Energy Efficient Explosive Motion with Compliant Actuation Arrangements in Articulated Robots*

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***Abstract*— This paper presents the motion optimization for a recently introduced asymmetric compliant actuator which provides energy efficient actuation for explosive motions such as jumping. Two actuation branches with significantly different stiffness and energy storage capacity properties driving a single joint make up the actuator design. An optimization problem is formulated to optimize the joint trajectories for energy efficient vertical jumping motions of a 2-DoF leg as proof-of-concept. Several configurations of the asymmetric compliant actuators have been investigated. Simulation studies of the optimized jumping motions demonstrate SOMETHING.**

I. INTRODUCTION

Introduction
sea's [1]
(parallel) compliant actuation [2] [3]
optimization in variable stiffness and compliance [4], [5]
biarticulated robot jumping [6].
biomechanics [7]
inverse kinematics recursive algorihm [8] [9] [10]
inverse kinematics hybrid dynamics algorithm, given desired joint accelerations, the base accelerations and joint torques are computed [11]
novel design Asymmetric Compliant Actuation [12] [13]
optimization B-spline trajectories [14] [6] objectives and constraints [6] [15] [16]
energy [17]

II. LEG DESIGN

A. Assymmetric Compliant Actuation

For the concept of Assymetric Compliant Actuation, or ACA, a combination of two parallel actuation branches with very different power and stiffness properties are used as shown in Fig. 1.

The Power Branch (*PB*) is a rotary Series Elastic Actuator (*SEA*) which consists of a high power motor *M1* in series with a torsional elastic element *SE*. The Energy Storage Branch (*ESB*) consists of a lower power motor *M2* with a high reduction linear transmission which transfers its power through a unidirectional series elastic element *PE*.

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Picture of ACA branches

Fig. 1. Caption

B. Configurations

In nature, bi-articulated muscle structures, muscle structures that actuate multiple joints, can be found on humans and animals. An example of a bi-articulated muscle structure are the hamstrings, which span both the hip and knee joint. ACA's can be utilized to realize bi-articulated actuation by letting the PB drive a joint directly, and letting the ESB tendon span the driven joint by a pulley and in turn drive a second joint. Bi-articulation is one of three configurations of the leg considered for the jumping optimization:

1) Firstly, the ACA's are not considered and jumping optimizations are performed for the leg with fixed compliance SEA's only.
2) Secondly, the ACA's are introduced to perform jumping optimizations on a mono-articulated version of the leg.
3) Lastly, a bi-articulated version of the leg is considered for optimization.

The leg actuation configurations are shown in Fig 2.

III. DYNAMIC MODEL

A. Forward Dynamics

The leg consists out of four links which are connected by the actuated ankle, knee and hip joints, denoted q_1, q_2, q_3, with torques τ_1, τ_2, τ_3 as shown in Fig. 3. The links have masses m_1, m_2, m_3, m_4 and rotational inertiae J_1, J_2, J_3, J_4. Their CoM is assumed to be located on the line connecting the proximal and distal joints at a distance of r_1, r_2, r_3, r_4 from the proximal joint for all links except for the foot; the model includes a floating base to allow for realistic modelling of the ground reaction forces (GRF). Together, the configurations of the bodies describe the system:

$$x = [x_1, y_1, \theta_1, x_2, y_2, \theta_2, x_3, y_3, \theta_3, x_4, y_4, \theta_4]^T. \quad (1)$$

Picture of configurations

Fig. 2. Caption

Picture leg

Fig. 3. Caption

The Euler-Lagrange formulism with generalised coordinates $\mathbf{q} \in \mathfrak{Q} \subset \Re^6$ is used to derive the dynamic equations for the system:

$$\mathbf{q} = [x_1, y_1, \theta_1, q_1, q_2, q_3]^T \quad (2)$$

leading to:

$$M(\mathbf{q})\ddot{\mathbf{q}} = \tau + \mathbf{G}(\mathbf{q}) - C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - D\dot{\mathbf{q}} + J_{GRF}^T \mathbf{F}_{GRF} \quad (3)$$

Here, the damping matrix is denoted by $D = \text{diag}(0, 0, 0, d_1, d_2, d_3)$, the generalised actuation forces are denoted by $\tau = [0, 0, 0, \tau_1, \tau_2, \tau_3]$, the generalised gravitational forces are denoted by $\mathbf{G}(\mathbf{q})$, the Coriolis matrix is denoted by $C(\mathbf{q}, \dot{\mathbf{q}})$ and the generalised inertia matrix is denoted by $M(\mathbf{q})$. The Jacobian for the heel and toe is denoted by J_{GRF}^T and the ground forces in generalised coordinates are subsequently expressed as $J_{GRF}^T \mathbf{F}_{GRF}$. Spring-dampers define $\mathbf{F}_{GRF}$ in the vertical direction and Coulomb and viscous friction define the horizontal component, proportional to the vertical forces.

