# Chapter 8 **(Oechslin: 8.5)**

# Hahn's globe clock in Stuttgart (1770)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

## 8.1 Introduction

The clock described here was constructed by Philipp Matthäus Hahn (1739-1790)[1] and is located in the Landesmuseum Württemberg in Stuttgart.[2] Another very similar clock is kept in this museum,[3] but it doesn't have a lunar globe on top of the celestial sphere.[4]

The clock has a square base of which three faces have dials, for the indications of time, for a tellurium and for an orrery. On top of this base, a celestial globe shows the geocentric motion of the Sun, the Moon and Venus, as well as the lunar nodes. On top of this sphere, a small sphere shows the phase of the Moon.

Although apparently simpler than other clocks and machines by Hahn, this clock display several idiosyncrasies which are found in only a few of Hahn's clocks. It is for instance unusual to have only Venus as a planet rotating around the celestial globe, although this is also found in the globe clock in Darmstadt (Oechslin 8.6). There is also a rather bad approximation of the motion of the

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [6, p. 408-409]. A description of this clock seems to be given by Hahn [3], but I haven't seen it.

[3]This clock is currently exhibited in the basement of the Landesmuseum.

[4]There are several similar clocks by Hahn, one of which was described by Hahn in 1770 [2]. Another similar clock was described by Hahn in 1774 [1]. The globe clock of Darmstadt (Oechslin 8.6) is also similar. Yet another similar clock was also reconstructed by Alfred Leiter [4].

Moon in the tellurium.

## 8.2 The clockwork

The clock is driven by a spring-driven pendulum clockwork, with the barrel wheel on arbor 1 making one turn in 36 hours. We have, from the front

$$\mathrm{T}_1^0 = \frac{24}{36} = \frac{2}{3} \tag{8.1}$$

Arbors 2 and 3 have the following velocities:

$$\mathrm{T}_2^0 = \mathrm{T}_1^0 \times \left(-\frac{54}{9}\right) = -4 \tag{8.2}$$

$$\mathrm{T}_3^0 = \mathrm{T}_2^0 \times \left(-\frac{60}{10}\right) = 24 \tag{8.3}$$

Hence, arbor 3 makes one turn in one hour and it carries the minute hand.

The escape wheel on arbor 5 has the velocity

$$\mathrm{T}_5^0 = \mathrm{T}_3^0 \times \left(-\frac{72}{6}\right) \times \left(-\frac{60}{6}\right) = 24 \times 120 = 2880 \tag{8.4}$$

It makes one turn clockwise in $\frac{86400}{2880} = 30$ seconds. It carries 36 teeth or pins and the pendulum makes half an oscillation in $\frac{30}{2\times 36} = \frac{5}{12}$ seconds. Its length is about 17 cm.

The time dial gives the hours on an large 24-hour dial, the minutes on an internal dial, and the day of the week and of the month on another internal dial. The calendar indications will not be described here. I will only explain the connection between the clockwork and the other parts of the clock.

The arbor 3 is used to make a motion of one turn in one day on tube 7, and then also on arbor 8:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{14}{56}\right) \times \left(-\frac{12}{72}\right) = 1 \tag{8.5}$$

$$\mathrm{T}_8^0 = \mathrm{T}_7^0 \tag{8.6}$$

The motion of arbor 8 is also used to obtain the days of the week on tube 10:

$$\mathrm{T}_{10}^0 = \mathrm{T}_8^0 \times \left(-\frac{24}{48}\right) \times \left(-\frac{16}{56}\right) = \frac{1}{7} \tag{8.7}$$

But arbor 8 is used in the back of the clockwork to produce the motion of arbor 18:

$$\mathrm{T}_{18}^0 = \mathrm{T}_8^0 \times \left(-\frac{35}{35}\right) \times \left(-\frac{35}{29}\right) \times \left(-\frac{38}{44}\right) \tag{8.8}$$

$$= -\frac{665}{638} \tag{8.9}$$

This motion is then used to obtain the input motion of the globe on the vertical arbor 19:

$$V_{19}^0 = T_{18}^0 \times \frac{76}{79} = \left(-\frac{665}{638}\right) \times \frac{76}{79} \tag{8.10}$$

$$= -\frac{25270}{25201} \tag{8.11}$$

This velocity is counted positively counterclockwise as seen from above. It is the usual ratio used by Hahn for the sidereal day.

