Chapter 25

(Oechslin: 3.2)

Neßtfell's *Planetenmaschine* in Munich (c1755-1761)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

25.1 Introduction

The astronomical machine described here was constructed around 1755-1761 by Johann Georg Neßtfell (1694-1762).[1] The machine was constructed for Adam Friedrich Graf von Seinsheim (1708-1779) who was the Prince-Bishop of Würzburg from 1755 to 1779 and Prince-Bishop of Bamberg from 1757 to 1779 and Neßtfell was given 4000 gulden for it.[2]

The clock described here was located in the *physikalische Kabinet* of the University of Würzburg. It entered the collections of the *Bayerisches Nationalmuseum* in Munich in 1877 where it is now exhibited (Inv.-Nr. Phys 254).[4]

This machine is very similar in appearance to Neßtfell's machine kept in Vienna and which was constructed in 1753. The Munich machine was badly damaged during WWII and was considered beyond repair, but thanks to the remarkable work of Ludwig Oechslin and others, it could be restored at the

[1]For a biographical notice of Neßtfell, see the description of the Vienna machine.

[2]For a summary of the history of this machine, see Hess [4, p. 73-77] and Oechslin [9, p. 223]. On possible persons who helped Neßtfell in the construction of his machines, see Oechslin [9, p. 219-220]. One of them was the mechanician Johann Georg Fellwöck (sometimes spelled Felbäch or Felbeck) (1728-1810).[3] The best sources for Neßtfell's machines are Hess [4] and Henck [3]. Besides Oechslin's work [9, p. 33, 36-37, 53-55, 199-203], see also King [5, p. 229-232] and Seelig's articles [13, 14, 17, 15, 16].

[4]Cf. [1, p. 19, 23] and [16, p. 441]. Röntgen confirms the year 1877 [11, p. 12], but in 1882 Schlereth still locates the machine in Würzburg [12, p. 44]. Seidl shows the location of Neßtfell's machine in Munich in 1902 [18, pl .XLVI]. This machine was exhibited at the 1989 Hahn exhibition [19, p. 60-61]. See also [9, p. 234].

end of the 1980s.[5] This was to a great extent possible because of the existence of the Vienna machine.

The most obvious difference between the two machines is that the base is decorated of simple columns in Munich, but of griffins in Vienna. There are also a number of sculpted figures in the Munich machine.[6] However, there are also some differences in the gears. The descriptions of the two machines are very similar and overlap to a large extent, but that way they can be read in any order.

It should moreover be observed that Frater Fridericus's machine in Bamberg, made with the help of Johann Georg Fellwöck in 1772 (Oechslin 4.1), is really a simplified version of Neßtfell's machines. It has the same general cylindrical appearance, there is no orrery, and the entire front Earth-Mercury system is laid out horizontally.

Both of Neßtfell's machines are about 2 meters tall, they have a diameter of about 1 meter, and have a prismatic glass case containing the planetary system. Due to this construction, almost all the gears can be seen. In the front of the machine, there is a large dial representing the Earth-Mercury system.

These two machines are both very daunting, especially because it is rather difficult to understand them at first sight. I am going to split the machine in three parts: the going work, the orrery and the Earth-Mercury system.

[5] On the restoration of the machine for the 1988-1989 exhibitions, see [13, 10, 8, 17].

[6] On these figures by Johann Peter Wagner (1730-1809), see Seelig [14, 15].

Figure 25.1: The location of Neßtfell's machine in 1902. (source: [18, pl .XLVI])

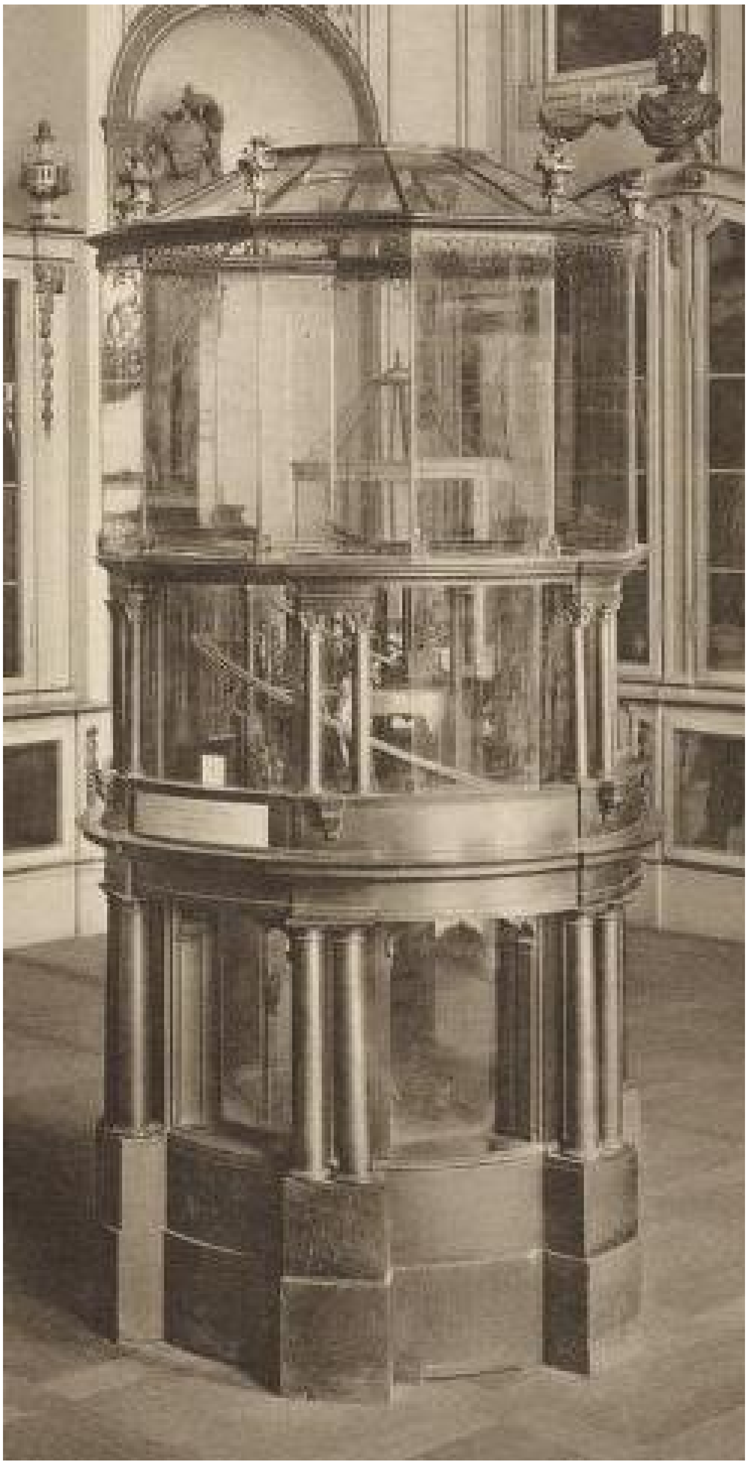

Figure 25.2: Neßtfell's machine in 1902 (detail). (source: [18, pl .XLVI])

Figure 25.3: Neßtfell's machine as it appears today. (photograph by the author)

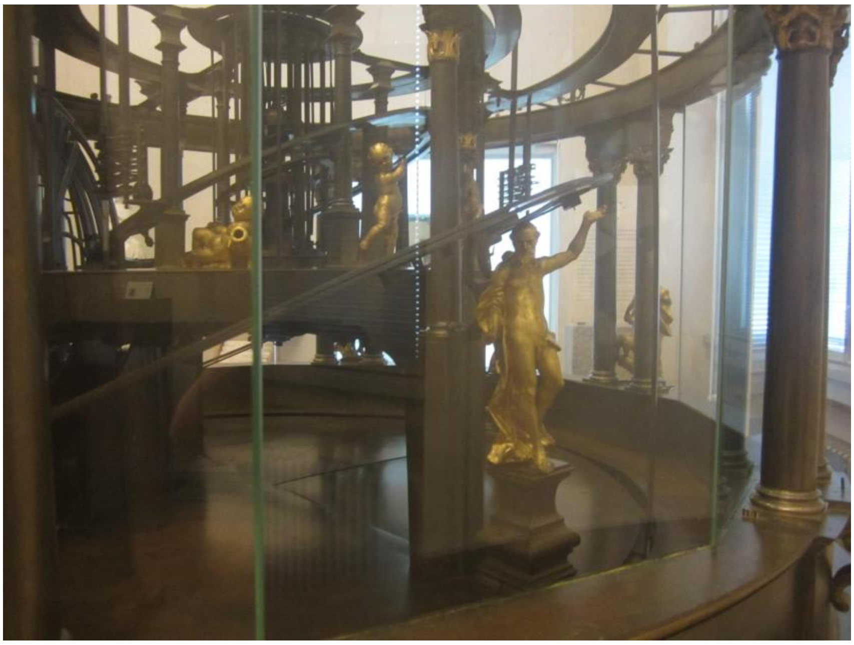

Figure 25.4: Detail of some of the figures inside Neßtfell's machine. (photograph by the author)

Figure 25.5: The gears from the front dial of Neßtfell's machine seen from the side. (photograph by the author)

Figure 25.6: The top of the case of Neßtfell's machine. (photograph by the author)

