



COMP 3 – 1 (RC)

S.E. (Computer Engg.) (Semester – III) Examination, May/June 2013
APPLIED MATHEMATICS – III
 (Revised Course)

Duration: 3 Hours

Total Marks: 100

- Instructions :** 1) Answer **five** questions and at least **one** from **each** Module.
 2) Figures to the **right** indicate **full** marks.
 3) Make suitable assumptions **wherever** required.
 4) **Use** statistical tables **wherever** required.

MODULE – 1

- I. a) i) Define adjoint of a square matrix. 6
 ii) If A is a non-singular $n \times n$ matrix then show that $|\text{adj } A| = |A|^{n-1}$.
 b) Reduce the matrix given below to its normal form and hence find its rank : 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

- c) Solve the following line an system of equations after testing for consistency : 8

$$x + y + z = 1$$

$$x + 3y + 2z = 2$$

$$2x + 2y + z = 3$$

- II. a) Find the minimum polynomial of the matrix,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

- b) For the real symmetric matrix A given below find the orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix : 8

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

P.T.O.



c) Verify the Cayley-Hamilton theorem for the matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

6

MODULE -2

III. a) The probability that an individual suffers a bad reaction on taking a certain drug is 0.002. Determine the probability that out of 1000 individuals

- Exactly 3 will suffer a bad reaction, and
- More than 2 will suffer a bad reaction. Use the Poisson distribution.

8

b) A continuous random variable X has the following probability density function :

$$f(x) = cx^2e^{-x}, x > 0$$

$$= 0, x \leq 0$$

- Find C, and
- Compute the mean of X.

c) In a random experiment, $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{3}$, $P(B/A) = \frac{3}{4}$. Find $P(A \cup B)$.

6

IV. a) Compute the moment generating function of a Binomial random variable X with probability mass function $P(X = r) = {}^nC_r p^r q^{n-r}$, $r = 0, \dots, n$, where $0 < p < 1$ and $q = 1 - p$. Hence find the mean and variance.

7

b) Find the coefficient of correlation for the following data :

6

X: 65 66 67 67 68 72 71 70

Y: 67 68 62 68 70 65 64 68

c) Random samples of 200 bolts manufactured by machine A and of 100 bolts manufactured by machine B, showed 19 and 5 defective bolts respectively. Test the hypothesis that machine B is performing better than A, at the 0.05 significance level.

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MODULE-3

V. a) Find the Laplace transform of the following :

9

i) $f(t) = e^{-2t} \cos^2 3t$

ii) $g(t) = \frac{\sinh t}{t}$

iii) $h(t) = \int_0^t u \sin 3u \, du.$

b) Find the inverse Laplace transform of the following :

6

i) $\bar{f}(s) = \frac{s}{s^2 + 4s + 5}$

ii) $\bar{g}(s) = \frac{1}{s} \log \left(\frac{s^2 + 4}{s^2 + 9} \right)$

c) Let $f(t)$ be periodic with period "p". Prove that,

$$L(f(t)) = \frac{1}{1 - e^{-sp}} \int_0^p e^{-st} f(t) \, dt.$$

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VI. a) Use the Laplace transform to evaluate $\int_0^\infty t e^{-2t} \sin 3t \, dt.$

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b) Use the convolution theorem to find the inverse Laplace transform of,

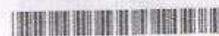
$$\bar{f}(s) = \frac{s}{(s^2 + 4)^2}.$$

6

c) Use the Laplace transform to solve the differential equation

$$y''(t) + y'(t) - 2y(t) = 3 \cos 3t - 11 \sin 3t, \quad y(0) = 0 \text{ and } y'(0) = 6.$$

8



MODULE – 4

VII. a) Find the Fourier transform of

$$f(x) = 1, \quad |x| \leq a$$

$$= 0, \quad |x| > a$$

Where $a > 0$. Hence find the value of $\int_0^{\infty} \frac{\sin x}{x} dx$. 8

b) Find the inverse Fourier transform of, $\hat{f}(s) = a - |s|$, $|s| \leq a$, where $a > 0$. 6

$$= 0, \quad |s| > a$$

c) Solve for $f(x)$ the integral equation,

$$\int_0^{\infty} f(x) \cos \lambda x dx = 1 - \lambda, \quad 0 < \lambda < 1 = 0, \lambda \geq 1. \quad \text{6}$$

VIII. a) Find the Z-transform of the following : 8

i) $f(n) = n$

ii) $g(n) = 2^n$.

b) Find the inverse Z-transform of $\bar{g}(z) = \frac{8z^2}{(2z-1)(4z+1)}$. 6

c) Use the Z-transform to solve the difference equation,

$$y(n+2) + 3y(n+1) - 4y(n) = 0, \quad n \geq 0, \text{ and } y(0) = 3, y(1) = -2. \quad \text{6}$$