comp 27/11/14 (M) Regular

COMP 3 – 1 (RC)

S.E. (Comp.) (Semester – III) Examination, November/December 2014
(Revised 2007-08)
APPLIED MATHEMATICS – III

Duration: 3 Hours Total Marks: 100

Instructions : 1) *Attempt* ***any five*** *questions. Atleast* ***one*** *from* ***each*** *Module.*
2) *Assume suitable data,* ***if necessary****.*
3) ***Use*** *of statistical table* ***permitted.***

MODULE – I

1. a) Define orthogonal matrix. Prove that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. **5**

b) Find the rank of the matrix by reducing it to its normal form.

$$\begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & -4 \\ 5 & 8 & -4 & 2 \end{bmatrix}$$ **7**

c) Determine for what value of λ and μ the following system of equations $x + y + z = 6$; $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has a

i) no solution

ii) unique solution

iii) more than one solution. **8**

2. a) Prove that the eigen vectors corresponding to distinct eigen values of a symmetric matrix are orthogonal. **4**

b) Find SinA given $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$. **6**

c) State and prove Cayley-Hamilton theorem for a square matrix. **6**

d) Show that if λ is the eigen value of an orthogonal matrix then $|\lambda| = 1$. **4**

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MODULE – II

3. a) Prove that $P(A \cup B) = 1 - P(B^c/A^c)P(A^c)$. 3

b) Define independent events. Show that if A and B are independent events than A^c and B are also independent. 6

c) The probability that a students will pass an engineering examination is 0.7. In order to appear for the examination the students ought to have 75% attendance. The probability that a student has 75% attendance is 0.85. What is the probability that a student will appear and pass the examination ? 3

d) Define moment generating function of a random variable. Find the moment generating function of the random variable X with distribution $P(X = i) = 2^{-i}$ $i = 1, 2, 3 \ldots$ 8

4. a) A telecom service company's subscribers monthly usage of telecom voice service follows normal distribution with mean $\mu = 240$ minutes and standard deviation $\sigma = 42$ minutes. Subscribers are charged Rs. 1.20 per minute of usage. Find the probability that a subscriber's bill will be more than Rs. 250/- per month ? If the company desires to reward 10% of its top subscribers with gift coupons, what should be the minimum billing to qualify for the reward ? 8

b) A firm sells oil cans that are supposed to contain 5 kgs of oil per can. A sample of 50 cans showed a mean of 4990 gms with a standard deviation of 20 gms. Find whether the mean weight differs significantly from 5 kg at 5% level of significance. 6

c) The two regression equations of the variable x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.5x$. Find the :

i) mean of x and y

ii) the correlation co-efficient of x and y. 6

MODULE – III

5. a) If $L(f(t)) = F(s)$, where L denotes the Laplace transform, prove the following

i) $L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s)\,ds$ ii) $L(t\,f(t)) = -\frac{d}{ds}(F(s))$. 6

b) Find the Laplace transform of

i) t Sint Cos3t ii) $\frac{e^{3t} - e^{2t}}{t}$. 6

c) Solve the ordinary differential equation, using Laplace transforms $2y''(t) + 5y'(t) - 3y(t) = e^{-2t}$, $y(0) = 1$, $y'(0) = 1$. 8

6. a) State and prove convolution theorem for Laplace transform. Use convolution theorem to find the inverse Laplace transform of 8

$$F(s) = \frac{3s}{(s^2+1)(s-3)}$$

b) Using Laplace transform evaluate $\int_0^{\infty} \frac{e^{-3t}\sin 2t}{t}\,dt$. 6

c) Find the Fourier transform of f(x) = $f(x) = \begin{cases} x^2 & |x| < 1 \\ 0 & |x| > 1. \end{cases}$ 6

MODULE – IV

7. a) If $F_c(f(x)) = F_c(s)$ and $F_s(f(x)) = F_s(s)$ are the Fourier Cosine and sine transform respectively. Show that 6

i) $F_s(f(x)\cos(ax)) = \frac{1}{2}\{F_s(s+a) + F_s(s-a)\}$

ii) $F_c(xf(x)) = \frac{d}{ds}(F_s(s))$.

b) Solve for f(x) the integral equation $\int_0^{\infty} f(x)\cos(sx)\,dx = e^{-2s}$. 6

c) Define convolution of two functions. Show that convolution is commutative. Find the convolution of the functions $f(x) = x,\ x > 0$ and $g(x) = e^{-2x},\ x > 0$. 8

8. a) Find the Z-transform of the following : 6

i) $2^n\cos(n\pi)$ ii) $(5n^2+1)/n$.

b) If $Z(f(n)) = F(z)$, is the Z-transform of $\{f(n)\}$ then show that :

i) $\text{Lim}_{n\to\infty} f(n) = \text{Lim}_{z\to 1}(z-1)F(z)$

ii) $Z(a^n f(n)) = F(z/a)$. 6

c) Solve the following difference equation using Z-transform

$y_{n+2} + 2y_{n+1} - 8y_n = 3^n,\ y_0 = 0,\ y_1 = 1.$ 8