Chapter 11

(Oechslin: 8.4)

Hahn's globe clock in Aschaffenburg (1776/1777)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

11.1 Introduction

The clock described here was constructed in 1776/1777 by Philipp Matthäus Hahn (1739-1790)[1] and is located in Schloss Johannisburg in Aschaffenburg.[2]

It has a base with four faces, three of which containing dials. One of the three dials shows the hour on a 24-hour scale, the minutes on an internal dial, the day of the month and of the week on another internal dial, and the year on four small 0-9 dials.[3] The second dial of the base shows a tellurium and the third shows an orrery with the planets from Mercury to Saturn.

The upper part is a celestial globe showing the geocentric motion of the planets, the Sun, the Moon and the lunar nodes.

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [2, p. 405-407]. This clock is also cited by Zinner [4, p. 353]. My photographs of the clock were taken on 24 July 2019.

[3] Oechslin's plate seems to imply that the units are given left, then the tens, then the hundreds, then the thousands, but this should be checked. A similar display of the year was used by Alexius Johann in 1796 (Oechslin 9.1).

Figure 11.1: General views of Hahn's clock in Aschaffenburg. (photographs by the author)

Figure 11.2: The time dial of Hahn's clock in Aschaffenburg. (photograph by the author)

Figure 11.3: The orrery of Hahn's clock in Aschaffenburg. (photograph by the author)

Figure 11.4: The tellurium of Hahn's clock in Aschaffenburg. (photograph by the author)

Figure 11.5: The side opposite the time dial on Hahn's clock in Aschaffenburg. (photograph by the author)

Figure 11.6: The celestial globe of Hahn's clock in Aschaffenburg. (photograph by the author)

11.2 The base motion

The clockwork has a wheel making one turn in 24 hours on arbor 1. We have (from the front)

$$T_1^0 = 1 \tag{11.1}$$
$$V_1^0 = -1 \tag{11.2}$$

because the motion is clockwise.

The motion of arbor 3 is then derived:

$$V_3^0 = V_1^0 \left(-\frac{76}{79}\right) \times \left(-\frac{57}{58}\right) = -\frac{2166}{2291} \tag{11.3}$$

Although Oechslin's plan lacks that connexion, a 35-teeth wheel on arbor 3 must mesh with a 33-teeth wheel on arbor 4, which is the vertical arbor driving the celestial globe. We then have

$$V_4^0 = -V_3^0 \times \left(-\frac{35}{33}\right) = -\left(-\frac{2166}{2291}\right) \times \left(-\frac{35}{33}\right) = -\frac{25270}{25201} \tag{11.4}$$

This motion is clockwise as seen from above. It corresponds to a period of

$$P_4^0 = -\frac{25201}{25270} \text{ days} = 86164.0838\ldots \text{ s} \tag{11.5}$$

Hence, the motion of arbor 4 is that of a sidereal day.

This ratio was also used by Hahn in the Gotha, Stuttgart and Nuremberg machines, as well as in the Darmstadt globe clock.

This velocity is then also used to drive the tellurium and the orrery.

I will first describe the gears of the orrery, then of the tellurium, and finally of the celestial globe.

11.3 The orrery

The orrery shows the motions of the planets from Mercury to Saturn around the Sun, as well as the Moon. Mercury, Mars, Jupiter and Saturn are given an irregular motion in order to account for their elliptical orbits.

The input to the orrery is the motion of arbor 5 which is clockwise as seen from the front of the orrery:

$$\mathrm{V}_5^0 = \mathrm{V}_4^0 = -\frac{25270}{25201} \tag{11.6}$$

11.3.1 The mean motions of the planets

The input motion is first used to produce the motion of tube 8, which is the mean motion of Mercury:

$$\mathrm{V}_8^0 = \mathrm{V}_5^0 \times \left(-\frac{32}{53}\right) \times \left(-\frac{6}{77}\right) \times \left(-\frac{20}{83}\right) \tag{11.7}$$

$$= \mathrm{V}_5^0 \times \left(-\frac{3840}{338723}\right) = \left(-\frac{25270}{25201}\right) \times \left(-\frac{3840}{338723}\right) = \frac{13862400}{1219451189} \tag{11.8}$$

$$\mathrm{P}_8^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{11.9}$$

This motion is counterclockwise as seen from the front. The same value is given by Oechslin.

