S.E. (Computer Engineering) (Semester – III) (RC) Examination, Nov./Dec. 2015
APPLIED MATHEMATICS – III

Duration : 3 Hours Total Marks : 100

Instructions : *1) Answer **any five** questions with atleast **one** from **each** Module.*
*2) **Assume** suitable data **if necessary**.*

MODULE – I

1. a) Prove the following : **6**

i) If A is an orthogonal matrix, then $|A| = \pm 1$.

ii) If A is a skew-symmetric matrix then diagonal elements of A are zeroes.

b) Define rank of a matrix. Find the rank of the following matrix by reducing it to normal form. **6**

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & -2 & 8 \end{bmatrix}$$

c) Find 'λ' so that the system

$2x - y + z = \lambda y$

$3x + 2y + z = 0$

$3x + 4y + z = \lambda x$

has a non trivial solution and solve the system for any one value of 'λ'. **8**

2. a) Find a matrix P that transforms the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form.

Hence find A^4. **9**

P.T.O.

b) Verify Cayley Hamilton theorem for the following matrix. **6**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Hence find the matrix represented by

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$

c) Find e^A, given $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$. **5**

MODULE – II

3. a) An urn contains nine balls, two of which are red, three blue and four black. Three balls are drawn from the urn at random without replacement. What is the probability that

   i) The three balls are of different colours ?

   ii) The three balls are of same colour ? **6**

   b) Define conditional probability. Show that for any two events A and B $P(\overline{A}/B) = 1 - P(A/B)$. **4**

   c) A box contains 2 silver and 4 gold coins and a second box contains 4 silver and 3 gold coins. If a coin is selected at random from one box, what is the probability that it is a gold coin ? **4**

   d) A binomial random variable X has mean 8/3 and variance 16/9. Find $P(X = 1)$. **6**

4. a) If a random variable X has the probability density function $f(x) = \begin{cases} kx\, e^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$ find k. **3**

   b) Define uniform distribution. Find its mean and variance. **5**

c) The height of 22 years old boys is distributed normally with mean 63 inches and standard deviation 2.5 inches. A boy is eligible if his height is between 62 and 65 inches. Find the expected number of boys out of 180 who will be eligible.
(The area of standard normal variable z between z = 0 and z = 0.4 is 0.1554 and that from z = 0 and z = 0.8 is 0.2881). **5**

d) The following regression equations and variances are obtained from a correlation table $20x - 9y - 107 = 0$ and $4x - 5y + 33 = 0$ and variance of x is 9. **(3+2+2)**

Find :

i) The mean value of x and y.

ii) Coefficient of correlation between x and y.

iii) The standard deviation of y.

MODULE – III

5. a) Find Laplace transform of

i) $\dfrac{e^{-2t} - e^{-3t}}{t}$ ii) $\int_0^t t^2 e^t dt$. **6**

b) Let $L\{f(t)\} = F(s)$, then prove that **6**

i) $L(t\,f(t)) = -\dfrac{d}{ds}F(s)$ ii) $L\left\{\int_0^t f(u)\,du\right\} = \dfrac{F(s)}{s}$.

c) Find the inverse Laplace transform of the following **8**

i) $\dfrac{s^2 + 4s + 4}{(s^2 + 4s + 8)(s^2 + 4s + 5)}$ ii) $\tan^{-1}\left(\dfrac{2}{s+1}\right)$

6. a) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} \sin t & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \end{cases}$$ Where $f(t+2\pi) = f(t)\ \forall t$. **6**

b) Using Laplace transforms, solve the following differential equation

$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=10\sin t$ given $y(0)=0$ and $y'(0)=-1$. **8**

c) State convolution theorem. Using convolution theorem, find inverse Laplace transform of $\frac{s^2}{(s^2+9)^2}$. **6**

MODULE – IV

7. a) Find the Fourier transform of $f(x)=e^{-4x^2}$. **8**

b) If $F\{f(x)\}=F(\lambda)$, then show that $F\{f(ax)\}=\frac{1}{|a|}F\left(\frac{\lambda}{a}\right); a \neq 0$. **4**

c) Find the Fourier sine and cosine transform of $f(x)=x^{n-1}; n>0$ and hence show that the function $f(x)=\frac{1}{\sqrt{x}}$ is self reciprocal under both these transforms. **8**

8. a) Find the Z-Transform of **6**

i) $3^n-\frac{4}{n!}$ ii) $5n^2+\frac{1}{n}$.

b) Using convolution theorem for Z transforms, find $Z^{-1}\left\{\frac{z^2}{(z-2)^2}\right\}$. **6**

c) Solve the following difference equation using Z transforms. **8**

$u_{n+2}-3u_{n+1}+2u_n=4^n$; given $u_0=0$ and $u_1=1$.
