SEM 2-1 (RC 07-08)

F.E. (Sem. – II) (Revised in 2007-08 Course) Examination, May/June 2012

APPLIED MATHEMATICS – II

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data **if necessary**.*

MODULE – I

1. a) Assuming the validity of differentiation under the integral evaluate

$$\int_0^\infty e^x \log_e(1+a^2e^{-zx})\,dx.$$ 8

b) Find the perimeter of the loop of the curve $x = t^2 - 5, y = \frac{t}{3}(3 - t^2)$. 6

c) The curve $r = 2a\cos\theta$ is revolved about the x-axis, find the surface area of the solid generated. 6

2. a) Show that $\bar{r}(t) = Ate^{2t}\bar{i} + Be^{3t}\,\bar{j}$, satisfies $\frac{d\bar{r}^2}{dt^2} - 4\frac{d\bar{r}}{dt} + 4\bar{r} = 0$. 6

b) Find the unit tangent vectors $\overline{T}$ and principal normal $\overline{N}$ for

$r(t) = (2t^2 + 3)\,\bar{i} + (5 - t^2)\,\bar{j}$ at $t = 1$. 8

c) State and prove Serret-Fernet formula. 6

MODULE – II

3. a) Evaluate $\int_0^1\int_0^1 ye^{xy+2y}\,dxdy$. 6

b) Write a single integral and integrate $\int_{-1}^{0}\int_{-y}^{1} 2y + 3dxdy + \int_{0}^{1}\int_{y^2}^{1} 2y + 3dxdy$. 8

c) Evaluate $\iint r\sin\theta + 3drd\theta$ over the region $1 \le r \le 2\cos\theta$. 6

P.T.O.

4. a) Find the volume of the solid generated by the revolution of the loop of the curve $y^2 = (x^2 - 4)(x^2 - 1)$ about the x-axis. 8

b) Evaluate $\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{x^2}^{1-z} 3x\,dy\,dx\,dz$. 6

c) Find the volume of the region bounded by the coordinate planes and the plane $2x + y + 3z = 6$. 6

MODULE – III

5. a) Define gradient of a scalar field. Show that the gradient at a point is the normal to the level surface of the scalar field. 6

b) Show that the vector field

$\bar{F} = (3z^2 \cos y + e^2 \cos x)\bar{i} - 3xz^2 \sin y\bar{j} + (6xz \cos y + e^2 \sin y)\bar{k}$ is irrotational. Find its potential function. 8

c) Evaluate $\int_c xy + 2z^2\, ds$ where c is the line from (1, 1, 0) and (2, 1, 3). 6

6. a) Verify Green theorem in the plane for $\oint (xy + 2)dx + (3x^2 + y)dy$ where C is the boundary of the region enclosed by $y = 2x$ and $y = 0$ and $x = 1$. 8

b) Verify Stoke's theorem for $F = 2x\bar{i} + (3y^2 + z)\bar{j} + 2yz\bar{k}$, over the surface of the tetrahedron bounded by the coordinate planes and the plane $2x + y + 3z = 6$ above the xy plane. 12

MODULE – IV

7. Solve the following differential equations : 20

a) $\dfrac{dy}{dx} = e^{2x-3y} + e^{-3y} \cos 2x$

b) $\dfrac{dy}{dx} + y \tan x = y^3 \cos x$

c) $\dfrac{dy}{dx} = \dfrac{2x - y + 4}{x - 3y + 1}$

d) $(xy + 2x^2y^2)y\,dx + (xy - x^2y^2)x\,dy = 0$.

8. Solve the following differential equations : **20**

a) $(D^2 + 4D + 5)y = 3e^{2x} + 5x^2$

b) $(D^3 + 4D^2 + D - 2)y = 3\sin^2 x + 2$

c) $(D^2 + 1)y = \sec x$

d) $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 4\cos(\log_e(1+x))$.
