Chapter 27

(Oechslin: 10.3)

Seige's metal orrery fragment in Prague (c1781?)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

27.1 Introduction

The orrery metal fragment described here was constructed by Engelbert Seige (1737-1811), probably around 1781. Before entering into the details of this fragment, a few words on Seige are in order.[1]

Engelbert Wenzel Seige was born on April 25, 1737 in Chlumec, near Ústí nad Labem, at about 20 km from the Osek monastery, in the Czech Republic. Seige entered this monastery at the age of 18 and became priest in 1764. He was also a professor of Greek between 1775 and 1783. He constructed many telescopes and astronomical clocks.

Seige worked on his most important clock for about ten years, between 1781 and 1791. This clock is now exhibited in the technical museum in Prague. The museum also keeps two fragments of astronomical clocks, the fragment described here, and a fragment in wood.

Seige was prior of the Osek monastery until 1808 and he died in 1811.

The present metal fragment features Uranus which was discovered in 1781, but with a very gross approximation of its period. It may well be the first machine showing the motion of Uranus. Hahn's *Weltmaschine* in Nuremberg (Oechslin 8.2) also shows Uranus, but with a more accurate period, and it must have been constructed a few years later.[2]

[1] Very little is known about the life of Seige, and I am drawing these few lines from Hrůšová's thesis [4] who seems to have borrowed it from [1].

[2] Faber seems to suggest that one of Seige's fragments is that of an unfinished clock at the time of his death [3, p. 9].

Seige's machine is made of a cubic base which contains the going work, as well as the gears for the four sides of the base. It is topped by an orrery in the middle, and four satellite systems at the corner, which is a somewhat unusual arrangement. The general structure is however very simple, only with many long gear trains. Many of the trains are independent from each other. The only more advanced construction is that of the Sun-Moon system at the base of the machine. I will first describe the base, then the upper parts.[3]

Seige's *Weltmaschine* from 1791 (Oechslin 10.1) seems to be the finalization of this fragment.

27.2 The base

27.2.1 The going work

The machine is weight driven and regulated by a pendulum and an anchor escapement. The drum is on arbor 1. If one computes the period of this drum from the period of arbor 8 which is indicated as 24 hours by Oechslin, then we reach a period of 64 hours. And eventually, we arrive at the escape wheel making a turn in 160 seconds and the pendulum being more than seven meters long. This is obviously wrong.

If Oechslin's drawing is correct, arbor 1 can't make a turn in 64 hours. The pendulum which is visible on some pictures seems to be about one meter long. We should therefore have the escape wheel making a turn in about a minute and arbor 1 making a turn in 24 hours. This also entails that arbor 8 should make a turn in nine hours and it does not fit with Oechslin writing that arbor 8 shows the motion of the Earth in 24 hours.

I will therefore assume that arbor 1 makes a turn in 24 hours:

$$\mathrm{V}_1^0 = 1 \tag{27.1}$$

I am measuring the velocities in the going work from the right in Oechslin's plate.

We can then deduce the velocities of arbors 2, 3, 4 and 5, the latter being that of the escape wheel:

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \left(-\frac{80}{10}\right) = -8 \tag{27.2}$$

[3]This fragment was also described by Oechslin, see [8, p. 34, 198].

This arbor makes a turn clockwise in three hours.

$$V_3^0 = V_2^0 \times \left(-\frac{60}{10}\right) = 48 \tag{27.3}$$

$$P_3^0 = \frac{1}{48} \text{ days} = 30 \text{ minutes} \tag{27.4}$$

$$V_4^0 = V_3^0 \times \left(-\frac{60}{16}\right) = -180 \tag{27.5}$$

$$P_4^0 = -\frac{1}{180} \text{ days} = -8 \text{ minutes} \tag{27.6}$$

$$V_5^0 = V_4^0 \times \frac{56}{7} = -1440 \tag{27.7}$$

$$P_5^0 = -\frac{1}{1440} \text{ days} = -60 \text{ seconds} \tag{27.8}$$

Note that on Oechslin's drawing the escapement arbor is shown vertically, but it is actually horizontal. My choices for the orientations rely on Oechslin's drawing.

If my assumption is correct, the escape wheel should make a turn in exactly one minute and the pendulum should be about one meter long.

However, as we will see below, the period of arbor 3 leads to a problem in all the upper mechanisms of the machine. The orrery and corner subsystems only turn as they should if arbor 3 makes a turn in one hour. Oechslin also observed some of these inconsistencies [8, p. 198].

