EE512 – Applied Biomedical Signal Processing

Practical session – SVD

Correction

Experiment 1: 12-leads ECG

QUESTION #1.1: Limb leads and augmented limb leads

a) *Explain the singular values of the six first columns.*

The SVD yields to :

0.4798 0.3122 0.0000 0.0000 0.0000 0.0000

Only two singular values are non-zero. Normal, in the standard ECG three of the signals are just linear combinations of three other ones (electrodes I, II, III), which are themselves dependent with respect to the potential reference.

a) *Describe the singular values of the last 6 columns and compute the effective rank with a threshold of 0.98.*

The same operation on the last 6 columns yields:

1.2328 0.7696 0.3568 0.2150 0.1709 0.1097

Which indicates a much weaker dependence. The effective rank test gives:

0.6532 0.9077 0.9624 0.9823 0.9948 1.0000

And thus an effective rank around 4.

Experiment 2: Singular values and process complexity

```
Singular values before: 1.000000, 0.000005, 0.000003, 0.000000
Singular values during: 1.000000, 0.102895, 0.073738, 0.067564
Singular values after: 1.000000, 0.208506, 0.187214, 0.088356
```

QUESTION #2.1: Data pre-processing

a) *Explain from their singular values the relationship between EEG leads before stimulation.*

Before stimulation, leads decomposition shows the 3 smallest singular values suggesting the signals between the leads are very correlated and so very similar.

After removing the mean and normalizing the variances of the EEG signals before stimulation, re-compute the singular value decomposition of the data matrix, and divide the singular values by the largest one.

```
Singular values before: 1.000000, 0.827413, 0.270988, 0.232586
Singular values during: 1.000000, 0.101275, 0.075675, 0.066337
Singular values after: 1.000000, 0.214136, 0.174770, 0.098511
```

b) Explain the difference observed in the singular values.

After zero-meaning and normalization, the singular values of the ECG signals before stimulation appears much higher suggesting that signals between the leads are very different and so independent from each other.

Subtracting the mean values allows one to cancel a common factor from the columns (which could have an influence on the singular values). Normalization (all signals scaled to unit variance) suppresses effects due to amplitude differences in the signals.

QUESTION #2.2: Correlation interpretation

a) Deduce from the singular values the signals are much more correlated during stimulation.

During stimulation, the singluar values are very low compared to the maximum singular value. When evaluating the effective rank of the matrix during the stimulation we see that the rank is 1, suggesting the 4 leads are strongly correlated.

b) Explain in terms of correlation the difference between each case ('before', 'during', 'after'), and that ˝after˝ is between ˝before˝ and ˝during˝, which suggests a remanence effect in the stimulation.

First we can notice that all singular values are significantly non zero prior to stimulation. This indicates the signals are not linearly dependent, and thus not strongly correlated (activities on 4 electrodes).
Then, the 3 smallest singular values are indeed much smaller during stimulation. The signals become a lot more similar during stimulation, because the latter influences all electrodes.
Finally, we see that the situation after stimulation is in between the two former situations. It indicates that the effect of stimulation is still partly present.

Experiment 3: Drift cancellation and frequency component extraction

QUESTION #1: Drift cancellation

Find a window length L, so that the first component obtained using the function SSA_decomposition corresponds to the long-term drift.

The SSA decomposition shows strong first component with a very low frequency corresponding to a long-term drift with a window length of 100 seconds (Figure 1). One can observe the first component signal follows the baseline of the raw signal (Figure 2).