This motion is also used to obtain one of the input motions of the tellurium on arbor 44 and the input motion of the orrery on arbor 20. I will therefore first describe the gears of the orrery.

## 8.3 The orrery

As mentioned above, the input motion of the orrery is arbor 20 and it makes one turn clockwise (seen from the front) in a sidereal day:

$$V_{20}^0 = -\frac{25270}{25201} \tag{8.12}$$

### 8.3.1 The mean motions of the planets

This motion is first used to obtain the mean motion of Mercury on tube 23:

$$V_{23}^0 = V_{20}^0 \times \left(-\frac{6}{53}\right) \times \left(-\frac{32}{77}\right) \times \left(-\frac{20}{83}\right) = V_{20}^0 \times \left(-\frac{3840}{338723}\right) \tag{8.13}$$

$$= \left(-\frac{25270}{25201}\right) \times \left(-\frac{3840}{338723}\right) = \frac{13862400}{1219451189} \tag{8.14}$$

$$P_{23}^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{8.15}$$

The same value is given by Oechslin. This motion is counterclockwise as seen from the front. This is exactly the same ratio as the one used in the Furtwangen orrery, in the Aschaffenburg globe clock, and in the Gotha and Nuremberg *Weltmaschinen*.

The remaining part of the orrery is also identical to the one used in the Aschaffenburg globe clock, so that my descriptions only differ by the arbor and tube numbers.

Mercury is actually given an irregular motion, in order to account for its elliptic orbit. I will get back to it below, but I will first deal with the mean motions.

Venus (tube 27) only has a mean motion which is obtained from the mean motion of Mercury:

$$V^0_{27} = V^0_{23} \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{8.16}$$

$$= V^0_{23} \times \frac{783}{2000} = \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{8.17}$$

$$P^0_{27} = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{8.18}$$

The same value is given by Oechslin.

The Earth (tube 29) also only has a mean motion which is obtained from the (mean) motion of Venus:

$$V^0_{29} = V^0_{27} \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = V^0_{27} \times \frac{3239}{5265} \tag{8.19}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{8.20}$$

$$P^0_{29} = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{8.21}$$

The same value is given by Oechslin. This is an approximation of the tropical year. This motion is also transferred to arbor 42 which is the input to the tellurium.

The motion of the Earth is then used to obtain the mean motion of Mars. The motion of Mars is used to obtain the motion of Jupiter, and the motion of Jupiter is used to obtain the motions of Saturn and of the Moon.

So, first we obtain the mean motion of Mars on tube 31:

$$V^0_{31} = V^0_{29} \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = V^0_{29} \times \frac{696}{1309} \tag{8.22}$$

$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{8.23}$$

$$P^0_{31} = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{8.24}$$

The same value is given by Oechslin.

Note that in the Aschaffenburg clock, there may be an error on the 33-teeth wheel, or perhaps it was Oechslin who made a typo.

From Mars, we obtain the mean motion of Jupiter on tube 34:

$$V^0_{34} = V^0_{31} \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = V^0_{31} \times \frac{595}{3751} \tag{8.25}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{8.26}$$

$$P^0_{34} = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{8.27}$$

The same value is given by Oechslin.

The mean motion of Jupiter is then used to obtain the mean motion of Saturn on tube 37:

$$\mathrm{V}_{37}^{0} = \mathrm{V}_{34}^{0} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) = \mathrm{V}_{34}^{0} \times \frac{363}{901} \tag{8.28}$$

$$= \frac{329647872}{1427553648235} \times \frac{363}{901} = \frac{988943616}{10629965595535} \tag{8.29}$$

$$\mathrm{P}_{37}^{0} = \frac{10629965595535}{988943616} = 10748.8085\ldots \text{ days} \tag{8.30}$$

The same value is given by Oechslin.

