Accuracy of original MPM

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Outline

- "Original" MPM
- Numerical accuracy
- Benchmarks
  - Vibrating bar
  - Oedometer
- Accuracy of FEM
- Grid crossing
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“Original” MPM

- Modified Lagrangian algorithm
- Euler-Cromer time integration
- Piecewise linear basis functions
- No additional interventions

"Original" MPM

- Modified Lagrangian algorithm
- Euler-Cromer time integration
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- No additional interventions
- Own MATLAB implementation

Numerical accuracy

Numerical Approximation

$$u_{ex} = u_{num} + \mathcal{O}(\Delta x^n) + \mathcal{O}(\Delta t)$$

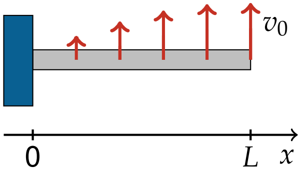
RMS Error

$$Error_{RMS} = \sqrt{\frac{1}{n_p}\left(\sum_{p=1}^{n_p} u_{num}(x_p, t) - u_{ex}(x_p, t)\right)^2}$$

Accuracy in displacement

For $\Delta t \to 0$, the order of accuracy is equal to n, i.e. the reduction of Δx by a factor of 2 decreases the RMS error by 2^n.

Vibrating bar



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}$$

Boundary conditions:

$$u(0,t) = 0$$

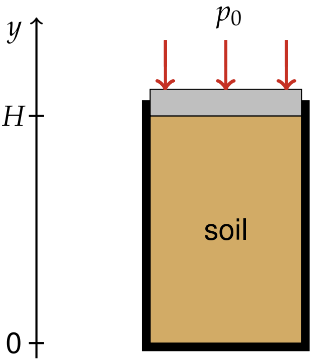
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial conditions:

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = v_0 \sin\left(\frac{\pi x}{2L}\right)$$

Oedometer



$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial^2 y} - g$$

Boundary conditions:

$$u(0,t) = 0$$

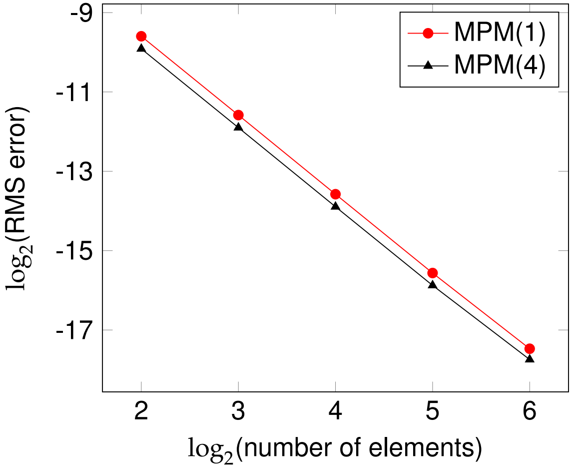
$$\frac{\partial u}{\partial y}(H,t) = \frac{p_0}{E}$$

Initial conditions:

$$u(y,0) = 0$$

$$\frac{\partial u}{\partial t}(y,0) = 0$$

Accuracy: vibrating bar



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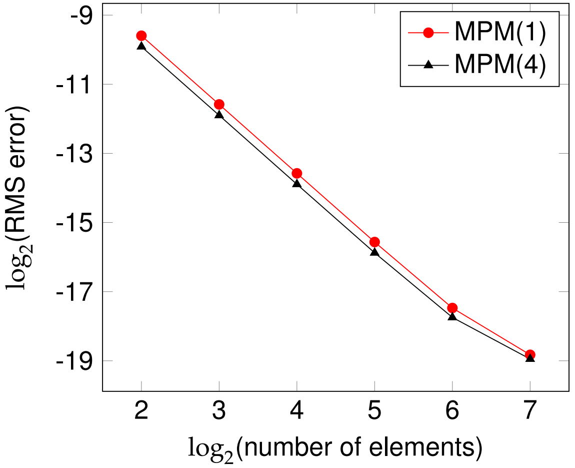

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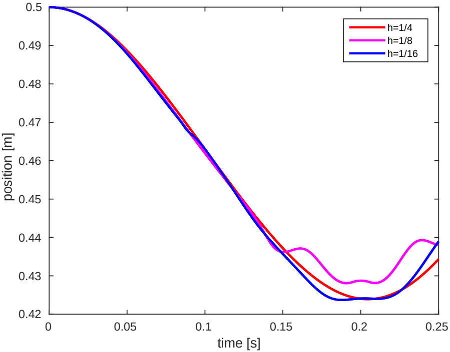
Accuracy: oedometer

Richardson's extrapolation

The order of accuracy n is obtained from

$$\frac{u_{num}(2h) - u_{num}(4h)}{u_{num}(h) - u_{num}(2h)} = 2^n.$$



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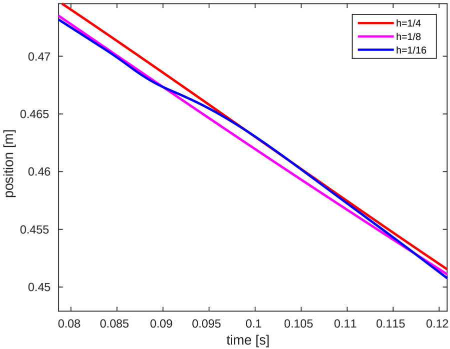