B. Inverse Dynamics

The jumping optimizations are guided with an objective function and its criteria. These objective criteria require information concerning the active torques, available through inverse dynamics calculations. In the past an inverse dynamics formulation for the open chain, fully actuated systems derived from the recursive Newton-Euler formulation has been used to implement a recursive hybrid dynamics algorithm to integrate the system forward in time [8] [9] [10]. However, for the optimization described in this paper we use an inverse dynamics calculation which revolves around the assumption that the motor torques from the previous time step are close to the motor commands computed at the current time step [11]. Using this assumption with the idea of a hybrid dynamics algorithm [18], the base accelerations and joint torques can be computed for given desired joint accelerations. The expression for $\mathbf{q}$ in equation (2) can be split in its passive joint and active joint components:

$$\mathbf{q_p} = [x_1, y_1, \theta_1]^T, \quad \mathbf{q_a} = [q_1, q_1, q_3]^T. \quad (4)$$

Incorporating these into the dynamic equations in equation (3) with the knowledge that $\boldsymbol{\tau_p} = 0$ leads to:

$$\begin{bmatrix} M_{pp} & M_{aa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_\mathbf{p} \\ \ddot{\mathbf{q}}_\mathbf{a} \end{bmatrix} + \begin{bmatrix} \mathbf{B_p}(\mathbf{q}, \dot{\mathbf{q}}) \\ \mathbf{B_a}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau_a} \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \mathbf{F}_{EXT}(\mathbf{q}, \dot{\mathbf{q}}) \quad (5)$$

Here, $\mathbf{B}$ denotes the gravitational, Coriolis and damping forces. Writing out the matrix multiplications to:

$$\begin{aligned} M_{pp}\ddot{\mathbf{q}}_\mathbf{p} + M_{pa}\ddot{\mathbf{q}}_\mathbf{a} + \mathbf{B_p}(\mathbf{q}, \dot{\mathbf{q}}) &= J_1 \mathbf{F}_{EXT}(\mathbf{q}, \dot{\mathbf{q}}) \\ M_{ap}\ddot{\mathbf{q}}_\mathbf{p} + M_{aa}\ddot{\mathbf{q}}_\mathbf{a} + \mathbf{B_a}(\mathbf{q}, \dot{\mathbf{q}}) &= J_2 \mathbf{F}_{EXT}(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{\tau_a} \end{aligned} \quad (6)$$

allows for an inverse formulation:

$$\begin{bmatrix} M_{pp} & 0 \\ M_{ap} & -I \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_\mathbf{p} \\ \boldsymbol{\tau_a} \end{bmatrix} = -\begin{bmatrix} M_{pa} \\ M_{aa} \end{bmatrix} \ddot{\mathbf{q}}_\mathbf{a} - \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}}) + J_{EXT}^T \mathbf{F}_{EXT}(\mathbf{q}, \dot{\mathbf{q}}). \quad (7)$$

An expression for the active torques is now easily achieved by inversion of the left-hand side mass matrix.

IV. Dynamic Optimisation

The search for an optimal motion requires finding the optimal active joint trajectories over time. Both the options in possible trajectories as the continuity of time pose an infinite amount of scenarios to investigate. Subsequently, the optimization we wish to perform needs to be reformulated from a a large scale trajectory optimal control problem into a finite-dimensional optimization problem.

