Chapter 2

(Oechslin: 11.1)

Adams' orrery in Munich (c1748-1757)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

Although Adams is not one of the *Priestermechaniker*, the analysis of Adams' orrery is included here because it was part of Oechslin's original work. It also sheds some light in the inspiration of Neßtfell's orreries.

2.1 Introduction

This chapter describes a grand orrery (figure 2.1) made by George Adams (1709-1772).

George Adams was an instrument maker based in Fleet Street, London [4]. After his death, his business was taken over by his son George Adams Jr. (1750-1795). Both George Adams father and son made a number of grand orreries, which are horizontal orreries on twelve-sided bases, with a half-hemisphere of colures and parallels. Among these orreries, we can mention those of the British Museum, of the Whipple museum, of the history of science museum in Geneva, and of the Vanderbilt Mansion National Historic Site.

Other grand orreries are the one extended by Thomas Wright in 1733 and exhibited in the Science Museum in London (see figures 2.4 and 2.5), Thomas Wright's orrery at Dunham Massey Hall, Benjamin Cole's grand orrery made c1758 or earlier, and exhibited in Dumfries House, and Joseph Pope's grand orrery (1776-1787) exhibited in Harvard.

The grand orrery kept in Munich was made by George Adams Sr. in London between 1748 and 1757.[1] It was acquired around 1758 by Charles

[1] On Adams's orreries, see Millburn [4]. King also has some pages on Adams [3, p. 204-205]. This orrery was also shown at the 1989 Hahn exhibition [10, p. 61-63].

Figure 2.1: General view of Adams' orrery (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`).

Figure 2.2: General view of Adams' orrery. (photography by the author)

Figure 2.3: The dial showing the time and the name of the constructor in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

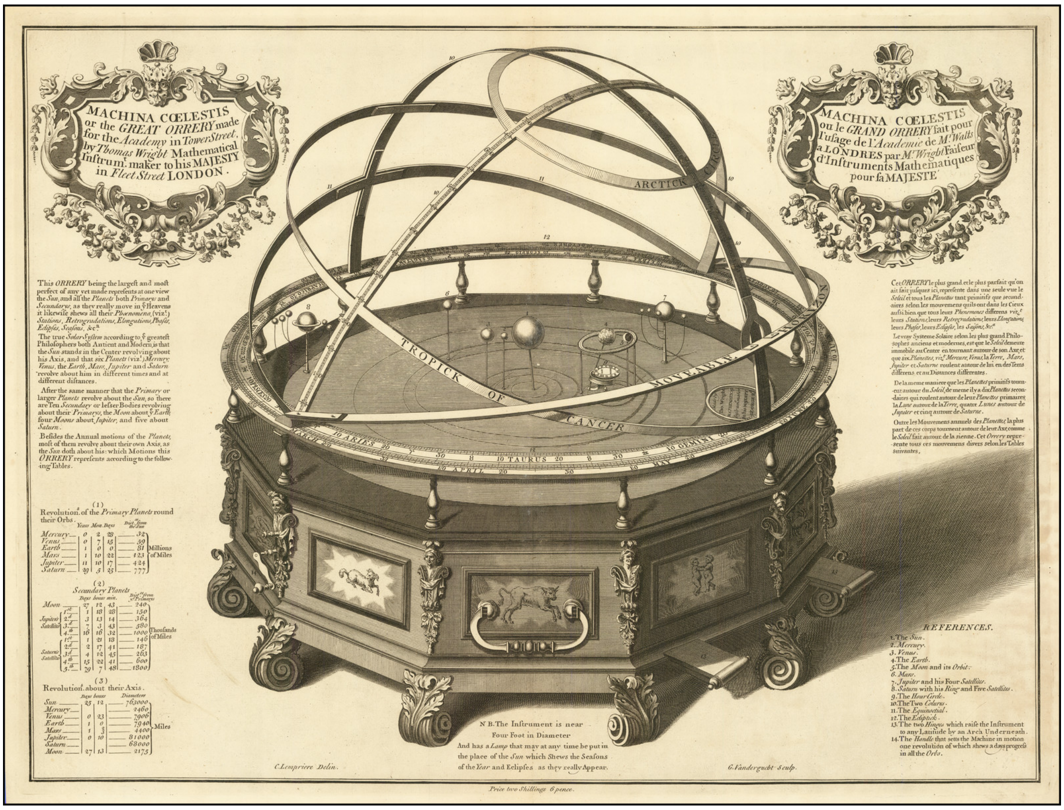

Figure 2.4: A woodcut of Thomas Wright's grand orrery.

Figure 2.5: Joseph Wright of Derby's painting "A Philosopher Lecturing on the Orrery" (1766). (source: Wikipedia)

Theodore (1724-1799), elector of Bavaria, for the court library of his palace in Mannheim. The piece came to Munich in about 1802/03 and is today kept in the *Bayerisches Nationalmuseum*. It appeared for the first time in the museum's guide in 1868.[2]

Although this is not an astronomical clock or mechanism made by a Priest-mechanic, Oechslin included it in its survey [5] as a comparison with the clocks and machines of Hahn and others.[3] Of course, Oechslin also happened to have worked on several clocks in the *Bayerisches Nationalmuseum*, and so he had access to Adams's orrery.[4]

[2]Cf. [8, p. 440].

[3]See [5, p. 34] for Oechslin's description.

[4]Adams' orrery was part of the exhibition of Neßtfell's restored Munich machine in 1988-1989 [9, p. 16].

Figure 2.6: The planets on Adams' orrery. Around the Sun in the center, we can see Mercury (behind the Sun), Venus (to the right), then the Earth (in white) with the Moon, then Mars (towards us), then Jupiter (at the top) and Saturn (at the bottom), with their satellites. (excerpt from the video https://www.youtube.com/watch?v=OL3G-lVG7Mw)

Figure 2.7: The large driving wheels of Adams' orrery, when the top cover is removed. Saturn is visible on the left. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