27.2.2 The outputs of the going work

The going work drives a number of arbors that were meant to display some motions. On one side, there is the arbor 8, supporting a wheel which according to the above calculations makes one turn in nine hours. However, Oechslin writes that it makes one turn in 24 hours.

Another side shows the motion of the Sun and the Moon. Its input is arbor 10 whose motion is derived from that of arbor 2:

$$V_{10}^0 = V_2^0 \times \left(-\frac{30}{60}\right) \times \left(-\frac{8}{32}\right) = -1 \tag{27.9}$$

(All directions are viewed from the right or the bottom on Oechslin's plate.)

So, arbor 10 makes one turn clockwise in one day.

On another side, we have the three arbors 110, 111 and 112. Their velocities (now measured from the left) are:

$$\mathrm{V}^0_{110} = \mathrm{V}^0_1 \times \frac{11}{56} \times \left(-\frac{10}{76}\right) = -\frac{55}{2128} \tag{27.10}$$

$$\mathrm{P}^0_{110} = -\frac{2128}{55} = -38.6909\ldots \text{ days} \tag{27.11}$$

$$\mathrm{V}^0_{111} = \mathrm{V}^0_2 \times \frac{40}{30} = -\frac{32}{3} \tag{27.12}$$

$$\mathrm{P}^0_{111} = -\frac{3}{32} = -2 \text{ hours } 15 \text{ minutes} \tag{27.13}$$

$$\mathrm{V}^0_{112} = \mathrm{V}^0_3 \times \frac{15}{60} = 12 \tag{27.14}$$

$$\mathrm{P}^0_{112} = \frac{1}{12} = 2 \text{ hours} \tag{27.15}$$

It isn't clear what these three arbors were meant to represent.

On the fourth side, we have the three arbors 113, 114, and 115 whose periods also remain clouded in mystery:

$$\mathrm{V}^0_{113} = \mathrm{V}^0_1 \times \left(-\frac{11}{56}\right) = -\frac{11}{56} \tag{27.16}$$

$$\mathrm{P}^0_{113} = -\frac{56}{11} = -5.0909\ldots \text{ days} \tag{27.17}$$

$$\mathrm{V}^0_{114} = \mathrm{V}^0_2 \times \left(-\frac{20}{40}\right) = 4 \tag{27.18}$$

$$\mathrm{P}^0_{114} = \frac{1}{4} = 6 \text{ hours} \tag{27.19}$$

$$\mathrm{V}^0_{115} = \mathrm{V}^0_2 \times \left(-\frac{45}{89}\right) = \frac{360}{89} \tag{27.20}$$

$$\mathrm{P}^0_{115} = \frac{89}{360} = 5.9333\ldots \text{ hours} \tag{27.21}$$

And finally, there are two outputs to the upper part of the machine, here measured as seen from above:

$$\mathrm{V}^0_{27} = \mathrm{V}^0_3 \times \left(-\frac{10}{60}\right) \times \left(-\frac{10}{40}\right) = 2 \tag{27.22}$$

$$\mathrm{P}^0_{27} = \frac{1}{2} = 12 \text{ hours} \tag{27.23}$$

$$\mathrm{V}^0_{83} = \mathrm{V}^0_3 \times \left(-\frac{20}{40}\right) \times \left(-\frac{12}{36}\right) = -8 \tag{27.24}$$

$$\mathrm{P}^0_{83} = -\frac{1}{8} = -3 \text{ hours} \tag{27.25}$$

However, if arbor 27 makes one turn in 12 hours, the periods in the upper central orrery, as well as in the two corner systems driven by that arbor are too small by a factor two. Obviously, these systems assume that arbor 27 makes one turn in 24 hours. I will therefore assume below that it is so.

The same is actually true for arbor 83 which should actually be making a turn in 6 hours and not 3 hours. Both of these problems go back to the velocity of arbor 3. I have computed above that this arbor makes a turn in 30 minutes. But if it did a turn in one hour, this would solve our problem. Perhaps it would make a turn in one hour and perhaps some teeth counts are incorrect. Perhaps the ratio between arbors 2 and 3 is 3 and not 6.

