Determinism and termination in the semantics of the Céu programming language

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1 The original formulation

The semantics discussed in this section follows as much as possible the original semantics of Céu presented in [**?**]. Any deviations from original definitions are duly noted in the text.

1.1 Abstract syntax

The *abstract syntax* of Céu programs is given by the following grammar:

(1) `skip` precisa aparecer na gramática já que aprece nos programas em P.
(2) Atribuição agora aparece explicitamente na gramática. Expressões aritméticas e booleanas também estão na gramática mas a sua estrutura interna é omitida.

$$
\begin{array}{rll}
p \in P ::= & \texttt{skip} & \text{do nothing} \\
| & v := a & \text{assignment} \\
| & \texttt{await}(e) & \text{await event} \\
| & \texttt{emit}(e) & \text{emit event} \\
| & \texttt{break} & \text{break innermost loop} \\
| & \texttt{if}\ b\ \texttt{then}\ p_1\ \texttt{else}\ p_2 & \text{conditional} \\
| & p_1 ; p_2 & \text{sequence} \\
| & \texttt{loop}\ p_1 & \text{repetition} \\
| & p_1\ \texttt{and}\ p_2 & \text{par/and} \\
| & p_1\ \texttt{or}\ p_2 & \text{par/or} \\
| & \texttt{fin}\ p_1 & \text{finalization} \\
| & \texttt{@awaiting}(e, n) & \text{awaiting } e \text{ since reaction } n \\
| & \texttt{@emitting}(e, n) & \text{emitting } e \text{ on stack level } n \\
| & p_1\ \texttt{@loop}\ p_2 & \text{unwinded loop}
\end{array}
$$

where $n \in N$ is an integer, $v \in V$ is a memory location (variable) identifier, $e \in E$ is an event identifier, $a \in A$ is an arithmetic expression, $b \in B$ is a boolean expression, and $p, p_1, p_2 \in P$ are programs. We assume the usual structure for arithmetic and boolean expressions, and omit their definition.

1.2 The reaction inner-step relation

The *state* of a Céu program within a reaction is represented by a stack of events $\alpha = e_1 e_2 \ldots e_n \in E^*$ together with a memory map $m : v \to N \in \mathcal{M}$. A *configuration* is a 4-tuple $\langle p, \alpha, m, n\rangle \in \Delta$ that represents the situation of program p waiting to be evaluated in state $\langle \alpha, m\rangle$ and reaction n. Given an initial configuration, each small-step within a program reaction is determined by the reaction-inner-step relation $\rightarrow \in \Delta \times \Delta$ such that $\langle p, \alpha, m, n\rangle \rightarrow \langle p', \alpha', m', n\rangle$ iff a reaction inner-step of program p in state $\langle \alpha, m\rangle$ and reaction number n evaluates to a modified program p' and a modified state $\langle \alpha', m'\rangle$ in the same reaction (n). Since relation $\rightarrow$ can only relate configurations with the same n, we shall write $\langle p, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha', m'\rangle$ for $\langle p, \alpha, m, n\rangle \rightarrow \langle p', \alpha', m', n\rangle$.

Relation $\rightarrow$ is defined inductively with the help of the auxiliary functions *eval*, *blocked*, and *clear*. The *eval* function evaluates arithmetic or boolean expressions on a given memory; we omit its definition and assume that such evaluation is deterministic and always terminates. The *blocked* function is a predicate that determines if all trails of a program p are blocked on a given event stack and reaction number. And the *clear* function extracts the body of `fin` blocks from a given program.

Definition 1.1. Function *blocked*: $P \times E^* \times N \rightarrow \{0, 1\}$ is defined inductively as follows.

Intuitivamente, um programa p está bloqueado se toda trilha de p está (i) aguardando (`@awaiting`) algum evento que não está no topo da pilha ou que foi emitido na reação atual, ou (ii) acabou de emitir um evento (`@emitting`) e esse evento ainda não foi consumido (i.e., está na pilha). É isso?

$$
\begin{aligned}
\mathit{blocked}(\texttt{skip}, e\alpha, n) &= 0\\
\mathit{blocked}(v := a, e\alpha, n) &= 0\\
\mathit{blocked}(\texttt{await}(e'), e\alpha, n) &= 0\\
\mathit{blocked}(\texttt{emit}(e'), e\alpha, n) &= 0\\
\mathit{blocked}(\texttt{break}, e\alpha, n) &= 0\\
\mathit{blocked}(\texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2, e\alpha, n) &= 0\\
\mathit{blocked}(p_1 ; p_2, e\alpha, n) &= \mathit{blocked}(p_1, e\alpha, n)\\
\mathit{blocked}(\texttt{loop}\, p, e\alpha, n) &= 0\\
\mathit{blocked}(p_1\, \texttt{and}\, p_2, e\alpha, n) &= \mathit{blocked}(p_1, e\alpha, n) \cdot \mathit{blocked}(p_2, e\alpha, n)\\
\mathit{blocked}(p_1\, \texttt{or}\, p_2, e\alpha, n) &= \mathit{blocked}(p_1, e\alpha, n) \cdot \mathit{blocked}(p_2, e\alpha, n)\\
\mathit{blocked}(\texttt{fin}\, p_1, e\alpha, n) &= 0\\
\mathit{blocked}(\texttt{@awaiting}(e', n'), e\alpha, n) &= \begin{cases} 1 & \text{if } e \neq e' \text{ or } n = n' \\ 0 & \text{otherwise} \end{cases}\\
\mathit{blocked}(\texttt{@emitting}(n'), e\alpha, n) &= \begin{cases} 1 & \text{if } |e\alpha| \neq n' \\ 0 & \text{otherwise} \end{cases}\\
\mathit{blocked}(p_1\, \texttt{@loop}\, p_2, e\alpha, n) &= \mathit{blocked}(p_1, e\alpha, n)
\end{aligned}
$$

Definition 1.2. Function *clear*: $P \rightarrow P'$ is defined inductively as follows.

Intuitivamente, dado um programa p, a função *clear* concatena em sequência o corpo das *primeiras* ocorrências de `fin` em cada trilha de p, e retorna a sequência resultante. A sintaxe do corpo de instruções `fin` está restrita à programas no conjunto restrito P', i.e., que não contenham instruções `await`, `@awaiting`, `emit`, `@emitting`, `and`, `or` e `fin`. É isso? Se sim, por que a chamada recursiva para *loops* mas não para *if-else*?

$$
\begin{aligned}
\mathit{clear}(\texttt{skip}) &= \texttt{skip}\\
\mathit{clear}(v := a) &= \texttt{skip}\\
\mathit{clear}(\texttt{await}(e')) &= \texttt{skip}\\
\mathit{clear}(\texttt{emit}(e')) &= \texttt{skip}\\
\mathit{clear}(\texttt{break}) &= \texttt{skip}\\
\mathit{clear}(\texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2) &= \texttt{skip}\\
\mathit{clear}(p_1; p_2) &= \mathit{clear}(p_1)\\
\mathit{clear}(\texttt{loop}\, p) &= \mathit{clear}(p)\\
\mathit{clear}(p_1\, \texttt{and}\, p_2) &= \mathit{clear}(p_1); \mathit{clear}(p_2)\\
\mathit{clear}(p_1\, \texttt{or}\, p_2) &= \mathit{clear}(p_1); \mathit{clear}(p_2)\\
\mathit{clear}(\texttt{fin}\, p) &= p\\
\mathit{clear}(\texttt{@awaiting}(e', n')) &= \texttt{skip}\\
\mathit{clear}(\texttt{@emitting}(n')) &= \texttt{skip}\\
\mathit{clear}(p_1\, \texttt{@loop}\, p_2) &= \mathit{clear}(p_1)
\end{aligned}
$$

Ainda na definição da *clear*, será que não é preciso chamar a função recursivamente também para a segunda metade da sequência? Do jeito que está

$$\mathit{clear}((v := a; \texttt{fin}\, p)\ \texttt{and}\ \texttt{skip}) = \texttt{skip}; \texttt{skip}.$$

Não deveria ser

$$\mathit{clear}((v := a; \texttt{fin}\, p)\ \texttt{and}\ \texttt{skip}) = p; \texttt{skip}?$$

Definition 1.3 (Reaction inner-step)**.** Relation $\rightarrow \subseteq \Delta \times \Delta$ is defined inductively as follows.

