Accuracy of original MPM

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Outline

- Numerical accuracy
- Benchmarks
  - Vibrating linear-elastic bar
  - Vibrating hyper-elastic bar
  - Oedometer

Numerical accuracy

Numerical Approximation

$$u_{ex} = u_{num} + \mathcal{O}(\Delta x^n) + \mathcal{O}(\Delta t)$$

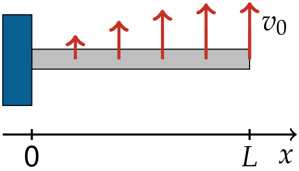
RMS Error

$$Error_{RMS} = \sqrt{\frac{1}{n_p}\left(\sum_{p=1}^{n_p} u_{num}(x_p, t) - u_{ex}(x_p, t)\right)^2}$$

Accuracy in displacement

For $\Delta t \rightarrow 0$, the order of accuracy is equal to n, i.e. the reduction of Δx by a factor of 2 decreases the RMS error by 2^n.

Vibrating linear-elastic bar



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}.$$

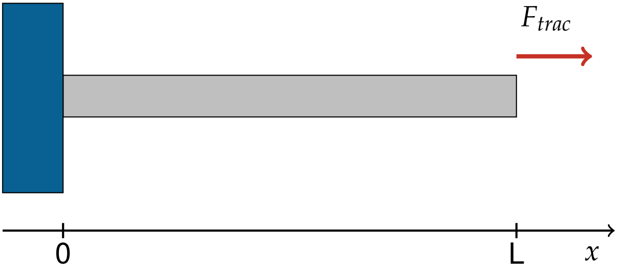
Boundary conditions:

$$u(0,t) = 0,$$
$$\frac{\partial u}{\partial x}(L,t) = 0.$$

Initial conditions:

$$u(x,0) = 0,$$
$$\frac{\partial u}{\partial t}(x,0) = v_0 \sin\left(\frac{\pi x}{2L}\right).$$

Vibrating hyper-elastic bar



Vibrating hyper-elastic bar: model

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}.$$

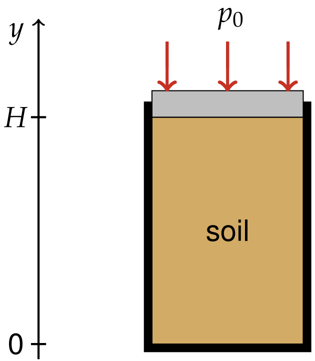
Boundary conditions:

$$u(0,t) = 0,$$
$$\frac{\partial u}{\partial x}(L,t) = \frac{\tau}{\rho}\sin\left(\frac{\pi t}{L}\right).$$

Initial conditions:

$$u(x,0) = 0,$$
$$\frac{\partial u}{\partial t}(x,0) = 0.$$

Oedometer



Oedometer: model

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial y^2} - g.$$

Boundary conditions:

$$u(0,t) = 0,$$
$$\frac{\partial u}{\partial y}(H,t) = -p_0/E.$$

Initial conditions:

$$u(y,0) = 0,$$
$$\frac{\partial u}{\partial t}(y,0) = 0.$$

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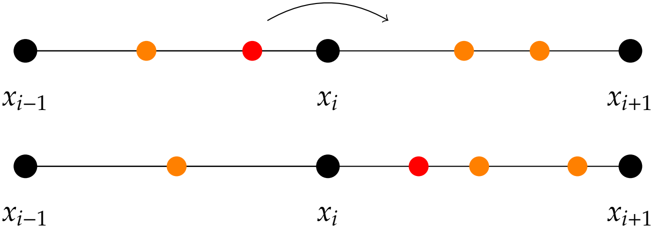
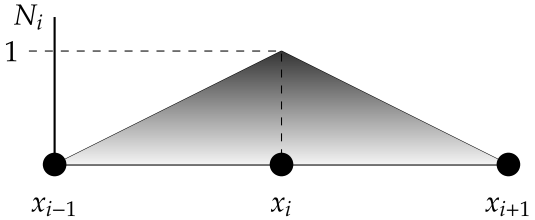
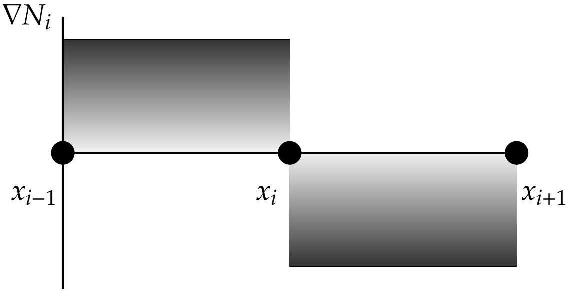
Grid-crossing



Grid-crossing: properties of shape functions





Grid crossing: internal force

$$F^i_{int} \approx \sum_{p=1}^{n_i} \sigma_p \Omega_p - \sum_{p=1}^{n_{i+1}} \sigma_p \Omega_p$$

$$F^i_{int} \approx \sigma \Omega (n_i - n_{i+1})$$

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