in the aftermath of the global financial crisis, as reflected in a rising REER. With domestic savings remaining high, this renewed external imbalance revived deflationary pressures. Abenomics marked a turning point by aiming to restore external absorption as a complementary mechanism. By deliberately depreciating the yen, reflected in the decline of Japan's REER shown in **Figure 6**, Abe's administration, that took office in December 2012, enhanced Japan's price competitiveness.

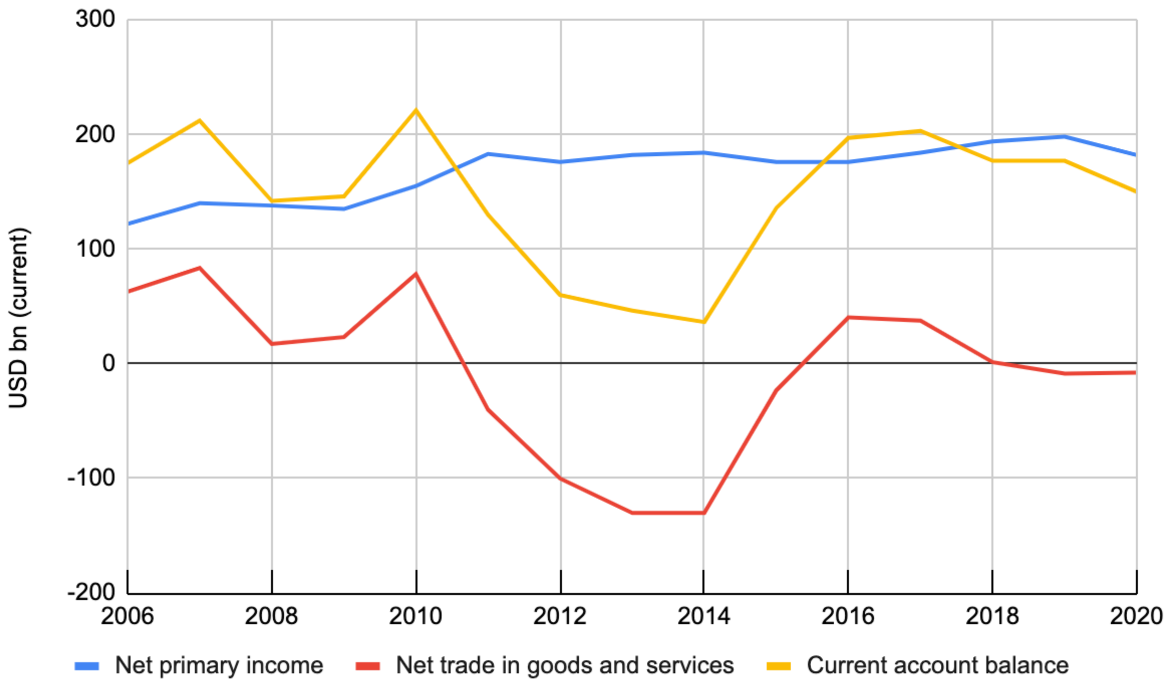
Figure 6 : Japan's Real Effective Exchange Rate (Index 2020 = 100)



Source : *Bank of International Settlements (2025)*

This contributed to a recovery in net exports, which in turn supported an increase in the current account surplus as shown in **Figure 7**.

Figure 7 : Japan's External Balances under Abenomics, 2012–2020



Source : *World Bank Data (2025)*

Since the current account is equal to net capital outflows[32], this implies that a portion of Japan's excess domestic savings was channeled abroad. This external absorption mechanism helped mitigate the need for further increases in public debt.

Why This Modifies Fiscal Space

The ability of governments to act as the borrower of last resort directly expands fiscal space. The logic at work is twofold.

First, in an environment of excess savings, **the government can borrow at low interest rates despite high debt** because these savings constantly seek a safe and liquid asset. Even with a debt-to-GDP ratio exceeding 250%, as in Japan, the government can maintain low borrowing costs because domestic savers continue to buy government bonds.

