COMP 3 – 1 (RC)

S.E. (Computer Engg.) (Semester – III) Examination, Nov./Dec. 2010
(Revised Course 2007-08)
APPLIED MATHEMATICS – III

Duration: 3 Hours | Total Marks: 100

Instructions : *1) Attempt **any five** questions and at least **one** from **each** Module.*
*2) Figures to the right indicate **full** marks.*
*3) Make suitable assumptions **wherever** required.*
*4) Use statistical tables **wherever** required.*

MODULE – 1

I. a) i) Let A be a non-singular $n \times n$ matrix. Prove that, $\text{adj}(\text{adj } A) = |A|^{n-2}.A$. **6**

ii) Let A be a $n \times n$ matrix such that rank of A is $n - 2$ $(\rho(A) = n - 2)$. Show that adj $A = 0$.

b) Determine $\alpha, \beta, \gamma,$ such that the matrix $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is an orthogonal matrix. **7**

c) Reduce the matrix given below to its normal form and hence find its rank :

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}.$$ **7**

II. a) Find the eigenvalues and eigenvectors of adj A, where the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ **7**

P.T.O.

b) Let A be a non-singular matrix and λ be an eigen-value of A. Then prove that, $\frac{1}{\lambda}$ is an eigen-value of A^{-1}. **6**

c) Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. Verify the Cayley-Hamilton theorem for A, and hence show that $A^{-1} = A^3$. **7**

MODULE – 2

III. a) Let X be a discrete random variable with the following probability distribution.

X = n :	1	2	3	4	5	6	7
P (X = n) :	C	2C	2C	3C	C^2	$2C^2$	$(7C^2 + C)$

i) Find the value of "C".

ii) Compute $P(1.5 < X < 4.5)$.

iii) Find the mean of X. **8**

b) Compute the moment generating function for the binomial distribution. Hence, compute its mean and variance. **8**

c) Let X be a random variable which is uniformly distributed over (–1, 1). Compute $P(0 < X \leq 4)$. **4**

IV. a) The probability that an individual suffers a bad reaction from injection of a certain serum is 0.002. Determine the probability that out of 1000 individuals,

i) Exactly 3 will suffer a bad reaction,

ii) More than 2 will suffer a bad reaction. **6**

b) A sample poll of 300 voters from district A and 200 voters from district B showed that 56% to and 48% respectively, were in favour of a given candidate. At a significance level of 0.05, test the hypothesis that there is a difference between the districts with regard to preference shown for the candidate. **8**

c) The equation of the regression line "y on x" is $8x - 10y + 66 = 0$. The mean of x is 13 and the standard deviations of x and y are 3 and 4 respectively. Find the equation of the regression line "x and y". **6**

MODULE – 3

V. a) Find the Laplace transform of the following :

i) $f(t) = e^{-2t}\, t^2 \sin 2t$,

ii) $g(t) = \dfrac{1-\cos t}{t}$,

iii) $h(t) = \left(\displaystyle\int_0^t e^{2u} \cos u \, du\right).t$ **9**

b) Prove the following :

i) $L\{f'(t)\} = s\bar{f}(s) - f(0)$,

ii) $L\{t^n f(t)\} = (-1)^n \dfrac{d}{ds}\bar{f}(s)$,

where $\bar{f}(s) = L\{f(t)\}$, denotes the Laplacetransform of f(t). **6**

c) Let f(t) be periodic with period "p". Prove that, $L\{f(t)\} = \dfrac{1}{1-e^{-sp}} \displaystyle\int_0^p e^{-st} f(t)\, dt$. **5**

VI. a) Find the inverse Laplace transform of the following :

i) $\bar{f}(s) = \dfrac{s^2+1}{(s+3)(s^2-3s+1)}$,

ii) $\bar{g}(s) = \tan^{-1}\left(\dfrac{1}{s}\right)$,

iii) $\bar{h}(s) = \dfrac{1}{s}\log\left(\dfrac{s^2+4}{s^2+9}\right)$. **10**

b) Solve the following initial value problem by using the Laplace transform :

$y''(t) + y(t) = t, \; y(0) = 1, \; y'(0) = -2$. **10**

MODULE – 4

VII. a) Find the Fourier transform of $f(x) = 1 - |x|, |x| \leq 1$ and hence prove that,
$= 0, |x| > 1$;

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}.$$ **8**

b) Prove that,

i) $F\{f(ax)\} = \frac{1}{a}\hat{f}\left(\frac{s}{a}\right), a > 0$;

ii) $F\{f(x-a)\} = e^{isa}\hat{f}(s)$, where $\hat{f}(s) = F\{f(x)\}$, denotes the Fourier transform of $f(x)$. **6**

c) Evaluate the following integral, using the Fourier transform :

$$\int_0^\infty \frac{1}{(x^2+a^2)^2} dx.$$ **6**

VIII. a) Find the Z-transform of the following :

i) $a_n = n^2 + 2^n \cos n\pi/2$,

ii) $b_n = 3^n (n+1)$. **6**

b) State and prove the final value theorem for Z-transform. **7**

c) Solve the difference equation $4_{n+2} + 4_n = 2^n$; with $4_0 = 1, 4_1 = 0$. **7**
