S.E. (Comp.) (Semester – IV) (RC) Examination, Nov./Dec. 2012
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours

Total Marks : 100

***Instructions**:* *1) Attempt **any five** questions choosing at least **one** question from **each** Module.*

*2) **Assume** suitable data **wherever** needed.*

*3) Figures to the **right** indicate marks allotted to that such question.*

MODULE – I

1. a) Let U be the universal set and A, B, C be subsets of U. Then prove that

   i) $A-(B\cap C)=(A-B)\cap(A-C)$

   ii) $(A-C)\cap(C-B)=\phi$; ϕ is the empty set. **6**

   b) Let $S=\{1, 2, 3\}$ and X be the collection of all bijective maps from S to S.

   For Fig. EX, define a relation R on X by f Rg iff $f(3)=g(3)$. Show that R is an equivalence relation. Find all equivalence classes of R. **6**

   c) Let $A=\{a, b, c\}$ and $S=\wp(A)$; power set of A. Prove that the relation R defined on S by XRY iff $X\subseteq Y$; $X, Y\in S$ is a partial ordering on S.

   Draw a Hasse diagram for the above poset. Is the above poset a lattice ? Justify. **8**

2. a) If 'a' , 'b' , 'c' , 'd' are integers and 'm' is a positive integer

   $a\equiv b(\text{mod } m)$, $C\equiv d$ (mod m) then show that

   i) $(a+c)\equiv(b+d)$ (mod M) .

   ii) $(a-c)\equiv(b-d)$ (mod M)

   iii) $ac\equiv bd$ (mod M). **(2+2+3=7)**

b) State the generalised Pigeon hole principle. Use the pigeon hole principle to prove that if 20 candidates appear in a competitive exam. Then there exists at least two among them whose roll numbers differ by a multiple of 19. 8

c) Find the number of warp that 9 toys can be divided between 4 children if the youngest is to receive 3 toys and the rest 2 toys each. 5

MODULE – II

3. a) Let $A \neq \phi$ and A^* be the collection of strings of finite length over A. Show that A^* under the operation 'concatination of two strings' is a monoid. 5

b) Let G be a group and a, b $\in$ G. Show that $(b^{-1} a b)^m = b^{-1} a^m b$ where 'm' is any integer. 7

c) Give an example of a non-Abelian (or non-commutative) group. 2

d) Prove that if $f : G \to G'$ is a group homomorphism with Kernel 'K' then f is injective if and only if $K = \{e\}$; where 'e' is the identity of G. 6

4. a) Show that a finite integral domain is a field. 7

b) Determine whether the set $S = \{(1,2,1), (3,1,2), (0, 1, 0)\}$ is a linearly independent set over the field $\mathbb{R}$ of real numbers. 5

c) i) If $T : U \to V$ is a linear transformation then prove that Kernel of T is a subspace of U. 3

d) Define $T : \mathbb{R}^2(\mathbb{R}) \to \mathbb{R}^2(\mathbb{R})$ by

$T(x_1, x_2) = (x_1 + x_2, x_2)$

show that T is a linear transformation. 5

MODULE – III

5. a) Let (B, +, . , – ,) be a Boolean Algebra where + , . , – denote disjunction, conjunction and complementation respectively. Prove that for all a, b $\in$ B. 6

i) $a + 1 = 1$

ii) $a + (a \cdot b) = a$

iii) $\overline{a \cdot b} = \overline{a} + \overline{b}$

b) Explain the 'Proof by contradiction' method. Use this method to prove $\neg q, p \to q \Rightarrow \neg p$. 5

c) Prove the following without using truth tables : 6

i) $\neg(p \vee (\neg p \wedge q)) \Leftrightarrow \neg p \wedge \neg q$

ii) $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$

d) Prove that $\{\uparrow\}$ is functionally complete. 3

6. a) State the principle of mathematical induction. Use this principle to show that $(n^3 - n)$ is an integral multiple of 3 where 'n' is a natural number. 6

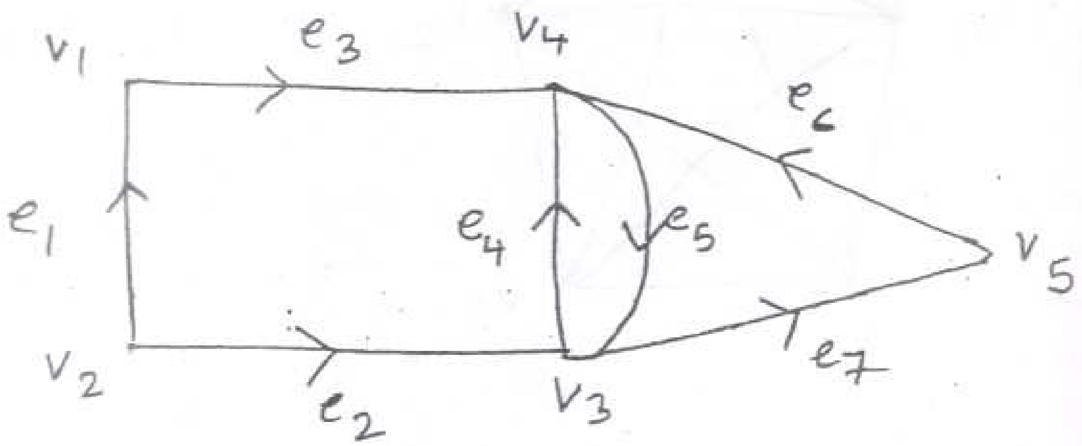
b) Find the recurrence relation for the number of sequences (binary) of length 'n' where the pattern 00 occurs for the first time at the end of the sequence. Also state the initial conditions. 6

