Preface

In the introduction to the chapter on 19th and 20th century astronomical clocks in the recent *General history of horology* [3], I wrote that in the 18th century, a number of remarkable astronomical clocks were made in Europe by Bernardo Facini (1665-1731), Johannes Klein (1684-1762), Johann Georg Neßtfell (1694-1762), Claude-Siméon Passemant (1702-69), Francesco Borghesi (1723-1802), David Ruetschmann (David a Sancto Cajetano, 1726-96), Michael Fras (Aurelius a San Daniele, 1728-82), Philipp Matthäus Hahn (1739-90), and many lesser figures, and this was a way to fill a gap, because in that general work, we unfortunately skipped the 17th and 18th centuries astronomical clocks.

At the time of publication, I was aware of all these clocks, and had seen several of them. I had also published an analysis of one of Passemant's clocks. But I had not been working in depth on most of the clocks I mentioned. The present work gives me an opportunity to have a closer look at these great clocks, those that Klaus Maurice called "*Die Uhrwerke der Theologe*" [1, p. 266-280], using Ludwig Oechslin's work on the "*Priestermechaniker*" as a basis. In fact, in that work Oechslin wrote that

> Die vorliegende Arbeit hofft Grundlagen bereitzustellen, die andere zu weiteren Forschungen anregen können [2, p. 6].

and this is exactly what it did in the present work.

[1] Klaus Maurice. *Die deutsche Räderuhr — Zur Kunst und Technik des mechanischen Zeitmessers im deutschen Sprachraum.* München: C. H. Beck, 1976. [2 volumes].

[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Denis Roegel. Clocks as astronomical models: The nineteenth and twentieth centuries. In Anthony Turner, James Nye, and Jonathan Betts, editors, *A general history of horology*, pages 273–287. Oxford: Oxford University Press, 2022.

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Chapter 1

Introduction

This introduction is a work in progress and is not yet finalized. Newer versions will be put online from time to time.

1.1 The foundation of this work

In 1996, Ludwig Oechslin published a remarkable work on the structure of 28 astronomical clocks or astronomical mechanisms made by "*Priestermechaniker*," that is priests who were also mechanicians, in Germany, Austria and elsewhere [19].[1] A 29th mechanism described by Oechslin is George Adams's orrery. Oechslin studied almost every aspect of these clocks, including their history, and he also disassembled and restored most of them.[2] This was done to a large extent as part of the preparation of the Hahn exhibitions in 1989/1990 [21, 22]. Oechslin counted the teeth of every gear, measured them, drew their layout and computed the gear ratios.

In addition to a general volume synthesizing the examination of all the clocks, Oechslin's work contains detailed drawings of each clock, summaries of gear ratios and calculation of periods, and some photographs.

1.1.1 Oechslin's drawings

Oechslin draws the wheels, arbors and tubes of the clocks all from the side, but only names the arbors and tubes with numbers. The wheels themselves are not named[3] but their teeth numbers are shown. These drawings were probably made with the AutoCAD software, and one can only be impressed by them, as Oechslin managed to fit a lot of information in a very rational way.

[1] On the overlap between theology and technology, see in particular Warnke's essay [23]. He lists all the *Priestermechaniker* mentioned here, except Frater David and Seige.

[2] The exceptions are Hahn's Gotha *Weltmaschine*, and the clocks of Klein and Rinderle, see [19, p. 5].

[3] To be correct, Oechslin gave names to wheels in only three cases, that is for Klein's clocks in Prague, because he added the letters used in Boehm's drawings [2].

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For instance, Oechslin draws the wheels in such a way that one can easily follow their rotation along a train. An example is given in figure 1.1 which corresponds to the system of Jupiter's satellites in Seige's 1791 clock. A number of arbors, tubes and wheels are shown. Usually, wheels that mesh touch each other, but in some cases Oechslin had to separate them in the layout and to use dotted lines to connect them. Moreover, each wheel is drawn as a rectangle with a missing side. Seen from that side, the wheel is assumed to rotate clockwise. In general, meshing wheels are drawn with the missing sides in alternating positions. This convention works for wheels with parallel axes or at other angles. Although Oechslin in general does not draw wheels from the front, it could be done, and we could then for instance add arrows.

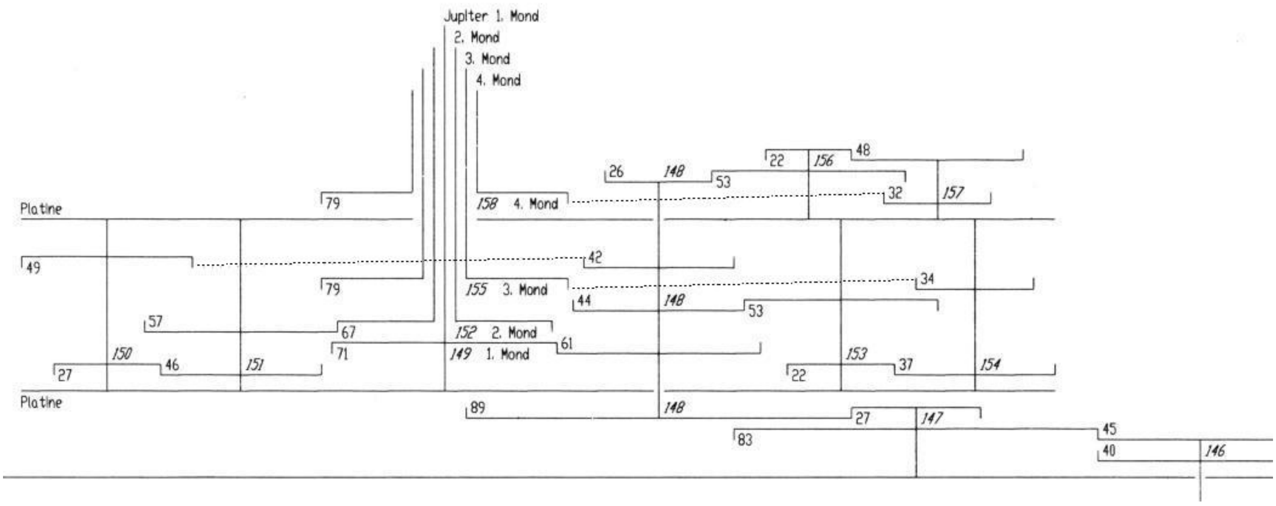

Figure 1.1: Excerpt of Oechslin's drawing for Seige's 1791 machine (Oechslin 10.1).

Oechslin's notation for worms is shown in figure 1.2. The vertical arbor on the right carries a worm (with one thread marked "1w") and this worm meshes with a 34-teeth wheel on arbor 30. Another worm on that arbor meshes with another 30-teeth wheel on the vertical arbor on the left, and so on. The convention for the rotation also applies to the worms.

Oechslin's convention for the wheels does not imply a vantage point, but it does imply that time increases. This is a reasonable assumption for a clock.

Oechslin also names reference frames. There is the absolute still frame, but any part is in fact a reference frame. Usually, a reference frame is some part that will serve as a reference, such as a an arbor, a tube, an arm, a globe, some plate, etc., and it is useful to name these parts, because some computations will make reference to them.

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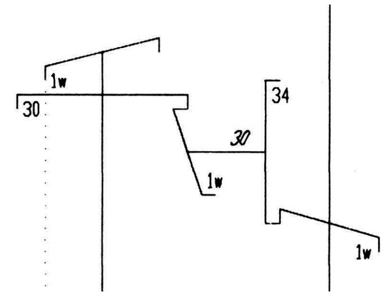

Figure 1.2: Excerpt of Oechslin's drawing for Neßtfell's machine (Oechslin 3.1).

1.1.2 Oechslin's notations

An important task when studying and describing a clock is to compute the motions of its parts. Different wheels rotate at different speeds.

Oechslin's idea is to count rotations, or more exactly ratios of rotations. For instance, for

Oechslin would consider that when arbor 151 makes a turn, arbor 150 makes $-\frac{46}{27}$ turns. This is the basic relation of two meshing wheels. In that case, Oechslin provides a negative ratio. His idea is that the two wheels are seen from the same direction, and when one rotates clockwise, the other one rotates counterclockwise.[4] Of course, this is not always true, depending how the wheels mesh, and also depending on the vantage point. For instance, the motion of arbor 151 (which turns clockwise from above) could be measured from above, and the motion of arbor 150 (which turns clockwise from below) could be measured from below. In that case, the ratio between the two motions should be positive, and not negative.

Most of the time, though, the arbors are vertical and one can agree on a simple viewpoint, such as from above. In some particular cases, though, we need to specify the vantage point.

Oechslin's notation for the above is U^c_{a-b} (U for *Umdrehungen*, or rotations) for the number of rotations of arbor a with respect to arbor b in the reference frame c. In other words, this expression gives the number of rotations of a when b makes one turn in the reference frame c. In the simple case above, if the reference frame is 0, we have $U^0_{150-151} = -\frac{46}{27}$. In figure 1.3, U^{33}_{22-24} is the number of turns of tube 22 when 24 makes one turn, with respect to frame

[4]This is not true if one wheel meshes inside another. In that case, Oechslin gave negative numbers to the wheel with interior teeth. This is a possibility, but it is practical only for parallel axes and if the point of view remains the same. Usually, this is the case. But if the point of view changes, the signs cannot be used blindly. In any case, every time there was an interior gearing, I have used a positive ratio and not a negative one.