### 8.3.2 The motion of the Moon

The mean motion of Jupiter is also used to obtain the motion of tube 40:

$$\mathrm{V}_{40}^{0} = \mathrm{V}_{34}^{0} \times \left(-\frac{121}{34}\right) \times \left(-\frac{3}{21}\right) \times \left(-\frac{73}{102}\right) = \mathrm{V}_{34}^{0} \times \left(-\frac{8833}{24276}\right) \tag{8.31}$$

$$= \frac{329647872}{1427553648235} \times \left(-\frac{8833}{24276}\right) = -\frac{2005357888}{23867281242805} \tag{8.32}$$

$$\mathrm{P}_{40}^{0} = -\frac{23867281242805}{2005357888} = 11901.7564\ldots \text{ days} \tag{8.33}$$

The same value is given by Oechslin.

The motion of tube 40 is used to rotate the Moon around the Earth. We can compute this motion precisely. We first compute the motion of the Moon with respect to the Earth. It is the motion of arbor 41. We have

$$\mathrm{V}_{41}^{29} = \mathrm{V}_{40}^{29} \times \left(-\frac{72}{6}\right) = -\left(\mathrm{V}_{40}^{0} - \mathrm{V}_{29}^{0}\right) \times 12 = \left(\mathrm{V}_{29}^{0} - \mathrm{V}_{40}^{0}\right) \times 12 \tag{8.34}$$

$$= \left(\frac{473632}{172990675} + \frac{2005357888}{23867281242805}\right) \times 12 \tag{8.35}$$

$$= \frac{336758509216}{119336406214025} \times 12 \tag{8.36}$$

$$= \frac{4041102110592}{119336406214025} \tag{8.37}$$

$$\mathrm{P}_{41}^{29} = \frac{119336406214025}{4041102110592} = 29.5306\ldots \text{ days} \tag{8.38}$$

As expected, we find an approximation of the synodic month. This value is not given by Oechslin who only gives the tropical month:

$$\mathrm{V}_{41}^{0} = \mathrm{V}_{41}^{29} + \mathrm{V}_{29}^{0} = \frac{4041102110592}{119336406214025} + \frac{473632}{172990675} \tag{8.39}$$

$$= \frac{4367833830368}{119336406214025} \tag{8.40}$$

$$\mathrm{P}_{41}^{0} = \frac{119336406214025}{4367833830368} = 27.3216\ldots \text{ days} \tag{8.41}$$

### 8.3.3 The anomalies of the planets

Finally, we examine how the anomalies of the planets Mercury, Mars, Jupiter and Saturn have been obtained. In each case, there is a central wheel fixed to the frame. Their teeth numbers are actually irrelevant, as they all mesh with identical wheels pivoting on the mean motion wheels. It therefore suffices to consider the case of Mercury.

The Mercury *satellite* wheel on arbor 25 rotates as the tube 23 rotates with the mean motion of Mercury. In the frame of tube 23, we have

$$\mathrm{V}_{25}^{23} = \mathrm{V}_{24}^{23} \times \left(-\frac{24}{24}\right) = -\mathrm{V}_{24}^{23} = \mathrm{V}_{23}^{24} = \mathrm{V}_{23}^{0} \tag{8.42}$$

$$\mathrm{P}_{25}^{23} = \mathrm{P}_{23}^{0} \tag{8.43}$$

Therefore, on the Mercury tube 23, the eccentric arbor of Mercury oscillates with the same period as the mean tropical motion of Mercury. This causes the acceleration and slowing down of the motion to occur at the same places in time, and therefore will account for the equation of center. Of course, in this simple case, the tropical and anomalistic motions of Mercury have been considered identical, which they are not exactly.

The same constructions are used for Mars, Jupiter and Saturn.

## 8.4 The tellurium

The tellurium occupies one side of the base of the clock. It shows the Earth rotating around the Sun and the Moon rotating around the Earth.