A. Trajectory Parametrization

One can use basis splines, or B-splines, combined with a time-scale factor to reduce the complex problem into a parameter optimization [14] [6] [8] [10]. A B-spline curve can be described by its basis functions $B_i(t)$ and control points $P = \{p_1, ..., p_n\}$ which are evenly spaced over time. The active joint trajectories can be expressed as:

$$q_a(t, p) = \sum_{i=1}^{n} B_i(t) p_i \quad (8)$$

Using the control points as optimization variables, an optimal set of control points can be determined and the corresponding B-splines yield optimal trajectories for the joint angle displacements, velocities and accelerations. However, the active torques pose a problem as the upper and lower bound constraints on the applied torques can become non-linear. Therefore it is chosen to express the torque limits by means of a penalty function in the objective criteria.

B. Pretension position

Explain relevance of pretension position to jumping and why it is added as an optimisation variable.

C. Objective criteria

The objective function is comprised out of three criteria which reward the performance of the leg, penalize excessive torque needed to complete a movement and maintain the postural stability of the leg. A concrete minimization of the objective functions with these criteria is represented by:

$$\min J = J_{performane} + J_{torque} + J_{stability} \quad (9)$$

In the objective criteria, c_1, c_2, c_3 and c_4 denote scaling constants.

1) Performance: For the performance of the leg we distinguish two different objectives:

1) Jumping a maximum height, where the height is defined as the y-coordinate of the centre of mass of the leg with respect to the ground:

$$J_{performane} = -c_1 \cdot y_{CoM}^2 \quad (10)$$

2) Jumping to a certain height efficiently, where the maximum y-coordinate reached by the centre of mass

of the leg is bound to an equality constraint and the energy use is defined as the power used for the jump:

$$J_{performane} = -c_1 \cdot P^2 \tag{11}$$

Here, P is defined as **define P**

2) Torque: The active torque τ_a is to be bounded within the maximum and minimum deliverable torque $[\underline{\tau}_a \; \bar{\tau}_a]$. This is enforced by means of a penalty function:

$$J_{torque} = c_2 \cdot \sum_{0}^{t_f} (\underline{\tau}_a - \tau_a)^T (\underline{\tau}_a - \tau_a) + (\tau_a - \bar{\tau}_a)^T (\tau_a - \bar{\tau}_a) \tag{12}$$

Here, t_f denotes the last time segment of the motion.

3) Stability: To ensure postural stability a stability criterion is introduced. The leg posture is considered stable when the x-coordinate of the centre of mass of the leg is equal to its initial x-coordinate, $x = 0$, at the end of the jump, i.e. when the centre of mass reaches its maximum height. Also, the motion is assumed to be stable when the mean value of the absolute x-coordinates of the centre of mass equals zero. This is achieved with the minimization of:

$$J_{stability} = c_3 \cdot x_{CoM}(t_h)^2 + c_4 \cdot \sum_{i=1}^{N} \frac{|x_{CoM}(t)_i|}{N} \tag{13}$$

Here, t_h denotes the point in time where the centre of mass reaches its maximum height. For succesful jumps $t_h = t_f$.

D. Algorithm

The objective criteria described above require information concerning the kinematic and dynamic state of the leg. Both forward and inverse dynamic calculations are to be performed while the leg states are set by adjusting the earlier described B-spline joint trajectories. This dependence is managed by the optimisation algorithm. The algorithm in words yields:

- Provide initial guess trajectory
- Create control points

◇ Create trajectory with B-splines
◇ Check joint angles, continue if limits are not exceeded else, vary control points and repeat ◇ steps
◇ Run simulation of motion through forward dynamics
◇ Calculate active torques through inverse dynamics
◇ Evaluate objective function

◇ Exit if local minimum is reached, else vary control points and repeat steps with ◇

Figure of motion sequence

Fig. 4. Caption

V. Results

Results, max height and energy cost for certain height.

The optimisation has been performed for the upward movement of the leg performing a jumping motion. The initial position of the leg yields a squatting posture and the optimisation is concluded when the centre of mass reaches its highest point. The initial guess **describe init guess add figure of motion sequence**.

TABLE I
MAXIMUM JUMP HEIGHT FOR DIFFERENT CONFIGURATIONS

Configuration	*Maximum jump height* [m]
Only SEA	
Mono-articulated	
Bi-articulated	

TABLE II
MINIMUM ENERGY USE FOR DIFFERENT CONFIGURATIONS

Configuration	*Minimum energy use* [J]
Only SEA	
Mono-articulated	
Bi-articulated	

VI. Discussion

Comparison of actuation topologies.

VII. Conclusions

ACKNOWLEDGMENT

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