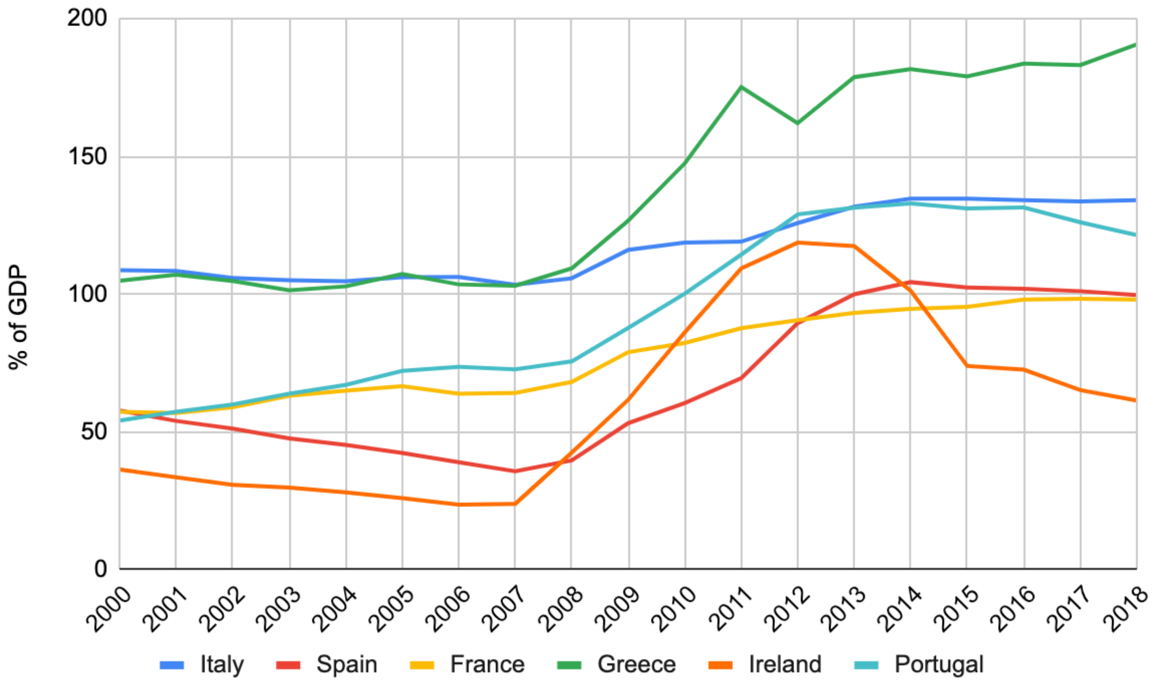
Second, **excess savings generate deflationary pressure, which supports the central bank's ability to maintain low interest rates without risking inflation**. In this context, the central

[32] If we assume error and omissions and capital account equal to 0.

bank can keep borrowing costs low for the government without undermining its credibility. For example, in Japan, despite massive public debt and extensive central bank intervention (through government bond purchases), inflation has remained consistently low.

One should note that **not acting as a borrower of last resort when it is needed can erode the fiscal space and further worsen the sustainability of a government**. The Eurozone's experience in the early 2010s provides an example of the risks of limiting government borrowing too soon. After the 2008 financial crisis, European governments initially used fiscal stimulus to support recovery but quickly shifted to austerity, constrained by the Maastricht criteria. This premature return to fiscal discipline triggered a recession because private demand was too weak to support growth without government spending. By cutting spending and raising taxes while the private sector remained cautious, European governments effectively withdrew support when it was most needed. This led to stagnant growth, rising unemployment. The attempt to contain debt paradoxically worsened it for most countries of the euro area because shrinking GDP increased the debt-to-GDP ratio ($r - g$ increased in the debt dynamics equation).

