S.E. (Comp.) (Sem. – III) Examination, November/December 2009 APP. MATHEMATICS – III (Revised 2007-08)

Duration: 3 Hours

Total Marks: 100

Note: i) Attempt any five questions.

- ii) Atleast one from each Module.
 - ii) Assume suitable data wherever required.

MODULE - I

1. a) Define:

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- i) a Hermitian matrix
- ii) a skew-Hermitian matrix.

Prove the following:

- i) The determinant of a Hermitian matrix is real
 - ii) The determinant of a skew-Hermitian matrix of odd order is zero.
- b) Find the rank of the following matrix:

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$$A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$$

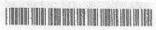
c) Test the following vectors for linear dependence or independence :

 $X_1, = [1, 1, 1, 3]^T, X_2 = [1, 2, 3, 1]^T, X_3 = [2, 3, 4, 1]^T.$

a) Find an non-singular matrix P such that P -1 AP is diagonal, whose diagonal elements are eigen values of 'A', given

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 6 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$
(O1 - GFR = [G)Y = (1)
$$\frac{1}{2} = \begin{bmatrix} \frac{1}{2} & \frac$$

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b) State and prove Cayley-Hamilton theorem.

c) Prove that eigen vectors corr. to distinct eigen values of a real symmetric matrix are orthogonal.

MODULE - II

3. a) State and prove Baye's theorem of probability.

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b) Two numbers x and y are chosen from {1, 2, 3n}. Find the probability that x + y will be divisible by 3. purply addressed section (a)

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c) A continuous random variable 'X' has probability density function :

$$f(x) = \begin{cases} Kx^3 e^{-x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find K, mean and variance.

4. a) A normal population has a mean 0.1 and standard deviation of 2.1. Find the probability that the mean of a sample of size 900 drawn from this population will be negative. To be minus nationally waste to manage and of

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b) A manufacturer claims that the mean breaking strength of safety belts for air passengers produced in his factory is 1275 kg. A sample of 100 belts was tested and the mean breaking strength and standard deviation were found to be 1258 and 90 kg resp. Test the manufacturers claim at 5% LOS.

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c) Obtain the lines of regression and find the coefficient of correlation from the following data:

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5. a) If L[f(t)] = F(s), prove the following:

1)
$$L[f'(t)] = sF(s) - f(0)$$

2)
$$L\left[\int_{0}^{t} f(u) du\right] = \frac{F(s)}{s}$$
.



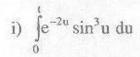
S u) Prove that:

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b) Find the Laplace transforms of:



ii) e^{3t}cos3t sint

iii)
$$\frac{e^{at} - \cos 6t}{t}$$

- c) If L [f(t)] = F(s) then prove that $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$.
- 6. a) State and prove convolution theorem for Laplace transforms.
 - b) Using Laplace transform solve :

$$\frac{d^2y}{dt^2} + 4y = \sin t, y(0) = 1, y'(0) = 0.$$

c) Find inverse Laplace transform of each of the following: Clause A. Shill Co.

1)
$$\frac{s}{(s^2+4)(s^2+9)}$$

2)
$$\log\left(\frac{s}{s-1}\right)$$
 to assent that $\frac{1}{1s}$ (i

MODULE – IV
$$\left(\frac{2\pi t}{\hbar}\right)^2 mis$$
 (if

7. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

and use it to evaluate
$$\int_{0}^{\infty} \left(\frac{x \cos x - \sin x}{x^{\frac{1}{3}}} \right) \cos \left(\frac{x}{2} \right) dx$$
.

b) Solve the integral equation $\int_{0}^{\infty} f(x) \cos \lambda x = e^{-\lambda}$.

c) If $\mathcal{I}[f(x)] = \overline{f}(\alpha)$ prove that :

i)
$$\mathcal{I}[f(ax)] = \frac{1}{a}\bar{f}\left(\frac{\alpha}{a}\right)$$

ii)
$$\mathcal{F}[f(x)\cos ax] = \frac{1}{2}[\bar{f}(\alpha+a)+\bar{f}(\alpha-a)]$$

where Idenotes Fourier transform.

8. a) Prove that:

i)
$$Z[n] = \frac{Z}{(Z-1)^2}$$

$$ii) Z[a^n] = \frac{Z_{-1}}{Z-a} \cdot -(a) Z^n = [aa] \cdot Z_{-1}$$

b) State convolution theorem and find inverse Z-transform of

.

$$\frac{8Z^2}{(2Z-1)(4Z+1)}$$

c) Find Z-transforms of:

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i)
$$\frac{1}{n!}$$
 and hence of $\frac{1}{(n+1)!}$

ii)
$$\sin^2\left(\frac{n\pi}{4}\right)$$
.

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T<|x| 1 0

ul use it to evaluate [[×