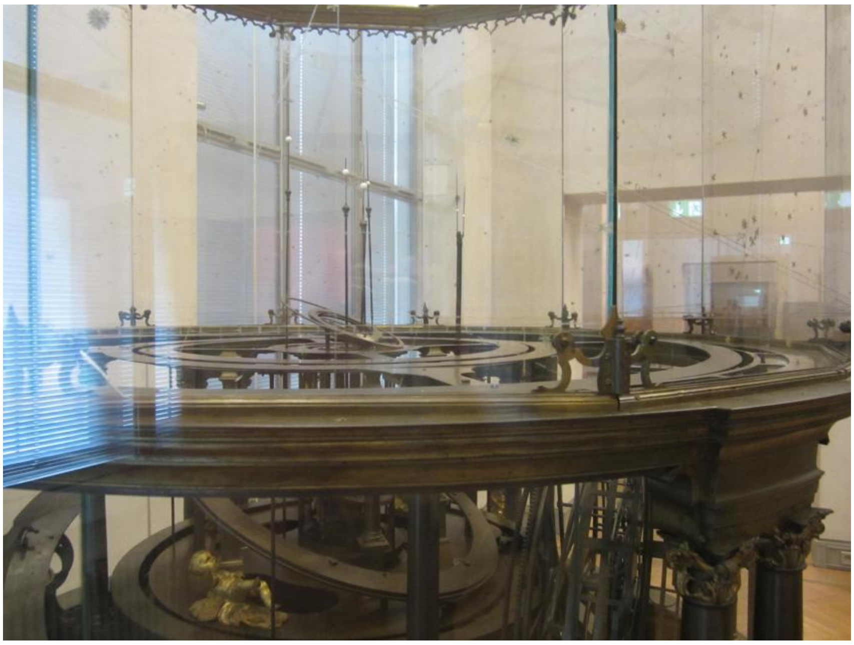

Figure 25.7: The upper level of Neßtfell's machine, which all the planets moving in a tilted plane. (photograph by the author)

25.2 The going work

The going work is set behind the Earth-Mercury system. It is weight-driven and regulated by a pendulum. The going work contains two winding drums, that add up their strengths for a unique output on arbor 9 which makes one turn clockwise (seen from the front) in 24 hours. We have

$$\mathrm{T}_9^0 = 1 \tag{25.1}$$

We can therefore deduce that the first drum (arbor 2) must have the velocity

$$\mathrm{T}_2^0 = \mathrm{T}_9^0 \times \left(-\frac{60}{15}\right) \times \left(-\frac{15}{90}\right) = \mathrm{T}_9^0 \times \frac{2}{3} = \frac{2}{3} \tag{25.2}$$

$$\mathrm{P}_2^0 = 1.5 \text{ days} \tag{25.3}$$

The second drum (arbor 8) has the same velocity:

$$\mathrm{T}_8^0 = \mathrm{T}_2^0 \times \left(-\frac{90}{15}\right) \times \left(-\frac{15}{90}\right) = \mathrm{T}_2^0 \tag{25.4}$$

$$\mathrm{P}_8^0 = 1.5 \text{ days} \tag{25.5}$$

Each drum makes a turn in 36 hours and they turn in the same directions. In the Vienna machine, the two drums make a turn in two days.

These two drums can be rewound using the winding arbors 1 (for the first drum) and 7 (for the second drum).

A gear train connects the drums to the escape wheel on arbor 6. We have

$$\mathrm{T}_6^0 = \mathrm{T}_2^0 \times \left(-\frac{90}{15}\right) \times \left(-\frac{60}{10}\right) \times \left(-\frac{60}{8}\right) \times \left(-\frac{56}{7}\right) \tag{25.6}$$

$$= \mathrm{T}_2^0 \times 2160 = 1440 \tag{25.7}$$

$$\mathrm{P}_6^0 = \frac{1}{1440} \text{ days} = 60 \text{ seconds} \tag{25.8}$$

The escape wheel having 30 teeth, we deduce that the pendulum should make a half swing in $\frac{60}{2\times 30} = 1$ seconds. Its length should be about 1 m.

The only output of the going work is arbor 9 which makes one turn clockwise in one day. This arbor is prolonged by arbor 11 through a coupling. These two arbors can be desynchronized using a shiftable 70-teeth wheel on an arbor also named 11 by Oechslin, but which I will call 11′. The arbor 11 immediately drives the Earth-Mercury system in the front. On arbor 11, there is also a 30-teeth wheel which drives a train towards the orrery. For now, I will merely assume that the going work directly drives the orrery.

The motion of arbor 11 is also transferred to arbor 46 which drives Saturn.

25.3 The Ptolemaic allegories

At the bottom of the machine, there are several allegories of the planets rotating around the Earth. This is a very simplified representation of the Ptolemaic system. The Earth is located is the middle and pictured as a wooden sculpture by Johann Peter Wagner (1730-1809).[7] It is surrounded by the Sun, Mars, Jupiter and Saturn. The planets Mercury and Venus do not seem to be there, and are perhaps missing.

The allegories move on a carrousel, all at the same speed. This motion is obtained from arbor $11'$ whose motion is transferred to the arbor 140, then to the arbor 141. All these arbors make one turn in 24 hours. The arbor 141 carries a 180-teeth wheel which meshes wish a similar wheel on tube 142. This apparently causes the motion of the allegories on arbor 145. This arbor has the following velocity:

$$V^0_{142} = 1 \tag{25.9}$$

$$V^0_{145} = V^0_{142} \times \frac{56}{14} \times \left(-\frac{14}{28}\right) \times \left(-\frac{14}{29}\right) = \frac{28}{29} \tag{25.10}$$

$$P^0_{145} = \frac{29}{28} = 24.8571\ldots \text{ hours} \tag{25.11}$$

which may be an approximation of the lunar day.

25.4 The orrery

The most unusual feature of this machine is that the planets do not move in a horizontal plane, or near a horizontal plane. In almost every orrery, the planets move either in or near a horizontal plane, or in or near a vertical plane. In Neßtfell's machine, the ecliptic is tilted by an angle of 23.5° and the planets move in or near that tilted plane.[8] Consequently, the plane of the equator is a horizontal plane, although it doesn't have any particular significance here. For instance, no horizontal plane is materialized through the Earth.

Moreover, the planets only move indirectly in the ecliptic. In fact, there are horizontal base motions, and the planets are on vertical arms that glide along tilted guides. This all gives a very unusual appearance to this machine.

As mentioned above, the input to the orrery is a gear train starting with the horizontal arbor 11 making one turn clockwise in a day. The motion of this arbor is transferred to arbor 46 which is used to drive the gears for Saturn. Then, the motion of arbor 11 is transferred to arbor 71 which drives the other planets:

$$V^0_{71} = V^0_{11} \times \left(-\frac{30}{78}\right) \times \left(-\frac{78}{30}\right) = V^0_{11} = -1 \tag{25.12}$$

[7] For more on these allegories and Johann Peter Wagner, see Seelig [14, 15].

[8] On this construction, see in particular [9, p. 146-147].

Arbor 71 also makes one turn clockwise in a day.

All the planets are driven similarly, sitting on carriages which are stuck between moving wheels and wheels that are not moving. Neßtfell used large wheels which are about the size of the orbits, and Oechslin assumes that he took his inspiration from a British orrery by John Rowley he had been working on and which had been bought in 1723 in England by Prince Eugene Francis of Savoy-Carignano (1663-1736) [21, p. 42]. This orrery may have been similar to the one by Adams described in this book [9, p. 154, 210].

I am first considering the mean motions of the planets, then the rotations of the planets and the satellites of Jupiter and the Moon. Finally, I am considering the motion of the Moon.

25.4.1 The mean motions and rotations of the planets

I am starting with Saturn and going backwards to Mercury. These systems are in fact all independent and could be described in any order, but I believe it is easier to look at them from the front to the back, and also from the outside to the inside. The descriptions below are very similar, because the structures are similar. However, I prefer to repeat the descriptions, so that the motions can be studied in any order.

All the planets are given a motion of rotation around their axis, but I will only analyze the rotation of Mercury, Venus, the Earth and Mars after each analysis of the mean motions. The rotations of Jupiter and Saturn will be examined when I describe their two satellite systems.

As mentioned above, Neßtfell first produces horizontal motions, and tilted slopes ensure that these motions are transferred near the ecliptic. I assume that each slope is adapted to its planet and that different planets therefore have different slopes. It is also possible that some of the orbits have been made eccentric, but this should be checked. In addition, one should keep in mind that the transfer from the horizontal plane to the tilted plane is a projection and that it does alter the motion.[9] But Neßtfell did not implement irregular motions, other than possibly those resulting from offset orbits.

The mean motions of the planets are obtained through epicyclical gear trains. In each case, there is an input motion and the rotation of a set of gears on a mobile carriage, this rotation being determined by a constraint on another wheel, in this case a fixed wheel. These constructions often lead to periods whose rational expression contains large primes:

- for Mercury, the period involves the prime number 317;
- for Venus, the period involves the prime number 2411;
- for the Earth, the period involves the prime number 296983;
- for Mars, the period involves the prime numbers 101 and 173;

[9] This was also observed by Oechslin, see [9, p. 201-202].

- for Jupiter, the period involves the prime numbers 131 and 389;
- for Saturn, the period involves the prime number 51817.

Neßtfell's construction makes use of gears whose teeth numbers contain only small primes, the largest being 109. It is therefore tempting to view Neßtfell's construction as a way to avoid the use of large primes in gear trains.

However, I do not believe that Neßtfell chose to reach the exact ratios he finally produced. His objectif must have been much more to have the planets on moving carriages, and the complex ratios that these constructions entailed were rather side effects. This is therefore entirely different from Frater David's methods which aimed at obtaining specific ratios without resorting to too large primes. I am giving some details of Frater David's methods in the introduction of this book.

Neßtfell also gives the revolution periods of the planets in his description of the clock [7],[10] and the periods implemented on the clock are only approximations of these values:

Mercury	87d	23h	14m	40s		87.9685...
Venus	224d	17h	44m 11/12			224.7395...
Earth	365d	5h	48m	58s 1/4		365.2423...
Mars	686d	23h	31m	57s		686.9805...
Jupiter	4332d	14h	49m	31s	56‴	4332.6177...
Saturn	10759d	5h				10759.2083...

Neßtfell does not give the actual periods on his machine.

