Chapter 29 **(Oechslin: 10.1)**

Seige's *Weltmaschine* in Prague (1791)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

29.1 Introduction

The machine described here was constructed by Engelbert Seige (1737-1811)[1] and is dated 1791.

According to Hrůšová, this machine was first located in the library of the Osek monastery, Czech Republic (at least around 1827), and transferred in 1847 to the room of natural sciences [4, 2].

It was restored in 1885 by Richard Swoboda and Hrůšová believes that Swoboda added the globes on each side of the clock. I find this statement doubtful.

Seige's machine was studied by Ludwig Oechslin around 1990 and later restored by Jaroslav Nový and included in the permanent exhibition of the technical museum in Prague [8].[2]

It is the most complex machine we know of Seige. It is however not that complex. It only happens that there are many dials, multiple celestial spheres, of which two are on sides. But what also adds to the difficulty of understanding this machine is that Oechslin's plan takes some time to assimilate, because there are many gear trains, and that the gears concerning some dials are not located on the plan as they are in reality. This is therefore perhaps the machine where this companion analysis will be the most needed.

[1]For some biographical information on Seige, see the chapter devoted to Seige's metal fragment.

[2]For a description of this machine, see [9, p. 34]. It was also briefly described by King [5, p. 243-244].

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Figure 29.1: Seige's machine (from Hrůšová [4]).

The machine is made of a cubic base which is surmounted by a central orrery encased in a kind of armillary sphere, and this orrery is surrounded by four smaller systems, like in the metal fragment (Oechslin 10.3). These smaller systems are also encased in armillary spheres. The remaining metal fragment is probably a prototype of this machine.

Two of the four faces of the base are sided by spheres within armillary spheres. One of these spheres shows the shadow of the Earth on a yearly basis. This side also contains seven smaller dials and an inscription plate.

The other sphere shows the Sun/Moon/Earth daily system. This side also contains seven smaller dials and an inscription plate. The small dials of these two opposite sides are clearly inspired by those of Frater David's 1769 clock (Oechslin 6.1). Seige must have seen this clock, and decided to copy its dials. This is not a mere hypothesis, when one considers the dial for the epacts.

The two other faces are simpler dials: the annual dial and the shadow of the Earth daily dial.

In my analysis below, I normally take the vantage point of the outside of the dials, but on the plate this can mean above, or below, or sideways. For the upper parts, the top of the plate coincides with the top of the systems.

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Figure 29.2: The inside of the machine seen from above. We can clearly see the two vertical arbors leading to the upper systems. (from Hrůšová [4]).

29.2 The going work

The clock is weight-driven and regulated by a pendulum. The drum on arbor 1 makes one turn on two days. Its velocity, measured from the right on Oechslin's plate, is:

$$V_1^0 = \frac{1}{2} \tag{29.1}$$
$$P_1^0 = 2 \text{ days} \tag{29.2}$$

The train to the escape wheel goes through arbors 2 and 3, as well as two unnumbered arbors which I am naming x and y. We have:

$$V_2^0 = V_1^0 \times \left(-\frac{80}{10}\right) = -4 \tag{29.3}$$
$$P_2^0 = -\frac{1}{4} = -6 \text{ hours} \tag{29.4}$$
$$V_3^0 = V_2^0 \times \left(-\frac{60}{10}\right) = 24 \tag{29.5}$$
$$P_3^0 = \frac{1}{24} = 1 \text{ hour} \tag{29.6}$$
$$V_x^0 = V_3^0 \times \left(-\frac{60}{6}\right) = -240 \tag{29.7}$$
$$P_x^0 = -6 \text{ minutes} \tag{29.8}$$
$$V_y^0 = V_x^0 \times \frac{48}{8} = -1440 \tag{29.9}$$
$$P_y^0 = -\frac{1}{1440} = -1 \text{ minute} \tag{29.10}$$

The 30-teeth wheel on arbor y is the escape wheel and its velocity is measured from above. It makes one turn in a minute and the pendulum makes a half-oscillation in one second.

Arbors 1 and 3 drive the Earth shadow system. Arbors 2 and 3 drive the Sun/Earth/Moon system on the opposite side.

Arbors 97 and 146 drive the upper part of the machine, both motions being derived from that of arbor 3.

29.3 The four sides of the base

29.3.1 The annual dial

This dial shows the yearly motion of the Sun, the Moon, the lunar nodes, and the lunar apsides The dial pictures a calendar, with the days and months running clockwise. Consequently, Seige chose to have the hands for the Sun, the Moon, the lunar nodes and the lunar apsides move clockwise. The long double hand is that of the lunar nodes and pictures the head and tail of a dragon.

The calendar is actually shown on four concentric rings. Each ring contains 365.25 days and the alternating black and white cells shift from one ring to the next. The first ring is for the first year in a 4-year cycle, the second ring is for the second year, and so on. There are a total of 1461 cells.

Within the months, a number of saints are shown. And the inner ring shows the various sundays and the moving feasts. This ring can be set each year so that Easter falls on the right date.

This dial is driven by arbor 80. The motion originates from arbor 2. If we measure the motion of arbor 80 from the left and arbor 17 from the above (on Oechslin's plan), we have:

$$V^0_{17} = V^0_2 \times \frac{30}{48} = -\frac{5}{2} \tag{29.11}$$

$$V^0_{80} = V^0_{17} \times \frac{16}{40} = \left(-\frac{5}{2}\right) \times \frac{2}{5} = -1 \tag{29.12}$$

$$P^0_{80} = -1 \text{ day} \tag{29.13}$$

All the remaining motions are measured from the front of the dial, that is from the left on Oechslin's drawing.

The motion of the Sun is that of tube 88. We first compute the velocity of arbor 83.

$$V^0_{83} = V^0_{80} \times \left(-\frac{30}{30}\right) \times \left(-\frac{10}{53}\right) \times \left(-\frac{14}{64}\right) = V^0_{80} \times \left(-\frac{35}{848}\right) \tag{29.14}$$

$$= \frac{35}{848} \tag{29.15}$$

$$V^0_{85} = V^0_{83} \times \left(-\frac{20}{62}\right) \times \left(-\frac{25}{37}\right) = V^0_{83} \times \frac{250}{1147} = \frac{35}{848} \times \frac{250}{1147} \tag{29.16}$$

$$= \frac{4375}{486328} \tag{29.17}$$

$$V^0_{88} = V^0_{85} \times \left(-\frac{21}{69}\right) = \frac{4375}{486328} \times \left(-\frac{7}{23}\right) = -\frac{30625}{11185544} \tag{29.18}$$

$$P^0_{88} = -\frac{11185544}{30625} = -365.2422 \text{ days} \tag{29.19}$$

This is an approximation of the tropical year.[3] The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the lunar apsides is that of tube 91. We first compute the motion of arbor 90 which is also the basis of the train for the lunar nodes. We

[3] Incidentally, Oechslin observed that Seige used five different values of the tropical year [9, p. 177]. Another one was used in the metal fragment (Oechslin 10.3).

