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SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (Revised in 2007-08) Examination, Nov./Dec. 2013
APPLIED MATHEMATICS – II

Duration : 3 Hours   Total Marks : 100

Instructions : *i) Attempt **any five** questions. At least **one** from **each** Module.*
*ii) **Assume** suitable data, **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^1 \frac{e^{2\sin\alpha} - 1}{\log_e x} dx$ assuming the validity of differentiation under the integral sign. **6**

b) Find the length of the curve $x(t) = 1 - \cos t + \frac{t}{\sqrt{10}}$ $y(t) = \frac{3}{\sqrt{10}} \sin t$ from $t = 0$ to $t = \frac{\pi}{z}$. **6**

c) The loop of the curve $9y^2 = (x + 5)(x + 2)^2$ is revolved about the x-axis. Find the surface area of the object generated. **8**

2. a) A particle moves on a cycloid in the xy plane in such a way that its position at time t is $\bar{r}(t) = (t - \sin t)\bar{c} + (1 - \cos t)\bar{j}$. Find the maximum and minimum values of $|v|$ and $|a|$. **6**

b) Define Torsion. If $\bar{r}(t)$ is the position vector of moving object then prove that its Torsion is

$$\frac{\left[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}\right]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}.$$ **5**

c) For the space curve $x = t + 1$, $y = t^2$, $z = 3t^2 + t$. Find the equation of tangent line and binomial line at $t = 1$. **6**

d) If $\overline{r(t)}$ has constant magnitude show that $\bar{r}(t)$ is perpendicular to its tangent $\frac{d\bar{r}}{dt}$. **3**

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MODULE – II

3. a) Evaluate $\iint xy + 5 \; dxdy$ over the region bounded by $x^2 = y$ and the line $y = 2x$. 6

b) Evaluate $\int_0^2 \int_{y^2}^{2+y} x + y \; dxdy$ by changing the order of integration. 8

c) Evaluate $\int_0^\infty \int_0^\infty \frac{xe^{-(x^2+y^2)}}{\sqrt{x^2+y^2}} \, dx\, dy$ by changing to polar co-ordinates. 6

4. a) The loop of the curve $y^2 = x(1 - x)$ is revolved about the x-axis. Find the volume of object generated. 6

b) Evaluate $\iiint x + z = dxdydz$ over region. 7

$R = \{(x, y, z) \mid x \geq 0, y \geq 0, 2x + y \leq 2, 0 \leq z \leq 2\}$.

c) Use triple integration to find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. 7

MODULE – III

5. a) Define Divergence of a vector field. If f is a scaler point function and $\overline{g}$ is a vector field show that div $(+ \overline{g}) = \nabla f . \overline{g} + fdw\overline{g}$. 6

b) Find the work done in moving a particle in a force field $F = 3x^2 i + 4yzj + z^2 \overline{k}$ along the curve $x = 2t^2$, $y = 3t + 1$, $z = t^2 - 1$ from $t = 0$ to $t = 1$. 6

c) Use Gauss Divergence theorem to evaluate $\int_s \overline{F}.\overline{n}dS$ where

$F = x^3 \overline{i} + y^3 \overline{j} + 3xy\overline{k}$, S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ and $\overline{n}$ is the unit normal vector to S. 8

6. a) Verify Green's theorem in the plane for $\oint (xy + 1)dx + 4x^2 dy$ e is the perimeter of the triangle having vertices (0,0), (1,0) and (1, 1). 8

b) Verify Stoke's theorem for the vector field $F = (x^2 + 1) \overline{c} + yz\overline{j} + 3z^2\overline{k}$ over surface of the cube bounded by the co-ordinate planes and the planes $x = 2$, $y = 2$, $z = 2$, excluding the surface in the xy plane. 12

MODULE – IV

7. Solve the following differential equations : **20**

i) $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$

ii) $(\sin x \cos y + e^{2x})\, dx + (\cos x \sin y + \tan y)\, dy = 0$

iii) $y(2xy + 1)\, dx + x(1 + 2xy - x^3y^3)\, dy = 0$

iv) $(3x - y + 4)\, dx + (4x + y + 1)\, dy = 0.$

8. Solve the following differential equations : **20**

1) $(D^2 - 3D + 2)\, y = 2x^2 + 3x$

2) $(D^3 + D^2 + 2D + 2)\, y = \sin 2x \cos x$

3) $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^2$

4) $\frac{d^2y}{dx^2} + y = \sec x.$
