[Total No. of Questions : 8]

S.E. (Computer) (Revised Course 2007 - 2008) (Semester - III) Examination, May 2011
APPLIED MATHEMATICS - III

Duration : 3 Hours **Total Marks : 100**

Instructions :
1) *Attempt any five questions and at least one from each module.*
2) *Figures to the right indicate full marks.*
3) *Make suitable assumptions wherever required.*
4) *Use statistical tables wherever required.*

MODULE - I

Q1) a) i) Let A and B be non - singular $n \times n$ matrices. Prove that, adj (AB) = adj B and adj A.

ii) Let A be an orthogonal matrix. Prove that $|A| = \pm 1$. **[6]**

b) Test for consistency the following non-homogeneous system of linear equations and find a solution if any exist: **[7]**

$$2x + 3y + 4z = -1$$
$$x + 2y - 2z = 2$$
$$3x - y + 3z = 1$$
$$x - y + 4z = 2$$

c) Investigate for what values of λ, the following homogeneous system of linear equations has a non-trivial solution. Hence solve the system for at least one of these values of λ. **[7]**

$$2x + 3\lambda y + (3\lambda + 4)z = 0$$
$$x + (\lambda + 4)y + (4\lambda + 2)z = 0$$
$$x + 2(\lambda + 1)y + (3\lambda + 4)z = 0.$$

Q2) a) Find the matrix P such that $P^{-1}AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$ **[7]**

b) Find the eigen - values and eigen - vectors of A^{-1}, where the matrix **[7]**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

c) Prove that the eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal. **[6]**

P.T.O.

MODULE - II

Q3) a) The cumulative distribution function of a random variable X is given by,

$$F(x) = 1-(1+x)e^{-x}, \quad x \geq 0$$
$$= 0, \quad x < 0$$

Find the probability density, mean and variance of X. **[6]**

b) Out of 800 families with 4 children each, how many families would be expected to have, **[7]**

i) 2 boys and 2 girls.

ii) at least one boy.

iii) at most two girls.

Assume equal probabilities for boys and girls.

c) Let X be a Poisson random variable such that $E(X^2) = 6$. Find, **[7]**

i) $P(X \leq 1)$ ii) $E(X)$

Q4) a) The diameter of screws manufactured by a company are normally distributed with mean 0.25 cm and standard deviation 0.02cm. A screw is considered defective if its diameter is less than 0.20 cm or greater than 0.28 cm. Find the percentage of defective screws manufactured by the company. **[6]**

b) A sample of 100 bulbs produced by a company A showed a mean life of 1190 hours and a standard deviation of 90 hours. Also, a sample of 75 bulbs produced by a company B showed a mean life of 1230 hours and a standard deviation of 120 hours. Is there a difference between the mean life times of the bulbs produced by the two companies at the 0.05 level of significance. **[6]**

c) The table below gives the heights and weights of a sample of 12 students. **[8]**

i) Find the lines of regression.

ii) Estimate the weight of a student whose height is known to be 63 inches.

Height (X inches)	70	63	72	60	66	70	74	65	62	67	65	68
Weight (Y pounds)	155	150	180	135	156	168	178	160	132	145	139	152

MODULE - III

Q5) a) Find the Laplace transform of the following: **[9]**

i) $f(t) = t\sqrt{1+\sin t}$ ii) $g(t) = \dfrac{e^{-2t} - \cos 3t}{t}$

iii) $h(t) = \dfrac{\sin\sqrt{t}}{\sqrt{t}}$

b) Prove that, [6]

i) $L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} \bar{f}(u)\, du$

ii) $L\left\{\int_0^t f(u)\, du\right\} = \frac{\bar{f}(s)}{s}$,

where $\bar{f}(s) = L\{f(t)\}$, denotes the Laplace transform of $f(t)$.

c) Find the Laplace transform of [5]

$$f(t) = \begin{cases} t, & 0 < t \le \pi \\ \pi - t, & \pi < t < 2\pi, \end{cases}$$

where $f(t + 2\pi) = f(t)\ \forall\, t$.

Q6) a) Find the inverse Laplace transform of the following: [11]

i) $\bar{f}(s) = \frac{s}{(s^2+4)^2}$, ii) $\bar{g}(s) = \tan^{-1}\left(\frac{2}{s^2}\right)$,

iii) $\bar{h}(s) = \frac{1}{s}\cos\frac{1}{s}$.

b) Using the Laplace transform solve the integral equation,

$$y(t) = t + \int_0^t y(u)\sin(t-u)\, du.$$ [9]

MODULE - IV

Q7) a) Find $f(x)$ such that,

$$\int_0^{\infty} f(x)\cos\lambda x\, dx = \begin{cases} \lambda - 1, & 0 \le \lambda < 1 \\ 0, & \lambda \ge 1. \end{cases}$$ [5]

b) Prove the following: [8]

i) $F\{f(x)\cos ax\} = \frac{1}{2}\left[\hat{f}(s+a) + \hat{f}(s-a)\right]$ where $\hat{f}(s) = F\{f(x)\}$ denotes the Fourier transform of $f(x)$.

ii) $F(f(x-a)) = e^{isa}\, \hat{f}(s)$,

where "a" is a constant.

c) Evaluate, using the Fourier transform, the integral $\int_0^{\infty} \frac{1}{(x^2+a^2)(x^2+b^2)}\, dx$, where "a" and "b" are constants. [7]

Q8) a) Find the Z-transform of the following : **[6]**

i) $a_n = \dfrac{1}{(n+1)!}$, ii) $b_n = \dfrac{2n+2}{(n+1)(n+2)}$

b) Use the convolution theorem to find the inverse Z-transform of, $\bar{u}(z) = \dfrac{z^2}{(z-a)^2}$ **[6]**

c) Solve the difference equation, $u_{n+2} + 6u_{n+1} + 8\,u_n = 1$, with $u_0 = 1$ and $u_1 = -1$. **[8]**

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