8. a) Find the Z-transform of the following :

   i) $2^n/(n+1)!$

   ii) $3n + 2^n$. **6**

   b) If $Z(f(n)) = F(z)$ then show that :

   i) $Z\left(\sum_1^n f(k)\right) = \frac{z}{z-1} F(z)$

   ii) $Z(n\, f(n)) = -z \frac{d}{dz}(F(z))$. **6**

   c) Solve the difference equation give below using Z-transform
   $y_{n+2} + 5y_{n+1} + 4y_n = 2^n$, $y_0 = 0$, $y_1 = 1$. **8**

____

b) Find the Laplace transform of

i) $\dfrac{\text{Cos}2t - \text{Cos}3t}{t}$

ii) $\int_0^t \text{Cos}(t-u)\,\text{Sin}2u\,du$. **6**

c) Solve the ordinary differential equation, using Laplace transforms

$y''(t) + 4y(t) = \text{Sint}, y(0) = 1, y'(0) = 1.s$ **8**

6. a) If f(t) is a periodic function having period p, then prove that

$L(f(t)) = \dfrac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\,dt$. **10**

Find the Laplace transform of rectified semi-wave function

$f(t) = \begin{cases} \text{Sinwt} & 0 < t < \pi/\omega \\ 0 & \pi/\omega < t < 2\pi/\omega \end{cases}$ and $f\left(t + \dfrac{2\pi}{\omega}\right) = f(t)$.

b) Solve the integro-differential equation using Laplace transform

$\dfrac{dy}{dt} + \int_0^t y(t-u)ue^u du = e^t,\ y(0) = 0.$ **5**

c) Using Laplace transform evaluate $\int_0^\infty \dfrac{e^{-t} - e^{-3t}}{t}\,dt$. **5**

MODULE – IV

7. a) Find the Fourier cosine transform of $f(x) = e^{-ax}\ a > 0$. **6**

b) If F(f(x)) = F(s) is the Fourier transform of f(x), show that

i) $F(f(x-a)) = e^{ias}F(s)$

ii) $F(f(x)) = \frac{1}{2}\{F(s+a) + F(s-a)\}$. **6**

c) Define convolution of two functions. Find the convolution of $f(t) = e^{-t}, t > 0$ and $f(t) = e^{-2t}, t > 0$. **8**

MODULE – II

3. a) Ten students are randomly assigned roll number between 1 and 10. There are 3 friends among them. What is the probability that the 3 friends roll numbers are consecutive numbers ? 6

   b) An online retail store has announced a discount sale of garments, wherein on purchase of 1, 2, 3 or 4 garments a discount of 5%, 10%, 15% and 20% respectively is offered. The number of garments purchased is a binomial distribution B(4, 2/3) and each garment cost Rs. 500/-. What is the probability that purchase of more than Rs. 950/- will be made by a customer ? What is the average amount spent on purchase by a customer ? 8

   c) Define independent random variables. Show that the sum of two independent Poisson random variables is Poisson. 6

4. a) The arrival of a train at a railway station is uniformly distributed between 9.00 a.m. and 9.30 a.m. What is the probability that the train will arrive before 9.20 a.m. ? Given that the train has not arrived till 9.10 a.m., what is the probability that it will arrive by 9.25 a.m. ? 6

   b) In an online auction, price quoted for a product was found to be normally distributed with mean Rs. 2000 and standard deviation Rs. 250. What is the probability that a bidder has bid more than Rs. 2200 ? If the online retailer has enough stock to cater to the demand of 5% of the number of bidders, then what will be the minimum price the product will fetch ? 8

   c) A manufacturer claimed that 95% of the equipment supplied to a factory confirmed to specification. An examination of sample of 200 units of equipment revealed that 14 were faulty. Test his claim at significance level of 0.01 and 0.05. 6

MODULE – III

5. a) If L(f(t)) = F(s), where L(f(t)) denotes the Laplace transform of f(t), prove the following :

   i) $L(t\,f(t)) = -\frac{d}{ds}(F(s))$

   ii) $L\left(\int_0^t f(t)\right) = \frac{1}{s}F(s)$ 6

3093 (R)

COMP 3 – 1 (RC)

S.E. (Comp.) Semester – III (Revised Course 2007-08)
Examination, May/June 2015
APPLIED MATHEMATICS – III

Duration : 3 Hours | Total Marks : 100

Instructions : 1) *Attempt **any five** questions. Atleast **one** from **each** Module.*
2) ***Assume** suitable data, **if necessary**.*
3) ***Use** of statistical table permitted.*

MODULE – I

1. a) Define orthogonal matrix. Show that the determinant of an orthogonal matrix is 1. **5**

   b) Find the rank of the matrix by reducing it to its normal form.

$$\begin{bmatrix} 1 & -1 & 3 & 6 \\ 2 & -1 & 2 & 5 \\ 5 & -3 & 7 & 16 \\ 6 & -4 & 10 & 22 \end{bmatrix}$$ **7**

   c) Find the condition on λ for which the following system of equations $3x - 2y + 4z = 3$, $x + 2y - 3z = -2$, $x + 2y + \lambda z = -3$ has unique solution. Find the solution for $\lambda = -5$. **8**

2. a) Diagonalize the matrix given below and obtain the modal matrix for

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$ **7**

   b) Find the eigen value and eigen vector of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$. **8**

   c) Verify Cayley-Hamilton theory for the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$. **5**

P.T.O.