(Copyrighted image not shown)

Figure 29.3: The annual dial (from Hrůšová [4]).

have

$$V_{90}^0 = V_{85}^0 \times \left(-\frac{19}{60}\right) \times \left(-\frac{19}{53}\right) = V_{85}^0 \times \frac{361}{3180} = \frac{4375}{486328} \times \frac{361}{3180} \tag{29.20}$$

$$= \frac{315875}{309304608} \tag{29.21}$$

Then

$$V_{91}^0 = V_{90}^0 \times \left(-\frac{20}{66}\right) = \frac{315875}{309304608} \times \left(-\frac{20}{66}\right) = -\frac{1579375}{5103526032} \tag{29.22}$$

$$P_{91}^0 = -\frac{5103526032}{1579375} = -3231.3579\ldots \text{ days} \tag{29.23}$$

which is about 8.85 years. The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the lunar nodes is that of tube 95. We have

$$V_{95}^0 = V_{90}^0 \times \left(-\frac{23}{22}\right) \times \left(-\frac{24}{38}\right) \times \left(-\frac{29}{41}\right) \times \left(-\frac{33}{107}\right) \tag{29.24}$$

$$= V_{90}^0 \times \frac{12006}{83353} \tag{29.25}$$

$$= \frac{315875}{309304608} \times \frac{12006}{83353} = \frac{33266625}{226153219216} \tag{29.26}$$

$$P_{95}^0 = \frac{226153219216}{33266625} = 6798.2014\ldots \text{ days} \tag{29.27}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

Finally, the mean motion of the Moon is that of frame 84. We have

$$V_{84}^0 = V_{83}^0 \times \left(-\frac{47}{53}\right) = \frac{35}{848} \times \left(-\frac{47}{53}\right) = -\frac{1645}{44944} \tag{29.28}$$

$$P_{84}^0 = -27.3215\ldots \text{ days} \tag{29.29}$$

This is an approximation of the tropical month. The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the Moon is corrected using a cam which is part of tube 91 and moves with the apsides. A feeler on this cam then moves the corrected Moon back and forth, depending on the position of the Moon with respect to the line of apsides.

29.3.2 The daily Earth shadow system

This dial is opposite to the one for the yearly system described in the previous section. It shows the Sun and the shadow of the Earth moving around a fixed Earth.

(Copyrighted image not shown)

Figure 29.4: The daily Earth shadow system (from Hrůšová [4]).

This system is driven by arbor 78. This motion also originates from arbor 2, with a similar train as in the case of arbor 80. We measure the motion from the side of the dial:

$$\mathrm{V}_{78}^{0} = \mathrm{V}_{2}^{0} \times \left(-\frac{30}{48}\right) \times \frac{16}{40} = \mathrm{V}_{2}^{0} \times \left(-\frac{1}{4}\right) = -4 \times \left(-\frac{1}{4}\right) = 1 \qquad (29.30)$$

$$\mathrm{P}_{78}^{0} = 1 \text{ day} \qquad (29.31)$$

The only difference with arbor 80 is that the motion is in the opposite direction.

The Sun and the shadow of the Earth move clockwise with frame 79 and makes a turn in one day. We have

$$\mathrm{V}^0_{79} = \mathrm{V}^0_{78} \times \left(-\frac{96}{96}\right) = -1 \tag{29.32}$$

The shadow always goes through the North pole and the inclination of the Earth was not taken into account in this dial.[4]

[4]See also [9, p. 149] on this construction.

29.3.3 The side of the yearly Earth shadow system

This side has seven dials around a central horizontal arbor leading to an armillary sphere containing the Earth shadow system.

29.3.3.1 The yearly Earth shadow system

In this system, the horizontal axis going through the armillary sphere represents the rotation axis of the Earth. The Earth has its North pole towards the center of the machine, and the South pole towards the outside. It makes one turn in a day. It seems that the shadow of the Earth as well as the Sun are pivoting as a consequence of a cam that rotates in one year. The Sun does not rotate around the Earth, but it moves either to the Northern latitudes or to the Southern latitudes, and the shadow of the Earth is always opposite to the Sun.

The motion of this system is derived from those of arbors 1 and 3. There are two tubes, tube 12 and tube 15. The motion of tube 12 is obtained from arbor 3. First, we have the motion of arbor 11, here measured from the outside:

$$\mathrm{V}_{11}^0 = \mathrm{V}_3^0 \times \frac{12}{48} = 24 \times \frac{1}{4} = 6 \tag{29.33}$$

Then

$$\mathrm{V}_{12}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{16}{96}\right) = 6 \times \left(-\frac{1}{6}\right) = -1 \tag{29.34}$$

$$\mathrm{P}_{12}^0 = -1 \tag{29.35}$$

Tube 12 makes a turn clockwise (seen from the outside) in one day. This corresponds to a counterclockwise motion seen from the North pole, which is indeed the motion of the Earth with respect to the Sun.

Similarly, we consider the motion of tube 13, also measured from the outside:

$$\mathrm{V}_{13}^0 = \mathrm{V}_1^0 \times \frac{11}{56} = \frac{1}{2} \times \frac{11}{56} = \frac{11}{112} \tag{29.36}$$

And

$$\mathrm{V}_{15}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{10}{76}\right) \times \left(-\frac{25}{118}\right) = \mathrm{V}_{13}^0 \times \frac{125}{4484} = \frac{11}{112} \times \frac{125}{4484} \tag{29.37}$$

$$= \frac{1375}{502208} \tag{29.38}$$

$$\mathrm{P}_{15}^0 = \frac{502208}{1375} = 365.2421\ldots \text{ days} \tag{29.39}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

Tube 15 also carries the equation of time cam which is used to show the true time in the time dial below.

(Copyrighted image not shown)

Figure 29.5: The yearly Earth shadow system (from Hrůšová [4]).

29.3.3.2 The other dials

There are seven dials and an inscription plate. This inscription plate recalls the restoration by Richard Swoboda in 1885.

From the bottom and going clockwise, we have 1) the inscription plate, 2) the day of the week, 3) the age of the Moon, 4) the golden number, the epacts and the indiction, 5) the time, 6) the solar cycle and the dominical letter, 7) a dial for the anomalistic month, and 8) a dial for the draconic month. However, it seems that some dials have been mixed up, as they do not all correspond to the gears which are underneath. I will however consider the dials in the apparent order, not in the order of the gears underneath.