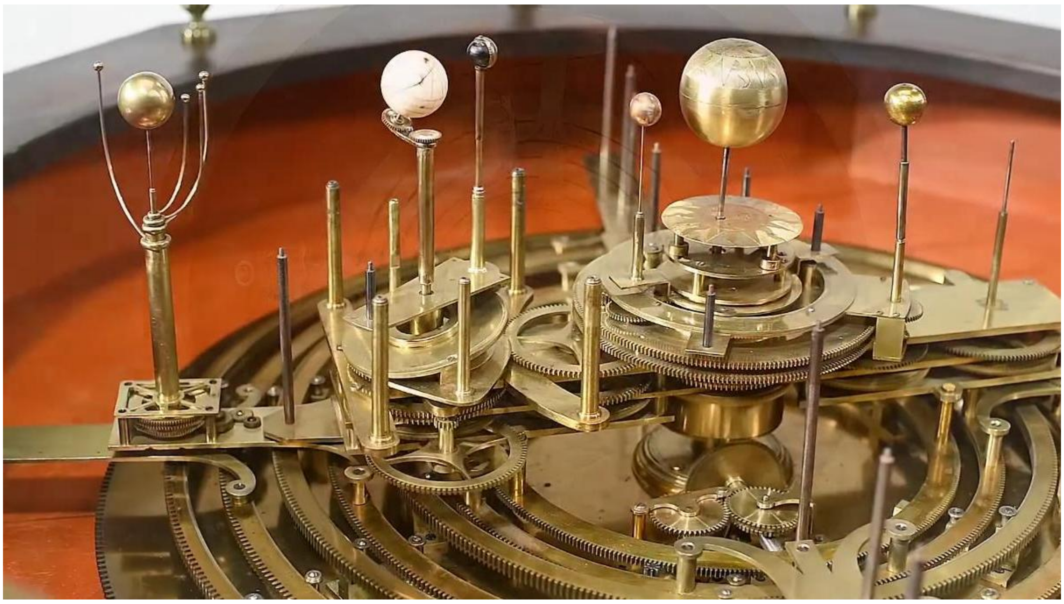

Figure 2.8: From left to right, the gears for Jupiter, the Earth/Moon system, Mercury and Venus in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

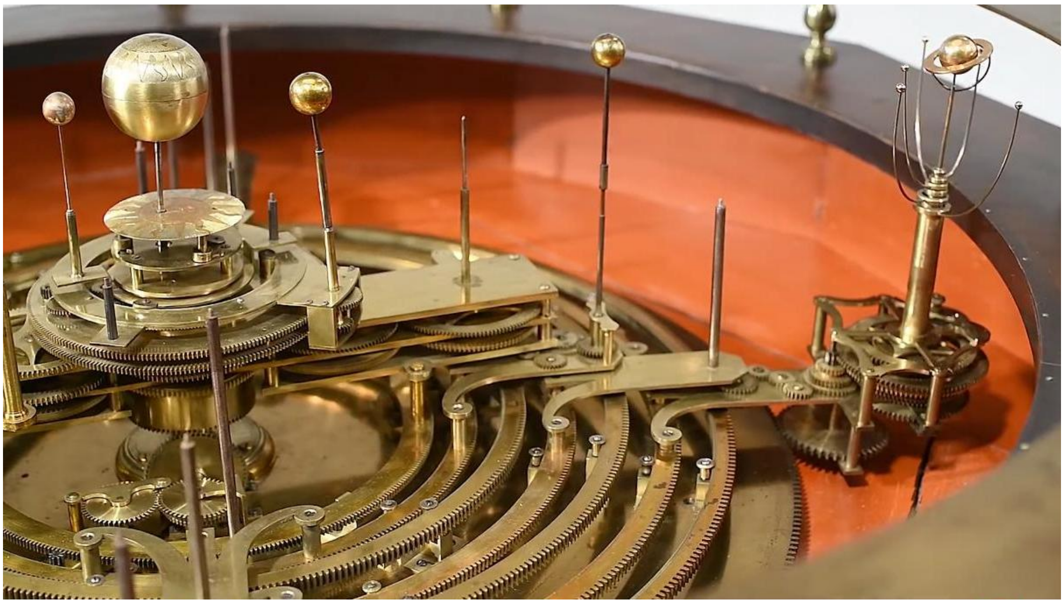

Figure 2.9: From left to right, the gears for Mercury, Venus, Mars and Saturn in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

Figure 2.10: Detail of the gears of Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

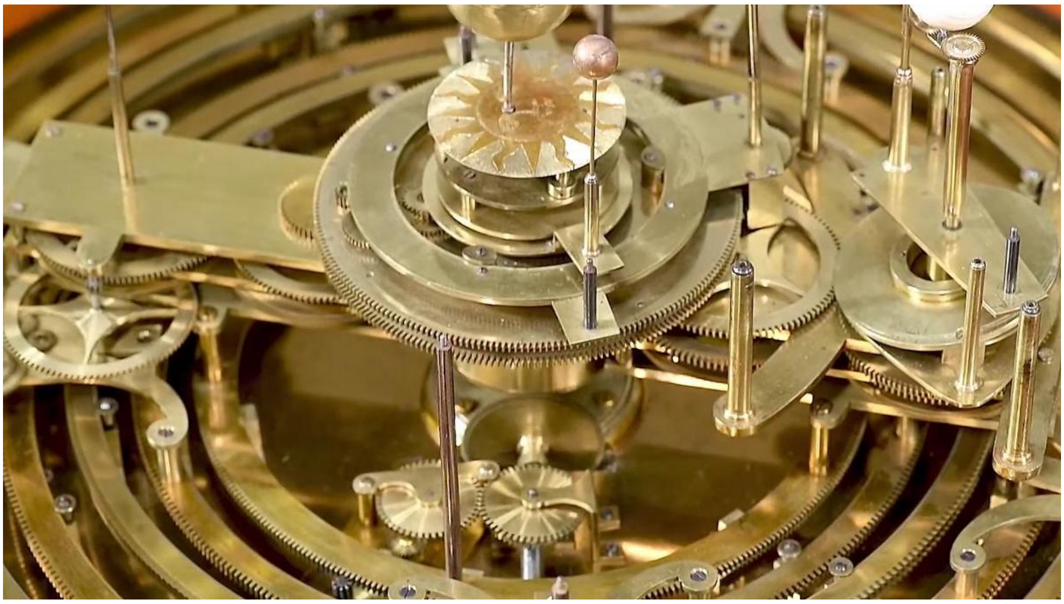

Figure 2.11: Detail of the gears of Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

2.2 The base motion

All the gears in Adams' orrery are driven by an arbor originating from the clockwork. However, it isn't immediately clear what motion this arbor has or should have. In order to get a better understanding of the gear ratios, we can in fact start with the dial set on the Earth platform, namely that of the hours on arbor 70 which is rotating on the moving frame 15. This arbor is supposed to make one turn in 24 hours on this frame. That is, we can start with

$$V^{15}_{70} = 1 \tag{2.1}$$

This arbor rotates counterclockwise.

If we start instead with the clockwork, we will have to make assumptions on the clockwork or the transmission to the orrery, and these assumptions will be entirely artificial and will only find their justifications later.