As mentioned above, the tellurium has two input motions. The first is that of arbor 44 and it makes one turn counterclockwise (seen from the front) in a sidereal day:

$$\mathrm{V}_{44}^{0} = \frac{25270}{25201} \tag{8.44}$$

The second one is arbor 42 which originates from the orrery. It provides the revolution of one tropical year and turns clockwise. The connection between the orrery and the tellurium is also found in the globe clock of Aschaffenburg (Oechslin 8.4) constructed in 1776/1777, and the structure of the tellurium is similar, except that there is no Moon in the Aschaffenburg tellurium.

$$\mathrm{V}_{42}^{0} = -\frac{473632}{172990675} \tag{8.45}$$

I am only considering the motion of arbor 42 from the tellurium side, hence the negative sign.

### 8.4.1 The motion of the Earth

The motion of arbor 42 is used to produce the counterclockwise motion of tube 43:

$$\mathrm{V}^0_{43} = \mathrm{V}^0_{42} \times \left(-\frac{56}{56}\right) = -\mathrm{V}^0_{42} = \frac{473632}{172990675} \tag{8.46}$$

Hence, tube 43 makes one turn counterclockwise in a tropical year. This tube supports the Earth frame.

The meridian of the Earth, which supports its axis, is fixed to a wheel on tube 48. The motion of this wheel replicates the motion of the central arbor 46 which is fixed. Therefore, the meridian, and also the axis of the Earth, always keep the same orientation with respect to the fixed frame, no matter where the Earth is located.

Finally, the arbor 50 replicates the motion of tube 45:

$$\mathrm{V}^0_{50} = \mathrm{V}^0_{45} = \mathrm{V}^0_{44} \times \left(-\frac{40}{40}\right) = -\mathrm{V}^0_{44} = -\frac{25270}{25201} \tag{8.47}$$

And arbor 51, which is that of the Earth's axis, has the velocity:

$$\mathrm{V}^0_{51} = \mathrm{V}^0_{50} \times \left(-\frac{20}{20}\right) = -\mathrm{V}^0_{50} = \frac{25270}{25201} \tag{8.48}$$

Consequently, the Earth rotates counterclockwise with the velocity of the sidereal day, as it should.

### 8.4.2 The motion of the Moon

The axis of the Moon is located on frame 53 which rotates on the Earth support 43. We can first compute the revolution of frame 53 with respect to the rotating support 43:

$$\mathrm{V}^{43}_{53} = \mathrm{V}^{43}_{45} \times \left(-\frac{60}{60}\right) \times \left(-\frac{3}{41}\right) \times \left(-\frac{24}{52}\right) = \mathrm{V}^{43}_{45} \times \left(-\frac{18}{533}\right) \tag{8.49}$$

$$= \left(\mathrm{V}^0_{45} - \mathrm{V}^0_{43}\right) \times \left(-\frac{18}{533}\right) = \left(\mathrm{V}^0_{44} + \mathrm{V}^0_{43}\right) \times \frac{18}{533} \tag{8.50}$$

$$= \left(\frac{25270}{25201} + \frac{473632}{172990675}\right) \times \frac{18}{533} = \frac{398491850662}{396321636425} \times \frac{18}{533} \tag{8.51}$$

$$= \frac{7172853311916}{211239432214525} \tag{8.52}$$

$$\mathrm{P}^{43}_{53} = \frac{211239432214525}{7172853311916} = 29.4498\ldots \text{ days} \tag{8.53}$$

Oechslin does not give the same value.

Then we have

$$\mathrm{V}_{53}^{0} = \mathrm{V}_{53}^{43} + \mathrm{V}_{43}^{0} = \frac{7172853311916}{211239432214525} + \frac{473632}{172990675} \tag{8.54}$$

$$= \frac{7751206768012}{211239432214525} \tag{8.55}$$

$$\mathrm{P}_{53}^{0} = \frac{211239432214525}{7751206768012} = 27.2524\ldots \text{ days} \tag{8.56}$$

This appears to be very bad approximations of the synodic and tropical months which are about 29.53 and 27.32 days. This is due to the ratio $\frac{18}{533}$ being a bad approximation of

$$x = \frac{s \cdot a}{m_t(s+a)} \approx 0.03367877 \tag{8.57}$$

where $s$ is the sidereal day, $a$ the tropical year and $m_t$ the tropical month. A much better approximation would have been obtained with $x = \frac{13}{386}$, but this fraction can't be expressed as a product of simple ratios. It is possible that Hahn sticked with the ratio $\frac{18}{533}$ and knew that it wasn't perfect.