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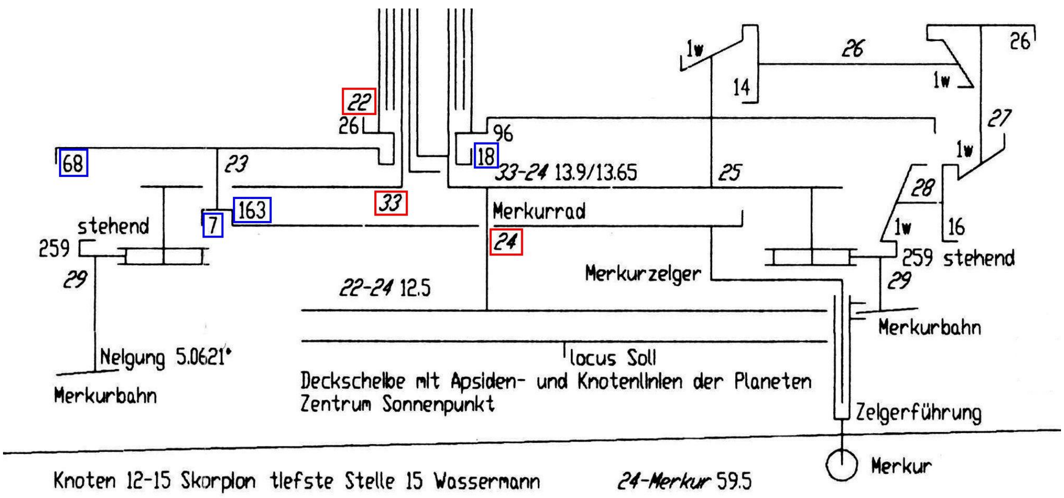
33. In the figure, the reference frames are marked in red (and italics) and the teeth numbers in blue.



$U_{13-15} = U_{13-20}U_{20-15}$			-0.997270	23h 56m 04.13, synodische zu siderischer Erdrotation, Sterntag
U^{20}_{15-22}	$\frac{179}{32}\,\frac{32}{170}$	1		1d, Verteiler
U^{33}_{22-24}	$\frac{68}{18}\,\frac{163}{7}$		87.968254	87d 23h 14m 17s, Merkur auf 33
U^{33}_{22-29}	$\frac{96}{26}\,\frac{14}{\pm1}\,\frac{26}{\pm1}\,\frac{16}{\pm1}\,\frac{259}{\pm1}$		5569536.000000	5569536d, 15248.8780a, Merkur Knoten auf 33

Figure 1.3: Excerpt of Oechslin's drawing (above) and calculations (below) for Neßtfell's machine (Oechslin 3.1). (Note that the expression U^{20}_{15-22} has a typo.)

1.1.3 Oechslin's photographs

Oechslin's book contains some photographs, but their small number is rather disappointing. Moreover, all the photographs, except those on the dust covers, are in black and white. This is probably explained by the need to reduce the publisher's expenses. However, there are certainly many more photographs of each of the clocks. In fact, when Oechslin worked on Bernardo Facini's clock located in the Vatican museums in 1982 [16], and perhaps already before, he developed the habit of carefully documenting the clocks, taking photographs of each part on graphpaper, taking measurements, etc. I know of no other horologist who does the same thing, but all horologists should actually do so. Only the restorers at the MIH (*Musée International d'Horlogerie*) seem to perpetuate this tradition, even to this day.

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1.2 A companion work

1.2.1 The motivation of my work

Unfortunately, although I consider Oechslin's work exceptional, I believe that it hasn't received the attention it deserves.

This is due to the very detailed nature of the work, to its small run (550 copies), to its cost, to its language (German), to its black and white iconography, to the small print of the plates, but also to the way the gear computations have been presented. Since the 1980s, Oechslin has been using his own notation illustrated above, which he masters, but which is almost a cypher for others.

Oechslin's notation is in fact twofold. There are drawing conventions, and Oechslin came up with a convention for side drawings that shows on which side of a wheel this wheel moves clockwise (in normal conditions). This is a very interesting innovation, and although it should also be supplemented by face drawings, I don't have something better to offer at the moment. In Oechslin's book, the gear trains are only shown from the side, at least on the plates. The worms could perhaps have be drawn differently, without sacrificing Oechslin's principles. In a number of cases, an additional view from the front (or from above) would also have been useful for the understanding. More photographs of details — and nowadays partial 3D models — would also have been useful for the understanding. Another possible improvement to Oechslin's drawings is the need to better distinguish the various tubes which are one in each other in many orreries. It is often difficult to follow the lines visually, and it would be better to use at least two different types of lines.

The convention for side drawings of the trains must have been elaborated around 1985-1989, because Oechslin doesn't use it in his work on Facini's clock [17], but he does in his brochure on Hahn's trains [18].[5] In fact, in 1985, Oechslin still only used front views of the trains, although he had already come up with his notations for ratios.

The other part of Oechslin's notation is that for the calculation of ratios, and these notations are very cryptic. He seems to have first used them in print in 1985 [17]. They often involve three indices such as U^c_{a-b}, which is further complicating the understanding. Moreover, Oechslin's periods are often negative (for example for Hahn's *Weltmaschine* in Nuremberg), because he measured the motions positively clockwise, when in fact most astronomical motions are counterclockwise.

As a testimony to how difficult Oechlin's book is, I want to mention that I own a (signed) copy of Oechslin's book since 2003, and I have also studied many clocks, some of them astronomical, but I have only started to go through Oechslin's computations in 2024, in part because the book seemed so daunting. It seemed such a precious book, the plates were so difficult to manipulate, the

[5]However, in that brochure, Oechslin numbered the wheels and not the arbors, but this is equivalent, since an arbor corresponds to a wheel. The only difference is that several numbers can then correspond to a same arbor.

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notations were so terse, that I never really made any use of Oechslin's work. For 20 years I have even kept Oechslin's book in its original packaging!

In fact, Oechslin seems to be the only one to use his notations. I am not aware of another work seriously making use of them.[6] And he seems to be faithful to his notations. For instance Ducommun's orrery was described by Oechslin in 2018 [20] with the same notations as in his 1996 book.

As mentioned above, Oechslin numbers arbors and frames, and not wheels. This could perhaps be improved, because it is often difficult to locate the arbor numbers in a complex drawing. The wheels could be named with letters, for instance and these numbers used instead of the arbors. On the other hand, I have been able to analyze all the clocks, even though I never had names for the wheels. Sometimes, however, when I needed to speak of this or that wheel, I had to write things like "the 72-teeth wheel on arbor such and such." Next to the wheels, the number of teeth are also given. Oechslin distinguishes the arbors by writing their number in italics. This is not always clear, a confusion with the teeth numbers may arise. More than once, I have mistakenly used an arbor number instead of the teeth number, and this is a good enough reason to banish numbers for arbors. Perhaps some additional marker should have been used, perhaps a line over the arbor number, or a bracket. But the best solution would probably be not to use numbers.

1.2.2 New notations

I had been considering the use of new notations for the description of gear trains around 2003 when I first saw the Türler clock in Zurich and Oechslin's 1985 book [17] which describes his notations, but I never pursued this project. In 2024, when I was studying Ducommun's orrery at the MIH in La Chaux-de-Fonds [20], I finally began to work out new notations and to see if it was possible to improve upon those of Oechslin.

In the meantime, I have often been struck by bad notations, cryptic notations that only horologists are accustomed to, or notations that are too language-dependent. For instance, Oechslin uses U for rotations (*Umdrehungen* in German), and other people sometimes use Z for teeth numbers (*Zahl* in German). It is true that some international standards such as ISO 701 define z as the number of teeth, but it is ridiculous to use such a notation when only teeth numbers are used, and not more advanced gear parameters. Moreover, some people use the z notation although they are not forced to do so. In the current work, the only gear parameter I use is the number of teeth. I am not

[6]The only short use of Oechslin's graphical representation of train I know of is in Marini's *Imago Cosmi* [14, p. 411]. Marini gives a figure for a tellurium using an *approximation* of Oechslin's conventions, but without respecting the rules for the rotation of the gears, which seems to indicate that Marini didn't understand Oechslin's conventions. In his book, Marini has also used some of the numerical periods given by Oechslin, but without actually describing how the clocks are working, and comparing the values with modern values, which is meaningless.

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even using the radius. So, if I had to name the number of teeth of a wheel, there would be no ambiguity to use something like N, perhaps subscripted by the wheel number. I encourage everyone who describes clocks with only radius and teeth numbers to use only R and N, at least in English, and not some unnecessary complicated notation, even if some standard prescribes it. Of course, if you are writing in German, using Z might be a good idea. I am not opposing the standard, but some people seem to believe that standards apply everywhere.

Now, regarding the computation of gear ratios, I believe that it is simpler to manipulate the velocity in a given frame of reference, and to make the units implicit, for instance the number of turns per day. In 2024, I have been able to redo all of Oechslin's computations for Ducommun's orrery, and this was an encouraging step. Shortly after my study of Ducommun's orrery, I had another look at Oechslin's 1996 book.

It then occurred to me that it is possible and useful to go again through all the computations first in order to check them (which had probably not yet been done for the 29 shown mechanisms), and then in order to present them in a more intelligible way, and also to make them accessible to a wider audience. This work has now been completed, and I believe that it demonstrates the feasibility of my new approach. It is in fact a lot simpler to use velocities instead of revolutions.[7] There are less indices, it is less prone to errors, and there are basically no special formulas to remember. It is also simpler to measure velocities in astronomical clocks positively counterclockwise, which is what I have done.

My notation (which is not entirely new) is the following: V_a^b is the velocity of arbor a with respect to frame b, measured from some conventional vantage point, and this velocity will always be measured in turns per day, a convenient unit in astronomical clocks. Moreover, the positive direction will be the mathematical "direct" one, which is counterclockwise. Almost all astronomical motions are counterclockwise, and this seems to be the best choice, to avoid carrying unnecessary negative signs.

It should be noted that the correspondence between a drawing and the computation of a ratio depends on the point of view. Oechslin's drawings do not specify the vantage point for the computation, only the vantage point corresponding to a clockwise motion. In general, unless otherwise mentioned, I am measuring the velocities on Oechslin's drawings from the top and from the right.

