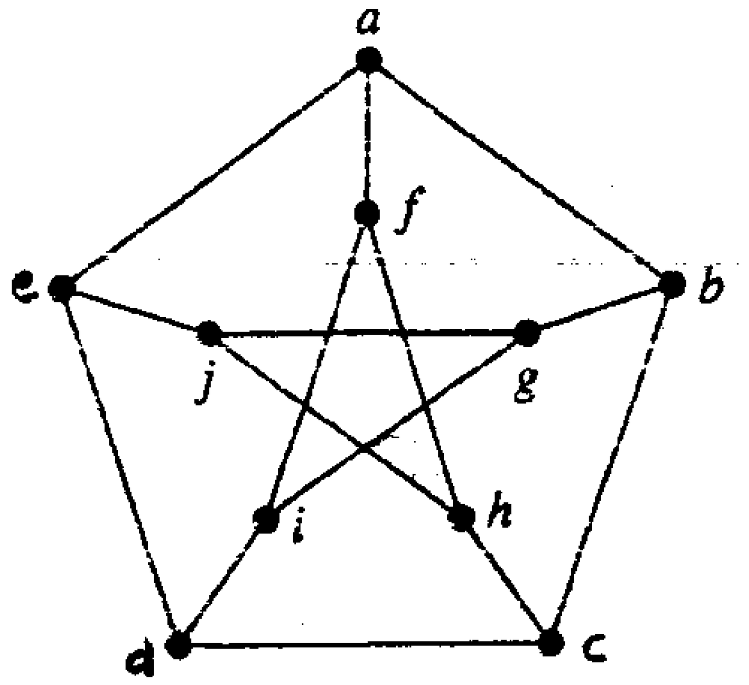
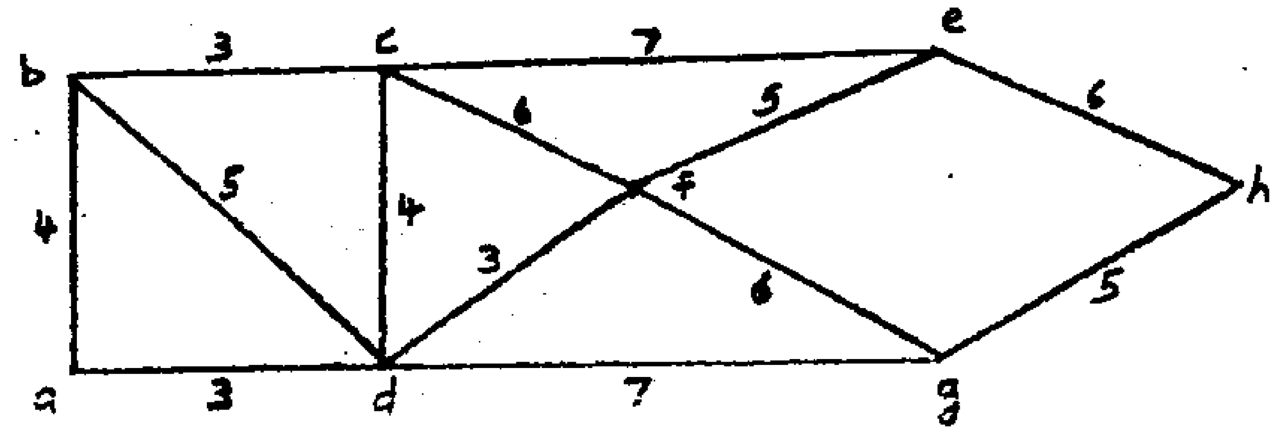
b) Explain Konisberg's Bridge problem and draw the Euler's graph representing the problem. 5

c) Using Kuratowski's theorem, show that the following graph is non planar. 5



d) Apply Dijkstra's algorithm to find the shortest path between a and h in the following weighted graph : 6



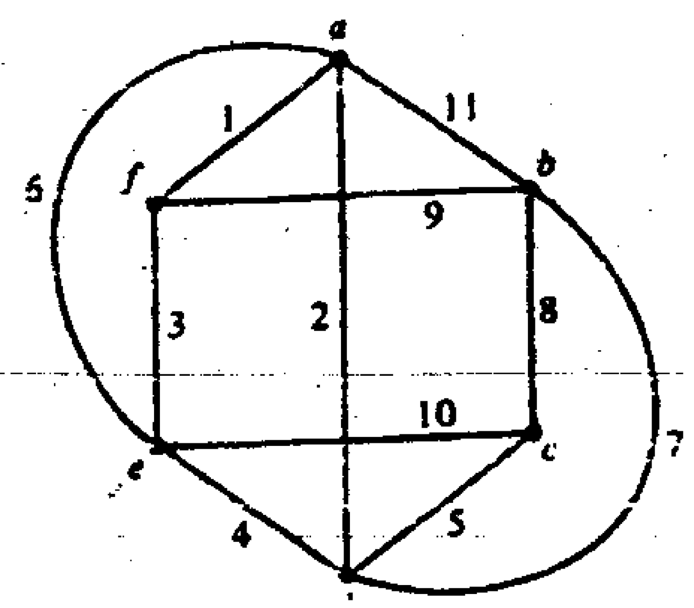
8. a) Show that a connected graph with n vertices and edges e = n – 1 is a tree. 6

