S.E. (Computer Engineering) (Revised 2007-08) Sem. – IV
Examination, May/June 2010
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours . Total Marks : 100

Instructions : 1) Answer any five questions with at least one from each Module.
2) Assume suitable data if necessary.

MODULE – I

1. a) Let A, B and C be any three non empty sets.
If $A \cap B = A \cap C$ and $A \cup B = A \cup C$ then show that $B = C$. 4

b) Let $(B, +, \cdot, ^{-}, 0, 1)$ be a Boolean algebra where +, · and ¯ are the AND, OR and NOT operators respectively for a, b ∈ B. Define a relation '≤' on B as $a \leq b$ iff $a \cdot \overline{b} = 0$. Show that (B, ≤) is a POSET. 7

c) Let $f : N \rightarrow N$ be defined as $f(n) = \begin{cases} \frac{n+1}{2} & ;\text{n odd} \\ \frac{n}{2} & ;\text{n even} \end{cases}$
If f is bijective, find its inverse. 6

d) Is $(Z - \{0\}, I)$ a poset, where 'I' denotes division. Justify. 3

2. a) State Pigeonhole principle.
Prove that at a party where there are at least two people, there are two people who know the same number of other people there. (Assume that knowing each other is a symmetric relation). 8

b) Find the remainder when $189 \times (491)^2 \times (592)^3$ is divided by 11. 6

c) How many integers between 1 to 300 (both inclusive) are
i) divisible by atleast one of 5 or 6 or 8.
ii) divisible by neither 5 nor 6 nor 8. 6

P.T.O.

MODULE – II

3. a) Every monoid is a group. Prove or Disprove. 2

b) Let $G = \{(a,b) : a, b \in R, a \neq 0\}$. Prove that $(G,*)$ is a commutative group under the operation '*' defined by (a, b) * (c, d) = (ac, b + d) for all (a, b), (c, d) ε G. 6

c) State and prove Lagrange's theorem for a group. 6

d) Let R be an algebraic system satisfying all the conditions for a ring with unity element with the possible exception of a + b = b + a. Prove that a + b = b + a must hold in R. 6

4. a) Show that the set $S = \{a + b\sqrt{2} : a, b \in Z\}$ for the operations +, x is an intergral domain but not a field. 7

b) Which of the following are vector subspaces of R^3 ? 6

i) $W_1 = \{(x, y, z) : x = 2\}$

ii) $W_2 = \{(x, y, z) : x + y + z = 0\}$

iii) $W_4 = \{(x, y, z) : z = x + y\}$

c) Consider the map $T : R^3 \to R^3$ given by T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z). Show that T is a linear transformation. Find the dimension of Kernel of T and Range of T. 7

MODULE – III

5. a) Define functionally complete set of connectives. 5

Show that $\{\downarrow\}$ is functionally complete.

b) Without using Truth tables prove that

$(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$. 4

c) Obtain the principal disjunctive normal form of the Boolean function

$f(x_1, x_2\ x_3) = (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + x_3)$ 6

d) Using the rules of inference, show that the premises $E \to S, S \to H, A \to H$ and the conclusion $E \wedge A$ is inconsistent. 5

6. a) State the principle of Mathematical Induction. Use mathematical induction to prove that

$$1+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{n^2}<2-\frac{1}{n} \text{ whenever } n>1.$$ 7

b) Solve the recurrence relation

$a_n - 6a_{n-1} + 9a_{n-2} = n^2 \cdot 2^n$ with $a_0 = 1$ and $a_1 = 0$; $n \geq 2$. 8

c) Find the recurrence relation for the number of ways of climbing n steps if a person can climb one or two or three steps at a time. Also give the initial conditions. 5