I will almost always only use simple relations such as

$$\mathrm{V}_a^b = -\mathrm{V}_b^a \tag{1.1}$$

[7]I am actually not the first one to use velocities. For instance, in 1993 Oestmann did use them in his book on the Strasbourg astronomical clock, but his notations are cumbersome and the derivations lack clarity. And Oechslin also defined it, but without really using it.

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and

$$\mathrm{V}_a^c = \mathrm{V}_a^b + \mathrm{V}_b^c \qquad \text{(Chasles' relation)} \tag{1.2}$$

Frame 0 is always the absolute frame and I have decided to make it always explicit, even if no frame moves. Making frame 0 implicit simplifies the notations, but breaks the symmetry of Chasles' relation.

Although the V notation is sufficient, a number of dials are time-related, and the hands have the usual clockwise motion. In order to avoid using negative values in this case, I am sometimes using a different velocity symbol for such indications. V will stand for an astronomical velocity, and T for the "time" motion.

Incidentally, in a few isolated cases, Oechslin uses notations such as U_{16}^{22} to denote the velocity of arbor 16 with respect to frame 22, and this is basically the same notation as mine, but Oechslin doesn't use it extensively.

1.2.2.1 Oechslin's notation redefined

Oechslin's notation U_{b-a}^c can be defined in terms of velocities. The number of turns of arbor b when arbor a makes a turn is the ratio of the velocities of b and a, or

$$U_{b-a}^c = \frac{\mathrm{V}_b^c}{\mathrm{V}_a^c} \tag{1.3}$$

When I first worked through Oechslin's calculations for Ducommun's orrery, I made a modified version of Oechslin's notation, and instead of U_{b-a}^c, I used $\mathrm{O}_{a\to b}^c$, where O stands for Oechslin, and where the arrow highlights the value represented, namely a number of turns of arbor b.

Within the same reference frame R, we have relations such as

$$U_{a-b}^R = U_{c-b}^R \times U_{a-c}^R \tag{1.4}$$

$$U_{a-b}^R = \frac{1}{U_{b-a}^R} \tag{1.5}$$

but the first relation would have been much clearer if Oechslin had used the modified notation:

$$\mathrm{O}_{b\to a}^R = \mathrm{O}_{b\to c}^R \times \mathrm{O}_{c\to a}^R \tag{1.6}$$

because it is similar to Chasles' relation for vectors.

We can also change reference frames. With the modified notation, we have for instance

$$\mathrm{O}_{b\to a}^c + \mathrm{O}_{c\to a}^b = \frac{\mathrm{V}_a^c}{\mathrm{V}_b^c} + \frac{\mathrm{V}_a^b}{\mathrm{V}_c^b} = \frac{\mathrm{V}_a^c}{\mathrm{V}_b^c} - \frac{\mathrm{V}_a^b}{\mathrm{V}_b^c} \tag{1.7}$$

$$= \frac{\mathrm{V}_a^c - \mathrm{V}_a^b}{\mathrm{V}_b^c} = \frac{\mathrm{V}_a^c + \mathrm{V}_b^a}{\mathrm{V}_b^c} = \frac{\mathrm{V}_b^c}{\mathrm{V}_b^c} = 1 \tag{1.8}$$

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and therefore

$$\mathrm{O}^c_{b\to a} = 1 - \mathrm{O}^b_{c\to a} \tag{1.9}$$

which is Oechslin's

$$U^c_{a-b} = 1 - U^b_{a-c} \tag{1.10}$$

From this equation, we can derive another one:

$$\begin{aligned}
\mathrm{O}^c_{b\to a} &= 1 - \mathrm{O}^b_{c\to a} \\
&= 1 - \mathrm{O}^b_{c\to d} \times \mathrm{O}^b_{d\to a} \\
&= 1 - \mathrm{O}^b_{d\to a} + \left(1 - \mathrm{O}^b_{c\to d}\right) \times \mathrm{O}^b_{d\to a} \\
&= 1 - \mathrm{O}^b_{d\to a} + \mathrm{O}^c_{b\to d} \times \mathrm{O}^b_{d\to a}
\end{aligned} \tag{1.11}$$

which is Oechslin's

$$U^c_{a-b} = 1 - U^b_{a-d} + U^c_{d-b} \times U^b_{a-d} \tag{1.12}$$

The latter is often used by Oechslin in his computations, but it is obviously a complex formula and using it is error-prone. It is best to avoid using it.[8]

1.2.2.2 The use of the new notation

In my opinion, using velocities instead of Oechslin's notation is simpler and more intuitive. The notation itself is simpler, as we always assume the same time basis. If we obtain a period in days, we can always convert it in years later. The numerical results are entirely equivalent.

Moreover, whereas Oechslin *multiplies* numbers of turns, we only have to *add* velocities.

[8]In the small guide he wrote for the 1989 Hahn exhibition [18], Oechslin considers a number of gear trains with increasing complexity. These trains can all be analyzed with the velocity approach, without having to memorize special formulas. I am drawing the attention of the reader to the fact that some of Oechslin's drawings in this brochure are ambigous. For instance, on page 9, one should stress that the wheel 2 is fixed. On page 10, the wheel 4 is fixed and not tied to wheel 1, contrary to the appearances. In this drawing, the wheels 2 and 3 are tied together. On page 11, the wheels 1 and 4 have separate motions, but the wheels 2 and 3 are tied together. The wheels 4 and 8 are also tied together. Once these observations are made, the computations of the velocities are straightforward. Oechslin gave general formulas for each case he described, and these formulas could easily be derived from a velocity analysis. There is however little point using these general formulas, as they lead to a very abstract perspective.

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1.3 Types of trains and velocities

This section enumerates the various types of trains and shows how the velocities are computed, using actual examples from the mechanisms described in this book. The velocities are always relative to a frame. This frame will very often be the absolute frame, which is that of the environment of the clock or mechanism. But it can also be that of a moving part of the mechanism.

I will first consider simple trains, with an input and an output, and the train may itself be put in motion from the outside. I will then look at frames whose motion is derived from the input and the output, this time taken as input. This will bring us to the topic of complex ratios and how they were obtained, in particular by Frater David.[9] Finally, I will consider the case of cranks on epicyclic wheels which play an important role for the uneven motions in the orreries and even more in the geocentric globes.

1.3.1 Simple trains

In this section, I am looking at a train from the point of view of its input, or globally. We can thus consider either a fixed train with an input, or a moving train with two inputs, the usual one, and that causing the motion of the entire frame.

I will not consider extensively how trains have been designed, as this is outside the scope of this book. For more on this topic, see Oechslin [19, v.1, p. 80-99], in particular regarding Hahn's methods and the sources in Wolff's *Anfangs-Gründe aller mathematischen Wissenschaften* [24] and Hahn's second *Werkstattbuch* [13]. Finding appropriate ratios sometimes took days to Hahn [19, v.1, p. 87].

1.3.1.1 The case of a fixed train in a frame

The simplest trains are those with arbors on fixed axes. Each arbor carries one or more wheels and each wheel meshes with a wheel on another arbor. There is usually a first and a last wheel, but we could actually have several "output" wheels.

An example is given below. The wheels are in a frame F. The frame itself may be in motion, but this will not concern us here, as all the velocities will be relative to F.

[9] On the use of this name, when others call him Cajetano, see section 1.4.3.

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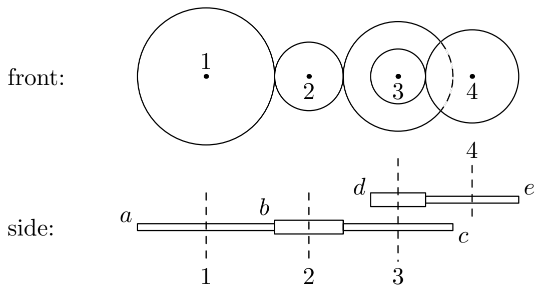
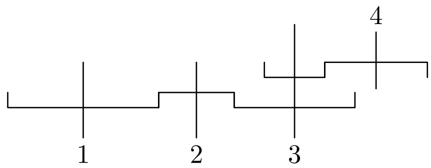


The arbors are numbered 1, 2, 3, and 4, as Oechslin does. The teeth counts are a, b, c, d, e.

Oechslin never shows the wheels from the front. Instead, he might have presented the previous example as



This assumes that the arbor 1 rotates clockwise as seen from above.

Arbor 1 turns at a certain speed V_1^F (number of turns/unit of time). This is an angular speed.

The ratio of speeds are merely equal to the inverse ratio of teeth counts.

(*) $\dfrac{\mathrm{V}_2^F}{\mathrm{V}_1^F} = \dfrac{a}{b}$: when arbor 1 makes one turn, arbor 2 makes $\frac{a}{b}$ turns;

(**) We can also write $\mathrm{V}_2^F = \mathrm{V}_1^F \times \dfrac{a}{b}$

(***) Or $\mathrm{V}_4^F = \mathrm{V}_1^F \times \dfrac{a}{b} \times \dfrac{b}{c} \times \dfrac{d}{e}$

When there is no ambiguity, the reference frame need not be given.

Since we also want to distinguish the direction of rotation, we introduce signs. We choose some *conventional* positive direction. Often it will be positive counterclockwise, because almost all astronomical motions are counterclockwise in the solar system.