Now, we have

$$V^{15}_{70} = V^{0}_{70} + V^{15}_{0} = V^{0}_{70} - V^{0}_{15} \tag{2.2}$$

And then

$$\begin{aligned} V^{0}_{15} &= V^{0}_{14} \times \left(-\frac{8}{366}\right) && (2.3)\\ &= V^{0}_{9} \times \frac{18}{48} \times \left(-\frac{48}{48}\right) \times \left(-\frac{20}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{80}\right) \times \left(-\frac{8}{366}\right) && (2.4)\\ &= -V^{0}_{9} \times \frac{1}{488} = -V^{0}_{7} \times \left(-\frac{20}{48}\right) \times \left(-\frac{48}{15}\right) \times \frac{1}{488} = -V^{0}_{7} \times \frac{1}{366} && (2.5) \end{aligned}$$

The 20-teeth wheel on arbor 7 meshes with an intermediate 48-teeth wheel which itself meshes with a 15-teeth wheel. On its arbor 9, there is a wheel with 60 teeth meshing with a wheel with 80 teeth on arbor/tube 56. This tube is the central mobile vertical tube of the orrery, rotating around the fixed vertical arbor 76. The resulting motion of tube 56 is obtained as follows:

$$V^{0}_{56} = V^{0}_{7} \times \left(-\frac{20}{48}\right) \times \left(-\frac{48}{15}\right) \times \left(-\frac{60}{80}\right) = -V^{0}_{7} \tag{2.6}$$

Then we have:

$$\begin{aligned} V^{15}_{70} &= V^{15}_{56} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) = V^{15}_{56} && (2.7)\\ V^{0}_{70} &= V^{15}_{70} + V^{0}_{15} = V^{15}_{56} + V^{0}_{15} = V^{0}_{56} = -V^{0}_{7} && (2.8) \end{aligned}$$

And now

$$V^{15}_{70} = -V^{0}_{7} + V^{0}_{7} \times \frac{1}{366} = -V^{0}_{7} \times \left(1 - \frac{1}{366}\right) = -V^{0}_{7} \times \frac{365}{366} \tag{2.9}$$

We obtain

$$\mathrm{V}_{70}^{15} = -\mathrm{V}_{7}^{0} \times \frac{365}{366} = 1 \tag{2.10}$$

and therefore

$$\mathrm{V}_{7}^{0} = -\frac{366}{365} \tag{2.11}$$

This is the velocity of the sidereal day. We conclude that arbor 7 must make one turn clockwise (from the right) in one sidereal day.

This derivation may seem contrived, but starting with the clockwork will be worse, because the period of the pendulum will have to be given without justification, or one will have to posit that arbor 7 has the velocity of a sidereal day, when in fact this is in no way obvious.

2.3 The period of the pendulum

Now that we know that arbor 7 must make one turn in a sidereal day, we can go backwards until the escapement. We find that the escape wheel E must make one turn in

$$T_E = \frac{365}{366} \times 86400 \times \frac{20}{40} \times \frac{10}{40} \times \frac{12}{72} \times \frac{8}{64} \times \frac{8}{60} \tag{2.12}$$

$$= \frac{1825}{61} = 29.9180\ldots \text{ s} \tag{2.13}$$

The pendulum (if there is a pendulum) makes a half-oscillation in $\frac{T_E}{2\times 30}$, that is a little less than 0.5 seconds, and is about 25 cm long.

2.4 The tropical year

The motion of tube 56 is transferred inside the mobile cage supporting the Earth-Moon system, the inner planets and the Sun. This cage rests on tube 15 which carries at its base a 366-teeth wheel. As we have already seen above, the motion of this tube is also obtained from the arbor 9 mentioned above. We first compute the velocities of arbor 14 and tube 15, which have not been fully explicited above. Arbor 14 will in particular later be used for the motion of Mars.

$$\mathrm{V}_{14}^{0} = \mathrm{V}_{9}^{0} \times \frac{18}{48} \times \left(-\frac{48}{48}\right) \times \left(-\frac{20}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{80}\right) \tag{2.14}$$

$$= \mathrm{V}_{9}^{0} \times \frac{3}{32} = \mathrm{V}_{7}^{0} \times \frac{4}{3} \times \frac{3}{32} = \left(-\frac{366}{365}\right) \times \frac{4}{3} \times \frac{3}{32} = -\frac{183}{1460} \tag{2.15}$$

$$\mathrm{V}_{15}^{0} = \mathrm{V}_{14}^{0} \times \left(-\frac{8}{366}\right) = \left(-\frac{183}{1460}\right) \times \left(-\frac{8}{366}\right) = \frac{1}{365} \tag{2.16}$$

In other words, the rotation of the cage 15 takes place in exactly 365 days. And since

$$\mathrm{V}_{15}^{0} = -\mathrm{V}_{7}^{0} \times \frac{1}{366} \tag{2.17}$$

it also takes place in 366 sidereal days. Or, put differently, Adams takes the ratio of the tropical year to the sidereal day to be exactly 366 which is an approximation, as the actual ratio is about 366.24. The motion of the cage is counterclockwise seen from above.

2.5 The motions inside cage 15

The motions inside cage 15 are entirely determined by the relative motion of tube 56 with respect to the cage:

$$\mathrm{V}_{56}^{15} = \mathrm{V}_{56}^{0} + \mathrm{V}_{0}^{15} = -\mathrm{V}_{7}^{0} - \mathrm{V}_{15}^{0} = \frac{366}{365} - \frac{1}{365} = 1 \tag{2.18}$$

With respect to frame 15, the tube 56 makes exactly one rotation counterclockwise in one day.

2.5.1 Tubes 70 and 72

And so does the arbor 70:

$$\mathrm{V}_{70}^{15} = \mathrm{V}_{56}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) = 1 \tag{2.19}$$

With respect to frame 15, the arbor 72 makes one rotation counterclockwise in 60 days:

$$\mathrm{V}_{72}^{15} = \mathrm{V}_{70}^{15} \times \left(-\frac{24}{96}\right) \times \left(-\frac{8}{120}\right) = \frac{1}{60} \tag{2.20}$$

2.5.2 The Earth and the Moon

The orientation of Earth's axis is determined by tube 67. We can compute the velocity of this tube in frame 15 and in the absolute frame 0:

