# **Energy Efficient Explosive Motion with Compliant Actuation Arrangements in Articulated Robots\***

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Abstract—This paper presents the motion optimization for a recently introduced asymmetric compliant actuator which provides energy efficient actuation for explosive motions such as jumping. Two actuation branches with significantly different stiffness and energy storage capacity properties driving a single joint make up the actuator design. An optimization problem is formulated to optimize the joint trajectories for energy efficient vertical jumping motions of a 2-DoF leg as proof-of-concept. Several configurations of the asymmetric compliant actuators have been investigated. Simulation studies of the optimized jumping motions demonstrate SOMETHING.

#### I. INTRODUCTION

Introduction Citation [1]

#### II. LEG DESIGN

A. Compliant Actuation

Description actuation principle

B. Configurations

SEA only, mono-articulated, bi-articulated

#### III. DYNAMIC MODEL

#### A. Forward Dynamics

The four links are connected by the actuated ankle, knee and hip joints, denoted  $q_1,q_2,q_3$ , with torques  $\tau_1,\tau_2,\tau_3$ . The links have masses  $m_1,m_2,m_3,m_4$  and rotational inertiae  $J_1,J_2,J_3,J_4$ . Their CoM is assumed to be located on the line connecting the proximal and distal joints at a distance of  $r_1,r_2,r_3,r_4$  from the proximal joint for all links except for the foot; the model includes a floating base to allow for realistic modelling of the ground reaction forces (GRF). Together, the configurations of the bodies describe the system:

$$x = [x_1, y_1, \theta_1, x_2, y_2, \theta_2, x_3, y_3, \theta_3, x_4, y_4, \theta_4]^T.$$
 (1)

The Euler-Lagrange formulism with generalised coordinates  $\mathbf{q} \in \mathfrak{Q} \subset \mathfrak{R}^{N=6}$  is used to derive the dynamic equations for the system:

$$\mathbf{q} = [x_1, y_1, \theta_1, q_1, q_2, q_3]^T \tag{2}$$

leading to:

$$M(\mathbf{q})\ddot{\mathbf{q}} = \tau + \mathbf{G}(\mathbf{q}) - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - D\dot{\mathbf{q}} + J_{GRF}^T \mathbf{F}_{GRF} \quad (3)$$

Here, the damping matrix is denoted by  $D=\mathrm{diag}(0,0,0,d_1,d_2,d_3)$ , the generalised actuation forces are denoted by  $\tau=[0,0,0,\tau_1,\tau_2,\tau_3]$ , the generalised gravitational forces are denoted by  $\mathbf{G}(\mathbf{q})$ , the Coriolis vector is denoted by  $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$  and the generalised inertia matrix is denoted by  $M(\mathbf{q})$ . The Jacobian for the heal and toe is denoted by  $J_{GRF}^T$  and the ground forces in generalised coordinates are subsequently expressed as  $J_{GRF}^T\mathbf{F}_{GRF}$ . Spring-dampers define  $\mathbf{F}_{GRF}$  in the vertical direction and Coulomb and viscous friction define the horizontal component, proportional to the vertical forces.

B. Inverse Dynamics

more math

$$\alpha + \beta = \chi$$

#### IV. DYNAMIC OPTIMISATION

Explain necessity problem downscaling

A. Trajectory Parametrization

$$\alpha + \beta = \chi$$

#### B. Pretension position

Explain relevance of pretension position to jumping and why it is added as an optimisation variable.

#### C. Objective criteria

The objective function is comprised out of three criteria which reward the performance of the leg, penalize excessive torque needed to complete a movement and maintain the postural stability of the leg. A concrete minimization of the objective functions with these criteria is represented by:

$$\min_{D} J(P) = -c_1 \cdot J_{performane} + c_2 \cdot J_{torque} + c_3 \cdot J_{stability}$$

Here,  $c_1$ ,  $c_2$  and  $c_3$  denote scaling constants.

- 1) Performance: For the performance of the leg we distinguish two different objectives:
  - 1) Jumping a maximum height, where the height is defined as the y-coordinate of the centre of mass of the leg with respect to the ground:

$$J_{performane} = y_{CoM}^2$$

2) Jumping to a certain height efficiently, where the maximum *y*-coordinate reached by the centre of mass

<sup>\*</sup>Supported by ???

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of the leg is bound to an equality constraint and the energy use is defined as the power used for the jump:

$$J_{performane} = -P^2$$

#### Here, P is defined as **define** P

2) Torque: The active torque  $\tau_a$  is to be bounded within the maximum and minimum deliverable torque  $[\tau_a \bar{\tau}_a]$ . This is enforced by means of a penalty function:

$$J_{torque} = \sum_{0}^{t_f} (\tau_a - \tau_a)^T (\tau_a - \tau_a) + (\tau_a - \bar{\tau}_a)^T (\tau_a - \bar{\tau}_a)$$

Here,  $t_f$  denotes the last time segment of the motion.

3) Stability: To ensure postural stability a stability criterion is introduced. The leg posture is considered stable when the x-coordinate of the centre of mass of the leg is equal to its initial x-coordinate, x=0, at the end of the motion, i.e. when the centre of mass reaches its maximum height. Also, the motion is assumed to be stable when the mean value of the x-coordinate of the centre of mass equals zero. This is achieved with the minimization of:

$$J_{stability} = |x_{CoM}(t_h) + \bar{x}_{CoM}|^2$$

Here,  $t_h$  denotes the point in time where the centre of mass reaches its maximum height.