Incidentally, the ratio $\frac{18}{533}$ is also used for the small lunar globe at the top of the celestial globe, but in that case the ratio serves to compute a good approximation of the synodic month.

## 8.5 The celestial globe and the lunar globe

The celestial globe is located above the base of the clock. It is contained in a frame representing the meridian and it rotates around the vertical axis representing the axis of the Earth. The globe shows the motions of the Sun, the Moon and its nodes, as well as Venus, as seen from a geocentric perspective. A small lunar globe is mounted on top of the celestial globe.

As mentioned above, the input motion of the globe is the vertical arbor 19 and it makes one turn clockwise (seen from above) in a sidereal day:

$$\mathrm{V}_{19}^{0} = -\frac{25270}{25201} \tag{8.58}$$

### 8.5.1 The celestial globe

The Sun, the Moon, the lunar nodes and Venus rotate around the celestial globe, but around the axis of the ecliptic, which is tilted by 23.5° on the axis of the Earth.

The input motion to the globe actually moves directly the globe, but through the frame supporting the axis 62 of the ecliptic. The globe is fixed to this central axis, to which are also attached three wheels that will be described later. It is the rotation of this tilted axis around the vertical axis that causes a motion within the sphere. This is done as follows.

The tube 57 is attached at the lower part of the fixed meridian of the globe. This tube carries a 71-teeth wheel. It meshes with a 34-teeth wheel on the arbor 58 which is pivoting on the frame supporting the ecliptic. When the tilted frame rotates, so does arbor 58.

Now, we have our first motion of the celestial globe, namely the clockwise (from above) rotation of the entire globe in one sidereal day, because of the motion of the vertical arbor 19. This motion should be clockwise, because it is the apparent motion of the sky and the Earth rotates counterclockwise as seen from above.

#### 8.5.1.1 The motion of the Moon

The rotation of arbor 58 causes the rotation of tube 61 which enters the globe in the inferior part. We can compute the velocity of tube 61 with respect to the frame/axis 62:

$$\mathrm{V}_{61}^{62} = \mathrm{V}_{57}^{62} \times \left(-\frac{71}{34}\right) \times \left(-\frac{34}{48}\right) \times \left(-\frac{28}{46}\right) \times \left(-\frac{3}{74}\right) = \mathrm{V}_{57}^{62} \times \frac{497}{13616} \tag{8.59}$$

$$= \mathrm{V}_{57}^{19} \times \frac{497}{13616} = -\mathrm{V}_{19}^{0} \times \frac{497}{13616} = \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{8.60}$$

$$\mathrm{P}_{61}^{62} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{8.61}$$

This is an approximation of the tropical month and tube 61 is actually the mean motion of the Moon. This value is positive, because the Moon revolves counterclockwise with respect to the stars. The same value is given by Oechslin, also in sidereal days.

The actual motion of the Moon shown around the celestial globe takes the equation of center into account. The "corrected Moon" is accelerated or slowed down with respect to the mean Moon. In principle, the period of the equation of center should be that of the anomalistic month, which is a bit longer than the tropical month. The motion of the corrected Moon is obtained using an eccentric pin on arbor 64. We can compute the period of arbor 64 with respect to frame 61:

$$\mathrm{V}_{64}^{61} = \mathrm{V}_{62}^{61} \times \left(-\frac{67}{43}\right) \times \left(-\frac{21}{33}\right) = -\mathrm{V}_{61}^{62} \times \frac{469}{473} \tag{8.62}$$

$$= -\frac{6279595}{171568408} \times \frac{469}{473} = -\frac{2945130055}{81151856984} \tag{8.63}$$

$$\mathrm{P}_{64}^{61} = -\frac{81151856984}{2945130055} = -27.5545\ldots \text{ days} \tag{8.64}$$

which is an excellent approximation of the anomalistic month. (The fact that the value is negative is irrelevant, as the oscillation of the corrected Moon would be equivalent whether the pins turns clockwise or counterclockwise.)