When two wheels mesh, the direction changes. So, instead of (*), (**), and (***), we will write

$$\frac{\mathrm{V}_2^F}{\mathrm{V}_1^F} = -\frac{a}{b} \tag{1.13}$$

$$\mathrm{V}_2^F = \mathrm{V}_1^F \times \left(-\frac{a}{b}\right) \tag{1.14}$$

$$\mathrm{V}_4^F = \mathrm{V}_1^F \times \left(-\frac{a}{b}\right) \times \left(-\frac{b}{c}\right) \times \left(-\frac{d}{e}\right) \tag{1.15}$$

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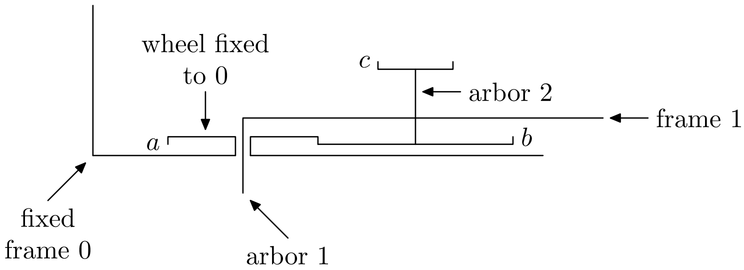
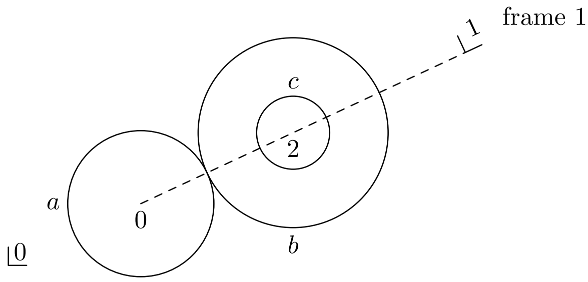
1.3.1.2 The case of a train on a frame which is in motion with respect to another frame

When one frame rotates around the other one, we have different speeds for a same wheel. For instance, consider this from Hahn's globe in the Nuremberg *Weltmaschine* (Oechslin 8.2):



Arbor 2 carries the two wheels b and c and is pivoting on frame 1. Frame 1 moves with arbor 1. When the arbor 1 rotates, it also rotates frame 1 and causes arbor 2 to turn, because of the fixed wheel on frame 0. From above, we have:



Notice here that the two axes are numbered 0 and 2, because the wheel a is on frame 0, and the two other wheels are on arbor 2.

If we assume that we are in frame 1, we have as in case 1:

$$\mathrm{V}_2^1 = \mathrm{V}_0^1 \times \left(-\frac{a}{b}\right) \tag{1.16}$$

In frame 0, the velocity of the frame 1 is V_1^0.

The angle by which arbor 2 has rotated with respect to frame 0 is in fact the sum of the angle of frame 1 with respect to frame 0 and of arbor 2 with respect to frame 1. Consequently, the velocities add up: $\mathrm{V}_2^0 = \mathrm{V}_2^1 + \mathrm{V}_1^0$.

These simple formulas suffice for all the mechanisms in this book. There is no need to remember anything more complex.

1.3.1.3 The more general case of the moving frame

In the previous example, the motion of arbor 1 (or frame 1) causes the motion of the arbor 2. More exactly, the motion within frame 1 is due to the motion of frame 1 with respect to frame 0, and to the relative motion of the wheel fixed to frame 0. That wheel might also be moving.

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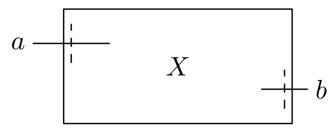
1.3.1.4 Generalization of the previous cases

It is easy to find many different cases of trains, and to derive formulas for each of these trains, but we should step back and try to find common patterns. In fact, as well as the basic (but not sole!) connection between two wheels is to have them meshing at a constant distance, the basic building block of a train is a series of meshing wheels on fixed arbors:



Such a train is represented as a box X. It has an input wheel a and an output wheel b, whose axes (dashed lines) are fixed within X. These axes may be parallel (as shown) or not, but when a turns, so does b. How the wheel b turns as a function of a depends on what is inside the box X. With a simple gear train, the motion of b is proportional to the motion of a.

With this simple train, we can have different configurations with b as the output:

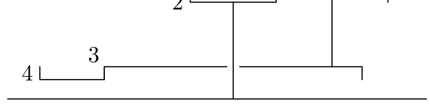
- X can be fixed and a turns;
- a can be fixed and X may turn;
- both a and X can turn.

Moreover, X can be mobile in translation or in rotation. The wheel a could for instant mesh with a rack.

The output wheel b may or may not mesh with another wheel. b could for instance only carry a hand or some other indicator.

However, if X rotates around a fixed point, it is necessary to have at least one input (a or b) mesh with a wheel with a fixed axis.

The two cases shown earlier are merely particular cases of this general case. For instance, the third case of Oechslin in his classification of gears [18] is the following:



Here the arbor 1 is at a fixed position on the wheel 3 which rotates. We can consider that the wheel 3 is a fixed element of the box X, and that the input is arbor 2 and the output is arbor 1:

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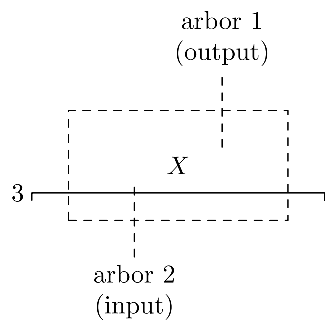

The box X rotates around the same axis as arbor 2, but this is a particular case. In general, X may rotate around a different axis as the input.

Oechslin's cases 4 and 5 can also be reinterpreted in terms of the train-box. Oechslin's case 5 is merely that where both X and the input are mobile. It doesn't seem necessary to call such a construction a "differo-epicyclic train."

1.3.2 Moving frames using the input and the output

Usually, either the box X is fixed and the input wheel a is moving, or both are moving, producing some motion for wheel b.

It is however also possible to have a and b as inputs, and this will cause the frame X to move. It is not a different construction, it is just that the inputs are not the same as usual.

Such constructions can be used to obtain motions which are difficult to obtain using the above methods. They have been used by Neßtfell in his two planetary machines (Oechslin 3.1 and 3.2) and by Frater David in his clock in the Palais Schwarzenberg (Oechslin 6.2), albeit not for the same reasons.

1.3.2.1 General principles

Of course, there is nothing really magic here, we merely have a function of two parameters $z = f(w, r)$, where w is the motion of some wheel, r the motion of some frame and z the motion of some output (which, admittedly, needs to be carefully defined). With certains trains, there exists a function g such that $r = g(z, w)$, in other words the motion of the frame r is determined by the motion of w and z.

However, in order to make this well-defined, we need to clarify what is z. In the example above, with arbor 2, z might be thought of as both the relative motion of arbor 2 with respect to frame 1, and the absolute position of the arbor 2. It is simplest to combine these two by introducing a new wheel having the same axis as the rotation axis of the frame and meshing with the output. Likewise, we introduce a new wheel having the same axis as the rotation axis of the frame and meshing with the input. This will define the two input motions, and the output motion which is that of the frame.

The most general case is thus illustrated here:

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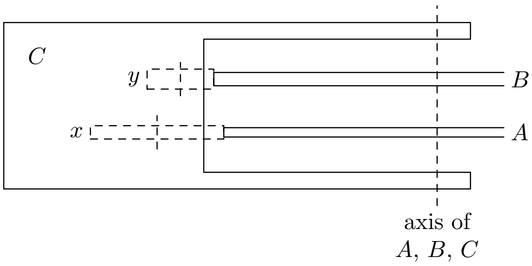

C is the rotating frame and the wheels A and B are the input motions. They all rotate around the same axis. The motions can be provided through, say, a central arbor for A and a tube for B, and the output can likewise be transferred through a tube fixed on C. It is thus possible to see this entire construction as a box having two inputs, A and B, and one output C. Such a box can be reused in other contexts.

It should also be noted that A and B may have interior gearings (as is in Neßtfell's constructions), but they are shown here with normal outside teeths. The two wheels x and y are on fixed axes on C, but usually not on the same one. The velocity of y with respect to C is a function of the velocity of x. In general, we have a linear rational function:

$$\mathrm{V}_y^C = \frac{p}{q} \times \mathrm{V}_x^C \text{ where } p, q \in \mathbb{Z}^* \tag{1.17}$$

We define $\alpha = \frac{n_A}{n_x}$ and $\beta = \frac{n_B}{n_y}$ where n_A, etc., are the number of teeth of A, etc. Some of these numbers may be negative in case of interior gearings. We also set $\gamma = \alpha/\beta$.

Let $a = \mathrm{V}_A^0$, $b = \mathrm{V}_B^0$ and $c = \mathrm{V}_C^0$ be the absolute velocities of A, B and C. Computing the motion of C is then straightforward. In the reference frame of C, we have

$$\mathrm{V}_B^C = (\mathrm{V}_A^C \times \alpha) \times \frac{p}{q} \times \frac{1}{\beta} = \mathrm{V}_A^C \times \frac{p}{q} \times \gamma \tag{1.18}$$

$$\mathrm{V}_B^C = \mathrm{V}_B^0 - \mathrm{V}_C^0 = b - c \tag{1.19}$$

$$\mathrm{V}_A^C = \mathrm{V}_A^0 - \mathrm{V}_C^0 = a - c \tag{1.20}$$

Therefore

$$b - c = (a - c) \times \frac{p}{q}\gamma \tag{1.21}$$

and

$$\frac{p}{q}\gamma = \frac{b - c}{a - c} = \frac{b/a - c/a}{1 - c/a} \tag{1.22}$$

We also have

$$c = \frac{b - a\frac{p}{q}\gamma}{1 - \frac{p}{q}\gamma} \tag{1.23}$$

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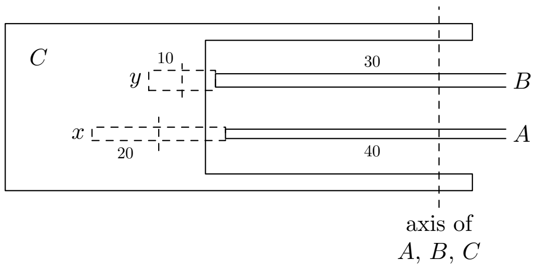
Given the details of the train, we can compute the velocity of C. Incidentally, it is clear that this construction can be used to add two velocities (take $\frac{p}{q}\gamma = -1$ and multiply by 2) and in fact to obtain any linear combination of two velocities.