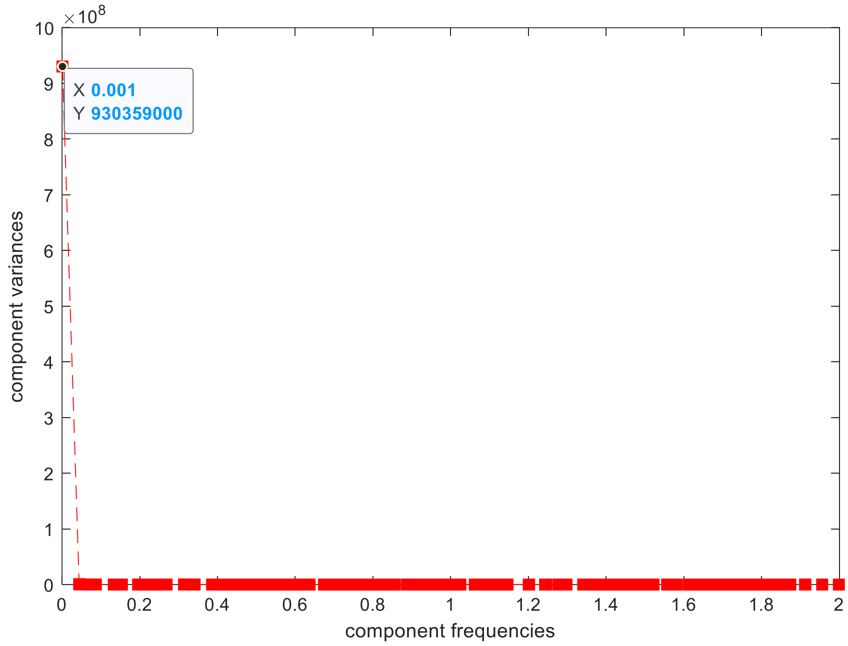

Figure 1. SSA components singular values and their corresponding main frequency.

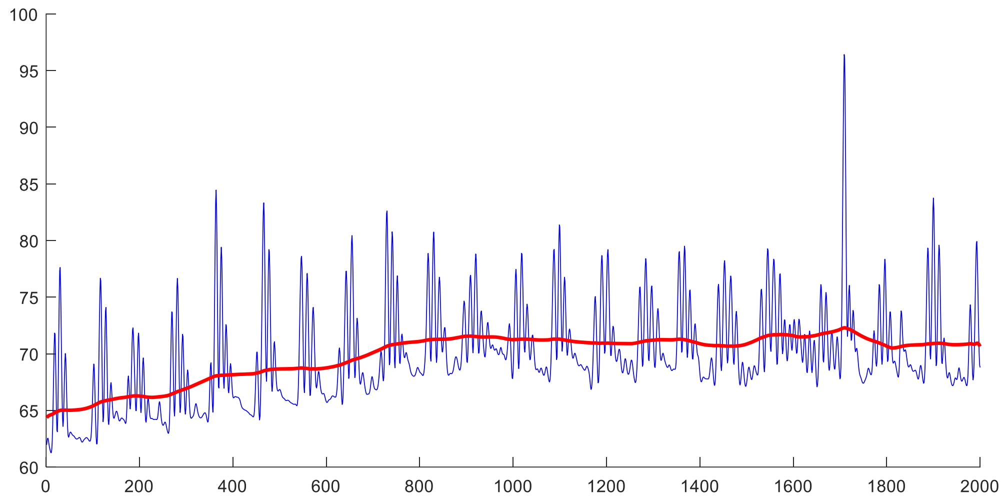

Figure 2. First component signal (red) and raw respiratory signal (blue).

QUESTION #2: Components extraction

Subtract this first component from the signal and decompose the resulting signal using SSA_decomposition with L=40.

- ***a)*** *Which are the components related to the respiration bursts?*

The two first components at 0.044 Hz and 0.045 Hz correspond to the respiration bursts (Figure 3), as we can observe when plotting their sum on the raw signal (Figure 4).

