01-06-15 CM

COMP 4 – 1 (RC)

S.E. (Computer Engineering) (Revised 2007-08) (Semester – IV) Examination, May/June 2015
DISCRETE MATHEMATICAL STRUCTURES

Duration : 3 Hours Total Marks : 100

Instructions : *1) Answer* ***any five*** *questions with atleast* ***one*** *from* ***each*** *Module.*
2) Assume suitable data if ***necessary****.*

MODULE – I

1. a) Let A, B and C be any three non empty sets. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. 3

   b) Let Z be the set of integers. Define a relation R on Z as a R b iff '5 divides a – b'. Show that R is an equivalence relation on Z. Express Z as a disjoint union of distinct equivalence classes. 6

   c) Let $f : A \to B$ and $g : B \to C$ be two bijective functions. Show that $g \circ f : A \to C$ is also bijective. 5

   d) Draw the Hasse diagram representing the partial ordering R on the set S = {1, 2, 3, 4, 6, 8, 12} given by a R b if 'a divides b'. Which are the maximal elements, upper bounds, lower bounds, supremum and infimum of the subset A = {2, 6, 8, 12} ? 6

2. a) State extended Pigeonhole principle. If 5 points are randomly chosen in a square of side 2 units, then show that atleast two of them are no more than $\sqrt{2}$ units apart. 7

   b) Find the number of positive integers not exceeding 400 which are 7
      i) divisible by 5 or 3 or 7.
      ii) divisible by 3 not by 5 nor by 7.

   c) Without actually carrying out multiplication, find the remainder when the integer $[9 \times 85 \times 89 \times (37)^2 \times (67)^2 \times 539 \times (1269)^3$ is divided by 16. 6

MODULE – II

3. a) Every monoid is a group. Prove or Disprove. 2

   b) Let Q-{1} be the set of all rational numbers except 1. Define an operation '*' on Q-{1} as $a * b = a + b - ab$. Show that (Q-{1},*) is an abelian group. 6

   c) If every element of a group other than identity element has order 2, then prove that the group is an abelian group. 5

   d) Prove that if H and K are subgroups of a group G, then HUK is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$. 7

P.T.O.

4. a) Let R be an algebraic system satisfying all the conditions for a ring with unity element with the possible exception of $a + b = b + a$. Prove that $a + b = b + a$ must hold in R. **7**

b) Which of the following are vector subspaces of R^3 : **6**

i) $W_1 = \{(x, y, z) : x = 2\}$

ii) $W_2 = \{(x, y, z) : x + y + z = 0\}$

iii) $W_4 = \{(x, y, z) : z = x + y\}$

c) Consider the map $T : R^3 \to R^3$ given by $T(x, y, z) = (3x, x-y, 2x + y + z)$. Show that T is a linear transformation. Find the dimension of Kernel of T and Range of T. **7**

MODULE – III

5. a) Let B be a Boolean Algebra. Prove that

i) $(a.b)' = a' + b'$ $\forall a, b \in B$. **4**

ii) $a + a = a$.

b) Without using truth tables prove that **5**

i) $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$.

ii) $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$.

c) Define functionally complete set of connectives. Show that the NOR connective $\{\downarrow\}$ is functionally complete.

d) Obtain the principal disjunctive normal form of the Boolean function. **6**

$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + x_2 + x_3)(x_1 + x_2 + x_3)$.

6. a) State the principle of mathematical induction. Use mathematical induction to prove that for all positive integers n, $2^{n+2} + 3^{2n+1}$ is divisible by 7. **7**

b) Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = 2^n$ with $a_0 = 1$ and $a_1 = 0$. **8**

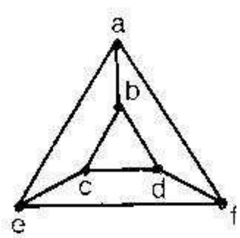
c) Find the recurrence relation for the number of ways of climbing n steps if a person can climb one or two or three steps at a time. Also give the initial conditions. **5**

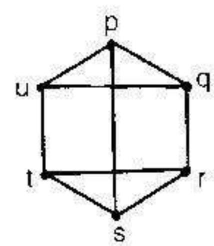
MODULE – IV

7. a) Define the following and give an example of each. **6**
   i) Complete graph
   ii) Regular graph
   iii) Complete bipartite graph

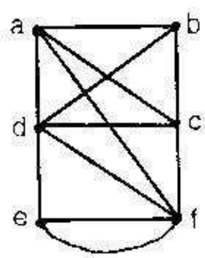
b) i) Show that an undirected graph has an even number of vertices of odd degree. **3**

ii) Define graph isomorphism. Check whether the following graphs are isomorphic or not. **4**

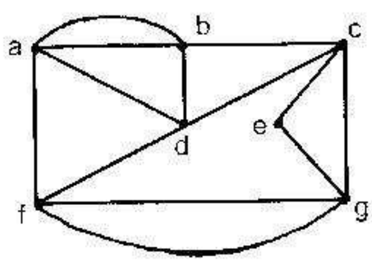




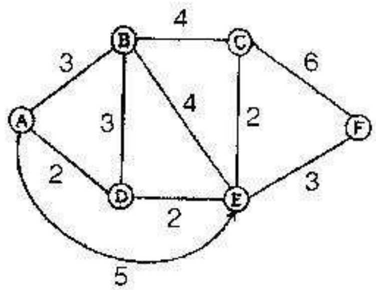
iii) Check whether the following graph is planar or not. **3**



c) Using Fleury's algorithm, find an Euler's circuit for the graph shown below. **4**

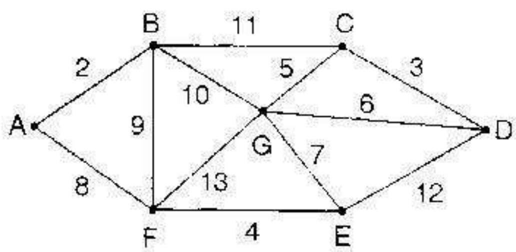


8. a) Apply Dijkstra's algorithm to find the shortest path between A and F in the following weighted graph. 6



b) Show that a connected graph with n vertices and edges e = n – 1 is a tree. 5

c) State Kruskal's algorithm for finding a minimum spanning trees. Usig Kruskal's algorithm, find the minimum spanning tree for the weighted graph shown below. 6



d) Show that a full (regular) m-ray tree with "i internal vertices contain n = mi + 1 vertices. 3