## S.E. (Computer Engg.) (Semester – III) Examination, Nov./Dec. 2012 (Revised Course 2007-08) APPLIED MATHEMATICS – III

Duration: 3 Hours

Total Marks: 100

Instructions: 1) Answer five questions and atleast one from each Module.

Figures to the right indicate full marks.

Make suitable assumptions wherever required.

4) Use statistical tables wherever required.

## MODULE-1

I. a) i) Define adjoint of a square matrix.

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ii) If A is a n×n matrix then show that adj (adj A) =  $|A|^{n-2}A$ , if  $|A| \neq 0$ .

b) Reduce the following matrix to its normal form and hence find its rank.

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$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

c) Test the linear system of equation given below for consistency and solve it. 7

$$x + y + z = 4$$
$$2x + 5y - 2z = 3$$

$$x + 7y - 7z = 5$$

a) Find the eigen values and the associated eigen vectors of the matrix.

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$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

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 Find the characteristic equation of the matrix given below and hence find its inverse by analog the Cayley Hamilton theorem.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

c) For the real symmetric matrix A given below find the orthogonal matrix P such that P-1 AP is a diagonal matrix.

$$A = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

MODULE-2

III. a) Let X be a discrete random variable with the following probability distribution: 8

$$X = n$$
: 1 2 3 4 5 6 7  
 $P(X = n)$ : C 2C 2C 3C  $C^2$  2 $C^2$  ( $7C^2 + C$ )

- i) Find the value of C
- ii) Compute P (1 < X < 4)
- iii) Find the mean of X.
- b) Compute the moment generating function for the exponential distribution with density function  $f(x)=2e^{-2x}$ , x>0

= 
$$0$$
 ,  $x \le 0$ .

c) Let 
$$P(A) = \frac{1}{3}$$
,  $P(B) = \frac{3}{4}$  and  $P(A \cup B) = \frac{11}{12}$ . Find  $P(A \mid B)$  and  $P(B \mid A)$ .

- IV. a) The life of a certain type of electrical lamps is normally distributed with mean 2040 hours and standard deviation 60 hours. In a consignment of 3000 lamps, how many would be expected to burn for:
  - i) More than 2150 hours, and
  - ii) Less than 1950 hours.

b) The breaking strengths of cables produced by a manufacturer have a mean of 1800 pounds and a standard deviation of 100 pounds by introducing a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test this claim a sample of 50 cables is tested and it is found that the mean breaking strength is 1850 lb. Can we support the claim at the 5% level of significance.

c) The equation of the two lines of regression one, x = 19.13 - 0.87 y and y = 11.4 - 0.50 x. Find:

- i) Mean of x values
- ii) Mean of y values and
- iii) The correlation coefficient between x and y.

## MODULE-3

V. a) Find the Laplace transform of the following:

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- i)  $f(t) = e^{3t} \sin 2t \sin t$ .
- ii)  $g(t) = te^{-4t} \sin 3t$ ,

iii) 
$$h(t) = \int_{0}^{t} \frac{e^{-u} \sin u}{u} du$$

b) Find the inverse Laplace transform of the following:

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i) 
$$\bar{f}(s) = \tan^{-1}\left(\frac{1}{s}\right)$$
,

ii) 
$$\overline{g}(s) = \frac{1}{s^2 - 2s - 3}$$

c) Let f(t) be periodic with period "p". Prove that,

5

$$L(f(t)) = \frac{1}{1 - e^{-sp}} \int_{0}^{P} e^{-st} f(t) dt$$

## COMP 3-1(RC)



- VI. a) Use the Laplace transform to evaluate  $\int te^{-3t} \cos 2t \, dt$ .
- 6

b) Use the convolution theorem to find the inverse Laplace transform of,

$$\bar{f}(s) = \frac{1}{(s+1)(s+2)}$$

c) Use the Laplace transform to solve the differential equation

$$y''(t) - 4y'(t) + 8y(t) = e^{2t}, y(0) = 2, y'(0) = -2$$

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MODULE-4

VII. a) Find the Fourier transform of  $f(x) = 1 - x^2$ ,  $|x| \le 1$ = 0, |x| > 1

Hence evaluate  $\int_{0}^{\infty} \left( \frac{\sin x - x \cos x}{x} \right) \cos \frac{x}{2} dx$ 

- b) Find the inverse Fourier transform of,  $\hat{f}(s) = 4 |s|, |s| \le 4$ = 0, |s| > 4
- c) Solve for f(n) the integral equation,  $\int_{0}^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda}$

VIII. a) Find the Z-transform of the following:

i) 
$$f(n) = \frac{1}{n(n-1)}$$
 ii)  $g(n) = \frac{2n+3}{(n+1)(n+2)}$ 

ii) 
$$g(n) = \frac{2n+3}{(n+1)(n+2)}$$

b) Find the inverse Z-transform of the following:

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$$f(z) = \frac{z^2}{(z+4)^2}$$

c) Use the Z-transform to solve the difference equation,

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$$y(n + 3) - 3y(n + 1) + 2y(n) = 0, n \ge 0$$

and 
$$y(0) = 4$$
,  $y(1) = 0$  and  $y(2) = 8$