Kevin

dilluns, 23 de maig de 2022 20:26

$L|K$ totally ramified of local fields

$n = [L:K] = ap^{\nu}$ $p \nmid a$

π_L, π_K uniformizer

We can write $\pi_K = a_0 \pi_L^n + a_1 \pi_L^{n+1} + \dots$ (*) $a_i \in K$ = Teichmuller reps

For $0 \leq j \leq \nu$ set

$$\tilde{i}_j = \min \{ r : a_r \neq 0 \quad v_p(r) \leq j \}$$

$$0 = \tilde{i}_\nu < \tilde{i}_{\nu-1} \leq \dots \leq \tilde{i}_0$$

(in char $p>0$ this is not affected if we change π_L, but in 0-char it might change)

Set $i_j = \min \{ \tilde{i}_{j'} + n v_K(p) : j \leq j' \leq \nu \} = \min \{ i_{j+1} + v_L(p), \tilde{i}_j \}$

Theorem (Heiermann) i_j is independent of the choices of π_L, π_K

$f(x) \in \mathcal{O}_K[x]$ Eisenstein with root π_L

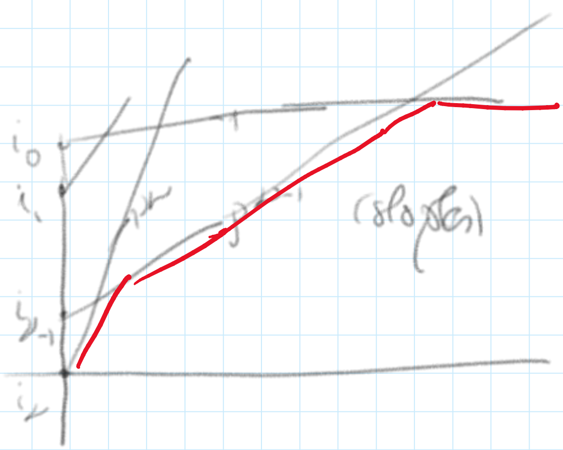
$$f(x) = x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

(it can be obtained from (*) by means of Weierstrass preparation thm)

Def: $\tilde{i}_j = \min\{ n v_K(a_i) + i - n : \quad 0 \le i \le n \quad v_p(i) \le j \}$

$i_j = \min\{ \tilde{i}_{j'} + n v_K(p) \quad j \le j' \le \nu \}$

$i_0 = v_L(\delta_{L/K}) - n + 1$



Minimum of these lines is

$n\, \phi_{L/K}(x)$ (Herbrand function)

Char $K = p$

Possible sequences of indices of insep.

$0 = i_\nu < i_{\nu-1} < \dots < i_0$

$v_p(i_j) = j$ unless $i_j = i_{j-1}$

Char $K = 0$ Need $i_j \le i_{j+1} + n v_K(p)$

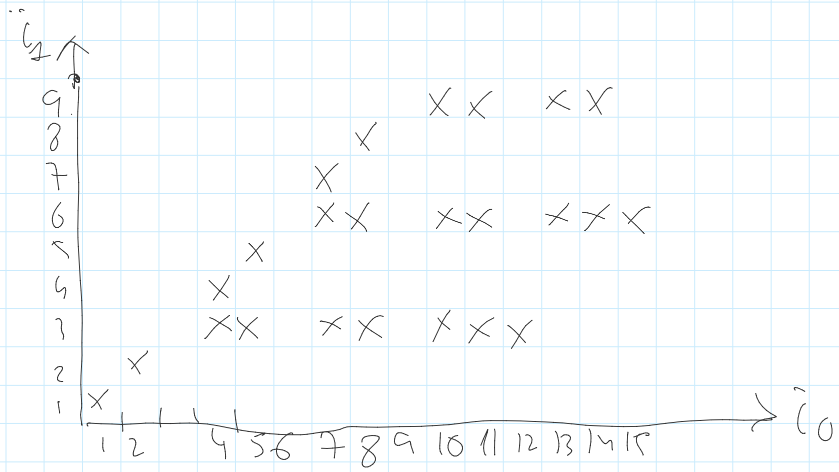
$v_p(i_j) = j$ unless either $i_j = i_{j-1}$ or $i_j = i_{j+1} + n v_K(p)$