Await and emit

$$\langle \texttt{await}(e), \alpha, m\rangle \xrightarrow{n} \langle \texttt{@awaiting}(e, n'), \alpha, m\rangle \quad \text{with } n' = n + 1 \tag{1.1}$$

$$\langle \texttt{@awaiting}(e, n'), e\alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, e\alpha, m\rangle \quad \text{if } n' \leq n \tag{1.2}$$

$$\langle \texttt{emit}(e), \alpha, m\rangle \xrightarrow{n} \langle \texttt{@emitting}(n'), e\alpha, m\rangle \quad \text{with } n' = |\alpha| \tag{1.3}$$

$$\langle \texttt{@emitting}(n'), \alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, \alpha, m\rangle \quad \text{if } n' = |\alpha| \tag{1.4}$$

Conditionals

$$\langle \texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1, \alpha, m\rangle \quad \text{if } \mathit{eval}(b, m) = 1 \tag{1.5}$$

$$\langle \texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2, \alpha, m\rangle \xrightarrow{n} \langle p_2, \alpha, m\rangle \quad \text{if } \mathit{eval}(b, m) = 0 \tag{1.6}$$

(1) Adicionamos o mapa de memória (m) à configuração e regras explícitas para atribuição. A avaliação de expressões aritméticas e booleanas está encapsulada na função *eval*.
(2) Adicionamos regras para consumir instruções `skip`.
(3) Adicionamos condições que garantem que a cada passo apenas uma regra é aplicável—não há escolha. Outra forma menos verbosa de fazer isso é dizer que elas devem ser avaliadas na ordem em que foram declaradas. Nesse caso, a primeira que for satisfeita deve ser aplicada. (Não fizemos isso para deixar explícitas as condições de cada regra.)

Sequences

$$\langle \texttt{skip}; p, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m\rangle \tag{1.7}$$

$$\langle v := a; p, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m'\rangle \qquad \text{with } m' = m[v/\mathit{eval}(a)] \tag{1.8}$$

$$\langle \texttt{break}; p, \alpha, m\rangle \xrightarrow{n} \langle \texttt{break}, \alpha, m\rangle \tag{1.9}$$

$$\frac{\langle p_1, \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha', m'\rangle}{\langle p_1; p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1'; p_2, \alpha', m'\rangle} \qquad \text{if } p_1 \neq \texttt{skip}, v := a, \texttt{break} \tag{1.10}$$

Loops

$$\langle \texttt{loop}\, p, \alpha, m\rangle \xrightarrow{n} \langle p \,\texttt{@loop}\, p, \alpha, m\rangle \tag{1.11}$$

$$\langle \texttt{skip}\,\texttt{@loop}\, p, \alpha, m\rangle \xrightarrow{n} \langle \texttt{loop}\, p, \alpha, m\rangle \tag{1.12}$$

$$\langle v := a \,\texttt{@loop}\, p, \alpha, m\rangle \xrightarrow{n} \langle \texttt{loop}\, p, \alpha, m'\rangle \qquad \text{with } m' = m[v/\mathit{eval}(a)] \tag{1.13}$$

$$\langle \texttt{break}\,\texttt{@loop}\, p, \alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, \alpha, m\rangle \tag{1.14}$$

$$\frac{\langle p_1, \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha', m'\rangle}{\langle p_1 \,\texttt{@loop}\, p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1' \,\texttt{@loop}\, p_2, \alpha', m'\rangle} \qquad \text{if } p_1 \neq \texttt{skip}, \; v := a, \texttt{break} \tag{1.15}$$

Par/and

Adicionamos a condição de o lado esquerdo estar bloqueado nas regras 1.17 e 1.19.

$$\langle \texttt{skip and}\; p, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m\rangle \tag{1.16}$$

$$\langle p \;\texttt{and skip}, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1 \tag{1.17}$$

$$\langle v := a \;\texttt{and}\; p, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m'\rangle \qquad \text{with } m' = m[v/\mathit{eval}(a)] \tag{1.18}$$

$$\langle p \;\texttt{and}\; v := a, \alpha, m\rangle \xrightarrow{n} \langle p, \alpha, m'\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1, \; \text{with } m' = m[v/\mathit{eval}(a)] \tag{1.19}$$

$$\langle \texttt{break and}\; p, \alpha, m\rangle \xrightarrow{n} \langle p'; \texttt{break}, \alpha, m\rangle \qquad \text{with } p' = \mathit{clear}(p) \tag{1.20}$$

$$\langle p \;\texttt{and break}, \alpha, m\rangle \xrightarrow{n} \langle p'; \texttt{break}, \alpha, m\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1, \; \text{with } p' = \mathit{clear}(p) \tag{1.21}$$

$$\frac{\langle p_1, \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha', m'\rangle}{\langle p_1 \;\texttt{and}\; p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1' \;\texttt{and}\; p_2, \alpha', m'\rangle} \qquad \text{if } \mathit{blocked}(p_1, \alpha, n) = 0 \;\text{and}\; p_1 \neq \texttt{skip}, \; v := a, \texttt{break} \tag{1.22}$$

$$\frac{\langle p_2, \alpha, m\rangle \xrightarrow{n} \langle p_2', \alpha', m'\rangle}{\langle p_1 \;\texttt{and}\; p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1 \;\texttt{and}\; p_2', \alpha', m'\rangle} \qquad \text{if } \mathit{blocked}(p_1, \alpha, n) = 1 \;\text{and}\; p_2 \neq \texttt{skip}, \; v := a, \texttt{break} \tag{1.23}$$

Par/or

$$(1.24)\qquad \langle \texttt{skip}\ \mathtt{or}\ p, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha, m\rangle \qquad \text{with } p' = \mathit{clear}(p)$$

$$(1.25)\qquad \langle p\ \mathtt{or}\ \texttt{skip}, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha, m\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1, \text{ with } p' = \mathit{clear}(p)$$

$$(1.26)\qquad \langle v := a\ \mathtt{or}\ p, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha, m'\rangle \qquad \text{with } m' = m[v/\mathit{eval}(a)] \text{ and } p' = \mathit{clear}(p)$$

$$(1.27)\qquad \langle p\ \mathtt{or}\ v := a, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha, m'\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1, \text{ with } m' = m[v/\mathit{eval}(a)] \text{ and } p' = \mathit{clear}(p)$$

$$(1.28)\qquad \langle \texttt{break}\ \mathtt{or}\ p, \alpha, m\rangle \xrightarrow{n} \langle p'; \texttt{break}, \alpha, m\rangle \qquad \text{with } p' = \mathit{clear}(p)$$

$$(1.29)\qquad \langle p\ \mathtt{or}\ \texttt{break}, \alpha, m\rangle \xrightarrow{n} \langle p'; \texttt{break}, \alpha, m\rangle \qquad \text{if } \mathit{blocked}(p, \alpha, n) = 1, \text{ with } p' = \mathit{clear}(p)$$

$$(1.30)\qquad \frac{\langle p_1, \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha', m'\rangle}{\langle p_1\ \mathtt{or}\ p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1'\ \mathtt{or}\ p_2, \alpha', m'\rangle} \qquad \text{if } \mathit{blocked}(p_1, \alpha, n) = 0 \text{ and } p_1 \neq \texttt{skip}, v := a, \texttt{break}$$

$$(1.31)\qquad \frac{\langle p_2, \alpha, m\rangle \xrightarrow{n} \langle p_2', \alpha', m'\rangle}{\langle p_1\ \mathtt{or}\ p_2, \alpha, m\rangle \xrightarrow{n} \langle p_1\ \mathtt{or}\ p_2', \alpha', m'\rangle} \qquad \text{if } \mathit{blocked}(p_1, \alpha, n) = 1 \text{ and } p_2 \neq \texttt{skip}, v := a, \texttt{break}$$

The next theorem establishes that the reaction inner-step relation is deterministic, i.e., that it is in fact a *partial* function.