25.4.1.1 The motion of Saturn

On arbor 46, there is a 25-teeth wheel which meshes with a 897-teeth contrate wheel on frame 47. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{47}^{0} = \mathrm{V}_{46}^{0} \times \left(-\frac{25}{897}\right) = \frac{25}{897} \qquad (25.13)$$

$$\mathrm{P}_{47}^{0} = \frac{897}{25} = 35.88 \text{ days} \qquad (25.14)$$

Above the 897-teeth wheel, there is a 900-teeth wheel with interior gearing which meshes with a 25-teeth wheel which is part of a carriage 54 of which one pinion meshes with a fixed 690-teeth wheel. This causes the entire carriage to move, and the carriage supports Saturn and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 48 on which the Saturn system moves up or down, as I will show later. The problem is to find the velocity of the carriage 54. I don't want to resort to some memorized formula. We need to be able to compute the motions without any cryptic

[10] The number of days for Jupiter is mistakenly given as 4323, an obvious typo.

magic. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 900-teeth wheel (frame 47) from that of the 690-teeth wheel (frame 53, fixed). We have

$$\mathrm{V}_{47}^{54} = \mathrm{V}_{53}^{54} \times \left(-\frac{690}{25}\right) \times \left(-\frac{25}{8}\right) \times \left(-\frac{109}{9}\right) \times \left(-\frac{103}{10}\right) \times \left(-\frac{25}{25}\right) \times \frac{25}{900} \tag{25.15}$$

$$= \mathrm{V}_{53}^{54} \times \left(-\frac{258221}{864}\right) = \mathrm{V}_{54}^{53} \times \frac{258221}{864} \tag{25.16}$$

It is important to note that the last ratio $\frac{25}{900}$ is positive, because the motion of arbor 48 is in the same direction as the frame 47, the 900-teeth wheel having an interior gearing.

Now, we have

$$\mathrm{V}_{47}^{54} = \mathrm{V}_{47}^{0} + \mathrm{V}_{0}^{54} = \mathrm{V}_{54}^{53} \times \frac{258221}{864} = \mathrm{V}_{54}^{0} \times \frac{258221}{864} \tag{25.17}$$

Therefore

$$\mathrm{V}_{47}^{0} = \mathrm{V}_{54}^{0} \times \frac{258221}{864} + \mathrm{V}_{54}^{0} = \mathrm{V}_{54}^{0} \times \left(\frac{258221}{864} + 1\right) = \mathrm{V}_{54}^{0} \times \frac{259085}{864} \tag{25.18}$$

And

$$\mathrm{V}_{54}^{0} = \mathrm{V}_{47}^{0} \times \frac{864}{259085} = \frac{25}{897} \times \frac{864}{259085} = \frac{1440}{15493283} \tag{25.19}$$

$$\mathrm{P}_{54}^{0} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{25.20}$$

This is an approximation of the sidereal orbital period of Saturn. The same value is given by Oechslin.

The motion of Saturn around its axis is described below in the section of Saturn's satellites.

25.4.1.2 The motion of Jupiter

The mean motion of Jupiter is obtained like that of Saturn.

On arbor 71, there is a 30-teeth wheel which meshes with a 360-teeth contrate wheel on frame 72. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{72}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{30}{360}\right) = \frac{30}{360} = \frac{1}{12} \tag{25.21}$$

$$\mathrm{P}_{72}^{0} = 12 \text{ days} \tag{25.22}$$

Above the 360-teeth wheel, there is a 361-teeth wheel which meshes with a 30-teeth wheel which is part of a carriage 77 of which one pinion meshes with a fixed 306-teeth wheel. This causes the entire carriage to move, and the carriage supports Jupiter and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 73 on which the Jupiter system moves up or down, as I will show later. The problem is to find the velocity of the carriage 77. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 361-teeth wheel (frame 72) from that of the 306-teeth wheel (frame 76, fixed). We have

$$\mathrm{V}_{72}^{77} = \mathrm{V}_{76}^{77} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \times \frac{30}{361} \tag{25.23}$$

$$= \mathrm{V}_{76}^{77} \times \left(-\frac{355725}{988}\right) = \mathrm{V}_{77}^{76} \times \frac{355725}{988} \tag{25.24}$$

The last ratio $\frac{30}{361}$ is positive for the same reason as in the case of Saturn. Now, we have

$$\mathrm{V}_{72}^{77} = \mathrm{V}_{72}^{0} + \mathrm{V}_{0}^{77} = \mathrm{V}_{77}^{76} \times \frac{355725}{988} = \mathrm{V}_{77}^{0} \times \frac{355725}{988} \tag{25.25}$$

Therefore

$$\mathrm{V}_{72}^{0} = \mathrm{V}_{77}^{0} \times \frac{355725}{988} + \mathrm{V}_{77}^{0} = \mathrm{V}_{77}^{0} \times \left(\frac{355725}{988} + 1\right) = \mathrm{V}_{77}^{0} \times \frac{356713}{988} \tag{25.26}$$

And

$$\mathrm{V}_{77}^{0} = \mathrm{V}_{72}^{0} \times \frac{988}{356713} = \frac{1}{12} \times \frac{988}{356713} = \frac{247}{1070139} \tag{25.27}$$

$$\mathrm{P}_{77}^{0} = \frac{1070139}{247} = 4332.5465\ldots \text{ days} \tag{25.28}$$

This is an approximation of the sidereal orbital period of Jupiter. The same value is given by Oechslin.

The motion of Jupiter around its axis is described below in the section of Jupiter's satellites.

25.4.1.3 The motion of Mars

The mean motion of Mars is obtained like those of Jupiter and Saturn.

On arbor 71, there is a 30-teeth wheel which meshes with a 202-teeth contrate wheel on frame 89. (In Vienna, we have the equivalent ratio 15/101.) This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{89}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{30}{202}\right) = \frac{15}{101} \tag{25.29}$$

$$\mathrm{P}_{89}^{0} = \frac{101}{15} = 6.7333\ldots \text{ days} \tag{25.30}$$

Above the 202-teeth wheel, there is a 204-teeth wheel which meshes with a 30-teeth wheel (102/15 in Vienna) which is part of a carriage 94 of which one pinion meshes with a fixed 194-teeth wheel (97 in Vienna). This causes the entire carriage to move, and the carriage supports Mars. All these motions take place in a horizontal plane, but the carriage itself has an arbor 97 on which Mars moves up or down, as I will show later. The problem is to find the velocity of the carriage 94. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 204-teeth wheel (frame 89) from that of the 194-teeth wheel (frame 93, fixed). We have

$$V_{89}^{94} = V_{93}^{94} \times \left(-\frac{194}{8}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \frac{30}{204} \tag{25.31}$$

$$= V_{93}^{94} \times \left(-\frac{879305}{8704}\right) = V_{94}^{93} \times \frac{879305}{8704} \tag{25.32}$$

The last ratio $\frac{30}{204}$ is positive for the same reason as in the cases of Jupiter and Saturn.

Now, we have

$$V_{89}^{94} = V_{89}^{0} + V_{0}^{94} = V_{94}^{93} \times \frac{879305}{8704} = V_{94}^{0} \times \frac{879305}{8704} \tag{25.33}$$

Therefore

$$V_{89}^{0} = V_{94}^{0} \times \frac{879305}{8704} + V_{94}^{0} = V_{94}^{0} \times \left(\frac{879305}{8704} + 1\right) = V_{94}^{0} \times \frac{888009}{8704} \tag{25.34}$$

And

$$V_{94}^{0} = V_{89}^{0} \times \frac{8704}{888009} = \frac{15}{101} \times \frac{8704}{888009} = \frac{43520}{29896303} \tag{25.35}$$

$$P_{94}^{0} = \frac{29896303}{43520} = 686.9554\ldots \text{ days} \tag{25.36}$$

This is an approximation of the sidereal orbital period of Mars. The same value is given by Oechslin.

Mars also rotates around its axis. In the (synodic) reference frame 94, we have

$$V_{97}^{94} = V_{93}^{94} \times \left(-\frac{194}{8}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \left(-\frac{13}{21}\right) \times \left(-\frac{22}{14}\right) \tag{25.37}$$

$$= V_{93}^{94} \times \left(-\frac{513227}{768}\right) = V_{94}^{0} \times \frac{513227}{768} \tag{25.38}$$

$$= \frac{43520}{29896303} \times \frac{513227}{768} = \frac{87248590}{89688909} \tag{25.39}$$

And in the absolute frame:

$$\mathrm{V}_{97}^{0} = \mathrm{V}_{97}^{94} + \mathrm{V}_{94}^{0} = \mathrm{V}_{94}^{0} \times \left(\frac{513227}{768} + 1\right) = \mathrm{V}_{94}^{0} \times \frac{513995}{768} \tag{25.40}$$

$$= \frac{43520}{29896303} \times \frac{513995}{768} = \frac{87379150}{89688909} \tag{25.41}$$

$$\mathrm{P}_{97}^{0} = \frac{89688909}{87379150} = 1.0264\ldots \text{ days} = 24 \text{ h } 38 \text{ mn } 3.87\ldots \text{ s} \tag{25.42}$$

The same value is given by Oechslin. This period is close to the actual sidereal rotation of Mars.

25.4.1.4 The motion of the Earth

The mean motion of the Earth is obtained like those of Mars, Jupiter and Saturn.

