S.E. (Comp.) (Semester – III) (Revised 2007 – 08 Course) Examination, May/June 2012
APPLIED MATHEMATICS – III

Duration : 3 Hours

Total Marks : 100

Instructions *: 1) Attempt **any five** questions. Atleast **one** from **each** Module.*

*2) **Assume** suitable data, **if** necessary.*

MODULE – I

1. a) Find the rank of the matrix by reducing it to its normal form.

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$ 6

b) For what values of λ and μ the system.

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

has (1) no solution (2) unique solution (3) infinitely many solutions. 8

c) Show that every square matrix can be uniquely expressed as sum of symmetric and skew-symmetric matrix. 6

2. a) Prove that two similar matrices have the same eigen value. 5

b) Find the characteristic and minimal polynomial of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

from the minimal equation, find A^{-1}. 8

P.T.O.

c) Diagonalise the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ 7

MODULE – II

3. a) A Honda car dealer, sells Honda city, Honda civic, and Honda Brio cars only. Of the cars sold 52% are Brio, 28% are city and 20% are civic cars. Of the next 8 car sales, what is the probability that 5 will be Brio, 1 will be city and 2 will be civic cars ? 7

b) A box contains tags ranked 1, 2, ..., n. Two tags are chosen at random without replacement. Find the probability that the number on the tags will be consecutive integers. 5

c) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group; what is the probability that atleast one of them would have scored above 75 ? 8

4. a) A machine produces 12 defective products in a sample of 500. After the machine is overhanded it puts out 2 defective articles in a sample of 100. Has the machine improved ? 6

b) The following marks have been obtained by a class of students in statistics :

Paper I	80	45	55	56	58	60	65	68	70	75	85
Paper II	81	56	50	48	60	62	64	65	70	74	90

Compute the coefficient of correlation for the data given above. Find the lines of regression. 8

c) Show that the sum of two independent Poisson random variable is also a Poisson random variable. 6

MODULE – III

5. a) Find the Laplace transform of the following functions : 6

1) $f(t) = \dfrac{e^{-4t}}{\sqrt{t}}$ 2) $f(t) = e^{-2t}\,\dfrac{(1-\cos t)}{t}$

b) Find the Laplace transform of the periodic function. 5

$$f(t) = \begin{cases} t\ , & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$$

$f(t + 2\pi) = f(t) \quad \forall t.$

c) State and prove convolution theorem for Laplace transform and hence find $L^{-1}\left\{\dfrac{s}{(s+1)\,(s^2+1)}\right\}$ 9

6. a) Use Laplace transform method to solve $\dfrac{dy}{dt} + 3\int_0^t y(u)du = 3e^t,\ y(0) = 1.$ 8

b) Solve the initial value problem

$2y'' + 5y' + 2y = e^{-2t},\ y(0) = 1,\ y'(0) = 1.$ 8

c) Using Laplace transform evaluate the integral. 4

$$\int_0^\infty e^{-4t}\ t\sin 3t\ \ dt.$$

MODULE – IV

7. a) Find the Fourier transform of 8

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases}.$$

Hence find the value of $\displaystyle\int_0^\infty \frac{\sin^4 t}{t^4}\,dt$.

b) Show that $f(x) = e^{\frac{-x^2}{2}}$ is a self-reciprocal function. 7

c) Solve the integral equation

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda}.$$ 5

8. a) Find the z-transform of 6

1) $\frac{1}{(n+1)!}$ 2) $\frac{1}{2}(n-1)(n+2)$

b) Using z-transform solve the equation

$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$. 6

c) State and prove convolution theorem and using convolution theorem, find . 8

$$Z^{-1}\left\{\frac{z^2}{(z-z)^2}\right\}.$$
