COMP 4-1 (RC)

S.E. (Comp.) (Semester – IV) Examination, May/June 2012
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours　　　　　　　　　　　　　　　　Total Marks : 100

Instructions : *1) Attempt **any five** questions choosing at least **one** question from **each** Module.*
*2) Assume suitable data **wherever** necessary.*
*3) Figures to the **right** indicate marks allocated to that sub-questions.*

MODULE – I

1. a) If A and B denote non empty sets then

   i) Prove $P(A) \cap P(B) = P(A \cap B)$

   ii) $P(A) \cup P(B) \subseteq P(A \cup B)$. Give an example to show that $P(A \cup B)$ need not be a subset of $P(A) \cup P(B)$.

   ($P(X)$: power set of X) **(3+4=7)**

   b) Let $\mathbb{Z}$ be the set of integers and 'n' be a fixed positive integer. Let R be a relation on $\mathbb{Z}$ defined by : **8**

   for a, $b \in \mathbb{Z}$; a R b iff $a \equiv b \pmod{n}$. Show that R is an equivalence relation on $\mathbb{Z}$. Express $\mathbb{Z}$ as a disjoint union of distinct equivalence classes of R.

   c) Give an example of a function which is injective but not subjective and a function which is subjective but not injective justify. **5**

2. a) Use the Pigeonhole principle to prove that if any five points are chosen at random within a square of length 2 then there are atleast two points whose distance apart is atmost $\sqrt{2}$. **6**

   b) Draw the Hasse diagram representing the partial ordering R on the set S when S = {1, 2, 3, 4, 6, 8, 12, 15, 20} and R is defined by a R b iff 'a' divides 'b'.

   i) Find the greatest and the least element (if they exist).

   ii) Find the maximal and the minimal elements.

   iii) Find the least upper bound and the greatest lower bound of A = {2, 3, 4}. **8**

   c) How many positive integers not exceeding 2000 are divisible by 7 or 13 ? **6**

P.T.O.

MODULE – II

3. a) Define a semi-group and a monoid. Give an example of a semi-group which is not a monoid and an example of a monoid which is not a group. 5

b) Define an Abelian group.

If (G, $*$) is a cyclic group then is it Abelian ? Justify. 4

c) Show that a group of prime order has only trivial subgroups. 5

d) Let $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} ; x \in \mathbb{R} \text{ and } x \neq 0 \right\}$. Note that (G,$*$), where $*$ denotes matrix multiplication, is a group. Define $f : (G, *) \to (\mathbb{R} - \{0\}, .)$ by $f\left(\begin{bmatrix} x & x \\ x & x \end{bmatrix}\right) = 2x; \forall \begin{bmatrix} x & x \\ x & x \end{bmatrix} \in G$. Prove that f is a homomorphism and find its kernel. ('.' denotes usual multiplication) 6

4. a) Let (R, +, .) be any ring with unity 1 and let $(x.y)^2 = x^2.y^2; \forall x, y \in R$. show that R is commutative. 6

b) Show that $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 / x_1 + x_2 = x_3\}$ is a subspace of $\mathbb{R}^3$ once $\mathbb{R}$. Also find a basis for W. 7

c) Define : $T : \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ by $T(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_2 - x_3)$. Show that T is a linear transformation. 7

MODULE – III

5. a) Without using truth tables ; show that

i) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

ii) $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q) \equiv T$ (Tautology) 6

b) Explain 'Rule of Conditional Proof' (Rule CP) use rule CP to show that $R \to S$ can be derived from $P \to (Q \to S)$, $\neg R \vee P$ and Q. 7

c) Let (B, +, ., –) be a Boolean algebra where + denotes disjunction, . denotes conjunction and – denotes complementation. Show that for all $a, b \in B$.

i) $a + a = a$ ii) $a . 0 = 0$ iii) $\overline{a \cdot b} = \overline{a} + \overline{b}$ 7

6. a) Obtain the Principal Conjunctional Normal Form (PCNF) of the Boolean expression $f(x_1, x_2, x_3) = x_1 + x_2\bar{x}_3 + x_1x_3$ 6

b) Use mathematical induction to prove that :

$$\frac{1}{1.3}+\frac{1}{3.5}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$ for all positive integers 'n'. 6

c) i) Find the recurrence relation for the number of ways of climbing 'n' steps if the person climbing the steps can take one, two or three steps at a time. Also state the initial conditions.

ii) Solve the recurrence relation. $a_n - 5a_{n-1} + 6a_{n-2} = 7;\ a_0 = 0,\ a_1 = 1$ (3+5)

