Assignment 1

Analytic Part

Continuous Signals

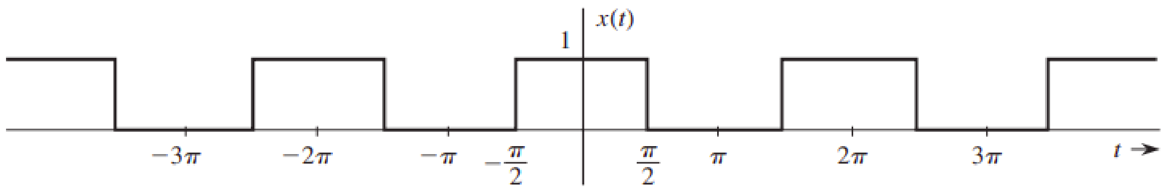
1. Fourier Transform:
   Annotations: The Fourier transform of $x(t)$ is $X(w)$
   a. Time convolution property:
      i. Given:
         1. $X_1(w)= \mathcal{F}(x_1(t))$, $X_2(w)= \mathcal{F}(x_2(t))$
      ii. Prove:
         1. $\mathcal{F}(x_1(t) * x_2(t)) = X_1(w)X_2(w)$, where * is the continuous convolution operator
   b. Linearity property:
      i. Given:
         1. $X_1(w) = \mathcal{F}(x_1(t))$, $X_2(w)= \mathcal{F}(x_2(t))$
      ii. Prove:
         1. $\mathcal{F}(ax_1(t) + bx_2(t)) = aX_1(w) + bX_2(w)$
   c. Scaling property:
      i. Given:
         1. $\mathcal{F}(x(t)) = X(w)$
      ii. Prove:
         1. For a > 0, $\mathcal{F}(x(at)) = \frac{1}{a} X(\frac{w}{a})$
   d. Time shifting property:
      i. Given:
         1. $\mathcal{F}(x(t)) = X(w)$
      ii. Prove:
         1. For a given t_0, $\mathcal{F}(x(t - t_0)) = X(w)e^{-jwt_0}$
         2. What is the effect of time shifting on the amplitude spectrum?
         3. What is the effect of time shifting on the phase spectrum?
   e. Fourier transform of unit gate (rect) function
      i. Given:

$$rect(t) = \begin{cases} 0 & |t| > 0.5 \\ 0.5 & |t| = 0.5 \\ 1 & |t| < 0.5 \end{cases}$$

      ii. Prove:
         1. $\mathcal{F}(rect(\frac{t}{\tau})) = \tau sinc(\frac{w\tau}{2})$

2. Draw $x(t) = rect(t)$
3. Draw $X(w)$, $|X(w)|$, $\angle X(w)$

2. Fourier Series:
   a. Fourier series of delta function
      i. Given the unit impulse train function $\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$
      ii. Find the values of D_n of the exponential form
      iii. Draw $x(t) = \delta_{T_0}(t)$ and $X(w)$
      iv. What is T_0
      v. Find the interval between D_n and D_{n+1}, and its relation to T_0
3. Using the properties of the transform and the results from 2.a and 1.d, draw the spectrum of the following continuous and periodic function:



4. Given $x(t) = e^{-at}u(t)$, where $a > 0$, and $u(t)$ is the unit step function.
   a. Find the $\mathcal{F}(x(t))$
   b. Draw its magnitude and phase
   c. What kind of filter can it be used for?

Discrete Signals

1. Given $F_s = 8000Hz\ (8KHz)$
   a. To which frequency $10KHz$ will be aliased to?
   b. How could you prevent the aliasing if we had the analogue signal? Explain shortly in words
2. Stereo hearing:
   a. Record yourself counting till 10 using your mobile device / phone - save it under the name ‘audio_r.wav’
   b. Make a copy of the file under ‘audio_l.wav’
   c. Open both files in [Audacity](#) / any other audio editing app that enables playing audio in stereo
   d. Wear headphones:
      i. Play both channels
      ii. Shift ‘audio_l.wav’ 2ms to the right w.r.t ‘audio_r.wav’ and play both channels
      iii. Shift ‘audio_r.wav’ 2ms to the right w.r.t ‘audio_l.wav’ and play both channels
   e. What do you hear? And why?