On arbor 71, there is a 13-teeth wheel which meshes with a 170-teeth contrate wheel on frame 98. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{13}{170}\right) = \frac{13}{170} \tag{25.43}$$

$$\mathrm{P}_{98}^{0} = \frac{170}{13} = 13.0769\ldots \text{ days} \tag{25.44}$$

Above the 170-teeth wheel, there is a 217-teeth wheel which meshes with a 16-teeth wheel which is part of a carriage 103 of which one pinion meshes with a fixed 83-teeth wheel. This causes the entire carriage to move, and the carriage supports the Earth. All these motions take place in a horizontal plane, but the carriage itself has an arbor 107 on which the Earth moves up or down, as I will show later. The problem is to find the velocity of the carriage 103. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 217-teeth wheel (frame 98) from that of the 83-teeth wheel (frame 102, fixed). We have

$$\mathrm{V}_{98}^{103} = \mathrm{V}_{102}^{103} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \frac{16}{217} \tag{25.45}$$

$$= \mathrm{V}_{102}^{103} \times \left(-\frac{286350}{10633}\right) = \mathrm{V}_{103}^{102} \times \frac{286350}{10633} \tag{25.46}$$

The last ratio $\frac{16}{217}$ is positive for the same reason as in the cases of Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{98}^{103} = \mathrm{V}_{98}^{0} + \mathrm{V}_{0}^{103} = \mathrm{V}_{103}^{102} \times \frac{286350}{10633} = \mathrm{V}_{103}^{0} \times \frac{286350}{10633} \tag{25.47}$$

Therefore

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{103}^{0} \times \frac{286350}{10633} + \mathrm{V}_{103}^{0} = \mathrm{V}_{103}^{0} \times \left(\frac{286350}{10633} + 1\right) = \mathrm{V}_{103}^{0} \times \frac{296983}{10633} \tag{25.48}$$

And

$$\mathrm{V}_{103}^{0} = \mathrm{V}_{98}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{25.49}$$

$$\mathrm{P}_{103}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{25.50}$$

This is an approximation of the sidereal year. The same value is given by Oechslin.

The Earth also rotates around its axis. In the (synodic) reference frame 103, we have

$$\mathrm{V}_{107}^{103} = \mathrm{V}_{102}^{103} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \tag{25.51}$$

$$\times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \tag{25.52}$$

$$= \mathrm{V}_{102}^{103} \times \left(-\frac{143175}{392}\right) = \mathrm{V}_{103}^{0} \times \frac{143175}{392} \tag{25.53}$$

$$= \frac{138229}{50487110} \times \frac{143175}{392} = \frac{80779335}{80779376} \tag{25.54}$$

$$\mathrm{P}_{107}^{103} = \frac{80779376}{80779335} = 1.0000005\ldots \text{ days} = 86400.0438\ldots \text{ seconds} \tag{25.55}$$

The Earth practically makes one turn around its axis in 24 hours with respect to the Sun. It should do so in exactly 24 hours, but it doesn't. In fact, Oechslin writes that the Earth makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}_{107}^{0} = \mathrm{V}_{107}^{103} + \mathrm{V}_{103}^{0} = \mathrm{V}_{103}^{0} \times \left(\frac{143175}{392} + 1\right) = \mathrm{V}_{103}^{0} \times \frac{143567}{392} \tag{25.56}$$

$$= \frac{138229}{50487110} \times \frac{143567}{392} = \frac{405002507}{403896880} \tag{25.57}$$

$$\mathrm{P}_{107}^{0} = \frac{403896880}{405002507} = 23 \text{ h } 56 \text{ mn } 4.1343\ldots \text{ s} \tag{25.58}$$

This period is close to the actual sidereal rotation of the Earth.

25.4.1.5 The motion of Venus

The mean motion of Venus is obtained like those of the Earth, Mars, Jupiter and Saturn.

On arbor 71, there is a 20-teeth wheel which meshes with a 96-teeth contrate wheel on frame 122. This wheel turns counterclockwise (from above) with the velocity

$$V^0_{122} = V^0_{71} \times \left(-\frac{20}{96}\right) = \frac{5}{24} \tag{25.59}$$

$$P^0_{122} = \frac{24}{5} = 4.8 \text{ days} \tag{25.60}$$

Above the 96-teeth wheel, there is a 103-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 127 of which one pinion meshes with a fixed 65-teeth wheel. This causes the entire carriage to move, and the carriage supports Venus. All these motions take place in a horizontal plane, but the carriage itself has an arbor 130 on which Venus moves up or down, as I will show later. The problem is to find the velocity of the carriage 127. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 103-teeth wheel (frame 122) from that of the 65-teeth wheel (frame 126, fixed). We have

$$V^{127}_{122} = V^{127}_{126} \times \left(-\frac{65}{8}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{44}{10}\right) \times \frac{21}{103} \tag{25.61}$$

$$= V^{127}_{126} \times \left(-\frac{4719}{103}\right) = V^{126}_{127} \times \frac{4719}{103} \tag{25.62}$$

The last ratio $\frac{21}{103}$ is positive for the same reason as in the cases of the Earth, Mars, Jupiter and Saturn.

Now, we have

$$V^{127}_{122} = V^0_{122} + V^{127}_0 = V^{126}_{127} \times \frac{4719}{103} = V^0_{127} \times \frac{4719}{103} \tag{25.63}$$

Therefore

$$V^0_{122} = V^0_{127} \times \frac{4719}{103} + V^0_{127} = V^0_{127} \times \left(\frac{4719}{103} + 1\right) = V^0_{127} \times \frac{4822}{103} \tag{25.64}$$

And

$$V^0_{127} = V^0_{122} \times \frac{103}{4822} = \frac{5}{24} \times \frac{103}{4822} = \frac{515}{115728} \tag{25.65}$$

$$P^0_{127} = \frac{115728}{515} = 224.7145\ldots \text{ days} \tag{25.66}$$

This is an approximation of the sidereal orbital period of Venus. The same value is given by Oechslin.

Venus also rotates around its axis. In the (synodic) reference frame 127, we have

$$\mathrm{V}_{130}^{127} = \mathrm{V}_{126}^{127} \times \left(-\frac{65}{8}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{44}{10}\right) \times \left(-\frac{20}{25}\right) \times \left(-\frac{25}{20}\right) \quad (25.67)$$

$$= \mathrm{V}_{126}^{127} \times \left(-\frac{1573}{7}\right) = \mathrm{V}_{127}^{0} \times \frac{1573}{7} \quad (25.68)$$

$$= \frac{515}{115728} \times \frac{1573}{7} = \frac{810095}{810096} \quad (25.69)$$

$$\mathrm{P}_{130}^{127} = \frac{810096}{810095} = 1.000001\ldots \text{ days} = 86400.1066\ldots \text{ seconds} \quad (25.70)$$

It would seem that Neßtfell attempted here too to have Venus make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. Like in the case of the Earth, Oechslin writes that Venus makes one turn with respect to the Sun in one day, but this is not true.

Incidentally, we can see that Nessfell uses a ratio $\frac{n+1}{n}$, and such ratios appear in several places, although there does not seem to be any good reason for it.

Now, in the absolute frame, we have:

$$\mathrm{V}_{130}^{0} = \mathrm{V}_{130}^{127} + \mathrm{V}_{127}^{0} = \mathrm{V}_{127}^{0} \times \left(\frac{1573}{7} + 1\right) = \mathrm{V}_{127}^{0} \times \frac{1580}{7} \quad (25.71)$$

$$= \frac{515}{115728} \times \frac{1580}{7} = \frac{203425}{202524} \quad (25.72)$$

$$\mathrm{P}_{130}^{0} = \frac{202524}{203425} = 23 \text{ h } 53 \text{ mn } 37.3213\ldots \text{ s} \quad (25.73)$$

The same value is given by Oechslin.

This period is very inaccurate, because Venus rotates much more slowly. It makes one turn in 243 days. But in the 18th century, Venus's motion around its axis was still a mystery, and some thought it was rotating in 23 hours, others in 25 days. The same problem occurs on the Vienna orrery.

25.4.1.6 The motion of Mercury

The mean motion of Mercury is obtained like those of Venus, the Earth, Mars, Jupiter and Saturn.

On arbor 71, there is a 21-teeth wheel which meshes with a 84-teeth contrate wheel on frame 131. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{131}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{21}{84}\right) = \frac{1}{4} \quad (25.74)$$

$$\mathrm{P}_{131}^{0} = 4 \text{ days} \quad (25.75)$$

Above the 84-teeth wheel, there is a 88-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 136 of which one pinion meshes with a fixed 60-teeth wheel. This causes the entire carriage to move, and the carriage supports Mercury. All these motions take place in a horizontal plane, but the carriage itself has an arbor 138 on which Mercury moves up or down, as I will show later. The problem is to find the velocity of the carriage 136. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 88-teeth wheel (frame 131) from that of the 60-teeth wheel (frame 135, fixed). We have

$$\mathrm{V}_{131}^{136} = \mathrm{V}_{135}^{136} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{7}\right) \times \left(-\frac{30}{19}\right) \times \frac{21}{88} \tag{25.76}$$

$$= \mathrm{V}_{135}^{136} \times \left(-\frac{8775}{418}\right) = \mathrm{V}_{136}^{135} \times \frac{8775}{418} \tag{25.77}$$

The last ratio $\frac{21}{88}$ is positive for the same reason as in the cases of Venus, the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{131}^{136} = \mathrm{V}_{131}^{0} + \mathrm{V}_{0}^{136} = \mathrm{V}_{136}^{135} \times \frac{8775}{418} = \mathrm{V}_{136}^{0} \times \frac{8775}{418} \tag{25.78}$$

Therefore

$$\mathrm{V}_{131}^{0} = \mathrm{V}_{136}^{0} \times \frac{8775}{418} + \mathrm{V}_{136}^{0} = \mathrm{V}_{136}^{0} \times \left(\frac{8775}{418} + 1\right) = \mathrm{V}_{136}^{0} \times \frac{9193}{418} \tag{25.79}$$

And

$$\mathrm{V}_{136}^{0} = \mathrm{V}_{131}^{0} \times \frac{418}{9193} = \frac{1}{4} \times \frac{418}{9193} = \frac{209}{18386} \tag{25.80}$$

$$\mathrm{P}_{136}^{0} = \frac{18386}{209} = 87.9712\ldots \text{ days} \tag{25.81}$$

This is an approximation of the sidereal orbital period of Mercury. The same value is given by Oechslin.