Except for the time dial, all of the motions of the dials are derived from that of arbor 18 which is obtained from the motion of arbor 2. The motion of the time dial is instead derived from the motion of arbor 3.

We have

$$\mathrm{V}_{18}^{0} = \mathrm{V}_{17}^{0} \times \left(-\frac{16}{40}\right) = \left(-\frac{5}{2}\right) \times \left(-\frac{2}{5}\right) = 1 \tag{29.40}$$

This arbor makes one turn counterclockwise (from the outside) in one day.

(Copyrighted image not shown)

Figure 29.6: The dials of the yearly Earth shadow system (from Hrůšová [4]).

(Copyrighted image not shown)

Figure 29.7: The gears for the dials of the yearly Earth shadow system. Note however that the dials do not perfectly correspond to the gears. For instance, the gears at the bottom left correspond to the day of the week, but the gears above correspond to the anomalistic month, not to the synodic month, as would be expected. Some of the dials have been misplaced. (from Hrůšová [4]).

29.3.3.2.1 The inscription plate

The inscription plate records the 1885 restoration of the machine by Richard Swoboda who was teacher in Hrob (Klostergrab in German).

(Copyrighted image not shown)

Figure 29.8: The inscription plate (from Hrůšová [4]).

29.3.3.2.2 The day of the week

The hand of this dial is on arbor 19 and it makes one turn clockwise (from the outside) in seven days:

$$\mathrm{V}_{19}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{12}{84}\right) = \mathrm{V}_{18}^{0} \times \left(-\frac{1}{7}\right) = -\frac{1}{7} \tag{29.41}$$

$$\mathrm{P}_{19}^{0} = -7 \text{ days} \tag{29.42}$$

The dial also shows the hours, but only every six hours.

(Copyrighted image not shown)

Figure 29.9: The day of the week (from Hrůšová [4]).

29.3.3.2.3 The age of the Moon

This dial shows the age of the Moon. The hand is carried by arbor 25.

(Copyrighted image not shown)

Figure 29.10: The age of the Moon (from Hrůšová [4]).

In order to compute the velocity of this arbor, we first compute the velocity of arbor 21, as it is also used for the anomalistic month:

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{41}{54}\right) \times \left(-\frac{8}{53}\right) = \mathrm{V}_{18}^{0} \times \frac{164}{1431} = \frac{164}{1431} \tag{29.43}$$

Then

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{21}^{0} \times \left(-\frac{34}{83}\right) \times \left(-\frac{44}{61}\right) \times \left(-\frac{40}{40}\right) = \mathrm{V}_{21}^{0} \times \left(-\frac{1496}{5063}\right) \tag{29.44}$$

$$= \frac{164}{1431} \times \left(-\frac{1496}{5063}\right) = -\frac{245344}{7245153} \tag{29.45}$$

$$\mathrm{P}_{25}^{0} = -\frac{7245153}{245344} = -29.5305\ldots \text{ days} \tag{29.46}$$

The hand makes a turn clockwise in one synodic month. The same value is given by Oechslin.

29.3.3.2.4 The golden number, the epacts and the indiction

This dial shows the values of the golden number, the epacts and the indiction. There are two series of epacts, the series for the 18th and 19th centuries on the outer circle, and the series for the 20th, 21st and 22nd centuries on the inner circle. This dial was obviously copied from the almost identical dial found in Frater David's clock from 1769 (Oechslin 6.1).

Like for the solar cycle dial, the two hands are driven by the arbor 31 which makes one turn in a year. So, we first consider the motion of arbor 31. This motion is itself derived from the motion of arbor 27 which we also compute, as it is used for the draconic month:

$$\mathrm{V}^0_{27} = \mathrm{V}^0_{18} \times \left(-\frac{12}{66}\right) \times \left(-\frac{43}{74}\right) = \mathrm{V}^0_{18} \times \frac{43}{407} = \frac{43}{407} \tag{29.47}$$

$$\mathrm{V}^0_{31} = \mathrm{V}^0_{27} \times \left(-\frac{17}{48}\right) \times \left(-\frac{12}{64}\right) \times \left(-\frac{16}{41}\right) = \mathrm{V}^0_{27} \times \left(-\frac{17}{656}\right) \tag{29.48}$$

$$= \frac{43}{407} \times \left(-\frac{17}{656}\right) = -\frac{731}{266992} \tag{29.49}$$

$$\mathrm{P}^0_{31} = -\frac{266992}{731} = -365.2421\ldots \text{ days} \tag{29.50}$$

The same value is given by Oechslin.

(Copyrighted image not shown)

Figure 29.11: The golden number, the epacts and the indiction (from Hrůšová [4]).

An arm on arbor 31 then makes a 19-teeth wheel on arbor 32 advance intermittently by one tooth. This wheel then moves two levers on arbor 35 which advance by one tooth the 19-teeth wheel on arbor 36 and the 15-teeth wheel on tube 37. The former carries the hand of the golden number and epacts, and the latter carries the hand of the indiction. Both hands move clockwise.

29.3.3.2.5 The time

The central arbor of this dial is arbor 8. We first obtain the velocity of arbor 5 (measured from the outside):

$$V_5^0 = V_3^0 \times \frac{24}{24} = 24 \tag{29.51}$$

Then

$$V_8^0 = V_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{36}{36}\right) \times \left(-\frac{40}{40}\right) = -24 \tag{29.52}$$

So, this arbor makes one turn clockwise (from above) in one hour and holds the minute hand.

(Copyrighted image not shown)

Figure 29.12: The time dial (from Hrůšová [4]).

The motion of tube 10 is derived from it and it makes one turn clockwise in 12 hours:

$$V_{10}^0 = V_8^0 \times \left(-\frac{15}{60}\right) \times \left(-\frac{20}{60}\right) = V_8^0 \times \frac{1}{12} = -2 \tag{29.53}$$

Finally, there is a moving hour minute dial which moves back and forth using a rack with a feeler on the equation cam on the tube 15 described earlier. This rack meshes with a 24-teeth wheel on the tube carrying the moving dial. When the minute hand is read on the moving dial, we get the minutes of true time. The hour may need to be corrected. For instance, if the mean time is 1:55 and the minutes of true time show 01 minute, then the true time is actually 2:01. In other cases, the hours may need to be decreased.

29.3.3.2.6 The solar cycle and the dominical letter

This dial shows the values of the solar cycle and the dominical letter. Its hand makes a turn clockwise in 28 years.

