Chapter 14 **(Oechslin: 8.11)**

Hahn's tellurium in Winterthur (1780)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

14.1 Introduction

Figure 14.1: General view of Hahn's tellurium in Winterthur. (photography by the author)

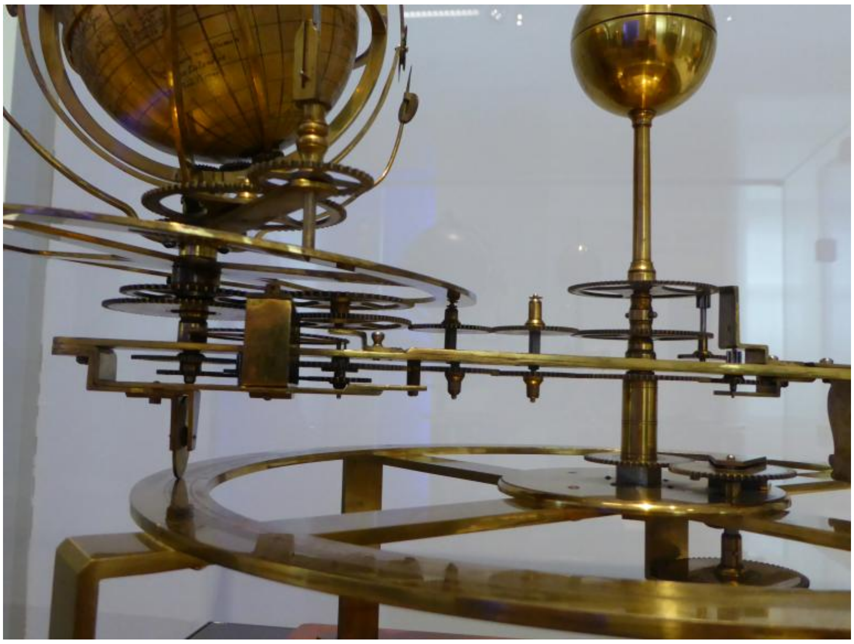

Figure 14.2: Detail of Hahn's tellurium in Winterthur. (photography by the author)

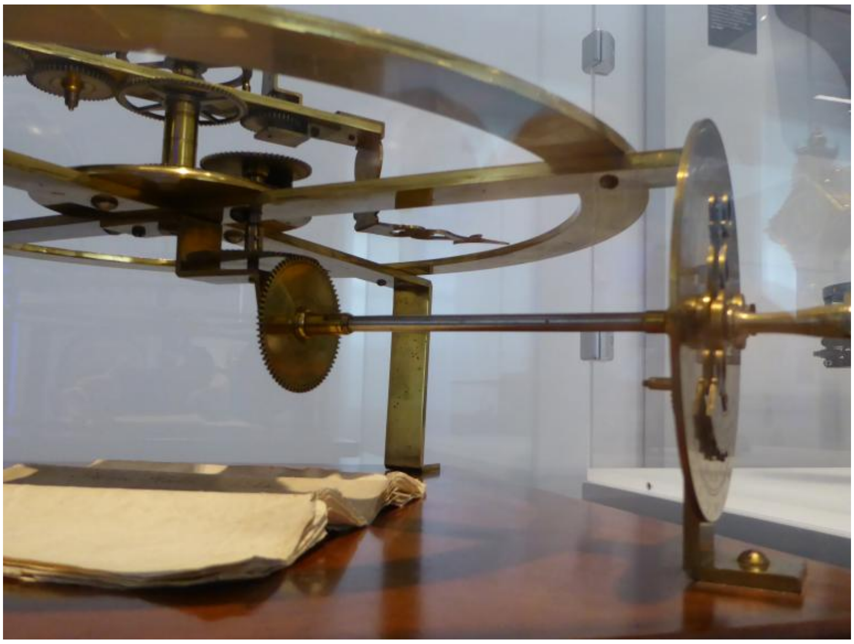

Figure 14.3: Detail of Hahn's tellurium in Winterthur. (photography by the author)

The tellurium described here was constructed in 1780 by Philipp Matthäus Hahn (1739-1790)[1] and is located in the Winterthur *Uhrenmuseum* (Switzerland).[2] It shows the motion of the Earth around the Sun and of the Moon around the Earth. It does not have a clockwork and is operated by a crank.

