

Accuracy of original MPM

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Numerical accuracy

Numerical Approximation

$$u_{ex} = u_{num} + \mathcal{O}(\Delta x^n) + \mathcal{O}(\Delta t)$$

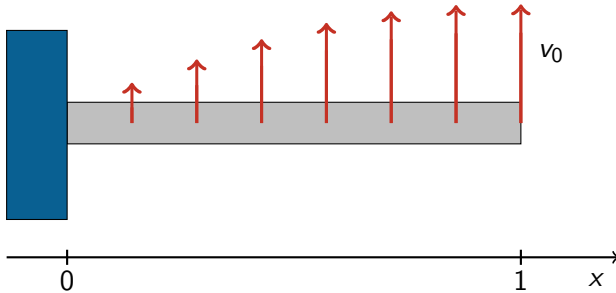
RMS Error

$$Error_{RMS} = \sqrt{\frac{1}{n_p} \left(\sum_{p=1}^{n_p} u_{num}(x_p, t) - u_{ex}(x_p, t) \right)^2}$$

Accuracy in displacement

For $\Delta t \rightarrow 0$, the order of accuracy is equal to n , i.e. the reduction of Δx by a factor of 2 decreases the RMS error by 2^n .

Vibrating linear-elastic bar



Vibrating linear-elastic bar: model

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}.$$

Boundary conditions:

$$u(0, t) = 0,$$

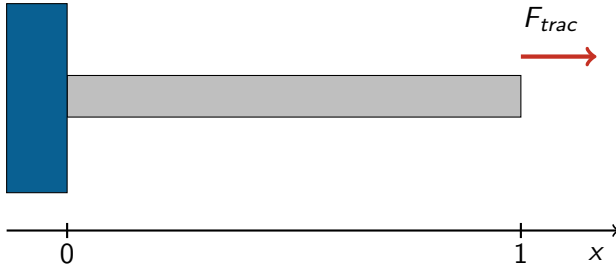
$$\frac{\partial u}{\partial x}(L, t) = 0.$$

Initial conditions:

$$u(x, 0) = 0,$$

$$\frac{\partial u}{\partial t}(x, 0) = v_0 \sin\left(\frac{\pi x}{2L}\right).$$

Vibrating hyper-elastic bar



Vibrating linear-elastic bar: model

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}.$$

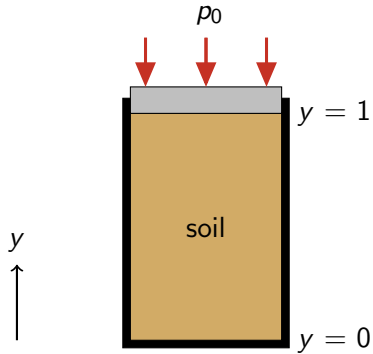
Boundary conditions:

$$\begin{aligned} u(0, t) &= 0, \\ \frac{\partial u}{\partial x}(L, t) &= \frac{\tau}{\rho} \sin\left(\frac{\pi t}{L}\right). \end{aligned}$$

Initial conditions:

$$\begin{aligned} u(x, 0) &= 0, \\ \frac{\partial u}{\partial t}(x, 0) &= 0. \end{aligned}$$

Oedometer



schematic representation

Oedometer: model

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial y^2} - g.$$

Boundary conditions:

$$u(0, t) = 0,$$

$$\frac{\partial u}{\partial y}(H, t) = -p_0/E.$$

Initial conditions:

$$u(y, 0) = 0,$$

$$\frac{\partial u}{\partial t}(y, 0) = 0.$$