

5/6/13  
SEM - II

ETC Dept (23)

## SEM 2 – 1 (RC 07-08)

### F.E. (Semester – II) Examination, May/June 2013 APPLIED MATHEMATICS – II (RC 07-08)

Duration : 3 Hours

Total Marks : 100

**Instructions :** 1) Attempt **any 5** questions, atleast **one** from **each** Module.  
2) Assume suitable data **if necessary**.

#### MODULE – I

1. a) Assuming the validity of differentiating under the integral sign prove that

$$\int_0^{\infty} e^{-\beta x} \frac{\sin \alpha x}{x} dx = \tan^{-1} \left( \frac{\alpha}{\beta} \right) \text{ where } \beta > 0. \quad 6$$

- b) Find the length of the curve  $x = \frac{1}{3} y^{3/2} - y^2$  from  $y = 1$  to  $y = 9$ . 7

- c) The loop of the curve  $x = t^2$ ,  $y = t - \frac{t^3}{3}$  is revolved about the y-axis. Find the surface area of the object generated. 7

2. a) A particle moving in space has constant acceleration  $i + 2j$ . If its initial displacement and velocity vector is  $3i$  and  $2i + k$  respectively, find its position vector at any time  $t$ . 6

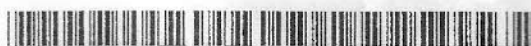
- b) Define curvature of a curve. Show that the curvature of any circle is constant and the curvature of a straight line is 0. 8

- c) For the space curve  $x = 2 \cos t$ ,  $y = 2 \sin t$  and  $z = 3t^2$ . Find the principal normal  $N$  and unit tangent  $T$ . 6

#### MODULE – II

3. a) Evaluate  $\iint_R (x + 2y) dx dy$ , where  $R$  is the triangular region with vertices  $(0, 0)$ ,  $(1, 1)$  and  $(1, -1)$ . 7

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- b) Evaluate  $\int_0^{\infty} \int_x^{\infty} e^{-y^2} dy dx$ . 6
- c) Find the volume of the object generated by the revolution of  $r = \cos 2\theta$  about the initial line. 7

4. a) Evaluate  $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} y e^{x^2+y^2} dx dy$  by changing to polar coordinates. 7

- b) Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ . 7

- c) Find the volume of the tetrahedron bounded by the co-ordinate plane and  $2x + 2y + 3z = 6$ . 6

## MODULE – III

5. a) Define gradient of a scalar field. Show that the gradient vector is normal to the level surface of the scalar field. 4

- b) Show that the vector field  $F = 4xy\bar{i} + (2x^2 + 4z)\bar{j} + 4y^2\bar{k}$  is irrotational. Find its potential function. 6

- c) Evaluate  $\int_c F ds$ , where  $F = x^2 + 2yz$  and  $c$  is the line from  $(0, 1, 1)$  to  $(1, 1, 2)$ . 4

- d) Use Gauss Divergence theorem to evaluate  $\int_s F \cdot \bar{n} ds$  where  $F = x^2\bar{i} + y\bar{j} + \bar{k}$ ,  $\bar{n}$  is the unit normal vector to  $S$  and  $S$  is the surface of the cube  $0 \leq x, y, z \leq 1$ . 6

6. a) Verify Green's theorem in the plane for  $\oint_c (3x^2 + y^2) dx + 2xy dy$  where  $c$  is the perimeter of the triangle having vertices  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$ . 8

- b) Verify Stoke's theorem for  $\bar{F} = x^2\bar{i} + 2yz\bar{j} + x\bar{k}$  and  $s$  are the three sides of the tetrahedron, bounded by the co-ordinate plane and the plane  $x + 2y + z = 2$ , excluding the side in the  $xy$  plane. 12



## MODULE – IV

7. Solve the following differential equation :

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i)  $e^y(1+x^2)\frac{dy}{dx} - 2x(1+e^y) = 0$

ii)  $x(1-x^2)\frac{dy}{dx} + (2x^2-1)y = x^3$

iii)  $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$

iv)  $(2x - y + 5) dx + (x + 3y + 1) dy = 0$

8. Solve the following differential equations :

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i)  $(D^2+2D+1)y = x e^x+2$

ii)  $(D^3 + 4D^2 + 3D) y = 3e^x \sin 3x$

iii)  $x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 12y = x^3 \log x$

iv)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

b) Derivations of a curve. Show that the derivative of  $\ln(x)$  is  $1/x$  and the derivative of  $\ln(x^2)$  is  $2/x$ .