Morning 6/6/2013 Sem - IV Comp Dept

COMP 4 – 1 (RC)

S.E. (Comp.) (Semester – IV) (RC) Examination, May/June 2013

DISCRETE MATHEMATICAL STRUCTURE

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, choosing at least **one** from **each** Module.*

*2) Assume **suitable** data if **necessary**.*

Module – I

1\. a) Let A, B & C be three non-empty sets. Prove that

i) $(A - C) \cap (B - C) = (A \cap B) - C$

ii) $(A - B) \times C = (A \times C) - (B \times C)$ 6

b) Let $\mathbb{Z}$ be the set of integers. Define a relation R on $\mathbb{Z}$ as follows. xRy if and only if 3 divides x-y. Show that R is an equivalence relation and also find its distinct equivalence classes. 8

c) Let $f : A \to B$ and $g : B \to C$ be two surjective functions. Show that $g \circ f : A \to C$ is also surjective. 6

2\. a) Let X be an non-empty set. Define a relation '$\leq$' on P(X) as follows : for any $A, B \in P(X)$, $A \leq B$ if and only if $A \subseteq B$. Prove that P(X) is a poset. 6

b) Draw Hasse diagram for the poset $(S, \leq)$ where S = {2, 4, 5, 8, 10, 12, 16, 20, 22, 25} and $a \leq b$ if and only if $a|b$ $\forall a, b \in S$. Also find minimal and maximal elements of S. 8

c) Show that if any 20 people are selected, then we can choose a subset of 3, so that all three are born on the same day of the week. 6

Module – II

3. a) Let $(G, *)$ be a group. Prove that

   i) $a * b = a * c \Rightarrow b = c$

   ii) $b * a = c * a \Rightarrow b = c$

   where a, b and $c \in G$. 6

   b) Prove that $(G, *)$ is an Abelian group if and only if $(a*b)^2 = a^2 * b^2$, where $a, b \in G$. 7

   c) State and prove Lagrange's theorem for groups. 7

4. a) If $(R, +, .)$ is a ring such that $a^2 = a \ \forall \ a \in R$, prove that

   i) $a + a = 0 \quad \forall\, a \in R$

   ii) $a + b = 0 \quad \Rightarrow a = b$

   iii) R is a commutative ring 7

   b) Show that $W = \{(x, y, z) : x - y + z = 0\}$ is a sub space of $\mathbb{R}^3$. Also find a basis for W. 7

   c) Show that $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x_1, x_2, x_3) = (x_1 + x_2, x_2, x_1 - x_3)$ is a linear transformation. 6

Module – III

5. a) Define a Boolean Algebra B. In a Boolean Algebra B, prove that :

   i) $a + a.b = a \qquad \forall\, a, b \in B$

   ii) $(a + b)' = a' \cdot b' \qquad \forall\, a, b \in B$ 6

   b) Define functionally complete set of connectives. Show that $\{\downarrow\}$ is functionally complete. 5

   c) Define conjunctive normal form. Express the following expression in the principal conjunctive normal form. $(\sim p \to r) \wedge (q \leftrightarrow p)$ where p, q and r are propositions. 5

   d) Using rules of inference, show that the premises $E \to S$, $S \to H$, $A \to H$ and the conclusion $E \wedge A$ is consistent. 4

6. a) State the principal of mathematical induction and use it to prove that

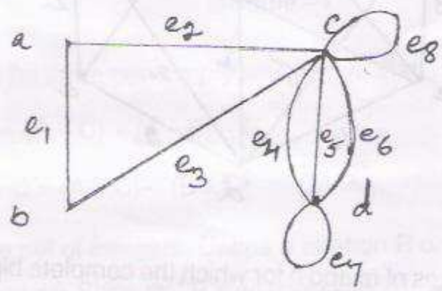
$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$ 7

b) A person invests Rs. 25,000/- @ 9% interest compounded annually. How much will be the total amount at the end of 17 years ? 6

c) Solve the recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 5$, $n \geq 2$, and $a_0 = 0$, $a_1 = 1$. 7

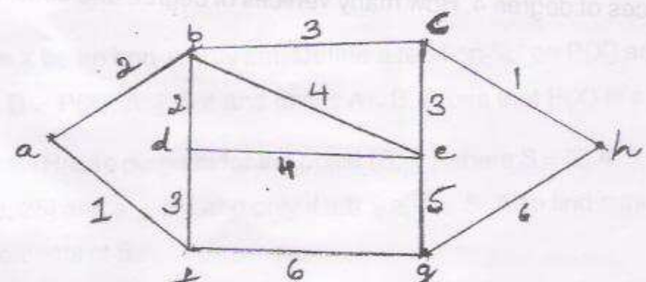
Module – IV

7. a) Define incidence matrix of an undirected graph. Represent the following graph by an incidence matrix. 5



b) Define a bipartite graph and a complete graph. Is it possible to draw a bipartite graph which is also complete graph ? Justify. 5

c) Apply Dijkstra's algorithm to find the shortest path between a and h in the following weighted graph. 6

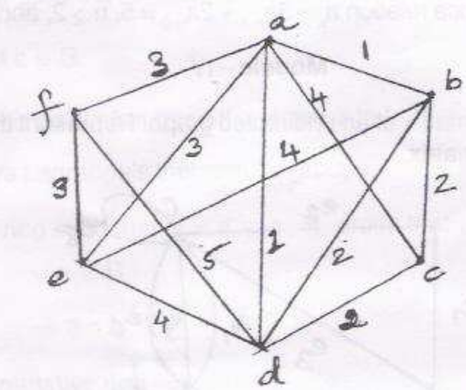


d) Is it possible to draw a graph with 7,7,7,7,6,6,5,5,5,4,4,3 degree sequence ? Justify. 4

8. a) i) Give three different definition of a tree. 3

ii) Draw a tree having three vertices of degree 3, one vertex of degree two and five vertices of degree five. 3

b) Use Kruskal's algorithm to obtain a minimal spanning tree for the graph. 7



c) i) State the values of m and n for which the complete bipartite graph $k_{m,n}$ is a tree. 1

ii) Show that a full (regular) m-ary tree with 'c' internal vertices contain $n = m, + 1$ vertices. 3

iii) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree one does it have ? 3