[Total No. of Questions : 8]

S.E. (Computer) (Semester - IV) Examination, May 2011

DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours **Total Marks : 100**

Instructions : *1) Attempt any five questions choosing at least one question from each Module.*

2) Assume suitable data wherever required.

3) Figures to the right indicate marks allocated to the sub-question.

MODULE - I

Q1) a) If P(X) denotes the power set of the set X, show that $P(A \cup B) \subseteq P(A) \cup P(B)$ for any two sets A and B. **[8]**

Give an illustration of two sets A and B for which $P(A \cup B) \neq P(A) \cup P(B)$. Justify your claim.

b) Give a non-trivial example of a relation R on the set S = {a, b, c, d} which is reflexive and transitive but not symmetric. Justify your claim briefly. **[3]**

c) Draw the Hasse-Diagram for the Poset $(S, \leq)$ where S = {2, 4, 5, 8, 10, 12, 16, 20, 25, 50} and where $a \leq b$ if, and only if, $a \mid b$; $\forall$ a, b $\in$ S. **[3]**

d) Show that the function $f : (R - \{7/5\}) \to (R - \{3/5\})$ given by $f(x) = [(3x + 4)/(5x - 7)]$ is injective on its domain $(R - \{7/5\})$. Further assuming that f is surjective, compute (the inverse of f) $f^{-1}(y)$ in terms of y. **[6]**

Q2) a) State Pigeon Hole Principle. Use the pigeonhole principle to show that among any five points placed in a square of side 4 cm, there are at least two points which are at most $2\sqrt{2}$ cm apart. **[5]**

b) Without actually carrying out the multiplication, find the remainder when the integer $[9 \times 85 \times 89 \times (37)^2 \times (67)^2 \times 539 \times (1269)^3]$ is divided by 16. **[5]**

c) State the Inclusion - Exclusion principle for three nonempty sets. Use the principle of Inclusion and Exclusion to find the number of integers between 1 and 9,000, including both 1 and 9,000 which are divisible by neither 5 nor by 6 nor by 15. Explain briefly the steps followed by you. **[7]**

d) If a, b, c and d are any four integers and n is any natural number > 3, show that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n} \Rightarrow ac \equiv bd \pmod{n}$. **[3]**

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MODULE - II

Q3) a) Define a Monoid and a Group. Give an example of a monoid which is not a group. Justify your claim briefly. [4

b) Let (G, *, e) be a group. In each of the following cases, show separately that the group is abelian. [6

i) $(a * b)^{-1} = a^{-1} * b^{-1}$ for each a and b in G;

ii) $(a * b)^2 = a^2 * b^2$ for each a and b in G.

c) Define a cyclic group. Show that every subgroup of a cyclic group is necessarily cyclic. [4]

d) Let (R, +, .) be a ring under consideration in which $x.x = x^2 = x$ for each x in R. Show that the ring is commutative. [6]

Q4) a) Explain what do you understand by a basis of a finite-dimensional vector-space V. Give two different bases of the vector-space R^3 each containing the vectors (3, 2, 1) and (–4, 3, 2). Justify your claim. [4]

b) Define a sub-space of a vector-space. Use your definition to prove that the intersection of any two sub-spaces of a vector-space V is again a sub-space of V. Give an example to show that the union of two sub-spaces of a vector-space V need not be a sub-space of V. [7]

c) $T : R^3 \to R^3$, is a linear transformation given by $T(x, y, z) = (x + y, y - z, x + z)$; $\forall (x, y, z) \in R^3$. Find a basis of the kernel of T and also a basis of the range of T. [5]

d) Examine whether the vectors (1, –2, 3), (–2, 3, –5) and (4, 5, –7) are linearly independent in the vector space R^3 over the usual field of real numbers R. [4]

MODULE - III

Q5) a) Let (B, +, ·, --, 0, 1) denote a Boolean Algebra. For any a in B, show that

i) $a + 1 = 1$ and ii) $a \cdot a = a$. [4]

b) Without using the truth tables, show that [5]

i) $\sim(p \vee (\sim p \wedge q)) \equiv \sim(p \vee q)$. ii) $(p \vee q) \to r \equiv (p \to q) \wedge (q \to r)$.

c) Define the principal conjunctive normal form for a logical expression involving the literals p, q and r. Obtain the principal conjunctive normal form for the logical expression. $p \wedge (\sim p \vee q)$. [5]

d) Express each of the following terms $\sim p$, $p \vee q$ and $p \wedge q$ using the literals p and q and only the connective NAND $\{\uparrow\}$. [6]

