S.E. (Computer Engineering) Semester – IV (Revised 2007-08) Examination, May/June 2009
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours | Total Marks : 100

Instructions : 1) *Answer* ***any five*** *questions with atleast* ***one*** *from* ***each*** *Module.*
2) *Assume* ***suitable*** *data if* ***necessary***.

MODULE – I

1. a) Let A, B and C be any three non-empty sets. 4
   i) Show that $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$
   ii) If $A\cap B=A\cap C$, is $B=C$? Justify.

   b) Draw the Hasse diagram representing the partial ordering $\leq$ on the set $S=\{1,2,3,4,6,8,12\}$ given by $a\leq b$ if 'a divides b'. Which are the maximal elements, upper bounds, lower bounds, supremum and infimum of the subset $A=\{2,6,8,12\}$? 5

   c) Let $f: N\to N$ be defined as $f(n)=\begin{cases}\dfrac{n+1}{2}; & n \text{ odd}\\ \dfrac{n}{2} ; & n \text{ even}\end{cases}$ 5

   If f is bijective, find its inverse.

   d) Let R be a transitive and reflexive relation on A. Let T be a relation on A such that $(a,b)\in T$ iff both (a,b) and $(b,a)\in R$. Show that T is an equivalence relation on A. 6

2. a) State Pigeonhole principle.
   Suppose 30 balls are numbered from 1 to 30 and placed in a large box, show that, if f 18 balls are drawn there must be a pair among them whose sum of the numbers appearing on the balls drawn is 35. 7

   b) Find the remainder when $4\times 12\times 27\times 35\times 41\times 527$ is divided by 13. 7

   c) Find the least number of ways of choosing three different numbers from 1 to 10, so that all choices have the same sum. 6

P.T.O.

MODULE – II

3. a) Define a submonoid generated by a set. Determine the submonoid generated by the set $S = \{p : p \text{ is a prime number}\}$ in the monoid $(N, \cdot)$ where N is the set of natural numbers and '$\cdot$' denotes multiplication. 5

b) Show that $(Z_6, +_6)$ is a cyclic group. Find one non-trival subgroup of $(Z_6, +_6)$. 7

c) If G is a group of prime order p, then show that G has no proper subgroup. 4

d) Let $(R^+, \cdot)$ be the multiplicative group of all positive real numbers. Define a function $f : R^+ \to R^+$ by $f(x) = x^2$ for all $x \in R^+$. Show that f is an automorphism. 4

4. a) Show that a finite integral domain is a field. 6

b) Which of the following are vector subspaces of R^3 : 6

i) $W_1 = \{(x,y,z) : x = 2\}$ ii) $W_2 = \{(x,y,z) : x = z = 0\}$

iii) $W_4 = \{(x,y,z) : z = x + y\}$?

c) Consider the map $T : R^3 \to R^3$ given by $T(x,y,z) = (x + z, x + y + 2z, 2x + y + 3z)$. Show that T is a linear transformation. Find the dimension of Kernel of T and Range of T. 8

MODULE – III

5. a) Define a Boolean Algebra. In a Boolean Algebra B, Prove that

i) $\forall a \in B; a'$ is unique ii) $\forall a, b \in B; (a \cdot b)' = a' + b'$ 6

b) i) Define conjunctive normal form. Express the following expression in the principal conjunctive normal form : 5

$(p \wedge q) \vee (\sim p \vee q) \vee (q \wedge r)$

ii) Without using truth tables prove that $(p \vee q) \wedge (\sim p \wedge (\sim p \wedge q)) \equiv \sim p \wedge q$ 4

c) Show that $r \wedge (p \vee q)$ is a valid conclusion from the premises $p \vee q$; $q \to r$; $p \to m$; $\sim m$. 5

6. a) State the second principle of Mathematical Induction.

Use mathematical induction to prove that for all positive integers n,

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

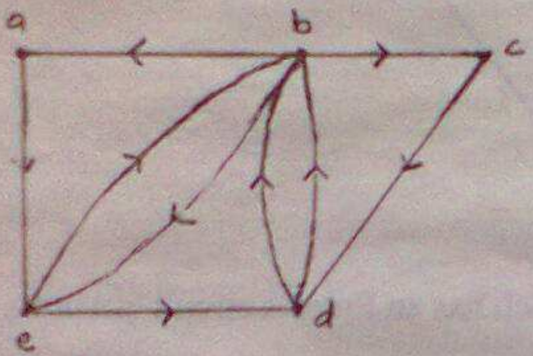
(The number of square roots is n). 7

b) Solve the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = 7$ with $a_0 = 1$ and $a_1 = 2$. 7