Figure 8 : Gross Public Debt of European Countries, Percent of GDP

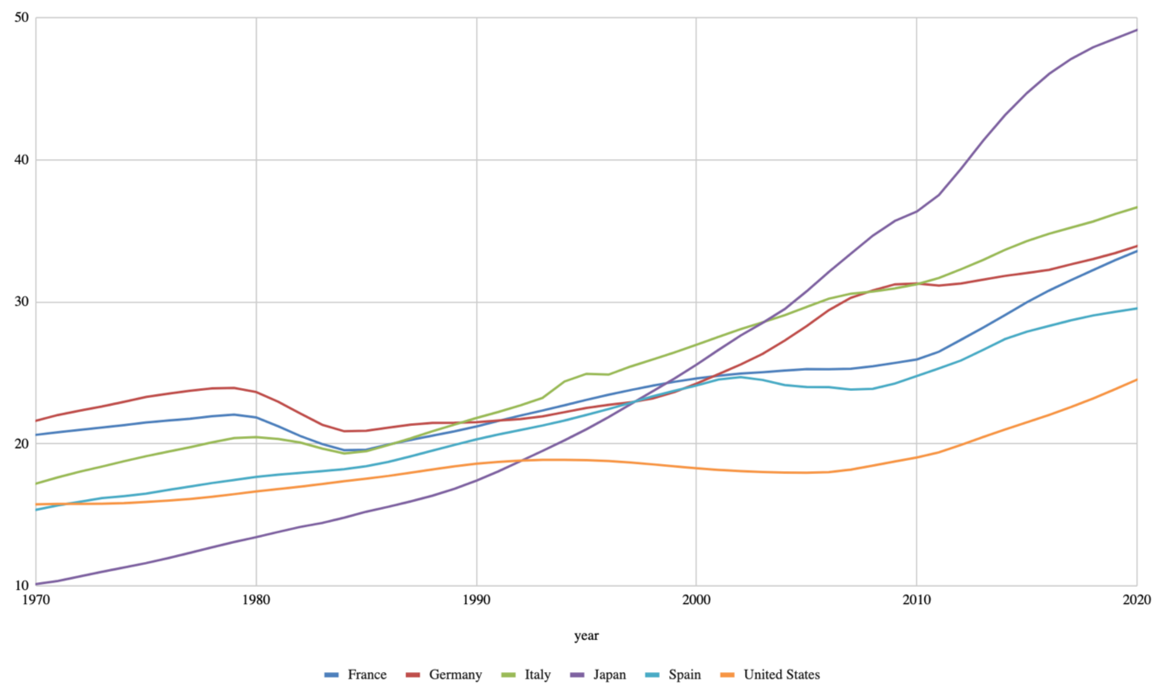


Source : *FMI, Global Debt Database (2025)*

2.4.3. Aging and Fiscal Sustainability: Evidence from the Euro Area's Four Largest Economies

This thesis aims to explore how demographic change, and in particular population aging, affects debt sustainability in advanced economies. Since the 1980s, most developed countries have experienced a steady rise in their **age dependency ratio** (see Figure 9)**, which defines the number of people aged 65+ relative to the working-age population,** largely due to the post-war baby-boom generation reaching retirement age.

Figure 9 : Evolution of the Age Dependency Ratio, 1970–2020



Source : *World Bank (2025)*

Aging is commonly viewed as a contingent liability, placing upward pressure on public finances through **increased health and long-term care spending, as well as rising pension expenditures**. In this context, the impact of aging is central to assessing fiscal space: if long-term demographic shifts structurally deteriorate fiscal balances, they may significantly reduce a country's fiscal room for manoeuvre.

However, this is a complex issue that must be addressed on a country-by-country basis, given the heterogeneity of pension systems and demographic profiles. In this paper, we choose to focus on the four largest euro area economies, Germany, Spain, France and Italy, drawing on the European Commission's 2021 *Ageing Report*, which projects the impact of demographic changes on public spending (pensions, health, care, education) as a share of GDP from 2019 to 2070.

Based on different assumptions about fertility, migration, life expectancy and pension reforms, the report reveals important cross-country divergences. Germany is the only one of the four to experience an overall increase in age-related public expenditure (+3.3 pps of GDP). Italy sees no net change (–0.1 pps), while Spain (–0.4 pps) and France (–0.8 pps) actually record a decline, despite rising age-related costs peaking between 2040 and 2050.

These findings raise the question: why might aging increase expenses and decrease fiscal space in Germany, but not in the other countries?

Spending on healthcare and long-term care is expected to rise across all countries. For healthcare, the increase reflects both demographic factors (greater longevity and years lived with age-related illness) and non-demographic drivers such as medical innovation and higher demand. Long-term care (LTC) costs also grow steadily due to increased dependency and greater use of formal care systems, especially in countries with aging populations and underdeveloped LTC infrastructures.

In contrast, education spending is projected to decline slightly as school-age cohorts shrink, except in Germany.

The most significant differences lie in public pension trends. Between 2019 and 2070, pension spending as a share of GDP is expected to fall by –2.2 pps in France, –2.1 in Spain, and –1.8 in Italy, while it increases by +2.1 pps in Germany.

To understand these dynamics, pension spending can be analytically decomposed into four structural drivers: the old-age dependency ratio, the coverage ratio, the benefit ratio, and the labour market effect.

$$\frac{\text{pension expenditure}}{\text{GDP}}$$

$$= \frac{\text{population} + 65}{\text{population}_{20-64}} \times \frac{\text{number of pensioners}}{\text{population} + 65} \times \frac{\text{average pension income}}{\frac{\text{GDP}}{\text{hours worked}_{20-74}}} \times \frac{\text{population}_{20-64}}{\text{hours worked}_{20-74}}$$

$$= (\text{dependency ratio}) \times (\text{coverage ratio}) \times (\text{benefit ratio}) \times (\text{labor market effect})$$

The **age dependency ratio,** is projected to increase substantially in all four countries. This is the only component that consistently puts upward pressure on pension spending: it contributes +7.1 pps in France, +9.5 in Italy, +9.2 in Spain, and +4.9 in Germany by 2070.

Yet this effect is offset in France, Italy, and Spain through reforms affecting the three other components:

The coverage ratio is reduced via delayed retirement and restricted early exit options.

- In Italy, the introduction of a Notional Defined Contribution (NDC) system and automatic indexation of the retirement age to life expectancy reduce coverage-driven spending by –3.5 pps.
- In France, progressive increases in the statutory retirement age and tighter early retirement eligibility lower it by –2.0 pps.