We have seen above that the 19-teeth wheel on arbor 32 advances intermittently by one tooth every year. Another lever on arbor 33 transmits this intermittent motion to the 28-teeth wheel on arbor 34. This wheel therefore advances by one tooth every year, and its arbor carries the hand for the solar cycle. The dial also shows the values of the corresponding dominical letters during the 18th century. This correspondence was no longer valid after 1800.

(Copyrighted image not shown)

Figure 29.13: The solar cycle and the dominical letter (from Hrůšová [4]).

29.3.3.2.7 The anomalistic month

This dial shows the number of days since the last mean perigee of the Moon, assuming it was set on 0 on passage of the mean Moon to a mean perigee.

(Copyrighted image not shown)

Figure 29.14: The anomalistic month (from Hrůšová [4]).

The hand of this dial is carried by arbor 22. Its motion is derived from the arbor 21 whose velocity was computed above. We have

$$V^0_{22} = V^0_{21} \times \left(-\frac{19}{60}\right) = \frac{164}{1431} \times \left(-\frac{19}{60}\right) = -\frac{779}{21465} \tag{29.54}$$

$$P^0_{22} = -\frac{21465}{779} = -27.5545\ldots \text{ days} \tag{29.55}$$

This is the period of the anomalistic month. The same value is given by Oechslin. The hand makes one turn clockwise in an anomalistic month.

29.3.3.2.8 The draconic month

This dial seems to show the angle since the last ascending node of the Moon, and therefore its hands on arbor 28 rotates in a draconic month.

(Copyrighted image not shown)

Figure 29.15: The draconic month (from Hrůšová [4]).

We have:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{27}^{0} \times \left(-\frac{24}{69}\right) = \frac{43}{407} \times \left(-\frac{24}{69}\right) = -\frac{344}{9361} \qquad (29.56)$$

$$\mathrm{P}_{28}^{0} = -\frac{9361}{344} = -27.2122\ldots \text{ days} \qquad (29.57)$$

The same value is given by Oechslin.

29.3.4 The side of the Sun/Earth/Moon daily system

This side has seven dials around a central horizontal arbor leading to an armillary sphere containing the Sun/Earth/Moon daily system.

29.3.4.1 The Sun/Earth/Moon daily system

The motion of this system is derived from that of tube 68 which makes one turn in a sidereal day. This motion is itself derived from that of arbor 2. We have

$$\mathrm{V}_{68}^{0} = \mathrm{V}_{2}^{0} \times \left(-\frac{45}{89}\right) \times \left(-\frac{59}{119}\right) = \mathrm{V}_{2}^{0} \times \frac{2655}{10591} \tag{29.58}$$

$$= -4 \times \frac{2655}{10591} = -\frac{10620}{10591} \tag{29.59}$$

$$\mathrm{P}_{68}^{0} = -\frac{10591}{10620} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{29.60}$$

The same value is given by Oechslin.

(Copyrighted image not shown)

Figure 29.16: The Sun/Earth/Moon system (from Hrůšová [4]).

29.3.4.2 The other dials

There are seven dials and an inscription plate. This inscription plate recalls the construction of the machine in 1791.

(Copyrighted image not shown)

Figure 29.17: The dials of the Sun/Earth/Moon system (from Hrůšová [4]).

From the bottom and going clockwise, we have 1) the inscription plate, 2) the dial of Venus, 3) the dial of Saturn, 4) the dial of Mars, 5) the mean and true time, 6) the dial of Uranus, 7) the dial of Jupiter, and 8) the dial of Mercury.

The motion of the hours is derived from that of arbor 3. The motions of the other dials are derived from that of arbor 43, which is itself derived from the motion of arbor 2. We have (measured from the side of dials)

$$\mathrm{V}_{43}^{0} = \mathrm{V}_{2}^{0} \times \left(-\frac{20}{40}\right) \times \left(-\frac{24}{48}\right) \times \left(-\frac{21}{42}\right) = \mathrm{V}_{2}^{0} \times \left(-\frac{1}{8}\right) \tag{29.61}$$

$$(-4) \times \left(-\frac{1}{8}\right) = \frac{1}{2} \tag{29.62}$$

The central arbor for the hours is arbor 40. Its velocity (also measured from the side of the dial) is

$$\mathrm{V}_{40}^{0} = \mathrm{V}_{3}^{0} \times \left(-\frac{24}{48}\right) \times \left(-\frac{16}{48}\right) \times \left(-\frac{20}{80}\right) = \mathrm{V}_{3}^{0} \times \left(-\frac{1}{24}\right) \tag{29.63}$$

$$= 24 \times \left(-\frac{1}{24}\right) = -1 \tag{29.64}$$

(Copyrighted image not shown)

Figure 29.18: The gears for the planet dials (from Hrůšová [4]).

29.3.4.2.1 The inscription plate

The inscription plate records the construction of the machine in 1791.

(Copyrighted image not shown)

Figure 29.19: The inscription plate (from Hrůšová [4]).

29.3.4.2.2 The Venus dial

This dial shows the longitude of Venus. The central arbor 51 carries the hand of Venus. We first compute the velocity of arbor 50 which is also used in the trains for Mars and Saturn. We have (measured from the front of the dial) is

$$\mathrm{V}^0_{50} = \mathrm{V}^0_{43} \times \left(-\frac{23}{76}\right) \times \left(-\frac{10}{60}\right) = \mathrm{V}^0_{43} \times \frac{23}{456} = \frac{1}{2} \times \frac{23}{456} = \frac{23}{912} \tag{29.65}$$

$$\mathrm{V}^0_{51} = \mathrm{V}^0_{50} \times \left(-\frac{12}{68}\right) = \frac{23}{912} \times \left(-\frac{12}{68}\right) = -\frac{23}{5168} \tag{29.66}$$

$$\mathrm{P}^0_{51} = -\frac{5168}{23} = -224.6956\ldots \text{ days} \tag{29.67}$$

The same value is given by Oechslin. The hand moves clockwise and makes one turn in the orbital period of Venus.

(Copyrighted image not shown)

Figure 29.20: The Venus dial (from Hrůšová [4]).

29.3.4.2.3 The Saturn dial

The trains of Mars and Saturn are both derived from arbor 53, and we first compute the motion of arbor 53. Then, the hand of Saturn is carried by arbor 57.