Oechslin doesn't give this value, but gives the period of precession of the apsides:

$$\mathrm{V}_{64}^{62} = \mathrm{V}_{64}^{61} + \mathrm{V}_{61}^{62} = -\frac{2945130055}{81151856984} + \frac{6279595}{171568408} \tag{8.65}$$

$$= \frac{6279595}{20287964246} \tag{8.66}$$

$$\mathrm{P}_{64}^{62} = \frac{20287964246}{6279595} = 3230.7759\ldots \text{ days} \tag{8.67}$$

#### 8.5.1.2 The motion of the Sun

The mean motion of the Sun is given by tube 68 and derived from the mean motion of the Moon:

$$\mathrm{V}_{68}^{62} = \mathrm{V}_{61}^{62} \times \left(-\frac{122}{27}\right) \times \left(-\frac{28}{28}\right) \times \left(-\frac{28}{43}\right) \times \left(-\frac{3}{118}\right) \tag{8.68}$$

$$= \mathrm{V}_{61}^{62} \times \frac{1708}{22833} = \frac{6279595}{171568408} \times \frac{1708}{22833} = \frac{2681387065}{979355364966} \tag{8.69}$$

$$\mathrm{P}_{68}^{62} = \frac{979355364966}{2681387065} = 365.2420\ldots \text{ days} \tag{8.70}$$

This is a good approximation of the tropical year and the same value is given by Oechslin. The value is positive, because the Sun moves counterclockwise with respect to the stars.

Hahn has also constructed the "corrected Sun," in that he added the equation of center. He did however assume that the tropical and anomalistic years are identical, that is he assumed the line of apsides to be fixed in the zodiac. The motion of the corrected Sun is obtained with an eccentric pin on an arbor rotating on frame 68. But the rotation of this arbor is such that it always replicates the motion of the central axis 62 of the sphere, therefore the pin is actually always oriented on the same direction. The direction of the pin is that at right angles of the line of apsides. This is a very simple construction that causes the mean Sun to be accelerated or slowed down depending how far it is from the line of apsides.

#### 8.5.1.3 The line of nodes

The corrected Sun (on a tube I will name 68′) is also used to obtain the motion of the line of nodes on tube 76. We have

$$\mathrm{V}_{76}^{62} = \mathrm{V}_{68'}^{62} \times \left(-\frac{70}{43}\right) \times \left(-\frac{20}{24}\right) \times \left(-\frac{4}{101}\right) = \mathrm{V}_{68'}^{62} \times \left(-\frac{700}{13029}\right) \tag{8.71}$$

$$\approx \mathrm{V}_{68}^{62} \times \left(-\frac{700}{13029}\right) \approx \frac{2681387065}{979355364966} \times \left(-\frac{700}{13029}\right) \tag{8.72}$$

$$\approx -\frac{938485472750}{6380010525071007} \tag{8.73}$$

$$\mathrm{P}_{76}^{62} \approx -\frac{6380010525071007}{938485472750} = -6798.1984\ldots \text{ days} \tag{8.74}$$

This is an excellent approximation of the period of precession of the lunar nodes. The value is negative, because the line of nodes is retrograding. The same value is given by Oechslin.

#### 8.5.1.4 The motion of Venus

The corrected Sun 68′ is also used to obtain the motion of Venus. The mean geocentric motion of Venus is actually that of the Sun. But because of the revolution of the Earth around the Sun, Venus appears to oscillate around the position of the Sun, and sometimes to retrograde. Ths oscillation results from the motion of an eccentric pin on arbor 73. The period of this oscillation should be that of the synodic revolution of Venus. We can compute this motion in the reference frame 68′, that is, that of the corrected Sun:

$$\mathrm{V}_{73}^{68'} = \mathrm{V}_{62}^{68'} \times \left(-\frac{31}{20}\right) \times \left(-\frac{26}{22}\right) \times \left(-\frac{14}{41}\right) = \mathrm{V}_{62}^{68'} \times \left(-\frac{2821}{4510}\right) \tag{8.75}$$

$$\approx \mathrm{V}_{62}^{68} \times \left(-\frac{2821}{4510}\right) = -\mathrm{V}_{68}^{62} \times \left(-\frac{2821}{4510}\right) \tag{8.76}$$

$$\approx \frac{2681387065}{979355364966} \times \frac{2821}{4510} = \frac{1512838582073}{883378539199332} \tag{8.77}$$

$$\mathrm{P}_{73}^{68'} = \frac{883378539199332}{1512838582073} = 583.9212\ldots \text{ days} \tag{8.78}$$

and this is an excellent approximation of the synodic period of Venus. (Like for other similar motions, the sign is irrelevant here, as the same motion would be obtained if arbor 73 were rotating clockwise.)

The ratio $\frac{2821}{4510}$ is also used in the globe clock of Aschaffenburg and in the *Weltmaschinen* of Stuttgart and Gotha.

Oechslin doesn't give the value of the synodic period of Venus, only the tropical one:

$$\mathrm{V}_{73}^{62} = \mathrm{V}_{73}^{68'} + \mathrm{V}_{68'}^{62} \approx \mathrm{V}_{73}^{68'} + \mathrm{V}_{68}^{62} \tag{8.79}$$

$$\approx \frac{1512838582073}{883378539199332} + \frac{2681387065}{979355364966} = \frac{3931449714703}{883378539199332} \tag{8.80}$$

$$\mathrm{P}_{73}^{62} = \frac{883378539199332}{3931449714703} = 224.6953\ldots \text{ days} \tag{8.81}$$

The Darmstadt globe clock (Oechslin 8.6) also displays the motion of Venus, but its irregular motion is obtained using the elongation between Venus and the Sun. That clock contains a wheel making one turn in the tropical period of Venus, which is not the case here.

### 8.5.2 The lunar globe

The frame causing the rotation of the celestial globe carries at the top a 3-leaves/3-pin pinion which is used to rotate the small lunar globe. This pinion makes one turn clockwise in one sidereal day:

$$\mathrm{V}_{19}^{0} = -\frac{25270}{25201} \tag{8.82}$$

The tube carrying the Moon is tube 79 and its velocity is

$$\mathrm{V}_{79}^{0} = \mathrm{V}_{19}^{0} \times \left(-\frac{3}{26}\right) \times \left(-\frac{18}{18}\right) \times \left(-\frac{12}{41}\right) \tag{8.83}$$

$$= \mathrm{V}_{19}^{0} \times \left(-\frac{18}{533}\right) = \frac{454860}{13432133} \tag{8.84}$$

$$\mathrm{P}_{79}^{0} = \frac{13432133}{454860} = 29.5302\ldots \text{ days} \tag{8.85}$$

The same value is given by Oechslin.

The globe makes one turn clock counterclockwise (as seen from above) in one synodic month. It is half lit and is moving within a hemisphere that hides half of the sphere (as in the current Strasbourg astronomical clock). This hemisphere is fixed to the central axis of the lunar globe, which is itself fixed to the meridian frame. The lunar globe therefore shows phases of the Moon. However, the phases seem to appear in the wrong order, and the sphere should in fact rotate clockwise, as it does in the Darmstadt globe clock (Oechslin 8.6) from 1785.

It is interesting to observe that we have again the ratio $\frac{18}{533}$ which was used in the tellurium. We now have a relatively good value of the synodic month, because the ratio $\frac{18}{533}$ is a good approximation of the ratio $s/m_s$ where $s$ is the sidereal day and $m_s$ is the synodic month, but it was not a good approximation for the ratio used in the tellurium. There may have been a mixup between the two ratios. This ratio $\frac{18}{533}$ was reused at least in the Darmstadt globe clock (Oechslin 8.6) and in the Zurich clock (Oechslin 8.7), both from 1785.

## 8.6 References

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