27.2.3 The base system of the Sun and the Moon

As mentioned above, the input to this system is arbor 10 which makes one turn clockwise (from the front) in one day:

$$V^0_{10} = -1 \tag{27.26}$$

27.2.3.1 The motion of the Sun

The motion of the Sun corresponds to tube 18. We can easily compute its velocity following the train from arbor 10 to tube 18:

$$V^0_{11} = V^0_{10} \times \left(-\frac{30}{30}\right) = 1 \tag{27.27}$$

$$V^0_{12} = V^0_{11} \times \left(-\frac{10}{53}\right) = -\frac{10}{53} \tag{27.28}$$

$$V^0_{13} = V^0_{12} \times \left(-\frac{14}{64}\right) = \left(-\frac{10}{53}\right) \times \left(-\frac{7}{32}\right) = \frac{35}{848} \tag{27.29}$$

$$V^0_{16} = V^0_{13} \times \left(-\frac{20}{62}\right) = \frac{35}{848} \times \left(-\frac{10}{31}\right) = -\frac{175}{13144} \tag{27.30}$$

$$V^0_{17} = V^0_{16} \times \left(-\frac{25}{37}\right) = \left(-\frac{175}{13144}\right) \times \left(-\frac{25}{37}\right) = \frac{4375}{486328} \tag{27.31}$$

$$V^0_{18} = V^0_{17} \times \left(-\frac{21}{69}\right) = \frac{4375}{486328} \times \left(-\frac{7}{23}\right) = -\frac{30625}{11185544} \tag{27.32}$$

$$P^0_{18} = -\frac{11185544}{30625} = -365.2422\ldots \text{ days} \tag{27.33}$$

The same value is given by Oechslin.

Note that the Sun moves clockwise here, but should move counterclockwise. In fact, all the motions are in the wrong directions, and it is possible that the input motion was meant to turn in the opposite direction. This would give correct motions of the Sun, of the Moon, and of the nodes and apsides.

27.2.3.2 The mean motion of the Moon

The mean motion of the Moon is given by frame 14 whose motion is derived from that of arbor 13:

$$\mathrm{V}_{14}^{0} = \mathrm{V}_{13}^{0} \times \left(-\frac{47}{53}\right) = \frac{35}{848} \times \left(-\frac{47}{53}\right) = -\frac{1645}{44944} \tag{27.34}$$

$$\mathrm{P}_{14}^{0} = -\frac{44944}{1645} = -27.3215\ldots \text{ days} \tag{27.35}$$

The same value is given by Oechslin. It is an approximation of the tropical month.

27.2.3.3 The lunar apsides

The motion of the lunar apsides is given by tube 21, obtained through a derivation from the motion of arbor 17. We have:

$$\mathrm{V}_{19}^{0} = \mathrm{V}_{17}^{0} \times \left(-\frac{19}{60}\right) = \frac{4375}{486328} \times \left(-\frac{19}{60}\right) = -\frac{16625}{5835936} \tag{27.36}$$

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{19}^{0} \times \left(-\frac{19}{53}\right) = \left(-\frac{16625}{5835936}\right) \times \left(-\frac{19}{53}\right) = \frac{315875}{309304608} \tag{27.37}$$

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{20}^{0} \times \left(-\frac{20}{66}\right) = \frac{315875}{309304608} \times \left(-\frac{10}{33}\right) = -\frac{1579375}{5103526032} \tag{27.38}$$

$$\mathrm{P}_{21}^{0} = -\frac{5103526032}{1579375} = -3231.3579\ldots \text{ days} \tag{27.39}$$

which is about 8.85 years. The same value is given by Oechslin.

27.2.3.4 The lunar nodes

The motion of the lunar nodes is given by tube 25, obtained through a long derivation from the motion of arbor 20. We have:

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{20}^{0} \times \left(-\frac{23}{22}\right) = \frac{315875}{309304608} \times \left(-\frac{23}{22}\right) = -\frac{7265125}{6804701376} \tag{27.40}$$

$$\mathrm{V}_{23}^{0} = \mathrm{V}_{22}^{0} \times \left(-\frac{24}{38}\right) = \left(-\frac{7265125}{6804701376}\right) \times \left(-\frac{12}{19}\right) = \frac{382375}{567058448} \tag{27.41}$$

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{23}^{0} \times \left(-\frac{29}{41}\right) = \frac{382375}{567058448} \times \left(-\frac{29}{41}\right) = -\frac{11088875}{23249396368} \tag{27.42}$$

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{24}^{0} \times \left(-\frac{33}{107}\right) = \left(-\frac{11088875}{23249396368}\right) \times \left(-\frac{33}{107}\right) \tag{27.43}$$

$$= \frac{33266625}{226153219216} \tag{27.44}$$

$$\mathrm{P}_{25}^{0} = \frac{226153219216}{33266625} = 6798.2014\ldots \text{ days} \tag{27.45}$$

which is an excellent approximation of the period of precession of the lunar nodes. The same value is given by Oechslin.