It was described by Oechslin who dated it to 1780 and attributed it to Hahn. However, the museum itself dates it to about 1725. The front dial has the inscription "Hahn Echterdingum," so that it was perhaps too quickly attributed to Hahn.

As I will demonstrate below, this tellurium embodies many problems and its periods have been very poorly calculated. I find it therefore hard to believe that this is the work of Philipp Matthäus Hahn, or of Hahn alone.

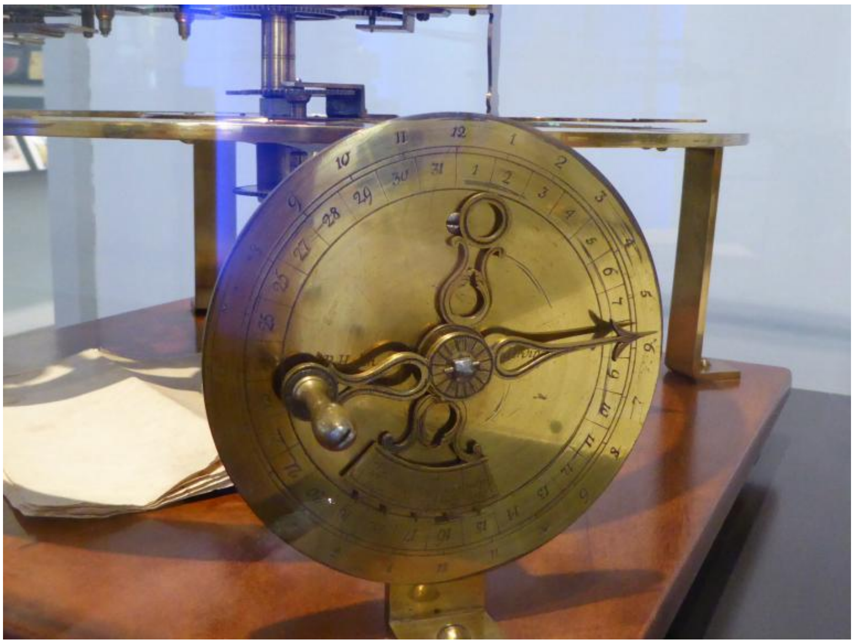

Figure 14.4: The dial of Hahn's tellurium in Winterthur. (photography by the author)

14.2 The driving mechanism

One turn of the crank corresponds to one day and the dial is divided into 24 hours (figure 14.4). When the crank is turned, a pin moves a 31-teeth wheel by one tooth and this wheel is linked to the hand for the day of the month.

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [2, p. 454-455] where Oechslin dates the tellurium after 1780. My photographs of the tellurium were taken on July 6, 2024.

The arbor 1 can then be thought of turning clockwise (seen from the front of the dial) in one day:

$$\mathrm{T}_1^0 = 1 \tag{14.1}$$

This motion is transferred to the vertical tube 5:

$$\mathrm{V}_5^0 = \mathrm{V}_1^0 \times \left(-\frac{b}{b}\right) \times \left(-\frac{61}{61}\right) = \mathrm{V}_1^0 = -1 \tag{14.2}$$

This tube makes one turn clockwise as seen from above.

Tube 5 surrounds a fixed vertical arbor 7 which goes through the Sun.