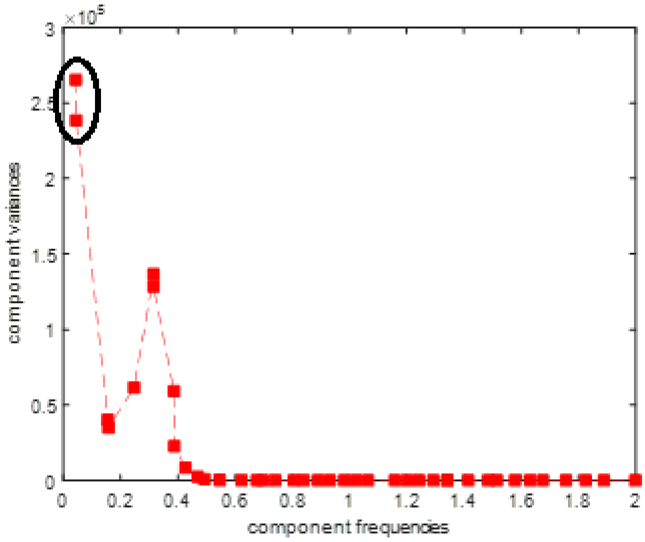

Figure 3. SSA components singular values and main frequency of signal without long term drift.The two first components correspond to the respiration bursts.

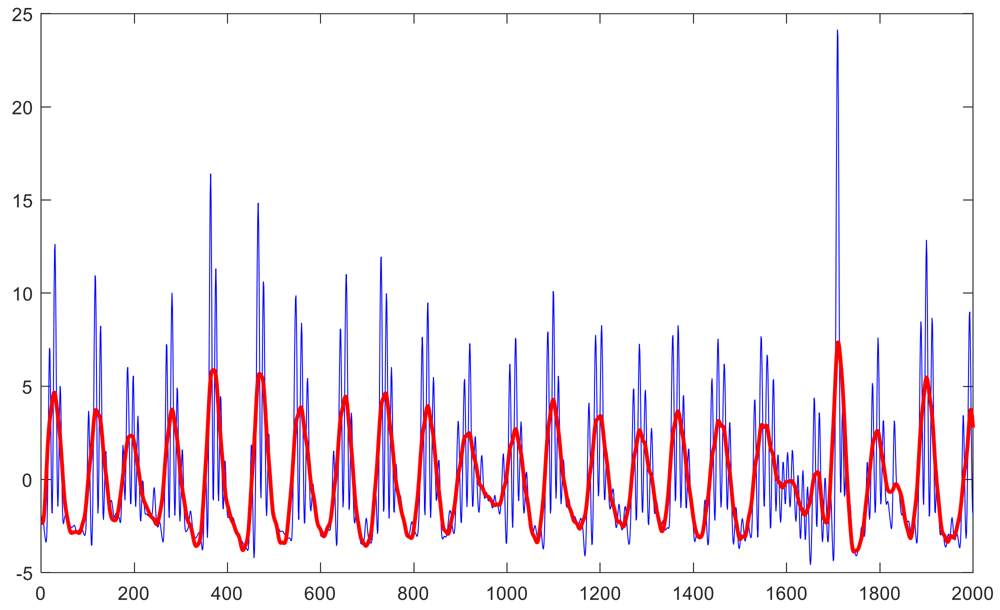

Figure 4. Respiratory signal without long term drift (blue), and the respiratory bursts signal (red).

b) *Which are the components related to respiration itself?*

The components corresponding to the respiration itself are at frequencies 0.314 Hz, 0.248 Hz, and 0.386 Hz (Figure 5). The sum of the components shows the respiratory fluctuations without baseline nor bursts when plotted (Figure 6).