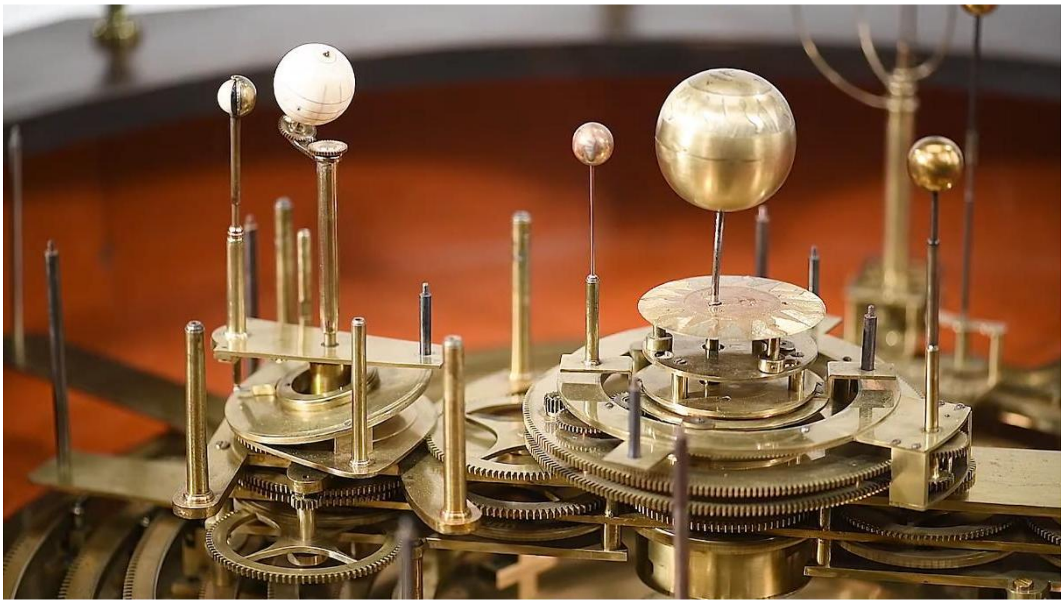

Figure 2.12: The Earth/Moon system and the tilted ramp for the latitude of the Moon in Adams' orrery. (excerpt from the video https://www.youtube.com/watch?v=OL3G-lVG7Mw)

$$V_{67}^{15} = V_{25}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) \tag{2.21}$$

The arbor 25 is the central arbor of the orrery and is fixed in the absolute frame. Its relative velocity in frame 15 is:

$$V_{25}^{15} = -V_{15}^{25} = -V_{15}^{0} = -\frac{1}{365} \tag{2.22}$$

Hence

$$V_{67}^{15} = -\frac{1}{365} \tag{2.23}$$

And

$$V_{67}^{0} = V_{67}^{15} + V_{15}^{0} = 0 \tag{2.24}$$

This is what we should have expected, namely that the Earth's axis has a fixed orientation in the absolute frame.

The rotation of the Earth in frame 15 is also easy to compute. The Earth is on arbor 60. We first compute the velocity of arbor 60 in the frame of tube 67:

$$\mathrm{V}_{60}^{67} = \mathrm{V}_{58}^{67} \times \left(-\frac{40}{10}\right) \times \left(-\frac{10}{40}\right) = \mathrm{V}_{58}^{67} \tag{2.25}$$

Then

$$\mathrm{V}_{58}^{67} = \mathrm{V}_{58}^{15} + \mathrm{V}_{15}^{67} = \mathrm{V}_{58}^{15} + \frac{1}{365} = \mathrm{V}_{56}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) + \frac{1}{365} \tag{2.26}$$

$$= 1 + \frac{1}{365} \tag{2.27}$$

$$\mathrm{V}_{60}^{67} = \mathrm{V}_{60}^{0} = 1 + \frac{1}{365} = \frac{366}{365} \tag{2.28}$$

This is the velocity of the sidereal day.

Then, the average velocity of the Earth's axis in frame 15:

$$\mathrm{V}_{60}^{15} = \mathrm{V}_{60}^{67} + \mathrm{V}_{67}^{15} = \frac{366}{365} - \frac{1}{365} = 1 \tag{2.29}$$

The Earth rotates on average in one day in frame 15. This is only an average, because the axis is wobbling in frame 15.

The direction of the Moon is determined by tube 62. Its velocity in frame 15 is

$$\mathrm{V}_{62}^{15} = \mathrm{V}_{58}^{15} \times \left(-\frac{24}{96}\right) \times \left(-\frac{16}{118}\right) = \mathrm{V}_{58}^{15} \times \frac{2}{59} = \frac{2}{59} \tag{2.30}$$

Hence, the synodic revolution of the Moon is $\frac{59}{2} = 29.5$ days. Oechslin obtains the same value.

The tropical revolution of the Moon is obtained from

$$\mathrm{V}_{62}^{0} = \mathrm{V}_{62}^{15} + \mathrm{V}_{15}^{0} = \frac{2}{59} + \frac{1}{365} = \frac{789}{21535} \tag{2.31}$$

$$\mathrm{P}_{62}^{0} = \frac{21535}{789} = 27.2940\ldots \text{ days} \tag{2.32}$$

$$= 27.3688\ldots \text{ sidereal days} \tag{2.33}$$

Oechslin gives the same two values.

The phase of the Moon is determined by tube 65. We expect the motion of tube 65 to be still in frame 15, so that the lit part of the Moon is always in the same direction in that frame. So, we compute

$$\mathrm{V}_{65}^{15} = \mathrm{V}_{65}^{62} + \mathrm{V}_{62}^{15} = \mathrm{V}_{65}^{62} + \frac{2}{59} \tag{2.34}$$

$$= \mathrm{V}_{63}^{62} \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) + \frac{2}{59} \tag{2.35}$$

$$= \mathrm{V}_{63}^{62} + \frac{2}{59} = \mathrm{V}_{15}^{62} + \frac{2}{59} = -\frac{2}{59} + \frac{2}{59} = 0 \tag{2.36}$$

The latitude of the Moon is determined by a tilted plane on tube 68. The velocity of this tube is computed as follows:

$$\mathrm{V}_{68}^{0} = \mathrm{V}_{68}^{15} + \mathrm{V}_{15}^{0} = \mathrm{V}_{25}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{100}{95}\right) + \mathrm{V}_{15}^{0} \tag{2.37}$$

$$= \mathrm{V}_{25}^{15} \times \frac{20}{19} + \frac{1}{365} \tag{2.38}$$

$$= \left(-\frac{1}{365}\right) \times \frac{20}{19} + \frac{1}{365} = \left(-\frac{1}{365}\right) \times \frac{1}{19} = -\frac{1}{6935} \tag{2.39}$$

This gives a draconic period of 6935 days, the actual value being about 6798.383 days. This value is negative, because there is a precession of the lunar nodes. Oechslin didn't give this value.

The velocity of tube 68 in frame 15 is

$$\mathrm{V}_{68}^{15} = \left(-\frac{1}{365}\right) \times \frac{20}{19} = -\frac{4}{1387} \tag{2.40}$$

and the associated period is

$$\mathrm{P}_{68}^{15} = -\frac{1387}{4} = -346.75 \text{ days} \tag{2.41}$$

which is the eclipse year. This period was also omitted by Oechslin.

2.6 The Sun, Mercury and Venus

The Sun's axis is offset from the center of the orbit of the Earth. Venus and Mercury's orbits are also offset. The centers of these two orbits, as well as the Sun's axis are located on the fixed plate 25. These offsets approximate the elliptic orbits of the planets. Arbor 25 actually represents the center of Earth's orbit.

