2915115 M.

SEM 2 – 1 (RC 07-08)

F.E. (Sem. – II) Revised 2007-08 Course Examination, May/June 2015
APPLIED MATHEMATICS – II

Time : 3 Hours | Max. Marks : 100

Instructions : *i) Attempt **any five** question, at least **one** from **each** module.*
*ii) Assume suitable data if **necessary**.*

MODULE – I

1. a) Evaluate $\int_0^{\infty} \frac{\cos \lambda x}{x}\left(e^{-ax} - e^{-bx}\right)dx$ applying differentiation under the integral sign. **6**

b) Find the length of the cycloid $x = 2(\theta + \sin\theta), y = 2(1 - \cos\theta)$ between two cusp. **6**

c) Find the curved surface area of the solid generated by the revolution about x-axis of $x(t) = 1 - \sin t + \frac{t}{\sqrt{5}}, y(t) = \frac{2}{\sqrt{5}}\cos t$, from $t = 0$ to $t = \pi/2$. **8**

2. a) A moving object starts it motion from the point (1, 1, 2) with speed 3 in the direction $\bar{i} + \bar{k}$. It has constant acceleration $2\bar{i} + \bar{j}$. Find the position vector of the moving object at time t. **6**

b) Find the principal normal N and the binomial B of $\bar{r}(t) = \sin t\,\bar{i} + (t+1)\bar{j} + \cos t\,\bar{k}$ at $t = \pi/2$. **6**

c) Evaluate $\int_0^{\pi/2} \cos^2 t\bar{i} + \sin t\,\bar{j} + \bar{k}dt$. **4**

d) Define curvature. Show that the curvature of $r(t) = 2\cos t\,\bar{i} + 2\sin t\,\bar{j}$ is constant. **4**

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MODULE – II

3. a) Evaluate $\int_1^2\int_0^y \frac{1}{x^2+y^2}\,dx\,dy$. **6**

b) Evaluate $\iint 3x+2\,dx\,dy$ over the region enclosed by $y = x$, $y = 2x - 2$ and $y = 0$. **8**

c) Change the order of integration $\int_1^3\int_0^x 2x+3y\,dx\,dy$ and then evaluate. **6**

4. a) The Loop of the curve $y^2 = x(2 - x)^2$ is revolved about the x-axis. Find the volume of the object generated. **6**

b) Evaluate the spherical coordinates integral $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 3r^3 \cos^2\theta\, dr\, d\theta\, d\varphi$. **6**

c) Find the volume of the region enclosed $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$. **8**

MODULE – III

5. a) Define Curl of a vector field. Show that $\text{Curl}\,(\nabla\phi) = 0$ where ϕ is a scalar point function. **6**

b) What is the greatest rate of change of $f(x, y, z) = 2x + 3z^2 + y^2$ at the point $(1, -2, 2)$? **4**

c) Evaluate $\iint_S \nabla \times \vec{F}\cdot\vec{n}\,ds$ where S is the triangle having vertices (1, 0, 0), (0, 2, 0) and (0, 0, 3). $\vec{n}$ is the unit normal vector to the S and $\vec{F} = (x^2 + yz)\vec{i} + (3z + x)\vec{j} + yx\vec{k}$. **10**

6. a) Verify Green's theorem in the plane for $\oint_C (x + 3y^2)\,dx + (2xy + 1)\,dy$ where C is the boundary of the region enclosed by $y^2 = 4x$ and $x = 1$. **8**

b) Verify Gauss divergence theorem for $F = (z^2 + 2x)\,\vec{i} + (x + 2z^2)\,\vec{j} - (y^2 + 3z)\vec{k}$, over the surface of the tetrahedron enclosed by the coordinate planes and the plane $x + y + z = 1$. **12**

MODULE – IV

7. Solve the following differential equations : **20**

a) $\frac{dy}{dx} - x^2e^y = e^{2x+y}$.

b) $\frac{dy}{dx} + y \cot x = \cos x$

c) $\frac{dy}{dx} = \frac{2y - x + 3}{4y - 2x + 2}$.

d) $(\sec x \tan x \tan y - e^{2x})\, dx + \sec x \sec^2 y dy = 0$.

8. Solve the following differential equations : **20**

a) $(D^2 + D - 12)y = 2 \sin^2 x + 3$.

b) $(D^2 + 4)y = 4 \mathrm{Tan}^2 x$.

c) $(D^3 + 6D - 7)y = 5xe^{3x}$.

d) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$.