Mercury also rotates around its axis. In the (synodic) reference frame 136, we have

$$\mathrm{V}_{138}^{136} = \mathrm{V}_{135}^{136} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{7}\right) \times \left(-\frac{30}{19}\right) \times \left(-\frac{20}{12}\right) \times \left(-\frac{12}{20}\right) \tag{25.82}$$

$$= \mathrm{V}_{135}^{136} \times \left(-\frac{11700}{133}\right) = \mathrm{V}_{136}^{0} \times \frac{11700}{133} \tag{25.83}$$

$$= \frac{209}{18386} \times \frac{11700}{133} = \frac{64350}{64351} \tag{25.84}$$

$$\mathrm{P}_{138}^{136} = \frac{64351}{64350} = 1.00001\ldots \text{ days} = 86401.3426\ldots \text{ seconds} \tag{25.85}$$

Again, it seems that Neßtfell attempted here too to have Mercury make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. And like in the case of the Earth and Venus, Oechslin writes that Mercury makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$V^{0}_{138} = V^{136}_{138} + V^{0}_{136} = V^{0}_{136} \times \left(\frac{11700}{133} + 1\right) = V^{0}_{136} \times \frac{11833}{133} \tag{25.86}$$

$$= \frac{209}{18386} \times \frac{11833}{133} = \frac{130163}{128702} \tag{25.87}$$

$$P^{0}_{138} = \frac{128702}{130163} = 23 \text{ h } 43 \text{ mn } 50.2128\ldots \text{ s} \tag{25.88}$$

Oechslin gives the same value, although there is a typo in his numerical evaluation.

This period is of course very inaccurate, because Mercury rotates much more slowly. It makes one turn in about 59 days. But in the 18th century, Mercury's motion around its axis was still a mystery. The same problem occurs in the Vienna machine.

25.4.2 The motion of the Sun

The motion of the Sun on tube 139 is derived from that of Mercury:

$$V^{136}_{139} = V^{136}_{135} \times \left(-\frac{60}{6}\right) \times \left(-\frac{20}{45}\right) \tag{25.89}$$

$$= V^{136}_{135} \times \frac{40}{9} = V^{135}_{136} \times \left(-\frac{40}{9}\right) = V^{0}_{136} \times \left(-\frac{40}{9}\right) \tag{25.90}$$

$$V^{0}_{139} = V^{136}_{139} + V^{0}_{136} = V^{0}_{136} \times \left(1 - \frac{40}{9}\right) \tag{25.91}$$

$$= V^{0}_{136} \times \left(-\frac{31}{9}\right) = \frac{209}{18386} \times \left(-\frac{31}{9}\right) = -\frac{6479}{165474} \tag{25.92}$$

$$P^{0}_{139} = -\frac{165474}{6479} = -25.5400\ldots \text{ days} \tag{25.93}$$

The same value is given by Oechslin. This is a good approximation of the period of rotation of the Sun which is about 25 days at the equator, but the direction of rotation is wrong. The rotation should have been counterclockwise (from above), and in fact Neßtfell did it correctly in the Vienna machine.[11]

[11] See also [9, p. 203].

25.4.3 The satellites of Jupiter and Saturn

25.4.3.1 The system of Jupiter

The gears for the rotation of Jupiter and its satellites are contained in a unnamed frame which is gliding along a tilted slope. I will call this frame J. The gears use exactly the same ratios as in the Vienna machine, although one wheel has a different teeth-count. The frame J therefore does have a fixed orientation with respect to the Sun. The motion of this frame is therefore exactly that of the carriage 77 seen earlier.

$$\mathrm{V}_J^0 = \mathrm{V}_{77}^0 \tag{25.94}$$

The input to frame J is arbor 73 whose motion is

$$\mathrm{V}_{73}^{77} = \mathrm{V}_{76}^{77} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \tag{25.95}$$

$$= \mathrm{V}_{76}^{77} \times \left(-\frac{450585}{104}\right) = \mathrm{V}_{77}^{76} \times \frac{450585}{104} = \mathrm{V}_{77}^{0} \times \frac{450585}{104} \tag{25.96}$$

$$= \frac{247}{1070139} \times \frac{450585}{104} = \frac{2853705}{2853704} \tag{25.97}$$

We can then easily compute the velocities of Jupiter and its four satellites in the rotating frame. All these motions are directly obtained from arbor 73, and not one from the other.

$$\mathrm{V}_{80}^J = \mathrm{V}_{73}^J \times \left(-\frac{29}{38}\right) \times \left(-\frac{38}{12}\right) = \mathrm{V}_{73}^J \times \frac{29}{12} \text{ (Jupiter)} \tag{25.98}$$

$$\mathrm{V}_{82}^J = \mathrm{V}_{73}^J \times \left(-\frac{31}{32}\right) \times \left(-\frac{28}{48}\right) = \mathrm{V}_{73}^J \times \frac{217}{384} \text{ (Io)} \tag{25.99}$$

$$\mathrm{V}_{84}^J = \mathrm{V}_{73}^J \times \left(-\frac{26}{42}\right) \times \left(-\frac{15}{33}\right) = \mathrm{V}_{73}^J \times \frac{65}{231} \text{ (Europa)} \tag{25.100}$$

$$\mathrm{V}_{86}^J = \mathrm{V}_{73}^J \times \left(-\frac{20}{43}\right) \times \left(-\frac{12}{40}\right) = \mathrm{V}_{73}^J \times \frac{6}{43} \text{ (Ganymede)} \tag{25.101}$$

$$\mathrm{V}_{88}^J = \mathrm{V}_{73}^J \times \left(-\frac{22}{43}\right) \times \left(-\frac{7}{60}\right) = \mathrm{V}_{73}^J \times \frac{77}{1290} \text{ (Callisto)} \tag{25.102}$$

And in the absolute frame:

$$\mathrm{V}_{80}^0 = \mathrm{V}_{80}^J + \mathrm{V}_J^0 = \mathrm{V}_{73}^J \times \frac{29}{12} + \mathrm{V}_J^0 = \frac{2853705}{2853704} \times \frac{29}{12} + \frac{247}{1070139} \tag{25.103}$$

$$= \frac{82765349}{34244448} \tag{25.104}$$

$$\mathrm{P}_{80}^0 = \frac{34244448}{82765349} = 9.9300\ldots \text{ hours} = 9 \text{ h } 55 \text{ mn } 48.2973 \text{ s} \tag{25.105}$$

This is an approximation of the rotation of Jupiter around its axis which is completed in about 9.8 hours. The same value is given by Oechslin.

$$\mathrm{V}_{82}^{0} = \mathrm{V}_{82}^{J} + \mathrm{V}_{J}^{0} = \mathrm{V}_{73}^{J} \times \frac{217}{384} + \mathrm{V}_{J}^{0} = \frac{2853705}{2853704} \times \frac{217}{384} + \frac{247}{1070139} \tag{25.106}$$

$$= \frac{619506913}{1095822336} \tag{25.107}$$

$$\mathrm{P}_{82}^{0} = \frac{1095822336}{619506913} = 1.7688\ldots \text{ days (Io)} \tag{25.108}$$

$$\mathrm{V}_{84}^{0} = \mathrm{V}_{84}^{J} + \mathrm{V}_{J}^{0} = \mathrm{V}_{73}^{J} \times \frac{65}{231} + \mathrm{V}_{J}^{0} = \frac{2853705}{2853704} \times \frac{65}{231} + \frac{247}{1070139} \tag{25.109}$$

$$= \frac{185642977}{659205624} \tag{25.110}$$

$$\mathrm{P}_{84}^{0} = \frac{659205624}{185642977} = 3.5509\ldots \text{ days (Europa)} \tag{25.111}$$

$$\mathrm{V}_{86}^{0} = \mathrm{V}_{86}^{J} + \mathrm{V}_{J}^{0} = \mathrm{V}_{73}^{J} \times \frac{6}{43} + \mathrm{V}_{J}^{0} = \frac{2853705}{2853704} \times \frac{6}{43} + \frac{247}{1070139} \tag{25.112}$$

$$= \frac{25725829}{184063908} \tag{25.113}$$

$$\mathrm{P}_{86}^{0} = \frac{184063908}{25725829} = 7.1548\ldots \text{ days (Ganymede)} \tag{25.114}$$

$$\mathrm{V}_{88}^{0} = \mathrm{V}_{88}^{J} + \mathrm{V}_{J}^{0} = \mathrm{V}_{73}^{J} \times \frac{77}{1290} + \mathrm{V}_{J}^{0} = \frac{2853705}{2853704} \times \frac{77}{1290} + \frac{247}{1070139} \tag{25.115}$$

$$= \frac{44116993}{736255632} \tag{25.116}$$

$$\mathrm{P}_{88}^{0} = \frac{736255632}{44116993} = 16.6887\ldots \text{ days (Callisto)} \tag{25.117}$$

The same values are given by Oechslin. The periods of the satellites are all good approximations of the actual ones.

25.4.3.2 The system of Saturn

The structure of the Saturn system is similar to that of Jupiter, except that it contains two worms for the slow motion of Saturn's fifth satellite, Iapetus. The ratios are the same as in the Vienna machine, except for Titan. There are also a few minor teeth-count differences, but which do not alter the ratios.