All the motions on plate 25 are obtained from wheel/tube 76 located under that plate. This tube 76 gets its motion from wheels located on frame 15. It has following velocity:

$$
\begin{aligned}
V_{76}^{15} &= V_{72}^{15} \times \left(-\frac{120}{100}\right) \times \left(-\frac{50}{36}\right) \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{73}\right) && (2.42)\\
&= V_{72}^{15} \times \frac{60}{73} = \frac{1}{60} \times \frac{60}{73} = \frac{1}{73} && (2.43)\\
V_{76}^{0} &= V_{76}^{15} + V_{15}^{0} = \frac{1}{73} + \frac{1}{365} = \frac{6}{365} && (2.44)
\end{aligned}
$$

The rotation of the Sun is computed as follows:

$$
\begin{aligned}
V_{85}^{0} &= V_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{58}{59}\right) = \frac{58}{1475} && (2.45)\\
P_{85}^{0} &= \frac{1475}{58} = 25.4310\ldots \text{ days} && (2.46)\\
&= 25.5007\ldots \text{ sidereal days} && (2.47)
\end{aligned}
$$

Oechslin gave the same two periods.

The actual rotation period of the Sun is 25.67 days at the equator, but it increases at higher latitudes.

Mercury is attached to an 88-teeth wheel 84 moving on the fixed plate 25. The velocity of Mercury is

$$
\begin{aligned}
V_{84}^{0} &= V_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{25}{88}\right) = V_{76}^{0} \times \frac{365}{528} = \frac{6}{365} \times \frac{365}{528} = \frac{1}{88} && (2.48)\\
P_{84}^{0} &= 88 \text{ days} && (2.49)\\
&= 88.2410\ldots \text{ sidereal days} && (2.50)
\end{aligned}
$$

Oechslin gave the same two periods.

Venus has both a motion of revolution around the Sun (or rather around a point near the Sun) and a motion of rotation around its axis. The revolution of Venus is that of wheel 80. It is obtained in a similar way as that of Mercury:

$$
\begin{aligned}
V_{80}^{0} &= V_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{25}{48}\right) \times \left(-\frac{48}{20}\right) \times \left(-\frac{20}{225}\right) && (2.51)\\
&= V_{76}^{0} \times \frac{73}{270} = \frac{6}{365} \times \frac{73}{270} = \frac{1}{225} && (2.52)\\
P_{80}^{0} &= 225 \text{ days} && (2.53)\\
&= 225.6164\ldots \text{ sidereal days} && (2.54)
\end{aligned}
$$

Oechslin gave the same two periods.

Venus is rotating around its axis 81.

The rotation about its axis is obtained as follows:

$$\begin{aligned}
V_{81}^{0} &= V_{81}^{80} + V_{80}^{0} && (2.55)\\
&= V_{25}^{80} \times \left(-\frac{122}{24}\right) + \frac{1}{225} && (2.56)\\
&= V_{0}^{80} \times \left(-\frac{61}{12}\right) + \frac{1}{225} && (2.57)\\
&= \left(-\frac{1}{225}\right) \times \left(-\frac{61}{12}\right) + \frac{1}{225} && (2.58)\\
&= \frac{73}{2700} && (2.59)\\
P_{81}^{0} &= \frac{2700}{73} = 36.9863\ldots \text{ days} && (2.60)\\
&= 37.0876\ldots \text{ sidereal days} && (2.61)
\end{aligned}$$

Oechslin gave the same value, but only in sidereal days.

It is not clear how Adams came to this value,[5] because at that time there were two conflicting values for the rotation of Venus: Cassini's 1666 estimation of about 23 hours[6] and Bianchini's estimation of about 24 days.[7]

There is also a tube 83 around the axis 81, that Oechslin claims to be unused. The velocity of this tube is the following:

$$\begin{aligned}
V_{83}^{0} &= V_{83}^{80} + V_{80}^{0} && (2.62)\\
&= V_{25}^{80} \times \left(-\frac{225}{25}\right) \times \left(-\frac{6}{54}\right) + V_{80}^{0} && (2.63)\\
&= V_{0}^{80} + V_{80}^{0} = 0 && (2.64)
\end{aligned}$$

We have $V_{25}^{80} = V_{0}^{80}$ because 25 is still.

The tube 83 is therefore not rotating in the absolute frame.

2.7 The motions of Mars

Mars has a motion of revolution and rotation. The revolution of Mars around the Sun is determined by wheel 18 which has 360 teeth. Its motion is obtained

[5]See Oechslin's brief mention of Venus in this context [5, p. 209].

[6]Cf. *Journal des sçavans*, 12 december 1667.

[7]cf. *Hesperi et Phosphori nova phaenomena sive observationes circa planetam Veneris* (Rome, 1728), p. 58.

similarly to that of the Earth frame 15. We have

$$V_{17}^{0} = V_{14}^{0} \times \left(-\frac{8}{18}\right) \times \left(-\frac{18}{16}\right) = V_{14}^{0} \times \frac{1}{2} = -\frac{183}{2920} \tag{2.65}$$

$$V_{18}^{0} = V_{17}^{0} \times \left(-\frac{8}{360}\right) = \frac{61}{43800} \tag{2.66}$$

$$P_{18}^{0} = \frac{43800}{61} = 718.0327\ldots \text{ days} \tag{2.67}$$

$$= 720 \text{ sidereal days} \tag{2.68}$$

Oechslin gave the same two periods.[8]

This value seems very inaccurate, the correct value being 687 days. Something must be wrong here. Perhaps the number of teeth of wheel 18 is not 360? The value 345 would for instance have been better.

The rotation of Mars' axis 55 is obtained from a wheel 51 rotating around the Sun, around the same excentric point as wheel 18 for Mars' revolution. The velocity of this wheel 51 is

$$V_{51}^{0} = V_{9}^{0} \times \left(-\frac{32}{360}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{32}{360}\right) = \frac{1952}{16425} \tag{2.69}$$

This wheel 51 then meshes with a wheel whose axis is on the Mars rotating frame 18:

$$V_{55}^{18} = V_{51}^{18} \times \left(-\frac{360}{16}\right) \times \left(-\frac{18}{27}\right) \times \left(-\frac{27}{48}\right) \times \left(-\frac{35}{36}\right) \tag{2.70}$$

$$= V_{51}^{18} \times \frac{525}{64} \tag{2.71}$$

and since

$$V_{51}^{18} = V_{51}^{0} + V_{0}^{18} = \frac{1952}{16425} - \frac{61}{43800} = \frac{15433}{131400} \tag{2.72}$$

therefore

$$V_{55}^{18} = \frac{15433}{131400} \times \frac{525}{64} = \frac{108031}{112128} \tag{2.73}$$

$$V_{55}^{0} = V_{55}^{18} + V_{18}^{0} = \frac{108031}{112128} + \frac{61}{43800} = \frac{2704679}{2803200} \tag{2.74}$$

$$P_{55}^{0} = \frac{2803200}{2704679} = 1.0364 \text{ days} \tag{2.75}$$

$$= 1.0392\ldots \text{ sidereal days} \tag{2.76}$$

Oechslin gave the same period, but only in sidereal days.

The actual period of the tropical rotation of Mars is 1.0259 days.

[8] See Oechslin's brief mention [5, p. 209].

