F.E. (Semester – II) (Revised Course 2007-08) Examination, May/June 2014
APPLIED MATHEMATICS – II

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **5** questions, atleast **one** from **each** Module.*
*2) **Assume** suitable data, **if necessary**.*
*3) Figures to the **right** indicate **full** marks.*

MODULE – I

1. a) Assuming the validity of differentiating under the integral sign prove that

$$\int_0^\infty \frac{\cos \lambda x}{x}\left(e^{-ax} - e^{-bx}\right)dx = \frac{1}{2}\log\left(\frac{b^2+\lambda^2}{a^2+\lambda^2}\right) \quad a > 0,\ b > 0.$$ **7**

b) Find the length of the curve x = a (2 cost – cos 2t) and y = a(2sint – sin 2t) from t = 0 to $t = \frac{\pi}{2}$. **6**

c) Find the area of the surface generated by the revolution of $x = \frac{y^3}{3}$; $0 \le y \le 1$ about the Y-axis. **7**

2. a) Find the unit tangent vector and unit acceleration vector of the curve x = 2 cost, y = 2 sint + 3, z = 4t at t = 1. **5**

b) State and prove Serret – Fernet formula. **8**

c) Define curvature of a point on a curve. If $\overline{\mu}(t)$ is any vector point function, show that the curvature is given by

$$k = \frac{\left|\dot{\overline{\mu}} \times \ddot{\overline{\mu}}\right|}{\left|\dot{\overline{\mu}}\right|^3}$$ **7**

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MODULE – II

3\. a) Evaluate $\iint_R x+y \, dxdy$ where 'R' is the region bounded by $y = 2x$, $y = x$ and $y = 1$. **7**

b) Write the following as one double integral and evaluate

$$\int_{-1}^{0}\int_{0}^{x+1} 2y + 3 \, dxdy + \int_{0}^{1}\int_{0}^{1} 2y + 3 \, dxdy.$$ **7**

c) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \cdot \sqrt{x^2+y^2} \, dxdy.$ **6**

4\. a) Find the area bounded by $x^2 + y^2 = 4$ and $x + y = 2$ in the 1^{st} quadrant. **6**

b) Evaluate $\iiint_V \frac{dx\,dy\,dz}{(1+x+y+z)^3}$ where 'V' is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. **7**

c) Use triple integration to find the volume of the region bounded by $x^2 + y^2 = 9$, $z = 0$ and $z = 2$. **7**

MODULE – III

5\. a) Find the rate of change of $f(x, y, z) = x^2y + 3z^2y$ at the point (1, 2, –1) in the direction of $3i + j - k$. **5**

b) Define Curl of a vector field. Prove that Curl (guad f) is 0. **5**

c) Show that $\overline{F} = (x^2 - y^2 + x)\, i - (2xy + y)\, j$ is irrotational and hence find its scalar potential. Also find the line integral of $\overline{F}$ from (1, 2) to (2, 1). **6**

d) Find the unit vector normal to $x^2 + 3y + z^2 = 4$ at (0, 1, –1). **4**

6\. a) Verify Green's theorem in the plane for $\oint_c [(xy + 4y^2)dx + (x^2 + 3)dy]$ where 'C' is the boundary of the region bounded by $x= 1$ and $y^2 = x$. **8**

b) Verify Stoke's theorem for $\overline{F} = xy\ i - 2yzj - zxk$ where 'S' is the open surface of the region bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 3$ above the XOY plane. **12**

MODULE – IV

7. Solve the following :

a) $(x + y + 1)^2 \dfrac{dy}{dx} = 1.$ **5**

b) $\left[y\left(1+\dfrac{1}{x}\right)+\cos y\right]dx + [x + \log x - x \sin y]\,dy = 0.$ **5**

c) $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0.$ **5**

d) $\sqrt{1-y^2}\,dx = (\sin^{-1} y - x)\,dy.$ **5**

8. Solve the following :

a) $(D^3 + D^2 + D)y = e^x \cosh(x).$ **5**

b) $(D^2 + 4)y = x^3 + \cos x.$ **5**

c) $(D^2 + 1)y = \tan x.$ **5**

d) $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = x^2 \log x.$ **5**
