



SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) Examination, Nov./Dec. 2014

(Rev. Course 2007-08)

APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

Instructions : 1) Attempt 5 questions, **atleast one** from **each Module**.2) **Assume** suitable data, if **necessary**.3) Figures to the **right** indicate **full marks**.

MODULE – I

1. a) Assuming the validity of differentiation under the integral sign prove that

$$\int_0^{\infty} \frac{\log(1 + \alpha x^2)}{x^2} dx = \pi\sqrt{\alpha}.$$

7

- b) Find the length of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 1$ to $y = 16$.

7

- c) Find the area of the surface generated by the revolution of

$$y = \sqrt{x+2}; 0 \leq x \leq 4 \text{ about the } x\text{-axis.}$$

6

2. a) If $\vec{r}(t) = \vec{a}e^{2t} + \vec{b}e^{3t}$; where \vec{a} and \vec{b} are constant vectors, then show that

$$\frac{d^2\vec{r}}{dt^2} - \frac{5d\vec{r}}{dt} + 6\vec{r} = 0.$$

6

- b) If $\vec{r}(t) = 2 \cos t \vec{i} + 3 \sin t \vec{j} + 4t \vec{k}$ is the position vector of a particle in space at time 't'. Find the particle velocity vector and acceleration vector at $t = \frac{\pi}{2}$.

5

- c) Define curvature of a curve. Show that the curvature of a circle is constant.

5

- d) Evaluate $\int_0^{\pi} \cos t \vec{i} + \sin^2 t \vec{j} + \vec{k} dt$.

4

P.T.O.



MODULE – II

3. a) Evaluate $\iint_R (x^2 - y^2) dx dy$ where 'R' is the triangle with vertices (0, 1), (1, 1) and (1, 2). 6

b) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$. 7

c) Change to polar co-ordinates and evaluate $\int_0^a \int_0^a \frac{x^2}{\sqrt{x^2 + y^2}} dx dy$. 7

4. a) Use double integration to find the area between $r = 1$ and $r = 3$ (polar co-ordinates). 6

- b) Convert the following to spherical polar co-ordinates and evaluate

$\iiint_V \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + z^2}} dx dy dz$ where 'V' is the region $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$. 7

- c) Find the volume of the region bounded by $x = 0$, $y = 0$, $z = 0$ and $3x + 2y + z = 6$. 7

MODULE – III

5. a) In what direction is the directional derivative of $f(x, y, z) = x^2y + 3xyz$ at the point $(-1, 2, 1)$ maximum and what is its magnitude? 5

- b) Define divergence of a vector field. Show that $\text{div}(\text{grad } f) = \nabla^2 f$. 5

- c) Find the work done in moving a particle in a force field

$\vec{F} = 3y^2 \mathbf{i} + (2yz + x) \mathbf{j} + xz \mathbf{k}$ along the curve $x = 3t + 1$, $y = 2t^2$, $z = 2t$ from $t = 0$ to $t = 1$. 5

- d) Use Gauss divergence theorem to prove $\iiint_V \vec{R} \cdot \hat{n} ds = 3V$; where 's' is any

closed surface, \hat{n} is the unit outward normal to s, 'V' is the volume enclosed by 's' and $\vec{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. 5



6. a) Verify Green's theorem in the plane for $\oint_c [(x^2 + 4y^2) dx + (y + 3x) dy]$

where c is the boundary of the region bounded by $y = 0$, $x = 0$ and $2x + y = 2$.

8

b) Use Stoke's theorem to evaluate $\iint_s (\nabla \times \vec{F}) \cdot \hat{n} \, ds$ where

$\vec{F} = (x^2 + 3y) \mathbf{i} + 2y \mathbf{j} + (3x + z) \mathbf{k}$; s is the surface of the tetrahedron having vertices $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 0)$, $(0, 0, 1)$ excluding the surface in the XY plane.

12

MODULE – IV

7. Solve the following :

a) $(\sec x \tan x \tan y - e^x) dx + (\sec x \sec^2 y) dy = 0$.

5

b) $\frac{dy}{dx} = \frac{-2x + y - 5}{x + 3y + 1}$.

5

c) $(x + y + 1)^2 \frac{dy}{dx} = 1$.

5

d) $\frac{dy}{dx} - y \tan x = y^4 \cdot \sec x$.

5

8. Solve the following :

a) $(D^2 + 2D + 1)y = xe^x + 2$.

5

b) $(D^4 + 2D^2 + 1)^2 y = \sin(2x)$.

5

c) $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$.

5

d) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x^2 \log x$.

5