Mercury is actually given an irregular motion, in order to account for its elliptic orbit. I will get back to it below, but will first deal with the mean motions.

Venus (tube 12) only has a mean motion which is obtained from the mean motion of Mercury:

$$\mathrm{V}_{12}^0 = \mathrm{V}_8^0 \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{11.10}$$

$$= \mathrm{V}_8^0 \times \frac{783}{2000} = \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{11.11}$$

$$\mathrm{P}_{12}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{11.12}$$

The same value is given by Oechslin.

The Earth (tube 14) also only has a mean motion which is obtained from the (mean) motion of Venus:

$$V^0_{14} = V^0_{12} \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = V^0_{12} \times \frac{3239}{5265} \tag{11.13}$$
$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{11.14}$$
$$P^0_{14} = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{11.15}$$

This is an approximation of the tropical year. The same value is given by Oechslin. This motion is also transferred to arbor 27 which is the input to the tellurium.

The motion of the Earth is then used to obtain the mean motion of Mars. The motion of Mars is used to obtain the motion of Jupiter, and the motion of Jupiter is used to obtain the motions of Saturn and of the Moon.

So, first we compute the mean motion of Mars on tube 16. The train as pictured on Oechslin's drawing contains a 34-teeth wheel and we obtain:

$$V^0_{16}? = V^0_{14} \times \left(-\frac{116}{34}\right) \times \left(-\frac{18}{119}\right) = V^0_{14} \times \frac{1044}{2023} \tag{11.16}$$
$$= \frac{473632}{172990675} \times \frac{1044}{2023} = \frac{494471808}{349960135525} \tag{11.17}$$
$$P^0_{16}? = \frac{349960135525}{494471808} = 707.7453\ldots \text{ days} \tag{11.18}$$

However, this value is not very accurate. We do in fact obtain a much better value for the orbital period of Mars if we replace the 34-teeth wheel by a 33-teeth one. And in fact, this is what Oechslin does in his calculations. We now have:

$$V^0_{16} = V^0_{14} \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = V^0_{14} \times \frac{696}{1309} \tag{11.19}$$
$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{11.20}$$
$$P^0_{16} = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{11.21}$$

and Oechslin has the same value.

This assumption is also supported by the use of the same ratio $\frac{696}{1309}$ in the *Weltmaschinen* of Gotha and Stuttgart, as well as in the Stuttgart globe clock (Oechslin 8.5).

Continuing with this assumption, from Mars, we obtain the mean motion of Jupiter on tube 19:

$$\mathrm{V}_{19}^{0} = \mathrm{V}_{16}^{0} \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = \mathrm{V}_{16}^{0} \times \frac{595}{3751} \tag{11.22}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{11.23}$$

$$\mathrm{P}_{19}^{0} = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{11.24}$$

The same value is given by Oechslin.

The mean motion of Jupiter is then used to obtain the mean motion of Saturn on tube 22:

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{19}^{0} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) = \mathrm{V}_{19}^{0} \times \frac{363}{901} \tag{11.25}$$

$$= \frac{329647872}{1427553648235} \times \frac{363}{901} = \frac{988943616}{10629965595535} \tag{11.26}$$

$$\mathrm{P}_{22}^{0} = \frac{10629965595535}{988943616} = 10748.8085\ldots \text{ days} \tag{11.27}$$

The same value is also given by Oechslin.

11.3.2 The motion of the Moon

The mean motion of Jupiter is also used to obtain the motion of tube 25:

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{19}^{0} \times \left(-\frac{121}{34}\right) \times \left(-\frac{3}{21}\right) \times \left(-\frac{73}{102}\right) = \mathrm{V}_{19}^{0} \times \left(-\frac{8833}{24276}\right) \tag{11.28}$$

$$= \frac{329647872}{1427553648235} \times \left(-\frac{8833}{24276}\right) = -\frac{2005357888}{23867281242805} \tag{11.29}$$

$$\mathrm{P}_{25}^{0} = -\frac{23867281242805}{2005357888} = 11901.7564\ldots \text{ days} \tag{11.30}$$

The same value is given by Oechslin.

