

MPM Project: Problem Description

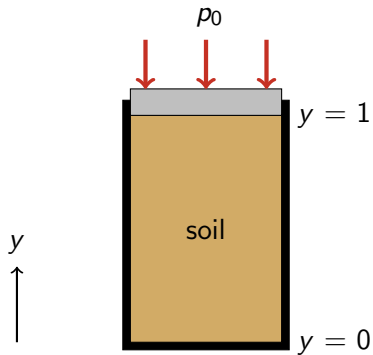
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Outline

- Oedometer problem
- Observations with 1 PPC

Oedometer problem



schematic representation

Oedometer: model

$$\rho \frac{\partial \hat{v}}{\partial t} = \frac{\partial \hat{\sigma}}{\partial y} - \rho g,$$

$$\frac{\partial \hat{\sigma}}{\partial t} = E \frac{\partial \varepsilon}{\partial t}.$$

Boundary conditions:

$$\hat{v}(0, t) = 0,$$

$$\hat{\sigma}(H, t) = -p_0.$$

Initial conditions:

$$\hat{v}(y, 0) = 0,$$

$$\hat{\sigma}(y, 0) = 0.$$

Oedometer: model

$$\frac{\partial^2 \hat{u}}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial y^2} - g.$$

Boundary conditions:

$$u(0, t) = 0,$$

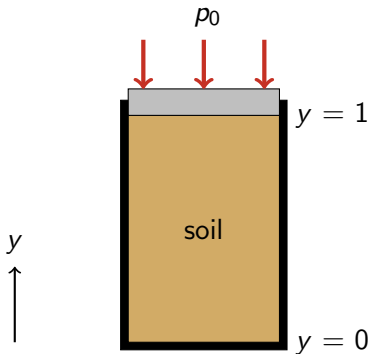
$$\frac{\partial u}{\partial y}(H, t) = -p_0/E.$$

Initial conditions:

$$u(y, 0) = 0,$$

$$\frac{\partial u}{\partial t}(y, 0) = 0.$$

Oedometer: discretization

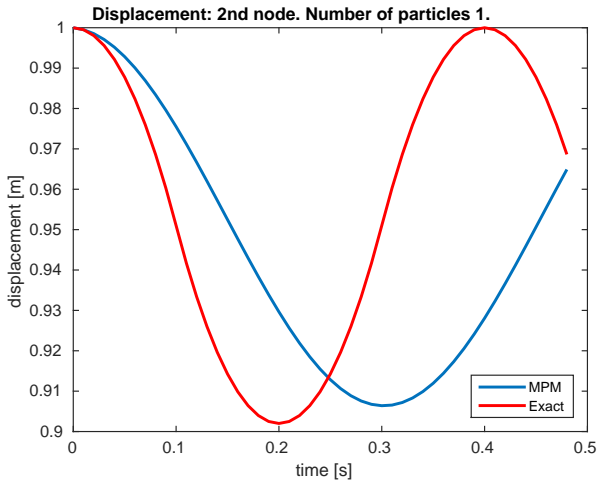


schematic representation

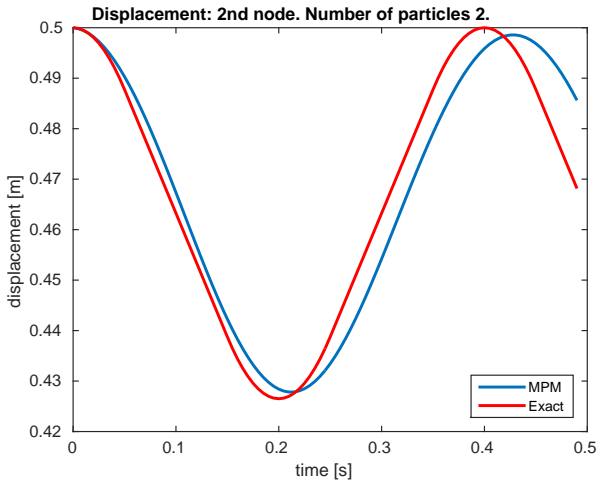


discretization

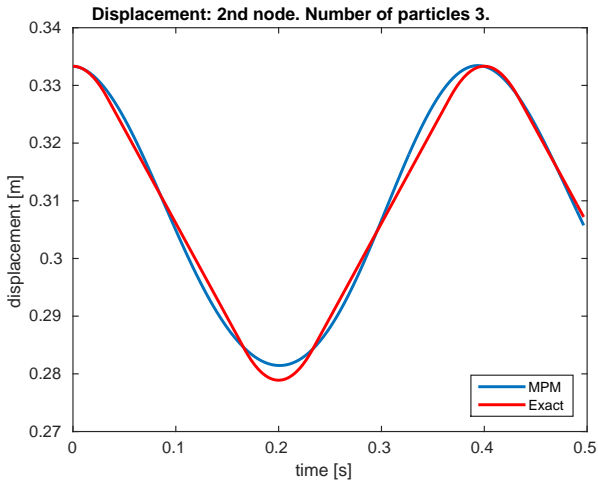
Oedometer: results



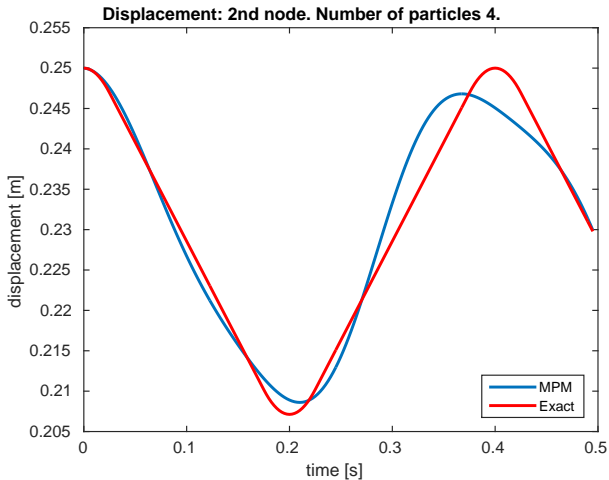
Oedometer: results



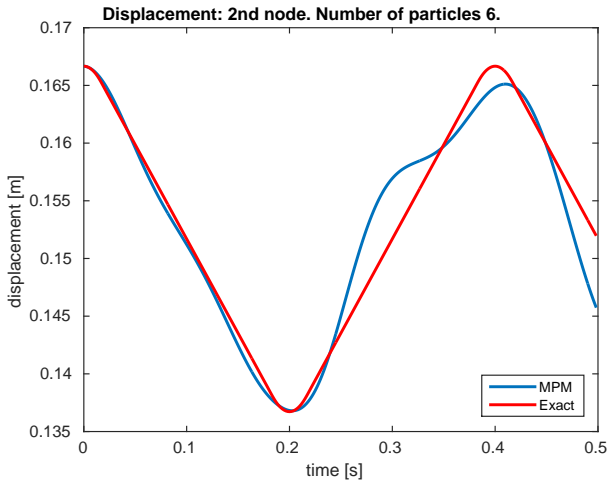
Oedometer: results



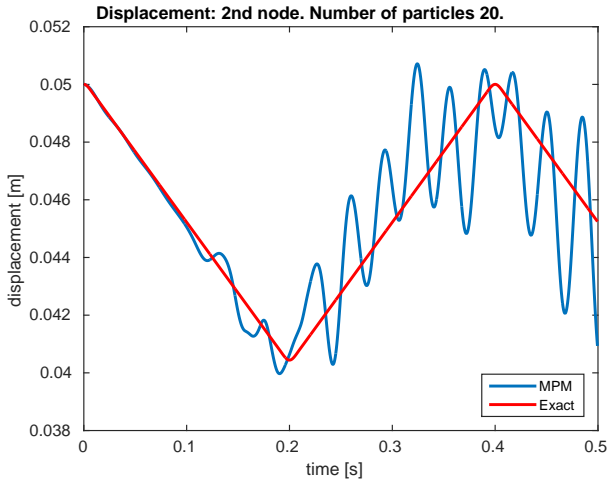
Oedometer: results



Oedometer: results



Oedometer: results



Observations

- Unexpected computational artifacts due to grid crossing / appearance of empty elements.
- Convergence only for very low number of grid nodes.

Similar observations by Bardenhagen et al.¹ and Steffen et al.²
Further research is required.

¹S. G. Bardenhagen, E. M. Kober. *The generalized interpolation material point method. Comput Model Engrg Sci*, 5 (2004), pp. 477-495.

²M. Steffen, R. M. Kirby, M. Berzins. *Analysis and reduction of quadrature errors in the material point method (MPM). Int. J. Numer. Methods. Engrg*, 76 (2008), pp. 922-948.