2.8 The motions of Jupiter and its satellites

Jupiter has a motion of revolution and rotation, and it also has four satellites. The revolution of Jupiter around the Sun is determined by wheel 21 which has 450 teeth. Its motion is obtained similarly to that of the Earth frame 15.

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{17}^{0} \times \left(-\frac{16}{24}\right) \times \left(-\frac{24}{77}\right) = \mathrm{V}_{17}^{0} \times \frac{16}{77} = \left(-\frac{183}{2920}\right) \times \frac{16}{77} \tag{2.77}$$

$$= -\frac{366}{28105} \tag{2.78}$$

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{20}^{0} \times \left(-\frac{8}{450}\right) = \left(-\frac{366}{28105}\right) \times \left(-\frac{8}{450}\right) = \frac{488}{2107875} \tag{2.79}$$

$$\mathrm{P}_{21}^{0} = \frac{2107875}{488} = 4319.4159\ldots \text{ days} \tag{2.80}$$

$$= 4331.25 \text{ sidereal days} \tag{2.81}$$

Oechslin gave the same two values of the period.

The actual orbital period is about 4332.6 days.

The rotation of Jupiter and of its satellites is obtained from the rotation of a 450-teeth wheel 40 driven from arbor 9:

$$\mathrm{V}_{40}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{34}{450}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{34}{450}\right) = \frac{8296}{82125} \tag{2.82}$$

The rotation of Jupiter's axis 46 is obtained as follows:

$$\mathrm{V}_{43}^{21} = \mathrm{V}_{40}^{21} \times \left(-\frac{450}{22}\right) \times \left(-\frac{24}{36}\right) \times \left(-\frac{36}{48}\right) = \mathrm{V}_{40}^{21} \times \left(-\frac{225}{22}\right) \tag{2.83}$$

$$\mathrm{V}_{46}^{21} = \mathrm{V}_{43}^{21} \times \left(-\frac{48}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{20}\right) = \mathrm{V}_{43}^{21} \times \left(-\frac{12}{5}\right) \tag{2.84}$$

$$= \mathrm{V}_{40}^{21} \times \left(-\frac{225}{22}\right) \times \left(-\frac{12}{5}\right) = \mathrm{V}_{40}^{21} \times \frac{270}{11} \tag{2.85}$$

Then

$$\mathrm{V}_{40}^{21} = \mathrm{V}_{40}^{0} + \mathrm{V}_{0}^{21} = \frac{8296}{82125} - \frac{488}{2107875} = \frac{637328}{6323625} \tag{2.86}$$

Hence

$$\mathrm{V}_{46}^{21} = \frac{637328}{6323625} \times \frac{270}{11} = \frac{3823968}{1545775} \tag{2.87}$$

$$\mathrm{V}_{46}^{0} = \mathrm{V}_{46}^{21} + \mathrm{V}_{21}^{0} = \frac{3823968}{1545775} + \frac{488}{2107875} = \frac{8194984}{3312375} \tag{2.88}$$

$$\mathrm{P}_{46}^{0} = \frac{3312375}{8194984} = 0.4041\ldots \text{ days} = 9.7006\ldots \text{ hours} \tag{2.89}$$

Oechslin made a mistake and gave an entirely different period of 2.025756 sidereal days.

The actual rotation period of Jupiter is about 9.92 hours.

The motion of the four satellites is obtained from four wheels on arbor 43:

$$V_{47}^{21} = V_{43}^{21} \times \left(-\frac{48}{84}\right) \text{ (Io)} \tag{2.90}$$

$$V_{48}^{21} = V_{43}^{21} \times \left(-\frac{24}{86}\right) \text{ (Europe)} \tag{2.91}$$

$$V_{49}^{21} = V_{43}^{21} \times \left(-\frac{12}{86}\right) \text{ (Ganymede)} \tag{2.92}$$

$$V_{50}^{21} = V_{43}^{21} \times \left(-\frac{6}{100}\right) \text{ (Callisto)} \tag{2.93}$$

We have seen above that

$$V_{43}^{21} = V_{40}^{21} \times \left(-\frac{225}{22}\right) = \frac{637328}{6323625} \times \left(-\frac{225}{22}\right) = -\frac{318664}{309155} \tag{2.94}$$

We can now compute the tropical revolutions of the satellites:

$$V_{47}^{0} = V_{47}^{21} + V_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{48}{84}\right) + \frac{488}{2107875} \tag{2.95}$$

$$= \frac{95636776}{162306375} \tag{2.96}$$

$$P_{47}^{0} = \frac{162306375}{95636776} = 1.6971\ldots \text{ days} \tag{2.97}$$

$$= 1.7017\ldots \text{ sidereal days} \tag{2.98}$$

$$V_{48}^{0} = V_{48}^{21} + V_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{24}{86}\right) + \frac{488}{2107875} \tag{2.99}$$

$$= \frac{287028424}{997024875} \tag{2.100}$$

$$P_{48}^{0} = \frac{997024875}{287028424} = 3.4736\ldots \text{ days} \tag{2.101}$$

$$= 3.4831\ldots \text{ sidereal days} \tag{2.102}$$

$$V_{49}^{0} = V_{49}^{21} + V_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{12}{86}\right) + \frac{488}{2107875} \tag{2.103}$$

$$= \frac{143629624}{997024875} \tag{2.104}$$

$$P_{49}^{0} = \frac{997024875}{143629624} = 6.9416\ldots \text{ days} \tag{2.105}$$

$$= 6.9606\ldots \text{ sidereal days} \tag{2.106}$$

$$\begin{aligned}
\mathrm{V}_{50}^{0} &= \mathrm{V}_{50}^{21} + \mathrm{V}_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{6}{100}\right) + \frac{488}{2107875} && (2.107)\\
&= \frac{1439356}{23186625} && (2.108)\\
\mathrm{P}_{50}^{0} &= \frac{23186625}{1439356} = 16.1090\ldots \text{ days} && (2.109)\\
&= 16.1531\ldots \text{ sidereal days} && (2.110)
\end{aligned}$$

Oechslin gave the same values of the periods.

The actual orbital periods of the four first satellites are 1.76 days, 3.53 days, 7.16 days and 16.69 days.

2.9 The motions of Saturn and its satellites

Saturn has a motion of revolution and rotation, and it also has five satellites. The revolution of Saturn around the Sun is determined by wheel 24 which has 450 teeth. Its motion is obtained similarly to that of the Earth frame 15.