There are currently no hands and the Moon seems to have an eccentric motion based on the motion of the lunar apsides.

27.3 The four corner systems

The upper part of the machine has a central orrery and four corner systems. Two of these systems are driven with the input from arbor 27 which should be making one turn in 24 hours. The other two systems are driven with the input from arbor 83 which should be making one turn in six hours.

$$\mathrm{V(h)}^0_{27} = 1 \tag{27.46}$$
$$\mathrm{P(h)}^0_{27} = 1 \text{ day} \tag{27.47}$$
$$\mathrm{V(h)}^0_{83} = -4 \tag{27.48}$$
$$\mathrm{P(h)}^0_{83} = -6 \text{ hours} \tag{27.49}$$

The motion of arbor 83 is first split into those of arbor 98 (daily Sun/Moon system) and 85 (satellites of Jupiter). And the motion of arbor 27 is split into those of arbor 52 (yearly Sun/Moon system) and 67 (satellites of Saturn). We therefore first compute the velocities of these four arbors:

$$\mathrm{V}^0_{98} = \mathrm{V}^0_{83} \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{80}\right) = \mathrm{V}^0_{83} \times \left(-\frac{1}{2}\right) = 2 \tag{27.50}$$
$$\mathrm{P}^0_{98} = \frac{1}{2} \text{ days} = 12 \text{ hours} \tag{27.51}$$
$$\mathrm{V}^0_{85} = \mathrm{V}^0_{83} \times \left(-\frac{45}{83}\right) \times \left(-\frac{27}{89}\right) = \mathrm{V}^0_{83} \times \frac{1215}{7387} = -\frac{4860}{7387} \tag{27.52}$$
$$\mathrm{P}^0_{85} = -\frac{7387}{4860} = -1.5199\ldots \text{ days} \tag{27.53}$$
$$\mathrm{V}^0_{52} = \mathrm{V(h)}^0_{27} \times \left(-\frac{30}{60}\right) \times \left(-\frac{19}{85}\right) \times \left(-\frac{27}{65}\right) \tag{27.54}$$
$$= \mathrm{V(h)}^0_{27} \times \left(-\frac{513}{11050}\right) = -\frac{513}{11050} \tag{27.55}$$
$$\mathrm{P}^0_{52} = -\frac{11050}{513} = -21.5399\ldots \text{ days} \tag{27.56}$$
$$\mathrm{V(h)}^0_{67} = \mathrm{V(h)}^0_{27} \times \left(-\frac{36}{58}\right) \times \left(-\frac{76}{72}\right) = \mathrm{V(h)}^0_{27} \times \frac{19}{29} = \frac{19}{29} \tag{27.57}$$
$$\mathrm{P(h)}^0_{67} = \frac{29}{19} = 1.5263\ldots \text{ days} \tag{27.58}$$

On the arbor 27, Oechslin's plate has a 50-teeth wheel, but this leads to incorrect periods for the satellites. Oechslin suggested in his calculations that the 50-teeth wheel be replaced by a 36-teeth wheel. However, if one wished to correct to periods, the 58-teeth wheel with which the 50-teeth wheel meshes should of course also be adapted, be it only for its size. Moreover, this is only one of several possible ways to fix the problem. For instance, the 76-teeth wheel could also have been replaced with a 55-teeth wheel for similar results.

27.3.1 The Sun/Moon daily system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the Earth. The Sun therefore makes one turn *clockwise* around the Earth in one day. There are four separate trains and they are all driven by the input on arbor 98.

