Chapter 10

(Oechslin: 8.10)

Hahn's astronomical clock in Basel (1775)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

10.1 Introduction

The clock described here was constructed in 1775 by Philipp Matthäus Hahn (1739-1790)[1] and has been acquired by the historical museum of Basel in 1913. This clock was commissioned in 1775 by the Basel tradesman Wilhelm Brenner and remained in his family until 1913.[2] The cost of this clock was 200 gulden.[3]

It is a longcase pendulum-clock with six dials. The central dial shows the minutes, the seconds, and the minutes corrected by the equation of time.

Four dials symmetrically surround the central dial and a sixth dial is positionned at the top. The lower left dial shows the hours, the lower right dial shows the day of the week and of the month, the upper right dial shows the motion of the Sun, the date in the year, and the length of the day. The upper left dial shows the motion and phase of the Moon, the sign of the zodiac, as well as the length of moonlight. Finally, the dial at the top is a planispheric representation of the sky.

An almost identical clock was made in 1774/1775 for the physician Diethelm Lavater (1743-1826) and is kept in the Museum of Music Automatons in Seewen (Switzerland), as a loan from the Swiss National Museum (*Landesmuseum Zürich*).[4]

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [5, p. 442-443]. See also Engelmann [1, fig. 31, 33], Reinhardt [4, p. 27] and Gutmann [2, p. 6].

[3] [3, p. 224]

[4] Cf. [6] and [2, p. 19] and especially the 1989 exhibition catalogue [5, p. 440-442]. This

Figure 10.1: Upper part of the dial of the Basel clock. (source: Wikimedia, Historisches Museum Basel)

10.2 The going work and the indication of time

The clock is driven by a weight of 23.171 kg[5] and the winding drum on arbor 1 makes one turn in 560 hours, that is in 23 days and 8 hours. Its motion is counterclockwise as seen from the front. The clock could work for one year without being wound.

We have

$$\mathrm{T}_1^0 = -\frac{24}{560} = -\frac{3}{70} \tag{10.1}$$

We then derive the motions of arbors 3 and 4:

$$\mathrm{T}_3^0 = \mathrm{T}_1^0 \times \left(-\frac{84}{12}\right) \times \left(-\frac{96}{12}\right) = -\frac{12}{5} \tag{10.2}$$

$$\mathrm{T}_4^0 = \mathrm{T}_3^0 \times \left(-\frac{120}{12}\right) = 24 \tag{10.3}$$

Arbor 4 makes 24 turns in a day, hence one turn clockwise in one hour.

This arbor drives the minute hand on the central dial of the clock.

The escape wheel is on arbor 6 and its velocity is

$$\mathrm{T}_6^0 = \mathrm{T}_4^0 \times \left(-\frac{64}{8}\right) \times \left(-\frac{60}{8}\right) = \mathrm{T}_4^0 \times 60 = 1440 \tag{10.4}$$

clock was exhibited in the Lavater house in Zurich in September 2025 as part of the exhibition "*Das Ticken der Uhren im Zeitalter der Aufklärung — Johann Caspar Lavater und Philipp Matthäus Hahn*."

[5] Cf. [2, p. 32].

