S.E. (Computer Engineering) (Revised 2007 – 08) (Semester – IV) (RC) Examination, November/December 2015
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

***Instructions** : 1) Answer **any five** questions with atleast **one** from **each** Module.*
*2) Assume suitable data if **necessary**.*

MODULE – I

1. a) Let A, B and C be any three non empty sets. Prove that 6

   i) $(A \cap B) - C = (A - C) \cap (B - C)$

   ii) $P(A) \cap P(B) = P(A \cap B)$ where P(A) denotes power set of A.

   b) Let $A = Z - \{0\}$ denote the set of all non-zero integers. Define a relation R on $A \times A$ as (a, b) R (c, d) iff ad = bc. Show that R is an equivalence relation on $A \times A$. Also find all the distinct equivalence classes. 8

   c) Let $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{3\}$ given by $f(x) = \dfrac{3x + 4}{x - 1}$. Check whether f is bijective or not. If yes, find f^{-1}. 6

2. a) State Pigeonhole principle. Suppose 30 balls are numbered from 1 to 30 and placed in a large box. Show that, if 18 balls are drawn randomly, there must be a pair among them whose sum of the numbers appearing on the balls drawn is 35. 7

   b) Among the first 500 positive integers, determine the number of integers that are divisible by 2 or 3 but not by 5. 6

   c) Let $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ where a, b, c, d $\in$ Z and m, k $\in Z^+$. Show that 7

   i) $ac \equiv bd \pmod{m}$

   ii) $a^k = b^k \pmod{m}$.

P.T.O.

MODULE – II

3. a) Give an example of a semigroup which is not a monoid and an example of a monoid which is not a group. Justify in both cases. 4

b) Show that the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where $a \neq 0$ and $b \neq 0$ are real numbers form a group under matrix multiplication. Is the group abelian ? 7

c) Show that every cyclic group is an abelian group. Is the converse true ? Justify your answer. 3

d) Let $(R^+, \cdot)$ be the multiplicative group of all positive real numbers. Define a function $f : R^+ \to R^+$ by $f(x) = x^2$ for all $x \in R^+$. Show that f is an automorphism. 6

4. a) Let $(R, +, \cdot)$ be a ring. If every element $a \in R$ satisfies $a^2 = a$, then prove that 8
   i) $a + a = 0$
   ii) $a + b = 0$ implies $a = b$
   iii) R is a commutative ring.

b) Show that $W = \{(x, y, z) : x + y - z = 0, x, y, z \in \mathbb{R}\}$ is a vector subspace of $\mathbb{R}^3$. 4

c) Consider the map $T : R^3 \to R^3$ given by $T(x, y, z) = (x - y, x + y, z)$. Show that T is a linear transformation. Find the matrix of the linear transformation with respect to the standard basis of R^3. 8

MODULE – III

5. a) Let B be a Boolean Algebra. Then prove that 5
   i) $a + (a \cdot b) = a \ \forall a, b \in B$
   ii) $(a + b)' = a' \cdot b' \ \forall a, b \in B$.

b) Without using Truth tables prove that 6
   i) $\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q) \equiv \sim p \vee q$
   ii) $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$.

c) Define conjunctive normal form. Express the following well formed formula in the principal conjunctive normal form. 5

$(\sim p \to r) \wedge (q \leftrightarrow p)$

d) Show that the NAND operator $\{\uparrow\}$ is functionally complete. 4

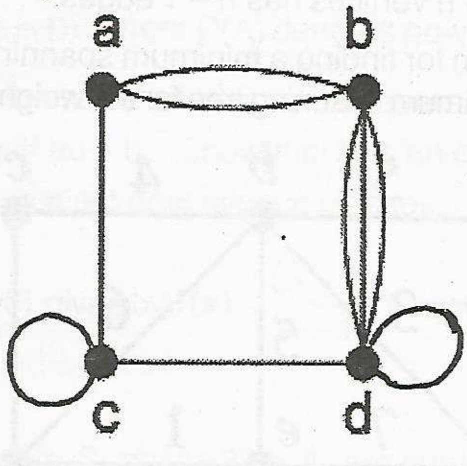
6\. a) State the first principle of Mathematical Induction. Use mathematical induction to prove that $1+\frac{1}{4}+\frac{1}{9}+...+\frac{1}{n^2}<2-\frac{1}{n}$ whenever $n \geq 2$. 7

b) Solve the recurrence relation $a_n - a_{n-1} - 2a_{n-2} = n^2$ with $a_0 = 1$ and $a_1 = 0$. 7

c) A restaurant serves three kinds of snacks A, B, C costing 1\$, 2\$ and 3\$ respectively. Find the recurrence relation for the number of ways of spending n dollars if a person eats one snack each day until the n dollars are exhausted. Also state the initial conditions. 6