Theorem 1.4 (Determinism of the inner-step relation)**.** *For all p, p_1, $p_2 \in P$, α, α_1, $\alpha_2 \in E^*$, m, m_1, $m_2 \in \mathcal{M}$, and $n \in N$,*

$$\text{if}\quad \langle p, \alpha, m\rangle \xrightarrow{n} \langle p_1, \alpha_1, m_1\rangle \quad \text{and} \quad \langle p, \alpha, m\rangle \xrightarrow{n} \langle p_2, \alpha_2, m_2\rangle,$$
$$\text{then}\quad \langle p_1, \alpha_1, m_1\rangle = \langle p_2, \alpha_2, m_2\rangle.$$

Proof. By induction on the structure of derivations. Suppose

$$d_1 \Vdash \langle p, \alpha, m\rangle \xrightarrow{n} \langle p_1, \alpha_1, m_1\rangle \quad \text{and} \quad d_2 \Vdash \langle p, \alpha, m\rangle \xrightarrow{n} \langle p_2, \alpha_2, m_2\rangle,$$

for some derivations d_1 and d_2. Then there are ten possibilities depending on the structure of p. (Note that p cannot be equal to `skip`, $v := a$, or `break`, as there are no rules to evaluate such programs.)

[Case 1] $p = \texttt{await}(e)$. Then d_1 and d_2 are instances of axiom (1.1), and as such, $p_1 = p_2 = \texttt{@awaiting}(e, n')$ with $n' = n + 1$, and $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 2] $p = \texttt{@awaiting}(e, n')$. Then d_1 and d_2 are instances of axiom (1.2), with $n' \leq n$. Thus $p_1 = p_2 = \texttt{skip}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 3] $p = \texttt{emit}(e)$. Then d_1 and d_2 are instances of axiom (1.3), and as such, $p_1 = p2 = \texttt{@emitting}(n')$ with $n' = |\alpha|$, and $\alpha_1 = \alpha_2 = e\alpha$ and $m_1 = m_2 = m$.

[Case 4] $p = \texttt{@emitting}(e, n')$. Then d_1 and d_2 are instances of axiom (1.4) with $n' = |\alpha|$. Thus $p_1 = p_2 = \texttt{skip}$, $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 5] $p = \texttt{if}\, b\, \texttt{then}\, p'\, \texttt{else}\, p''$.

[Case 5.1] $\mathit{eval}(b, m) = 1$. Then d_1 and d_2 are instances of axiom (1.5), and as such, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 5.2] $\mathit{eval}(b, m) = 0$. Then d_1 and d_2 are instances of axiom (1.6), and as such, $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 6] $p = p'; p''$.

[Case 6.1] $p' = \texttt{skip}$. Then d_1 and d_2 are instances of axiom (1.7), and as such, $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 6.2] $p' = v := a$. Then d_1 and d_2 are instances of axiom (1.8), and as such, $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$ and, as eval is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 6.3] $p' = \texttt{break}$. Then d_1 and d_2 are instances of axiom (1.9), and as such, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 6.4] $p' \neq \texttt{skip}, v := a, \texttt{break}$. Then d_1 and d_2 are instances of rule (1.10). Thus there are derivations d_1' and d_2' such that

$$d_1' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_1', \alpha_1, m_1 \rangle \quad \text{and} \quad d_2' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_2', \alpha_2, m_2 \rangle,$$

for some $p_1', p_2' \in P$. Since $d_1' \prec d_1$ and $d_2' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha_2$, $m_1 = m_2$, and $p_1' = p_2'$, which implies

$$p_1 = p_1'; p'' = p_2'; p'' = p_2.$$

[Case 7] $p = \texttt{loop}\, p'$. Then d_1 and d_2 are instances of axiom (1.11), and as such, $p_1 = p_2 = p'\, \texttt{@loop}\, p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8] $p = p'\, \texttt{@loop}\, p''$.

[Case 8.1] $p = \texttt{skip}\, \texttt{@loop}\, p'$. Then d_1 and d_2 are instances of axiom (1.12), and as such, $p_1 = p_2 = \texttt{loop}\, p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8.2] $p = v := a\, \texttt{@loop}\, p'$. Then d_1 and d_2 are instances of axiom (1.13), and as such, $p_1 = p_2 = \texttt{loop}\, p'$, $\alpha_1 = \alpha_2 = \alpha$, and as eval is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 8.3] $p = \texttt{break}\, \texttt{@loop}\, p'$. Then d_1 and d_2 are instances of axiom (1.14), and as such, $p_1 = p_2 = \texttt{skip}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8.4] $p = p'\, \texttt{@loop}\, p''$ with $p' \neq \texttt{skip}, v := a, \texttt{break}$. Then d_1 and d_2 are instances of rule (1.15). Thus there are derivations d_1' and d_2' such that

$$d_1' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_1', \alpha_1, m_1 \rangle \quad \text{and} \quad d_2' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_2', \alpha_2, m_2 \rangle,$$

for some $p_1', p_2' \in P$. Since $d_1' \prec d_1$ and $d_2' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha$, $m_1 = m_2$, and $p_1' = p_2'$, which implies

$$p_1 = p_1'\, \texttt{@loop}\, p'' = p_2'\, \texttt{@loop}\, p'' = p_2.$$

[Case 9] $p = p' \texttt{ and } p''$.

[Case 9.1] $p' = \texttt{skip}$. Then d_1 and d_2 are instances of axiom (1.16), and as such, $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.2] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = \texttt{skip}$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 9.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, d_1 and d_2 are instances of axiom (1.17), and as such, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.3] $p' = v := a$. Then d_1 and d_2 are instances of axiom (1.18), and as such, $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and as eval is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 9.4] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = v := a$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 9.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of axiom (1.19), and as such, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and as eval is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 9.5] $p' = \texttt{break}$. Then d_1 and d_2 are instances of axiom (1.20), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as clear is a total function, $p_1 = p_2 = \mathit{clear}(p''); \texttt{break}$.

[Case 9.6] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = \texttt{break}$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 9.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of axiom (1.21), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as clear is a total function, $p_1 = p_2 = \mathit{clear}(p'); \texttt{break}$.

[Case 9.7] $p', p'' \neq \texttt{skip}, v := a, \texttt{break}$. If $\mathit{blocked}(p', \alpha, m) = 0$, d_1 and d_2 are instances of (1.22). Thus there are derivations d_1' and d_2' such that

$$d_1' \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha_1, m_1\rangle \quad \text{and} \quad d_2' \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p_2', \alpha_2, m_2\rangle,$$

for some p_1', $p_2' \in P$. Since $d_1' \prec d_1$ and $d_2' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha_2$, $m_1 = m_2$, and $p_1' = p_2'$, which implies

$$p_1 = (p_1' \texttt{ and } p'') = (p_2' \texttt{ and } p'') = p_2.$$

If, however, $\mathit{blocked}(p', \alpha, m) = 1$, d_1 and d_2 are instances of (1.23). Thus there are derivations d_1'' and d_2'' such that

$$d_1'' \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p_1'', \alpha_1, m_1\rangle \quad \text{and} \quad d_2'' \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p_2'', \alpha_2, m_2\rangle,$$

for some p_1'', $p_2'' \in P$. Since $d_1'' \prec d_1$ and $d_2'' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha_2$, $m_1 = m_2$, and $p_1'' = p_2''$, which implies

$$p_1 = (p' \texttt{ and } p_1'') = (p' \texttt{ and } p_2'') = p_2.$$

[Case 10] $p = p' \texttt{ or } p''$.

[Case 10.1] $p' = \texttt{skip}$. Then d_1 and d_2 are instances of axiom (1.24), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as clear is a total, $p_1 = p_2 = \mathit{clear}(p'')$.

[Case 10.2] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = \texttt{skip}$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 10.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of axiom (1.25), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as *clear* is a total function, $p_1 = p_2 = \mathit{clear}(p')$.

[Case 10.3] $p' = v := a$. Then d_1 and d_2 are instances of axiom (1.26), and as such, $\alpha_1 = \alpha_2 = \alpha$, and as *eval* and *clear* are total functions, $m_1 = m_2 = m[v/\mathit{eval}(a)]$ and $p_1 = p_2 = \mathit{clear}(p'')$.