The benefit ratio, which captures pension generosity, declines significantly in countries implementing structural reforms:

- France reduced its replacement rate, which refers to the ratio of an individual's pension benefit to their pre-retirement earnings.
- Spain extended the contributory period and introduced a sustainability factor applied on pension benefits.
- Italy's NDC model automatically adjusts future pensions downward. These changes are projected to reduce pension spending by –8.3 pps in Spain and –5.9 in France, compared to only –1.4 in Germany.

The labour market effect, contributes to lower pension spending in Italy (–2.9 pps), Spain (–2.1), and to a lesser extent France (–1.0), largely due to declining unemployment and better employment outcomes for older workers. In Germany, this effect is minimal.

Taken together, these adjustments explain how France, Italy, and Spain manage to offset most of the fiscal pressure from population aging, notably by reducing the benefit ratio, and to a lesser extent the coverage ratio and through improved labour market conditions. Germany, by contrast, fails to counteract the demographic drag sufficiently.

However, these developments raise concerns over pension adequacy, particularly in Spain, where benefit generosity was already low. **Further cuts to public pensions could jeopardize income sufficiency in retirement**. In this regard, strengthening private pension systems could offer a complementary solution for countries like France, Italy, and Spain.

Scenario-Based Sensitivities: The Impact of Demographic and Macroeconomic Assumptions on Public Pension Spending

Scenario-based projections highlight how structural shocks, both demographic and macroeconomic, can significantly reshape long-term pension expenditure trajectories. In line with the findings of Ajovalasit, Consigli, Provenzano (2024), the report suggests that declining fertility, reduced net migration, total factor productivity negative shock, or post-COVID scarring effects on productivity tend to reduce GDP growth and increase ageing-related expenses. France appears particularly sensitive to fertility and productivity shocks, while Spain and Italy are more vulnerable to migration-related scenarios due to their reliance on immigration for sustaining contribution bases.

Conversely, favourable scenarios, including higher employment rates among older workers, productivity gains, and reforms that link the statutory retirement age to life expectancy, can ease fiscal pressures. Such measures not only delay pension eligibility but also reinforce the contribution base and boost output. Among these, automatic indexation of retirement age to longevity stands out as the most impactful lever, particularly in France, where no such mechanism currently exists.

Overall, evaluating whether a country implements measures to mitigate the fiscal impact of aging is crucial to assessing debt sustainability, as it provides insight into how demographic change will affect the burden of age-related spending on the primary balance, and thus how fiscal space is likely to evolve in the future.

Empirical Test: Searching for a Long-Term Link Between Aging and Fiscal Balance

Building on the observation that population aging in France has been a persistent trend since the 1980s, we conducted an empirical test to assess whether this demographic shift has exerted a structural influence on the country's primary fiscal balance over the long term. To this end, we applied the Engle–Granger cointegration methodology to test for a stable long-run relationship between the age dependency ratio and the primary balance.

The results were statistically inconclusive: the balance was found to be negative trend-stationary (I(0)), while the age dependency ratio displayed unit root behaviour, precluding standard cointegration analysis. Even when working with the first difference of age dependency, which is positive trend-stationary (I(0)), no robust statistical relationship emerged. These inconclusive findings may reflect the offsetting effect of pension and healthcare reforms, which could have mitigated the fiscal pressures typically associated with demographic aging.

However, due to the limited statistical power of our tests, we refrain from making any definitive claims regarding the existence of a structural relationship.
Full methodological details are provided in Appendix 4.

Conclusion

This thesis argues that sovereign debt sustainability in advanced economies cannot be reduced to a general rule.
Theoretical approaches, such as those based on the IGBC offer conceptual clarity but are often difficult to operationalize, particularly when interest rates fall below growth rates, as it is the case in many developed economies.
Bohn's fiscal reaction approach is more operational, even if it may sometimes appear overly simplistic for complex fiscal realities.
The IMF's Debt Sustainability Analysis represents a significant step forward, combining a common structure adaptable across countries.
However, it is difficult to identify a universal approach or framework that can fully capture the wide array of constraints shaping fiscal trajectories in practice. This is why no debt-to-GDP ratio limit can be universally applied, and a rising debt-to-GDP ratio should not be automatically interpreted as a sign of unsustainability, as has often been argued in the case of Japan, for instance.

In that sense, we showed that evaluating fiscal space can provide valuable insights to complement sustainability assessments based on cross-country frameworks.
A country with significant fiscal space is less exposed to sudden shifts in market sentiment and is more resilient to the materialization of contingent liabilities.
It also has greater leeway to address structural challenges that may otherwise compromise its debt trajectory. By contrast, a country that has exhausted its fiscal space should raise greater sustainability concerns, as it becomes increasingly vulnerable to market stress, investor panic, and potential loss of market access, developments that may ultimately lead to default, a materialization of debt unsustainability.
In practice, we showed that these liquidity-related risks can be mitigated not only by preserving ample fiscal space, but also by lengthening the maturity structure of public debt, reducing

exposure to foreign investors, and, most importantly, by securing the support of a credible central bank acting as lender of last resort.