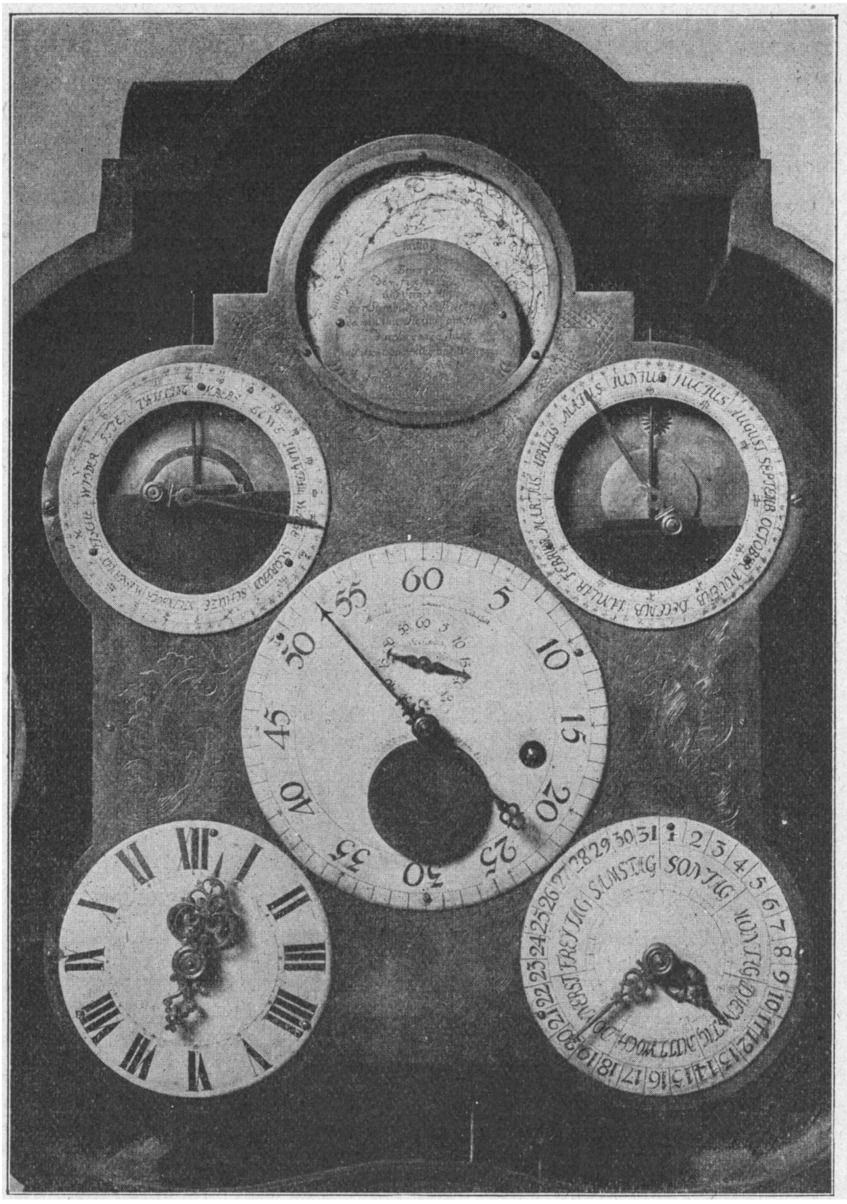

Figure 10.2: Hahn's clock in Basel. (source: [1])

It does therefore make one turn in one minute. It has 30 pins and the pendulum makes a half-swing in one second. Its length is about one meter.

The arbor 6 carries the second hand which is located on the small upper dial within the large central dial.

The large minute hand is also replicated on a smaller dial within the large central dial:

$$\mathrm{T}_8^0 = \mathrm{T}_4^0 \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) = \mathrm{T}_4^0 \tag{10.5}$$

These two hands therefore should always be parallel to each other.

The smaller hand on arbor 8 actually moves on an oscillating dial so that this smaller dial shows the minutes corrected for the equation of time, that is the minutes of true (solar) time, and not mean time. This correction is obtained by a toothed segment meshing with a wheel on tube 38. This segment is controlled by the equation of time cam which is located on the central arbor of the solar dial, which makes one turn in a year.

The motion of arbor 3 is also used to obtain the indication of the hour on tube 11. This tube controls the hour hand on the lower left dial. We have

$$\mathrm{T}_{11}^0 = \mathrm{T}_3^0 \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{54}\right) \times \left(-\frac{54}{72}\right) \tag{10.6}$$

$$= \mathrm{T}_3^0 \times \left(-\frac{5}{6}\right) = \left(-\frac{12}{5}\right) \times \left(-\frac{5}{6}\right) = 2 \tag{10.7}$$

In other words, the tube 11 makes two turns clockwise in one day, hence one turn in twelve hours.