[Case 10.4] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = v := a$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 10.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of axiom (1.27), and as such, $\alpha_1 = \alpha_2 = \alpha$, and as *eval* and *clear* are total functions, $m_1 = m_2 = m[v/\mathit{eval}(a)]$ and $p_1 = p_2 = \mathit{clear}(p')$.

[Case 10.5] $p' = \texttt{break}$. Then d_1 and d_2 are instances of axiom (1.28), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as *clear* is a total function, $p_1 = p_2 = \mathit{clear}(p''); \texttt{break}$.

[Case 10.6] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = \texttt{break}$. If $\mathit{blocked}(p', \alpha, m) = 0$, this case becomes Case 10.7. Otherwise, if $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of axiom (1.29), and as such, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and as *clear* is a total function, $p_1 = p_2 = \mathit{clear}(p'); \texttt{break}$.

[Case 10.7] $p', p'' \neq \texttt{skip}, v := a, \texttt{break}$. If $\mathit{blocked}(p', \alpha, m) = 0$, d_1 and d_2 are instances of (1.30). Thus there are derivations d_1' and d_2' such that

$$d_1' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_1', \alpha_1, m_1 \rangle \quad \text{and} \quad d_2' \Vdash \langle p', \alpha, m \rangle \xrightarrow{n} \langle p_2', \alpha_2, m_2 \rangle,$$

for some p_1', $p_2' \in P$. Since $d_1' \prec d_1$ and $d_2' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha_2$, $m_1 = m_2$, and $p_1' = p_2'$, which implies

$$p_1 = (p_1' \ \texttt{or} \ p'') = (p_2' \ \texttt{or} \ p'') = p_2.$$

If, however, $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of (1.31). Thus there are derivations d_1'' and d_2'' such that

$$d_1'' \Vdash \langle p'', \alpha, m \rangle \xrightarrow{n} \langle p_1'', \alpha_1, m_1 \rangle \quad \text{and} \quad d_2'' \Vdash \langle p'', \alpha, m \rangle \xrightarrow{n} \langle p_2'', \alpha_2, m_2 \rangle,$$

for some p_1'', $p_2'' \in P$. Since $d_1'' \prec d_1$ and $d_2'' \prec d_2$, by induction hypothesis, $\alpha_1 = \alpha_2$, $m_1 = m_2$, and $p_1'' = p_2''$, which implies

$$p_1 = (p' \ \texttt{or} \ p_1'') = (p' \ \texttt{or} \ p_2'') = p_2. \qquad \square$$

The next lemma establishes that given a nontrivial program p either it is possible to advance p by an inner-step or all its trails are blocked, but not both.

Lemma 1.5. *For all $p \in P$, $\alpha \in E^*$, $m \in \mathcal{M}$, and $n \in N$, if $p \neq \texttt{skip}, v := a, \texttt{break}$ then either*

$$\exists \delta \in \Delta(\langle p, \alpha, m \rangle \xrightarrow{n} \delta) \quad \text{or} \quad \mathit{blocked}(p, \alpha, n) = 1,$$

but not both.

Proof. By induction on the structure of programs.

[Case 1] $p = \texttt{await}(e)$. Then by axiom (1.1),

$$\langle \texttt{await}(e), \alpha, m\rangle \xrightarrow{n} \langle \texttt{@awaiting}(e, n'), \alpha, m\rangle = \delta,$$

where $n' = n + 1$. And by Definition 1.1, $\mathit{blocked}(\texttt{await}(e), \alpha, n) = 0$.

[Case 2] $p = \texttt{@awaiting}(e, n')$.

[Case 2.1] $n' < n$. If e is the top-of-stack event in α, in symbols $e = \alpha_{[1]}$, then by axiom (1.2),

$$\langle \texttt{@awaiting}(e, n'), \alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, \alpha, m\rangle = \delta,$$

and by Definition 1.1, $\mathit{blocked}(\texttt{@awaiting}(e, n'), \alpha, n) = 0$.
If, however, $e \neq \alpha_{[1]}$, then there is no such δ, as no rule is applicable. And by Definition 1.1, $\mathit{blocked}(\texttt{@awaiting}(e, n'), \alpha, n) = 1$.

[Case 2.2] $n' = n$. [FIXME: Pelo axioma (1.2),

$$\langle \texttt{@awaiting}(e, n'), \alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, \alpha, m\rangle.$$

E $\mathit{blocked}(\texttt{@awaiting}(e, n'), \alpha, n) = 1$. Ou seja, ambos os lados do "ou" deram verdadeiro, o que invalida o lema.]

[Case 2.3] $n' > n$. If $e = \alpha_{[1]}$ then [FIXME: Não existe tal δ e

$$\mathit{blocked}(\texttt{@awaiting}(e, n'), \alpha, n) = 0.$$

O que, novamente, invalida o lema.]
If, however, $e \neq \alpha_{[1]}$, then there is no such δ (no rule is applicable) and, by Definition 1.1, $\mathit{blocked}(\texttt{@awaiting}(e, n'), \alpha, n) = 1$.

[Case 3] $p = \texttt{emit}(e)$. Then by axiom (1.3),

$$\langle \texttt{emit}(e), \alpha, m\rangle \xrightarrow{n} \langle \texttt{@emitting}(n'), e\alpha, m\rangle = \delta,$$

where $n' = |\alpha|$. And by Definition 1.1, $\mathit{blocked}(\texttt{emit}(e'), e\alpha, n) = 0$.

[Case 4] $p = \texttt{@emitting}(e, n')$.

[Case 4.1] $n' = |\alpha|$. By axiom (1.4),

$$\langle \texttt{@emitting}(e, n'), \alpha, m\rangle \xrightarrow{n} \langle \texttt{skip}, \alpha, m\rangle = \delta.$$

And by Definition 1.1, $\mathit{blocked}(\texttt{@emitting}(e, n'), \alpha, n) = 0$.

[Case 4.2] $n' \neq |\alpha|$. Then there is no such δ (no rule is applicable) and, by Definition 1.1, $\mathit{blocked}(\texttt{@emitting}(e, n'), \alpha, n) = 1$.

[Case 5] $p = \texttt{if}\, b\, \texttt{then}\, p'\, \texttt{else}\, p''$. By axioms (1.5) and (1.6), if $eval(b, m) = 1$, $\delta = \langle p', \alpha, m\rangle$, otherwise $\delta = \langle p'', \alpha, m\rangle$. And by Definition 1.1,

$$blocked(\texttt{if}\, b\, \texttt{then}\, p'\, \texttt{else}\, p'', e\alpha, n) = 0.$$

[Case 6] $p = p'; p''$.

[Case 6.1] $p' = \texttt{skip}$. By axiom (1.7), $\delta = \langle p'', \alpha, m\rangle$, and by Definition 1.1,

$$blocked(\texttt{skip}; p'', \alpha, n) = blocked(\texttt{skip}, \alpha, n) = 0.$$

[Case 6.2] $p' = v \coloneqq a$. By axiom (1.8), $\delta = \langle p'', \alpha, m[v/eval(a)]\rangle$, and by Definition 1.1,

$$blocked(v \coloneqq a; p'', \alpha, n) = blocked(v \coloneqq a, \alpha, n) = 0.$$

[Case 6.3] $p' = \texttt{break}$. By axiom (1.9), $\delta = \langle \texttt{break}, \alpha, m\rangle$, and by Definition 1.1,

$$blocked(\texttt{break}; p'', \alpha, n) = blocked(\texttt{break}, \alpha, n) = 0.$$

[Case 6.4] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $\langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_1, \alpha', m'\rangle$, for some $p'_1 \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$, then by rule (1.10),

$$\langle p'; p'', \alpha, m\rangle \xrightarrow{n} \langle p'_1; p'', \alpha', m'\rangle,$$

and by Definition 1.1,

$$blocked(p'; p'', \alpha, n) = blocked(p', \alpha, n) = 0.$$

If, however, $blocked(p', \alpha, n) = 1$, then there is no such d (no rule is applicable) and by Definition 1.1,

$$blocked(p'; p'', \alpha, n) = blocked(p', \alpha, n) = 1.$$

[Case 7] $p = \texttt{loop}\, p'$. By axiom (1.11), $\delta = \langle p'\, \texttt{@loop}\, p', \alpha, m\rangle$. And by Definition 1.1, $blocked(\texttt{loop}\, p', \alpha, m) = 0$.

