Mandelbrot Plot Lab

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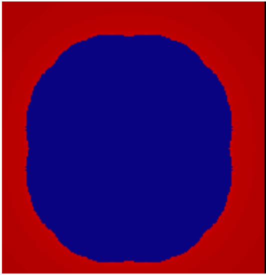
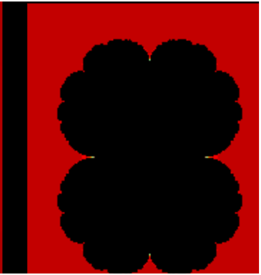
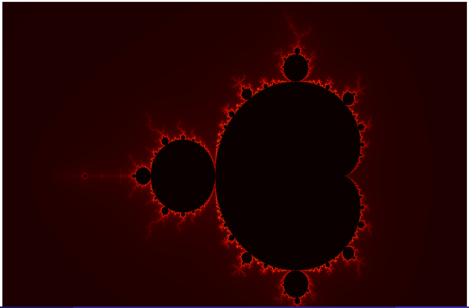
Introduction

The Mandelbrot set M_c associated with a complex constant c is the set of all points (z_0, c) such that after many iterations of the function $z_{n+1} = z_n^2 + c$, z_n stays bounded. An example of such a point is $(-1, 0)$. The Mandelbrot set, along with the related Julia set, is just one type of fractal. Such figures are self similar when zoomed in on. No matter how much one zooms into the Mandelbrot set, they will see more new and interesting shapes appear alongside repetitions of the main shape shown in future slides. These fractals are very useful in describing natural phenomena and have lots of practical applications.

Images







Mandelbrot vs Julia sets

The Mandelbrot set and the Julia sets are closely related. In M, we fix z_0 such that $z_0 = 0$ and vary c. Moving c around the complex plane gives us our set. Interestingly, all the points in M are at most a distance 2 away from the origin. The Julia sets on the other hand occur when you fix c to some constant such that $c = k$ and vary z_0. In this case, we change the initial conditions. This change affects the shape of the figures. As you could see in the previous slide, the Julia sets are more rounded and circular while the Mandelbrot set is more exotic and interesting. Another aspect of the Julia set is that the restriction that all points must be within 2 units of the origin is lifted.

Mandelbrot set and the Logistic Map

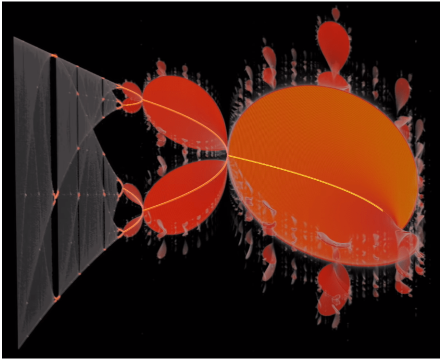
We know that the logistic map is as follows: $x_{n+1} = rx_n(1 - x_n)$. The Mandelbrot set, M, intersects with the real line along the interval $[-2, \frac{1}{4}]$. The values along this interval have a one to one correspondence with the parameters in the logistic map. They are as follows,

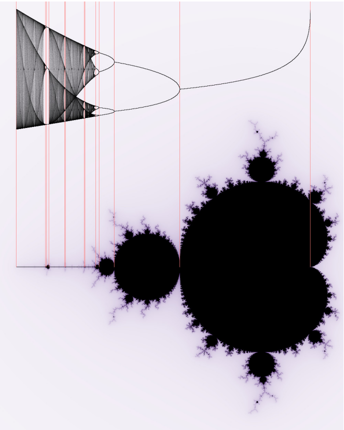
$$z = r(\frac{1}{2} - x)$$

$$c = \frac{r}{2}(1 - \frac{r}{2})$$

Another connection is that plotting the Mandelbrot set in 3-d gives us the logistic map when viewed from the side, complete with the bifurcations.

Mandelbrot and Logistic Map: Visualized





Applications

The Mandelbrot set is part of a larger part of math called Fractal Geometry. This field deals with the figures like fractals. These figures can be found in nature all over the place. Because a fractal is a shape with self similarity, things across the world from the Eifell Tower to the vessels making up your lungs can all be better understood through the lens of Fractal Geometry. One particular application of Fractal Geometry is the idea of image compression. Satellites take extremely high resolution images and in order to transmit them back to land, they must be compressed. One must decompress them to view them and get any sort of useful data and fractal geometry has made that much easier.

Plotting

The plotting made use of torch and matplotlib. First, we created two line vectors, one for x coordinates and another for y. Then we created a matrix filled with their intersection points and these were the ones plugged into our function. Then we actually plugged these points in as our c into the function $z_{n+1} = z_n^2 + c$. If the point diverged after a few iterations, we saved the value and if it converged, we also saved the value in a separate list. Then we plotted the results with all convergent points filled black and divergent ones colored.