# Chapter 17 (Oechslin: 8.7)

# Hahn's globe clock in Zurich (1785)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

## 17.1 Introduction

The clock described here was constructed in 1785 by Philipp Matthäus Hahn (1739-1790)[1] and was located in Zurich.[2] It is one of a number of double-globe clocks made by Hahn. This clock has a simple base with a large dial showing the hours on a 24-hours scale, the minutes on on inner dial, and the day of the week and of the month on another inner dial.

Two globes are mounted on top of the base. The globe on the left shows the Earth and its position with respect to the Sun. The globe on the right shows the celestial sphere and is itself surmounted by a small lunar globe.

## 17.2 The going work

The clock is driven by a spring and regulated by a pendulum. The barrel wheel on arbor 1 makes one turn clockwise (viewed from the front) in 56 hours:

$$\mathrm{T}_1^0 = \frac{24}{56} = \frac{3}{7} \tag{17.1}$$

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [3, p. 416-417]. I believe that this clock was auctioned by Koller in 2008, as the description of the auction perfectly fits the descriptions by Oechslin, both in 1989 and 1996 [2, p. 51]. This clock was originally located in the Geras monastery in Austria.



See at the beginning the rights associated to this document.

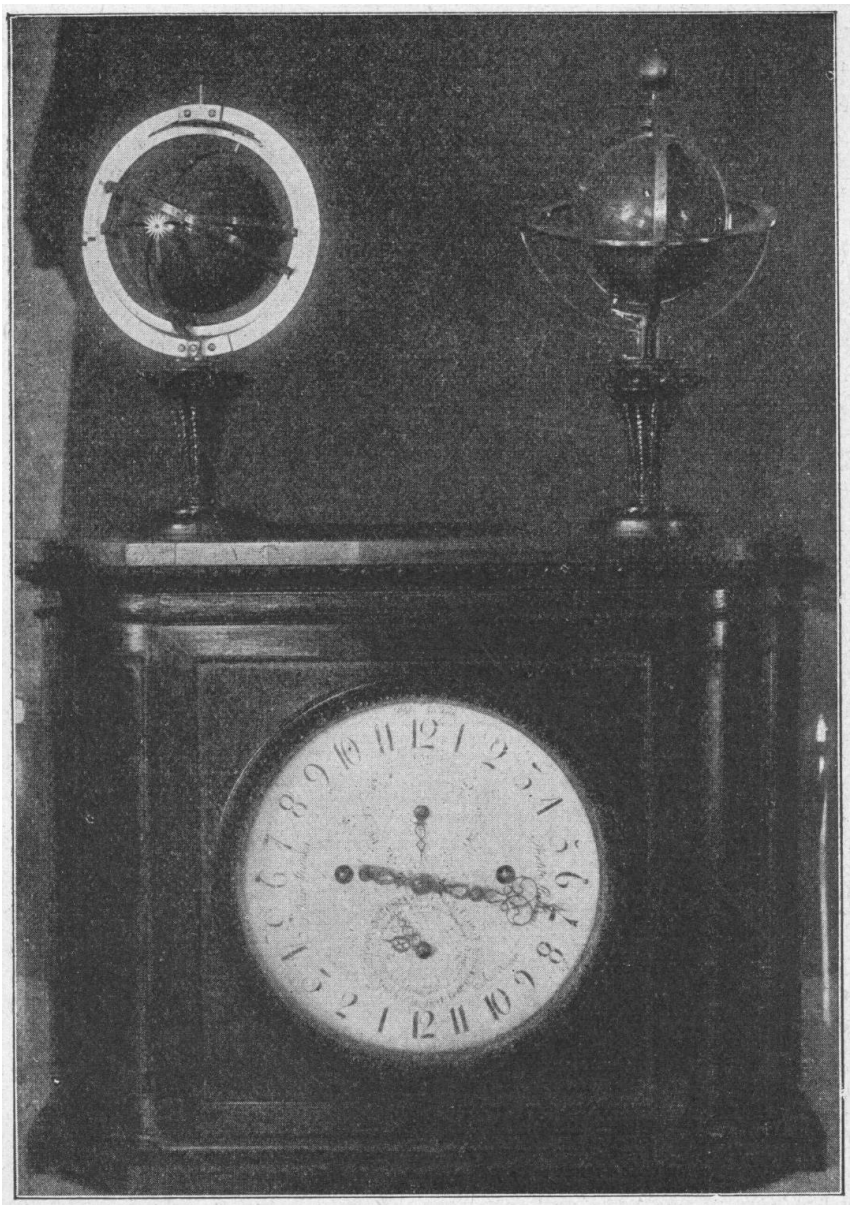

Figure 17.1: Hahn's clock from the Geras monastery in Austria. It is very similar (or identical?) to the clock described here. (source: [1])



See at the beginning the rights associated to this document.