c) Solve the recurrence relation. 8

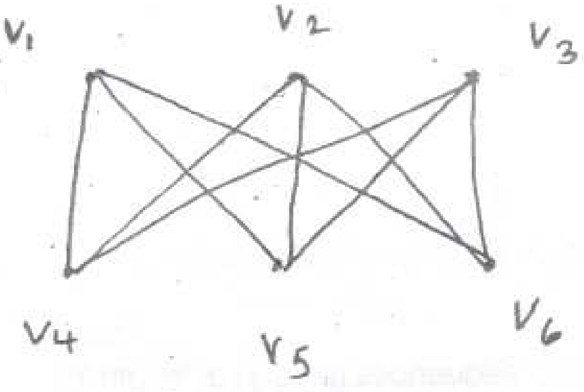
$a_{n+2} - a_{n+1} - 2a_n = n$; $a_0 = 1$, $a_1 = 2$.

MODULE – IV

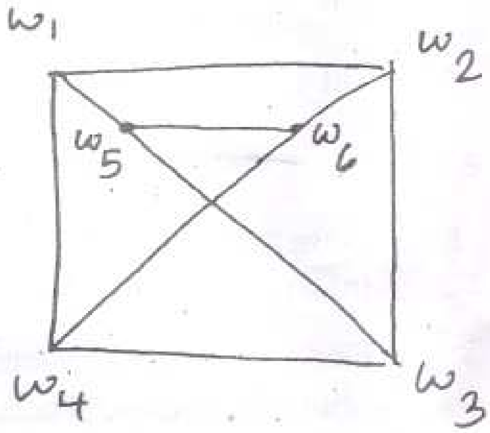
7. a) Define Adjacency matrix for a directed graph. Find the adjacency matrix for the following : 6



b) Define isomorphism of two graphs . Examine whether the following graphs are isomorphic. Justify. 6

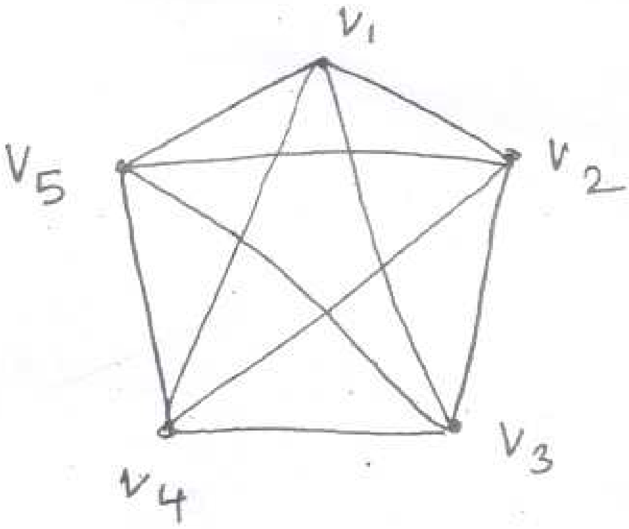


G_1



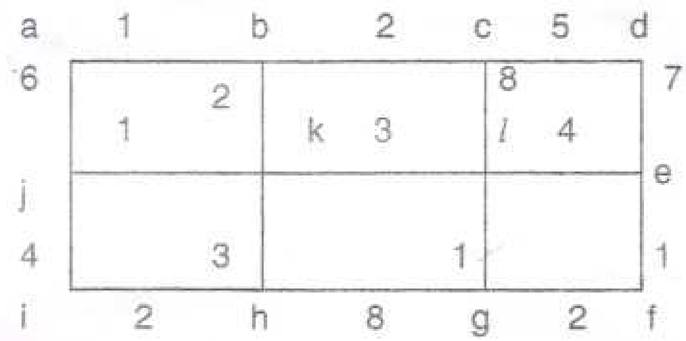
c) State Dijkstra's algorithm to find the shortest path between two vertices of an undirected graph. 4

d) Examine whether the graph given below is planar. Justify. 4



8. a) Prove that if G is a connected and planar graph then $V - e + r = 2$; where 'V' is the number of vertices, 'e' is the number of edges and 'r' is the number of regions. **7**

b) Define a 'Minimal Spanning Tree'. Use the Kruskal's algorithm to find a minimal spanning tree for the following : **7**



c) i) Give an example of bipartite graph for which the partition of the matrix set is net unique.

ii) Give three different definitions of a tree. **6**