[Case 8] $p = p'\, \texttt{@loop}\, p''$.

[Case 8.1] $p' = \texttt{skip}$. By axiom (1.12), $\delta = \langle \texttt{loop}\, p'', \alpha, m\rangle$, and by Definition 1.1,

$$blocked(\texttt{skip}\, \texttt{@loop}\, p'', \alpha, n) = blocked(\texttt{skip}, \alpha, n) = 0.$$

[Case 8.2] $p' = v \coloneqq a$. By axiom (1.13), $\delta = \langle \texttt{loop}\, p'', \alpha, m[v/eval(a)] \rangle$, and by Definition 1.1,

$$blocked(v \coloneqq a\, \texttt{@loop}\, p'', \alpha, n) = blocked(v \coloneqq a, \alpha, n) = 0.$$

[Case 8.3] $p' = \texttt{break}$. By axiom (1.14), $\delta = \langle \texttt{skip}, \alpha, m \rangle$, and by Definition 1.1,

$$blocked(\texttt{break}\, \texttt{@loop}\, p'', \alpha, n) = blocked(\texttt{break}, \alpha, n) = 0.$$

[Case 8.4] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m \rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $\langle p', \alpha, m \rangle \xrightarrow{n} \langle p'_1, \alpha', m' \rangle$, for some $p'_1 \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$, then by rule (1.15),

$$\langle p'\, \texttt{@loop}\, p'', \alpha, m \rangle \xrightarrow{n} \langle p'_1\, \texttt{@loop}\, p'', \alpha', m' \rangle,$$

and by Definition 1.1,

$$blocked(p'\, \texttt{@loop}\, p'', \alpha, n) = blocked(p', \alpha, n) = 0.$$

If, however, $blocked(p', \alpha, n) = 1$, then there is no such d (no rule is applicable) and by Definition 1.1,

$$blocked(p'\, \texttt{@loop}\, p'', \alpha, n) = blocked(p', \alpha, n) = 1.$$

[Case 9] $p = p'$ and p''.

[Case 9.1] $p' = \texttt{skip}$. By axiom (1.16), $\delta = \langle p'', \alpha, m \rangle$, and by Definition 1.1,

$$\begin{aligned} blocked(\texttt{skip and}\ p'', \alpha, n) &= blocked(\texttt{skip}, \alpha, n) \cdot blocked(p'', \alpha, n) \\ &= 0. \end{aligned}$$

[Case 9.2] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$ and $p'' = \texttt{skip}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m \rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $blocked(p', \alpha, n) = 0$, this case becomes Case 9.7.
If, however, $blocked(p', \alpha, n) = 1$, then by axiom (1.17), $\delta = \langle p', \alpha, m \rangle$, and by Definition 1.1,

$$\begin{aligned} blocked(p'\ \texttt{and skip}, \alpha, n) &= blocked(p', \alpha, n) \cdot blocked(\texttt{skip}, \alpha, n) \\ &= 1 \cdot 0 = 0. \end{aligned}$$

[Case 9.3] $p' = v := a$. By axiom (1.18), $\delta = \langle p'', \alpha, m[v/eval(a)]\rangle$, and by Definition 1.1

$$\begin{aligned} blocked(v := a \texttt{ and } p'', \alpha, n) &= blocked(v := a, \alpha, n) \cdot blocked(p'', \alpha, n) \\ &= 0. \end{aligned}$$

[Case 9.4] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = v := a$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $blocked(p', \alpha, n) = 0$, this case becomes Case 9.7.
If, however, $blocked(p', \alpha, n) = 1$, then by axiom (1.19), $\delta = \langle p', \alpha, m[v/eval(a)]\rangle$, and by Definition 1.1,

$$\begin{aligned} blocked(p' \texttt{ and } v := a, \alpha, n) &= blocked(p', \alpha, n) \cdot blocked(v := a, \alpha, n) \\ &= 1 \cdot 0 = 0. \end{aligned}$$

[Case 9.5] $p' = \texttt{break}$. By axiom (1.20), $\delta = \langle clear(p''), \alpha, m\rangle$, and by Definition 1.1,

$$\begin{aligned} blocked(\texttt{break and } p'') &= blocked(\texttt{break}) \cdot blocked(p'', \alpha, n) \\ &= 0. \end{aligned}$$

[Case 9.6] $p' \neq \texttt{skip}, v := a, \texttt{break}$ and $p'' = \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $blocked(p', \alpha, n) = 0$, this case becomes Case 9.7.
If, however, $blocked(p', \alpha, n) = 1$, then by axiom (1.21), $\delta = \langle clear(p'), \alpha, m\rangle$, and by Definition 1.1,

$$\begin{aligned} blocked(p' \texttt{ and } \texttt{break}) &= blocked(p') \cdot blocked(\texttt{break}, \alpha, n) \\ &= 1 \cdot 0 = 0. \end{aligned}$$

[Case 9.7] $p', p'' \neq \texttt{skip}, v := a, \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $\langle p', \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha', m'\rangle$, for some $p_1' \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$, then by rule (1.22),

$$\langle p' \texttt{ and } p'', \alpha, m\rangle \xrightarrow{n} \langle p_1' \texttt{ and } p'', \alpha', m'\rangle.$$

And by Definition 1.1,

$$\begin{aligned} blocked(p' \texttt{ and } p'', \alpha, n) &= blocked(p', \alpha, n) \cdot blocked(p'', \alpha, n) \\ &= 0 \cdot blocked(p'', \alpha, n) = 0. \end{aligned}$$

If, however, $blocked(p', \alpha, n) = 1$ then, again, by induction hypothesis,

$$\exists d'' \in \Delta(\langle p'', \alpha, m\rangle \xrightarrow{n} d'') \quad \text{or} \quad blocked(p'', \alpha, n) = 1,$$

but not both. Suppose $\langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_2, \alpha', m'\rangle$, for some $p''_2 \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$. Then by rule (1.23),

$$\langle p' \texttt{ and } p'', \alpha, m\rangle \xrightarrow{n} \langle p' \texttt{ and } p''_2, \alpha', m'\rangle,$$

and by Definition 1.1,

$$\begin{aligned} blocked(p' \texttt{ and } p'', \alpha, n) &= blocked(p', \alpha, n) \cdot blocked(p'', \alpha, n) \\ &= 1 \cdot 0 = 0. \end{aligned}$$

Suppose $blocked(p'', \alpha, n) = 1$. Then there is no such δ (no rule is applicable) and by Definition 1.1,

$$\begin{aligned} blocked(p' \texttt{ and } p'', \alpha, n) &= blocked(p', \alpha, n) \cdot blocked(p'', \alpha, n) \\ &= 1 \cdot 1 = 1. \end{aligned}$$

[Case 10] $p = p' \texttt{ or } p''$, for some $p' \in P$, By Definition 1.1

$$blocked(p' \texttt{ or } p'', \alpha, n) = blocked(p', \alpha, n) \cdot blocked(p'', \alpha, n)$$

[Case 10.1] $p' = \texttt{skip}$. By axiom (1.24), $\delta = \langle clear(p''), \alpha, m\rangle$, and by Definition 1.1, $blocked(\texttt{skip}, \alpha, n) = 0$.

[Case 10.2] $p'' = \texttt{skip}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $blocked(p', \alpha, m) = 0$, then the derivation of δ is similar to Case 10.7.
If, however, $blocked(p', \alpha, n) = 1$, then by axiom (1.25), $\delta = \langle clear(p'), \alpha, m\rangle$.
Either way, $blocked(\texttt{skip}, \alpha, n) = 0$.

[Case 10.3] $p' = v := a$, for some $v \in V$ and $a \in A$. By axiom (1.26), $\delta = \langle clear(p''), \alpha, m[v/eval(a)]\rangle$, and by Definition 1.1, $blocked(v := a, \alpha, n) = 0$.