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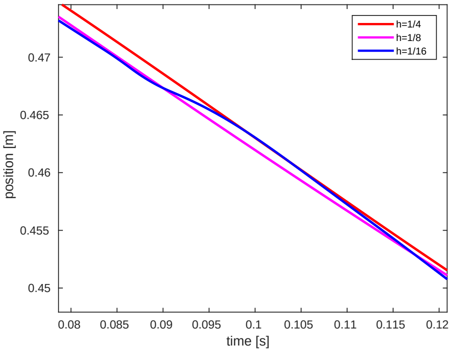
Conclusion

Lack of convergence.



FEM: oedometer

Theoretical order of accuracy

$k+1$, where k is the order of the interpolating polynomials.

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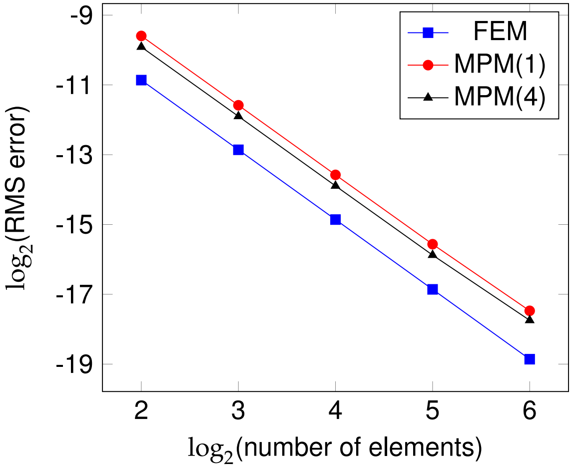
Observations

- Lack of convergence
- Problems arise due to external forces

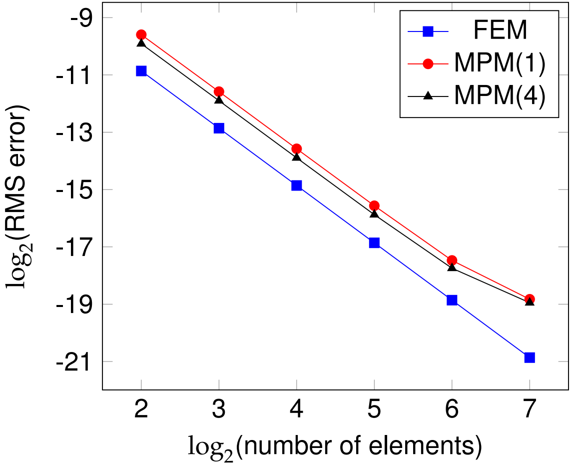
$$\mathbf{M}\frac{d\mathbf{v}}{dt} = \mathbf{F}_{ext} - \mathbf{F}_{int},$$

$$\text{where } \mathbf{F}_{ext} = \mathbf{N}(H)^T p_0 - \int_0^H \mathbf{N}^T \rho g dy$$

