Accuracy of original MPM

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Outline

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Numerical accuracy

Numerical Approximation

$$u_{ex} = u_{num} + \mathcal{O}(\Delta x^n) + \mathcal{O}(\Delta t)$$

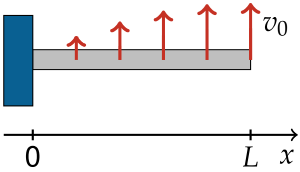
RMS Error

$$Error_{RMS} = \sqrt{\frac{1}{n_p}\left(\sum_{p=1}^{n_p} u_{num}(x_p, t) - u_{ex}(x_p, t)\right)^2}$$

Accuracy in displacement

For $\Delta t \rightarrow 0$, the order of accuracy is equal to n, i.e. the reduction of Δx by a factor of 2 decreases the RMS error by 2^n.

Vibrating bar



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}$$

Boundary conditions:

$$u(0,t) = 0$$

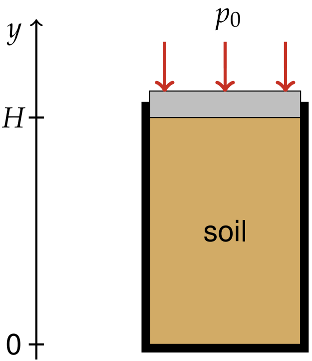
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial conditions:

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = v_0 \sin\left(\frac{\pi x}{2L}\right)$$

Oedometer



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial^2 x} - g$$

Boundary conditions:

$$u(0,t) = 0,$$

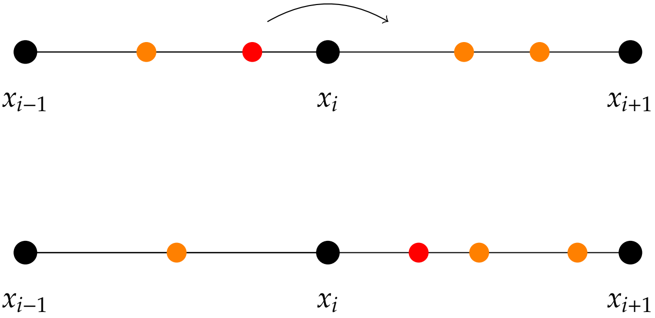
$$\frac{\partial u}{\partial x}(L,t) = \frac{p_0}{E}$$

Initial conditions:

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = v_0 \sin\left(\frac{\pi x}{2L}\right)$$

Grid-crossing



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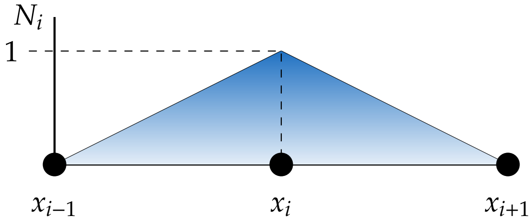
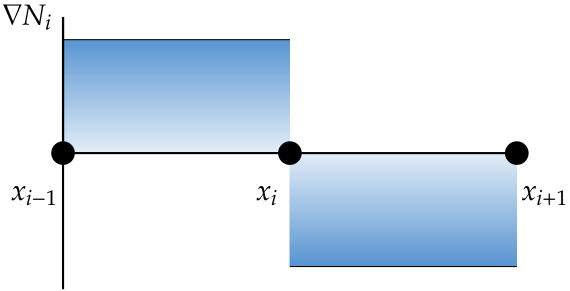
Grid-crossing: properties of shape functions





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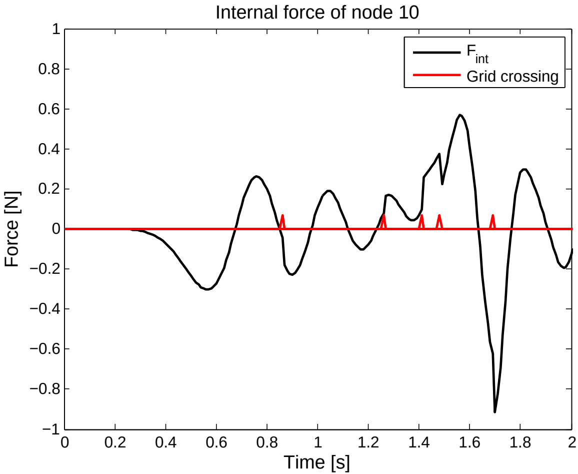
Grid crossing: internal force

$$F_{i+1}^{int} \approx \sum_{p=1}^{n_i} \nabla N_i(\xi_p)\sigma_p\Omega_p + \sum_{p=1}^{n_{i+1}} \nabla N_i(\xi_p)\sigma_p\Omega_p$$

$$F_{i+1}^{int} \approx \sigma\Omega(n_i - n_{i+1})$$

$$\begin{cases} F_{i+1}^{int} = 0, & \text{if } n_i = n_{i+1} \\ F_{i+1}^{int} \neq 0, & \text{otherwise} \end{cases}$$

Grid crossing: internal force



Grid crossing: vibrating bar

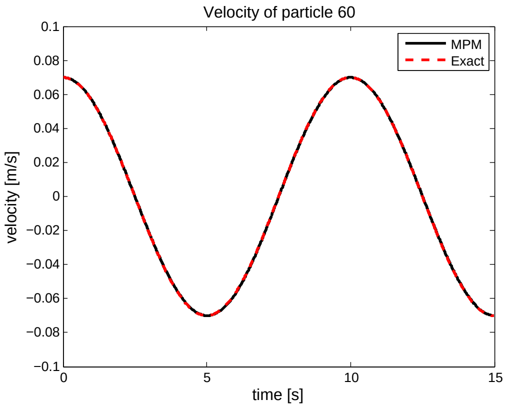


Figure: No grid crossing (30 elements).

Grid crossing: vibrating bar

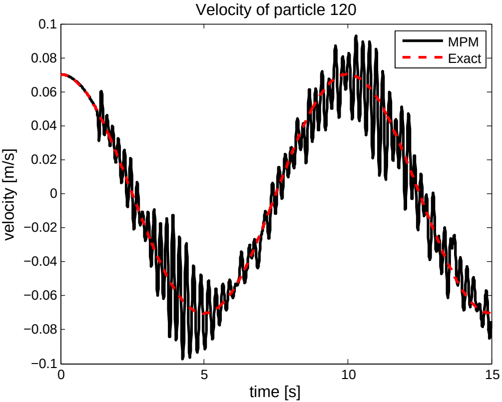


Figure: Grid crossing (60 elements).

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Grid crossing: oedometer

Error versus number of elements

	FEM	MPM(1)	MPM(4)
4	$5.3698 \cdot 10^{-4}$	$1.2918 \cdot 10^{-3}$	$1.0374 \cdot 10^{-3}$
8	$1.3456 \cdot 10^{-4}$	$3.2595 \cdot 10^{-4}$	$2.6167 \cdot 10^{-4}$
16	$3.3657 \cdot 10^{-5}$	$8.1795 \cdot 10^{-5}$	$6.5694 \cdot 10^{-5}$
32	$8.4138 \cdot 10^{-6}$	$2.0632 \cdot 10^{-5}$	$1.6625 \cdot 10^{-5}$
64	$2.1019 \cdot 10^{-6}$	$5.4969 \cdot 10^{-6}$	$4.5505 \cdot 10^{-6}$

Table: Vibrating bar: RMS Error versus number of elements.

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Table: Vibrating bar: RMS Error versus number of elements.

Order of accuracy

All three methods are second order accurate.

Accuracy: vibrating bar