# RAMS

Reliability, Availability,
Maintainability, and Safety

# Compering the ACER and POT MCMC (acronyms?) Extreme Value Statistics Methods Through Analysis of Commodities Data

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MASTER THESIS

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# Preface

Here, you give a brief introduction to your work. What it is (e.g., a Master's thesis in RAMS at NTNU as part of the study program xxx and...), when it was carried out (e.g., during the autumn semester of 2021). If the project has been carried out for a company, you should mention this and also describe the cooperation with the company. You may also describe how the idea to the project was brought up.

It is suggested that the reader have some statistical knowledge, especially within extreme value and computational statistical. Some financial knowledge and experience with value at risk is desirable.

"When your done: https://www.scribbr.com/thesis/preface-thesis/"

Trondheim, 2016-04-25

(Your signature)

Kristoffer Kofoed Rødvei

# Summary and Conclusions

Here you give a summary of your work and your results. This is like a management summary and should be written in a clear and easy language, without many difficult terms and without abbreviations. Everything you present here must be treated in more detail in the main report. You should not give any references to the report in the summary – just explain what you have done and what you have found out. The Summary and Conclusions should be no more than two pages.

You may assume that you have got three minutes to present to the Rector of NTNU what you have done and what you have found out as part of your thesis. (He is an intelligent person, but does not know much about your field of expertise.)

# Contents

# Chapter 1

# Theory

This chapter gives an introduction of/to the theory behind this thesis/paper. Some of the following sections only give a brief description of the theory whit sources for more in depth explanation. The main focus of this chapter is describing the theory behind the Markov chain Mote Carlo method for solving extreme value problem developed for this paper.

## 1.1 Extreme value theory

peak over threshold..

## 1.2 Average Condition Exceedance Rate

blabla

## 1.3 Bayesian Inference

## 1.4 Markov Chain Monte Carlo Method

Markov Chain Monte Carlo (MCMC) is a powerful iterative method used to sample from probability distributions, which analytically or through other simulation methods, could be difficult and impracticable to sample from. The algorithm is constructed by converging the desired

probability distribution to an irreducible and aperiodic Markov Chain with limiting distribution equal the target distribution. Independent of starting position the Markov Chain will then converge towards the desired probability distribution in the limit as the numbers of iterations goes to infinity. The first numbers of realizations from the Markov Chain until converging is called the burn-in period, and is discarded for further analyses. The remaining realizations, approximately follows the desired probability distribution, where the accuracy increase as the numbers of realizations increase. Monte Carlo method can then be used to calculate the quantity of interest like mean, expected value, future prediction, credible interval etc.

### 1.4.1 Gibbs Sampling

In situations where it is difficult to sample from the joint distribution, but applicable from the conditional distribution, Gibbs sampler is preferable. The principle of Gibbs sampling is to construct the Markov Chain, by repeatedly sample each parameter with the rest of the parameters as the condition. The Gibbs sampler start with an initial guess $\boldsymbol{X}^0 = [X_1^0, \cdots, X_n^0]$ for the parameters, then iteratively update each as follows

$$
\begin{aligned}
X_1^{t+1} &\sim f(X_1 | X_2^t, \cdots, X_n^t), \\
X_2^{t+1} &\sim f(X_2 | X_1^{t+1}, X_3^t, \cdots, X_n^t), \\
&\vdots \\
X_n^{t+1} &\sim f(X_n | X_1^{t+1}, \cdots, X_{n-1}^{t+1}).
\end{aligned}
\tag{1.1}
$$

where.... The iterative process is repeated until enough realizations are generated for sufficient accuracy.

REFFF!!

### 1.4.2 Metropolis–Hastings Algorithm

Metropolis–Hastings algorithm is another method for constructing a suitable Markov Chain. The algorithm is preferable for situations where a proportional distribution is simple to evaluate, while the target probability distribution is difficult. Bayesian inference (see chapter 1.3), often result in a distribution where the normalizing constant cannot analytically be calculated.

While possible numerically, the normalizing constant often becomes computationally hard, which make it impracticable for iterative simulations. Using Metropolis–Hastings algorithm on a proportional distribution whiteout normalizing constant, results in samples from the target distributions. The Metropolis–Hastings algorithm start with an initial guess for the parameters. For each iteration a new parameters $X^*$ are suggested from a proposal distribution $g(X^*|X^t)$, given the last accepted parameter $X^t$. The new parameter is then evaluated against the last accepted by

$$R(X^*, X^t) = \frac{f(X^*)g(X^t|X^*)}{f(X^t)g(X^*|X^t)} \tag{1.2}$$

where $f(x)$ is the target distribution, or a distribution proportional to the target distribution. The parameter $X^{t+1}$ takes value $X^*$ with probability $\min\{1, R(X^*, X^t)\}$, if rejected, $X^{t+1} = X^t$ instead.

The Metropolis–Hastings algorithm could in situations be necessary for some of the steps in the Gibbs sampler, equation (1.1). A combination of Metropolis-Hastings algorithm and Gibbs sampler is referred to as a Hybrid Gibbs sampler.

REFFF!!

hybrid as in (1.1). REFFF!!

### 1.4.3 Adaptive Metropolis–Hastings Algorithm

### 1.4.4 Effective Sample Size

The realizations of the simulated Markov Chain will often be correlated, and dependent on the future and past iterations. The correlation implies that the information hold by the realizations is actually less than the numbers of realizations. The effective sample size gives a method of calculating the theoretical size of an equally informative independent and identical distributed, realization set. The effective sample size is estimated as

$$L_{eff} = \frac{L}{1 + 2\sum_{k=1}^{K} \hat{\rho}(k)}, \tag{1.3}$$

Where $L$ is the realizations of the sample size, $\hat{\rho}(k)$ is the estimated $k$ step autocorrelation between realizations and $K$ is chosen as the first $k$ where $\hat{\rho}(k) < 0.1$ ref(BOK COMP STAT).

## 1.5 Applying Markov Chain Monte Carlo to Peak Over Threshold

The left side of equation **??** can be written as

$$\Pr(X > x | X > u) = \frac{\Pr(X > x, X > u)}{\Pr(X > u)}, \tag{1.4}$$

where the nominator $\Pr(X > x, X > u) = \Pr(X > x)$, since the conditional distribution require $x > u$. Reorganizing **??** gives

$$\Pr(X > x) = \Pr(X > u) \cdot \left[ 1 + \xi \left( \frac{x - u}{\sigma} \right) \right]^{-\frac{1}{\xi}}. \tag{1.5}$$

Usefull to have as much knowledge as possible abouth the target distribution as possible=> better simmulation ish 23% acceptance.

Prior distribution hard to choose.... future improvements!! irreducible if its state space is a single communicating class; in other words, if it is possible to get to any state from any state. check. aperiodic if there exists n such that for all n'>n $\Pr(X_{n'} = i | X_0 = i) > 0$

### 1.5.1 Multivariate Random Normal Generator

lkajsd yous tla blaa

## 1.6 Value at Risk

mer bla bla

## 1.7 ARCH/GARCH

is this needed????

# Chapter 2

# Data

hei

## 2.1 Synthetic Data

blabla

### 2.1.1 Pareto

sim sim

### 2.1.2 Commodity Generated Data

sam sam

## 2.2 Commodity Data

blabla