26-5-16

COMP 4 – 1 (RC)

S.E. (Computer Engineering) (Semester – IV) (RC)
Examination, May/June 2016
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours — Total Marks : 100

Instructions : *1) Answer **any five** questions with atleast **one** from **each** Module.*
*2) Assume suitable data **if necessary**.*

MODULE – I

1. a) If X and Y are non empty sets then P.T. $P(X) \cup P(Y) \subseteq P(X \cup Y)$. Give an example to show that $P(X \cup Y)$ need not be a subset of $P(X) \cup P(Y)$; P(A) is the power set of A. **8**

   b) If R be the relation in a set of integers Z defined by $R = (x, y) : x \in Z, y \in Z$, $(x - y)$ is divisible by 6. Then prove that R is an equivalence relation. **6**

   c) Let $f : A \to B$ and $g : B \to C$ be two surjective functions. Show that $g_{\circ} f : A \to C$ is also surjective. **6**

2. a) How many positive integers not exceeding 2000 are divisible by 7 or 13 ? **6**

   b) State Pigeonhole principle. Suppose 14 students having random seat numbers are answering an examination. Prove that there are atleast two among them whose seat numbers differ by a multiple of 13. **6**

   c) If $p \equiv q(\text{mod } n)$ and $r \equiv s(\text{mod } n)$, prove that **8**

      i) $pr \equiv qs(\text{mod } n)$

      ii) $p^k \equiv q^k(\text{mod } n)$

      where $p, q, r, s \in Z$ and $n, k \in N$.

MODULE – II

3. a) Q is the set of rational nos. and $*$ is a binary operation defined by $a * b = a + b - ab$, $\forall a, b$ in Q. Show that $(Q, *)$ is a group. **6**

   b) Let Q – 1 be the set of all rational nos. except 1. Define an operation $*$ on Q – 1 as $a * b = a + b - ab$ s.t. $(Q - 1, *)$ is an abelian group. **6**

   c) State and prove Lagrange's theorem for groups. **8**

P.T.O.

4. a) P.T. a ring R has no zero divisors if and only if it satisfies the cancellation laws for multiplication in R. 6

b) Consider the map $T : R^3 \rightarrow R^3$ given by $T(x,y,z) = (3x, x - y, 2x + y + z)$. S.T. T is a linear transformation. Find the dimension of the kernel of T and the range of T. 6

c) Define a subspace of a vector space. Use it to prove that the intersection of two subspaces of a vector space V is again a subspace of V. Give an example to show that the union of two subspaces of a vector space V need not be a subspace of V. 8

MODULE – III

5. a) Let B be a Boolean algebra. P.T. (a) the inverse of an element unique ; (b) if $l + n = m + n$ and $l.n = m.n$ then $l = m \ \forall l, m \in B$. 8

b) Obtain the principle disjunctive normal form of the Boolean Algebra $f(x_1, x_2, x_3) = (\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)$. 6

c) Without using truth tables, P.T. : 6

i) $q \vee (P \wedge \neg q) \vee (\neg p \wedge \neg q) \equiv T$

ii) $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$.

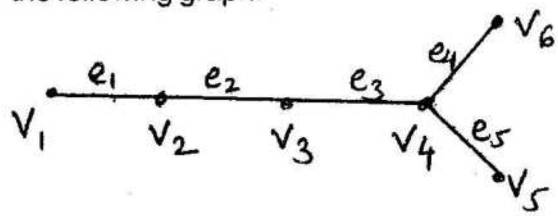
6. a) State the principle of Mathematical Induction and use it to prove that $1^3 + 2^3 + 3^3 + \ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$. 8

b) Solve recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 5;\ n \geq 2;\ a_0 = 0,\ a_1 = 1$. 6

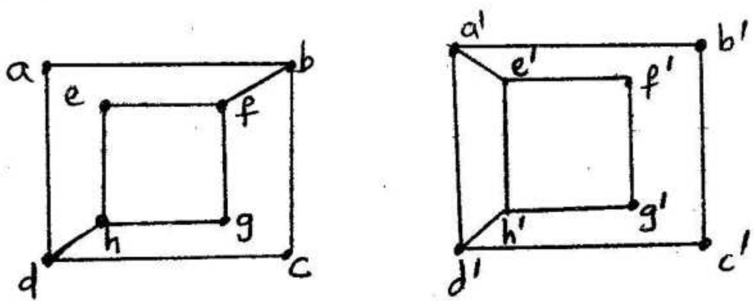
c) A person invests Rs. 25,000 @ 9% interest compounded annually. How much will be the total amount at the end of 17 years ? 6

MODULE – IV

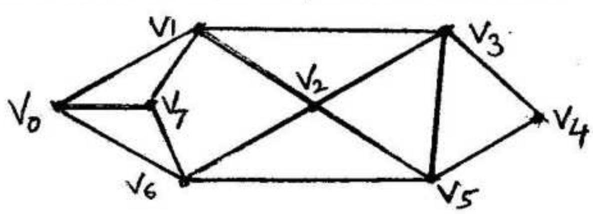
7. a) Define incidence matrix of an undirected graph. Obtain incidence matrix for the following graph. **6**



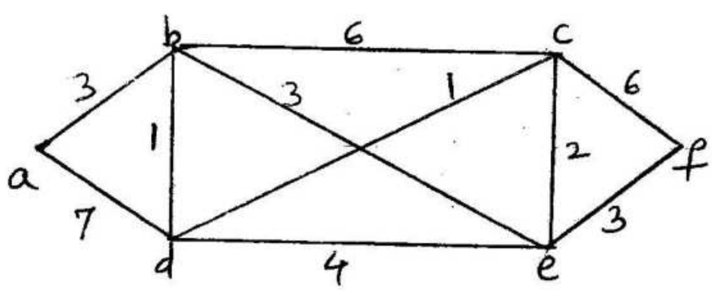
b) Define graph isomorphism. Check whether following graphs are isomorphic or not. **8**



c) Define Hamiltonian graph. Determine whether following graph has Hamiltonian circuit or not ? If it does, find such a circuit. **6**



8. a) Use Dijkstra's algorithm to find the shortest path between the vertices a and f in the given weighted graph. **8**



b) i) Prove that a tree T with n vertices has $n - 1$ edges. **6**

ii) Draw a tree with four internal vertices and six terminal vertices.

c) Use Kruskal's algorithm to find minimum spanning tree for the given weighted graph. **6**