We have

$$\mathrm{V}_{53}^{0} = \mathrm{V}_{50}^{0} \times \left(-\frac{20}{72}\right) \times \left(-\frac{67}{73}\right) = \mathrm{V}_{50}^{0} \times \frac{335}{1314} \tag{29.68}$$

$$= \frac{23}{912} \times \frac{335}{1314} = \frac{7705}{1198368} \tag{29.69}$$

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{53}^{0} \times \left(-\frac{12}{46}\right) \times \left(-\frac{15}{77}\right) \times \left(-\frac{20}{68}\right) = \mathrm{V}_{53}^{0} \times \left(-\frac{450}{30107}\right) \tag{29.70}$$

$$= \frac{7705}{1198368} \times \left(-\frac{450}{30107}\right) = -\frac{8375}{87147984} \tag{29.71}$$

$$\mathrm{P}_{57}^{0} = -\frac{87147984}{8375} = -10405.7294\ldots \text{ days} \tag{29.72}$$

This seems a bad approximation of the orbital period of Saturn, as the expected value is about 10747 days, but the same value was obtained by Oechslin.

Now, since the periods of Venus and Mars are very accurate, it is likely that the Saturn train contains an error, but the error is not merely that of a single ratio. If the three ratios from arbor 53 to arbor 57 were replaced by 15/41, 15/72 and 15/79, we would for instance have obtained:

$$\mathrm{V(h)}_{57}^{0} = \mathrm{V}_{53}^{0} \times \left(-\frac{15}{41}\right) \times \left(-\frac{15}{72}\right) \times \left(-\frac{15}{79}\right) \tag{29.73}$$

$$\mathrm{P(h)}_{57}^{0} = -10746.9994\ldots \text{ days} \tag{29.74}$$

which would have been much better.

(Copyrighted image not shown)

Figure 29.21: The Saturn dial (from Hrůšová [4]).

29.3.4.2.4 The Mars dial

The train for Mars is derived from the motion of arbor 53. The hand for Mars is carried by arbor 54. We have

$$V^0_{54} = V^0_{53} \times \left(-\frac{12}{53}\right) = \frac{7705}{1198368} \times \left(-\frac{12}{53}\right) = -\frac{7705}{5292792} \tag{29.75}$$

$$P^0_{54} = -\frac{5292792}{7705} = -686.9295\ldots \text{ days} \tag{29.76}$$

The same value is given by Oechslin. The Mars hand moves clockwise and makes a turn in the orbital period of Mars. The period found here is a good approximation of the actual orbital period.

(Copyrighted image not shown)

Figure 29.22: The Mars dial (from Hrůšová [4]).

29.3.4.2.5 The mean and true time

This dial actually has a fixed outer 24-hours dial for the display of the mean time, and a moving inner 24-hours dial for the display of the true time. The latter is moved by a equation cam on tube 47, which is itself located around tube 68. We therefore first need to compute the motion of this cam. We have

$$\mathrm{V}^0_{47} = \mathrm{V}^0_{43} \times \left(-\frac{12}{25}\right) \times \left(-\frac{14}{42}\right) \times \left(-\frac{14}{69}\right) \times \left(-\frac{14}{83}\right) \tag{29.77}$$

$$= \mathrm{V}^0_{43} \times \frac{784}{143175} = \frac{1}{2} \times \frac{784}{143175} = \frac{392}{143175} \tag{29.78}$$

$$\mathrm{P}^0_{47} = \frac{143175}{392} = 365.2423\ldots \text{ days} \tag{29.79}$$

The same value is given by Oechslin. The equation cam therefore makes a turn in a tropical year. This cam has only been located around the center of this side of the machine for convenience and there is no relationship with the daily Sun/Earth/Moon system.

The cam moves a rack which moves a 30-teeth wheel on the tube 48 carrying the moving inner dial.

The central arbor 40 carries the hour hand and makes a turn in one day, as described above.

(Copyrighted image not shown)

Figure 29.23: The mean and true time (from Hrůšová [4]).

29.3.4.2.6 The Uranus dial

For the dials of Mercury, Jupiter and Uranus, we first compute the velocities of arbors 59 and 62:

$$\mathrm{V}_{59}^0 = \mathrm{V}_{43}^0 \times \left(-\frac{10}{51}\right) \times \left(-\frac{25}{87}\right) = \mathrm{V}_{43}^0 \times \frac{250}{4437} = \frac{1}{2} \times \frac{250}{4437} = \frac{125}{4437} \tag{29.80}$$

$$\mathrm{V}_{62}^0 = \mathrm{V}_{59}^0 \times \left(-\frac{14}{78}\right) \times \left(-\frac{8}{79}\right) = \mathrm{V}_{59}^0 \times \frac{56}{3081} \tag{29.81}$$

$$= \frac{125}{4437} \times \frac{56}{3081} = \frac{7000}{13670397} \tag{29.82}$$

We can then compute the motion of arbor 66 which carries the hand of Uranus:

$$\mathrm{V}_{66}^0 = \mathrm{V}_{62}^0 \times \left(-\frac{16}{109}\right) \times \left(-\frac{31}{53}\right) \times \left(-\frac{29}{39}\right) = \mathrm{V}_{62}^0 \times \left(-\frac{14384}{225303}\right) \tag{29.83}$$

$$= \frac{7000}{13670397} \times \left(-\frac{14384}{225303}\right) = -\frac{3472000}{106206257079} \tag{29.84}$$

$$\mathrm{P}_{66}^0 = -\frac{106206257079}{3472000} = -30589.3597\ldots \text{ days} \approx -83.75 \text{ years} \tag{29.85}$$

The same value is given by Oechslin. This is again an excellent approximation of Uranus' orbital period which is about 30588.74 days.

(Copyrighted image not shown)

Figure 29.24: The Uranus dial (from Hrůšová [4]).

29.3.4.2.7 The Jupiter dial

The hand of Jupiter is carried by the arbor 63 whose velocity is

$$V^0_{63} = V^0_{62} \times \left(-\frac{23}{51}\right) = \frac{7000}{13670397} \times \left(-\frac{23}{51}\right) = -\frac{161000}{697190247} \tag{29.86}$$

$$P^0_{63} = -\frac{697190247}{161000} = -4330.3742\ldots \text{ days} \tag{29.87}$$

The same value is given by Oechslin. This is an excellent approximation of Jupiter's orbital period.

(Copyrighted image not shown)

Figure 29.25: The Jupiter dial (from Hrůšová [4]).

29.3.4.2.8 The Mercury dial

The hand of Mercury is carried by the arbor 60 whose velocity is

$$\mathrm{V}_{60}^{0} = \mathrm{V}_{59}^{0} \times \left(-\frac{23}{57}\right) = \frac{125}{4437} \times \left(-\frac{23}{57}\right) = -\frac{2875}{252909} \tag{29.88}$$

$$\mathrm{P}_{60}^{0} = -\frac{252909}{2875} = -87.9683\ldots \text{ days} \tag{29.89}$$

The same value is given by Oechslin. This is an excellent approximation of Mercury's orbital period.