The gears for the rotation of Saturn and its satellites are contained in a frame that I call S which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame S is therefore exactly that of the carriage 54 seen earlier:

$$\mathrm{V}_{S}^{0} = \mathrm{V}_{54}^{0} \tag{25.118}$$

The input to frame S is arbor 48 whose motion is

$$\mathrm{V}_{48}^{54} = \mathrm{V}_{53}^{54} \times \left(-\frac{690}{25}\right) \times \left(-\frac{25}{8}\right) \times \left(-\frac{109}{9}\right) \times \left(-\frac{103}{10}\right) \times \left(-\frac{25}{25}\right) \tag{25.119}$$

$$= \mathrm{V}_{53}^{54} \times \left(-\frac{258221}{24}\right) = \mathrm{V}_{54}^{53} \times \frac{258221}{24} = \mathrm{V}_{54}^{0} \times \frac{258221}{24} \tag{25.120}$$

$$= \frac{1440}{15493283} \times \frac{258221}{24} = \frac{673620}{673621} \tag{25.121}$$

We can then easily compute the velocities of Saturn and its five satellites in the rotating frame. All these motions are directly obtained from arbor 48, and not one from the other.

$$\mathrm{V}_{57}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{30}{52}\right) \times \left(-\frac{52}{10}\right) = \mathrm{V}_{48}^{S} \times 3 \text{ (Saturn)} \tag{25.122}$$

$$\mathrm{V}_{59}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{24}{19}\right) \times \left(-\frac{26}{62}\right) = \mathrm{V}_{48}^{S} \times \frac{312}{589} \text{ (Tethys)} \tag{25.123}$$

$$\mathrm{V}_{61}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{19}{31}\right) \times \left(-\frac{31}{52}\right) = \mathrm{V}_{48}^{S} \times \frac{19}{52} \text{ (Dione)} \tag{25.124}$$

$$\mathrm{V}_{63}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{47}{30}\right) \times \left(-\frac{12}{86}\right) = \mathrm{V}_{48}^{S} \times \frac{47}{215} \text{ (Rhea)} \tag{25.125}$$

$$\mathrm{V}_{65}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{27}{41}\right) \times \left(-\frac{8}{84}\right) = \mathrm{V}_{48}^{S} \times \frac{18}{287} \text{ (Titan)} \tag{25.126}$$

$$\mathrm{V}_{69}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{2}{8}\right) \times \left(-\frac{2}{40}\right) \times \left(-\frac{53}{11}\right) \times \left(-\frac{9}{43}\right) \tag{25.127}$$

$$= \mathrm{V}_{48}^{S} \times \frac{477}{37840} \text{ (Iapetus)} \tag{25.128}$$

And in the absolute frame:

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{57}^{S} + \mathrm{V}_{S}^{0} = \mathrm{V}_{48}^{S} \times 3 + \mathrm{V}_{S}^{0} = \frac{673620}{673621} \times 3 + \frac{1440}{15493283} \tag{25.129}$$

$$= \frac{46481220}{15493283} \tag{25.130}$$

$$\mathrm{P}_{57}^{0} = \frac{15493283}{46481220} = 7.9997\ldots \text{ hours (Saturn)} \tag{25.131}$$

$$= 7 \text{ h } 59 \text{ m } 59.1505\ldots \text{ s} \tag{25.132}$$

The same value is given by Oechslin. The actual period of rotation of Saturn is about 10.65 hours.

$$V^0_{59} = V^S_{59} + V^0_S = V^S_{48} \times \frac{312}{589} + V^0_S = \frac{673620}{673621} \times \frac{312}{589} + \frac{1440}{15493283} \quad (25.133)$$

$$= \frac{4834745280}{9125543687} \quad (25.134)$$

$$P^0_{59} = \frac{9125543687}{4834745280} = 1.8874\ldots \text{ days (Tethys)} \quad (25.135)$$

$$V^0_{61} = V^S_{61} + V^0_S = V^S_{48} \times \frac{19}{52} + V^0_S = \frac{673620}{673621} \times \frac{19}{52} + \frac{1440}{15493283} \quad (25.136)$$

$$= \frac{73611705}{201412679} \quad (25.137)$$

$$P^0_{61} = \frac{201412679}{73611705} = 2.7361\ldots \text{ days (Dione)} \quad (25.138)$$

$$V^0_{63} = V^S_{63} + V^0_S = V^S_{48} \times \frac{47}{215} + V^0_S = \frac{673620}{673621} \times \frac{47}{215} + \frac{1440}{15493283} \quad (25.139)$$

$$= \frac{145698564}{666211169} \quad (25.140)$$

$$P^0_{63} = \frac{666211169}{145698564} = 4.5725\ldots \text{ days (Rhea)} \quad (25.141)$$

$$V^0_{65} = V^S_{65} + V^0_S = V^S_{48} \times \frac{18}{287} + V^0_S = \frac{673620}{673621} \times \frac{18}{287} + \frac{1440}{15493283} \quad (25.142)$$

$$= \frac{279291960}{4446572221} \quad (25.143)$$

$$P^0_{65} = \frac{4446572221}{279291960} = 15.9208\ldots \text{ days (Titan)} \quad (25.144)$$

$$V^0_{69} = V^S_{69} + V^0_S = V^S_{48} \times \frac{477}{37840} + V^0_S \quad (25.145)$$

$$= \frac{673620}{673621} \times \frac{477}{37840} + \frac{1440}{15493283} \quad (25.146)$$

$$= \frac{372238731}{29313291436} \quad (25.147)$$

$$P^0_{69} = \frac{29313291436}{372238731} = 78.7486\ldots \text{ days (Iapetus)} \quad (25.148)$$

The same values are given by Oechslin. These period are good approximations of the actual sidereal periods of the satellites of Saturn. Only the period of Titan slightly differs from that in the Vienna machine.

25.4.4 The motion of the Moon

25.4.4.1 The mean motion

The main motion of the Moon is that of frame 111 which rotates around the axis of the Earth. Its motion is obtained through arbor 99 which is located on the Earth carriage 103. We can compute the motion of frame 111 in frame

103:

$$\begin{aligned}
V_{111}^{103} &= V_{99}^{103} \times \left(-\frac{37}{49}\right) \times \left(-\frac{12}{39}\right) \times \left(-\frac{12}{26}\right) \times \left(-\frac{6}{19}\right) &(25.149)\\
&= V_{99}^{103} \times \frac{5328}{157339} &(25.150)
\end{aligned}$$

and

$$V_{99}^{103} = V_{102}^{103} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) = V_{102}^{103} \times \left(-\frac{143175}{392}\right) \qquad (25.151)$$

and therefore

$$\begin{aligned}
V_{111}^{103} &= V_{102}^{103} \times \left(-\frac{143175}{392}\right) \times \frac{5328}{157339} = V_{102}^{103} \times \left(-\frac{95354550}{7709611}\right) &(25.152)\\
&= V_{103}^{102} \times \frac{95354550}{7709611} = V_{103}^{0} \times \frac{95354550}{7709611} &(25.153)\\
&= \frac{138229}{50487110} \times \frac{95354550}{7709611} = \frac{295599105}{8729221319} &(25.154)\\
P_{111}^{103} &= \frac{8729221319}{295599105} = 29.5306\ldots \text{ days} &(25.155)
\end{aligned}$$

The same value is given by Oechslin. This is the motion of the Moon with respect to the Sun, and an excellent approximation of the synodic month. It is however a slightly different ratio than the one used in the Vienna machine.

We can also compute the motion of the Moon in an absolute (sidereal) frame:

$$\begin{aligned}
V_{111}^{0} &= V_{111}^{103} + V_{103}^{0} = \frac{295599105}{8729221319} + \frac{138229}{50487110} = \frac{3194988991}{87292213190} &(25.156)\\
P_{111}^{0} &= \frac{87292213190}{3194988991} = 27.3216\ldots \text{ days} &(25.157)
\end{aligned}$$

The same value is given by Oechslin. This is an approximation of the sidereal month. It is also a slightly different ratio than the one used in the Vienna machine.

25.4.4.2 The nodes and the apsides

Neßtfell's construction for the nodes and apsides in the Munich machine is much simpler than in the Vienna machine. Here, we have a frame 113 which rotates with the Earth. This frame carries a number of gears, two of which mesh with a 138-teeth wheel on a fixed frame 112. One of these wheels drives the train to the nodes tube 117, and the other one the train to the apsides tube 121.

We can compute the motion of tube 117 with respect to frame 113:

$$\mathrm{V}_{117}^{113} = \mathrm{V}_{112}^{113} \times \left(-\frac{138}{29}\right) \times \left(-\frac{31}{35}\right) \times \left(-\frac{10}{30}\right) \times \left(-\frac{30}{40}\right) \tag{25.158}$$

$$= \mathrm{V}_{112}^{113} \times \frac{2139}{2030} = \mathrm{V}_{113}^{112} \times \left(-\frac{2139}{2030}\right) = \mathrm{V}_{103}^{0} \times \left(-\frac{2139}{2030}\right) \tag{25.159}$$

$$= \frac{138229}{50487110} \times \left(-\frac{2139}{2030}\right) = -\frac{42238833}{14641261900} \tag{25.160}$$

$$\mathrm{P}_{117}^{113} = -\frac{14641261900}{42238833} = -346.6303\ldots \text{ days} \tag{25.161}$$

This is is the period of revolution of the nodes with respect to the Sun, the so-called eclipse year. The same ratio was used in the Vienna machine.

We can then compute the motion of the nodes in the absolute frame:

$$\mathrm{V}_{117}^{0} = \mathrm{V}_{117}^{113} + \mathrm{V}_{113}^{0} = \mathrm{V}_{117}^{113} + \mathrm{V}_{103}^{0} \tag{25.162}$$

$$= -\frac{42238833}{14641261900} + \frac{138229}{50487110} = -\frac{2152423}{14641261900} \tag{25.163}$$

$$\mathrm{P}_{117}^{0} = -\frac{14641261900}{2152423} = -6802.2233\ldots \text{ days} \tag{25.164}$$

which is a good approximation of the actual value of 6798 days. The same value is given by Oechslin. This value is negative, because the lunar nodes are retrograding.

Second, the 138-teeth wheel of frame 112 meshes with a 16-teeth wheel for the revolution of the apsides. This wheel meshes in turn with a 12-teeth wheel, whose arbor carries a 2-threaded worm which meshes with a 11-teeth wheel. And the arbor of this wheel carries another 2-threaded worm which meshes with a 37-teeth wheel which moves the apsides on tube 121.