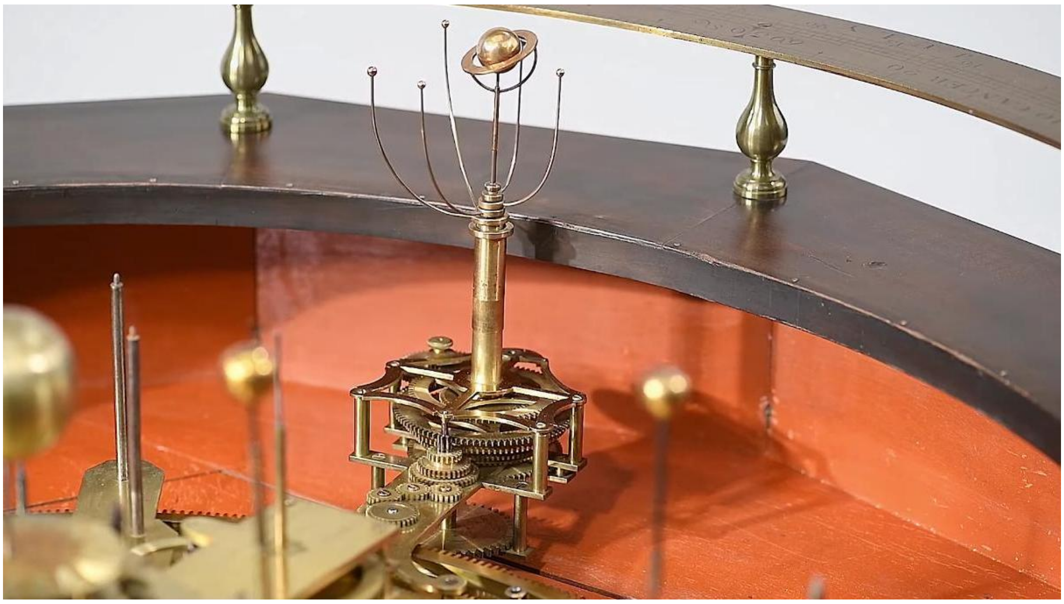

Figure 2.13: Detail of the gears for Saturn in Adams' orrery. The largest wheel at the bottom is the 450-teeth wheel which is used for the rotation of Saturn. This wheel is fixed, and the wheels on arbor 8 and 9 and going through an opening of this wheel. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

$$\begin{aligned}
\mathrm{V}_{24}^{0} &= \mathrm{V}_{20}^{0} \times \left(-\frac{31}{18}\right) \times \left(-\frac{18}{77}\right) \times \left(-\frac{8}{450}\right) && (2.111)\\
&= \mathrm{V}_{20}^{0} \times \left(-\frac{124}{17325}\right) = \left(-\frac{366}{28105}\right) \times \left(-\frac{124}{17325}\right) && (2.112)\\
&= \frac{15128}{162306375} && (2.113)\\
\mathrm{P}_{24}^{0} &= \frac{162306375}{15128} = 10728.8719\ldots \text{ days} && (2.114)\\
&= 10758.2661\ldots \text{ sidereal days} && (2.115)
\end{aligned}$$

Oechslin gave the same two values of the period.

The actual orbital period is about 10755.7 days.

The rotation of Saturn on its axis 27 is obtained using a fixed 450-teeth wheel and the intermediate arbor 26 located on frame 24. We first compute

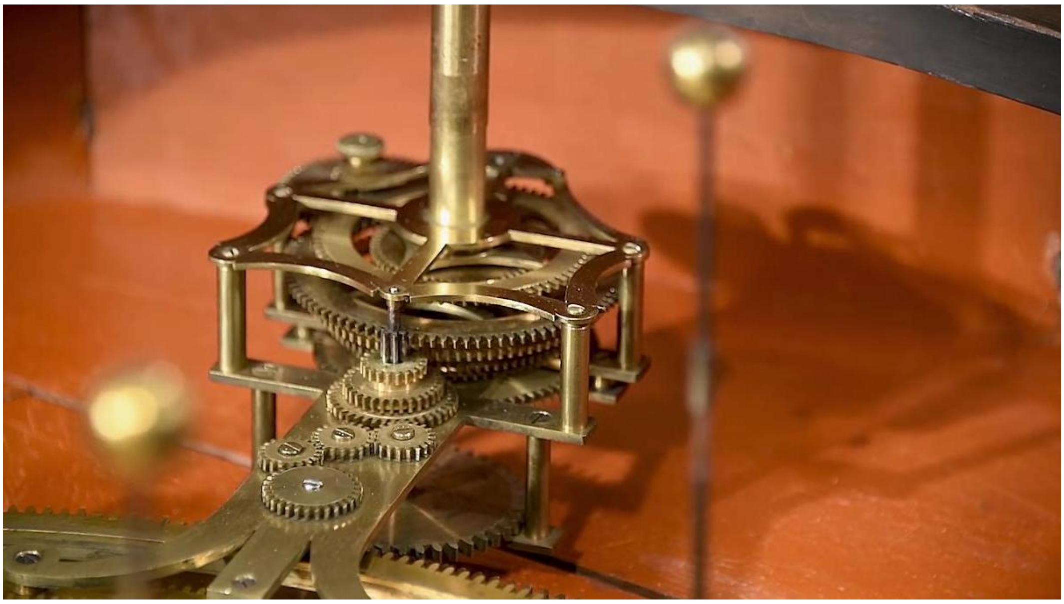

Figure 2.14: Detail of the gears for Saturn in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=0L3G-lVG7Mw`)

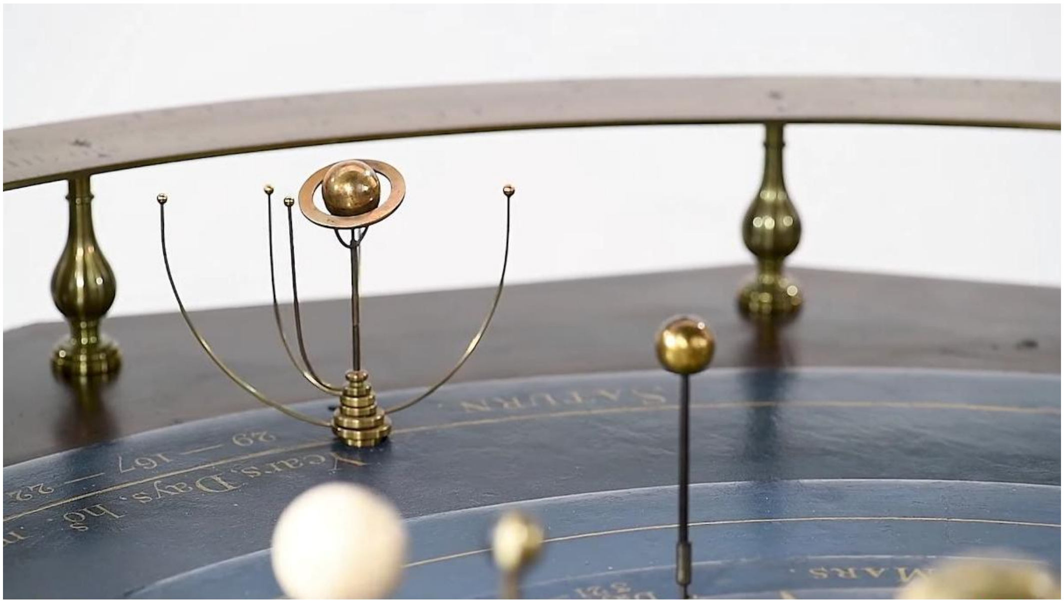

Figure 2.15: Saturn and its satellites when the top cover of Adams' orrery conceals the gears. (excerpt from the video `https://www.youtube.com/watch?v=0L3G-lVG7Mw`)

the velocity of Saturn's axis with respect to wheel 24:

$$\mathrm{V}_{27}^{24} = \mathrm{V}_{0}^{24} \times \left(-\frac{450}{45}\right) \times \left(-\frac{8}{89}\right) = \mathrm{V}_{0}^{24} \times \frac{80}{89} \tag{2.116}$$