As an application, consider for instance the train



with $p/q = 25/17$, $a = 1$ and $b = 3$. We obtain $c = 103$.

Conversely, with $a = 1$, $b = 3$ and $c = 103$, we find

$$\frac{p}{q}\gamma = \frac{3-103}{1-103} = \frac{50}{51} \tag{1.24}$$

and we can choose for instance $p/q = 25/17$ and $\gamma = 2/3$. Other solutions are possible.

In general, given a value of $r = c/a$, we can choose b/a such that $\frac{b-c}{a-c}$ only contains small enough prime factors, in which case we can easily design the train within C (ratio p/q) and choose the ratios for x and y. But one should not take any random value for a and b. For instance, in the previous example, with $a = 2$ and $b = 6$, we still end up with primes 97 and 101.

1.3.2.2 The reduction of primes

If we start with an input motion of (say) one turn per day, can we obtain any rational output period using only small primes? For instance, can we get a wheel making a turn in 991/613 days, where both numerator and denominator are prime, using, say, only wheels with less than 50 teeth? An answer to this question may not seem obvious at first.

This question was certainly considered by many authors. Oechslin cites for instance the case of Hahn who gave an (incorrect) train to obtain a period of 179 days (179 is prime), without using a 179-teeth wheel.[10]

1.3.2.2.1 A theoretical answer

It is however easy to give a theoretical answer. We can first observe that if we can get a motion in 991 days and another one in 613 days, these two motions can be combined to obtain the fraction 991/613. This follows from

[10] See [19, p. 92-95].

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the fact that a train corresponding to a multiplication by 613 also corresponds to a division by 613 when reversed, that is when we exchange the input and the output. Chaining a train for 991 and another one for 1/613, produces a train for the ratio 991/613. This is certainly not the best way to obtain this ratio, but it is a theoretical construction. It shows that in a theoretical approach, we only need to consider integer velocities. In the above, we can also assume $a = 1$ and $b, c \in \mathbb{N}$. In that case, we have

$$\frac{p}{q}\gamma = \frac{b-c}{1-c} \tag{1.25}$$

If c is a prime number, taking $b > 1$ always results in a ratio with smaller prime factors. For instance, if $c = 991$ and $b = 2$, we have

$$\frac{b-c}{1-c} = \frac{991-2}{990} = \frac{23 \times 43}{2 \times 3^2 \times 5 \times 11} \tag{1.26}$$

Even if we take $c = 3$ and $b = 2$, we have

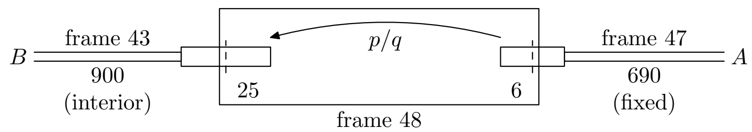
$$\frac{b-c}{1-c} = \frac{1}{2} \tag{1.27}$$

and this shows that in fact we can even get rid of the factor 3. It is possible to implement any ratio using only the prime factor 2. We could build an astronomical clock with whatever complex motion using only wheels with say 16 and 32 teeth. This wouldn't be efficient, but it would be possible.

1.3.2.2.2 Neßtfell's construction

Neßtfell and Frater David used configurations such as the above, but their aims were not the same. I will first consider Neßtfell's constructions which he implemented in his two planetary machines starting in 1753 (Oechslin 3.1 and 3.2).

Here is for instance his configuration for the revolution of Saturn:



Saturn is held by a carriage (frame 48) which is stuck between a fixed wheel A and a moving interior wheel B. The 900-teeth wheel on frame 43 rotates with the velocity $\mathrm{V}^0_{43} = \frac{25}{897}$ and meshes with a 25-teeth pinion in the carriage. A 6-teeth pinion of the carriage meshes with the fixed 690-teeth wheel (frame 47). Finally, the velocity of the 25-teeth pinion is $p/q = 11227/120$ times that of the 6-leaves pinion. This ratio only involves primes up to 109.

With this configuration, we have $a = 0$, $b = 25/897$, $\gamma = -\frac{115}{36}$ and

$$c = \frac{b - a\frac{p}{q}\gamma}{1 - \frac{p}{q}\gamma} = \frac{25/897}{1 + 11227/120 \times 115/36} = \frac{1440}{15493283} \tag{1.28}$$

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and the period is

$$P^0_{48} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{1.29}$$

Now, although the numerator 15493283 contains the large prime 51817, the above construction led to this period without using a wheel with that many teeth.

In the light of the later work of Frater David, it is tempting to consider that Neßtfell also tried to reach exact ratios with epicyclic gears. But this assumption is difficult to defend, first because the periods implemented by Neßtfell are not exactly those that he gave in his description of the machines [15], and second because the implemented ratios do not derive from simple periods in seconds or other units. For instance, for Jupiter, Neßtfell produces a revolution in 1070139/247 days, and this value does not have a simple expression in seconds. This applies to all the planets, except Saturn.

Oechslin did no try to analyze Neßtfell's methods, but I believe that the epicyclic constructions used by Neßtfell do have another purpose. His aim was clearly to have the planets move along rails, and for this to work, Neßtfell decided to use large gears, two of which are moving, and the third one being fixed. For instance, for Saturn, the fixed wheel has 690 teeth and the moving wheels have 897 and 900 teeth. Neßtfell clearly understood that he could set the planets on moving carriages located between these large gears.

Neßtfell obviously computed the motions of the planets, but his purpose was not to obtain the exact values given in his description of the clock. Perhaps he would have done so if he knew that he could reach these values, but I believe this was not clear to him.

I rather think that Neßtfell arranged each train so that the revolution period of the planet was sufficiently close to the one he gave in his description, and so that the train obtained could be constructed. However, in no case has he obtained exactly the value that he thought was the most accurate.

1.3.2.2.3 Frater David's method

In his clock in the Palais Schwarzenberg (Oechslin 6.2) constructed around 1793, Frater David used a more systematic method, always expressing the periods sought in integer numbers of seconds, minutes, hours or days. He explained his technique in his books [4, 5, 6] and a series of articles published shortly before his death [7, 8, 9, 10, 11]. This technique was summarized by Oechslin [19, p. 95-99, 153-154]. Frater David may have been inspired by Neßtfell's planetary machine in Vienna, which he restored at about the same time,[11] but his purpose was different. Whereas Neßtfell certainly used epicyclic

[11]See [19, p. 227-228]. This connection was not put forward by Bertele when he wrote about this clock, and he still viewed the origin of epicyclic gears in Frater David's clock as a mystery we would never be able to elucidate [1, p. 24].

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gears because the planets were located on moving carriages, Frater David used epicyclic gears for the specific purpose of getting some definite ratios.

Moreover, Frater David's also wanted to use only wheels with less than 100 teeth, but as Oechslin observed [19, p. 176], this was not that efficient, because it came at the expense of a greater number of wheels and pinions.

The figure 1.4 shows the arrangement of Frater David's gears. A central arbor carries a fixed wheel B, a moving wheel C and a rotating frame A. This frame carries three wheels, two which are also named A, and one idle wheel W (for *Wechselrad* in German).

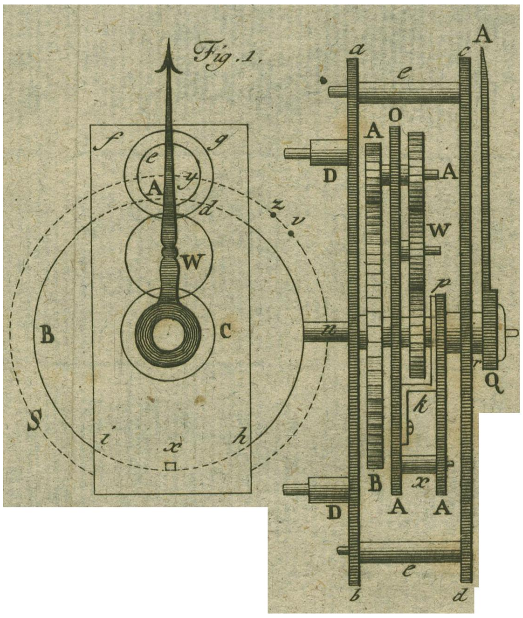

Figure 1.4: Frater David's epicyclic box as illustrated in 1791 [4]. The figure was redrawn in the 1793 edition, but the structure is similar [5].

Although the notations are a bit clumsy, the idea is to have an input motion on wheel C, and that this input motion causes the frame A to rotate at a certain speed. This is in fact exactly a case of the general structure given earlier, but with different names.

But what is interesting is that Frater David explained how he obtained the trains. He does not manipulate velocities, but the periods of rotation (the inverses of the velocities). Let p be the period of rotation of frame A, say in hours. And let r be the period of rotation of the input wheel C. And let s be the ratio between the velocity of B and that of C with respect to the frame A. That is, we have

$$\mathrm{V}_B^A = s \times \mathrm{V}_C^A \tag{1.30}$$

In general, p is what we are trying to achieve, and for that purpose we can carefully choose the input period r and the train inside the epicyclic box, with the velocity ratio s.

