SEM - III COMP - 2015|13

COMP 3 – 1 (RC)

S.E. (Computer Engg.) (Semester – III) Examination, May/June 2013
APPLIED MATHEMATICS – III
(Revised Course)

Duration: 3 Hours Total Marks: 100

Instructions : 1) *Answer* ***five*** *questions and at least* ***one*** *from* ***each*** *Module.*
2) *Figures to the* ***right*** *indicate* ***full*** *marks.*
3) *Make suitable assumptions* ***wherever*** *required.*
4) ***Use*** *statistical tables* ***wherever*** *required.*

MODULE – 1

I. a) i) Define adjoint of a square matrix.

ii) If A is a non-singular $n \times n$ matrix then show that $|\text{adj } A| = |A|^{n-1}$. 6

b) Reduce the matrix given below to its normal form and hence find its rank : 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

c) Solve the following line an system of equations after testing for consistency : 8

$$x + y + z = 1$$
$$x + 3y + 2z = 2$$
$$2x + 2y + z = 3$$

II. a) Find the minimum polynomial of the matrix,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ 6

b) For the real symmetric matrix A given below find the orthogonal matrix P such that $P^{-1}AP$ is a diagonal-matrix : 8

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

P.T.O.

c) Verify the Cayley-Hamilton theorem for the matrix, 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

MODULE – 2

III. a) The probability that an individual suffers a bad reaction an taking a certain drug is 0.002. Determine the probability that out of 1000 individuals

i) Exactly 3 will suffer a bad reaction, and

ii) More than 2 will suffer a bad reaction. Use the Poisson distribution. 8

b) A continuous random variable X has the following probability density function : 6

$$f(x) = cx^2 e^{-x}, \ x > 0$$
$$= 0, \quad x \le 0$$

i) Find C, and

ii) Compute the mean of X.

c) In a random experiment, $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{3}$, $P(B/A) = \frac{3}{4}$. Find $P(A \cup B)$. 6

IV. a) Compute the moment generating function of a Binomial random variable X with probability mass function $P(X = r) = {}^nc_r\, p^r q^{n-r}$, $r = 0, \ldots, n$, where $0 < p < 1$ and $q = 1 - p$. Hence find the mean and variance. 7

b) Find the coefficient of correlation for the following data : 6

X :	65	66	67	67	68	72	71	70
Y :	67	68	62	68	70	65	64	68

c) Random samples of 200 bolts manufactured by machine A and of 100 bolts manufactured by machine B, showed 19 and 5 defective bolts respectively. Test the hypothesis that machine B is performing better than A, at the 0.05 significance level. 7

MODULE – 3

V. a) Find the Laplace transform of the following : 9

i) $f(t) = e^{-2t} \cos^2 3t$

ii) $g(t) = \frac{\sinh t}{t}$

iii) $h(t) = \int_0^t u \sin 3u \, du$.

b) Find the inverse Laplace transform of the following : 6

i) $\bar{f}(s) = \frac{s}{s^2 + 4s + 5}$

ii) $\bar{g}(s) = \frac{1}{s} \log\left(\frac{s^2 + 4}{s^2 + 9}\right)$

c) Let f(t) be periodic with period "p". Prove that,

$L(f(t)) = \frac{1}{1 - e^{-sp}} \int_0^p e^{-st} f(t) \, dt$. 5

VI. a) Use the Laplace transform to evaluate $\int_0^\infty t\, e^{-2t} \sin 3t \, dt$. 6

b) Use the convolution theorem to fond the inverse Laplace transform of ,

$\bar{f}(s) = \frac{s}{(s^2 + 4)^2}$. 6

c) Use the Laplace transform to solve the differential equation

$y''(t) + y'(t) - 2y(t) = 3 \cos 3t - 11 \sin 3t$, $y(0) = 0$ and $y'(0) = 6$. 8

MODULE – 4

VII. a) Find the Fourier transform of

$$f(x) = 1 \ , \ |x| \le a$$
$$= 0 \ , \ |x| > a$$

Where $a > 0$. Hence find the value of $\int_0^\infty \frac{\sin x}{x} dx$. **8**

b) Find the inverse Fourier transform of , $\bar{f}(s) = a - |s|, \ |s| \le a$, where $a > 0$. **6**
$= 0, \ |s| > a$

c) Solve for f(x) the integral equation,

$\int_0^\infty f(x) \cos \lambda x \, dx = 1 - \lambda, 0 < \lambda < 1 = 0, \lambda \ge 1$. **6**

VIII. a) Find the Z-transform of the following : **8**

i) $f(n) = n$

ii) $g(n) = 2^n$.

b) Find the inverse Z-transform of $\bar{g}(z) = \frac{8z^2}{(2z-1)(4z+1)}$. **6**

c) Use the Z-transform to solve the difference equation,

$y(n + 2) + 3y(n + 1) - 4y(n) = 0, n \ge 0$, and $y(0) = 3$, $y(1) = -2$. **6**

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