S.E. (Comp.) Sem. III Examination, May 2009
APPLIED MATHEMATICS – III (Revised 2007-08)

Duration : 3 Hours — Total Marks : 100

Instructions : 1) *Attempt **any five** questions.*
2) *At least **one** from **each** module.*
3) *Assume suitable data **whenever** required.*

MODULE – I

1. a) If $A = [a_{ij}]_{n\times n}$ then prove that $A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,I_n$. 6

   b) By computing the rank of the augmented matrix and the coefficient matrix, test for consistency and solve if possible the system of equations. 8

   $2x_1 + 3x_2 + 4x_3 = 4$

   $5x_1 + 6x_2 + 7x_3 = 10$

   $8x_1 + 9x_2 + 10x_3 = 16$

   c) Prove the following : 6

   i) A square matrix 'A' is singular iff zero is an eigen value of A.

   ii) If 'A' and B are similar matrices then $|A| = |B|$.

2. a) Prove that eigen vectors corresponding to different eigen values are linearly independent. 6

   b) Use Cayley-Hamilton theorem to find $A^4 - 3A^2 + 2A - I$ where, 6

   $$A = \begin{bmatrix} 1 & 4 & -1 \\ -2 & 0 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

   c) For the matrix 'A' given below find matrix P such that $P^{-1}AP$ is a diagonal matrix. 8

   $$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

P.T.O.

MODULE – II

3. a) Define independent events. 6

If A and B are independent, prove $P(A \cup B) = 1 - P(\overline{A})\, P(\overline{B})$.

b) In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, betn. 45% and 60%, betn. 60% and 75% and above 75% resp. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Find the percentage of the students who have got first class and second class. (Assume normal distribution of marks.) 8

c) Find Moment generating function for Binomial distribution, hence find mean and variance of Binomial distribution. 6

4. a) Using Poisson distribution find the probability that ace of spades will be drawn from a pack of well shuffled cards at least once in 104 consecutive trails. 6

b) The standard deviation of a random sample of 900 members is 4.6 and that of another independent sample of 1600 members is 4.8. Examine if the 2 samples could have been drawn from a population with standard deviation 4. 7

c) Find the correlation coefficient from the following data : 7

x:	2	4	5	6	8	11	12
y:	18	12	10	8	7	5	6

MODULE – III

5. a) Prove the following : 9

i) If $L[f(t)] = F(s)$ then $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.

ii) If $L[f(t)] = F(s)$ then $L[f'(t)] = SF(s) - f(o)$.

iii) If $L[f(t)] = F(s)$ then $L\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{S}$.

b) Using Laplace transform show that $\int_0^{\infty} e^{-3t}\, t \sin t\, dt = \frac{3}{50}$. 5

c) Find the Laplace transforms of : 6

i) $\frac{e^{2t} - e^{-3t}}{t}$ ii) $te^{-2t}\cos 4t$

6. a) Find inverse Laplace transforms of : 6

i) $\frac{s^3}{(s-2)^3}$ ii) $\frac{s+2}{s^2+4s+5}$

b) Find inverse Laplace transform of 4

$\frac{10}{(s+1)(s^2+4)}$ using convolution theorem.

c) Using Laplace transform solve $x(t) + \int_0^t x(t)\, dt = t^2 + 2t$. 10

MODULE – IV

7. a) Prove that : 5

i) $F[f'(t)] = -i\alpha F[f(t)]$

ii) $F[f''(t)] = -\alpha^2 F[f(t)]$, F - Stands for Fourier Transform.

b) Find the Fourier transform of 8

$$f(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

Hence deduce that $\int_0^{\infty} \frac{\sin^2 t}{t^2}\, dt = \pi/2$

c) State and prove convolution theorem for Fourier transforms. 7

8. a) Find Z- transform of : 8

i) $\dfrac{2n+3}{(n+1)(n+2)}$

ii) $\dfrac{1}{n(n+1)}$

b) If $Z\left[f(n)\right] = F(z)$ then prove that :

i) $\lim\limits_{z \to \infty} F(z) = f(o)$

ii) $\lim\limits_{n \to \infty} f(n) = \lim\limits_{z \to 1} \{(z-1)\,F(z)\}$

c) Using Z - transform solve the difference equation $y_{n+1} + 4y_{n+1} + 3y_n = 2^n$. 7