For the motion of the Sun on tube 105, we have

$$\mathrm{V}^0_{105} = \mathrm{V}^0_{98} \times \left(-\frac{30}{60}\right) = -1 \tag{27.59}$$

$$\mathrm{P}^0_{105} = -1 \text{ day} \tag{27.60}$$

The motion of the Moon is given by the central arbor 101:

$$\mathrm{V}^0_{101} = \mathrm{V}^0_{98} \times \left(-\frac{21}{23}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{42}{74}\right) \tag{27.61}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{2058}{4255}\right) = -\frac{4116}{4255} \tag{27.62}$$

$$\mathrm{P}^0_{101} = -\frac{4255}{4116} = -24 \text{ h } 48 \text{ m } 37.7842\ldots \text{ s} \tag{27.63}$$

The same value is given by Oechslin. It is an approximation of the apparent diurnal motion of the Moon, which is also clockwise.

The motion of the sky is given by tube 108:

$$\mathrm{V}^0_{108} = \mathrm{V}^0_{98} \times \left(-\frac{27}{34}\right) \times \left(-\frac{40}{42}\right) \times \left(-\frac{59}{89}\right) \tag{27.64}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{5310}{10591}\right) = -\frac{10620}{10591} \tag{27.65}$$

$$\mathrm{P}^0_{108} = -\frac{10591}{10620} = -0.9972\ldots \text{ days} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{27.66}$$

This is an approximation of the sidereal day. The same value is given by Oechslin.

Finally, the motion of the lunar nodes is given on tube 104:

$$\mathrm{V}^0_{104} = \mathrm{V}^0_{98} \times \left(-\frac{28}{27}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{43}{83}\right) \tag{27.67}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{16856}{33615}\right) = -\frac{33712}{33615} \tag{27.68}$$

$$\mathrm{P}^0_{104} = -\frac{33615}{33712} = -0.9971\ldots \text{ days} = -23 \text{ h } 55 \text{ m } 51.4000\ldots \text{ s} \tag{27.69}$$

The same value is given by Oechslin.

27.3.2 The satellites of Jupiter

This system is driven by the input on arbor 85.

The velocities of the four satellites are:

$$V_{86}^0 = V_{85}^0 \times \left(-\frac{61}{71}\right) = \left(-\frac{4860}{7387}\right) \times \left(-\frac{61}{71}\right) = \frac{296460}{524477} \tag{27.70}$$

$$P_{86}^0 = \frac{524477}{296460} = 1.7691\ldots \text{ days (Io)} \tag{27.71}$$

$$V_{89}^0 = V_{85}^0 \times \left(-\frac{42}{49}\right) \times \left(-\frac{27}{46}\right) \times \left(-\frac{57}{67}\right) = V_{85}^0 \times \left(-\frac{4617}{10787}\right) \tag{27.72}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{4617}{10787}\right) = \frac{22438620}{79683569} \tag{27.73}$$

$$P_{89}^0 = \frac{79683569}{22438620} = 3.5511\ldots \text{ days (Europa)} \tag{27.74}$$

$$V_{92}^0 = V_{85}^0 \times \left(-\frac{44}{53}\right) \times \left(-\frac{22}{37}\right) \times \left(-\frac{34}{79}\right) = V_{85}^0 \times \left(-\frac{32912}{154919}\right) \tag{27.75}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{32912}{154919}\right) = \frac{159952320}{1144386653} \tag{27.76}$$

$$P_{92}^0 = \frac{1144386653}{159952320} = 7.1545\ldots \text{ days (Ganymede)} \tag{27.77}$$

$$V_{95}^0 = V_{85}^0 \times \left(-\frac{26}{53}\right) \times \left(-\frac{22}{48}\right) \times \left(-\frac{32}{79}\right) = V_{85}^0 \times \left(-\frac{1144}{12561}\right) \tag{27.78}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{1144}{12561}\right) = \frac{1853280}{30929369} \tag{27.79}$$

$$P_{95}^0 = \frac{30929369}{1853280} = 16.6889\ldots \text{ days (Callisto)} \tag{27.80}$$

The same values are given by Oechslin. These are good approximations of the actual periods of the satellites. The satellites all move counterclockwise as they should. Note however that the 26-teeth wheel used in Callisto's train is mistakenly marked as having 72 teeth in Oechslin's drawing, but his calculations are correct.

The 1791 machine (Oechslin 10.1) uses the same ratios.

27.3.3 The satellites of Saturn

This system is driven by the input on arbor 67. I am here assuming that the motion of arbor 67 is the one taking into account the correction suggested by Oechslin, but other corrections are possible.

