Online Supplementary Materials

This section provides in depth coverage of the illustrative example described in the text. In the following section, I present details about the meta-analytic dataset, then present SPSS syntax and annotated output of the results of three analyses: A fixed-effects model of the linear effect of Lag, a fixed-effects model of the nonlinear effect of Lag, and then a mixed-effects model of the nonlinear effect of Lag. The goal of this supplement is to provide readers a usable model for performing the LAMMA described in the main manuscript.

The Meta-Analytic Database

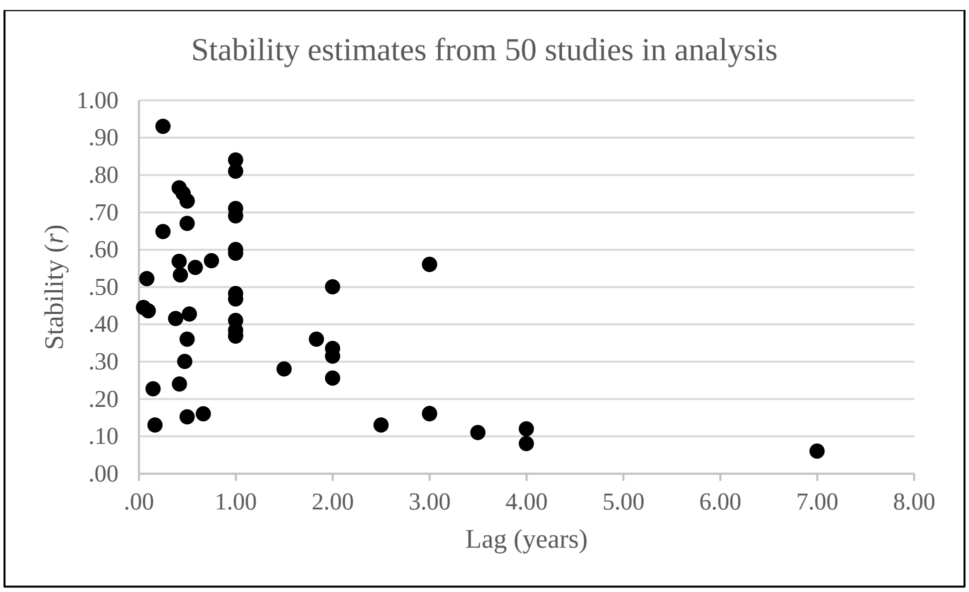
The meta-analytic database used in this illustrative example comes from a larger series of meta-analyses on peer victimization (Card, 2003). The larger meta-analysis searched within computerized and paper-based databases using a variety of keywords (e.g., "victim*", "harass*", "bully*") and backward searches, and included studies presenting data on peer victimization among samples of children or adolescents. A total of 222 empirical reports containing 205 distinct studies, with a total sample size of about 375,000 children and adolescents, were included in the larger meta-analysis.

From this larger pool, I re-analyzed a subset of 51 studies reporting the inter-individual stability of victimization. For these studies, Card (2003) had coded the stability as *r*, the sample size (for weighting), and the time lag between measurement occasions. One study was removed because it was a retrospective design with an unclear duration of time lag. Therefore, the data analyzed in this illustrative example are as follows, arranged from shortest to longest time lag (represented as number of years):

Supplemental Table: *Studies used as data for illustrative example*.

Lag	N	W	r	Zr	Lag	N	W	r	Zr
.05	18	15	.44	.48	1.00	774	771	.71	.89
.08	26	23	.52	.58	1.00	294	291	.48	.53
.10	18	15	.44	.47	1.00	40	37	.38	.40
.15	18	15	.23	.23	1.00	85	82	.59	.68
.17	68	65	.13	.13	1.00	428	425	.81	1.13
.25	165	162	.93	1.66	1.00	392	389	.47	.51
.25	40	37	.65	.77	1.00	197	194	.37	.39
.38	18	15	.42	.44	1.00	106	103	.37	.39
.42	40	37	.57	.65	1.00	1054	1051	.41	.44
.42	45	42	.77	1.01	1.00	254	251	.60	.69
.42	200	197	.24	.24	1.50	388	385	.28	.29
.43	18	15	.53	.59	1.83	82	79	.36	.38
.46	189	186	.75	.97	2.00	1953	1950	.31	.33
.47	18	15	.30	.31	2.00	392	389	.33	.35
.50	73	70	.73	.93	2.00	197	194	.26	.26
.50	388	385	.36	.38	2.00	96	93	.50	.55
.50	68	65	.67	.81	2.50	388	385	.13	.13
.50	186	183	.15	.15	3.00	214	211	.56	.63
.50	47	44	.73	.93	3.00	201	198	.56	.63
.52	18	15	.43	.46	3.00	388	385	.16	.16
.58	40	37	.55	.62	3.00	197	194	.16	.16
.67	68	65	.16	.16	3.50	388	385	.11	.11
.75	40	37	.57	.65	4.00	197	194	.08	.08
1.00	229	226	.84	1.22	4.00	1316	1313	.12	.12
1.00	533	530	.69	.85	7.00	71	68	.06	.06