The motion of tube 25 is used to rotate the Moon around the Earth. We can compute this motion precisely. We first compute the motion of the Moon with respect to the Earth. It is the motion of arbor 26. We have[4]

$$\mathrm{V}_{26}^{14} = \mathrm{V}_{25}^{14} \times \left(-\frac{72}{6}\right) = -\left(\mathrm{V}_{25}^{0} - \mathrm{V}_{14}^{0}\right) \times 12 = \left(\mathrm{V}_{14}^{0} - \mathrm{V}_{25}^{0}\right) \times 12 \tag{11.31}$$

$$= \left(\frac{473632}{172990675} + \frac{2005357888}{23867281242805}\right) \times 12 \tag{11.32}$$

$$= \frac{336758509216}{119336406214025} \times 12 \tag{11.33}$$

$$= \frac{4041102110592}{119336406214025} \tag{11.34}$$

$$\mathrm{P}_{26}^{14} = \frac{119336406214025}{4041102110592} = 29.5306\ldots \text{ days} \tag{11.35}$$

As expected, we find an approximation of the synodic month. This value is not given by Oechslin.

We can also compute the tropical motion of the Moon:

$$\mathrm{V}_{26}^{0} = \mathrm{V}_{26}^{14} + \mathrm{V}_{14}^{0} = \frac{4041102110592}{119336406214025} + \frac{473632}{172990675} = \frac{4367833830368}{119336406214025} \tag{11.36}$$

$$\mathrm{P}_{26}^{0} = \frac{119336406214025}{4367833830368} = 27.3216\ldots \text{ days} \tag{11.37}$$

The same value is given by Oechslin.

11.3.3 The anomalies of the planets

Finally, we examine how the anomalies of the planets Mercury, Mars, Jupiter and Saturn have been obtained. In each case, there is a central wheel fixed to the frame. Oechslin doesn't give the teeth numbers of these wheels, but they are actually irrelevant, as they all mesh with identical wheels pivoting on the mean motion wheels. It therefore suffices to consider the case of Mercury.

The Mercury *satellite* wheel on arbor 10 rotates as the tube 8 rotates with the mean motion of Mercury. In the frame of tube 8, we have

$$\mathrm{V}_{10}^{8} = \mathrm{V}_{9}^{8} \times \left(-\frac{a}{a}\right) = -\mathrm{V}_{9}^{8} = \mathrm{V}_{8}^{9} = \mathrm{V}_{8}^{0} \tag{11.38}$$

$$\mathrm{P}_{10}^{8} = \mathrm{P}_{8}^{0} \tag{11.39}$$

Therefore, on the Mercury tube 8, the eccentric arbor of Mercury oscillates with the same period as the mean tropical motion of Mercury. This causes the acceleration and slowing down of the motion to occur at the same places in time, and therefore will account for the equation of center. Of course, in this simple case, the tropical and anomalistic motions of Mercury have been considered identical, which they are not exactly.

The same constructions are used for Mars, Jupiter and Saturn.

[4] Instead of 72, Oechslin gives the number of teeth of the upper wheel on tube 25 as 30/72. The teeth of this wheel are possibly uneven, but I am not sure that this is what Oechslin meant.

11.4 The tellurium

The tellurium shows the motion of the Earth around the Sun. The axis of the Earth is tilted and always points to the same direction. The structure of the tellurium is similar to that of the tellurium in a globe clock now kept in Stuttgart (Oechslin 8.5) and constructed in 1770, except that the latter also has the Moon revolving around the Earth.

Like in the Stuttgart globe clock mentioned above, this tellurium has two inputs. The first input is the motion of arbor 29 which is that of a sidereal day. It is counterclockwise as seen from the front of the tellurium:

$$\mathrm{V}^0_{29} = -\mathrm{V}^0_4 = \frac{25270}{25201} \tag{11.40}$$

This motion is used to produce the rotation of the Earth with respect to a fixed frame.