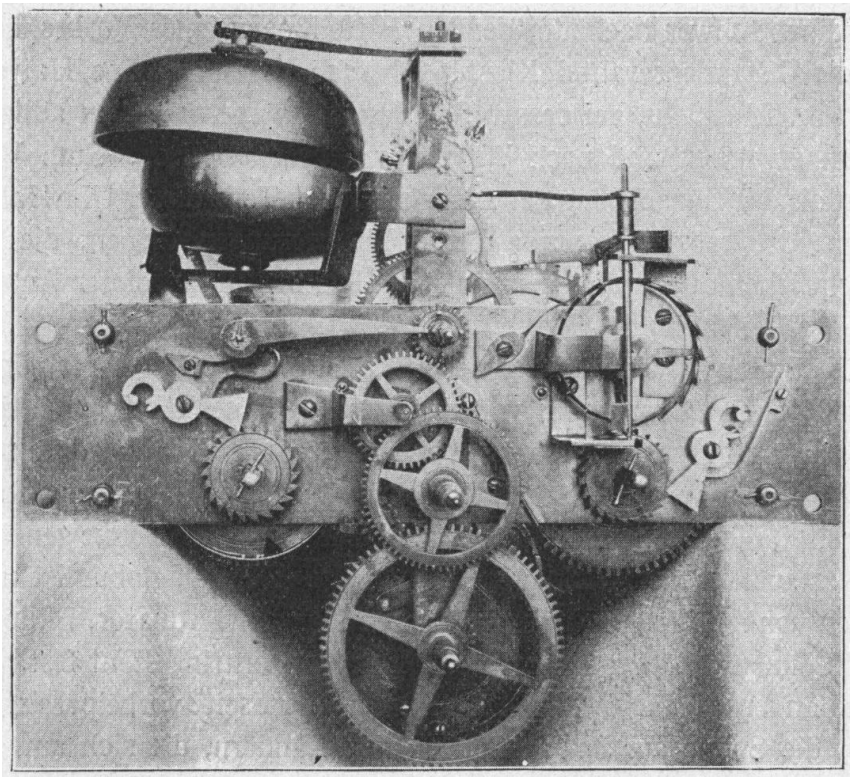

Figure 17.2: The mechanism of Hahn's clock from the Geras monastery in Austria. (source: [1])



See at the beginning the rights associated to this document.

Figure 17.3: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)



See at the beginning the rights associated to this document.

Figure 17.4: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)



See at the beginning the rights associated to this document.

Figure 17.5: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)



See at the beginning the rights associated to this document.

Figure 17.6: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)



See at the beginning the rights associated to this document.

Figure 17.7: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)



See at the beginning the rights associated to this document.

This leads to arbor 3 which carries the wheel of the minute hand:

$$\mathrm{T}_3^0 = \mathrm{T}_1^0 \times \left(-\frac{70}{10}\right) \times \left(-\frac{72}{9}\right) = \mathrm{T}_1^0 \times 56 = 24 \tag{17.2}$$

Tube 7 carries the hour hand:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{24}{48}\right) \times \left(-\frac{6}{72}\right) = \mathrm{T}_3^0 \times \frac{1}{24} = 1 \tag{17.3}$$

This tube makes one turn in one day. It is coupled with the central arbor 8 which also has the same period.

$$\mathrm{T}_8^0 = \mathrm{T}_7^0 = 1 \tag{17.4}$$

This is used to obtain the input motions of the two globes.

## 17.3 The Earth globe

The Earth globe shows the motion of the Earth with respect to the Sun. The Sun is at a fixed position, the Earth rotates around its axis in 24 hours, and its axis also rotates around the vertical axis. This revolution must be clockwise as seen from above.

The input of the Earth globe is the vertical arbor 17. Its velocity (from above) is

$$\mathrm{V}_{17}^0 = \mathrm{V}_8^0 \times \frac{57}{57} \times \left(-\frac{57}{57}\right) = -\mathrm{V}_8^0 = \mathrm{T}_8^0 = 1 \tag{17.5}$$

(the motion of arbor 8 is still viewed from the front in the above)

The arbor 17 makes one turn counterclockwise in one day as seen from above.