MODULE – IV

7\. a) i) Define adjacency matrix of an undirected graph. Represent the following graph by an adjacency matrix. 3



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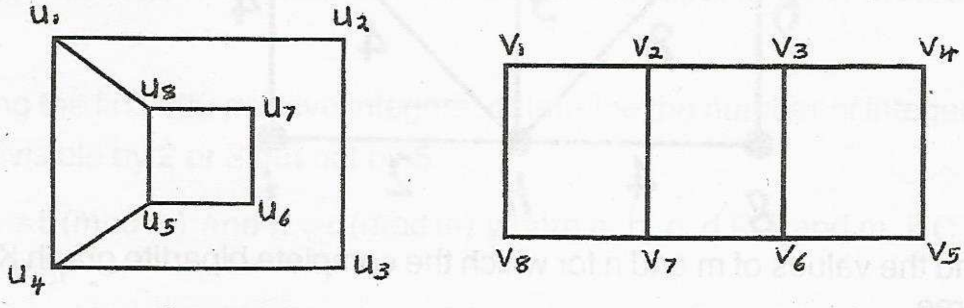
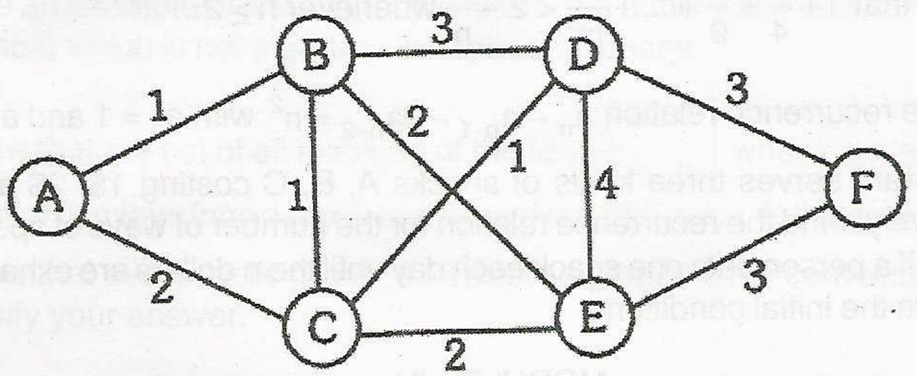
ii) Determine whether the following graphs are isomorphic or not. 4



b) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. 5

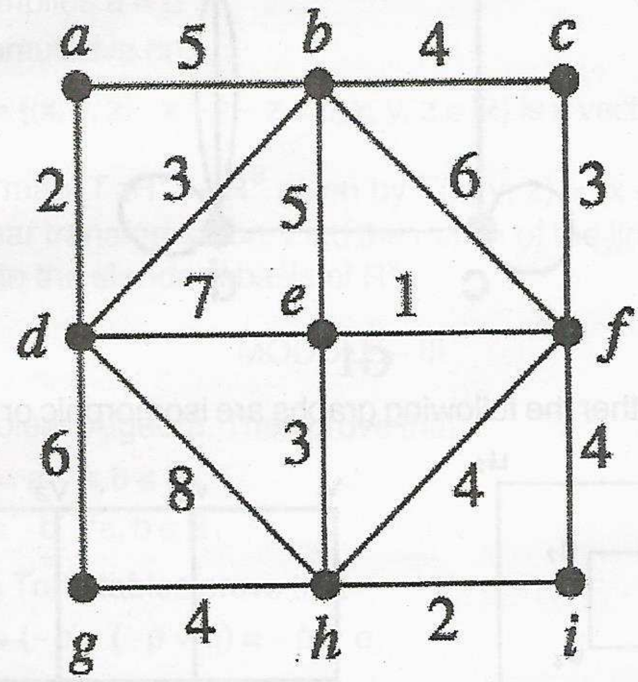
c) Apply Dijkstra's algorithm to find the shortest path between A and F in the following weighted graph. 6



d) Give an example of a graph which has a Hamiltonian circuit but not an Euler's circuit. 2

8. a) Prove that a tree with n vertices has n – 1 edges. 6

b) State Prim's algorithm for finding a minimum spanning tree. Using Prim's algorithm, find the minimum spanning tree for the weighted graph shown below. 7



c) i) Find the values of m and n for which the complete bipartite graph $K_{m,n}$ is a tree. 4

ii) Show that a full (regular) m-ary tree with "i" internal vertices contain n = mi + 1 vertices.

d) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have ? 3