

Accuracy of original MPM

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Outline

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 - Vibrating linear-elastic bar
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 - Oedometer
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Numerical accuracy

Numerical Approximation

$$u_{ex} = u_{num} + O(\Delta x^n) + O(\Delta t)$$

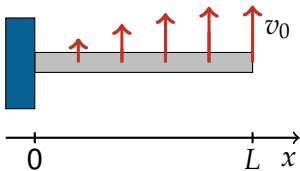
RMS Error

$$Error_{RMS} = \sqrt{\frac{1}{n_p} \left(\sum_{p=1}^{n_p} u_{num}(x_p, t) - u_{ex}(x_p, t) \right)^2}$$

Accuracy in displacement

For $\Delta t \rightarrow 0$, the order of accuracy is equal to n , i.e. the reduction of Δx by a factor of 2 decreases the RMS error by 2^n .

Vibrating linear-elastic bar



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}$$

Boundary conditions:

$$u(0, t) = 0$$

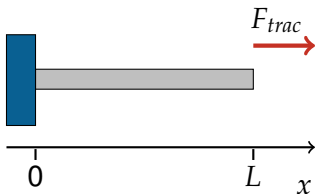
$$\frac{\partial u}{\partial x}(L, t) = 0$$

Initial conditions:

$$u(x, 0) = 0$$

$$\frac{\partial u}{\partial t}(x, 0) = v_0 \sin\left(\frac{\pi x}{2L}\right)$$

Vibrating hyper-elastic bar



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}$$

Boundary conditions:

$$u(0, t) = 0$$

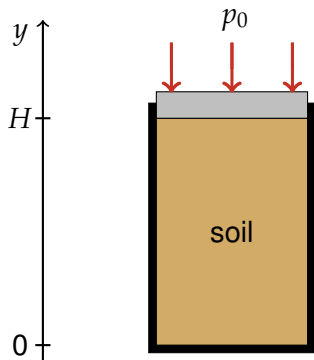
$$\frac{\partial u}{\partial x}(L, t) = \frac{\tau}{\rho} \sin\left(\frac{\pi t}{L}\right)$$

Initial conditions:

$$u(x, 0) = 0$$

$$\frac{\partial u}{\partial t}(x, 0) = 0$$

Oedometer



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial y^2} - g$$

Boundary conditions:

$$u(0, t) = 0$$

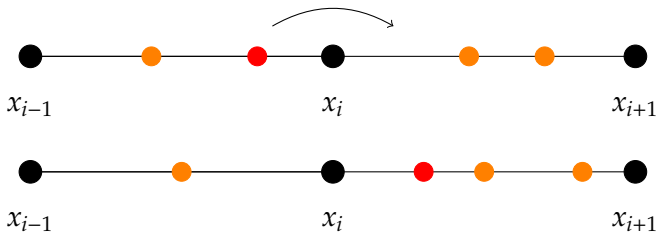
$$\frac{\partial u}{\partial x}(L, t) = \frac{\tau}{\rho} \sin\left(\frac{\pi t}{L}\right)$$

Initial conditions:

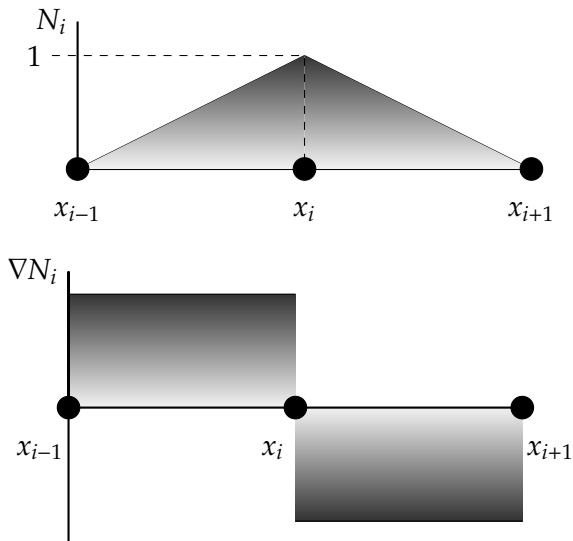
$$u(y, 0) = 0,$$

$$\frac{\partial u}{\partial t}(y, 0) = 0$$

Grid-crossing



Grid-crossing: properties of shape functions



Grid crossing: internal force

$$F_i^{int} \approx \sum_{p=1}^{n_i} \nabla N_i(\epsilon_p) \sigma_p \Omega_p + \sum_{p=1}^{n_{i+1}} \nabla N_i(\epsilon_p) \sigma_p \Omega_p$$

$$F_i^{int} \approx \sigma \Omega (n_i - n_{i+1})$$

$$\begin{cases} F_i^{int} = 0, & \text{if } n_i = n_{i+1} \\ F_i^{int} \neq 0, & \text{otherwise} \end{cases}$$