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We have

$$\begin{aligned} \mathrm{V}_B^A &= \mathrm{V}_C^A \times s = -\mathrm{V}_A^B = -\mathrm{V}_A^0 && (1.31)\\ &= \left(\mathrm{V}_C^0 + \mathrm{V}_0^A\right) \times s && (1.32)\end{aligned}$$

Hence

$$\begin{aligned} \mathrm{V}_0^A(s+1) &= -\mathrm{V}_C^0 s && (1.33)\\ \frac{\mathrm{V}_A^0}{\mathrm{V}_C^0} &= \frac{s}{s+1} && (1.34)\end{aligned}$$

and

$$\begin{aligned} \frac{\mathrm{P}_A^0}{\mathrm{P}_C^0} &= \frac{p}{r} = \frac{s+1}{s} && (1.35)\\ \frac{1}{s} &= \frac{p-r}{r} && (1.36)\end{aligned}$$

For instance, Frater David considers the case where the input wheel C makes a turn in an hour ($r = 1$) and the frame A makes a turn in 23 hours ($p = 23$) [4, p. 24]. He then finds $1/s = 22$ and he chooses the train $1/s = \frac{8}{36} \times \frac{9}{44}$. Instead of the prime 23, Frater David now only has the prime 11.

In another example [4, p. 25], he takes $p = 101$ and $r = 1$ and finds $1/s = 100$.

Frater David also considers the case where the frame A and the input wheel C turn in opposite directions, but it suffices here to add signs to the periods.

In a further example [4, p. 32], Frater David considers the case of $p = 23$ and $r = 6$. The idea is always to choose r such that $\frac{p-r}{r}$ contains only small primes. If p is a prime, any positive integer value of r will lead to a smaller prime.

In another more realistic example [4, p. 100], Frater David considers the tropical revolution of the Sun. Taking $p = 31556928$ seconds, he first chooses $r = 32928$ and obtains a first train whose greatest prime is 71. He also considered $r = 63072$ and then the greatest prime is 73. The first solution is the one he took for the Palais Schwarzenberg clock. The period of arbor 42 in that clock is exactly r.

Frater David also explains how he obtained the tropical motion of the Moon in the Palais Schwarzenberg clock ($p = 2360584$ and $r = 10584$) [4, p. 185].

And finally, Frater David has also used this construction for the motion of the apsides and the lunar nodes in the Palais Schwarzenberg clock. The details are given in his *Praktische Anleitung für Künstler* [6, p. 40 and 59]. In these two trains, he took p and r to be periods expressed in minutes (as he does in some of his examples), namely $p = 4670432$ and $r = 42432$ for the apsides, and $p = 9789403$ and $r = 29403$ for the nodes.

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1.3.3 Cranks on epicyclic wheels

A number of constructions have cranks on epicyclic wheels, that is, cranks on wheels whose center is rotating. These cranks are used to produce oscillatory motions and not merely rotating motions. These motions are not particularly difficult to compute and usually all we need to compute is the velocity of the arbor supporting the crank.

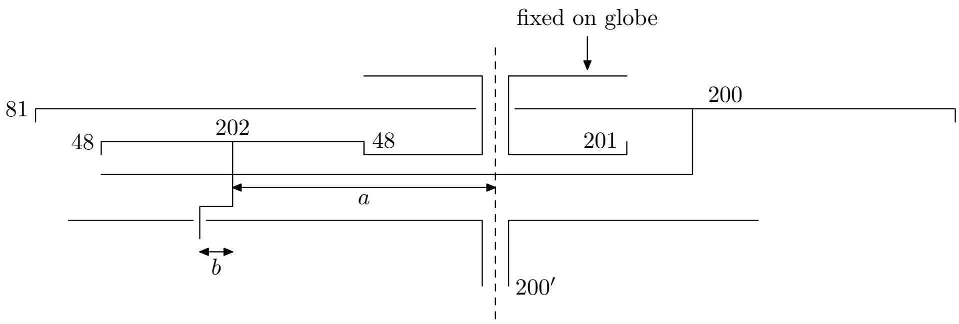
The oscillatory motions are mainly used to account for the uneven motion of the planets around the Sun, and for the retrogradations of the planets in geocentric representations. A number of constructions have been used in the clocks described here, and I will review the main ones.

1.3.3.1 Elliptical motions

The first category of oscillatory motions are meant to accelerate or slow down the motions of the planets or the Moon to account for the main anomaly in their motion, due to their non circular orbit.

1.3.3.1.1 The motion of the Sun in the Nuremberg *Weltmaschine*

For instance, in Hahn's Nuremberg *Weltmaschine* (Oechslin 8.2), we have the following configuration:



A 81-teeth wheel on tube (frame) 200 rotates on a fixed tube 201 tied to globe G and carrying a 48-teeth wheel. The tube 200 also carries a support on which arbor 202 pivots. This arbor carries a 48-teeth wheel meshing with the previously mentioned 48-teeth wheel, and a crank which moves tube 200′ which carries the Sun. Tube 200′ has the same mean motion as tube 200, but because of the crank, it is sometimes ahead and sometimes behind 200.

Tube 200 makes one turn counterclockwise in one tropical year and corresponds to the mean motion of the Sun. Its velocity is V_{200}^{G} where $G = 201$ is the globe.

The question is: what is the motion of tube 200′? We could actually compute it very precisely using the offsets a and b which are given by Oechslin. But we are mainly interested here in the period of oscillation and we want to know if Hahn's construction is correct.

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If $b = 0$, then tubes 200 and 200′ would have the same motion. It is also clear that if $b > 0$, tube 200′ has an oscillating motion around a mean position which is that of tube 200.

In order to compute this oscillation, we should imagine that we are on frame 200 and look at the motion of arbor 202. We can therefore compute

$$\mathrm{V}_{202}^{200} = \mathrm{V}_{G}^{200} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{G}^{200} = \mathrm{V}_{200}^{G} \tag{1.37}$$

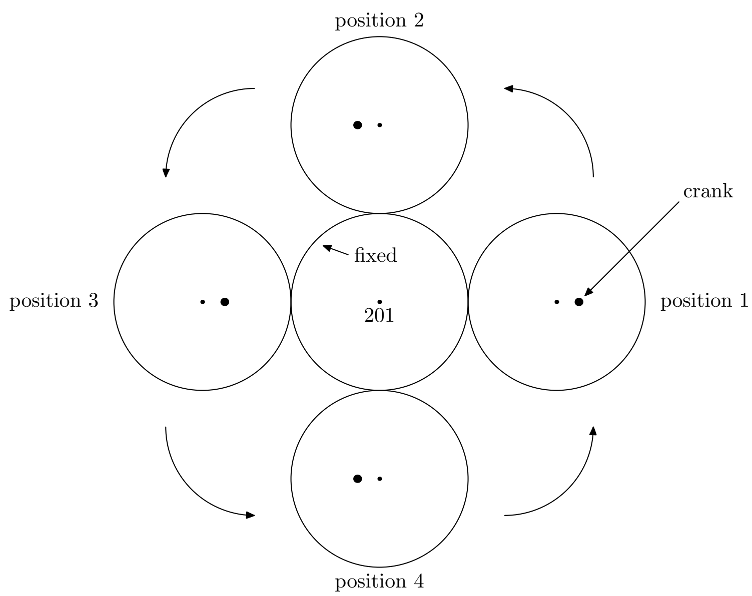
and we find that the period of oscillation is exactly the tropical year.

This is actually what is expected, because the acceleration and slowing down take place at one year intervals. To be fair, the period of oscillation should be the anomalistic year and not the tropical year, but these two years are very close to each other.

But we can also take a different perspective and compute the (rotation) motion of arbor 202 with respect to G. In other words, we can examine the orientation of the crank with respect to the globe. How does it move? Is it in fact moving at all? It is easy to compute:

$$\mathrm{V}_{202}^{G} = \mathrm{V}_{202}^{200} + \mathrm{V}_{200}^{G} = \mathrm{V}_{200}^{G} + \mathrm{V}_{200}^{G} = 2\mathrm{V}_{200}^{G} \tag{1.38}$$

So, in fact, arbor 202 moves twice as fast as wheel 200 in the reference frame of G. We can look at the two 48-teeth wheels from above:



From position 1 to position 3, the motion of 200′ will be ahead of that of 200. And from position 3 to position 1, it will be behind.

Similar constructions are used to produce the oscillatory motions of the Moon, of Mars, Jupiter and Saturn.

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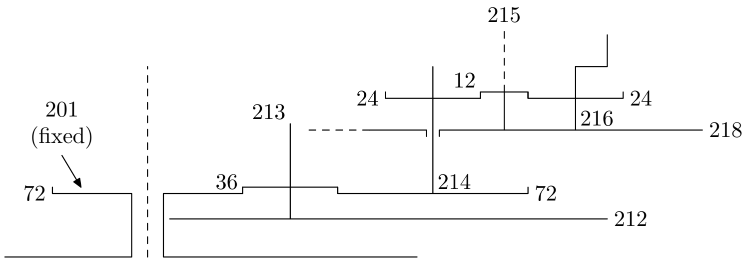
1.3.3.1.2 The motion of Mercury in the Nuremberg *Weltmaschine*

In the case of Mercury, Hahn is attempting to take into account both the elliptic orbit of Mercury and the retrogradations due to the Earth orbiting around the Sun. Hahn's construction is the following one, where I am only focusing on the elliptic orbit:



The fixed frame 201 is that of the globe. The frame 212 represents the corrected motion of the Sun, which is the mean motion of Mercury from a geocentric perspective. This is the same motion as the one of tube 200′ above. The arbor 214 is the mean direction of Mercury on this machine. Arbor 216 represents the motion of Mercury corrected for the geocentric perspective and I will examine it later. The frame 218 is only partly shown and rotates around the arbor 214. It is part of a train not shown. Finally, the crank on arbor 216 is meant to account for the elliptic motion of Mercury.

The question we have to ask is how does the crank alter the direction of arbor 216? The crank provides a correction to the direction of arbor 216 and this corrected direction is sometimes ahead and sometimes behind that of arbor 216.