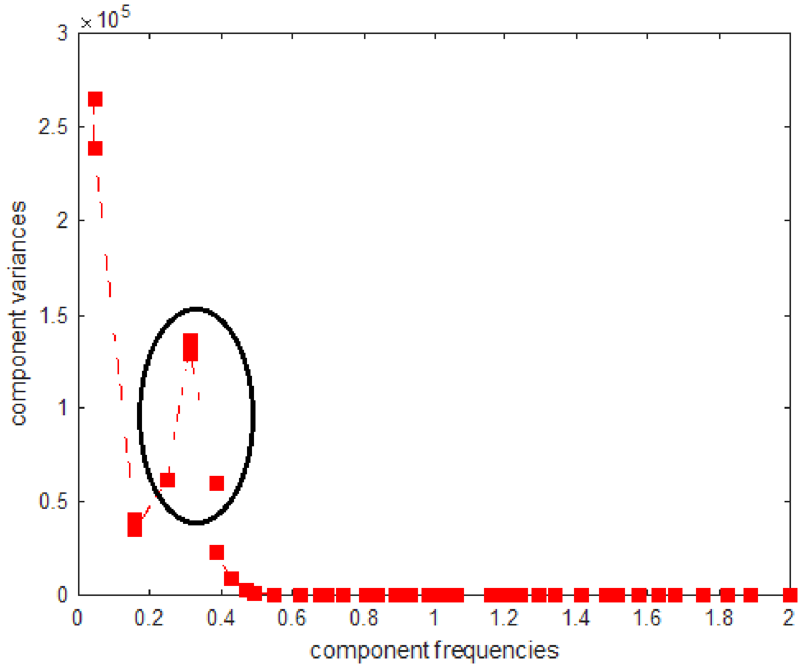

Figure 5. SSA components singular values and main frequency of signal without long term drift.The 3rd, 4th, 5th, and 6th components correspond to the respiration itself.

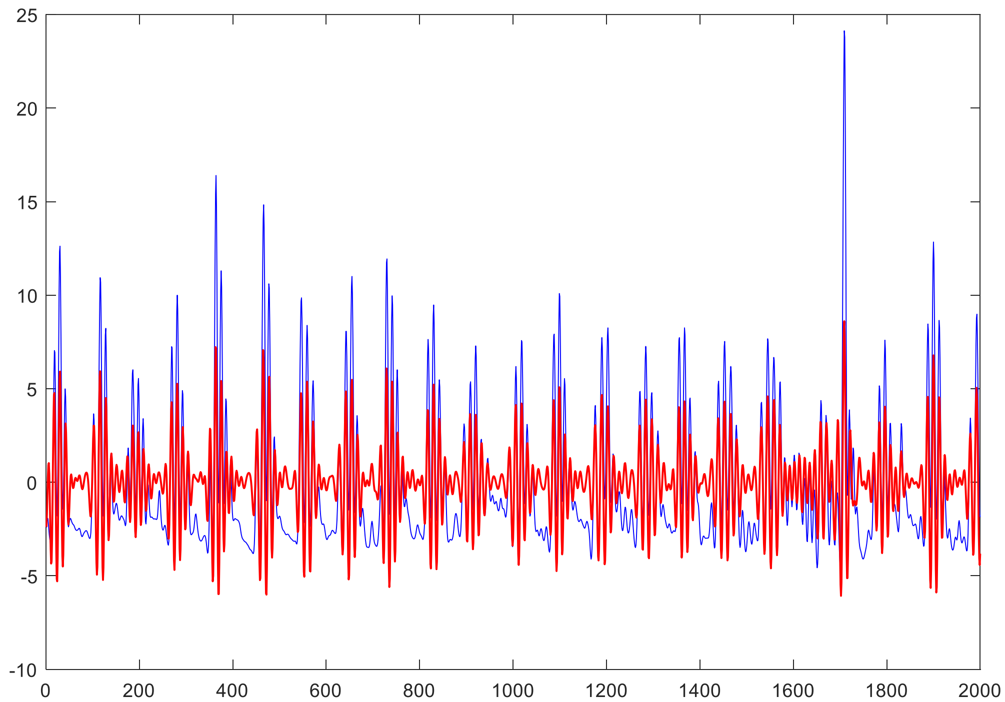

Figure 6. Respiratory signal without long term drift (blue), and the respiratory bursts signal (red).