14.3 The motion of the main support arm

The motion of the main support arm 6 carrying the Earth-Moon system is obtained as follows. A 103-teeth wheel is fixed to the central arbor going to the Sun. This wheel is therefore not moving. Then, there is a gear train on superior part of the support arm, whose first wheel meshes with the above mentioned 103-teeth wheel. There is also another train on the inferior part of the support arm, whose first wheel meshes with a 80-teeth wheel fixed on tube 5. These two trains meet at arbor 12. (Oechslin's drawing is misleading, as the arbors 8 and 14 on one hand, and 9 and 13 on the other hand are distinct.) We can now compute the motion of tube 5 in the reference frame of the support arm 6:

$$\mathrm{V}_{12}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{103}{6}\right) \times \left(-\frac{64}{3}\right) \tag{14.3}$$

$$= \mathrm{V}_7^6 \times \frac{3296}{9} \tag{14.4}$$

$$\mathrm{V}_5^6 = \mathrm{V}_{12}^6 \times \left(-\frac{80}{c}\right) \times \left(-\frac{c}{c}\right) \times \left(-\frac{c}{80}\right) \tag{14.5}$$

$$= \mathrm{V}_{12}^6 = \mathrm{V}_7^6 \times \frac{3296}{9} \tag{14.6}$$

Then

$$\mathrm{V}_6^0 = \mathrm{V}_6^5 + \mathrm{V}_5^0 = -\mathrm{V}_7^6 \times \frac{3296}{9} + \mathrm{V}_5^0 \tag{14.7}$$

$$= \mathrm{V}_6^0 \times \frac{3296}{9} + \mathrm{V}_5^0 \tag{14.8}$$

Therefore

$$\mathrm{V}_5^0 = \mathrm{V}_6^0 \times \left(1 - \frac{3296}{9}\right) = -\mathrm{V}_6^0 \times \frac{3287}{9} \tag{14.9}$$

and

$$\mathrm{V}_6^0 = -\mathrm{V}_5^0 \times \frac{9}{3287} = \frac{9}{3287} \tag{14.10}$$

$$\mathrm{P}_6^0 = \frac{3287}{9} = 365.2222\ldots \text{ days} \tag{14.11}$$

This is an approximation of the tropical year, but it is surprisingly a rather bad approximation. The same value is given by Oechslin. This may raise some doubts about the attribution of the clock, but the ratio $\frac{9}{3296}$ and the value of the tropical year are also found in the double-globe astronomical clock of Zurich (Oechslin 8.7), so that things are not so clear.

14.4 The orientation of Earth's axis

It is easy to see that the axis of the Earth (tube 17) keeps a fixed orientation when the Earth revolves around the Sun. We have in the reference frame 6:

$$\mathrm{V}_{17}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{103}{6}\right) \times \left(-\frac{21}{21}\right)\left(-\frac{6}{103}\right) \tag{14.12}$$

$$= \mathrm{V}_7^6 \tag{14.13}$$

and therefore

$$\mathrm{V}_{17}^0 = \mathrm{V}_7^0 = 0 \tag{14.14}$$

14.5 The motion of the lunar nodes

Let us first assume that the main frame 6 makes one turn counterclockwise in 365.25 days. We can now compute the revolution of the lunar nodes (tube 15) in the reference frame 6:

$$\mathrm{V}_{15}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{56}{53}\right) = \mathrm{V}_7^6 \times \frac{56}{53} \tag{14.15}$$

$$= -\mathrm{V}_6^0 \times \frac{56}{53} \tag{14.16}$$

$$\mathrm{V}_{15}^0 = \mathrm{V}_{15}^6 + \mathrm{V}_6^0 = \mathrm{V}_6^0 \times \left(1 - \frac{56}{53}\right) = -\mathrm{V}_6^0 \times \frac{3}{53} = -\frac{9}{3287} \times \frac{3}{53} \tag{14.17}$$

$$= -\frac{27}{174211} \tag{14.18}$$

$$\mathrm{P}_{15}^0 = -\frac{174211}{27} = -6452.2592\ldots \text{ days} \tag{14.19}$$

This value is negative because the nodes are retrograding, but it is a very bad approximation of the period of revolution of the nodes which is about 6798 days. The same value is given by Oechslin.