It is easy to see that when the support 218 rotates around arbor 214, it also causes the crank to turn, and therefore that there is an oscillation. But what is the period of this oscillation? The arbor 216 actually replicates the motion of arbor 214, no matter the orientation of support 218. How much the crank moves Mercury on one side or another of arbor 216 therefore does not depend on where arbor 216 is located, or does not depend much on it. There is admittedly a small parallax effect, but we will ignore it here. We therefore do not need to care about how 218 moves, that is on how the retrogradations are taken care of.

Similarly, the arbor 214 replicates the motion of tube 201, no matter the orientation of the frame 212. As a consequence, the crank on arbor 216 has the same orientation as tube 201 which is fixed. The crank 216 therefore has a fixed orientation. It always points in the same direction.

Now, as Mercury revolves around the sky, we have the following configuration:

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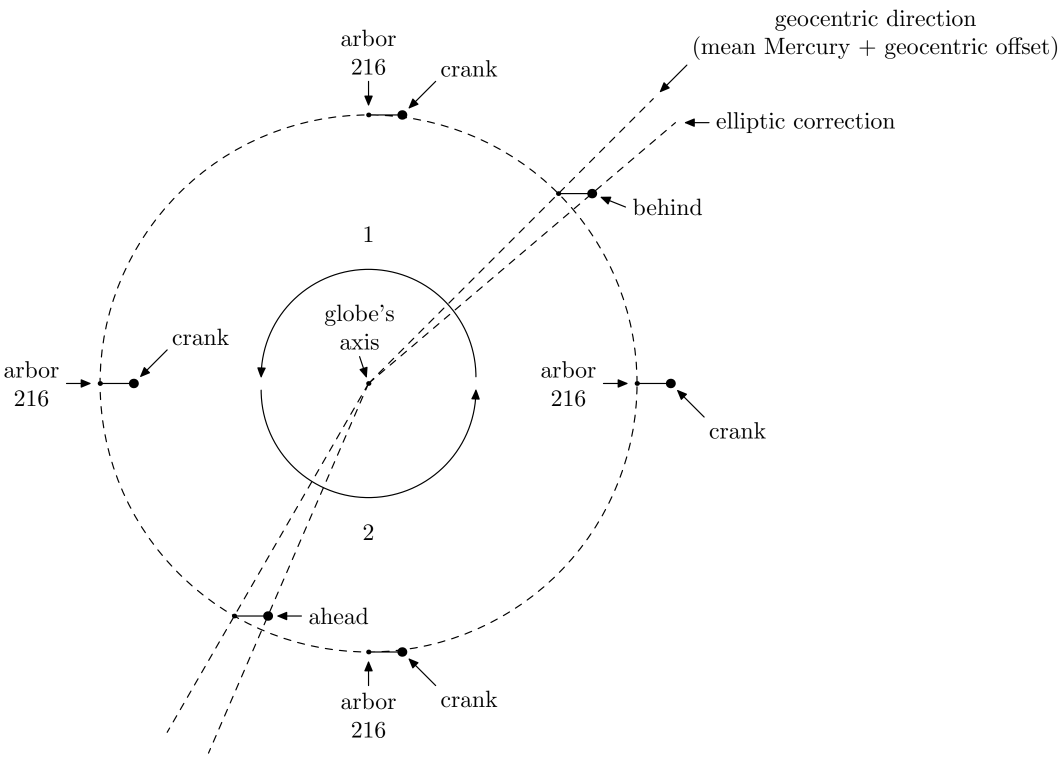

Here I have shown the (counterclockwise) revolution of arbor 216 around the globe, that is the revolution of the apparent position of Mercury assuming that there is no correction for the elliptic motion.

On the first half of the revolution, Mercury's motion is slowed down. On the second half, it is accelerated.

In this case, the crank actually points towards Mercury's aphelion and the aphelion is at a fixed position on the celestial sphere.

This construction is different from the one seen above, but in the first one, there is also an anchor to the celestial sphere. Although the crank doesn't have a fixed orientation for the Sun, Mars, Jupiter, Saturn and the Moon, there is a position for the epicyclic wheel (the one carrying the crank) where the crank is farthest from the center, this position corresponds to a fixed direction and this direction is the perihelion.

1.3.3.2 The geocentric cranks in the Nuremberg *Weltmaschine*

Now, it remains to see how the geocentric cranks for Mercury, Venus, Mars, Jupiter and Saturn work. I am only considering the case of the Nuremberg *Weltmaschine*. These cranks are used to produce the retrogradations of the planets.[12]

1.3.3.2.1 The cases of Mercury and Venus

In the case of Venus, we have the following configuration:

[12]On the parameters for the retrogradations, see in particular [19, p. 185-189].

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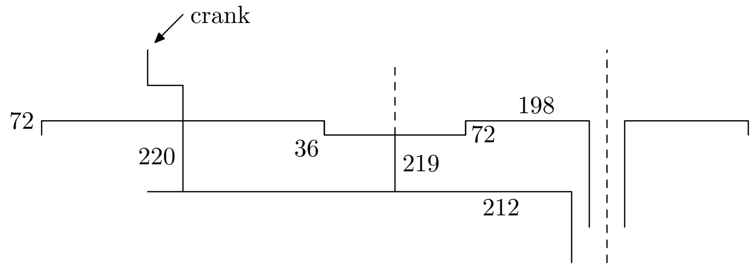


Tube 198 moves with the tropical period of Venus. Tube 212 is the corrected motion of the Sun. In the case of Mercury, we have a similar configuration, with tube 196 replacing tube 198.

The crank should represent the synodic motion of Mercury or Venus, that is, it should make one turn whenever the angular difference between tubes 198 and 212 increases by one turn. And indeed, we have

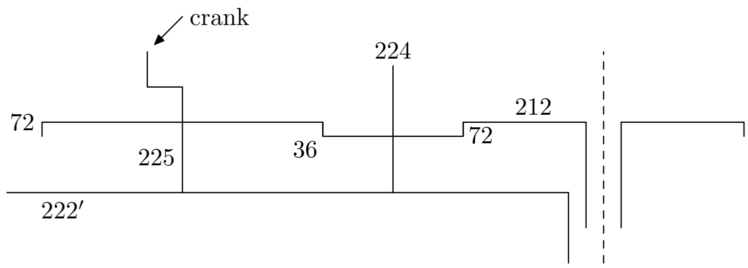
$$\mathrm{V}_{220}^{212} = \mathrm{V}_{198}^{212} \times \left(-\frac{72}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{198}^{212} \tag{1.39}$$

Hence, the motion of the crank on plate 212 is that of Venus with respect to the Sun.

The case of Mercury is similar. The geocentric correction is applied first, before the elliptic correction (see above).

1.3.3.2.2 The case of Mars

In the case of Mars, we have the configuration



where 222′ is the corrected motion of Mars (for the eccentricity). In this case, the elliptical correction is applied first, then the geocentric one. (For Mercury, it was the opposite.)

In the case of Mercury, the eccentricity should have been applied first, but it doesn't change much.

Like for Venus, the crank's motion is in par with the difference between 212 and 222′, that is between the corrected motion of Mars and the corrected motion of the Sun.

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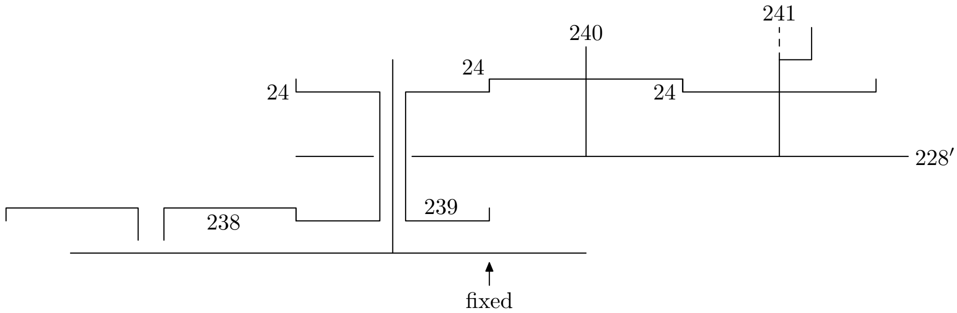
1.3.3.2.3 The cases of Jupiter and Saturn

The cases of Jupiter and Saturn are similar. Hahn first corrects the mean motions for the eccentricities (see above). Then he has the following configuration for Jupiter:



The direction of arbor 241, that is the frame 228′, corresponds to the true longitude of Jupiter. This arbor rotates like tube 239. This tube makes one turn clockwise in a tropical year.

In summary, the motion of the crank is the difference between the true motion of Jupiter (228′) and the motion of the Sun (239).

The main difference with Mars is that the motions are flipped, and that they still need to be reversed, which is done when the 90-teeth wheel on arbor 228″ meshes with the 90-teeth wheel on tube 244.

The case of Saturn is similar.

1.3.4 Periods of rotation

Once we have the velocity of a wheel, it is easy to compute its period of rotation. The period of rotation is merely the inverse of the velocity. For instance, if a wheel (or arbor) has the velocity $V = 0.5$ turns/day, the period if $P = 1/V = 2$ days/turn.

Of course, if the velocity is not measured in turns per day, but for instance in degrees per second, then the period must be expressed differently. In the latter case, if $V = 20$ degrees/s, then $1/V = 1$ second for 20 degrees and $P = \frac{360}{20} \times 1$ second $= 18$ seconds.

It is therefore simplest to restrict the velocities to turns/day and to express the periods in days. Given these assumptions, the units can be made implicit. The periods can always be converted to other units later.

1.4 The extent and limits of this work

1.4.1 Methodology

As I wrote before, Oechslin's work is daunting, and even deterring. Probably only very few people have worked through all of its details, and I am not one of

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them. My plan, when working through Oechslin's book, was to do something else. I wanted to redo all the calculations in an axiomatic way and in particular to compute the exact ratios involved in the motions of the planets, something that Oechslin had not done. This is in fact also seldom done by other authors, and it is the cause of many approximations and sometimes serious errors.