Beyond liquidity risks, assessing debt sustainability also requires identifying the key factors that determine a government's intrinsic debt limit and compare it with the current debt level.
As illustrated by the stylized model, a negative interest-growth differential and a historically strong primary surplus response to rising debt can increase this limit.
However, rising investor risk aversion in the face of adverse shocks can reduce it, making a previously stable debt trajectory suddenly vulnerable.
In advanced economies, a negative interest-growth differential has in many cases allowed governments to expand their fiscal space or to ease the required adjustment in primary balances.
Yet, understanding debt sustainability requires going further: it is essential to identify the factors that have led to a negative $r- g$.
Monitoring the evolution of these factors will enable a more forward-looking assessment of $r- g$ and, by extension, of debt sustainability itself.

In that context, the perception of government debt as a safe and liquid asset in advanced economies plays a crucial role. This perception gives debt a service value to investors, lowering the yield governments need to offer.
It is strongly reinforced by the credibility of the central bank, which not only ensures financial stability, but also intervenes actively, purchasing sometimes long-term government securities at low yields, without undermining its credibility in low-inflation a environment. Central bank credibility thus becomes a key policy asset: it supports market confidence and enables the government to finance itself at lower interest rates.
By reducing the interest-growth differential and reinforcing the safe and liquid status of government debt, credible monetary intervention effectively reduces the primary surplus required to stabilize debt dynamics.
This mechanism alleviates pressure on fiscal authorities and supports higher debt limits, but its effectiveness remains conditional on preserving monetary credibility and anchoring low inflation expectations.

Persistent excess savings in some advanced economies can also justify a greater role for government borrowing, particularly as a tool to prevent deflationary pressures.
In such contexts, higher debt levels may remain sustainable precisely because government

issuance helps absorb excess liquidity and supports aggregate demand without triggering any inflationary pressure.

The case of Japan is especially illustrative: despite very high public debt ratios, strong domestic demand for safe assets and low inflation have enabled sustained public borrowing at low cost.

Accurately identifying contingent liabilities is also essential to assess whether debt sustainability can be maintained over time. In this thesis, we focused on aging-related pressures, which are likely to weaken the primary balance response by increasing public spending on pensions and healthcare. Our analysis of four euro area countries showed that various reforms can mitigate these fiscal pressures and help preserve long-term fiscal space. However, attention must be paid to the social implications of such reforms, especially when reduced pension generosity risks becoming politically or socially untenable.

Beyond aging, other emerging challenges, though not addressed in this thesis, are likely to increasingly influence fiscal space and debt sustainability.

Climate-related risks and natural disasters are set to raise uncertainty and generate significant contingent liabilities for governments.

Political and geopolitical risks may also undermine institutional trust, particularly if central bank credibility is called into question.

In the case of the United States, recent increases in long-term US treasuries risk premiums[33] suggest that markets are beginning to reassess political and institutional risks.

Concerns have emerged regarding the credibility of the Federal Reserve, particularly following episodes of political pressure around its Chairman that questioned its independence.

Moreover, the geopolitical dimension of sovereign debt holdings has become increasingly relevant. China and Japan, the two largest foreign holders of U.S. Treasuries, have occasionally leveraged this position in trade negotiations, raising the prospect, however remote, of strategic divestments. Such dynamics underscore the potential vulnerability of even the most established issuers when debt sustainability becomes entangled with geopolitical tensions.

[33] One of the factors pushing up term premiums is the growing unpredictability of US economic policy. An index of such uncertainty surged toward a record in April 2025 after President Donald Trump announced sweeping tariffs and then backtracked on some of them. Proposals for tax cuts and a potential need to increase the US government debt limit are also inflating Treasury term premiums.

Bibliography

- Reinhart, C. M. and Rogoff, K. S. (2010). Growth in a time of debt. *American Economic Review*, 100(2), pp. 573–578.
- Herndon, T., Ash, M. and Pollin, R. (2013). Does high public debt consistently stifle economic growth? A critique of Reinhart and Rogoff. *Cambridge Journal of Economics*, 38(2), pp. 257–279.
- Abiad, A. and Ostry, J.D. 2005. *Primary Surpluses and Sustainable Debt Levels in Emerging Market Countries*. IMF Policy Discussion Paper No. 05/6. Washington, D.C.: International Monetary Fund.
- Mendoza, E.G. and Ostry, J.D. 2008. *International Evidence on Fiscal Solvency: Is Fiscal Policy "Responsible"? Journal of Monetary Economics*, 55(6), pp.1081–1093.
- Alogoskoufis, G. 2012. *Greece's Sovereign Debt Crisis: Retrospect and Prospect*. Hellenic Observatory Paper No. 54. London: London School of Economics and Political Science.
- Bohn, Henning. 1998. "The Behavior of U.S. Public Debt and Deficits." *Quarterly Journal of Economics* 113(3): 949–963.
- International Monetary Fund. 2018. *Argentina: Request for Stand-By Arrangement*. IMF Country Report No. 18/219. Washington, D.C.: IMF.
- Blanchard, Olivier. 2019. *Public Debt and Low Interest Rates*. Cambridge, MA: MIT Press.
- International Monetary Fund. 2016. *Arab Republic of Egypt: Request for Extended Arrangement Under the Extended Fund Facility*. IMF Country Report No. 16/337. Washington, D.C.: IMF.
- European Commission. 2015. *Greece – Compliance Report: Fourth Review*. June.
- IMF. 2015. *Greece: Preliminary Draft Debt Sustainability Analysis*. IMF Country Report No. 15/165, June 26.
- Nenova, T. 2025. *Global or regional safe assets: Evidence from bond substitution patterns.* BIS Working Paper No. 1254. Basel: Bank for International Settlements.
- Habib, M.M., Stracca, L. and Venditti, F. 2020. *The fundamentals of safe assets.* ECB Working Paper No. 2355. Frankfurt: European Central Bank