5. $\mathcal{Z}$ Transform:
   Annotations: The $\mathcal{Z}$ transform of $x[n]$ is $X(z)$, marked as $\mathcal{Z}(x[n]) = X(z)$
   a. Proof the following property (time convolution → frequency multiplication):
      i. Given:
         1. $\mathcal{Z}(x_1[n]) = X_1(z)$, and $\mathcal{Z}(x_2[n]) = X_2(z)$
      ii. Prove:
         1. $\mathcal{Z}(x_1[n] * x_2[n]) = X_1(z)X_2(z)$, where * is the discrete convolution operator
   b. Scaling property:
      i. Given:
         1. $\mathcal{Z}(x_1[n]) = X_1(z)$
      ii. Prove:
         1. For a > 0, $\mathcal{Z}(x[an]) = \frac{1}{a} X(\frac{z}{a})$
6. DTFS:
   a. Given the signal $x[n] = cos(0.1\pi n)$:
      i. How many samples are there in one period (what is N_0)?
      ii. What is the discrete time fourier series of $x[n]$?

Technical Part - Python3.10

1. Record yourself speaking for 10 seconds - 5 seconds when you are 20cm to the microphone and 5 seconds when you are 3m. The goal is that you'll have soft and loud speech segments in your recording.
   a. Load the audio file.
      i. If the audio was recorded in stereo, keep only a single channel.
      ii. What is the sampling frequency of the audio?
   b. Set the sampling rate of the signal to 32KHz using scipy.signal.resample function (make sure that you cast the audio to np.float32)
   c. Let's **downsample** the audio to 16KHz:
      i. using 2 methods:
         1. Take every even sample from the audio
         2. Resample the audio using scipy.signal.resample function (make sure that you cast the audio to np.float32)
   d. Write a function (you can use librosa/matplotlib) that given an input audio and its sampling frequency it **plots** a figure containing 4 subplots:
      i. Audio
      ii. Spectrogram. The spectrogram should also contain:
         1. Validate that you see F_{max}
         2. Pitch contour on top of the spectrogram. You can use Praat or pyworld python package
            a. Why are there missing timeframes in the pitch contour?
      iii. Mel-Spectrogram

iv. Energy and RMS
v. Notes:
1. The energy, Spectrogram and Mel-Spectrogram should be calculated using window_size of 20ms, and hop_size of 10ms.
2. Make sure that axes have labels specifying the units of measurements (x axis- time [sec], y axis- frequency [Hz])

e. Apply this function on the resampled audios from 1.b, and listen to both outputs-
i. Which one is better?
ii. Why?

2. Adding noise:
    a. Load the stationary_noise.wav audio file, and resample it to 16KHz.
    b. Add the noise to the audio from Q1.c.2 using ‘+’ operator. If you need to truncate it, do so.
    c. Plot the audio, noise, and noisy audio signals.
3. Implement and apply **spectral subtraction** to enhance the noisy signal from Q2.b
    a. Find the speech parts (voice activity detection) using a threshold on the energy level.
        i. Set up the threshold and plot its value over the energy contour .
    b. For every time-frame, find its noise estimation (‘noise footprint’) and subtract it from the signal. Apply this in a sequential manner
    c. Plot the output using the function from Q1.d
4. The audio segment from Q1.c.2 contains loud and soft speech segments, recorded when the speakers were close /distant from the microphone, respectively.
    a. Apply Auto Gain Control (AGC) on the audio from Q1.c.2
        i. Determine the desired RMS in dB.
        ii. Determine the noise floor threshold.
        iii. For every time-frame, find its relevant gain and amplify/attenuate accordingly (using statistics based on a window of ~1s). Apply this in a sequential manner
        iv. Make sure you don’t have overflow in the audio after amplifying. You can use a sigmoid function to avoid clipping.
        v. Plot the output using the function from Q1.d.
        vi. Plot the scaling factors vs time.
5. Using the audio from Q1.c.2, increase the speed of the audio by factor of x1.5, while preserving the pitch.
    a. Apply a time stretching algorithm using the phase vocoder.
        i. Set the mapping between the input and output.
        ii. Apply STFT on the audio signal
        iii. Calculate the magnitude and phase values of the output
        iv. Apply iSTFT and listen to the audio.
        v. Plot the signals in the time domain and spectral domain.