All the clocks and mechanisms described by Oechslin have been reanalyzed directly from Oechslin's excellent plates of the gear layouts. In general, studying a clock or mechanism amounts to break it down into parts, and this is also the case with Oechslin's plans. I have often viewed these plans as forests, or labyrinths, and I had to find a beginning, and then slowly walk my way through them. This sometimes took time. When I made calculations, it was not unusual for me to get an obviously incorrect result, and I had to see if the error was mine, if it was Oechslin who got it wrong, or the constructor of the clock. More than once, the error was mine, and Oechslin was my safeguard. But the other possibilities occurred too.

I didn't use Oechslin's calculations, because I found them too difficult to follow, and because they were lacking the explanations I wished to provide. As a result, my computations are longer than Oechslin's, because I have strived to explain them and I wanted to give systematically the exact ratios.[13] In each case, I have compared my final ratios or values with those given by Oechslin, and, in case of discrepancies, I have sought the origin of the discrepancies and tried to correct them. My work thus provides an independent recalculation of the gears. By providing a detailed step-by-step derivation of the periods, I hope to make Oechslin's work more accessible and also more useful.

The exact ratios often involve very large integers but these integers were of course not used by Hahn or others. It is however important to do the exact computations, and not to carry approximate values from one equation to another. It is only in the final stages that I give numerical decimal values, usually to four decimal places, truncated (and not rounded). This is intentional. In some cases, I give the values to more decimal places, if there appears a need, for instance if some ambiguity would have arisen with only four places.

As far as the revolution periods are concerned, I have mainly compared those of the clocks or mechanisms with the then-known values, but I did not go so far as quantifying the errors. And of course, it would be meaningless to compare the values chosen by Hahn or someone else with those of modern almanacs.[14] The only meaningful comparison is between the periods on the clocks and those that were known and available at the time of the construc-

[13] For the exact fractions, I have not given the decompositions in prime factors, as these decompositions are not relevant here. It might be done in the future, but it would ne anecdotical. For tables of primes and factors, I refer to the numerous tables given in my LOCOMAT project (`https://locomat.loria.fr`).

[14] I find it anecdotical and of little interest to evaluate the accuracy of gear trains by the deviations after many years, as many sensationalist writers do (an example is Hahn's article [12]). Astronomical clocks never work uninterrupted, they always end up neglected, and arguing about a few seconds here or there is pointless. These discussion often miss the real points of a clock.

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tion of these clocks. But even these comparisons are objectionable, because different constructors may regard the accuracies in different ways. Some constructors may have contented themselves with crude approximations, and may not have wanted to go the trouble of complex trains, when it suffices to set the mechanism once in a while.

As mentioned above, Oechslin's plates are in general very good, and there is not much that could be improved, except for a clearer naming of the arbors and frames. Perhaps his notation for worms might be changed, but at this point I do not have a replacement, and this would be a minor problem, because there are not that many worms. They mainly appear in Neßtfell's clocks. It is in any case clear that Oechslin gave a lot of thought to his graphical notations. The main improvement that would have been useful, would have been to split many large plates into smaller parts, and this would have made the mechanisms much easier to study. It is also sometimes difficult to relate the plans to the actual clocks, especially when there are many dials (such as in Frater David's 1769 clock, or in Seige's 1791 clock).

I ended up making enlarged copies of many plates, but these enlargements were then often even more difficult to manipulate. In several cases, it took me some time to understand the plates, because I didn't have any guide. For instance, on the plate for Frater David's clock in Vienna, the gears are divided in three groups, and it is not readily obvious that these three groups are actually more or less superimposed. Oechslin split them for clarity, but if you only look at the plates, which is how I approached the clocks (I only read the first volume after I studied all the plates), this may be misleading. Finally, another problem in Oechslin's plates has to do with the dotted lines which are too light, and sometimes escape the attention.

Regarding the use of complex formulas, it is fitting here to recall that a serious horologist should not use rules of thumb. Some horologists compute the frequency of a pendulum by dividing the product of some numbers of teeth by the product of other numbers of teeth, and this may give the expected result in some cases, but it is easy to get it wrong. Horologists should use simple relations that they understand. In fact, one should try as much as possible to stick to the two following basic rules:

1. two wheels on the same arbor have the same speed;
2. the ratio of the speeds of two meshing wheels with parallel and fixed axes is the inverse of the ratio of the numbers of teeth.

In addition, there is sometimes the need to add velocities or to change reference frames, but this has to be done with caution.

Horologists should not use complex formulas, they should not use Willis' formula for epicyclic gears, or anything like that. There is no need for it! A general formula such as Willis' formula is only useful when it will simplify the analysis, in particular when many similar planetary constructions occur. This is seldom the case in the analysis of clocks. And I didn't find it necessary for any of the 29 clocks and mechanisms described here.

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1.4.2 The limits of this work

In my analyses of Oechslin's drawings, I have mainly concentrated on computing the exact ratios and on understanding and explaining how the various machines are obtaining their displays. I have, however, not expanded on how these constructions were obtained, although I have given some general principles in this chapter. For further details, I refer to Oechslin's first volume where he explains the various constructions used by the *Priestermechaniker*. However, in most cases it is easy to see how the ratios have been obtained.

In some cases, one can have the feeling that I am giving huge (or even monstruous) gear ratios. However, such ratios are commonplace in astronomical clocks and they help understand the real motions, even if the authors of the clocks have almost never used them. It is better to manipulate the exact ratios than numerical approximations of these ratios.

1.4.3 How to use this work

After this introduction, the book is organized in 29 chapters, one for each of the mechanism or clock described by Oechslin. The clocks are not given in the same order as Oechslin. Oechslin more or less organized his clocks chronologically, but with notable exceptions. He often described larger or complex machines first, and Hahn's clocks, for instance, are not described chronologically. I have tried to put the clocks and machines of each author in chronological order, so that it is easier to see what has been reused. Hahn, Neßtfell, Seige, and others, have reused ratios from one machine to another, and this can only well be perceived if the clocks are shown in chronological order. I have also decided to depart from how Frater David a Sancto Cajetano, Pater Aurelius a San Daniele and Frater Fridericus a S. Christophoro, are often called in various articles, namely Cajetano, Daniele and Christophoro. I see no reason to call them that way, and I have chosen to call them Frater David, Pater Aurelius and Frater Fridericus. This should not give rise to any ambiguity in this work. Moreover, I am in no way proposing a radical change, as others, such as Czermak [3], have already adopted this convention.

However, in order to facilitate the location of a clock, I have decided to list the authors alphabetically, thus starting with Adams and ending with Stuart. It is only within a given author that the clocks have been shown in chronological order, or in the best chronological order I could think of.

This, however, does not mean that the chapters need to be read in that order. The chapters are in fact all independent and can be read in any order. It would, however, seem more interesting to study the mechanisms of an author in chronological order. It should also be noted that each chapter may have references to other mechanisms, but the notations are independent. A velocity such as V_5^0 may appear in several chapters, with entirely different meanings. I will never reuse the velocity given in one chapter in another chapter. It should also be noted that there is some overlap between the chapters of a same author,

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and I have in particular repeated most of the bibliographic references.

Now, even though the chapters are independent from eachother, the book is not self-contained. It is meant to be a companion to Oechslin's book, and especially to his plates. When reading this book, it is best to have Oechslin's plates at hand (after having them enlarged), and to follow my derivations at the same time. Reading my derivations without the plates makes little sense. Unfortunately, at this point, I cannot include Oechslin's plates, and redrawing them would take months, if not years. They might be included in a future version.

1.4.4 Future versions

This work is a preliminary work. It is a work in progress which is intentionally made available in that state, not entirely polished, with few or no illustrations, and minimal historical notices, although I have tried to give the main bibliographical references in each cases. Some of the photographs are currently hidden, because they are not free to use. The plans of the gear trains are also not shown, for the same reasons.

This work is of course above all a companion to Oechslin's work, and I hope that it will ease the study of these 29 mechanisms by others, and also provide some impetus for new studies. In order to understand how these mechanisms work, and in particular the periods involved, it is in fact not necessary to read the first volume of Oechslin's work, and one can very well limit oneself to Oechslin's plates and my complements. But in order to have more insight on these mechanisms, or on the constructions used by their authors, Oechslin's text remains very useful.

I have also intentionally left aside everything that concerns the calendar, and in particular how the lengths of the months are taken into account. This is a different topic that I might tackle elsewhere some day. I have also not analyzed in detail the irregular motions caused by the eccentric orbits, or by the retrogradation of the planets, but they too might be described in more detail in the future.[15] In the meantime, I refer the reader to Oechslin's work. Oechslin took precise measurements of the eccentricities, he analyzed and compared them, both between the clocks, and with respect to the actual astronomical motions.

I am of course interested by any feedback, comments, corrections, etc., which may help me to improve my work. If some of the computations are not clear enough, I can try to improve them.

[15] I have always given the mean motions or the mean periods, but it is possible to provide more accurate expressions for the irregular motions, for instance the retrogradations in geocentric motions, although these expressions tend to become very complex. These analyses are interesting, but they are mainly exercises in theoretical kinematics, not really anything that brings an insight in the thoughts of the makers. None of these analyses were made by Oechslin. Incidentally, many years ago I worked out such expression for a complex clock by Antide Janvier, and those to whom I showed my work didn't understand what I was doing (which doesn't mean it was useless).

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I am also perfectly aware of many shortcomings. To take only one example, I am far from happy with the descriptions of Neßtfell's machines. There is much more to say, on the construction, on the dials, etc. In front of these machines, I still feel some frustration. In fact, an entire book should be devoted to them, but I cannot do it, because I do not have the necessary data and photographs. Moreover, I believe that this should be a team work, such great is the task.

In the future, I hope to complete my descriptions, in particular with photographs, but this will probably take place rather irregularly.

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