10.3 The lunar dial

The hand on tube 11 actually moves another hand which is fixed on the central arbor 12 of the hours dial. The motion of this arbor is transferred to tube 14 on the lunar dial:

$$\mathrm{V}_{14}^0 = \mathrm{V}_{12}^0 \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{12}^0 \times \frac{1}{2} \tag{10.8}$$

and since

$$\mathrm{V}_{12}^0 = -\mathrm{T}_{12}^0 = -\mathrm{T}_{11}^0 \tag{10.9}$$

we have

$$\mathrm{V}_{14}^0 = -\mathrm{T}_{11}^0 \times \frac{1}{2} = -1 \tag{10.10}$$

Tube 14 thus makes one turn clockwise in one day. This tube moves the ring with the age of the Moon. We will see below that its motion should indeed be that of one turn per day.

The motion of tube 14 is used to move the central arbor 20:

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{14}^{0} \times \left(-\frac{28}{54}\right) \times \left(-\frac{6}{85}\right) = -\frac{28}{765} \tag{10.11}$$

$$\mathrm{P}_{20}^{0} = -\frac{765}{28} = -27.3214\ldots \text{ days} \tag{10.12}$$

This is the period of the tropical month. The same value is given by Oechslin. It is the period of the large hand which is also responsible for the oscillation of the horizon, but it isn't clear why the horizon should move with this period.

Tube 14 also carries an arm on which pivots arbor 16. This arbor carries a 42-teeth wheel which meshes with a fixed 16-teeth wheel on tube 15. The rotation of tube 14 thus produces a rotation of arbor 16. We can compute this motion in the reference frame of tube 14:

$$\mathrm{V}_{16}^{14} = \mathrm{V}_{15}^{14} \times \left(-\frac{16}{42}\right) = -\mathrm{V}_{14}^{15} \times \left(-\frac{8}{21}\right) = \mathrm{V}_{14}^{0} \times \frac{8}{21} = -\frac{8}{21} \tag{10.13}$$

A 8-leaves pinion on arbor 16 meshes with two wheels: a 90-teeth wheel on tube 18 and a 80-teeth wheel on tube 17. Tube 18carries the disk with the hole, which represents the Moon. We have:

$$\mathrm{V}_{17}^{14} = \mathrm{V}_{16}^{14} \times \left(-\frac{8}{80}\right) = \left(-\frac{8}{21}\right) \times \left(-\frac{1}{10}\right) = \frac{4}{105} \tag{10.14}$$

$$\mathrm{V}_{18}^{14} = \mathrm{V}_{16}^{14} \times \left(-\frac{8}{90}\right) = \left(-\frac{8}{21}\right) \times \left(-\frac{4}{45}\right) = \frac{32}{945} \tag{10.15}$$

$$\mathrm{P}_{18}^{14} = \frac{945}{32} = 29.53125 \text{ days} \tag{10.16}$$

The same value is given by Oechslin. After one synodic month, the Moon has made one turn with respect to the interior ring (on tube 14) showing the age of the Moon. And this motion is counterclockwise, which is why the ring is numbered counterclockwise.

Finally, we can compute the absolute motion of tube 18:

$$\mathrm{V}_{18}^{0} = \mathrm{V}_{18}^{14} + \mathrm{V}_{14}^{0} = \frac{32}{945} - 1 = -\frac{913}{945} \tag{10.17}$$

$$\mathrm{P}_{18}^{0} = -\frac{945}{913} \approx -24 \text{ h } 50 \text{ mn} \tag{10.18}$$

The Moon apparently revolves around the Earth in 24 hours and 50 minutes.

