(Pages : 04)

COMP 4 - 1 (RC)

B - 600

[Total No. of Questions : 8]

S.E. (Comp.) (Semester - IV) (Revised Course) Examination, Nov. - 2011

DISCRETE MATHEMATICAL STRUCTURES

Total Marks : 100

Duration : 3 Hours

Instructions : 1) *Attempt any five questions choosing at least one question from each Module.*

2) *Assume suitable data wherever required.*

3) *Figures to the right indicate marks allocated to the sub - question.*

MODULE - I

Q1) a) i) If $A \cap B \subseteq A \cap C$ for three sets, does it imply $B = C$? Justify your claim fully. [8]

ii) If P (X) denotes the power set of the set X, show that $P(A \cap B) = P(A) \cap P(B)$ for any two sets A and B.

b) Give a non - trivial example of a relation R on the set $S = \{a, b, c, d\}$ which is reflexive and symmetric but not transitive. Justify your claim briefly. [3]

c) Draw the Hasse - Diagram for the Poset $(S, \leq)$ where $S = \{2, 3, 4, 5, 6, 8, 9, 12, 18, 72\}$ and where $a \leq b$ if, and only if, $a \mid b$; $\forall$ a, b $\in$ S. [3]

d) Show that the function $f : (R - \{3/5\}) \to (R - \{7/5\})$ given by $f(x) = [(7x + 4) / (5x - 3)]$ is injective on its domain $(R - \{3/5\}]$.
Further assuming that f is surjective, compute (the inverse of f) $f^{1}(y)$ in terms of y. [6]

Q2) a) State Pigeon Hole Principle.
Use the pigeonhole principle to show that among any five points placed in an equilateral triangle of side 2 cm, there are at least two points which are at most 1cm apart. [5]

b) Without actually carrying out the multiplication, find the remainder when the integer $[7 \times 75 \times 29 \times (37)^2 \times (53)^3 \times 539 \times (1269)^2]$ is divided by 12. [5]

c) State the Inclusion - Exclusion principle for three nonempty sets. Use the principle of Inclusion and Exclusion to find the number of integers between 1 and 8,000, including both 1 and 8,000 which are divisible by neither 6 nor by 8 nor by 15. Explain briefly the steps followed by you. [7]

d) If a, b, c and d are any four integers and n is any natural number> 3, show that $a \equiv b \pmod n$ and $c \equiv d \pmod n \Rightarrow ac \equiv bd \pmod n$. [3]

P.T.O.

MODULE - II

Q3) a) Define a monoid and a semigroup. Give an example of a semigroup which is not a monoid and also give an example of a monoid which is not a group. Justify your claim in each case. **[5]**

b) If (G, *, e) is any group, for any a, b and c in G show that

i) $(a * b)^{-1} = b^{-1} * a^{-1}$ and

ii) $a * b = a * c \Rightarrow b = c$. **[4]**

c) Show that the intersection of anytwo subgroups of a group is again a subgroup. Show with an example that the union of any two subgroups of a group need not be a subgroup. Justify your claim. **[5]**

d) Let (R, + , .) be a ring under consideration in which $x.x = x^2 = x$ for each x in R. Show that the ring is commutative. **[6]**

Q4) a) Examine whether the vectors (1, 2, –3), (–2, 3, 4) and (3, –4, 5) are linearly independent in the vector - space R^3 over the usual field of the real numbers R. **[4]**

b) Extend the set of vectors {(2, 3, 5), (–3, 5, 2)} as basis of the vector - space R^3 (over the usual field of the real numbers R) in two different ways. **[4]**

c) Which of the following subsets of the vector - space R^3 (over the usual field of the real numbers R) are the sub-spaces of R^3? Give reasons in each case. **[7]**

i) $W = \{(x, y, z) / x, y, z \in R, x \leq y\}$,

ii) $X = \{(x, y, z) / x, y, z \in R, x - y = z\}$ and

iii) $Y = \{(x, y, z) / x, y, z \in R, x + y = 7\}$.

d) $T : R^3 \rightarrow R^3$ is a linear transformation given by

$T(x, y, z) = (x - y, y - z, x - z); \forall (x, y, z) \in R^3$.

Find a basis of the kernel of T and also a basis of the range of T. **[5]**

MODULE - III

Q5) a) i) Explain what do you understand from the terms Tautology and Contradiction. Give one illustrative example of each. Also justify your claim briefly in each case.

ii) Without using the truth tables, show that

$(p \rightarrow q) \rightarrow q \equiv (p \vee q)$. **[5]**

b) Define functionally complete set of connectives. Show that the set S = $\{\downarrow\}$ consisting of only one connective NOR $\downarrow$ is functionally complete. **[7]**

c) Let (B, +, •, --, 0, 1) be a Boolean Algebra.

for any a in B, show that

i) $a \bullet a = a$ and ii) $a \bullet 0 = 0$. **[4]**

d) Obtain the principal conjunctive normal form of the Boolean function $f(x, y, z) = x + x \bullet y + y \bullet z$. **[4]**

