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COMP 3 – 1 (RC)

S.E. (Comp.) (Semester – III) (Revised Course)
Examination, May/June 2014
APPLIED MATHEMATICS – III

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions. Atleast **one** from **each Module**.*
*2) **Assume** suitable data, if **necessary**.*

MODULE – I

1. a) Define a Hermitian matrix. If A and B are Hermitian matrices show that AB-BA is skew Hermitian. 4

b) Find the rank of the matrix by reducing it to its normal form : 6

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

c) Find the values of a and b for which the equations $x + ay + z = 3$; $x + 2y + 2z = b$; $x + 5y + 3z = 9$ are consistant. When will these equations have a unique solution ? 6

d) Are the following vectors $x = (3, 2, 7)$, $y = (2, 4, 1)$, $z = (1, -3, 6)$ linearly dependent. If so find a relation between them. 4

2. a) Find the eigen value and eigen vector of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. 8

b) A real symmetric matrix A has eigen values 6, 3, 2 with eigen vectors $[1, 1, 2]'$ and $[1, 1, -1]'$ corresponding to 6 and 3 determine matrix A. 6

c) Find e^A given $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$. 6

P.T.O.

MODULE – II

3. a) A missile can be launched if two relays A and B both have failed. The probability of A failing is 0.01 and B failing is 0.04. It is also known that B is more likely to fail, probability of 0.08, if A has failed : **(3+3+2)**

   i) What is the probability of an accidental launch ?

   ii) What is the probability that A will fail if B fails ?

   iii) Are the events "A fails" and "B fails" statistical independent ?

   b) A students is known to arrive late to class 40% of the time. If the class meets once on each of the five days of the week : **6**

   a) Find the probability that the student is late for at least 3 classes of the week.

   b) What is the probability that the student is late for the second time of the week on Thursday ?

   c) Define independent random variables. Show that the sum of two Poisson independent random variables is Poisson. **6**

4. a) The average amount of time (in minutes) it takes to be served at a cafeteria is a random variable with probability density function $f(x) = \frac{1}{3}e^{\frac{x}{3}}$ $x > 0$. **6**

   i) Find the average amount of time it takes to be served.

   ii) Find the probability that a person will be served in 5 minutes.

   b) Find the moment generating function of a normal distribution $N(\mu, \sigma)$. Use it find the mean. **8**

   c) A manufacturer claimed that 95% of the equipment supplied to a factory confirmed to specification. An examination of sample of 200 units of equipment revealed that 15 were faulty. Test his claim at significance level of 0.01 and 0.05. **6**

MODULE – III

5. a) If L (f(t) = F (s), where L (f(t)) denotes the Laplace transform of f (t), prove the following : 6

i) $L(f'(t)) = sF(s) - f(0)$

ii) $L\left(\int_0^t f(t)\,dt\right) = 1/s\,F(s)$.

b) Find the Laplace transform of : 6

i) t Cos 2t Sint

ii) $\int_0^t \text{Cos}2(t-u)\,\text{Sin}u\,du$.

c) Solve the ordinary differential equation, using Laplace transforms $y''(t) + y(t) = \text{Sin}(4t)$, $y(0) = 1$, $y(\pi/2) = -2$. 8

6. a) If f (t) is a periodic function having period p, then prove that

$L(f(t)) = \frac{1}{1-e^{-ps}}\int_0^p e^{-st} f(t)\,dt$. Find the Laplace transform of

$f(t) = 3t + 2 \quad 0 < t < 2,\ f(t+2) = f(t)$. 10

b) Solve the integro-differential equation using Laplace transform

$\frac{dy}{dt} + \int_0^t y(t-u)\,e^u\,du = e^t,\ y(0) = 0$ 5

c) Using Laplace transform evaluate $\int_0^\infty \frac{1-\text{Cos}3t}{t}\,dt$. 5

MODULE – IV

7. a) Find the Fourier transform of $f(t) = \begin{cases} 4 - t^2 & 0 < t < 2 \\ 0 & t \geq 2 \end{cases}$. Hence show that

$$\int_0^\infty \frac{\text{Sin } 2x - 2x \text{ Cos } 2x}{x^3} dx = \pi$$ **8**

b) If F (f (x)) = F (s) is the Fourier transform of f (x), show that : **6**

i) $F(f(x-a)) = e^{ias} F(s)$

ii) $F(f'(x)) = -is F(s)$ if $f(x) \to 0$ as $x \to \pm\infty$.

c) Find the Fourier Sine transform of $f(x) = e^{-2x}$. **6**

8. a) Find the Z-transform of the following : **6**

i) $2^n / (n+1)!$

ii) $3n + 2^n$.

b) If Z (f (n) = F (z) then show that : **6**

i) $Z\left(\sum_1^n f(k)\right) = \frac{z}{z-1} F(z)$

ii) $Z(n f(n)) = -z \frac{d}{dz}(F(z))$.

c) Solve the difference equation give below using Z-transform
$y_{n+2} + 5y_{n+1} + 4y_n = 2^n$, $y_0 = 0$, $y_1 = 1$. **8**

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