COMP 4 – 1 (RC)

S.E. (Comp.) (Semester – IV) (R.C.) Examination, May/June 2014
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data, **if necessary**.*

MODULE – I

1. A) $\{A_k : k = 1, 2, ...\}$ be collection of subsets of some universal set U then show that

$$\left(\bigcap_{k \in I} A_k\right)' = \bigcup_{k \in I} A_k'.$$ **6**

B) If A and B are two non-empty subsets of a Universal set, prove that if $A \subset B$ and $C \subset D \Rightarrow (A \times C) \subset (B \times D)$. **6**

C) N is a set of natural numbers. In $N \times N$ show that the relation R defined by $(a, b)\ R\ (c, d)$ if and only if $a + d = b + c$ is an equivalence relation. Give an example of a relation on a set which is symmetric and transitive but not reflexive. **8**

2. A) Set A is the set of factors of particular positive integer m and $<$ be the relation

$\leq = \{(x, y) \mid x \in A, y \in A, x \text{ divides } y\}$.

Draw an Hasse diagram for $m = 45$. **7**

B) Among the first 1000 positive integers, determine the integers which are not divisible by 5, nor by 7, nor by 9. **6**

C) State Pigeon Hole Principle. Using Pigeon Hole Principle show that, if any 5 integers from 1 to 8 are chosen, then at least two of them have a sum 9. **7**

P.T.O.

MODULE – II

3. A) $(G_1, *)$ and $(G_2, *)$ are semigroups then $(G_1 \times G_2, *)$ is a semigroup where $*$ is defined by $(x_1, y_1) * (x_2, y_2) = (x_1 * x_2, y_1 * y_2)$. 5

B) G is the set of all non-zero real numbers and $*$ is a binary operation defined by $a * b = \frac{ab}{4}$. Show that $(G, *)$ is an abelian group. 5

C) $(G, \circ)$ is a group. Show that $(G, \circ)$ is an abelian group if and only if $(a \circ b)^2 = a^2 \circ b^2$ for all $a, b \in G$. 5

D) Let G_1 and G_2 be sub-groups of G. Show that $G_1 \cap G_2$ is also a subgroup of G. 5

4. A) Let R be the field of real numbers. Show that $W = \{(x, x, x), x \in R\}$ is a subspace of R^3. 7

B) Show that the vector $(2, -5, 3)$ is not in subspace of R^3 generated by the vectors $(1, -3, 2)$, $(2, -4, -1)$, $(1, -5, 7)$. 6

C) Show that the union of two subspaces is a subspace if and only if one is contained in the other. 7

MODULE – III

5. A) State Principle of Mathematical Induction. Use it to prove that $\forall n \in N$ $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$ is a natural number. 7

B) A person invests Rs. 40,000 @ 9% interest compounded annually. How much will be the total amount at the end of 18 years ? 6

C) Solve the recurrence relation $a_{r+2} - 2a_{r+1} + a_r = 3r + 5$. 7

6. A) Define a Boolean Algebra B. Prove that

i) $a.a = a \ \forall \ a \in B$ ii) $(a \cdot b)' = a' + b' \ \forall a, b \in B$. 6

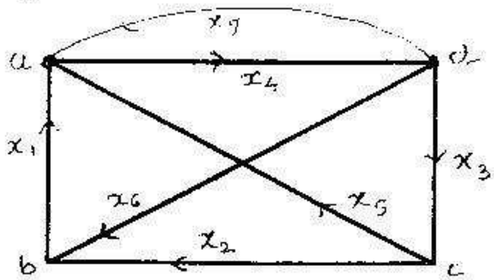
B) Simplify the Boolean expression $x + x'(x + y) + y \cdot z$. 5

C) Define disjunctive normal form. Obtain the principle disjunctive normal form for $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$. 4

D) Prove that $p \to (q \to r) \Leftrightarrow p \to (\sim q \vee r) \Leftrightarrow (p \wedge q) \to r$. 5