We can also compute the motion of tube 17 with respect to tube 18:

$$\mathrm{V}_{17}^{0} = \mathrm{V}_{17}^{14} + \mathrm{V}_{14}^{0} = \frac{4}{105} - 1 = -\frac{101}{105} \tag{10.19}$$

$$\mathrm{V}_{17}^{18} = \mathrm{V}_{17}^{0} - \mathrm{V}_{18}^{0} = -\frac{101}{105} + \frac{913}{945} = \frac{4}{945} \tag{10.20}$$

$$\mathrm{P}_{17}^{18} = \frac{945}{4} = 8\mathrm{P}_{18}^{14} \tag{10.21}$$

We can see that in one synodic month (P^{14}_{18}), tube 17 rotates of an eighth turn with respect to tube 18. This is why tube 17 is tied with a disk contained eight figures of the Moon. This disk revolves counterclockwise behind the disk moved by tube 18.

This dial also has an outer ring with the signs of the zodiac, but its use is not clear, given that no hand in this dial makes a turn in one year.

10.4 The solar dial

The motion of tube 14 in the lunar component is used to derive the motion of tube 22 in the solar dial. We have

$$V^0_{22} = V^0_{14} \times \frac{72}{36} \times \frac{36}{72} = V^0_{14} = -1 \tag{10.22}$$

This corresponds to a clockwise rotation in one day and is used to move the disk carrying the Sun.

The motion of tube 22 is used to provide the motion to the calendar part (see below). It causes in particular the rotation of arbor 28 in one day:

$$V^0_{28} = V^0_{22} \times \frac{72}{36} \times \left(-\frac{36}{72}\right) = -V^0_{22} = 1 \tag{10.23}$$

This motion is used to move the central arbor 30 of the solar dial with the help of a wormgear:[6]

$$V^0_{30} = V^0_{28} \times \frac{21}{65} \times \left(-\frac{1}{118}\right) = -\frac{21}{7670} \tag{10.24}$$

$$P^0_{30} = -\frac{7670}{21} = -365.2380\ldots \text{ days} \tag{10.25}$$

which is an approximation of the tropical year. The same value is given by Oechslin.

This arbor moves the hand showing the months in one year. It also carries the moving horizon.

Finally, this arbor carries on its rear the cam of the equation of the time which is used to display the corrected minutes on the central dial.

10.5 The calendar

I will not describe the calendar dial in detail.[7] It has two hands, one for the day of the week, and another for the day of the month. The motion of the hand for the day of the week is merely obtained from the daily motion of arbor 28 mentioned above.

The day of the month takes the lengths of the months into account, including that of leap years (but not the secular exceptions).

[6]Wormgears have apparently seldom been used by Hahn, cf. [2, p. 32].

[7]For more information on the calendar gears, see [4, p. 30-31] and [2, p. 33, 39].

10.6 The celestial map

The motion of tube 22 in the solar component is also used to derive the motion of the sky which is that of arbor 26. We have

$$\mathrm{V}_{26}^{0} = \mathrm{V}_{22}^{0} \times \left(-\frac{72}{36}\right) \times \left(-\frac{35}{29}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{38}{79}\right) = \mathrm{V}_{22}^{0} \times \frac{25270}{25201} \tag{10.26}$$

$$= -\frac{25270}{25201} \tag{10.27}$$

We find here the familiar value for the sidereal day. This value is negative, because the stars apparently move clockwise, from East to West.

10.7 References

[1] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

[2] Veronika Gutmann. *Die astronomische Uhr von Philipp Matthäus Hahn (1775).* Number 16 in Basler Kostbarkeiten. Basel: Baumann & Cie, Banquiers, 1995.

[3] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[4] Hans Franz Reinhardt. Die Basler astronomische Uhr von Philipp Matthäus Hahn. In *Jahresbericht des Historischen Museums Basel 1964*, pages 25–31. 1964.

[5] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.

[6] Rudolf Wolf. Astronomische Uhr von Hahn. — Geschenkt von Herrn Blass-Lavater in Zürich. In *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, volume 28, pages 146–152. 1883.