27/11/2013

COMP 3 – 1 (RC)

S.E. (Comp.) (Semester – III) (Revised Course)

Examination, November/December 2013

APPLIED MATHEMATICS – III

Duration : 3 Hours Total Marks : 100

Instructions : 1) *Attempt* ***any five*** *questions.* ***At least one*** *from* ***each*** *Module.*

2) ***Assume*** *suitable data, if* ***necessary.***

MODULE – I

1. a) Show that 5

$$\begin{bmatrix} \sin\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1}$$

b) Find the rank of the matrix by reducing it to its normal form. 5

$$\begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & -4 \\ 5 & 8 & -4 & 2 \end{bmatrix}$$

c) Prove that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. 5

d) Are the following vectors $x = (1, 3, 2)$; $y = (5, -2, 1)$; $z = (-7, 13, 4)$ linearly dependent ? If so, find the relation between them. 5

2. a) Prove that the eigen vectors corresponding to distinct eigen values of a symmetric matrix are orthogonal. 4

b) Find Sin A given $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$. 6

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c) State and prove Cayley-Hamilton theorem for a square matrix. 6

d) Show that if λ is the eigen value of an orthogonal matrix, then $|\lambda| = 1$. 4

MODULE – II

3. a) Find the probability that a year chosen at random has 53 Sundays. 6

b) If $P(A) = 0.4$, $P(A \cup B) = 0.6$ and A & B are independent, find P(B). 4

c) State and prove theorem of total probability. 4

d) Define moment generating function of a random variable. Find the moment generating function of a Poisson random variable. 6

4. a) The army is holding trails to select Jawans. 1500 have reported and their heights are normally distributed with mean 67.12 inches and standard deviation of 3.174 inches. How many will be selected if the army decided to have the minimum height as 68.5 inches ? What should be the minimum height so that the tallest 300 will be short listed ? (Area under the standard normal curve between $z = 0$ and $z = 0.43$ is 0.1664 and between $z = 0$ and $z = 0.84$ is 0.30). 8

b) A firm owning trucks is suspicious of the claim that the average lifetime of a certain tyre is at least 28,000 miles. To check the claim, the firm puts 40 of these tyres on its trucks and gets a mean life time of 27,463 miles with standard deviation of 1348 miles. What can be concluded if the probability of a type I error is to be 0.05 ? 6

c) If the regression lines of Y on X and X on Y are respectively given by $y = a_0 + a_1x$ and $x = b_0 + b_1y$, prove that $a_1b_1 = r^2$ where r is the correlation coefficient ? 6

MODULE – III

5. a) If $L(f(t)) = F(s)$, where L denotes the Laplace transform, prove the following : 6

i) $L\left(\dfrac{f(t)}{t}\right) = \int_s^{\infty} F(s)\, ds$

ii) $L\left(t\, f(t)\right) = -\dfrac{d}{ds}\left(F(s)\right)$.

b) Find the Laplace transform of 6

i) t Sin t Cos 3t

ii) $\frac{e^{3t} - e^{2t}}{t}$.

c) Solve the ordinary differential equation, using Laplace transforms
$2y''(t) + 5y'(t) - 3y(t) = e^{-2t}$, $y(0) = 1$, $y'(0) = 1$. 8

6. a) State and prove convolution theorem for Laplace transform. Use convolution theorem to find the inverse Laplace transform of

$$F(s) = \frac{3s}{(s^2+1)(s-3)}.$$ 8

b) Using Laplace transform evaluate $\int_0^\infty \frac{e^{-3t} \text{ Sin } 2t}{t} dt$. 6

c) Find the Fourier transform of $f(x) = \begin{cases} x^2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$. 6

MODULE – IV

7. a) If $F_c(f(x)) = F_c(s)$ and $F_s(f(x)) = F_s(s)$ are the Fourier Cosine and Sine transform respectively show that 6

i) $F_s(f(x) \text{ Cos}(ax)) = \frac{1}{2}\{F_s(s+a) + F_s(s-a)\}$

ii) $F_c(x f(x)) = \frac{d}{ds}(F_s(s))$.

b) Solve for f (x) the integral equation $\int_0^\infty f(x)\, Cos(sx)\, dx = e^{-2s}$. **6**

c) Define convolution of two functions. Show that convolution is commutative. Find the convolution of the functions $f(x) = 2x,\ x > 0$ and $g(x) = e^x,\ x > 0$. **8**

8. a) Find the Z-transform of the following : **6**

i) $2^n \, Cos\,(n\pi)$

ii) $(5n^2 + 1)/n$.

b) If $Z\,(f(n)) = F(z)$, is the Z-transform of $\{f(n)\}$ then show that **6**

i) $Lim_{n\to\infty}\ f(n) = Lim_{z\to\infty}\,(z-1)\,F(z)$

ii) $Z\,(f(n+k)) = z^k\, F(z)$.

c) Solve the following difference equation using Z-transform $y_{n+2} - 3y_{n+1} + 2y_n = 3^n,\ y_0 = 0,\ y_1 = 1$. **8**

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