Readers interested in the specific studies and other coded study characteristics can refer to the original report (Card, 2003). A scatter diagram showing these stability estimates in relation to lag is shown in the following figure:



As expected, the stability estimates tend to go down with longer time lag. Beyond this observation, it is challenging to draw conclusions from mere visual inspection of the scatter diagram. One challenge it that there is an abundance of studies with shorter lags, which might be expected in many fields given the challenges of longer-term studies. If any studies had both extreme lags and stability estimates far off of predicted values, it might be worth performing sensitivity analyses to understand if results change markedly with the presence or removal of single studies.

Linear Effect of Lag on Stability

The first analysis performed is a simple regression of transformed stability estimates (Z_r) onto lag (time span between measurement occasion, in years), weighted by W (= N – 3). The following SPSS syntax was used (note: this analysis can be performed with any software that can perform weighted regression, including SAS, Stata, and several packages within R):

```
REGRESSION
  /REGWGT=W
  /STATISTICS COEFF OUTS R ANOVA
  /NOORIGIN
  /DEPENDENT Zr
  /METHOD=ENTER Lag.
```

After applying this procedure to the 50 studies of the example meta-analytic database, the following output was produce (partial display of output):

ANOVA[a,b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	498.446	1	498.446	28.210	.000[c]
	Residual	848.127	48	17.669		
	Total	1346.573	49			

a. Dependent Variable: Zr

b. Weighted Least Squares Regression - Weighted by W

c. Predictors: (Constant), Lag

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.764	.068		11.163	.000
	Lag	-.166	.031	-.608	-5.311	.000

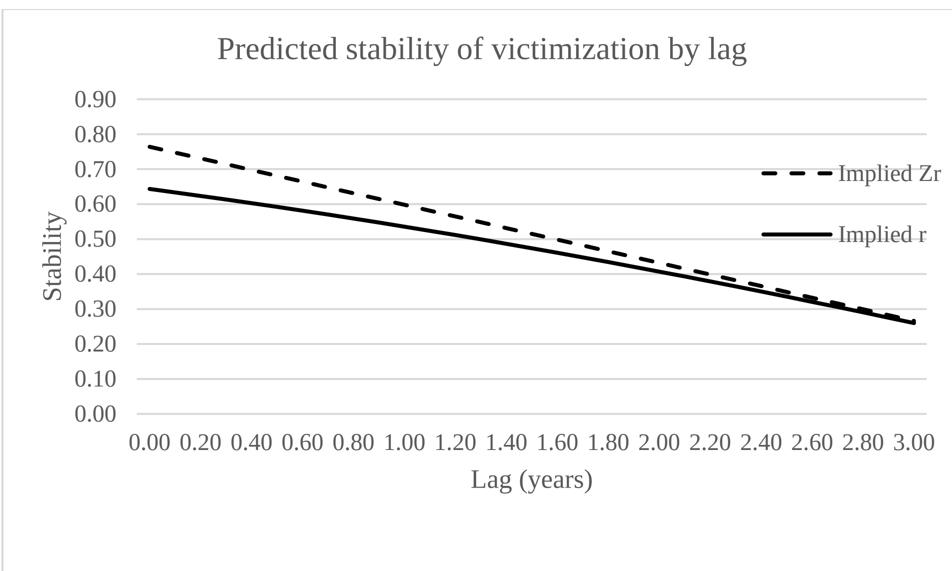
a. Dependent Variable: Zr

b. Weighted Least Squares Regression - Weighted by W

From the ANOVA table, we see that the heterogeneity across these 50 studies was significant, $Q_{(49)} = 1346.57$, $p < .001$ (note: the significance is evaluated as χ^2 distribution separate from this output; the F ratio reported in this output is inaccurate, see Lipsey & Wilson, 2001). The regression model with Lag predicts a significant amount of this heterogeneity, $Q_{(1)} = 498.45$, $p < .001$ (note that this value is evaluated as with df = number of predictors, or 1 in this case). To gain a sense of the magnitude, we could compute a percent of heterogeneity accounted for by Lag, $R^2 = 498.45 / 1346.57 = .37$, or 37% of the heterogeneity.