[Case 10.4] $p'' = v := a$, for some $v \in V$ and $a \in A$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad blocked(p', \alpha, n) = 1.$$

If $blocked(p', \alpha, m) = 0$, then the derivation of δ is similar to Case 10.7.
If, however, $blocked(p', \alpha, n) = 1$, then by axiom (1.27), $\delta = \langle clear(p'), \alpha, m[v/eval(a)]\rangle$.
Either way, $blocked(v := a, \alpha, n) = 0$.

[Case 10.5] $p' = \texttt{break}$. By axiom (1.28), $\delta = \langle clear(p''), \alpha, m\rangle$, and by Definition 1.1, $blocked(\texttt{break}, \alpha, n) = 0$.

[Case 10.6] $p'' = \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad \mathit{blocked}(p', \alpha, n) = 1.$$

If $\mathit{blocked}(p', \alpha, n) = 0$, then the derivation of δ is similar to Case 10.7.
If, however, $\mathit{blocked}(p', \alpha, n) = 1$, then by axiom (1.29), $\delta = \langle \mathit{clear}(p'), \alpha, m\rangle$.
Either way, $\mathit{blocked}(\texttt{break}, \alpha, n) = 0$.

[Case 10.7] $p' \neq \texttt{skip}, v := a, \texttt{break}$. By induction hypothesis, exactly one of the following hold:

$$\exists d' \in \Delta(\langle p', \alpha, m\rangle \xrightarrow{n} d') \quad \text{or} \quad \mathit{blocked}(p', \alpha, n) = 1.$$

Suppose $\langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_1, \alpha', m'\rangle$, for some $p'_1 \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$. Then by rule (1.30),

$$\langle p' \;\texttt{or}\; p'', \alpha, m\rangle \xrightarrow{n} \langle p'_1 \;\texttt{or}\; p'', \alpha', m'\rangle.$$

And by Definition 1.1,

$$\mathit{blocked}(p' \;\texttt{or}\; p'', \alpha, n) = \mathit{blocked}(p', \alpha, n) \cdot \mathit{blocked}(p'', \alpha, n) = 0.$$

Now suppose $\langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_1, \alpha', m'\rangle$, for some $p''_1 \in P$, $\alpha' \in E^*$, and $m' \in \mathcal{M}$. Then by rule (1.31),

$$\langle p' \;\texttt{or}\; p'', \alpha, m\rangle \xrightarrow{n} \langle p' \;\texttt{and}\; p''_1, \alpha', m'\rangle.$$

And by Definition 1.1,

$$\mathit{blocked}(p' \;\texttt{or}\; p'', \alpha, n) = \mathit{blocked}(p', \alpha, n) \cdot \mathit{blocked}(p'', \alpha, n) = 0.$$

□

1.3 The reaction outer-step relation

From the previous inner-step relation we define an outer-step relation ($\Rightarrow$) that when necessary pops the event stack and advances blocked programs. [TODO: Improve this description.]

Definition 1.6 (Reaction outer-step)**.**

$$\frac{\langle p, \alpha, m\rangle \xrightarrow{n} \langle p', \alpha', m'\rangle}{\langle p, \alpha, m\rangle \overset{n}{\Rightarrow} \langle p', \alpha', m'\rangle} \quad \text{if } \mathit{blocked}(p, \alpha, n) = 0 \tag{1.32}$$

$$\langle p, e\alpha, m\rangle \overset{n}{\Rightarrow} \langle p, \alpha, m\rangle \quad \text{if } \mathit{blocked}(p, \alpha, n) = 1 \tag{1.33}$$

Theorem 1.7 (Determinism of the outer-step relation)**.** *For all p, p_1, $p_2 \in P$, α, α_1, $\alpha_2 \in E^*$, $\alpha = \alpha_{[1]}...\alpha_{[n]}$ ($\alpha_{[1]}$ is in the top of stack α), m, m_1, $m_2 \in \mathcal{M}$, and $n \in N$,*

$$\begin{aligned} \textit{if} \quad \langle p, \alpha, m\rangle \overset{n}{\Rightarrow} \langle p_1, \alpha_1, m_1\rangle \quad \textit{and} \quad \langle p, \alpha, m\rangle \overset{n}{\Rightarrow} \langle p_2, \alpha_2, m_2\rangle, \\ \textit{then} \quad \langle p_1, \alpha_1, m_1\rangle = \langle p_2, \alpha_2, m_2\rangle. \end{aligned}$$

Proof. By induction on the structure of derivations. Suppose

$$d_1 \Vdash \langle p, \alpha, m\rangle \overset{n}{\Rightarrow} \langle p_1, \alpha_1, m_1\rangle \quad \text{and} \quad d_2 \Vdash \langle p, \alpha, m\rangle \overset{n}{\Rightarrow} \langle p_2, \alpha_2, m_2\rangle,$$

for some derivations d_1 and d_2. Then there are ten possibilities depending on the structure of p. (Note that p cannot be equal to `skip`, $v \coloneqq a$, or `break`, as there are no rules to evaluate such programs.)

[Case 1] $p = \mathtt{await}(e)$, for some $e \in E$. By Definition 1.1 $\mathit{blocked}(\mathtt{await}(e), \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.1), $p_1 = p_2 = \mathtt{@awaiting}(e, n')$ with $n' = n + 1$, and $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 2] $p = \mathtt{@awaiting}(e, n')$, for some $e \in E$ and $n' \in N$.

Precisa revisar esse caso

[Case 2.1] $e \neq \alpha_{[1]}$ or $n' \neq |\alpha|$. By Definition 1.1, $\mathit{blocked}(\mathtt{@awaiting}(e, n'), \alpha, m) = 1$, therefore d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = \mathtt{@awaiting}(e, n')$ and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \ldots \alpha_{[n]}$, and $m_1 = m_2 = m$.

[Case 2.2] otherwise, $\mathit{blocked}(\mathtt{@awaiting}(e, n'), \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.2), $p_1 = p_2 = \mathtt{skip}$ and $\alpha_1 = \alpha_2 = \alpha$ and $m_1 = m_2 = m$.

[Case 3] $p = \mathtt{emit}(e)$, for some $e \in E$. By Definition 1.1, $\mathit{blocked}(\mathtt{emit}(e), \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.3), $p_1 = p_2 = \mathtt{@emitting}(n')$, with $n' = |\alpha|$, $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = e\alpha_{[1]} \ldots \alpha_{[n]}$, and $m_1 = m_2 = m$.

[Case 4] $p = \mathtt{@emitting}(e, n')$, for some $e \in E$ and $n' \in N$.

[Case 4.1] $n = |\alpha|$. By Definition 1.1, $\mathit{blocked}(\mathtt{@emitting}(e, n'), \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.4), $p_1 = p_2 = \mathtt{skip}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 4.2] $n' \neq |\alpha|$. By Definition 1.1, $\mathit{blocked}(\mathtt{@emitting}(e, n'), \alpha, m) = 1$, therefore d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = \mathtt{@emitting}(e, n')$ and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \ldots \alpha'_{[n]}$, and $m_1 = m_2 = m$.

[Case 5] $p = \mathtt{if}\, b\, \mathtt{then}\, p'\, \mathtt{else}\, p''$, for some $b \in B$ and $p', p'' \in P$.

[Case 5.1] $\mathit{eval}(b, m) = 1$. By Definition 1.1, $\mathit{blocked}(\mathtt{if}\, b\, \mathtt{then}\, p'\, \mathtt{else}\, p'', \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.5), $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 5.2] $\mathit{eval}(b, m) = 0$. By Definition 1.1, $\mathit{blocked}(\mathtt{skip}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.6), $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 6] $p = p'; p''$, for some $p', p'' \in P$. By Definition 1.1, $\mathit{blocked}(p'; p'', \alpha, m) = \mathit{blocked}(p', \alpha, m)$.