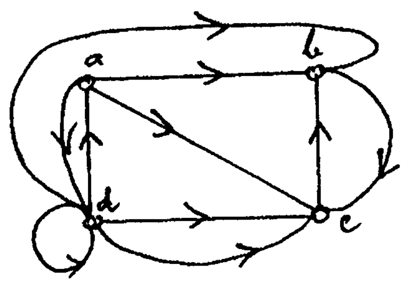
Q6) a) State the Principle of Mathematical Induction. Use the Principle of Mathematical Induction to show that $(2^{4n} - 1)$ is an integral multiple of 15; $\forall$ n $\in$ N. **[6]**

b) Find the recurrence relation for a number of n-digit binary sequence having no pair of consecutive (successive) 0's. State the initial conditions. If a_n denotes the number of different binary sequences of length n satisfying the condition that there are no two consecutive zeros, find a_5 and a_6. **[7]**

c) Solve the recurrence relation $a_n + 3\, a_{n-1} + 2\, a_{n-2} = 5 - 6n$; $a_0 = 2$, and $a_1 = 3$. **[7]**

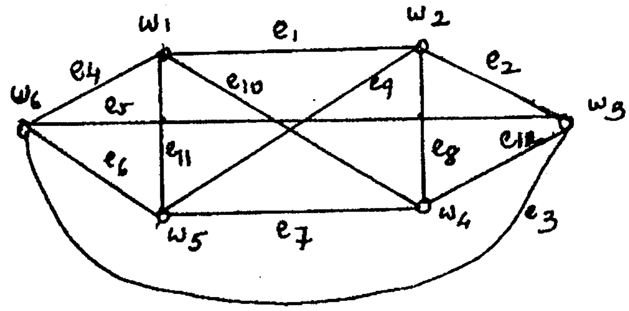
MODULE - IV

Q7) a) Define an Adjacency Matrix for a directed graph. Obtain the Adjacency matrix for the following directed graph. **[4]**

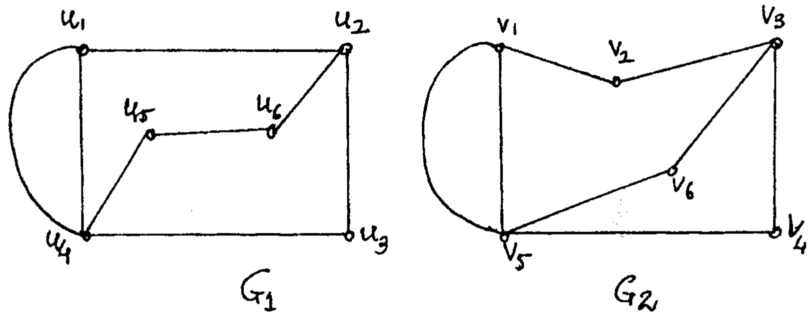


b) Define a bi-partile and a complete bi-partile graphs. Give an illustrative example of a bipartile graph which not a complete bi-partile graph. Give also an illustrative example of a complete bi-partile graph. Justify your claims in each case. **[6]**

c) Define an Eulerian Graph. Use appropriate algorithm to obtain an Eulerian Circuit for the following Eulerial Graph. Explain briefly and sequentially steps followed by you. **[5]**



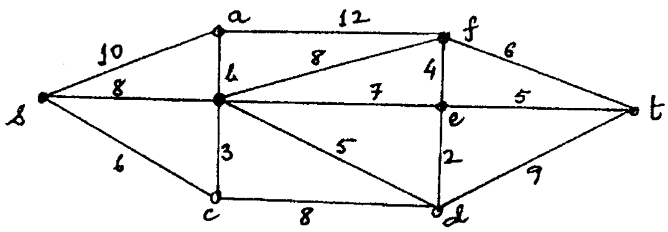
d) Define Isomorphism of two graphs. Use your definition to examine whether the following graphs G_1 and G_2 are isomorphic. **[5]**



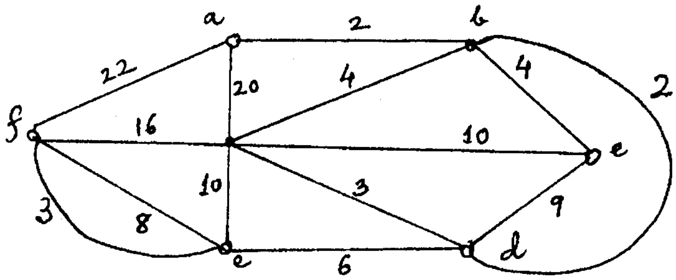
Q8) a) i) Define a tree in three different equivalent ways.

ii) A tree has five vertices of degree two, two vertices of degree three, and four vertices of degree four. Find the number of vertices, if any, of degree one this tree has. **[6]**

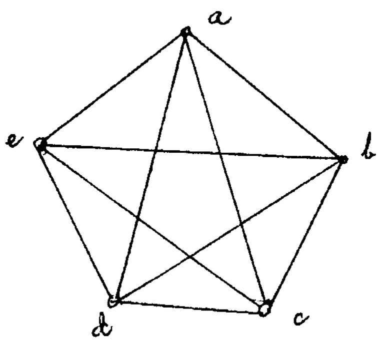
b) Use only Dijkstra's shortest path algorithm to find the shortest path and the corresponding shortest distance between the points s and t in the following network: **[5]**



c) Use a suitable algorithm to obtain a minimal spanning tree for the following connected graph. Explain briefly and sequentially the steps followed by you. **[5]**



d) Define a planar graph. Examine whether the following graph is planar. **[4]**



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