With this caveat, the velocities of the five satellites are then:

$$V^0_{68} = V(h)^0_{67} \times \left(-\frac{38}{47}\right) = \frac{19}{29} \times \left(-\frac{38}{47}\right) = -\frac{722}{1363} \tag{27.81}$$

$$P^0_{68} = -\frac{1363}{722} = -1.8878\ldots \text{ days (Tethys)} \tag{27.82}$$

$$V^0_{71} = V(h)^0_{67} \times \left(-\frac{21}{26}\right) \times \left(-\frac{18}{21}\right) \times \left(-\frac{33}{41}\right) = V(h)^0_{67} \times \left(-\frac{297}{533}\right) \tag{27.83}$$

$$= \frac{19}{29} \times \left(-\frac{297}{533}\right) = -\frac{5643}{15457} \tag{27.84}$$

$$P^0_{71} = -\frac{15457}{5643} = -2.7391\ldots \text{ days (Dione)} \tag{27.85}$$

$$V^0_{74} = V(h)^0_{67} \times \left(-\frac{38}{62}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{30}{47}\right) = V(h)^0_{67} \times \left(-\frac{5415}{16027}\right) \tag{27.86}$$

$$= \frac{19}{29} \times \left(-\frac{5415}{16027}\right) = -\frac{102885}{464783} \tag{27.87}$$

$$P^0_{74} = -\frac{464783}{102885} = -4.5175\ldots \text{ days (Rhea)} \tag{27.88}$$

$$V^0_{77} = V(h)^0_{67} \times \left(-\frac{37}{39}\right) \times \left(-\frac{12}{29}\right) \times \left(-\frac{10}{41}\right) = V(h)^0_{67} \times \left(-\frac{1480}{15457}\right) \tag{27.89}$$

$$= \frac{19}{29} \times \left(-\frac{1480}{15457}\right) = -\frac{28120}{448253} \tag{27.90}$$

$$P^0_{77} = -\frac{448253}{28120} = -15.9407\ldots \text{ days (Titan)} \tag{27.91}$$

$$V^0_{81} = V(h)^0_{67} \times \left(-\frac{13}{30}\right) \times \left(-\frac{8}{46}\right) \times \left(-\frac{12}{47}\right) = V(h)^0_{67} \times \left(-\frac{104}{5405}\right) \tag{27.92}$$

$$= \frac{19}{29} \times \left(-\frac{104}{5405}\right) = -\frac{1976}{156745} \tag{27.93}$$

$$P^0_{81} = -\frac{156745}{1976} = -79.3243\ldots \text{ days (Iapetus)} \tag{27.94}$$

The same values are given by Oechslin.

In this system, the satellites move clockwise, but they should actually move counterclockwise. The 1791 machine (Oechslin 10.1) uses the same ratios, but the satellites move correctly.

27.3.4 The Sun/Moon annual system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the zodiac. There are four separate trains and they are all driven by the input on arbor 52.

The motion of the Sun is that of tube 65. We have

$$\mathrm{V}^0_{65} = \mathrm{V}^0_{52} \times \left(-\frac{20}{74}\right) \times \left(-\frac{67}{69}\right) \times \left(-\frac{20}{89}\right) \tag{27.95}$$

$$= \mathrm{V}^0_{52} \times \left(-\frac{13400}{227217}\right) = \left(-\frac{513}{11050}\right) \times \left(-\frac{13400}{227217}\right) \tag{27.96}$$

$$= \frac{45828}{16738319} \tag{27.97}$$

$$\mathrm{P}^0_{65} = \frac{16738319}{45828} = 365.2421\ldots \text{ days} \tag{27.98}$$

The same value is given by Oechslin.

The motion of the Moon is that of the central arbor 55. We have

$$\mathrm{V}^0_{55} = \mathrm{V}^0_{52} \times \left(-\frac{22}{23}\right) \times \left(-\frac{28}{29}\right) \times \left(-\frac{35}{41}\right) = \mathrm{V}^0_{52} \times \left(-\frac{21560}{27347}\right) \tag{27.99}$$

$$= \left(-\frac{513}{11050}\right) \times \left(-\frac{21560}{27347}\right) = \frac{1106028}{30218435} \tag{27.100}$$

$$\mathrm{P}^0_{55} = \frac{30218435}{1106028} = 27.3215\ldots \text{ days} \tag{27.101}$$

The same value is given by Oechslin.