But we have found:

$$\mathrm{V}_{0}^{24} = -\frac{15128}{162306375} \tag{2.117}$$

Hence

$$\mathrm{V}_{27}^{24} = \left(-\frac{15128}{162306375}\right) \times \frac{80}{89} = -\frac{242048}{2889053475} \tag{2.118}$$

$$\mathrm{V}_{27}^{0} = \mathrm{V}_{27}^{24} + \mathrm{V}_{24}^{0} = -\frac{242048}{2889053475} + \frac{15128}{162306375} \tag{2.119}$$

$$= \frac{45384}{4815089125} \tag{2.120}$$

$$\mathrm{P}_{27}^{0} = \frac{4815089125}{45384} = 106096.6227\ldots \text{ days} \tag{2.121}$$

$$= 106387.2983\ldots \text{ sidereal days} \tag{2.122}$$

Oechslin gave the same period, but only in sidereal days.

The value produced by Adams' orrery is entirely incorrect, because Saturn rotates around its axis in about 10 hours, and this was already known by Huygens. It is however not clear where Adams erred and it would be interesting to compare the gears of this grand orrery with other orreries made by Adams. Perhaps some other orreries could help understand Adams' error.

The rotation of Saturn's satellites is obtained from another 450-teeth wheel 28 driven from arbor 9, as in the case of Jupiter:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{15}{450}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{15}{450}\right) = \frac{244}{5475} \tag{2.123}$$

$$\mathrm{V}_{33}^{24} = \mathrm{V}_{28}^{24} \times \left(-\frac{450}{15}\right) \times \left(-\frac{30}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{40}\right) \tag{2.124}$$

$$= \mathrm{V}_{28}^{24} \times \left(-\frac{45}{2}\right) = (\mathrm{V}_{28}^{0} + \mathrm{V}_{0}^{24}) \times \left(-\frac{45}{2}\right) \tag{2.125}$$

$$= \left(\frac{244}{5475} - \frac{15128}{162306375}\right) \times \left(-\frac{45}{2}\right) \tag{2.126}$$

$$= \frac{2406084}{54102125} \times \left(-\frac{45}{2}\right) = -\frac{10827378}{10820425} \tag{2.127}$$

The motion of the five satellites is obtained from five wheels on frame 24:

$$V_{34}^{24} = V_{33}^{24} \times \left(-\frac{40}{75}\right) = \frac{28873008}{54102125} \text{ (Tethys)} \tag{2.128}$$

$$V_{35}^{24} = V_{33}^{24} \times \left(-\frac{32}{88}\right) = \frac{43309512}{119024675} \text{ (Dione)} \tag{2.129}$$

$$V_{36}^{24} = V_{33}^{24} \times \left(-\frac{20}{90}\right) = \frac{2406084}{10820425} \text{ (Rhea)} \tag{2.130}$$

$$V_{37}^{24} = V_{33}^{24} \times \left(-\frac{6}{96}\right) = \frac{5413689}{86563400} \text{ (Titan)} \tag{2.131}$$

$$V_{39}^{24} = V_{33}^{24} \times \left(-\frac{6}{96}\right) \times \left(-\frac{58}{59}\right) \times \left(-\frac{16}{78}\right) = V_{33}^{24} \times \left(-\frac{29}{2301}\right) \tag{2.132}$$

$$= \frac{104664654}{8299265975} \text{ (Iapetus)} \tag{2.133}$$

And in the absolute frame:

$$V_{34}^{0} = V_{34}^{24} + V_{24}^{0} = \frac{28873008}{54102125} + \frac{15128}{162306375} = \frac{7875832}{14755125} \tag{2.134}$$

$$P_{34}^{0} = \frac{14755125}{7875832} = 1.8734\ldots \text{ days (Tethys)} \tag{2.135}$$

$$= 1.8786\ldots \text{ sidereal days} \tag{2.136}$$

$$V_{35}^{0} = V_{35}^{24} + V_{24}^{0} = \frac{43309512}{119024675} + \frac{15128}{162306375} = \frac{649809088}{1785370125} \tag{2.137}$$

$$P_{35}^{0} = \frac{1785370125}{649809088} = 2.7475\ldots \text{ days (Dione)} \tag{2.138}$$

$$= 2.7550\ldots \text{ sidereal days} \tag{2.139}$$

$$V_{36}^{0} = V_{36}^{24} + V_{24}^{0} = \frac{2406084}{10820425} + \frac{15128}{162306375} = \frac{36106388}{162306375} \tag{2.140}$$

$$P_{36}^{0} = \frac{162306375}{36106388} = 4.4952\ldots \text{ days (Rhea)} \tag{2.141}$$

$$= 4.5075\ldots \text{ sidereal days} \tag{2.142}$$

$$V_{37}^{0} = V_{37}^{24} + V_{24}^{0} = \frac{5413689}{86563400} + \frac{15128}{162306375} = \frac{81326359}{1298451000} \tag{2.143}$$

$$P_{37}^{0} = \frac{1298451000}{81326359} = 15.9659\ldots \text{ days (Titan)} \tag{2.144}$$

$$= 16.0096\ldots \text{ sidereal days} \tag{2.145}$$

$$V_{39}^{0} = V_{39}^{24} + V_{24}^{0} = \frac{104664654}{8299265975} + \frac{15128}{162306375} = \frac{225938998}{17784141375} \tag{2.146}$$

$$P_{39}^{0} = \frac{17784141375}{225938998} = 78.7121\ldots \text{ days (Iapetus)} \tag{2.147}$$

$$= 78.9277\ldots \text{ sidereal days} \tag{2.148}$$

Oechslin has the same periods, except for Dione, where his values are slightly different from mine.

The actual periods are 1.9 days (Tethys), 2.7 days (Dione), 4.5 days (Rhea), 16 days (Titan) and 79 days (Iapetus).

2.10 Conclusion

Using velocities, we have found the same periods as Oechslin, except for the rotation of Jupiter and the orbital period of Dione, where Oechslin made mistakes. Sometimes Oechslin also gives the periods only expressed in sidereal days.

A huge discrepancy was also identified in the rotation of Saturn, and perhaps the comparison with other grand orreries by Adams could help understand its origin.

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