[Case 6.1] $p' = \texttt{skip}$. By Definition 1.1, $\mathit{blocked}(\texttt{skip}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.7), $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 6.2] $p' = v \coloneqq a$, for some $v \in V$ and $a \in A$. By Definition 1.1, $\mathit{blocked}(v \coloneqq a, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.8), $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and, as *eval* is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 6.3] $p' = \texttt{break}$. By Definition 1.1, $\mathit{blocked}(\texttt{break}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.9), $p_1 = p_2 = \texttt{break}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 6.4] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$. By Lemma 1.5, p' either has an applicable rule or $\mathit{blocked}(p', \alpha, m) = 1$.
If $\mathit{blocked}(p', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.10 there are derivations d_1' and d_2' such that

$$d_1' \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p_1', \alpha_1', m_1'\rangle \quad \text{and} \quad d_2' \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p_2', \alpha_2', m_2'\rangle,$$

for some $p_1', p_2' \in P, \alpha_1', \alpha_2' \in E$ and $m_1', m_2' \in \mathcal{M}$. By Theorem 1.4 $d_1' = d_2'$, therefore $\alpha_1' = \alpha_2'$, $m_1' = m_2'$, and $p_1' = p_2'$, which implies

$$p_1 = p_1'; p'' = p_2'; p'' = p_2.$$

If, otherwise, $\mathit{blocked}(p', \alpha, m) = 1$, then d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = p'; p''$ and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \dots \alpha_{[n]}$, and $m_1 = m_2 = m$.

[Case 7] $p = \texttt{loop}\, p'$. By Definition 1.1, $\mathit{blocked}(\texttt{loop}\, p', \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.11), $p_1 = p_2 = p' \,\texttt{@loop}\, p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8] $p = p' \,\texttt{@loop}\, p''$. By Definition 1.1, $\mathit{blocked}(p' \,\texttt{@loop}\, p'', \alpha, m) = \mathit{blocked}(p', \alpha, m)$.

[Case 8.1] $p' = \texttt{skip}$. By Definition 1.1, $\mathit{blocked}(\texttt{skip}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.12), $p_1 = p_2 = \texttt{loop}\, p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8.2] $p' = v \coloneqq a$, for some $a \in A, v \in V$. By Definition 1.1, $\mathit{blocked}(v \coloneqq a, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.13), $p_1 = p_2 = \texttt{loop}\, p''$, $\alpha_1 = \alpha_2 = \alpha$, and, as *eval* is a total function, $m_1 = m_2 = m[v/\mathit{eval}(a)]$.

[Case 8.3] $p' = \texttt{break}$. By Definition 1.1, $\mathit{blocked}(\texttt{break}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.14), $p_1 = p_2 = \texttt{skip}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 8.4] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$ and $p'' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$. By Lemma 1.5, p' either has an applicable rule or $\mathit{blocked}(p', \alpha, m) = 1$.

If $blocked(p', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.15 there are derivations d'_1 and d'_2 such that

$$d'_1 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_1, \alpha'_1, m'_1\rangle \quad \text{and} \quad d'_2 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_2, \alpha'_2, m'_2\rangle,$$

for some p'_1, $p'_2 \in P, \alpha'_1, \alpha'_2 \in E$ and $m'_1, m'_2 \in \mathcal{M}$. By Theorem 1.4 $d'_1 = d'_2$, therefore $\alpha'_1 = \alpha'_2$, $m'_1 = m'_2$, and $p'_1 = p'_2$, which implies

$$p_1 = p'_1 \,\texttt{@loop}\, p'' = p'_2 \,\texttt{@loop}\, p'' = p_2.$$

If, otherwise, $blocked(p', \alpha, m) = 1$, then d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = p' \,\texttt{@loop}\, p''$ and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \ldots \alpha_{[n]}$, and $m_1 = m_2 = m$.

[Case 9] $p = p' \,\texttt{and}\, p''$. By Definition 1.1, $blocked(p' \,\texttt{and}\, p'', \alpha, m) = blocked(p', \alpha, m) \cdot blocked(p'', \alpha, m)$.

[Case 9.1] $p' = \texttt{skip}$. By Definition 1.1, $blocked(\texttt{skip}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.16), $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.2] $p'' = \texttt{skip}$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(\texttt{skip}, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 9.7. Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.17, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.3] $p' = v \coloneqq a$, for some $v \in V$ and $a \in A$. By Definition 1.1, $blocked(v \coloneqq a, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by rule (1.18), $p_1 = p_2 = p''$, $\alpha_1 = \alpha_2 = \alpha$, and, as $eval$ is a total function, $m_1 = m_2 = m[v/eval(a)]$.

[Case 9.4] $p'' = v \coloneqq a$, for some $v \in V$, $a \in A$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(v \coloneqq a, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 9.7. Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.19, $p_1 = p_2 = p'$, $\alpha_1 = \alpha_2 = \alpha$, and, as $eval$ is a total function, $m_1 = m_2 = m[v/eval(a)]$.

[Case 9.5] $p' = \texttt{break}$. By Definition 1.1, $blocked(\texttt{break}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by rule (1.20), $p_1 = p_2 = clear(p''); \texttt{break}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.6] $p'' = \texttt{break}$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(\texttt{break}, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 9.7. Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.21, $p_1 = p_2 = clear(p'); \texttt{break}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 9.7] $p' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$ and $p'' \neq \texttt{skip}, v \coloneqq a, \texttt{break}$. By Lemma 1.5, p either has an applicable rule or $blocked(p, \alpha, m) = 1$.
If $blocked(p', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.22 there are derivations d'_1 and d'_2 such that

$$d'_1 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_1, \alpha'_1, m'_1\rangle \quad \text{and} \quad d'_2 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_2, \alpha'_2, m'_2\rangle,$$

for some $p'_1,\ p'_2 \in P, \alpha'_1, \alpha'_2 \in E$ and $m'_1, m'_2 \in \mathcal{M}$. By Theorem 1.4 $d'_1 = d'_2$, therefore $\alpha'_1 = \alpha'_2$, $m'_1 = m'_2$, and $p'_1 = p'_2$, which implies

$$p_1 = p'_1 \text{ and } p'' = p'_2 \text{ and } p'' = p_2.$$

If, otherwise, $blocked(p', \alpha, m) = 1$, then there are two cases. Either $blocked(p'', \alpha, m) = 0$ or $blocked(p'', \alpha, m) = 1$.
If $blocked(p'', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.23 there are derivations d''_1 and d''_2 such that

$$d''_1 \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_1, \alpha''_1, m''_1\rangle \quad \text{and} \quad d''_2 \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_2, \alpha''_2, m''_2\rangle,$$

for some $p''_1,\ p''_2 \in P, \alpha''_1, \alpha''_2 \in E$ and $m''_1, m''_2 \in \mathcal{M}$. By Theorem 1.4 $d''_1 = d''_2$, therefore $\alpha''_1 = \alpha''_2$, $m''_1 = m''_2$, and $p''_1 = p''_2$, which implies

$$p_1 = p' \text{ and } p''_1 = p' \text{ and } p''_2 = p_2.$$

If, otherwise, $blocked(p'', \alpha, m) = 1$, then d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = p'$ and p'' and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \ldots \alpha_{[n]}$, and $m_1 = m_2 = m$.

[Case 10] $p = p'$ $\texttt{or}$ p''. By Definition 1.1, $blocked(p'\ \texttt{or}\ p'', \alpha, m) = blocked(p', \alpha, m) \cdot blocked(p'', \alpha, m)$.

[Case 10.1] $p' = \texttt{skip}$. By Definition 1.1, $blocked(\texttt{skip}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by axiom (1.24), $p_1 = p_2 = clear(p'')$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 10.2] $p'' = \texttt{skip}$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(\texttt{skip}, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 10.7. Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.25, $p_1 = p_2 = clear(p')$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 10.3] $p' = v \coloneqq a$, for some $v \in V$ and $a \in A$. By Definition 1.1, $blocked(v \coloneqq a, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by rule (1.26), $p_1 = p_2 = clear(p'')$, $\alpha_1 = \alpha_2 = \alpha$, and, as $eval$ is a total function, $m_1 = m_2 = m[v/eval(a)]$.

[Case 10.4] $p'' = v := a$, for some $v \in V$, $a \in A$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(v := a, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 10.7.
Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.27, $p_1 = p_2 = clear(p')$, $\alpha_1 = \alpha_2 = \alpha$, and, as *eval* is a total function, $m_1 = m_2 = m[v/eval(a)]$.