The motion of the lunar nodes is that of tube 59. We have

$$\mathrm{V}^0_{59} = \mathrm{V}^0_{52} \times \left(-\frac{43}{57}\right) \times \left(-\frac{10}{72}\right) \times \left(-\frac{10}{75}\right) \times \left(-\frac{22}{97}\right) \tag{27.102}$$

$$= \mathrm{V}^0_{52} \times \frac{473}{149283} \tag{27.103}$$

$$= \left(-\frac{513}{11050}\right) \times \frac{473}{149283} = -\frac{473}{3215550} \tag{27.104}$$

$$\mathrm{P}^0_{59} = -\frac{3215550}{473} = -6798.2029\ldots \text{ days} \tag{27.105}$$

This value is negative because the lunar nodes are retrograding. The same value is given by Oechslin.

Finally, the motion of the lunar apsides is that of tube 62. We have

$$\mathrm{V}^0_{62} = \mathrm{V}^0_{52} \times \left(-\frac{19}{82}\right) \times \left(-\frac{10}{79}\right) \times \left(-\frac{20}{88}\right) = \mathrm{V}^0_{52} \times \left(-\frac{475}{71258}\right) \tag{27.106}$$

$$= \left(-\frac{513}{11050}\right) \times \left(-\frac{475}{71258}\right) = \frac{9747}{31496036} \tag{27.107}$$

$$\mathrm{P}^0_{62} = \frac{31496036}{9747} = 3231.3569\ldots \text{ days} \tag{27.108}$$

or about 8.85 years. This is the period of precession of the lunar apsides. The same value is given by Oechslin.

27.4 The central orrery

This system is driven by the input on arbor 28 whose motion is derived from the motion of arbor 27. I am assuming that arbor 27 makes one turn in a day:

$$\mathrm{V}^0_{28} = \mathrm{V(h)}^0_{27} \times \left(-\frac{20}{40}\right) = -\frac{1}{2} \tag{27.109}$$

This arbor makes one turn in two days. The motion of this arbor is used to obtain the motion of arbor 30:

$$\mathrm{V}^0_{30} = \mathrm{V}^0_{28} \times \left(-\frac{10}{51}\right) \times \left(-\frac{25}{87}\right) = \mathrm{V}^0_{28} \times \frac{250}{4437} = -\frac{125}{4437} \tag{27.110}$$

This motion is then used to derive the motions of all the planets.

First, the motion of Mercury is given by the central arbor 31 whose velocity is

$$\mathrm{V}^0_{31} = \mathrm{V}^0_{30} \times \left(-\frac{46}{114}\right) \tag{27.111}$$

$$= \left(-\frac{125}{4437}\right) \times \left(-\frac{23}{57}\right) = \frac{2875}{252909} \tag{27.112}$$

$$\mathrm{P}^0_{31} = \frac{252909}{2875} = 87.9683\ldots \text{ days} \tag{27.113}$$

The same value is given by Oechslin. It is a good approximation of the orbit period of Mercury, and it turns counterclockwise as it should.

The motions of the other planets are computed similarly. Venus is on tube 34, but we first compute the motion of arbor 33, which is also useful for Mars. We have:

$$\mathrm{V}^0_{33} = \mathrm{V}^0_{30} \times \left(-\frac{59}{67}\right) \times \left(-\frac{37}{75}\right) \tag{27.114}$$

$$= \mathrm{V}^0_{30} \times \frac{2183}{5025} = \left(-\frac{125}{4437}\right) \times \frac{2183}{5025} = -\frac{10915}{891837} \tag{27.115}$$

$$\mathrm{V}^0_{34} = \mathrm{V}^0_{33} \times \left(-\frac{32}{88}\right) = \left(-\frac{10915}{891837}\right) \times \left(-\frac{32}{88}\right) = \frac{43660}{9810207} \tag{27.116}$$

$$\mathrm{P}^0_{34} = \frac{9810207}{43660} = 224.6955\ldots \text{ days} \tag{27.117}$$

The same value is given by Oechslin.

For the Earth which is on tube 46, we first compute the motion of arbor 45 which is useful later for Jupiter:

$$\mathrm{V}^0_{45} = \mathrm{V}^0_{30} \times \left(-\frac{33}{59}\right) \times \left(-\frac{29}{76}\right) = \mathrm{V}^0_{30} \times \frac{957}{4484} \tag{27.118}$$

$$= \left(-\frac{125}{4437}\right) \times \frac{957}{4484} = -\frac{1375}{228684} \tag{27.119}$$

$$\mathrm{V}^0_{46} = \mathrm{V}^0_{45} \times \left(-\frac{51}{112}\right) = \left(-\frac{1375}{228684}\right) \times \left(-\frac{51}{112}\right) = \frac{1375}{502208} \tag{27.120}$$

$$\mathrm{P}^0_{46} = \frac{502208}{1375} = 365.2421\ldots \text{ days} \tag{27.121}$$

The same value is given by Oechslin.

