5/6/13
Sem - II
ETC Dept (23)

SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) Examination, May/June 2013
APPLIED MATHEMATICS – II (RC 07-08)

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt* ***any 5*** *questions, atleast* ***one*** *from* ***each*** *Module.*
2) Assume suitable data ***if necessary.***

MODULE – I

1. a) Assuming the validity of differentiating under the integral sign prove that
$\int_0^\infty e^{-\beta x}\frac{\sin\alpha x}{x}\,dx = \tan^{-1}\left(\alpha/\beta\right)$ where $\beta > 0$. 6

b) Find the length of the curve $x = \frac{1}{3}y^{3/2} - y^{\frac{1}{2}}$ from y = 1 to y = 9. 7

c) The loop of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is revolved about the y-axis. Find the surface area of the object generated. 7

2. a) A particle moving in space has constant acceleration i + 2j. If its initial displacement and velocity vector is 3 i and $\overline{2i} + \overline{k}$ respectively, find its position vector at any time t. 6

b) Define curvature of a curve. Show that the curvature of any circle is constant and the curvature of a straight line is 0. 8

c) For the space curve x = 2 cos t, y = 2 sin t and z = $3t^2$. Find the principal normal N and unit tangent T. 6

MODULE – II

3. a) Evaluate $\iint_R x + 2y\,dxdy$, where R is the triangular region with vertices (0, 0) (1, 1) and (1, –1). 7

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b) Evaluate $\int_0^{\infty}\int_x^{\infty} e^{-y^2} dydx$. 6

c) Find the volume of the object generated by the revolution of $r = \cos 2\theta$ about the initial line. 7

4. a) Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} y\, e^{x^2+y^2}\, dxdy$ by changing to polar coordinates. 7

b) Evaluate $\int_0^{1}\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. 7

c) Find the volume of the tetrahedron bounded by the co-ordinate plane and $2x + 2y + 3z = 6$. 6

MODULE – III

5. a) Define gradient of a scaler field. Show that the gradient vector is normal to the level surface of the scaler field. 4

b) Show that the vector field $F = 4xy\bar{i} + (2x^2 + 4z)\bar{j} + 4y^2\bar{k}$ is irrotational. Find its potential function. 6

c) Evaluate $\int_c F ds$, where $F = x^2+2yz$ and c is the line from (0, 1, 1) to (1, 1, 2). 4

d) Use Gauss Divergence theorem to evaluate $\int_s F.\bar{n}ds$ where $F = x^2\bar{i} + y\bar{j} + \bar{k}$, $\bar{n}$ is the unit normal vector to S and S is the surface of the cube $0 \leq x, y, z \leq 1$. 6

6. a) Verify Green's theorem in the plane for $\oint_c (3x^2 + y^2)dx + 2xydy$ where c is the perimeter of the triangle having vertices (0, 0) (1, 0) and (0, 1). 8

b) Verify Stoke's theorem for $\bar{F} = x^2\bar{i} + 2yz\bar{j} + x\bar{k}$ and s are the three sides of the tetrahedron, bounded by the co-ordinate plane and the plane $x + 2y + z = 2$, excluding the side in the xy plane. 12

MODULE – IV

7. Solve the following differential equation : **20**

i) $e^y(1+x^2)\frac{dy}{dx}-2x(1+e^y)=0$

ii) $x(1-x^2)\frac{dy}{dx}+(2x^2-1)y=x^3$

iii) $(xy^2+2x^2y^3)\,dx+(x^2y-x^3y^2)\,dy=0$

iv) $(2x-y+5)\,dx+(x+3y+1)\,dy=0$

8. Solve the following differential equations : **20**

i) $(D^2+2D+1)y=x\,e^x+2$

ii) $(D^3+4D^2+3D)\,y=3e^x\sin 3x$

iii) $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=x^3\log x$

iv) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$
