Lab 8 Report

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Abstract

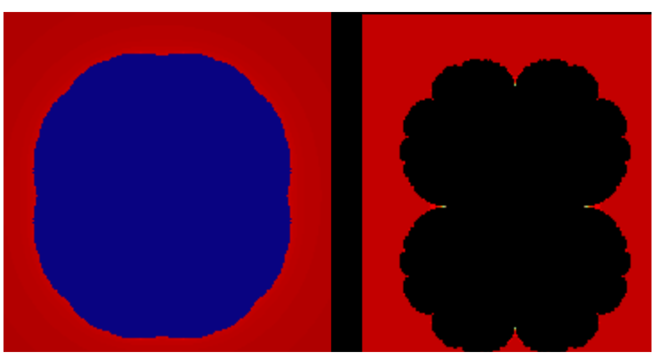
In this lab we explore the Mandelbrot set and how to plot it. It can be constructed by the recursive function $z_{n+1} = z_n^2 + c$. Where converging points are colored back and diverging points are colored in a spectrum of colors depending how fast they reach divergence.

Introduction

The Mandelbrot set is perhaps the most famous fractal and it was discovered by Benoit B. Mandelbrot in 1980. Although it only takes simple mathematics to algebraically understand the equation that constructs the fractal, one can only observe its remarkable nature after a large number of iterations, where many peculiar characteristics appear.

Julia Sets

The Julia set for a constant k is the set where you allow z_0 to vary and for all such initial values, set $c = k$. It produces geometrically simple pictures. These were studied by Dr. Mandelbrot before the discovery and extensive study of the set named after him.



Mandelbrot Set

The Mandelbrot set is the set of points $c \in C$ such that when setting $z_0 = 0$, the function doesn't diverge. For example, consider the two values, $c = 1$ and $c = -1$. Applying the rule to $c = 1$, we get 0,1,2,5,26.... It is clear that this sequence is not bounded and diverges to infinity. Thus we know 1 is not in the set. Applying the rule to -1 we get: 0,-1,0,1,0,.... It is clear that this sequence doesn't diverge to infinity so the -1 is in the set. Another interesting fact about the set is that all points within the set are at most a distance of 2 from the origin. The amazing aspect of the set is the self similarity that occurs as one zooms in. Imagine for instance, zooming in on the boarder of the points in the set and the points outside. No matter how far in you go, you would see smaller versions of the whole set, often called islands, along with other wonderful shapes. One could zoom in infinitely and the whole time see new and clear details.

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Connection to the Logistic Map We can recall that the logistic map is as follows: $x_{n+1} = rx_n(1 - x_n)$. The Mandelbrot set, M, intersects with the real line along the interval $[-2, \frac{1}{4}]$. The values along this interval have a one to one correspondence with the parameters in the logistic map. They are as follows,

$$z = r(\frac{1}{2} - x)$$

$$c = \frac{r}{2}(1 - \frac{r}{2})$$

Another connection is that when plotting the function on the vertical axis, one sees what is essentially the logistic map! It bifurcates at the points where the function is finite.

Plotting

First, we created two vectors, one of x coordinates and another of y coordinates. Then, using the meshgrid function, we created a matrix of the intersection points. Then we applied the iteration function to the set of intersection points. For points that were outside the set, meaning the ones that diverged to ∞, we stored the iteration at which they grew past a boundary such that we could plot the color correctly. The coloring of a point is correlated to the speed of divergence. Then we plotted the function. The figure below is the result.