#### D. Algorithm

The objective criteria described above require information concerning the kinematic and dynamic state of the leg. Both forward and inverse dynamic calculations are to be performed while the leg states are set by adjusting the earlier described B-spline joint trajectories. This dependence is managed by the optimisation algorithm. The algorithm in words yields:

- Provide initial guess trajectory
- Create control points
- $\diamond$  Create trajectory with B-splines
- Check joint angles, continue if limits are not exceeded else, vary control points and repeat <> steps
- Run simulation of motion through forward dynamics
- ♦ Calculate active torques through inverse dynamics
- ♦ Evaluate objective function
- ♦ Exit if local minimum is reached, else vary control points and repeat steps with ♦

Textbox to insert figure

Fig. 1. Caption

#### V. RESULTS

Results, max height and energy cost for certain height.

The optimisation has been performed for the upward movement of the leg performing a jumping motion. The initial position of the leg yields a squatting posture and the optimisation is concluded when the centre of mass reaches its highest point **add figure of motion sequence**.

### TABLE I MAXIMUM JUMP HEIGHT FOR DIFFERENT CONFIGURATIONS

Configuration	Maximum jump height [m]
Only SEA	
Mono-articulated	
Bi-articulated	

### TABLE II MINIMUM ENERGY USE FOR DIFFERENT CONFIGURATIONS

Configuration	Minimum energy use [J]
Only SEA	
Mono-articulated	
Bi-articulated	

#### VI. DISCUSSION

Comparison of actuation topologies.

## VII. CONCLUSIONS APPENDIX

Appendixes should appear before the acknowledgment.

#### **ACKNOWLEDGMENT**

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

#### REFERENCES

- G. O. Young, Synthetic structure of industrial plastics (Book style with paper title and editor), in Plastics, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 1564.
- [2] W.-K. Chen, Linear Networks and Systems (Book style). Belmont, CA: Wadsworth, 1993, pp. 123135.
- [3] H. Poor, An Introduction to Signal Detection and Estimation. New York: Springer-Verlag, 1985, ch. 4.
- [4] B. Smith, An approach to graphs of linear forms (Unpublished work style), unpublished.
- [5] E. H. Miller, A note on reflector arrays (Periodical styleAccepted for publication), IEEE Trans. Antennas Propagat., to be publised.
- [6] J. Wang, Fundamentals of erbium-doped fiber amplifiers arrays (Periodical styleSubmitted for publication), IEEE J. Quantum Electron., submitted for publication.
- [7] C. J. Kaufman, Rocky Mountain Research Lab., Boulder, CO, private communication, May 1995.
- [8] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, Electron spectroscopy studies on magneto-optical media and plastic substrate interfaces(Translation Journals style), IEEE Transl. J. Magn.Jpn., vol. 2, Aug. 1987, pp. 740741 [Dig. 9th Annu. Conf. Magnetics Japan, 1982, p. 301].

- [9] M. Young, The Techincal Writers Handbook. Mill Valley, CA: University Science, 1989.
- [10] J. U. Duncombe, Infrared navigationPart I: An assessment of feasibility (Periodical style), IEEE Trans. Electron Devices, vol. ED-11, pp. 3439, Jan. 1959.
- [11] S. Chen, B. Mulgrew, and P. M. Grant, A clustering technique for digital communications channel equalization using radial basis function networks, IEEE Trans. Neural Networks, vol. 4, pp. 570578, July 1993.
- [12] R. W. Lucky, Automatic equalization for digital communication, Bell Syst. Tech. J., vol. 44, no. 4, pp. 547588, Apr. 1965.
- [13] S. P. Bingulac, On the compatibility of adaptive controllers (Published Conference Proceedings style), in Proc. 4th Annu. Allerton Conf. Circuits and Systems Theory, New York, 1994, pp. 816.
- [14] G. R. Faulhaber, Design of service systems with priority reservation, in Conf. Rec. 1995 IEEE Int. Conf. Communications, pp. 38.
- [15] W. D. Doyle, Magnetization reversal in films with biaxial anisotropy, in 1987 Proc. INTERMAG Conf., pp. 2.2-12.2-6.
- [16] G. W. Juette and L. E. Zeffanella, Radio noise currents n short sections on bundle conductors (Presented Conference Paper style), presented at the IEEE Summer power Meeting, Dallas, TX, June 2227, 1990, Paper 90 SM 690-0 PWRS.
- [17] J. G. Kreifeldt, An analysis of surface-detected EMG as an amplitude-modulated noise, presented at the 1989 Int. Conf. Medicine and Biological Engineering, Chicago, IL.
- [18] J. Williams, Narrow-band analyzer (Thesis or Dissertation style), Ph.D. dissertation, Dept. Elect. Eng., Harvard Univ., Cambridge, MA, 1993.
- [19] N. Kawasaki, Parametric study of thermal and chemical nonequilibrium nozzle flow, M.S. thesis, Dept. Electron. Eng., Osaka Univ., Osaka, Japan, 1993.
- [20] J. P. Wilkinson, Nonlinear resonant circuit devices (Patent style), U.S. Patent 3 624 12, July 16, 1990.