Q6) a) State the Principle of Mathematical Induction. Use the Principle of Mathematical Induction to show that $(n^3 + 2n)$ is an integral multiple of 3; $\forall n \in N$. **[6]**

b) A person climbs a stair - case climbing the either one stair or two stairs at a time. If he decides to climb a staircase containing n stairs and if a_n denotes the number of different ways he can do the climbing, find the recurrence relation in terms of a_n. Justify your claim fully. State also the initial conditions. Compute the values of a_4 and a_5. **[7]**

c) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2n + 3$ with $a_0 = 0$ and $a_1 = 1$. **[7]**

MODULE - IV

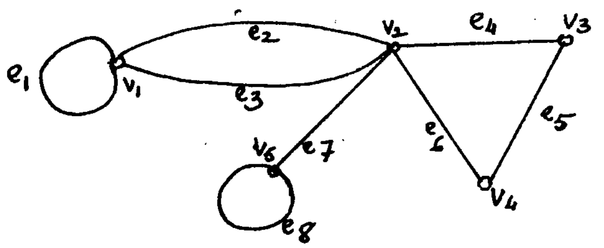
Q7) a) Define

i) an Eulerian graph and

ii) a Hamiltonian graph.

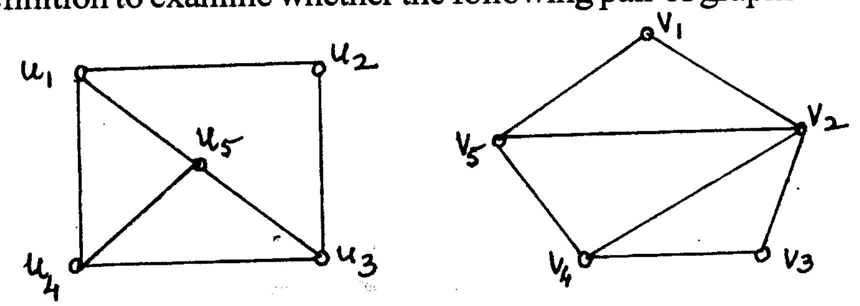
Give an illustrative example of a Hamiltonian graph which is not Eulerian. Also give an illustrative example of an Eulerian graph which is not Hamiltonian. Justify your claim in each case. **[7]**

b) Explain what do you understand by an Incidence Matrix of a finite labeled graph. Obtain an incidence matrix of the following finite labeled graph: **[4]**

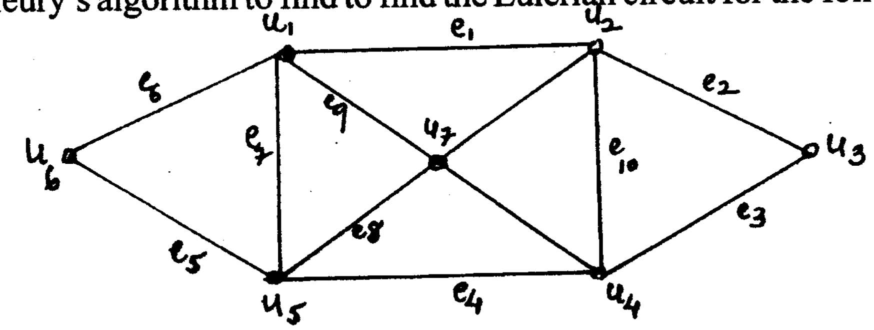


c) Define : graph isomorphism.

Use your definition to examine whether the following pair of graphs are isomorphic: **[5]**



d) Use Fleury's algorithm to find to find the Eulerian circuit for the following graph: **[4]**



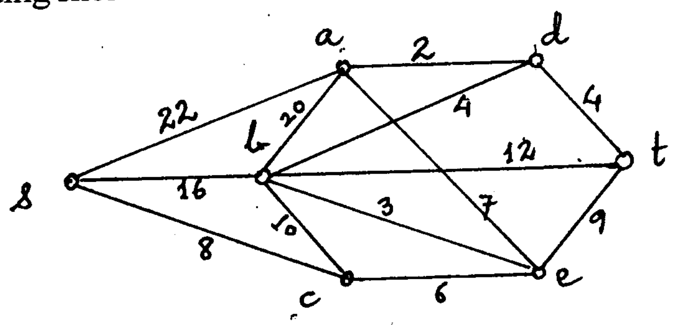
Q8) a) i) Give three different definitions of a tree. [6]

ii) A tree has three vertices of degree two, two vertices of degree three, and five vertices of degree four. Find the number of vertices, if any, of degree one this tree has.

b) Define a bi-partile graph. [4]

Give an example of a bi-partile graph for which the bi-partition of the vertex set is not unique. Justify your claim.

c) Use only Dijkstra's shortest path algorithm to find the shortest path and the corresponding shortest distance between the points s and t in the following weighted graph. [5]



d) Use Kruskal's algorithm to obtain a minimal spanning tree for the following connected graph. Explain briefly and sequentially the steps followed by you. [5]