S.E. (Comp.) (Sem. – III) Examination, November/December 2009
APP. MATHEMATICS – III
(Revised 2007-08)

Duration : 3 Hours Total Marks : 100

Note : i) Attempt any five questions.
ii) Atleast one from each Module.
ii) Assume suitable data wherever required.

MODULE – I

1. a) Define : 8

i) a Hermitian matrix

ii) a skew-Hermitian matrix.

Prove the following :

i) The determinant of a Hermitian matrix is real

ii) The determinant of a skew-Hermitian matrix of odd order is zero.

b) Find the rank of the following matrix : 8

$$A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$$

c) Test the following vectors for linear dependence or independence : 4

$X_1, = [1, 1, 1, 3]^T$, $X_2 = [1, 2, 3, 1]^T$, $X_3 = [2, 3, 4, 1]^T$.

2. a) Find an non-singular matrix P such that $P^{-1}AP$ is diagonal, whose diagonal elements are eigen values of 'A', given 8

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 6 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

b) State and prove Cayley-Hamilton theorem. 8

c) Prove that eigen vectors corr. to distinct eigen values of a real symmetric matrix are orthogonal. 4

MODULE – II

3. a) State and prove Baye's theorem of probability. 5

b) Two numbers x and y are chosen from {1, 2, ... 3n}. Find the probability that x + y will be divisible by 3. 8

c) A continuous random variable 'X' has probability density function : 7

$$f(x) = \begin{cases} Kx^3 e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find K, mean and variance.

4. a) A normal population has a mean 0.1 and standard deviation of 2.1. Find the probability that the mean of a sample of size 900 drawn from this population will be negative. 5

b) A manufacturer claims that the mean breaking strength of safety belts for air passengers produced in his factory is 1275 kg. A sample of 100 belts was tested and the mean breaking strength and standard deviation were found to be 1258 and 90 kg resp. Test the manufacturers claim at 5% LOS. 7

c) Obtain the lines of regression and find the coefficient of correlation from the following data : 8

x :	1	2	3	4	5	6	7
y :	9	8	10	12	11	13	14

MODULE – III

5. a) If L[f(t)] = F(s), prove the following : 6

1) $L[f'(t)] = sF(s) - f(0)$

2) $L\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{s}$.

b) Find the Laplace transforms of : 9

i) $\int_0^t e^{-2u} \sin^3 u \, du$

ii) $e^{3t} \cos 3t \sin t$

iii) $\dfrac{e^{at} - \cos 6t}{t}$

c) If $L[f(t)] = F(s)$ then prove that 5

$L[f''(t)] = s^2 F(s) - sf(0) - f'(0)$.

6. a) State and prove convolution theorem for Laplace transforms. 8

b) Using Laplace transform solve : 6

$\dfrac{d^2y}{dt^2} + 4y = \sin t, \; y(0) = 1, \; y'(0) = 0$.

c) Find inverse Laplace transform of each of the following: 6

1) $\dfrac{s}{(s^2+4)(s^2+9)}$ 2) $\log\left(\dfrac{s}{s-1}\right)$

MODULE – IV

7. a) Find the Fourier transform of 8

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1 \end{cases}$$

and use it to evaluate $\int_0^\infty \left(\dfrac{x\cos x - \sin x}{x^3}\right) \cos\left(\dfrac{x}{2}\right) dx$.

b) Solve the integral equation $\int_0^\infty f(x) \cos \lambda x = e^{-\lambda}$. 7

c) If $\mathcal{F}[\, f(x) \,] = \bar{f}(\alpha)$ prove that : 5

i) $\mathcal{F}[\, f(ax)] = \frac{1}{a}\bar{f}\left(\frac{\alpha}{a}\right)$

ii) $\mathcal{F}[\, f(x)\cos ax] = \frac{1}{2}\left[\bar{f}(\alpha+a)+\bar{f}(\alpha-a)\right]$

where $\mathcal{F}$ denotes Fourier transform.

8. a) Prove that : 6

i) $Z[n] = \frac{Z}{(Z-1)^2}$ ii) $Z[a^n] = \frac{Z}{Z-a}$.

b) State convolution theorem and find inverse Z-transform of 6

$$\frac{8Z^2}{(2Z-1)(4Z+1)}$$

c) Find Z-transforms of : 8

i) $\frac{1}{n!}$ and hence of $\frac{1}{(n+1)!}$

ii) $\sin^2\left(\frac{n\pi}{4}\right)$.

____