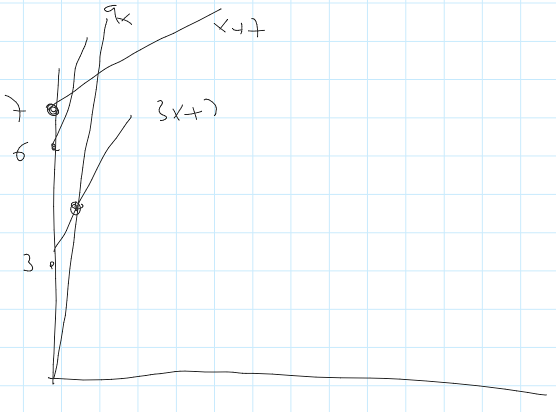
Example

$K = \mathbb{Q}_3 \quad n = 9 = 3^2 \quad \nu = 2$

$i_2 = 0 \qquad 0 \leq i_1 \leq 9$

i_1	i_0
1	1
2	2
3	$3 < i_0 \leq 12$, $v_3(i_0) = 0$ or $i_0 = 12$
4	4
5	5
6	$6 < i_1 < 15$ $v_3(i_0) = 0$ or $i_0 = 15$
7	7
8	8
9	$9 < i_0 < 18$ $v_3(i_0) = 0$ or $i_0 = 18$





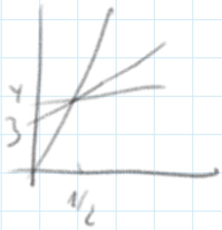
$x + 7 = 9x \longrightarrow x = 7/8$

$3x + 3 = 9x \longrightarrow x = 1/2$

$3x + 7 = x \longrightarrow x = 2$

$i_0 = 4 \quad i_1 = 3$

$4 + x = 8x \longrightarrow x = 1/2$



$f = x^9 + 3x^7 + 6x^6 + 6 \qquad$ ("x is π_L")

$\tilde{i}_2 = 9 = i_2$

$\tilde{i}_0 = 7 \qquad \tilde{i}_1 = 6$

$i_0 = 9 \qquad i_1 = 6 \quad i_0 = 7$

$f = x^9 + 3x^7 + 3 \qquad \tilde{i}_2 = 0 \quad \tilde{i}_1 = 1 \quad \tilde{i}_0 = 7$

$i_0 = i_1 = 7 \quad i_2 = 0$

$[L:K] = p^{\mu}$ totally ramified

Fix π_K. Then there is an Eisenstein polynomial

$$f(X) = X^{p^{\mu}} + a_{p^{\mu}-1} X^{p^{\mu}-1} + \dots + a_1 X + a_0$$

such that $\pi_K \equiv a_0 \ (\pi_K^2)$

Suppose there is i such that $0 \le j \le \mu$ and $v_p(i) = j$

and $i = p^{\mu} v(e) - p^{\mu} + r$

and $i_j = p^{\omega} v_K(a_r) - p^{\nu} + r$

Then $v_K(a_r) = 1 + \frac{i_j - r}{p^{\nu}} =: t$

<u>Theorem</u> $a_r \pmod{\pi_K^{t+1}}$ is uniquely determined by L/K and π_K

$f(x) = x^8 + 4x^5 + 4x^4 + 6$

$\tilde{i}_3 = 0 \quad \tilde{i}_2 = 8+4 = 12 \quad \tilde{i}_1 = 8+5 = 13 = \tilde{i}_0$

$i_3 = 0 \quad i_2 = \min\{i_3 + 8, \tilde{i}_2\} = \min\{8, 12\} = 8$

$i_1 = 13 = i_0$