(Copyrighted image not shown)

Figure 29.26: The Mercury dial (from Hrůšová [4]).

29.4 The upper part of the machine

The upper part of the machine derives its motions from the two arbors 97 and 146. The motion of arbor 97 is derived from that of arbor 3:

$$\mathrm{V}_{97}^{0} = \mathrm{V}_{3}^{0} \times \left(-\frac{12}{48}\right) \times \left(-\frac{12}{48}\right) \times \left(-\frac{12}{36}\right) \tag{29.90}$$

$$= \mathrm{V}_{3}^{0} \times \left(-\frac{1}{48}\right) = 24 \times \left(-\frac{1}{48}\right) = -\frac{1}{2} \tag{29.91}$$

The motion of arbor 146 is also derived from that of arbor 3:

$$\mathrm{V}^0_{146} = \mathrm{V}^0_3 \times \left(-\frac{24}{48}\right) \times \frac{12}{36} = \mathrm{V}^0_3 \times \left(-\frac{1}{6}\right) = 24 \times \left(-\frac{1}{6}\right) = -4 \qquad (29.92)$$

(Copyrighted image not shown)

Figure 29.27: The upper part of the machine (from Hrůšová [4]).

29.4.1 The Sun/Moon annual system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the zodiac. There are four separate trains and they are all driven by the arbor 118. This arbor takes its motion from arbor 97 which makes one turn in two days. We have

$$V^0_{118} = V^0_{97} \times \left(-\frac{30}{106}\right) \times \left(-\frac{28}{96}\right) = V^0_{97} \times \frac{35}{424} \tag{29.93}$$

$$= \left(-\frac{1}{2}\right) \times \frac{35}{424} = -\frac{35}{848} \tag{29.94}$$

The motion of the Sun is that of tube 129. We have

$$V^0_{129} = V^0_{118} \times \left(-\frac{21}{31}\right) \times \left(-\frac{10}{37}\right) \times \left(-\frac{25}{69}\right) \tag{29.95}$$

$$= V^0_{118} \times \left(-\frac{1750}{26381}\right) = \left(-\frac{35}{848}\right) \times \left(-\frac{1750}{26381}\right) \tag{29.96}$$

$$= \frac{30625}{11185544} \tag{29.97}$$

$$P^0_{129} = \frac{11185544}{30625} = 365.2422\ldots \text{ days} \tag{29.98}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment, but not in the Sun/Moon annual system.

The motion of the Moon is that of the central arbor 119. We have

$$V^0_{119} = V^0_{118} \times \left(-\frac{47}{53}\right) = \left(-\frac{35}{848}\right) \times \left(-\frac{47}{53}\right) = \frac{1645}{44944} \tag{29.99}$$

$$P^0_{119} = \frac{44944}{1645} = 27.3215\ldots \text{ days} \tag{29.100}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment, but not in the Sun/Moon annual system.

The motion of the lunar nodes is that of tube 123. We have

$$V^0_{123} = V^0_{118} \times \left(-\frac{19}{44}\right) \times \left(-\frac{8}{50}\right) \times \left(-\frac{9}{51}\right) \times \left(-\frac{19}{65}\right) \tag{29.101}$$

$$= V^0_{118} \times \frac{1083}{303875} \tag{29.102}$$

$$= \left(-\frac{35}{848}\right) \times \frac{1083}{303875} = -\frac{7581}{51537200} \tag{29.103}$$

$$P^0_{123} = -\frac{51537200}{7581} = -6798.2060\ldots \text{ days} \tag{29.104}$$

The same value is given by Oechslin. This value is negative because the lunar nodes are retrograding. A slightly different ratio was used in the metal fragment.

Finally, the motion of the lunar apsides is that of tube 126. We have

$$\mathrm{V}^0_{126} = \mathrm{V}^0_{118} \times \left(-\frac{16}{75}\right) \times \left(-\frac{8}{71}\right) \times \left(-\frac{17}{109}\right) \tag{29.105}$$

$$= \mathrm{V}^0_{118} \times \left(-\frac{2176}{580425}\right) \tag{29.106}$$

$$= \left(-\frac{35}{848}\right) \times \left(-\frac{2176}{580425}\right) = \frac{952}{6152505} \tag{29.107}$$

$$\mathrm{P}^0_{126} = \frac{6152505}{952} = 6462.7153\ldots \text{ days} \tag{29.108}$$

This should be the period of precession of the lunar apsides. Oechslin obtained the same value, but it is of course incorrect.[5] There is obviously a missing factor 2, as the correct value is about 3231.36 days, or about 8.85 years.

[5] See [9, p. 205] on this error.

29.4.2 The Sun/Moon daily system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the Earth. The Sun therefore makes one turn *clockwise* around the Earth in one day. There are four separate trains and they are all driven by the input on arbor 161. The motion of this arbor is in turn derived from that of arbor 146 which makes one turn in 6 hours. We have

$$V^0_{161} = V^0_{146} \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{80}\right) \tag{29.109}$$

$$= V^0_{146} \times \left(-\frac{1}{2}\right) = (-4) \times \left(-\frac{1}{2}\right) = 2 \tag{29.110}$$

Arbor 161 therefore makes one turn counterclockwise (from above) in 12 hours.

For the motion of the Sun on tube 168, we have

$$V^0_{168} = V^0_{161} \times \left(-\frac{30}{60}\right) = -1 \tag{29.111}$$

$$P^0_{168} = -1 \text{ day} \tag{29.112}$$

The same ratio was used in the metal fragment.

The motion of the Moon is given by the central arbor 164:

$$V^0_{164} = V^0_{161} \times \left(-\frac{21}{23}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{42}{74}\right) \tag{29.113}$$

$$= V^0_{161} \times \left(-\frac{2058}{4255}\right) = -\frac{4116}{4255} \tag{29.114}$$

$$P^0_{164} = -\frac{4255}{4116} = -24 \text{ h } 48 \text{ m } 37.7842\ldots \text{ s} \tag{29.115}$$

The same value is given by Oechslin.

This is an approximation of the apparent diurnal motion of the Moon, which is also clockwise. The same ratio was used in the metal fragment.

The motion of the sky is given by tube 171:

$$V^0_{171} = V^0_{161} \times \left(-\frac{27}{34}\right) \times \left(-\frac{40}{42}\right) \times \left(-\frac{59}{89}\right) \tag{29.116}$$

$$= V^0_{161} \times \left(-\frac{5310}{10591}\right) = -\frac{10620}{10591} \tag{29.117}$$

$$P^0_{171} = -\frac{10591}{10620} = -0.9972\ldots \text{ days} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{29.118}$$

The same value is given by Oechslin. This is an approximation of the sidereal day. The same ratio was used in the metal fragment.