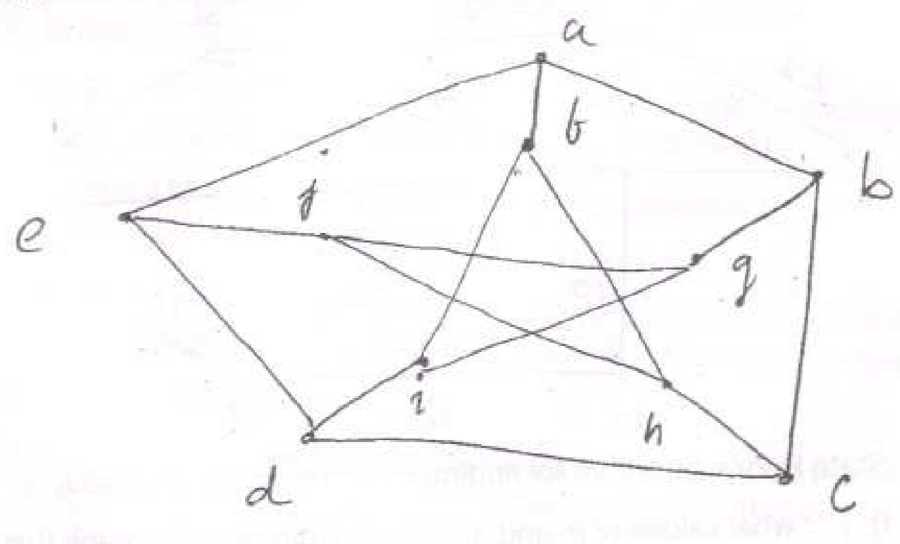
MODULE – IV

7. a) Define the following and give one example of each

i) Bipartite graph

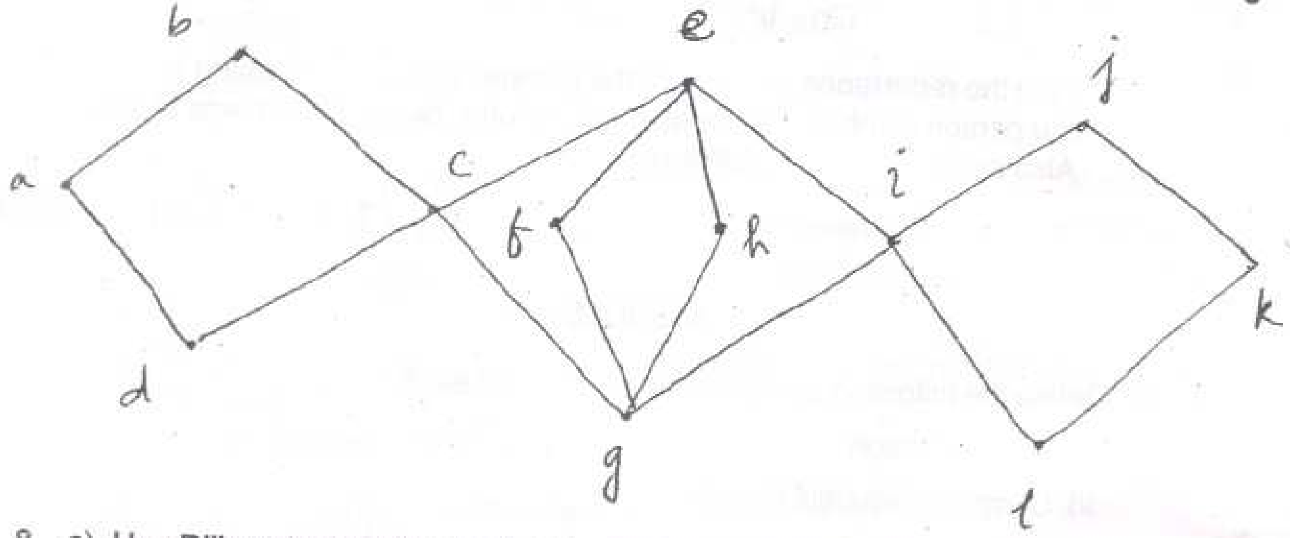
ii) Complete Bipartite Graph 4

b) Determine, using a suitable theorem, whether the following graph is non planar. 6

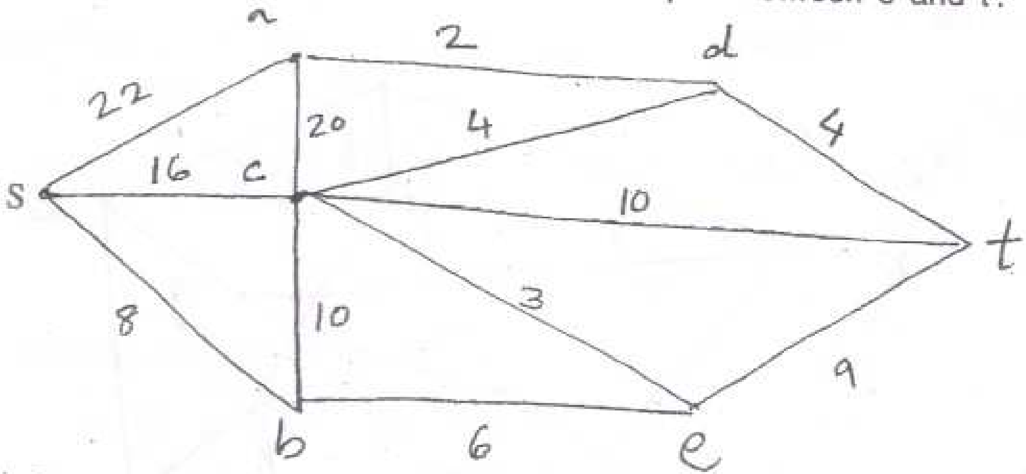


c) Explain Konigsberg's bridge problem and draw the graph representing the problem. 5

d) Use Fleury's algorithm to find an Euler path for the graph given below. 5



8. a) Use Dijkstra's algorithm to find the shortest path between 's' and 't'. 8



b) State Prim's algorithm for finding minimum spanning trees. 5

c) i) For what values of m and n, the complete bipartite graph Km, n is a tree ?

ii) Show that a full (regular) m – any three with 'i' internal vertices contains n = mi + 1 vertices.

iii) A tree has 2 vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have ? 7