This arbor then moves the arbor 20 which is tilted and the actual axis of the Earth. The motion of arbor 20 is on average the same as that of arbor 17. This is the apparent motion of the Earth with respect to the Sun, resulting in a day of 24 hours.

But at the North pole, arbor 20 carries a pinion which is part of a train ending with the fixed 103-teeth wheel on the frame carrying the day-night boundary. The motion of arbor 17 causes a rotation of arbor 20 around the vertical axis. In order to understand this motion, we can examine it in the reference frame of the Earth's meridian, which I call $M$, but is also named 18 by Oechslin. This meridian is the circle going through the vertical axis and the axis of the Earth. Since both arbors 17 and 22 are at fixed locations on $M$, we have

$$\mathrm{V}_{22}^M = \mathrm{V}_{20}^M \times \left(-\frac{3}{64}\right) \times \left(-\frac{6}{103}\right) = \mathrm{V}_{20}^M \times \frac{9}{3296} \tag{17.6}$$

$$\mathrm{V}_{17}^M = \mathrm{V}_{20}^M \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{36}\right) = \mathrm{V}_{20}^M \tag{17.7}$$



See at the beginning the rights associated to this document.

Hence

$$\mathrm{V}_{22}^{M} = \mathrm{V}_{17}^{M} \times \frac{9}{3296} \tag{17.8}$$

$$\mathrm{V}_{M}^{22} = \mathrm{V}_{M}^{0} = \mathrm{V}_{M}^{17} \times \frac{9}{3296} \tag{17.9}$$

The ratio $\frac{9}{3296}$ also appears in the Winterthur tellurium (Oechslin 8.11).

Now, we have

$$\mathrm{V}_{M}^{17} = \mathrm{V}_{M}^{0} - \mathrm{V}_{17}^{0} \tag{17.10}$$

and therefore

$$\mathrm{V}_{M}^{0} = \left(\mathrm{V}_{M}^{0} - \mathrm{V}_{17}^{0}\right) \times \frac{9}{3296} \tag{17.11}$$

$$\mathrm{V}_{M}^{0} \times \left(1 - \frac{9}{3296}\right) = -\mathrm{V}_{17}^{0} \times \frac{9}{3296} \tag{17.12}$$

$$\mathrm{V}_{M}^{0} \times \frac{3287}{3296} = -\mathrm{V}_{17}^{0} \times \frac{9}{3296} \tag{17.13}$$

and thus

$$\mathrm{V}_{M}^{0} = -\mathrm{V}_{17}^{0} \times \frac{9}{3287} = -\frac{9}{3287} \tag{17.14}$$

$$\mathrm{P}_{M}^{0} = -\frac{3287}{9} = -365.2222\ldots \text{ days} \tag{17.15}$$

The meridian circle $M$ makes one turn in one year. The same value is given by Oechslin. The motion is clockwise (from above), as expected. It is however a rather bad approximation of the tropical year.

Incidentally, it is also the same value as the one used in the Winterthur tellurium.

## 17.4 The celestial globe

The celestial globe as a structure very similar to that of the Earth globe. It shows the motion of the sky, and there are two hands rotating around the globe, one for the Sun and one for the Moon. Moreover, a small lunar globe is fixed on top of the celestial globe.

### 17.4.1 The general motion of the globe

The input of the celestial globe is the vertical arbor 25. Its velocity (from above) is

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{8}^{0} \times \left(-\frac{57}{58}\right) \times \frac{70}{66} \times \left(-\frac{76}{79}\right) = \mathrm{V}_{8}^{0} \times \frac{25270}{25201} = -\frac{25270}{25201} \tag{17.16}$$

This is the familiar velocity used by Hahn for the sidereal day. Thus, arbor 25 makes one turn clockwise (seen from above) in one sidereal day.

This arbor represents the axis of the Earth and the entire structure of the globe rotates with arbor 25 like the apparent sky.



See at the beginning the rights associated to this document.

