28/11/14 Repeat (M) comp

COMP 4 – 1 (RC)

S.E. (Comp.) (Sem. – IV) (RC) Examination, Nov./Dec. 2014
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, atleast **one** from **each** Module.*
*2) Assume suitable data, **if necessary**.*

MODULE – I

1. A) $\{A_k : k = 1, 2, ...\}$ be collection of subsets of some universal set U then show that $\left(\bigcup_{k \in I} A_k\right)' = \bigcap_{k \in I} A'_k$. 6

   B) If A and B are two non-empty subsets of a Universal set, prove that $A - B = A$ if and only if $A \cap B = \varphi$. 6

   C) N is a set of natural numbers. In $N \times N$ show that the relation R defined by (a, b) R (c, d) if and only if ad = bc is an equivalence relation.
   Give an example of a relation on a set which is reflexive and symmetric but not transitive. 8

2. A) Set A is the set of factors of particular positive integer m and $\leq$ be the relation $\leq = \{(x, y) \mid x \in A, y \in A, x \text{ divides } y\}$.
   Draw an Hasse diagram for m = 12. 7

   B) Among the first 500 positive integers, determine the integers which are not divisible by 2, nor by 3, nor by 5. 6

   C) State Pigeon Hole Principle. Using Pigeon Hole Principle show that, in a group of 13 children, there must be at least two children who were born in the same month. 7

P.T.O.

MODULE – II

3\. A) Let (A, $*$) be a semigroup. Show that for a, b, c in A, if $a * c = c * a$ and $b * c = c * b$ then $(a * b) * c = c * (a * b)$. Justify every step of your answer. 5

B) Q is the set of rational numbers and $*$ is a binary operation defined by $a * b = a + b - ab$, for all a, b in Q. Show that (Q, $*$) is a group. 5

C) Z is the set of integers and operation $*$ is defined by $x * y$ = maximum (x, y). Determine whether (Z, $*$) is a monoid or a group or an abelian group. 5

D) $G = \{1, 5, 7, 11, 13, 17\}$ under multiplication modulo 18. Construct the multiplication table of G. Find 5^{-1}, 7^{-1}, 17^{-1}. Is G Cyclic ? Justify your answer. 5

4\. A) Let R be the field of real numbers. Show that $W = \{(x, 2y, 3z) \mid x, y, z \in R\}$ is a subspace of R^3. 7

B) Prove that if two vectors are linearly dependent, one of them is scalar multiple of other. 6

C) Let T be linear transformation on R^3 defined by $T(a, b, c) = (3a, a - b, 2a + b + c)$, $\forall a, b, c \in R^3$. Show that T is invertible. 7

MODULE – III

5\. A) State Principle of Mathematical Induction. Use it to show that $11^{n+2} + 12^{2n+1}$ is divisible by 133. 7

B) A person invests Rs. 35,000 @ 9.5% interest compounded annually. How much will be the total amount at the end of 17 years ? 6

C) Solve the recurrence relation $a_{r+2} - 4a_r = r^2 + r - 1$. 7

6\. A) Define a Boolean Algebra. B) Prove that : 6

i) The inverse of an element is unique.

ii) If $x + z = y + z$ and $x.z = y.z$ then $x = y$ for all $x, y \in B$.

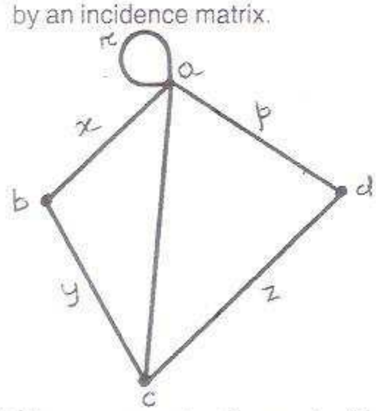
B) Simplify the Boolean expression $(y \cdot z + x) \cdot (x' \cdot y' + z') + x' \cdot y' \cdot z'$. 5

C) Define Conjunctive Normal form. Obtain the principle Conjunctive Normal form for $p \rightarrow [(p \rightarrow q) \wedge \sim (\sim q \vee p)]$ 4

D) Prove that : $p \rightarrow (q \rightarrow r) \Leftrightarrow p \rightarrow (\sim q \vee r) \Leftrightarrow (p \wedge q) \rightarrow r$. 5

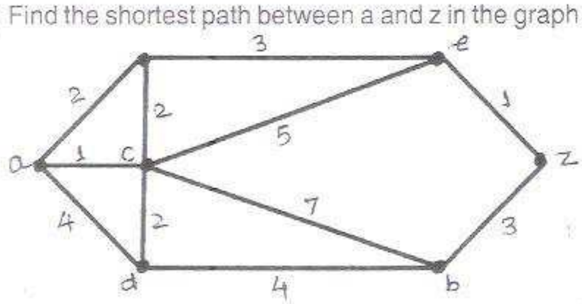
MODULE – IV

7. A) Define incidence matrix of a undirected graph. Represent the following graph by an incidence matrix. 6

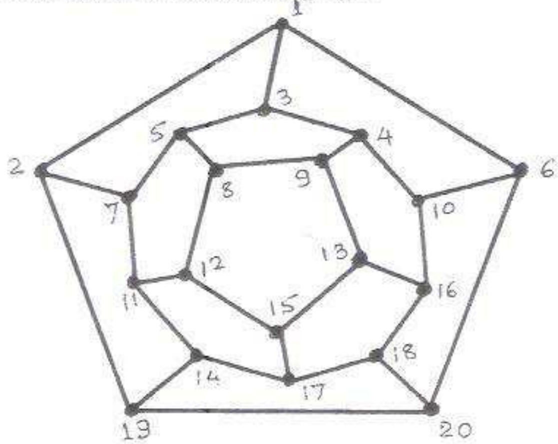


B) Give an example of a graph with six vertices that has exactly two cut points. 4

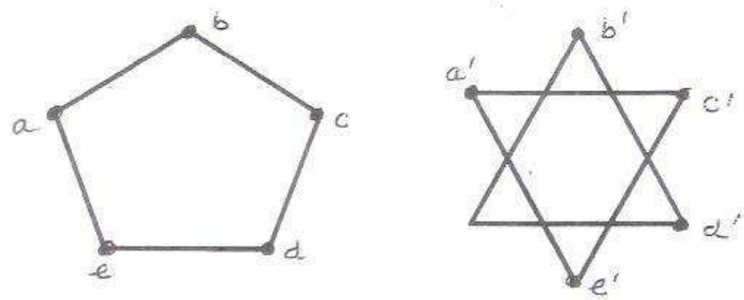
C) Find the shortest path between a and z in the graph shown below. 5



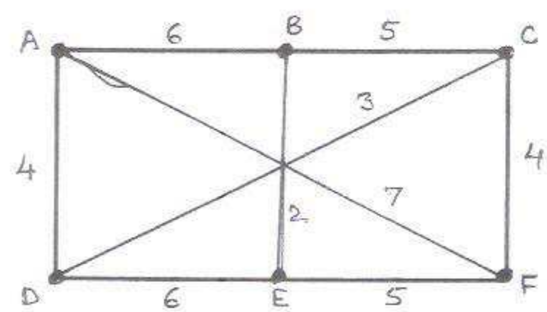
D) Define Hamiltonian graph. The graph below is Hamiltonian graph. Determine Hamiltonian circuit for this graph. 5



8. A) Show that the given pair of graph below is isomorphic. 7



B) Using Kruskal's algorithm determine the minimum spanning tree of the weighted graph below. 8



C) Prove that the maximum number of vertices in a Binary tree of height x is $(2^{x+1} - 1)$. 5