

Property: b

In a lattice $\{L, \leq\}$, P.T

- (i) $(a \wedge b) \vee (c \wedge d) \leq (a \vee c) \wedge (b \vee d)$
- (ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) \quad \forall a, b, c \in L$

Proof:

(i)	$a \vee c = \text{LUB}\{a, c\}$		$b \vee d = \text{LUB}\{b, d\}$
ii)	$a \leq a \vee c$		$b \leq b \vee d$
	$c \leq a \vee c$		$d \leq b \vee d$

$$a \wedge b \leq (a \vee c) \wedge (b \vee d) \rightarrow \textcircled{1}$$

$$c \wedge d \leq (a \vee c) \wedge (b \vee d) \rightarrow \textcircled{2}$$

From $\textcircled{1}$ and $\textcircled{2}$,

$$(a \wedge b) \vee (c \wedge d) \leq (a \vee c) \wedge (b \vee d)$$

$$(ii) (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$a \vee b = \text{LUB}\{a, b\}$	$b \vee c = \text{LUB}\{b, c\}$	$c \vee a = \text{LUB}\{c, a\}$
$a \leq a \vee b$	$b \leq b \vee c$	$c \leq c \vee a$
$b \leq a \vee b$	$c \leq b \vee c$	$a \leq c \vee a$

$a \wedge b \leq (a \vee b) \wedge (b \vee c)$	$b \wedge c \leq (a \vee b) \wedge (b \vee c)$
$b \wedge c \leq (b \vee c) \wedge (c \vee a)$	$c \wedge a \leq (b \vee c) \wedge (c \vee a)$
$c \wedge a \leq (a \vee b) \wedge (c \vee a)$	$a \wedge b \leq (c \vee a) \wedge (a \vee c)$

$$a \wedge b \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$$b \wedge c \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

$$c \wedge a \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

\Rightarrow

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) //$$