14.6 The motion of the Moon

Let us now compute the motion of the Moon. The phase of the Moon is merely obtained by a train of gears that ensures that the Moon always keeps the same orientation towards the Sun.

The direction of the Moon is given by tube 22. We can compute the motion of this tube with respect to the moving frame 6:

$$
\begin{aligned}
\mathrm{V}_{22}^{6} &= \mathrm{V}_{5}^{6} \times \left(-\frac{80}{c}\right) \times \left(-\frac{c}{c}\right) \times \left(-\frac{c}{80}\right) \times \left(-\frac{32}{63}\right) \times \left(-\frac{6}{90}\right) && (14.20)\\
&= \mathrm{V}_{5}^{6} \times \left(-\frac{32}{945}\right) = \left(\mathrm{V}_{5}^{0} - \mathrm{V}_{6}^{0}\right) \times \left(-\frac{32}{945}\right) && (14.21)\\
&= \left(-1 - \frac{9}{3287}\right) \times \left(-\frac{32}{945}\right) && (14.22)\\
&= \frac{105472}{3106215} && (14.23)\\
\mathrm{P}_{22}^{6} &= \frac{3106215}{105472} = 29.4506\ldots \text{ days} && (14.24)
\end{aligned}
$$

This value is also rather bad, because the correct value is the synodic month of about 29.53 days.

Oechslin finds a period of 29.531250 days, but his derivation is wrong.

We can also compute the tropical motion of the Moon:

$$
\begin{aligned}
\mathrm{V}_{22}^{0} &= \mathrm{V}_{22}^{6} + \mathrm{V}_{6}^{0} = \frac{105472}{3106215} + \frac{9}{3287} = \frac{113977}{3106215} && (14.25)\\
\mathrm{P}_{22}^{0} &= \frac{3106215}{113977} = 27.2529\ldots \text{ days} && (14.26)
\end{aligned}
$$

This value is also bad, but agrees with the one given by Oechslin.

14.7 The motion of the Earth around its axis

Finally, we can compute the motion of the Earth around its axis (arbor 19). The motion of this axis actually is the opposite of the motion of tube 5, which is making one turn clockwise in 24 hours.

It would actually seem that tube 5 should have the motion of the sidereal day, and not make one turn in 24 hours.

If we instead assume that tube 5 makes one turn in one sidereal day. the motion of the Earth around its axis is correct.

14.8 The rotation of the Sun

The Sun is mounted on tube 28 and its rotation is merely derived from the motion of tube 5:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{5}^{0} \times \left(-\frac{80}{40}\right) \times \left(-\frac{8}{32}\right) \times \left(-\frac{6}{84}\right) = \mathrm{V}_{5}^{0} \times \left(-\frac{1}{28}\right) = \frac{1}{28} \tag{14.27}$$

$$\mathrm{P}_{28}^{0} = 28 \text{ days} \tag{14.28}$$

The Sun makes one turn in 28 days. This is also a crude approximation of the rotation of the Sun, worse than the one for instance used in the Nuremberg *Weltmaschine*.

Oechslin found a period of 25.94 days which is incorrect.

14.9 Conclusion

This tellurium is obviously rather puzzling, and, as I wrote in the introduction, I find it hard to attribute it to Philipp Matthäus Hahn, or at least to Hahn alone. There are too many errors, the sidereal day is not computed, no well-known ratio of Hahn appears here, the tropical year is bad, the revolution of the nodes is bad, and the synodic month is bad. On the other hand, the ratios used for the tropical year are also found in the double-globe astronomical clock of Zurich (Oechslin 8.7).

In light of the many errors of this mechanism, Oechslin assumed in 1996 that it must have been one of the first works by Hahn [1, p. 198], but this does not fit with the announced construction date of 1780 which he himself put forward in the catalogue for the 1989 Hahn exhibition. This machine is possibly older than 1780, and may have been finalized in 1780. Perhaps Hahn was only partly involved in its construction.

14.10 References

[1] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[2] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.