The coefficients table from this output also provides meaningful information. The regression coefficient for Lag is -.17, meaning that the values of the effect size (Z_R) decrease .17 per one year longer lag. The standard errors as reported in this table are inaccurate, and must be adjusted; as described by Lipsey and Wilson (2001), the adjusted standard error is $SE_{Adjusted} = \frac{SE_{Output}}{\sqrt{MS_{Residual}}}$. In this example, $SE_{Adjusted} = \frac{.031}{\sqrt{17.67}} = .0073$. The significance of the effect of Lag can then be evaluated using a Wald test: $Z = \frac{B}{SE_{Adjusted}} = \frac{-.17}{.0073} = -23.15$, $p < .001$ (note that the significance of the single predictor is identical, within rounding imprecision, to the significance of the overall regression model).

The combination of intercept and regression coefficient from this output allows us to compute model-implied effect sizes across a range of Lags, using the prediction equation $\widehat{Z_r} = .764 - .166 * Lag$. Substituting the values of 0.5, 1, and 2 for lag yields the model-implied 6-month stability $Z_r = .68$ with backtransformed $r = .59$, model-implied values for one-year lag $Z_r = .60$ and $r = .54$, and two-year model-implied values of $Z_r = .43$ and $r = .41$. A more general representation of the effect of lag might be displayed in a figure such as the following:



This type of chart would potentially be useful for readers planning future studies of various time lags to understand the likely stability of peer victimization in their study.

Nonlinear Effect of Lag on Stability

As described in the text, it might be expected that the effect of Lag on stability is nonlinear, and that a quadratic function might be a reasonable model for the relation of Lag with stability. In order to reduce collinearity between the linear and quadratic aspects of Lag, it is desirable to center Lag. However, it is useful to keep in mind that the linear and quadratic Lag predictors will be used within a weighted regression, so to center Lag we should first estimate its weighted mean. This can be performed within a weighted empty regression model, in which we regress Lag onto a constant 1, weighted by *W*. The SPSS syntax in the illustrative example is:

```
COMPUTE Constant=1.
EXECUTE.
REGRESSION
  /REGWGT=W
  /STATISTICS COEFF OUTS R ANOVA
  /ORIGIN
  /DEPENDENT Lag
  /METHOD=ENTER Constant.
```

This syntax creates a variable called "constant" which has the value of 1 for all studies, and then regresses each studies' Lag onto this constant value (note that the syntax statement "/ORIGIN" removes the SPSS default of automatically estimating the intercept). This non-informative estimate yields a regression coefficient equal to the weighted average lag, in this illustrative example equal to 1.83 years:

Coefficients[a,b,c]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	1.827	.171	.836	10.655	.000

a. Dependent Variable: Lag

b. Linear Regression through the Origin

c. Weighted Least Squares Regression - Weighted by W

I then create a new variable for Lag by subtracting this average from each study's actual lag, calling this variable "Centered Lag". I then square values of this centered lag to create a "Quadratic Lag" variable for each study:

```
COMPUTE Centered_Lag = Lag - 1.827.
COMPUTE Quadratic_Lag = Centered_Lag *Centered_Lag.
EXECUTE.
```

The correlation between the Quadratic and Centered Lag (weighted $r = .51$) is far less than that between the original uncentered Lag and a squared term computed from this variable (weighted $r = .93$). Centering has eliminated what is termed "nonessential collinearity" (Aiken & West, 1991) but leaves behind "essential collinearity" due to Lag being positively skewed in this example, $skew = 2.09$, $SE_{skew} = .337$, $Z = 6.20$, $p < .001$. If this remaining essential collinearity was considered problematic, one could alternatively orthoganalize the quadratic and Lag terms (see Little et al., 2006).