We can compute the motion of tube 121 with respect to frame 113:

$$\mathrm{V}_{121}^{113} = \mathrm{V}_{112}^{113} \times \left(-\frac{138}{16}\right) \times \left(-\frac{16}{12}\right) \times \frac{2}{11} \times \frac{2}{37} = \mathrm{V}_{112}^{113} \times \frac{46}{407} \tag{25.165}$$

$$= \mathrm{V}_{113}^{112} \times \left(-\frac{46}{407}\right) = \mathrm{V}_{103}^{0} \times \left(-\frac{46}{407}\right) \tag{25.166}$$

$$= \frac{138229}{50487110} \times \left(-\frac{46}{407}\right) = -\frac{3179267}{10274126885} \tag{25.167}$$

$$\mathrm{P}_{121}^{113} = -\frac{10274126885}{3179267} = -3231.6024\ldots \text{ days} \tag{25.168}$$

This is a good approximation of the revolution of the apsides, *but with respect to the absolute frame*. And therefore it is false.[12] Moreover, the value is negative, and it should have been positive, which it is in the Vienna machine.

We should actually have found a period of about 411 days, which is the period of revolution of the apsides with respect to the Sun. It would then

[12] About the error in the motion of the apsides, see also [9, p. 203].

seem that Neßtfell mixed up his computations, and used a gear train meant for the absolute motion of the apsides in the context of a synodic motion of the apsides. The problem is not a mere typo or one wheel that could be easily fixed.

If we now compute the motion of the apsides in the absolute frame, we obtain

$$\mathrm{V}_{121}^{0} = \mathrm{V}_{121}^{113} + \mathrm{V}_{113}^{0} = \mathrm{V}_{121}^{113} + \mathrm{V}_{103}^{0} \tag{25.169}$$

$$= -\frac{3179267}{10274126885} + \frac{138229}{50487110} = \frac{49900669}{20548253770} \tag{25.170}$$

$$\mathrm{P}_{121}^{0} = \frac{20548253770}{49900669} = 411.7831\ldots \text{ days} \tag{25.171}$$

but this is the value that should have been obtained with respect to the Sun.

25.5 The Earth-Mercury system

The front part of the machine is the Earth-Mercury system and is focused on displaying the apparent motion of Mercury. It is similar to the Earth-Mercury system in Vienna's machine. Both the Earth and Mercury rotate around the central axis which represents the Sun. Such a main display with Mercury on an astronomical clock is unusual, but the choice of that display is certainly a consequence of the interest for the transits of Mercury in the 18th century. Neßtfell certainly did not choose to display the Earth-Venus system, because transits of Venus are much less frequent than those of Mercury. This representation also has a pedogogical function, it serves to highlight a particular part of the solar system, as stressed by Meier.[13] Finally, it is also a way to illustrate the retrogradations of the planets, as Neßtfell explained himself, and as Oechslin observed [9, p. 36].

The central axis of the panel represents the position of the Sun and the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves around the Sun as I will describe below.

The input to the Earth-Mercury system is arbor 11 which comes from the going work. This arbor makes one turn clockwise in one day:

$$\mathrm{V}_{11}^{0} = -\mathrm{T}_{11}^{0} = -1 \tag{25.172}$$

This arbor is also used to derive several other auxiliary motions which will be described later.

25.5.1 The mean orbital motion of the Earth

Arbor 11 carries a 26-teeth wheel which meshes with a 340-teeth wheel on frame 12. The axis of this frame is the central axis of the front part. The

[13]Cf. [6, p. 44-45].

velocity of frame 12 (seen from the front) is

$$V^0_{12} = V^0_{11} \times \left(-\frac{26}{340}\right) = \frac{13}{170} \tag{25.173}$$

As I note below, it is possible that the distance between the axes of the 26-teeth wheel and the 340-teeth wheel varies, because of the motion of the line of apsides.

Frame 12 also carries an interior wheel of 434 teeth, which meshes with a 32-teeth wheel which is part of a train located on frame 18 and ending with a 32-teeth wheel on tube 20. This tube is not fixed, although one might expect it to be fixed. The frame 18 also carries the Earth's meridian as well as an arbor for the rotation of the Earth. We might therefore expect frame 18 to rotate in a tropical year.

The motion of frame 18 is derived from a train laid between the meridian's axis and a fixed 249-teeth wheel on frame 17. We can compute the motion of frame 18 by first considering the relative motion of frame 17 with respect to that of frame 12:

$$V^{18}_{17} = V^{18}_{12} \times \frac{434}{32} \times \left(-\frac{14}{50}\right) \times \left(-\frac{7}{30}\right) \times \left(-\frac{30}{69}\right) \times \left(-\frac{24}{249}\right) \tag{25.174}$$

$$= V^{18}_{12} \times \left(-\frac{10633}{286350}\right) = \left(V^0_{18} + V^{12}_0\right) \times \frac{10633}{286350} \tag{25.175}$$

Hence

$$V^0_{12} \times \frac{10633}{286350} = V^0_{18} \times \left(1 + \frac{10633}{286350}\right) = V^0_{18} \times \frac{296983}{286350} \tag{25.176}$$

And

$$V^0_{18} = V^0_{12} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{25.177}$$

$$P^0_{18} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{25.178}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

25.5.2 The rotation of the Earth around its axis

We can now also compute the velocity of the central tube 20:

$$V^{18}_{20} = V^{18}_{13} = V^{18}_{12} \times \frac{434}{32} = \left(V^0_{12} - V^0_{18}\right) \times \frac{217}{16} \tag{25.179}$$

$$= \left(\frac{13}{170} - \frac{138229}{50487110}\right) \times \frac{217}{16} = \frac{372255}{5048711} \times \frac{217}{16} = \frac{80779335}{80779376} \tag{25.180}$$

$$V^0_{20} = V^{18}_{20} + V^0_{18} = \frac{80779335}{80779376} + \frac{138229}{50487110} = \frac{405002507}{403896880} \tag{25.181}$$

$$P^0_{20} = \frac{403896880}{405002507} = 0.9972\ldots \text{ days} = 86164.1343\ldots \text{ seconds} \tag{25.182}$$

The central tube 20 actually rotates with the velocity of the sidereal day.

Moreover, arbor 13 which carries the 32-teeth wheel meshing with the interior gearing also carries the frame for the Earth's meridian. Because of the two 32-teeth wheels, the one on the central fixed tube 20, and the one on tube 13, arbor 13 replicates the motion of the central tube. The Earth therefore rotates around a vertical axis (and not a tilted one) and makes one turn in one sidereal day in the absolute frame.

25.5.3 The line of apsides

As mentioned above, the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves like the line of the apsides. However, since the motion of the Earth is obtained through the 26-teeth wheel on arbor 11 meshing with the 340-teeth wheel on frame 12, this implies that there is a variable distance between the two axes. If this is the case, then the teeth are probably such that the meshing is maintained, even though the distance varies.[14]

The motion of the line of apsides is that of tube 34. The slow motion of tube 34 is obtained from that of arbor 11, with four intermediate worms:

$$\mathrm{V}_{34}^{0} = \mathrm{V}_{11}^{0} \times \left(-\frac{1}{61}\right) \times \left(-\frac{1}{26}\right) \times \left(-\frac{1}{11}\right) \times \frac{1}{447} \tag{25.183}$$

$$= \mathrm{V}_{11}^{0} \times \left(-\frac{1}{7798362}\right) = \frac{1}{7798362} \tag{25.184}$$

$$\mathrm{P}_{34}^{0} = 7798362 \text{ days} \approx 21351 \text{ years} \tag{25.185}$$

The same value is given by Oechslin. This is a good approximation of the period of tropical apsidal precession which is about 21000 years. The ratios used here are slightly different from those used in the Vienna machine, resulting in a slightly less accurate period.

25.5.4 A front gear

The front panel carries one large ring with 352 teeth (frame 38). The velocity of this frame is:

$$\mathrm{V}_{38}^{0} = \mathrm{V}_{11}^{0} \times \left(-\frac{1}{103}\right) \times \left(-\frac{1}{16}\right) \times \left(-\frac{1}{16}\right) \times \left(-\frac{1}{352}\right) \tag{25.186}$$

$$= \mathrm{V}_{11}^{0} \times \frac{1}{9281536} = -\frac{1}{9281536} \tag{25.187}$$

$$\mathrm{P}_{38}^{0} = -9281536 \text{ days} \approx -25412 \text{ years} \tag{25.188}$$

The same value is given by Oechslin. This is the period of precession of the equinoxes. Oechslin adds a question mark to the value 352, and perhaps he

[14]See Oechslin's observations on this problem [9, p. 154-155, 199-200].

meant that the wheel is missing. It appears in any case that with a 352-teeth wheel, we obtain almost the same period as in the Vienna machine, and perhaps this is what Oechslin has been trying to achieve, if this is a reconstitution. Yet, the period obtained is somewhat off the more accurate value of 26000 years. If this wheel has been missing, then 352 is perhaps not the value that Neßtfell had used. Perhaps Neßtfell had actually used a 360-teeth wheel, in which case he would have obtained a period of about 25990 years, much closer to what was then thought to be the period of precession of equinoxes.

However, given that the front panel has the Earth rotate in a tropical year, which is confirmed by the motion of the apsides (which otherwise would not take place in about 21000 years), the zodiac is actually fixed. The actual constellations should then move *counterclockwise* and this is presumably what Neßtfell attempted to display, in order to show where in the sky Mercury is located. But on Neßtfell's machine, frame 38 moves clockwise, as if he wanted to show a precession.

In any case, frame 38 is only used to position the actual axis of the Earth, if I understand it correctly.

25.5.5 The motion of Mercury

The main front reference frame for Mercury is frame 34. We have seen above that this frame moves with the motion of the line of apsides of the orbit of the Earth. The Mercury wheel is a wheel on arbor 24 which pivots eccentrically on frame 34. This will account for the elliptical orbit of Mercury, but it does assume that the line of apsides of Mercury coincides with that of the Earth, which is not the case.