Finally, the motion of the lunar nodes is given on tube 167:

$$\mathrm{V}^0_{167} = \mathrm{V}^0_{161} \times \left(-\frac{28}{27}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{43}{83}\right) \tag{29.119}$$

$$= \mathrm{V}^0_{161} \times \left(-\frac{16856}{33615}\right) = -\frac{33712}{33615} \tag{29.120}$$

$$\mathrm{P}^0_{167} = -\frac{33615}{33712} = -0.9971\ldots \text{ days} = -23 \text{ h } 55 \text{ m } 51.4000\ldots \text{ s} \tag{29.121}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment.

29.4.3 The satellites of Jupiter

This system takes its input from arbor 148 whose motion is itself derived from that of arbor 146 which makes one turn in 6 hours. We have

$$\mathrm{V}^0_{148} = \mathrm{V}^0_{146} \times \left(-\frac{45}{83}\right) \times \left(-\frac{27}{89}\right) \tag{29.122}$$

$$= \mathrm{V}^0_{146} \times \frac{1215}{7387} = (-4) \times \frac{1215}{7387} = -\frac{4860}{7387} \tag{29.123}$$

The same ratio was used in the metal fragment.

(Copyrighted image not shown)

Figure 29.28: The satellites of Jupiter (from Hrůšová [4]).

The velocities of the four satellites are:

$$V^0_{149} = V^0_{148} \times \left(-\frac{61}{71}\right) = \left(-\frac{4860}{7387}\right) \times \left(-\frac{61}{71}\right) = \frac{296460}{524477} \tag{29.124}$$

$$P^0_{149} = \frac{524477}{296460} = 1.7691\ldots \text{ days (Io)} \tag{29.125}$$

$$V^0_{152} = V^0_{148} \times \left(-\frac{42}{49}\right) \times \left(-\frac{27}{46}\right) \times \left(-\frac{57}{67}\right) = V^0_{148} \times \left(-\frac{4617}{10787}\right) \tag{29.126}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{4617}{10787}\right) = \frac{22438620}{79683569} \tag{29.127}$$

$$P^0_{152} = \frac{79683569}{22438620} = 3.5511\ldots \text{ days (Europa)} \tag{29.128}$$

$$V^0_{155} = V^0_{148} \times \left(-\frac{44}{53}\right) \times \left(-\frac{22}{37}\right) \times \left(-\frac{34}{79}\right) = V^0_{148} \times \left(-\frac{32912}{154919}\right) \tag{29.129}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{32912}{154919}\right) = \frac{159952320}{1144386653} \tag{29.130}$$

$$P^0_{155} = \frac{1144386653}{159952320} = 7.1545\ldots \text{ days (Ganymede)} \tag{29.131}$$

$$V^0_{158} = V^0_{148} \times \left(-\frac{26}{53}\right) \times \left(-\frac{22}{48}\right) \times \left(-\frac{32}{79}\right) = V^0_{148} \times \left(-\frac{1144}{12561}\right) \tag{29.132}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{1144}{12561}\right) = \frac{1853280}{30929369} \tag{29.133}$$

$$P^0_{158} = \frac{30929369}{1853280} = 16.6889\ldots \text{ days (Callisto)} \tag{29.134}$$

The same values are given by Oechslin. These are good approximations of the actual periods of the satellites. The satellites all move counterclockwise as they should. The exact same trains were used in the metal fragment.

29.4.4 The satellites of Saturn

This system takes its input from arbor 131 whose motion is itself derived from that of arbor 97 which makes one turn in two days. We have

$$\mathrm{V}^0_{131} = \mathrm{V}^0_{97} \times \left(-\frac{72}{58}\right) \times \left(-\frac{76}{72}\right) \tag{29.135}$$

$$= \mathrm{V}^0_{97} \times \frac{38}{29} = \left(-\frac{1}{2}\right) \times \frac{38}{29} = -\frac{19}{29} \tag{29.136}$$

This is the same ratio as the one used in the metal fragment, except that it moves in the opposite direction.

(Copyrighted image not shown)

Figure 29.29: The satellites of Saturn (from Hrůšová [4]).

The velocities of the five satellites are then:

$$V_{132}^0 = V_{131}^0 \times \left(-\frac{38}{47}\right) = \frac{19}{29} \times \left(-\frac{38}{47}\right) = -\frac{722}{1363} \tag{29.137}$$

$$P_{132}^0 = -\frac{1363}{722} = -1.8878\ldots \text{ days (Tethys)} \tag{29.138}$$

$$V_{135}^0 = V_{131}^0 \times \left(-\frac{21}{26}\right) \times \left(-\frac{18}{21}\right) \times \left(-\frac{33}{41}\right) = V_{131}^0 \times \left(-\frac{297}{533}\right) \tag{29.139}$$

$$= \frac{19}{29} \times \left(-\frac{297}{533}\right) = -\frac{5643}{15457} \tag{29.140}$$

$$P_{135}^0 = -\frac{15457}{5643} = -2.7391\ldots \text{ days (Dione)} \tag{29.141}$$

$$V_{138}^0 = V_{131}^0 \times \left(-\frac{38}{62}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{30}{47}\right) = V_{131}^0 \times \left(-\frac{5415}{16027}\right) \tag{29.142}$$

$$= \frac{19}{29} \times \left(-\frac{5415}{16027}\right) = -\frac{102885}{464783} \tag{29.143}$$

$$P_{138}^0 = -\frac{464783}{102885} = -4.5175\ldots \text{ days (Rhea)} \tag{29.144}$$

$$V_{141}^0 = V_{131}^0 \times \left(-\frac{37}{39}\right) \times \left(-\frac{12}{29}\right) \times \left(-\frac{10}{41}\right) = V_{131}^0 \times \left(-\frac{1480}{15457}\right) \tag{29.145}$$

$$= \frac{19}{29} \times \left(-\frac{1480}{15457}\right) = -\frac{28120}{448253} \tag{29.146}$$

$$P_{141}^0 = -\frac{448253}{28120} = -15.9407\ldots \text{ days (Titan)} \tag{29.147}$$

$$V_{144}^0 = V_{131}^0 \times \left(-\frac{13}{30}\right) \times \left(-\frac{8}{46}\right) \times \left(-\frac{12}{47}\right) = V_{131}^0 \times \left(-\frac{104}{5405}\right) \tag{29.148}$$

$$= \frac{19}{29} \times \left(-\frac{104}{5405}\right) = -\frac{1976}{156745} \tag{29.149}$$

$$P_{144}^0 = -\frac{156745}{1976} = -79.3243\ldots \text{ days (Iapetus)} \tag{29.150}$$

The same values are given by Oechslin. We have here the exact same ratios as in the metal fragment, and the satellites move counterclockwise, as they should. The error in the metal fragment, where the satellites would have moved clockwise, has been corrected.