The second input is that of arbor 27 stemming from the orrery. This motion is that of the tropical year and it rotates clockwise as seen from the front of the tellurium.

$$\mathrm{V}^0_{27} = -\frac{473632}{172990675} \tag{11.41}$$

I am only considering the motion of arbor 27 from the tellurium side, hence the negative sign.

The motion of arbor 27 is used to produce the counterclockwise motion of tube 28:

$$\mathrm{V}^0_{28} = \mathrm{V}^0_{27} \times \left(-\frac{80}{80}\right) = -\mathrm{V}^0_{27} \tag{11.42}$$

Hence, tube 28 makes one turn counterclockwise in a tropical year. This tube supports the Earth frame.

The meridian of the Earth, which supports its axis, is fixed to a wheel on tube 33. The motion of this wheel replicates the motion of the central arbor 31 which is fixed. Therefore, the meridian, and also the axis of the Earth, always keep the same orientation with respect to the fixed frame, no matter where the Earth is located.

Finally, the arbor 35 replicates the motion of tube 30:

$$\mathrm{V}^0_{35} = \mathrm{V}^0_{30} = \mathrm{V}^0_{29} \times \left(-\frac{66}{66}\right) = -\mathrm{V}^0_{29} = -\frac{25270}{25201} \tag{11.43}$$

And arbor 36, which is that of the Earth's axis, has the velocity:

$$\mathrm{V}^0_{36} = \mathrm{V}^0_{35} \times \left(-\frac{30}{30}\right) = -\mathrm{V}^0_{35} = \frac{25270}{25201} \tag{11.44}$$

Consequently, the Earth rotates counterclockwise with the velocity of the sidereal day, as it should.

11.5 The celestial globe

The celestial globe on the top of the clock shows the geocentric motion of the sky, the planets, the Sun and the Moon. The globe is rotating inside a fixed frame representing the meridian. The vertical axis is the axis of the Earth, but the Sun, the Moon and the planets approximately move in the ecliptic, whose axis is tilted by 23.5° with respect to the equator. This axis rotates with the celestial globe.

A comparison with the gears of the globes of the *Weltmaschinen* of Stuttgart (Oechslin 8.1) (1769) and Gotha (Oechslin 8.3) (1780) shows that the structures of these three globes are very similar, and there are mainly small differences in teeth counts, but without altering the ratios.[5] In fact, the structure of the present globe is almost that of the Stuttgart machine, except for Mercury where the ratio is different. Moreover, in the Gotha machine, the motion of the lunar apsides and nodes is fraught with an error (either in the machine itself, or in Oechslin's account).

As mentioned earlier, the input to the globe is the motion of the vertical arbor 4 which makes one turn clockwise (seen from above) in a sidereal day:

$$\mathrm{V}_4^0 = -\frac{25270}{25201} \tag{11.45}$$

The central axis 47 of the globe is tied to three fixed wheels which will be used to obtain some of the motions displayed.

11.5.1 The motions of the Moon and the Sun

Within the celestial globe, all the motions are produced from the motion of tube 39. The velocity of this tube in the globe reference frame 47 is:

$$\mathrm{V}_{39}^{47} = \mathrm{V}_{37}^{47} \times \left(-\frac{71}{92}\right) \times \left(-\frac{7}{148}\right) = \mathrm{V}_{37}^{47} \times \frac{497}{13616} \tag{11.46}$$

where the tube/wheel 37 is a fixed part on the meridian frame of the globe. Now, since

$$\mathrm{V}_{37}^{47} = \mathrm{V}_{0}^{47} = -\mathrm{V}_{47}^{0} = -\mathrm{V}_4^0 = \frac{25270}{25201} \tag{11.47}$$

it follows that

$$\mathrm{V}_{39}^{47} = \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{11.48}$$

$$\mathrm{P}_{39}^{47} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{11.49}$$

[5] A photograph of the gears of the globe is shown in the 1989 exhibition catalogue [3, p. 528].

The tube 39 actually corresponds to the mean motion of the Moon and the period of its rotation around the globe is one tropical month. The same value is given by Oechslin, also in sidereal days.

