Logistic Map Lab 4

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Abstract

We examine the recursive equation of the logistic map, formulated by $x_{n+1} = rx_n(1 - x_n)$ where the variable r is commonly referred as the growth rate and $x_n \in R|0 \leq x_n \leq 1$. The logistic map can be observed extensively in nature, from population modeling to many other dynamical systems. The logistic map is a simple case of a nonlinear system, and its properties can be closely understood throughout many methods. In this lab we studied how changing initial conditions affect the final evolved system. We have found that modifying the initial population x converges the map similarly but changing the growth rate can affect it deeper.

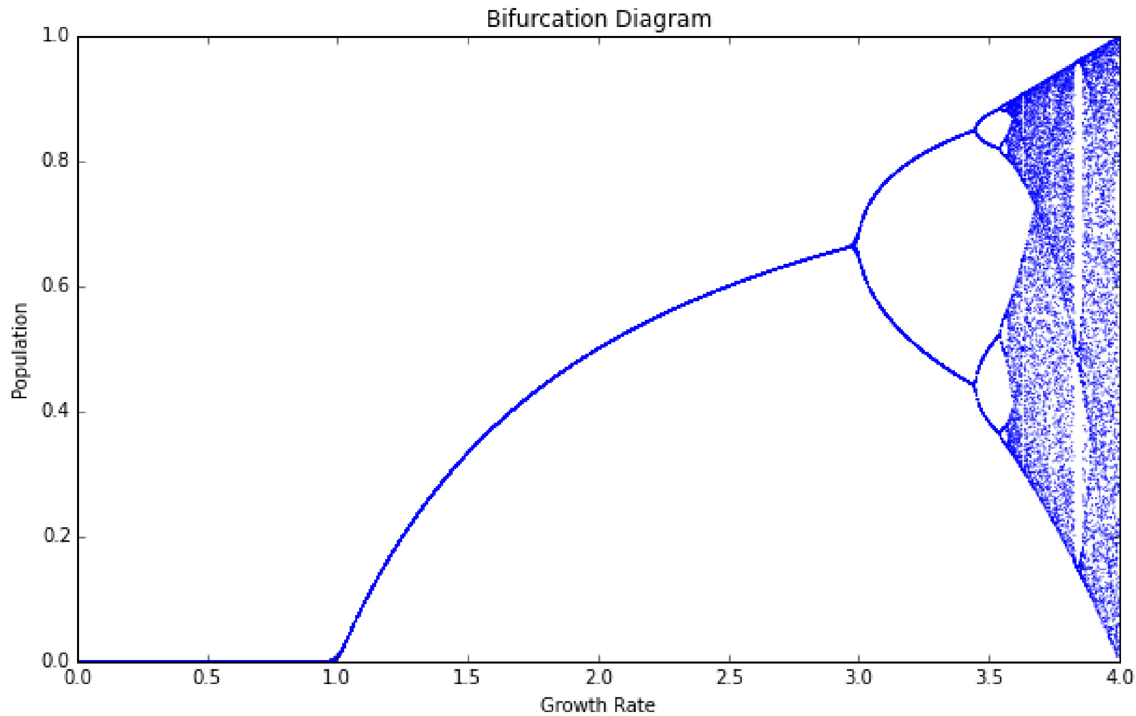
Introduction and Theory

$$x_{n+1} = rx_n(1 - x_n)$$

. For $r < 1$, the map clearly converges to 0. Interpreting it directly, if the growth rate is below 1 for a population, we get that the population is shrinking, which leads to an end value of 0. For $0 \leq r \leq 3$ the value of the map will converge to a number. The resultant graph clearly has a lower and upper bound. Then around 3 is periodic and bifurcates, varying between two values. This specific pattern can be observed as a function of real life human eye flickering.After a certain point both branches bifurcate once again leading to 4 possible values. This process happens up until about 3.57, where the behavior cannot be predicted. This further has applications in building pseudo-random number generators.

Looking at the splits we can find that the ratio of the lengths of any two successive splits are around 4.6, named the Feigenbaum constant.

Below we graph the growth rate versus the total population. The bifurcations can be clearly observed, as well as the chaotic behavior.



Methods

The main objective of the lab was to observe how changes in the initial conditions affect the overall output of the map. We analysed the effects of a wide range of initial growth rate values, and observed the output after about 30 iterations. We followed this by looping through a vector of 10 different r values from 2.5 to 4 and graphing each, where we observe the bifurcations and chaos.

Conclusions

In conclusion, we can see how a simple looking equation can lead to very chaotic and unpredictable outputs. Our lab gave us a closer inspection into how changing the initial conditions of the logistic map affects the final population. We can observe this equation in many places in the natural world, and thus analysis its intricacies is of great interest.