28/11/2013

COMP 4 – 1 (RC)

S.E. (Comp.) (Semester – IV) (RC) Examination, Nov./Dec. 2013
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

Instructions : 1) *Attempt **any five** questions, atleast **one** from **each** Module.*
2) *Assume suitable data **if necessary**.*

MODULE – I

1. a) Prove that for the non-empty sets A and B : 6

i) $A \oplus B = (A - B) \cup (B - A)$

ii) $A \oplus (A \oplus B) = B$

Where $\oplus$ denotes symmetric difference.

b) Give an example of a relation on the set of positive integers which is 6

i) Symmetric and reflexive but not transitive.

ii) Reflexive and transitive but not symmetric.

c) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as 8

$$f(x) = \begin{cases} 2x^2 + 1 & x \geq 0 \\ 2x + 1 & x < 0 \end{cases}$$

Test whether f is bijective ? If yes find f^{-1}.

2. a) Let Z^+ denote set of all positive integers. Define a relation '$\leq$' on Z^+ as $a \leq b$ iff a/b. Prove that '$\leq$' is a partial order on $\mathbb{Z}^+$. 6

b) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9; 10} with partial order '/'. Let B= {2, 3, 4, 5} $\subset$ A. 7

i) Draw Hasse diagram of A.

ii) Find the upper bound, lower bound, least upper bound, greatest lower bound of Set B.

c) In how many ways can 6 different toys be distributed among three different children, if each child has to get atleast one toy. 7

P.T.O.

MODULE – II

3. a) Consider the monoid Z_5 under multiplication modulo 5. Find the submonoid generated by $S = \{3, 4\}$. 6

   b) Let G be a group. Prove that G is Abelian if $(ab)^m = a^m b^m$ for three consecutive integers. 7

   c) Prove that $S = \{3n = n \in \mathbb{Z}\}$ is a subgroup of $(\mathbb{Z}, +)$. 7

4. a) Prove that a ring R has no zero divisors if and only if it satisfies the cancellation laws for multiplication in R. 6

   b) Prove that every finite integral domain is a field. 6

   c) Let $W = \{(x_1, x_2, x_3) : \; x_1 + x_2 = x_3\}$

   Prove that :

   i) W is a subspace of $\mathbb{R}^3$. 8

   ii) Find the dimension of W.

MODULE – III

5. a) Explain what do you understand from the terms Tautology and contradiction. Give one illustrative example of each. 5

   b) Without using truth tables prove that 5

   $(\sim P \wedge (\sim Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$

   c) Let (B, +, ., ,, 0, 1) be a Boolean Algebra for any $a \in B$ show that 4

   i) $a \cdot a = a$

   ii) $a \cdot o = o$.

   d) Obtain the principal disjunctive normal form of the Boolean function. 6

   $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + x_3)$

6. a) State the principal of mathematical induction and use it to prove that 3 divides $n^3 + 2n$ for n.a non-negative integer. 7

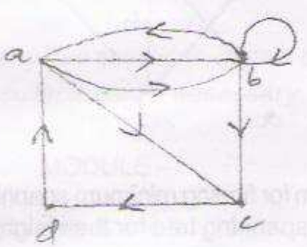
   b) Find a recurrence relation for the number of bit string of length n that contain a pair of consecutive zero's. What are the initial condition ? 6

   c) Solve the recurrence relation : 7

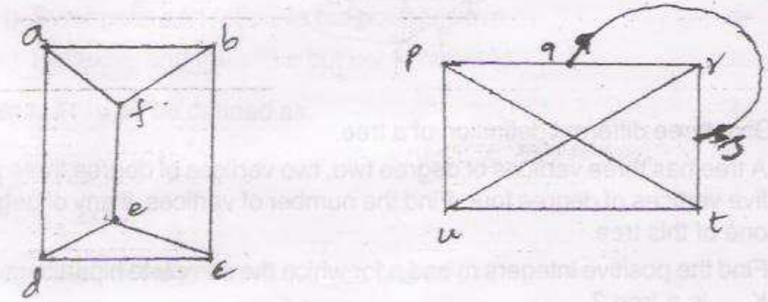
   $a_r - 8a_{r-1} + 15a_{r-2} = 4^r$ given $a_0 = 0, a_1 = 1, r \geq 2$.

MODULE – IV

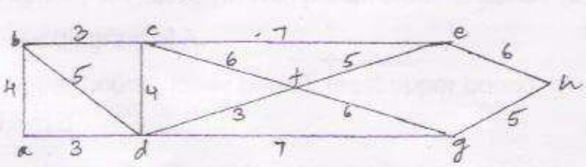
7. a) Define adjacency matrix of a directed graph. Represent the following graph by an adjacency matrix. 5



b) Define graph isomorphism. Check whether the following graphs are isomorphic. 5

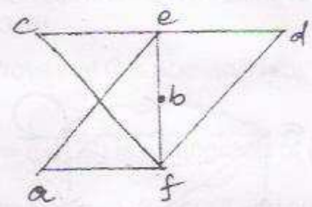


c) Apply Dijkstra's Algorithm to find the shortest path between a and h in the following weighted graph : 6

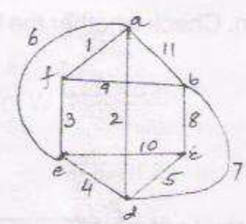


d) Prove that in a undirected graph, there are always even number of odd degree vertices. 4

8. a) Define a bipartite graph. Is the following graph bipartite. **5**



b) State Prim's algorithm for finding minimum spanning tree. Use Prim's algorithm to find the minimum spanning tree for the weighted graph shown below : **7**



c) i) Give three different definition of a tree. **3**

ii) A tree has three vertices of degree two, two vertices of degree three and five vertices of degree four. Find the number of vertices, if any of degree one of this tree. **3**

iii) Find the positive integers m and n for which the complete bipartite graph $K_{m,n}$ is a tree ? **2**