This motion is used to obtain the mean motion of the Sun on tube 41:

$$\mathrm{V}_{41}^{47} = \mathrm{V}_{39}^{47} \times \left(-\frac{112}{53}\right) \times \left(-\frac{4}{113}\right) = \mathrm{V}_{39}^{47} \times \frac{448}{5989} \tag{11.50}$$

$$= \frac{6279595}{171568408} \times \frac{448}{5989} = \frac{351657320}{128440399439} \tag{11.51}$$

$$\mathrm{P}_{41}^{47} = \frac{128440399439}{351657320} = 365.2430\ldots \text{ days} \tag{11.52}$$

The same value is given by Oechslin, also in sidereal days.

The actual motion of the Sun is on tube 68. This tube has the same motion as another tube S' (erroneously numbered 41 by Oechslin), being connected through axis 61 and the same gear ratios.

The motion of this tube S' is itself the same as that of an unnumbered mobile frame containing a number of gears and which I am calling S.[6] This frame has an oscillating motion around the position of frame 41 which is the mean Sun. This oscillation is produced by an excentric pin on a wheel on arbor 49 located on frame 41. The oscillation period is the time it takes for this wheel to rotate on frame 41:

$$\mathrm{V}_{49}^{41} = \mathrm{V}_{47}^{41} \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) = \mathrm{V}_{47}^{41} = -\mathrm{V}_{41}^{47} \tag{11.53}$$

Therefore

$$\mathrm{P}_{49}^{41} = -\mathrm{P}_{41}^{47} \tag{11.54}$$

(The negative sign is irrelevant here.)

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. In reality, the apsides are not fixed, but move at a rate of one turn in about 21000 years with respect to the zodiac. That would be the period that Hahn would have had to implement.

Now, the mean motion of the Sun is used to obtain the apsidal precession of the Moon on tube 43:

[6] Oechslin's plate is slightly incorrect as it mistakenly links a 116-teeth wheel on tube S' with the 113-teeth wheel on tube 41. See the Gotha plate which is better in that respect.

$$V_{43}^{47} = V_{41}^{47} \times \left(-\frac{28}{78}\right) \times \left(-\frac{23}{73}\right) \tag{11.55}$$
$$= V_{41}^{47} \times \frac{322}{2847} \tag{11.56}$$
$$= \frac{351657320}{128440399439} \times \frac{322}{2847} = \frac{4923202480}{15898687704471} \tag{11.57}$$
$$P_{43}^{47} = \frac{15898687704471}{4923202480} = 3229.3385\ldots \text{ days} \tag{11.58}$$

The same value is given by Oechslin, also in sidereal days.

The actual value is about 3233 days (8.85 years). It should be noted that in the Gotha *Weltmaschine*, a ratio of $\left(-\frac{28}{81}\right)$ is used instead of $\left(-\frac{28}{78}\right)$, and this causes the motions of the lunar apsides and nodes to be wrong.

The motion of the apsides is then used to obtain the apparent motion of the Moon, namely that of the mean Moon plus the equation of center. The equation of center is computed based on the position of the perigee. Now, whereas the equation of center for the Sun rested on a fixed wheel on axis 47, because the perihelion/aphelion were considered fixed on the celestial sphere, the equation of center for the Moon rests on the wheel corresponding to the apsidal precession, because the perigee/apogee are fixed with respect to that wheel. The period of the oscillation caused by the equation of center of the Moon is however not the tropical month:

$$V_{44}^{39} = V_{43}^{39} \times \left(-\frac{48}{48}\right) = -V_{43}^{39} = V_{39}^{43} \tag{11.59}$$
$$= V_{39}^{47} + V_{47}^{43} = \frac{6279595}{171568408} - \frac{4923202480}{15898687704471} \tag{11.60}$$
$$= \frac{106165514457065}{2925358537622664} \tag{11.61}$$
$$P_{44}^{39} = \frac{2925358537622664}{106165514457065} = 27.5546\ldots \text{ days} \tag{11.62}$$

This is the anomalistic month. This value is not given by Oechslin.