For Mars which is on tube 37, we first compute the motion of arbor 36 which is reused later in the Saturn train. We have:

$$\mathrm{V}^0_{36} = \mathrm{V}^0_{33} \times \left(-\frac{42}{62}\right) \times \left(-\frac{22}{71}\right) = \mathrm{V}^0_{33} \times \frac{462}{2201} \tag{27.122}$$

$$= \left(-\frac{10915}{891837}\right) \times \frac{462}{2201} = -\frac{1680910}{654311079} \tag{27.123}$$

$$\mathrm{V}^0_{37} = \mathrm{V}^0_{36} \times \left(-\frac{51}{90}\right) = \left(-\frac{1680910}{654311079}\right) \times \left(-\frac{51}{90}\right) = \frac{168091}{115466661} \tag{27.124}$$

$$\mathrm{P}^0_{37} = \frac{115466661}{168091} = 686.9294\ldots \text{ days} \tag{27.125}$$

The same value is given by Oechslin.

For Jupiter, we have:

$$\mathrm{V}^0_{49} = \mathrm{V}^0_{45} \times \left(-\frac{19}{68}\right) \times \left(-\frac{18}{59}\right) \times \left(-\frac{41}{91}\right) = \mathrm{V}^0_{45} \times \left(-\frac{7011}{182546}\right) \tag{27.126}$$

$$= \left(-\frac{1375}{228684}\right) \times \left(-\frac{7011}{182546}\right) = \frac{169125}{732374552} \tag{27.127}$$

$$\mathrm{P}^0_{49} = \frac{732374552}{169125} = 4330.3742\ldots \text{ days} \tag{27.128}$$

The same value is given by Oechslin.

For Saturn which is on tube 40, we first compute the motion of arbor 39 which is also used in the train for Uranus:

$$\mathrm{V}^0_{39} = \mathrm{V}^0_{36} \times \left(-\frac{25}{42}\right) \times \left(-\frac{25}{67}\right) = \mathrm{V}^0_{36} \times \frac{625}{2814} \tag{27.129}$$

$$= \left(-\frac{1680910}{654311079}\right) \times \frac{625}{2814} = -\frac{75040625}{131516526879} \tag{27.130}$$

$$\mathrm{V}^0_{40} = \mathrm{V}^0_{39} \times \left(-\frac{15}{92}\right) = \left(-\frac{75040625}{131516526879}\right) \times \left(-\frac{15}{92}\right) \tag{27.131}$$

$$= \frac{375203125}{4033173490956} \tag{27.132}$$

$$\mathrm{P}^0_{40} = \frac{4033173490956}{375203125} = 10749.3067\ldots \text{ days} \tag{27.133}$$

The same value is given by Oechslin.

Finally, Uranus is on tube 43 and its motion is computed as follows:

$$V_{43}^0 = V_{39}^0 \times \left(-\frac{12}{73}\right) \times \left(-\frac{47}{78}\right) \times \left(-\frac{59}{82}\right) = V_{39}^0 \times \left(-\frac{2773}{38909}\right) \tag{27.134}$$

$$= \left(-\frac{75040625}{131516526879}\right) \times \left(-\frac{2773}{38909}\right) = \frac{208087653125}{5117176544335011} \tag{27.135}$$

$$P_{43}^0 = \frac{5117176544335011}{208087653125} = 24591.4472\ldots \text{ days} \tag{27.136}$$

This is a period of about 67.3 years. Oechslin obtains the same value. The actual orbital period of Uranus is about 84 years. Given that the period of Uranus was found to be about 83 years already in 1782, it is possible that Seige's construction is a very early one and based on an early account of the discovery of Uranus (then not yet called Uranus), and this warrants attributing the present machine to about 1781. It is possibly the first machine showing the motion of Uranus. Hahn's *Weltmaschine* in Nuremberg (Oechslin 8.2) also shows Uranus, but with a more accurate period, and it must have been constructed a few years later.

27.5 References

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