b) i) For what values of m and n, the complete bipartite graph $K_{m,n}$ is a tree ? 2

ii) Show that a full (regular) m-ary tree with "i" internal vertices contain $n = mi + 1$ vertices. 3

iii) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree I does it have ? 3

c) State Prim's algorithm for finding minimum spanning trees. Using Prim's Algorithm find the minimum spanning tree for the weighted graph shown below. 6



S.E. (Computer Engineering) (Revised 2007-08) Semester – IV Examination, November 2010
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

*Instructions : 1) Answer **any five** questions with at least **one** from each Module.*
*2) **Assume** suitable data **if necessary**.*

MODULE – I

1. a) Let A, B and C be any three non empty sets.

i) Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

ii) If $A \cap B = A \cap C$, is $B = C$? Justify. (3+2)

b) Give an example of a relation which is :

i) reflexive and symmetric but not transitive

ii) symmetric but no antisymmetric.

Justify your answer in both cases. 6

c) Let $f : A \to B$ and $g : B \to C$ be two onto (surjective) functions. Show that $g \circ f : A \to C$ is also onto (surjective). 5

d) Define a lattice. Is (N, |) a lattice, where '|' denotes division ? 4

2. a) State Pigeonhole principle.
Suppose 14 students having random seat numbers are answering an examination Prove that there are at least two among them whose seat numbers differ by a multiple of 13. 6

b) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$, prove that 8

i) $ac \equiv bd \pmod n$

ii) $a^k \equiv b^k \pmod n$

where a, b, c, d ∈ Z and n, k ∈ N.

c) Find how many integers between 1 and 60 are not divisible by 2 nor by 3 and nor by 5. 6

MODULE – II

3. a) Determine the submonoid generated by the set $S = \{ p : p \text{ is a prime number} \}$ in the monoid $(N, .)$ where N is the set of natural numbers and "." denotes multiplication. **3**

b) Let $Q - \{1\}$ be the set of all rational numbers except 1. Define an operation '*' on $Q - \{1\}$ as $a * b = a + b - ab$. Show that $(Q - \{1\}, *)$ is an abelian group. **5**

c) Show that every subgroup of a cyclic group is cyclic. **6**

d) Let $(G_1, *_1)$ and $(G_2, *_2)$ be two groups and let $f : G_1 \to G_2$ be a homomorphism from G_1 to G_2. Show that

i) $f(e_1) = e_2$ where e_1 and e_2 are the identity elements of G_1 and G_2 respectively.

ii) $f(a^{-1}) = (f(a))^{-1}\ \forall a \in G_1$. **6**

4. a) Let $(R, +, .)$ be a ring such that $(xy)^2 = x^2 y^2$ for all $x, y \in R$. Prove that R is a commutative ring. **6**

b) Prove that a subset of a set of linearly independent vectors is linearly independent. **5**

c) Check whether the following subset is a vector subspace of R^3. **3**

$W_1 = \{(x, y, z) : y = x + z\}$

d) Given the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$.

Find the linear transformation $T : R^2 \to R^3$ relative to the standard bases. **6**

MODULE – III

5. a) Define a Boolean Algebra. In a Boolean Algebra B, prove that

i) $\forall a, b \in B : a + a \cdot b = a$

ii) $\forall a, b \in B : (a + b)' = a' \cdot b'$. **6**

b) Without using Truth tables prove that

$q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv T$. **3**

c) Define conjunctive normal form. Express the following expression in the principal conjunctive normal form.

$(\sim p \to r) \wedge (q \leftrightarrow p)$. **5**

d) Determine the validity of the following argument :

My father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore, if father praises me, then I do not study well. **6**

6. a) State the principle of Mathematical Induction.

Use mathematical induction to prove that for all positive integers n,

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots + \sqrt{2}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right).$$

(The number of square roots is n). **7**

b) Find the recurrence relation for the number of binary sequences of length n where the pattern 00 occurs for the first time at the end of the sequence. Also state the initial conditions and determine the number of such sequences of length 5. **5**

c) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + 3n$ with $a_0 = 0$ and $a_1 = 1$. **8**

MODULE – IV

7. a) i) Define incidence matrix of a undirected graph. Represent the following graph by an incidence matrix. **4**