As far as the Moon is concerned, the mean motion of the Sun is also used to obtain the precession of the lunar nodes on tube 46:

$$V_{46}^{47} = V_{41}^{47} \times \left(-\frac{28}{78}\right) \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{39}\right) \tag{11.63}$$
$$= V_{41}^{47} \times \left(-\frac{2975}{55341}\right) = \frac{351657320}{128440399439} \times \left(-\frac{2975}{55341}\right) \tag{11.64}$$
$$= -\frac{1046180527000}{7108020145353699} \tag{11.65}$$

This value is negative, because the nodes have a retrograde motion.

$$P_{46}^{47} = -\frac{7108020145353699}{1046180527000} = -6794.2577\ldots \text{ days} \tag{11.66}$$

The same value is given by Oechslin. This corresponds to about 18.6 tropical years.

11.5.2 The motions of the planets

The motion of the true Sun is used to obtain the apparent motions of Mercury and Venus.

11.5.2.1 Venus

I am first considering the motion of Venus, as it is the simplest. Hahn merely has Venus oscillating around the position of the Sun with the period of a wheel on arbor 59. The velocity of this wheel is

$$\mathrm{V}_{59}^{S} = \mathrm{V}_{47}^{S} \times \left(-\frac{31}{20}\right) \times \left(-\frac{26}{44}\right) \times \left(-\frac{28}{41}\right) = \mathrm{V}_{47}^{S} \times \left(-\frac{2821}{4510}\right) \tag{11.67}$$

Now, since

$$\mathrm{V}_{47}^{S} \approx \mathrm{V}_{47}^{41} \tag{11.68}$$

we have

$$\mathrm{V}_{59}^{S} \approx \mathrm{V}_{47}^{41} \times \left(-\frac{2821}{4510}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{2821}{4510}\right) \tag{11.69}$$

$$\approx \frac{99202529972}{57926620146989} \tag{11.70}$$

and

$$\mathrm{P}_{59}^{S} \approx \frac{57926620146989}{99202529972} = 583.9228\ldots \text{ days} \tag{11.71}$$

which is the synodic period of Venus. This value is not given by Oechslin. The above value is an average, because it is based on the motion of the true Sun. The oscillation of Venus makes it possible to show the retrogradations of this planet.

From this period, we can obtain the tropical orbit period of Venus which is[7]

$$\text{tropical orbit period} = \frac{1}{\frac{1}{\text{tropical year}} + \frac{1}{\text{synodic period}}} \tag{11.72}$$

$$= \frac{1}{\frac{1}{\mathrm{P}_{41}^{47}} + \frac{1}{\mathrm{P}_{59}^{S}}} \tag{11.73}$$

$$\approx \frac{57926620146989}{257799981292} = 224.6959\ldots \text{ days} \tag{11.74}$$

The same value is given by Oechslin, also in sidereal days.

[7]The derivation of this formula can for instance be found in section 22.4.3.1, in the chapter on Klein's Tychonic clock.

11.5.2.2 Mercury

The motion of Mercury is more complex because Hahn both takes into account the retrogradations of Mercury and the excentricity of its orbit (but not the precession of the apsides). If we do not take the excentricity into account, the position of Mercury is that of arbor 56 and it is also merely oscillating around the true Sun.

For Mercury, Hahn did not use the same ratios as in Stuttgart and Gotha, but the period obtained is very close.

The velocity of Mercury's axis 56 around its average position (tube 54) with respect the mean Sun is

$$\mathrm{V}_{54}^{S} = \mathrm{V}_{47}^{S} \times \left(-\frac{69}{28}\right) \times \left(-\frac{55}{43}\right) \times \left(-\frac{43}{43}\right) = \mathrm{V}_{47}^{S} \times \left(-\frac{3795}{1204}\right) \tag{11.75}$$

As before, since

$$\mathrm{V}_{47}^{S} \approx \mathrm{V}_{47}^{41} \tag{11.76}$$

we have

$$\mathrm{V}_{54}^{S} \approx \mathrm{V}_{47}^{41} \times \left(-\frac{3795}{1204}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{3795}{1204}\right) \tag{11.77}$$

$$\approx \frac{188387850}{21829791209} \tag{11.78}$$

and

$$\mathrm{P}_{54}^{S} \approx \frac{21829791209}{188387850} = 115.8768\ldots \text{ days} \tag{11.79}$$

which is the synodic period of Mercury. Like for Venus, the above value is an average, because it is based on the motion of the true Sun. The oscillation of Mercury makes it possible to show the retrogradations of this planet.