Appendix

Appendix 1. Derivation of Equation (3)

We start from equation (*2*), the government's nominal flow budget constraint:

$$D_t = (1 + r_t)D_{t-1} - PB_t$$

To express this equation in terms of ratios to GDP, we divide both sides by nominal output $P_t Y_t$, where P_t denotes the price level and Y_t the real GDP:

$$\frac{D_t}{P_t Y_t} = \frac{(1 + r_t)D_{t-1}}{P_t Y_t} - \frac{PB_t}{P_t Y_t}$$

The term involving the lagged debt stock is then rewritten using the identity:

$$\frac{(1 + r_t)D_{t-1}}{P_t Y_t} = \frac{1 + r_t}{1 + g_t} \cdot \frac{D_{t-1}}{P_{t-1} Y_{t-1}}$$

where g_t is the nominal growth rate of GDP, defined by:

$$1 + g_t = \frac{P_t Y_t}{P_{t-1} Y_{t-1}}$$

Substituting this expression back into the equation yields:

$$\frac{D_t}{P_t Y_t} = \frac{1 + r_t}{1 + g_t} \cdot \frac{D_{t-1}}{P_{t-1} Y_{t-1}} - \frac{PB_t}{P_t Y_t}$$

Finally, defining the debt-to-GDP ratio as $d_t = D_t/(P_t Y_t)$, and the primary balance-to-GDP ratio as $pb_t = PB_t/(P_t Y_t)$, we obtain the debt dynamics equation for the debt ratio:

$$d_t = \frac{1 + r_t}{1 + g_t} d_{t-1} - pb_t$$

which corresponds to equation (3) in the main text.

Appendix 2. Derivation of Equation (4)

We start from equation (*3*), which expresses the evolution of the debt-to-GDP ratio from one period to the next:

$$d_t = \frac{(1 + r_t)}{(1 + g_t)} d_{t-1} - pb_t$$

To obtain a forward-looking expression for the debt-to-GDP ratio, we iterate this equation forward nnn periods. Applying the equation recursively, we get:

$$d_{t+1} = \frac{(1 + r_{t+1})}{(1 + g_{t+1})} d_t - pb_{t+1}$$

$$d_{t+2} = \frac{1 + r_{t+1}}{1 + g_{t+1}} (\mathrm{d}_{t+1}) - pb_{t+2}$$

$$d_{t+2} = \frac{1 + r_{t+1}}{1 + g_{t+1}} \left(\frac{1 + r_t}{1 + g_t} d_t - pb_{t+1} \right) - pb_{t+2}$$

$$d_{t+3} = \frac{1 + r_{t+2}}{1 + g_{t+2}} \left(\frac{1 + r_{t+1}}{1 + g_{t+1}} \left(\frac{1 + r_t}{1 + g_t} d_t - pb_{t+1} \right) - pb_{t+2} \right) - pb_{t+3}$$

$$d_{t+3} = \left(\prod_{j=0}^{2} \frac{1 + r_{t+j}}{1 + g_{t+j}} \right) d_t - \sum_{j=1}^{3} \left(\prod_{i=j}^{2} \frac{1 + r_{t+i}}{1 + g_{t+i}} \right) pb_{t+j}$$

Generalizing this pattern, we obtain:

$$d_{t+n} = \left(\prod_{j=0}^{n-1} \frac{1 + r_{t+j}}{1 + g_{t+j}} \right) d_t - \sum_{j=1}^{n} \left(\prod_{i=j}^{n-1} \frac{1 + r_{t+i}}{1 + g_{t+i}} \right) pb_{t+j}$$

