S.E. (Comp.) (Semester – III) (RC) Examination, May 2010
APPLIED MATHEMATICS – III

Duration : 3 Hours Total Marks : 100

Instructions : 1) Attempt five questions, at least one from each Module.
2) Assume suitable data if required.

MODULE – I

1. a) Prove that $(AB)^T = B^T A^T$. 4

b) By using elementary row transformation find A^{-1}, where

$$A = \begin{bmatrix} 3 & 2 & -2 \\ 2 & 2 & 1 \\ 4 & 2 & 3 \end{bmatrix}_{3 \times 3}.$$ 6

c) Find Rank of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 & 1 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & 2 & 4 \\ 5 & 3 & 1 & -4 \end{bmatrix}_{4 \times 4}.$$ 5

d) For what values of a and b, the equations

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + az = b$ have

i) A unique solution

ii) Infinitely many solutions

iii) No solution. 5

2. a) Let $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}_{3\times 3}$, Find eigen values and eigen vectors of the matrix 'A'. 6

b) Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ use Cayley-Hamilton theorem to prove that $A^3 = A^{-1}$. 7

c) Find P s.t $P^{-1}AP$ is a diagonal matrix, $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}_{3\times 3}$. 7

MODULE – II

3. a) In a large consignment of electric lamps 5% are defective. A random sample of eight lamps is taken for inspection. What is the probability that this sample has one or more defective lamps. 6

b) If X is a Poisson distribution such that $P(X = 1) = P(X = 2)$ find $E(X)$ and $E(X^2)$. 5

c) Define exponential distribution. Use its moment generating function to find its mean. 4

d) The life of a semiconductor lazer at constant power is normally distributed with a mean of 7000 hrs. and S.D. 600 hr. What is the probability that a lazer fails before 6000 hr ? 5

4. a) The following table gives the heights and weights of a sample of 12 students obtain the lines of regression, estimate the weight of a student whose height is known to be 63 inches and estimate the height of a student whose weight is known to be 168 lb. 10

H(t) (X")	70	63	72	60	66	70	74	65	62	67	65	68
wt (Y lb)	155	150	180	135	156	168	178	160	132	145	139	152

b) Experience has shown that 20% of a manufactured product is of top quality. In one days production of 400 articles only 50 are of top quality. Show that either the production of the day choosen was not a representative sample or the hypothesis of 20% was wrong. Use 5% LOS. Also based on the particular day's production find the 95% confidence limits for the percentage of top quality product. 10

MODULE – III

5. a) Find Laplace transform of the following : 6

i) $t^2 \cdot \cos^2 t - e^{-2t} - \cos t$

ii) $\dfrac{e^{at} - \cos bt}{t}$

b) Prove that :

i) $L\{f'(t)\} = SF(s) - f(0)$

ii) $L\{f''(t)\} = S^2F(s) - Sf(0) - f'(0)$

iii) $L\left\{\int_0^t f(t)\,dt\right\} = \dfrac{F(s)}{S}$

Where $F(s) = L\{f(t)\}$. 9

c) Find Laplace transform of $f(t) = \begin{cases} E\sin\omega t, & 0 < t \le \pi/\omega \\ 0, & \pi/\omega < t \le \frac{2\pi}{\omega} \end{cases}$ $f\left(t + \frac{2\pi}{\omega}\right) = f(t)\ \forall\, t$. 5

6. a) Find Inverse Laplace transform of the following : 9

i) $\dfrac{S^2 + 2S}{(S-3)(S^2+2S+5)}$

ii) $\dfrac{1}{2}\log\left(\dfrac{S^2+4}{S^2-16}\right)$

iii) $\dfrac{1}{S}\cdot\sin\left(\dfrac{1}{S}\right)$

b) State convolution theorem for Laplace transform and hence use it to find inverse Laplace transform of $\frac{1}{(S^2+3S+1)^2}$. 6

c) Use Laplace transform method to solve the integral equation $3y + 2\int_0^t y\,dt = t$. 5

MODULE – IV

7. a) Prove that :

i) $F_c\{f(x)\cdot \cos ax\} = \frac{1}{2}\left[\bar{f}_c(s+a) + \bar{f}_c(s-a)\right]$ where $\bar{f}_c(s) = F_c\{f(x)\}$

ii) $F_s\{f(x)\cdot \cos ax\} = \frac{1}{2}\left[\bar{f}_s(s+a) + \bar{f}_s(s-a)\right]$ where $f_s(s) = F_s\{f(x)\}$. 6

b) Solve for f(x), The integral equation $\int_0^\infty f(x)\sin sx\,dx = \begin{cases} 1, & 0 \le s \le 1 \\ 2, & 1 \le s \le 2 \\ 0, & s \ge 2 \end{cases}$. 6

c) Find Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence prove that

$\int_0^\infty \frac{\sin^4 x}{x^4}\,dx = \pi/3$. 8

8. a) Prove that

i) $Z_T\{u(n-k)\} = z^{-k}\cdot \bar{u}(z)$ where $\bar{u}(z) = Z_T\{u(n)\}$

ii) $Z_T\{n\cdot u(n)\} = -z\cdot \frac{d}{dz}\bar{u}(z)$

$\bar{u}(z) = Z_T\{u(n)\}$.

Here Z_T denotes Z-transform. 6

b) Use convolution theorem for Z-transform to find inverse Z-transform of $\frac{Z^2}{(Z+a)(Z+b)}$. 6

c) Use Z-transform method to solve the difference equation $u_{n+2} - 3y_{n+1} + 2y_n = -2$, $u_0 = u_1 = 0$. 8