Again, we can also deduce the tropical orbit period of Mercury which is

$$\frac{5522937175877}{62783390810} = 87.9681\ldots \text{ days} \tag{11.80}$$

The same value is given by Oechslin, also in sidereal days.

Hahn does however add a second oscillation on top of the first one, in that the pin driving Mercury is excentered from arbor 56. Mercury rotates around the axis 56.

Arbor 56 replicates tube 54 which replicates arbor 47. Consequently, the crank has a fixed orientation with respect to the globe.

11.5.2.3 Mars

The motions of Mars, Jupiter and Saturn are based on the true motion of the Sun. The motion of the Sun is replicated in three 116-teeth wheels which have the same motion as they are connected with identical gears to arbor 61. The intermediate of these three 116-teeth is the one on tube 68 and is the one carrying the actual Sun, as was already mentioned.

The mean motion of Mars is obtained first on tube 63:

$$\mathrm{V}_{63}^{47} = \mathrm{V}_{S'}^{47} \times \left(-\frac{116}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{12}{32}\right) \times \left(-\frac{32}{119}\right) \tag{11.81}$$

$$\approx \mathrm{V}_{S'}^{47} \times \frac{696}{1309} = \frac{351657320}{128440399439} \times \frac{696}{1309} = \frac{1205682240}{828219127417} \tag{11.82}$$

$$\mathrm{P}_{63}^{47} \approx \frac{828219127417}{1205682240} = 686.9298\ldots \text{ days} \tag{11.83}$$

This is Mars' orbital period. The same value is given by Oechslin, also in sidereal days. Like for Mercury and Venus, the above value is an average, because it is based on the motion of the true Sun.

Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. This is done exactly in the same way as the equation of center for the Sun. There are two wheels of 48 teeth, one of them located on the frame for the mean motion of Mars, the other fixed inside the celestial globe (frame 47). We have

$$\mathrm{V}_{64}^{63} = \mathrm{V}_{47}^{63} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{47}^{63} = \mathrm{V}_{63}^{47} \tag{11.84}$$

Therefore

$$\mathrm{P}_{64}^{63} = \mathrm{P}_{63}^{47} \tag{11.85}$$

The oscillation period is exactly that of the tropical orbit period, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Mars is on a frame that Oechslin doesn't seem to number, but that we will name m.

For the geocentric view of the motion of Mars, Hahn proceeds as for Venus, where he corrected the position of Venus using the elongation with the true Sun S. In the case of Mars, the correction is done with respect to S', and we have

$$\mathrm{V}_{66}^{S'} = \mathrm{V}_{66}^{m} + \mathrm{V}_{m}^{S'} = \mathrm{V}_{S'}^{m} \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) + \mathrm{V}_{m}^{S'} \tag{11.86}$$

$$= \mathrm{V}_{S'}^{m} + \mathrm{V}_{m}^{S'} = 0 \tag{11.87}$$

We could also have seen immediately that the motion of arbor 66 replicates that of S', and therefore that $\mathrm{V}_{66}^{S'} = 0$.

The synodic period of Mars corresponds to the velocity

$$\mathrm{V}_{S'}^{m} \approx \mathrm{V}_{41}^{m} \approx \mathrm{V}_{41}^{47} - \mathrm{V}_{m}^{47} \approx \mathrm{V}_{41}^{47} - \mathrm{V}_{63}^{47} \tag{11.88}$$

$$\approx \frac{351657320}{128440399439} - \frac{1205682240}{828219127417} \tag{11.89}$$

$$\approx \frac{30795133880}{24018354695093} \tag{11.90}$$

and therefore

$$\mathrm{P}_{S'}^{m} \approx \frac{24018354695093}{30795133880} = 779.9399\ldots \text{ days} \tag{11.91}$$

This value is not given by Oechslin.