We now define the **growth-adjusted discount factor** between *t* and *t+j* as:

$$f_{t,t+j} = \prod_{b=t}^{t+j-1} \frac{1 + g_b}{1 + r_b}$$

Note that this is the inverse of the product of interest-growth ratios, such that:

$$\prod_{b=t}^{t+j-1} \frac{1 + r_b}{1 + g_b} = \frac{1}{f_{t,t+j}}$$

Using this notation, the equation for d_{t+n} simplifies to:

$$d_{t+n} = \frac{1}{f_{t,t+n}} d_t - \frac{1}{f_{t,t+n}} \sum_{j=1}^{n} f_{t,t+j}\ pb_{t+j}$$

Solving for d_t, we obtain the forward-looking debt equation (4):

$$d_t = f_{t,t+n}\, d_{t+n} + \sum_{j=1}^{n} f_{t,t+j}\ pb_{t+j}$$

Appendix 3. Proof of Bohn's Sustainability Condition

Bohn starts from the government budget constraint defined as follows:

$$D_t = (D_{t-1} - PB_{t-1}) * (1 + r_t)$$

This equation is a slightly different interpretation of equation (2). In that case, the primary surplus of the previous year is used to repay a part of the debt of the previous year. However, this variation has no major implications for the analysis; it is simply another way of interpreting debt dynamics.

The above equation can be expressed relative to the GDP:

$$d_t = x_t(d_{t-1} - pb_{t-1})$$

Where: $(x_t \approx 1 + r_t - g_t)$.

The change in the debt-GDP ratio is expressed as:

$$\Delta d_t = d_t - d_{t-1} = x_t(d_{t-1} - pb_{t-1}) - d_{t-1}$$

Substituting pb_{t-1} with $(\rho d_{t-1} + \mu_{t-1})$:

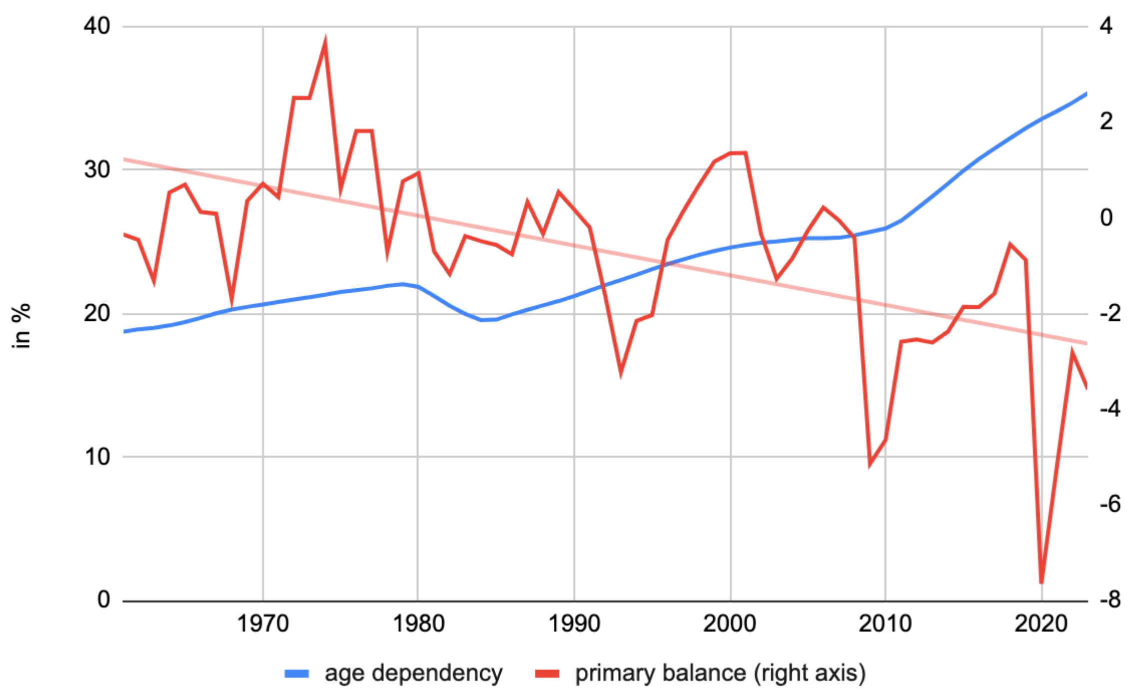
$$\Delta d_t = x_t(d_{t-1} - \rho d_{t-1} - \mu_{t-1}) - d_{t-1} \leftrightarrow \Delta d_t = -[1 - x_t(1-\rho)]d_{t-1} - x_t\mu_{t-1}$$

Assuming stationary x_t $\left(noted\ \underline{x}\right)$ and μ_{t-1}, the debt-to-GDP ratio should be a stationary, mean-reverting process, if $\underline{x}(1-\rho) < 1$. Thus, if $(r < g)$, then $\left(\underline{x} < 1\right)$ and $(\rho > 0)$ is not a necessary but is a **sufficient condition** for sustainability, ensuring the Bohn test remains consistent.