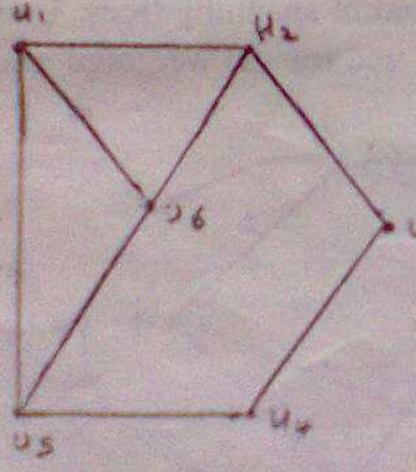
c) A Restaurant serves three kinds of snacks A, B, C costing 1\$, 2\$ and 3\$ respectively. Find the recurrence relation for the number of ways of spending n dollars if a person eats one snack each day until the n dollars are exhausted. Also state the initial conditions. 6

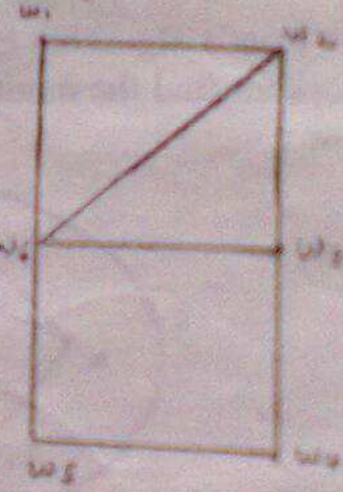
MODULE – IV

7. a) i) Define incidence matrix of a directed graph. Represent the following graph by an incidence matrix : 3

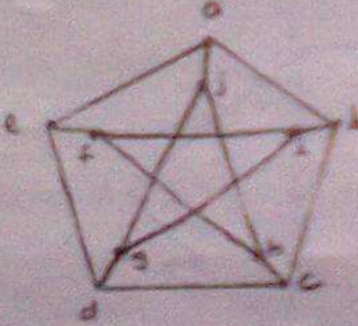


ii) Define graph isomorphism.
Cheek whether the following graphs are isomorphic or not. 4

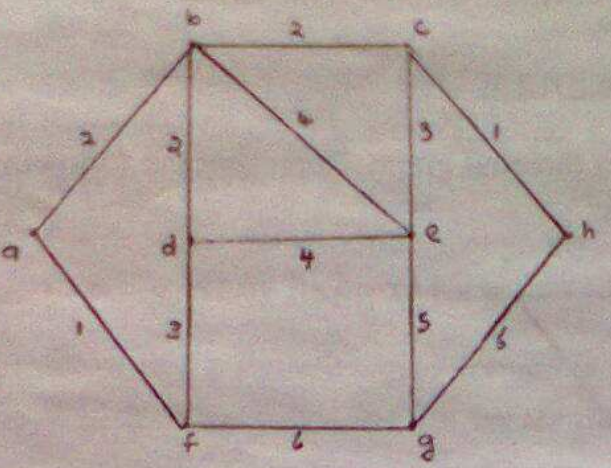




b) Using Kuratowski's theorem, show that the following graph is non-planar : 5



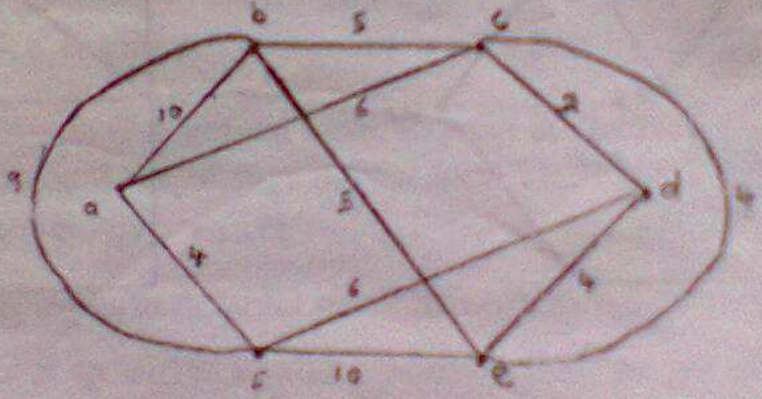
c) Apply Dijkstra's algorithm to find the shortest path between a and h in the following weighted graph : 6



d) Give an example of a graph which has an Euler's circuit but not a Hamiltonian circuit. 2

8. a) Show that a connected graph with n vertices and edges e = n – 1 is a tree. 6

b) State Prim's algorithm for finding minimum spanning trees. Using Prim's Algorithm find the minimum spanning tree for the weighted graph shown below : 6



c) Using Ford-Fulkerson's algorithm, find the maximum flow in the transport network. 8