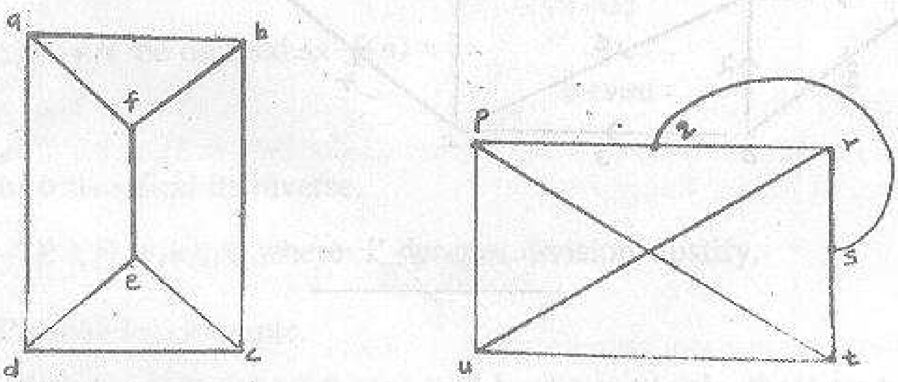
MODULE – IV

7. a) i) Give an example of an undirected graph with degree sequence 1, 3, 3, 4, 5, 6. 2

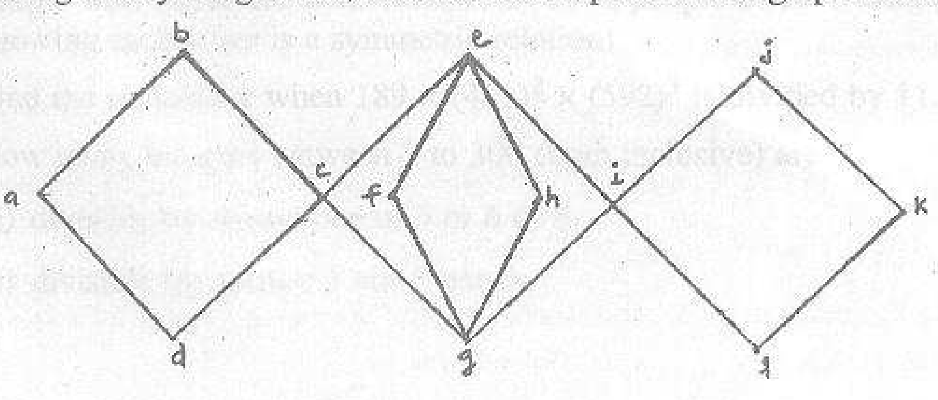
ii) Is the cycle C_7 bipartite ? Justify. 2

iii) Define graph isomorphism. 4

Check whether the following graphs are isomorphic or not.



b) Using Fleury's algorithm, find an Euler's path for the graph shown below. 4



c) State and prove Euler's theorem for a connected planar graph. 6

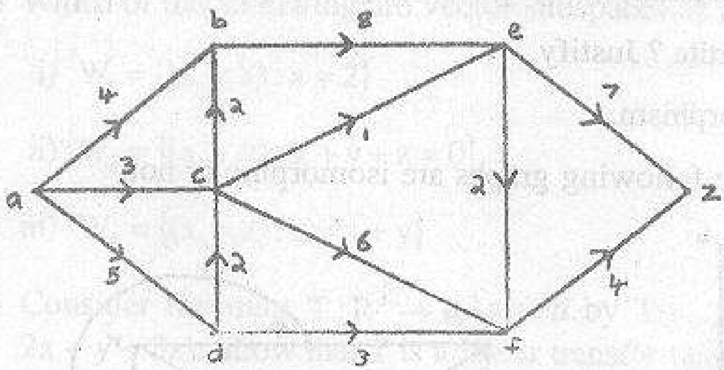
d) Give an example of graph which has an Euler's circuit but not a Hamiltonian circuit. 2

8. a) Show that a graph with n – 1 edges and n vertices that has no circuit is a tree. 6

b) i) Show that a full (regular) m-ary tree with "i" internal vertices contain n = mi + 1 vertices. 6

ii) How many vertices and internal vertices does a full (regular) 4-ary tree with 100 leaves have ?

c) Using Ford-Fulkerson's algorithm, find the maximum flow in the transport network shown below. 8



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