25.5.5.1 The mean motion of Mercury

The mean motion of Mercury is given by the Mercury wheel on arbor 24. This motion is derived from that of tube 20. We have

$$\mathrm{V}_{24}^{34} = \mathrm{V}_{20}^{34} \times \left(-\frac{36}{18}\right) \times \left(-\frac{18}{36}\right) \times \left(-\frac{18}{68}\right) \times \left(-\frac{7}{163}\right) \tag{25.189}$$

$$= \mathrm{V}_{20}^{34} \times \frac{63}{5542} \tag{25.190}$$

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{24}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{20}^{34} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} = \left(\mathrm{V}_{20}^{0} - \mathrm{V}_{34}^{0}\right) \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \tag{25.191}$$

$$= \mathrm{V}_{20}^{0} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \times \left(1 - \frac{63}{5542}\right) = \mathrm{V}_{20}^{0} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \times \frac{5479}{5542} \tag{25.192}$$

$$= \frac{405002507}{403896880} \times \frac{63}{5542} + \frac{1}{7634700} \times \frac{5479}{5542} \tag{25.193}$$

$$= \frac{572949380244583}{50263193608696800} \tag{25.194}$$

$$\mathrm{P}_{24}^{0} = \frac{50263193608696800}{572949380244583} = 87.7271\ldots \text{ days} \tag{25.195}$$

This is a good approximation of the orbital period of Mercury. The same value was used in the Vienna machine.

Of course, we could have ignored the term $\frac{1}{7634700}$, and this would only have slightly altered the orbital period of Mercury. This is actually what Oechslin did, and he gave the period as 87.728107 days. But the exact ratio resulting from Neßtfell's construction is the one given above, not the one given by Oechslin.

25.5.5.2 The inclination of Mercury's orbit

Neßtfell has Mercury move radially depending on its distance from its line of nodes. The orbit of Mercury is tilted by about 7 degrees, but on Neßtfell's panel for the Vienna machine, this angle is about 5 degrees, according to Oechslin. This tilted orbital plane is represented by frame 30 which carries a 259-teeth wheel. This is rotated using gears located on frame 34. We first compute the velocity of frame 30 with respect to frame 34:

$$\mathrm{V}_{30}^{34} = \mathrm{V}_{20}^{34} \times \left(-\frac{52}{96}\right) \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{26}\right) \tag{25.196}$$

$$\times \left(-\frac{1}{16}\right) \times \frac{1}{16} \times \left(-\frac{8}{259}\right) \tag{25.197}$$

$$= \mathrm{V}_{20}^{34} \times \frac{1}{5569536} \tag{25.198}$$

$$\mathrm{V}_{30}^{0} = \mathrm{V}_{30}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{20}^{34} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \tag{25.199}$$

$$= \left(\mathrm{V}_{20}^{0} - \mathrm{V}_{34}^{0}\right) \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \tag{25.200}$$

$$= \mathrm{V}_{20}^{0} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \times \left(1 - \frac{1}{5569536}\right) \tag{25.201}$$

$$= \mathrm{V}_{20}^{0} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \times \frac{5569535}{5569536} \tag{25.202}$$

$$= \frac{405002507}{403896880} \times \frac{1}{5569536} + \frac{1}{7634700} \times \frac{5569535}{5569536} \tag{25.203}$$

$$= \frac{3142112029261}{10102586296593530880} \tag{25.204}$$

$$\mathrm{P}_{30}^{0} = \frac{10102586296593530880}{3142112029261} = 3215221.5460\ldots \text{ days} \approx 8800 \text{ years} \tag{25.205}$$

This is the period of the revolution of the nodes resulting from Neßtfell's construction, but it is possible that Neßtfell aimed at having the nodes fixed and that his gears merely provide a very long period. The exact same period was used in the Vienna machine, although with a slightly different train. The period of the revolution of the nodes of Mercury was actually not well known in the 18th century.

Oechslin, instead, found a wrong period of about 15207 years.

25.5.5.3 The viewpoint from the Earth

Finally, there is a guide linking the Earth to Mercury, and showing the backdrop of the sky where Mercury is located. Neßtfell did however not attempt to show the transits of Mercury.

25.5.6 The calendar part

The front panel also has a calendar display, which I am not describing here. It shows the year in an opening like the Vienna machine. The Vienna machine has a mention about the year 1828 not being a leap year. However, Gutwein's engraving of the front of the Vienna machine (figure 24.15) mentions the year 1826. I don't know if a similar mention appears here.[15]

[15] See in particular Fowler's brief description [2, p. 26-27].

Neßtfell's description of the clock also mentions that a day should be removed every 126 years [7, p. 54]. After that time, the excess of the Julian year to the tropical year accumulates to about a day, but it is not totally clear how Neßtfell arrived at that value. In any case, on Gutwein's engraving from 1754, this is indicated by the two years 1826 and 1952.

25.6 Conclusion

Neßtfell's clock follows a very original construction, and one can only be in awe of his imagination, and also in admiration of Oechslin's efforts to render the structure of the machine in a plan, which is certainly not easy.

25.7 References

[1] Renate Eikelmann. *Hundertfünfzig Jahre Bayerisches Nationalmuseum.* München: Bayerisches Nationalmuseum, 2005. [Neßtfell's machine is mentioned on p. 19 and 23].

[2] Ian David Fowler. Die sogenannten „Gutwein-Uhren". *Klassik-Uhren*, 21(3):16–29, 1998.

[3] Herbert Henck. Planetenmaschinen. Eine Bestandsaufnahme der Schriften zu vier fränkischen Planetenmaschinen des 18. Jahrhunderts aus dem Kreis um Johann Georg Neßtfell unter besonderer Berücksichtigung der Beiträge Johann Ludwig Frickers und Johann Zicks. Mit einer Bibliographie zu Johann Georg Neßtfell. *Blätter für württembergische Kirchengeschichte*, 79:62–139, 1979.

[4] Wilhelm Hess. *Johann Georg Nesstfell. Ein Beitrag zur Geschichte des Kunsthandwerkes und der physikalischen Technik des XVIII. Jahrhunderts in den ehemaligen Hochstiftern Würzburg und Bamberg*, volume 98 of *Studien zur deutschen Kunstgeschichte.* Strasbourg: J. H. Ed. Heitz, 1908. [also describes Frater Fridericus's machine].

[5] Henry Charles King and John Richard Millburn. *Geared to the stars — The evolution of planetariums, orreries, and astronomical clocks.* Toronto: University of Toronto Press, 1978.

[6] Ludwig Meier. *Der Himmel auf Erden: die Welt der Planetarien.* Leipzig: Johann Ambrosius Barth, 1992.

[7] Johann Georg Neßtfell. *Kurz-gefaßte, doch gründliche Beschreibung Der - von mir Johann Georg Neßtfell Erfunden- und verfertigten accuraten Copernicanischen Planeten-Machine. Nebst einer Erklärung Des vielfältigen Gebrauches, und Nutzens Derselben In der Astronomie, Geographie, Und Chronologie.* Bamberg: Georg Andreas Gertner, 1761.

[8] Ludwig Oechslin. Planetenmaschine von Johann Georg Neßtfell am BNM in München: Technische Anmerkungen. *Alte Uhren und moderne Zeitmessung*, 11(5):73, 1988.

[9] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[10] Egidius Roidl. Die Restaurierung der Planetenmaschine. *Alte Uhren und moderne Zeitmessung*, 11(6):50–52, 1988.

[11] Wilhelm Conrad Röntgen. *Zur Geschichte der Physik an der Universität Würzburg.* Würzburg: H. Stürtz, 1894.

[12] Heinrich W. Schlereth. *Die Kreis- Haupt- & Universitätsstadt Würzburg und ihre Umgebungen.* Würzburg: Heinrich W. Schlereth, 1882.

[13] Lorenz Seelig. Planetenmaschine von Johann Georg Neßtfell am BNM in München: Kunsthistorische Anmerkungen. *Alte Uhren und moderne Zeitmessung*, 11(5):74–76, 1988.

[14] Lorenz Seelig. » Die Sonne steht und die Erde bewegt sich « — Johann Georg Neßtfells Planetenmaschine im Bayerischen Nationalmuseum. *Kunst & Antiquitäten*, 1988(5):32–39, 1988.

[15] Lorenz Seelig. Der Figurenschmuck der Planetenmaschine Johann Georg Neßtfells. Zum Frühwerk des Würzburger Bildhauers Johann Peter Wagner. In Rainer Kahsnitz and Peter Volk, editors, *Skulptur in Süddeutschland 1400-1770. Festschrift für Alfred Schädler*, pages 327–346. München: Deutscher Kunstverlag, 1998.

[16] Lorenz Seelig. Uhren und wissenschaftliche Instrumente. In Ingolf Bauer, Birgitta Heid, and Lorenz Seelig, editors, *Das Bayerische Nationalmuseum 1855-2005 — 150 Jahre Sammeln, Forschen, Ausstellen*, pages 433–447. München: Hirmer, 2006.

[17] Lorenz Seelig and Egidius Roidl. Die Würzburger Planetenmaschine Johann Georg Nesstfells 1755-1761. Ein Meisterwerk des wissenschaftlichen Uhrenbaus. (Studioausstellung im Bayerischen Nationalmuseum München, 19. Oktober 1988 bis 8. Januar 1989), 1988. [reprints Seelig's article [14] and Roidl's report on the restoration [10], as well as a 5-pages catalogue of the exhibition].

[18] Gabriel von Seidl. *Der Neubau des Bayerischen Nationalmuseums in München.* München: F. Bruckmann A.-G., 1902.

[19] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6

of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

[20] Franz Wawrik. Andreas Spitzer — Schneider, Jesuit, Instrumentenbauer. *Der Globusfreund*, 38/39:87–96, 1990. [and two plates; Neßtfell's planetarium is briefly described on pages 92-93].

[21] Ernst Zinner. *Der Sternenmantel Kaiser Heinrichs — Himmelskunde und Rechenkunst im alten Bamberg*. Bamberg: Bamberger Verlagshaus Meisenbach, 1939. [p. 42 on Neßtfell and Frater Fridericus].