[Case 10.5] $p' = \mathtt{break}$. By Definition 1.1, $blocked(\mathtt{break}, \alpha, m) = 0$, therefore d_1 and d_2 are instances of rule (1.32) and, as such, by rule (1.28), $p_1 = p_2 = clear(p''); \mathtt{break}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 10.6] $p'' = \mathtt{break}$. There are two cases. Either $blocked(p', \alpha, m) = 0$ or $blocked(p', \alpha, m) = 1$. By Definition 1.1, $blocked(\mathtt{break}, \alpha, m) = 0$, therefore, in both cases, d_1 and d_2 are instances of rule (1.32).
If $blocked(p', \alpha, m) = 0$, then the derivations of d_1 and d_2 are similar to Case 10.7.
Otherwise, if $blocked(p', \alpha, m) = 1$, then by axiom 1.29, $p_1 = p_2 = clear(p'); \mathtt{break}$, $\alpha_1 = \alpha_2 = \alpha$, and $m_1 = m_2 = m$.

[Case 10.7] $p' \neq \mathtt{skip}, v := a, \mathtt{break}$ and $p'' \neq \mathtt{skip}, v := a, \mathtt{break}$. By Lemma 1.5, p either has an applicable rule or $blocked(p, \alpha, m) = 1$.
If $blocked(p', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.30 there are derivations d'_1 and d'_2 such that

$$d'_1 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_1, \alpha'_1, m'_1\rangle \quad \text{and} \quad d'_2 \Vdash \langle p', \alpha, m\rangle \xrightarrow{n} \langle p'_2, \alpha'_2, m'_2\rangle,$$

for some p'_1, $p'_2 \in P$, $\alpha'_1, \alpha'_2 \in E$ and $m'_1, m'_2 \in \mathcal{M}$. By Theorem 1.4 $d'_1 = d'_2$, therefore $\alpha'_1 = \alpha'_2$, $m'_1 = m'_2$, and $p'_1 = p'_2$, which implies

$$p_1 = p'_1 \;\mathtt{or}\; p'' = p'_2 \;\mathtt{or}\; p'' = p_2.$$

If, otherwise, $blocked(p', \alpha, m) = 1$, then there are two cases. Either $blocked(p'', \alpha, m) = 0$ or $blocked(p'', \alpha, m) = 1$.
If $blocked(p'', \alpha, m) = 0$, then d_1 and d_2 are instances of rule (1.32), and, as such, by rule 1.31 there are derivations d''_1 and d''_2 such that

$$d''_1 \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_1, \alpha''_1, m''_1\rangle \quad \text{and} \quad d''_2 \Vdash \langle p'', \alpha, m\rangle \xrightarrow{n} \langle p''_2, \alpha''_2, m''_2\rangle,$$

for some p''_1, $p''_2 \in P$, $\alpha''_1, \alpha''_2 \in E$ and $m''_1, m''_2 \in \mathcal{M}$. By Theorem 1.4 $d''_1 = d''_2$, therefore $\alpha''_1 = \alpha''_2$, $m''_1 = m''_2$, and $p''_1 = p''_2$, which implies

$$p_1 = p' \;\mathtt{or}\; p''_1 = p' \;\mathtt{or}\; p''_2 = p_2.$$

If, otherwise, $blocked(p'', \alpha, m) = 1$, then d_1 and d_2 are instances of rule (1.33) and, as such, $p_1 = p_2 = p' \;\mathtt{or}\; p''$ and $\alpha_1 = \alpha_2 = \alpha'$, with $\alpha' = \alpha_{[2]} \ldots \alpha_{[n]}$, and $m_1 = m_2 = m$.

□

Theorem 1.8 (Termination of the outer-step relation)**.** *For all $p \in P$, $\alpha \in E$ and $m \in \mathcal{M}$, if $p \neq$ `skip`, $v := a$, `break` then*

$$\exists \delta \in \Delta(\langle p, \alpha, m \rangle \overset{n}{\Rrightarrow} \delta).$$

Proof. [TODO: Directly from Lemma 1.5.] □

1.4 The reaction relation

From the reflexive-transitive closure of the outer-step relation ($\overset{*}{\Rrightarrow}$) we define the reaction relation $\vDash \subseteq \Delta \times (P \times \mathcal{M} \times N)$, which computes a full program reaction. Given an initial configuration, the reaction relation evaluates it until the event stack becomes empty.

Definition 1.9 (Reaction)**.** Let p, $p' \in P$, $\alpha \in E^*$, m, $m' \in \mathcal{M}$. Then

$$\langle p, \alpha, m \rangle \overset{n}{\vDash} \langle p', m' \rangle \quad \text{iff} \quad \langle p, \alpha, m \rangle \overset{*}{\Rrightarrow} \langle p', \varepsilon, m' \rangle.$$

The next two theorems establish, respectively, that reactions are deterministic and always terminate (for the nontrivial programs $p \neq$ `skip`, $v := a$, `break`).

Theorem 1.10 (Determinism of the reaction relation)**.** *For all p, p_1, $p_2 \in P$, $\alpha \in E^*$, m, m_1, $m_2 \in \mathcal{M}$, and $n \in N$,*

$$\begin{aligned} \textit{if} \quad \langle p, \alpha, m \rangle \overset{n}{\vDash} \langle p_1, m_1 \rangle \quad \textit{and} \quad \langle p, \alpha, m \rangle \overset{n}{\vDash} \langle p_2, m_2 \rangle, \\ \textit{then} \quad \langle p_1, m_1 \rangle = \langle p_2, m_2 \rangle. \end{aligned}$$

Proof. [TODO: ?] □

Theorem 1.11 (Termination of the reaction relation)**.** *For all $p \in P$, $\alpha \in E$, and $m \in \mathcal{M}$, if $p \neq$ `skip`, $v := a$, `break` then*

$$\langle p, \alpha, m \rangle \overset{n}{\vDash} \langle p', m' \rangle,$$

for some $p' \in P$ and $m' \in \mathcal{M}$.

Proof. [TODO: ?] □

2 Big-step version of the original formulation

[TODO: Minha ideia aqui é fazer uma versão big-step da formulação original. E no final comparar as duas versões, i.e., mostrar que são equivalentes.]

Definition 2.1. [TODO: Parcial e provavelmente incorreta.]
Empty program

$$\langle \varepsilon, \alpha, m, n \rangle \rightsquigarrow \langle \alpha, m, n \rangle \tag{2.1}$$

Assignment

$$\langle v \coloneqq a, \alpha, m, n\rangle \rightsquigarrow \langle \alpha, m[v/eval(a)], n\rangle \tag{2.2}$$

Conditionals

$$\frac{\langle p_1, \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle}{\langle \texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2, \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle} \quad \text{if } eval(b, m) = 1 \tag{2.3}$$

$$\frac{\langle p_2, \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle}{\langle \texttt{if}\, b\, \texttt{then}\, p_1\, \texttt{else}\, p_2, \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle} \quad \text{if } eval(b, m) = 0 \tag{2.4}$$

Await

$$\frac{\langle \texttt{@awaiting}(e, n+1), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle}{\langle \texttt{await}(e), \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle} \tag{2.5}$$

$$\langle \texttt{@awaiting}(e', n'), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha, m, n\rangle \quad \text{if } e' = e \text{ and } n' < n \tag{2.6}$$

$$\frac{\langle \texttt{@awaiting}(e', n'), \alpha, m, n\rangle \rightsquigarrow \langle \alpha'', m'', n''\rangle}{\langle \texttt{@awaiting}(e', n'), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha'', m'', n''\rangle} \quad \text{if } e' \neq e \text{ or } n' \geq n \tag{2.7}$$

Emit

$$\frac{\langle \texttt{@emitting}(|\alpha|), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle}{\langle \texttt{emit}(e), \alpha, m, n\rangle \rightsquigarrow \langle \alpha', m', n'\rangle} \tag{2.8}$$

$$\langle \texttt{@emitting}(n'), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha, m, n\rangle \quad \text{if } |e\alpha| = n' \tag{2.9}$$

$$\frac{\langle \texttt{@emitting}(n'), \alpha, m, n\rangle \rightsquigarrow \langle \alpha'', m'', n''\rangle}{\langle \texttt{@emitting}(n'), e\alpha, m, n\rangle \rightsquigarrow \langle \alpha'', m'', n''\rangle} \quad \text{if } |e\alpha| \neq n' \tag{2.10}$$