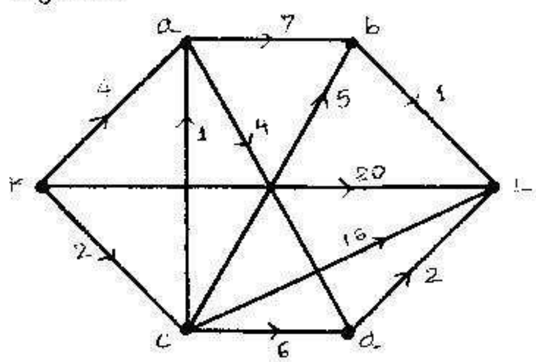
MODULE – IV

7. A) Define incidence matrix of a directed graph. Represent the following graph by an incidence matrix. **6**

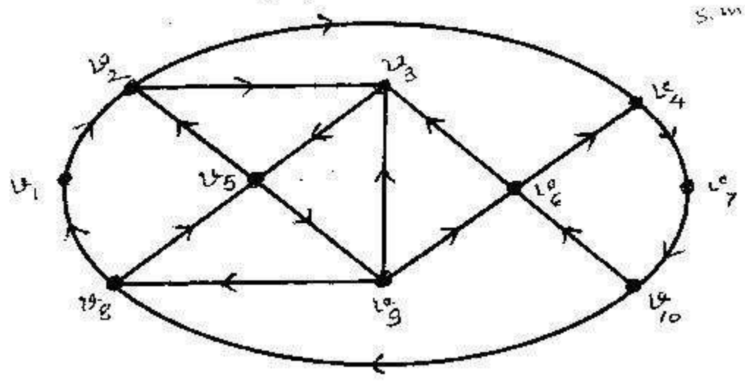


B) Give an example of a graph with six vertices that has no cut points. **4**

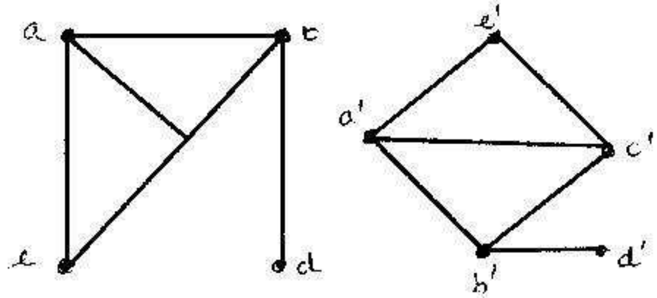
C) Find the shortest path between K and L the graph below by using Dijkstra's Algorithm. **5**



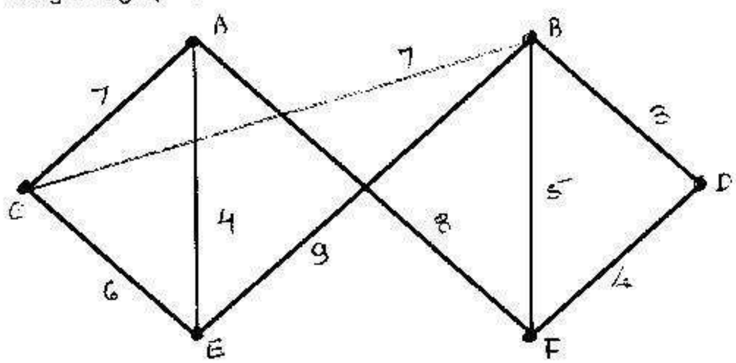
D) Define an Eulerian graph. Graph shown below is an Euler's graph. Determine Euler's circuit for this graph. **5**



8. A) Show that the given pair of graph below is isomorphic. 7



B) Using Kruskal's Algorithm determine the minimum spanning tree of the weighted graph below. 8



C) Prove that even non-trivial tree T has at least two vortices of degree 1. 5