### 17.4.2 The motion of the Moon

A gear train at the bottom of the globe is used to move the hand of the Moon. This motion is obtain using a fixed 53-teeth wheel on the tube 26 supporting the meridian. Arbor 25 goes through this tube. We can compute the motion of the Moon (tube 28) with respect to the globe (frame 25):

$$\mathrm{V}_{28}^{25}? = \mathrm{V}_{26}^{25} \times \left(-\frac{53}{66}\right) \times \left(-\frac{4}{66}\right) = \mathrm{V}_{26}^{25} \times \frac{53}{1089} = -\mathrm{V}_{25}^{26} \times \frac{53}{1089} \tag{17.17}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{53}{1089} = \frac{25270}{25201} \times \frac{53}{1089} = \frac{1339310}{27443889} \tag{17.18}$$

$$\mathrm{P}_{28}^{25}? = \frac{27443889}{1339310} = 20.4910\ldots \text{ days} \tag{17.19}$$

This is obviously wrong. Oechslin observed that the 4-leaves/4-pin pinion should actually have had only 3 pins. In that case, we have

$$\mathrm{V}_{28}^{25} = \mathrm{V}_{26}^{25} \times \left(-\frac{53}{66}\right) \times \left(-\frac{3}{66}\right) = \mathrm{V}_{26}^{25} \times \frac{53}{1452} = -\mathrm{V}_{25}^{26} \times \frac{53}{1452} \tag{17.20}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{53}{1452} = \frac{25270}{25201} \times \frac{53}{1452} = \frac{669655}{18295926} \tag{17.21}$$

$$\mathrm{P}_{28}^{25} = \frac{18295926}{669655} = 27.3214\ldots \text{ days} \tag{17.22}$$

This is now a good approximation of the tropical month, the motion of the Moon with respect to the stars. This ratio was also used in the double-globe clock in Karlsruhe (Oechslin 8.8), also from 1785.

Incidentally, this value is also given by Oechslin, in sidereal days, and then in days, hours, minutes, and seconds, but with an incorrect conversion (27.425075) to its decimal representation. The same error appears in the Karlsruhe double globe clock (Oechslin 8.8), but the conversion was correctly made for the double globe clock in Villingen (Oechslin 8.9).

### 17.4.3 The motion of the Sun

Similarly, a gear train at the top of the globe is used to move the hand of the Sun. This motion is obtain using a fixed 3-teeth/3-pin wheel on the meridian frame. We can compute the motion of the Sun (tube 30) with respect to the globe (frame 25):

$$\mathrm{V}_{30}^{25} = \mathrm{V}_{26}^{25} \times \left(-\frac{3}{64}\right) \times \left(-\frac{6}{103}\right) = \mathrm{V}_{26}^{25} \times \frac{9}{3296} = -\mathrm{V}_{25}^{26} \times \frac{9}{3296} \tag{17.23}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{9}{3296} = \frac{25270}{25201} \times \frac{9}{3296} = \frac{113715}{41531248} \tag{17.24}$$

$$\mathrm{P}_{30}^{25} = \frac{41531248}{113715} = 365.2222\ldots \text{ days} \tag{17.25}$$

This is an approximation of the tropical year and the Sun does indeed move counterclockwise with respect to the stars. The same value was given by



See at the beginning the rights associated to this document.

Oechslin. Interestingly, though, this value is close but not identical to the one used in the Earth globe.

### 17.4.4 The motion of the lunar globe

Finally, we consider the motion of the small lunar globe on top of the celestial sphere. This motion is obtained through a 12-teeth pinion located at the top of the meridian ring, but which moves with the entire globe frame. This pinion thus makes one turn clockwise in one sidereal day:

$$\mathrm{V}^0_{25} = -\frac{25270}{25201} \tag{17.26}$$

The lunar globe is on arbor 32 and its velocity is

$$\mathrm{V}^0_{32} = \mathrm{V}^0_{25} \times \left(-\frac{12}{26}\right) \times \left(-\frac{3}{41}\right) = \mathrm{V}^0_{25} \times \frac{18}{533} \tag{17.27}$$

$$= -\frac{25270}{25201} \times \frac{18}{533} = -\frac{454860}{13432133} \tag{17.28}$$

$$\mathrm{P}^0_{32} = -\frac{13432133}{454860} = -29.5302\ldots \text{ days} \tag{17.29}$$

The lunar globe makes one turn clockwise (from above) in one synodic month. Half of the Moon is hidden by a hemisphere and thus the phases of the Moon are shown.

Oechslin only gave the synodic month in sidereal days, but without clearly saying so.

It should be observed that the ratio $\frac{18}{533}$ and this synodic month value were also used in the globe clocks of Stuttgart (Oechslin 8.5, 1770) and Darmstadt (Oechslin 8.6, 1785).

## 17.5 References

[1] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.



See at the beginning the rights associated to this document.