11.5.2.4 Jupiter

The motion of Jupiter is obtained in a similar way as that of Mars. The mean motion of Jupiter is obtained first on tube 72, from the mean motion of Mars:

$$\mathrm{V}_{72}^{47} = \mathrm{V}_{63}^{47} \times \left(-\frac{119}{31}\right) \times \left(-\frac{31}{31}\right) \times \left(-\frac{5}{34}\right) \times \left(-\frac{34}{121}\right) \tag{11.92}$$

$$= \mathrm{V}_{63}^{47} \times \frac{595}{3751} \approx \frac{1205682240}{828219127417} \times \frac{595}{3751} \tag{11.93}$$

$$\approx \frac{42198878400}{182744114525951} \tag{11.94}$$

$$\mathrm{P}_{72}^{47} \approx \frac{182744114525951}{42198878400} = 4330.5443\ldots \text{ days} \tag{11.95}$$

This is Jupiter's orbital period. The same value is given by Oechslin. Like for Mercury, Venus and Mars, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}_{73}^{72} = \mathrm{V}_{47}^{72} \times \left(-\frac{60}{60}\right) = -\mathrm{V}_{47}^{72} = \mathrm{V}_{72}^{47} \tag{11.96}$$

Therefore

$$\mathrm{P}_{73}^{72} = \mathrm{P}_{72}^{47} \tag{11.97}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Jupiter is on a frame that Oechslin doesn't seem to number, but that we will name j. We have

$$\mathrm{V}_{75}^{S} = \mathrm{V}_{75}^{j} + \mathrm{V}_{j}^{S} = \mathrm{V}_{S}^{j} \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) + \mathrm{V}_{j}^{S} \tag{11.98}$$

$$= \mathrm{V}_{S}^{j} + \mathrm{V}_{j}^{S} = 0 \tag{11.99}$$

In other words

$$\mathrm{V}_{75}^{47} = \mathrm{V}_{75}^{S} + \mathrm{V}_{S}^{47} = \mathrm{V}_{S}^{47} \tag{11.100}$$

We could likewise compute the synodic period of Jupiter, as we did for Mars.

11.5.2.5 Saturn

The motion of Saturn is also obtained similarly. The mean motion of Saturn is obtained first on tube 78, from the mean motion of Jupiter:

$$\mathrm{V}_{78}^{47} = \mathrm{V}_{72}^{47} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) \tag{11.101}$$

$$= \mathrm{V}_{72}^{47} \times \frac{363}{901} \approx \frac{42198878400}{182744114525951} \times \frac{363}{901} \tag{11.102}$$

$$\approx \frac{126596635200}{1360764026346131} \tag{11.103}$$

$$\mathrm{P}_{72}^{47} \approx \frac{1360764026346131}{126596635200} = 10748.8166\ldots \text{ days} \tag{11.104}$$

This is Saturn's orbital period. The same value is given by Oechslin. Like for Mercury, Venus, Mars and Jupiter, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars and Jupiter, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}_{79}^{78} = \mathrm{V}_{47}^{78} \times \left(-\frac{60}{60}\right) = -\mathrm{V}_{47}^{78} = \mathrm{V}_{78}^{47} \tag{11.105}$$

Therefore

$$\mathrm{P}_{79}^{78} = \mathrm{P}_{78}^{47} \tag{11.106}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Saturn is on a frame that Oechslin doesn't seem to number, but that we will name s. We have

$$\mathrm{V}_{81}^{S} = \mathrm{V}_{81}^{s} + \mathrm{V}_{s}^{S} = \mathrm{V}_{S}^{s} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) + \mathrm{V}_{s}^{S} \tag{11.107}$$

$$= \mathrm{V}_{S}^{s} + \mathrm{V}_{s}^{S} = 0 \tag{11.108}$$

In other words

$$\mathrm{V}_{81}^{47} = \mathrm{V}_{81}^{S} + \mathrm{V}_{S}^{47} = \mathrm{V}_{S}^{47} \tag{11.109}$$

We could likewise compute the synodic period of Saturn, as we did for Mars.

11.6 References

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