29.4.5 The central orrery

At the center of the upper part of the machine, we have an orrery showing the mean motions of the planets from Mercury to Uranus. This orrery is mounted inside an armillary sphere.

This orrery takes its motion from arbor 98. This motion is itself derived from arbor 97 which also drives the Sun/Moon annual system and the satellites of Saturn.

We have

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{97}^{0} \times \left(-\frac{23}{57}\right) = \left(-\frac{1}{2}\right) \times \left(-\frac{23}{57}\right) = \frac{23}{114} \tag{29.151}$$

We next compute the motion of arbor 99 which is used in several trains:

$$\mathrm{V}_{99}^{0} = \mathrm{V}_{98}^{0} \times \left(-\frac{10}{51}\right) = \frac{23}{114} \times \left(-\frac{10}{51}\right) = -\frac{115}{2907} \tag{29.152}$$

The motion of Mercury is that of arbor 100:

$$\mathrm{V}_{100}^{0} = \mathrm{V}_{99}^{0} \times \left(-\frac{25}{87}\right) = \left(-\frac{115}{2907}\right) \times \left(-\frac{25}{87}\right) = \frac{2875}{252909} \tag{29.153}$$

$$\mathrm{P}_{100}^{0} = \frac{252909}{2875} = 87.9683\ldots \text{ days} \tag{29.154}$$

The same value is given by Oechslin. The exact same ratio was used in the metal fragment.

The motion of Venus is that of tube 101:

$$\mathrm{V}_{101}^{0} = \mathrm{V}_{99}^{0} \times \left(-\frac{9}{80}\right) = \left(-\frac{115}{2907}\right) \times \left(-\frac{9}{80}\right) = \frac{23}{5168} \tag{29.155}$$

$$\mathrm{P}_{101}^{0} = \frac{5168}{23} = 224.6956\ldots \text{ days} \tag{29.156}$$

The same value is given by Oechslin.

This ratio was not used in the metal fragment.

The motion of the Earth is that of tube 110, but we first compute the motion of arbor 109 which is useful for Jupiter and Uranus. We have

$$\mathrm{V}_{109}^{0} = \mathrm{V}_{99}^{0} \times \left(-\frac{21}{51}\right) \times \left(-\frac{13}{33}\right) = \mathrm{V}_{99}^{0} \times \frac{91}{561} \tag{29.157}$$

$$= \left(-\frac{115}{2907}\right) \times \frac{91}{561} = -\frac{10465}{1630827} \tag{29.158}$$

$$\mathrm{V}_{110}^{0} = \mathrm{V}_{109}^{0} \times \left(-\frac{32}{75}\right) = \left(-\frac{10465}{1630827}\right) \times \left(-\frac{32}{75}\right) = \frac{66976}{24462405} \tag{29.159}$$

$$\mathrm{P}_{110}^{0} = \frac{24462405}{66976} = 365.2413\ldots \text{ days} \tag{29.160}$$

This is a somewhat less accurate value of the tropical year, but Oechslin obtained it too.

The motion of Mars is that of tube 104, but we first compute the motion of arbor 103 which is also useful for Saturn. We have

$$V^0_{103} = V^0_{99} \times \left(-\frac{12}{59}\right) \times \left(-\frac{34}{58}\right) = V^0_{99} \times \frac{204}{1711} \tag{29.161}$$

$$= \left(-\frac{115}{2907}\right) \times \frac{204}{1711} = -\frac{460}{97527} \tag{29.162}$$

And then

$$V^0_{104} = V^0_{103} \times \left(-\frac{25}{81}\right) = \left(-\frac{460}{97527}\right) \times \left(-\frac{25}{81}\right) = \frac{11500}{7899687} \tag{29.163}$$

$$P^0_{104} = \frac{7899687}{11500} = 686.9293\ldots \text{ days} \tag{29.164}$$

The same value is given by Oechslin.

The motion of Jupiter is that of tube 113, but we first compute the motion of arbor 112 which is also useful for Uranus. We have

$$V^0_{112} = V^0_{109} \times \left(-\frac{12}{61}\right) \times \left(-\frac{17}{54}\right) = V^0_{109} \times \frac{34}{549} \tag{29.165}$$

$$= \left(-\frac{10465}{1630827}\right) \times \frac{34}{549} = -\frac{20930}{52666119} \tag{29.166}$$

$$V^0_{113} = V^0_{112} \times \left(-\frac{43}{74}\right) = \left(-\frac{20930}{52666119}\right) \times \left(-\frac{43}{74}\right) = \frac{449995}{1948646403} \tag{29.167}$$

$$P^0_{113} = \frac{1948646403}{449995} = 4330.3734\ldots \text{ days} \tag{29.168}$$

The same value is given by Oechslin.

The motion of Saturn is that of tube 107. We have

$$V^0_{107} = V^0_{103} \times \left(-\frac{17}{59}\right) \times \left(-\frac{14}{56}\right) \times \left(-\frac{23}{84}\right) = V^0_{103} \times \left(-\frac{391}{19824}\right) \tag{29.169}$$

$$= \left(-\frac{460}{97527}\right) \times \left(-\frac{391}{19824}\right) = \frac{44965}{483343812} \tag{29.170}$$

$$P^0_{107} = \frac{483343812}{44965} = 10749.3341\ldots \text{ days} \tag{29.171}$$

The same value is given by Oechslin.

Finally, the motion of Uranus is that of tube 116. We have

$$V^0_{116} = V^0_{112} \times \left(-\frac{17}{41}\right) \times \left(-\frac{19}{30}\right) \times \left(-\frac{26}{83}\right) = V^0_{112} \times \left(-\frac{4199}{51045}\right) \tag{29.172}$$

$$= \left(-\frac{20930}{52666119}\right) \times \left(-\frac{4199}{51045}\right) = \frac{54418}{1664608077} \tag{29.173}$$

$$P^0_{116} = \frac{1664608077}{54418} = 30589.2917\ldots \text{ days} \approx 83.75 \text{ years} \tag{29.174}$$

The same value is given by Oechslin.

All these motions are counterclockwise as seen from above.

29.5 References

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