

Property : b

In a lattice $\{L, \leq\}$, P.T

$$(i) (a \wedge b) \vee (c \wedge d) \leq (a \vee c) \wedge (b \vee d)$$

$$(ii) (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) \quad \forall a, b, c \in L$$

Proof:

$$(i) a \vee c = \text{LUB}\{a, c\} \quad | \quad b \vee d = \text{LUB}\{b, d\}$$

$$a \leq a \vee c \quad | \quad b \leq b \vee d$$

$$c \leq a \vee c \quad | \quad d \leq b \vee d$$

$$a \wedge b \leq (a \vee c) \wedge (b \vee d) \rightarrow ①$$

$$c \wedge d \leq (a \vee c) \wedge (b \vee d) \rightarrow ②$$

From ① and ②,

$$(a \wedge b) \vee (c \wedge d) \leq (a \vee c) \wedge (b \vee d)$$

$$(iii) (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$$a \vee b = \text{LUB}\{a, b\} \quad | \quad b \vee c = \text{LUB}\{b, c\} \quad | \quad c \vee a = \text{LUB}\{c, a\}$$

$$a \leq a \vee b$$

$$b \leq b \vee c$$

$$c \leq c \vee a$$

$$b \leq a \vee b$$

$$c \leq b \vee c$$

$$a \leq c \vee a$$

$$a \wedge b \leq (a \vee b) \wedge (b \vee c)$$

$$b \wedge c \leq (b \vee c) \wedge (c \vee a)$$

$$b \wedge c \leq (b \vee c) \wedge (c \vee a)$$

$$c \wedge a \leq (a \vee b) \wedge (c \vee a)$$

$$a \wedge b \leq (c \vee a) \wedge (a \vee c)$$

$$a \wedge b \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$$b \wedge c \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$$c \wedge a \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

\Rightarrow

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) //.$$