Appendix 4. Investigating the Long-Term Relationship Between Demographic Aging and the Government Primary Balance in France

This appendix explores whether the long-term increase in the old-age dependency ratio has contributed to the decline in France's government primary balance observed since the 1980s. The dependency ratio, defined as the share of individuals aged 65 and over relative to the working-age population (15–64), has followed a clear upward trend since the early 1980s, while the government primary balance has shown a downward trend over the same period.

Figure 10 : Evolution of the Government Primary Balance and the Age Dependency Ratio for France, 1961–2023



Source : *World Bank, IMF (2025)*

The objective is to examine whether these opposing dynamics are statistically related, over the period 1980–2023. If so, it would support the hypothesis that aging puts pressure on public finances and limits fiscal space by weakening the primary balance response.

The first step consists in testing the order of integration of each series. The government primary balance is found to be stationary both with and without trend according to ADF tests (p-value = 0.0121 with no trend; p-value = 0.0050 with trend), and is therefore classified as I(0). The old-age dependency ratio, in contrast, is non-stationary in level, but its first difference (denoted dage) is stationary when a trend is included (ADF p-value = 0.0060; KPSS p-value = 0.1). This suggests that dage is trend-stationary, although for KPSS the high p-value implies that stationarity is not rejected rather than firmly established. Overall, the two variables are not integrated in the same order, so the conditions for a standard Engle-Granger cointegration test are not satisfied.

Table 1 : Results of Stationarity Tests

Variable	Regression Type	ADF Stat	ADF p-value	ADF Conclusion	KPSS Stat	KPSS p-value	KPSS Conclusion
gvt_primary_balance	none	-2.49	0.0121	**Stationary at 5%**	—	—	Test failed (invalid regression type)
	constant	-2.81	0.0559	**Non-stationary at 5%**	0.7703	0.0100	**Non-stationary at 5%**
	constant + trend	-4.16	0.0050	**Stationary at 5%**	0.0761	0.1000	**Stationary at 5%**
dage	none	-0.07	0.6590	**Non-stationary at 5%**	—	—	Test failed (invalid regression type)
	constant	-0.81	0.8141	**Non-stationary at 5%**	0.5903	0.0235	**Non-stationary at 5%**
	constant + trend	-4.11	0.0006	**Stationary at 5%**	0.0672	0.1000	**Stationary at 5%**

Despite this, we attempt to estimate a possible long-run relationship between demographic aging and the government primary balance using an OLS regression with dage as the main explanatory variable and the output gap as a control. In the first regression without time trend, the coefficient on dage is negative and highly significant (–1.7643, $p < 0.001$), which would support the view that aging erodes the primary balance. However, the residuals of the regression are non-stationary (ADF p-value = 0.2645), indicating that the result is likely spurious and does not reflect a stable long-term relationship.

Table 2 : OLS Regression Results (Without Trend) and Residual Stationarity Test

Section	Statistic	Value
Regression Overview	Dependent Variable	gvt_primary_balance
	Number of Observations	44
	R-squared	0.391
	Adjusted R-squared	0.368
	F-statistic	13.08
	Prob (F-statistic)	0.00005
	Durbin-Watson	0.856
	Covariance Type	Robust (HC0)
	Condition Number (Cond. No.)	4.61
Regression Coefficients	Variable	Coefficient
	Constant	-0.7034
	dage	-1.7643
	output_gap	0.5612
ADF Test on Residuals	Test Statistic	-2.0514
	p-value	0.2645
	Conclusion	Residuals **not stationary** at 5%

We then estimate a second specification that includes a linear time trend. In this case, the coefficient on dage becomes statistically insignificant (0.5609, p = 0.355), and the trend absorbs most of the explanatory power (coefficient on trend = –0.9968, p = 0.021). But strong collinearity could exist between dage and the trend, a point confirmed by a high condition number (97.4). Once again, the residuals are non-stationary (ADF p-value = 0.1511), and thus the regression is spurious.

Table 3 : OLS Regression Results (With Trend) and Residual Stationarity Test

Section	Statistic	Value
Regression Overview	Dependent Variable	`gvt_primary_balance`
	Number of Observations	44
	R-squared	0.592
	Adjusted R-squared	0.561
	F-statistic	22.09
	Prob (F-statistic)	8.12e-09
	Durbin-Watson	0.511

	Covariance Type	Robust (HC0)
	Condition Number (Cond. No.)	97.4
Regression Coefficients	Variable	Coefficient
	Constant	0.6607
	`dage`	0.5609
	`output_gap`	0.6208
	`trend`	-0.0968
ADF Test on Residuals	Test Statistic	-2.3675
	p-value	0.1511
	Conclusion	Residuals **not stationary** at 5%

These tests are statistically inconclusive. Although the economic intuition remains plausible, the limitations of our tests do not allow for a robust empirical validation.