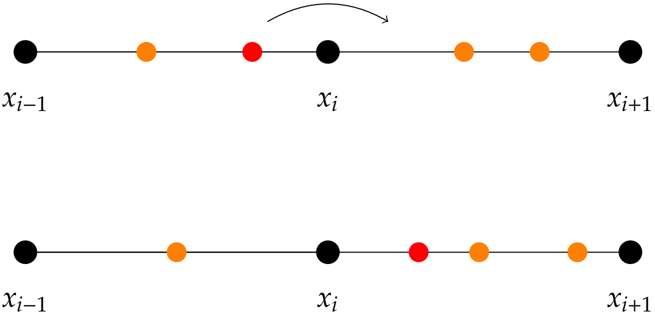
FEM: vibrating bar



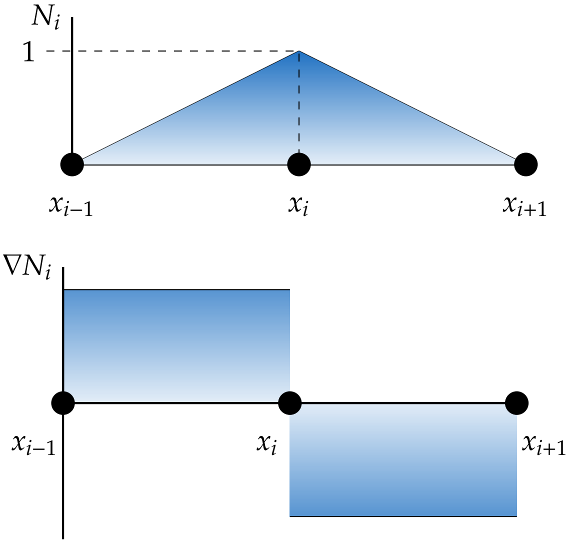
FEM: vibrating bar



Grid-crossing



Grid-crossing: properties of shape functions



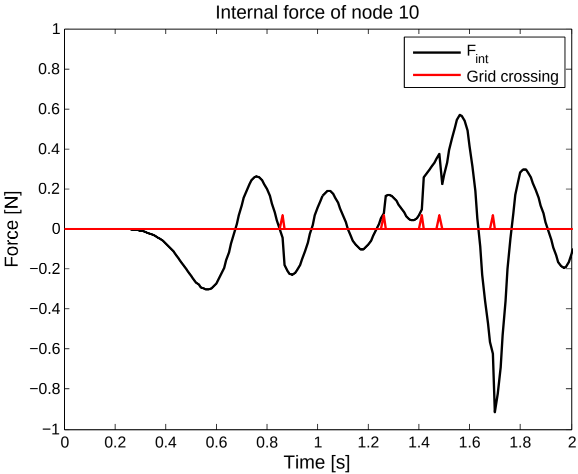
Grid crossing: internal force

$$F_{i+1}^{int} \approx \sum_{p=1}^{n_i} \nabla N_i(\xi_p)\sigma_p\Omega_p + \sum_{p=1}^{n_{i+1}} \nabla N_i(\xi_p)\sigma_p\Omega_p$$

$$F_{i+1}^{int} \approx \sigma\Omega(n_i - n_{i+1})$$

$$\begin{cases} F_{i+1}^{int} = 0, & \text{if } n_i = n_{i+1} \\ F_{i+1}^{int} \neq 0, & \text{otherwise} \end{cases}$$

Grid crossing: internal force



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Grid crossing: vibrating bar

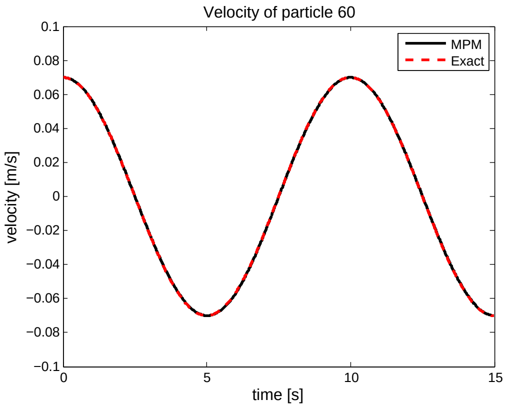


Figure: No grid crossing (30 elements).

Grid crossing: vibrating bar

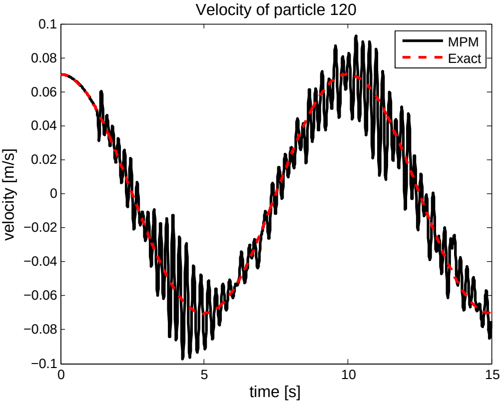
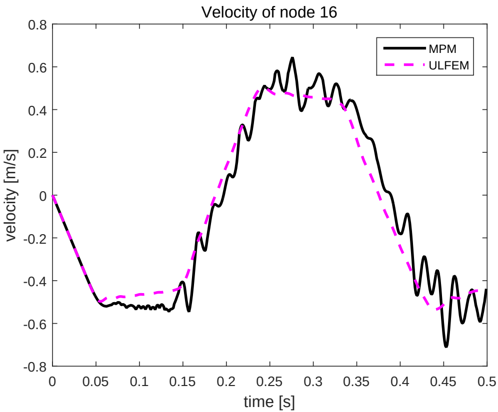


Figure: Grid crossing (60 elements).

Oedometer: velocity



Settings: vibrating bar

	Symbol	Value	Unit
Length	L	25	m
Tension	E	100	Pa
Density	ρ	1	kg/m^3
Maximum velocity	v_0	0.1	m/s
Time step	Δt	$1 \cdot 10^{-3}$	s
Measurement time[1]	t	0.5	s
PPC[2]		4	

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Settings: oedometer

	Symbol	Value	Unit
Height	L	1	m
Young's modulus	E	$1 \cdot 10^5$	Pa
Density	ρ	$1 \cdot 10^3$	kg/m^3
Load	p_0	0	Pa
Gravitational acceleration	g	9.81	m/s^2
Time step	Δt	$1 \cdot 10^{-3}$	s
Measurement time[1]	t	0.5	s
Position particle[1]	x_p	≈ 0.5	m
Number of elements[2]		30	
PPC[2]		10	

Settings: Steffen

	Symbol	Value	Unit
Length	L	1	m
Tension	E	100	Pa
Density	ρ	100	kg/m^3
Load	p_0	0.7	Pa
Gravitational acceleration	g	0	m/s^2
Time step	Δt	$1 \cdot 10^{-2}$	s
Domain		1.15	m
Number of elements		20	
PPC		10	