After creating variables for the linear and quadratic functions of Lag, we can regress stability effect sizes onto these two predictors using the following syntax, and yielding the following output:

```
REGRESSION
  /REGWGT=W
  /STATISTICS COEFF OUTS R ANOVA
  /NOORIGIN
  /DEPENDENT Zr
  /METHOD=ENTER Centered_Lag Quadratic_Lag.
```

ANOVA[a,b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	522.071	2	261.036	14.880	.000[c]
	Residual	824.502	47	17.543		
	Total	1346.573	49			

a. Dependent Variable: Zr

b. Weighted Least Squares Regression - Weighted by W

c. Predictors: (Constant), Quadratic_Lag, Centered_Lag

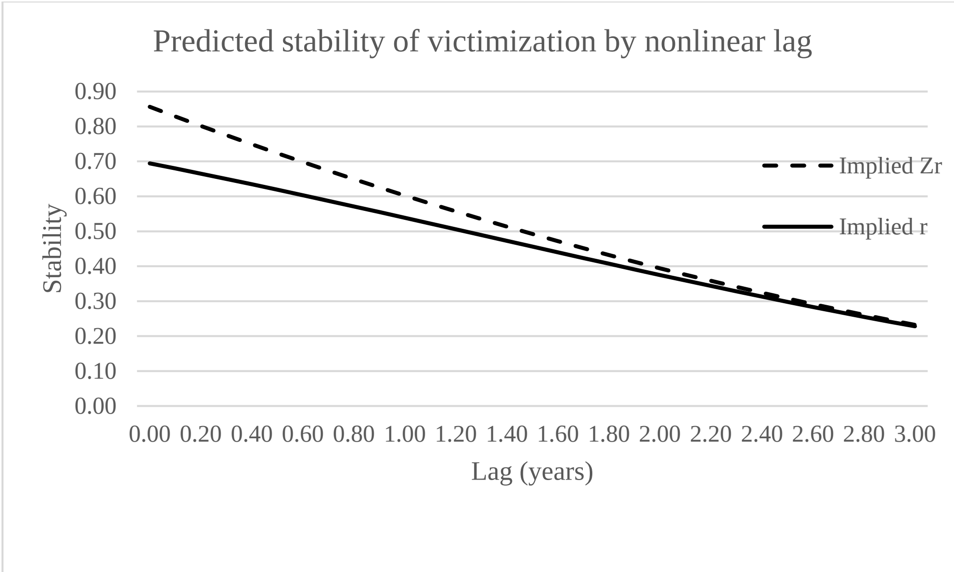
Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.427	.047		9.103	.000
	Centered_Lag	-.193	.039	-.706	-4.977	.000
	Quadratic_Lag	.023	.020	.165	1.160	.252

a. Dependent Variable: Zr

b. Weighted Least Squares Regression - Weighted by W

The regression equation now uses two predictors, linear and quadratic lag, of stability. The overall regression model is significant ($Q_{(2)} = 522.07$, $p < .001$) and now predicts $R^2 = .39$. Examination of the individual regression coefficients shows that effect sizes decrease linearly with Lag (-.19, $SE_{Adjusted} = .0093$, $Z = -20.41$, $p < .001$), as found earlier. However, there is also a positive quadratic effect of Lag (.023, $SE_{Adjusted} = .0048$, $Z = 4.82$, $p < .001$). Plotting the model-implied stabilities at various lags using $\widehat{Z_r} = .427 - .193 * CenteredLag\ + .023 * QuadraticLag$ yields the values shown in the following figure:



Note that although centered lag (and squared centered lag) were included in the regression model, the model-implied stabilities are plotted relative to Lag in its original, more understandable, metric.

Mixed-Effects Model of Lag Predicting Stability

In both the linear and nonlinear models of Lag predicting stability in peer victimization, there remained significant residual heterogeneity ($Q_{(48)} = 848.13$, $p < .001$ and $Q_{(47)} = 824.50$, $p < .001$ for linear and quadratic models, respectively). This residual heterogeneity indicates that the study stability estimates deviated around the regression lines more than expectable by sampling fluctuation alone. Therefore, a more appropriate model for evaluating moderation by Lag is one termed a mixed-effects model, in which the prediction of stability (Zr) by Lag (and potentially Lag squared) are estimated as fixed-effects (i.e., one regression coefficient) whereas the intercept is allowed to randomly vary. This model is displayed as Equation 6 in the main text of this manuscript.

The underlying matrix algebra of the mixed-effects model is described in the main text. Here, I note that there exist two computational approaches to estimating a mixed-effects

LAMMA. This first approach is to perform the matrix algebra within a standard software package like SPSS (or SAS, Stata, or R). A set of useful macros for doing this can be obtained via David Wilson's website: http://mason.gmu.edu/~dwilsonb/ma.html. A second approach is to use a Structural Equation Modeling (SEM) representation (Cheung, 2008) using a software package that allows randomly-varying intercepts such as Mplus (Muthén & Muthén, 1998). For transparency of presentation, I proceed with the illustrated example showing Mplus syntax.

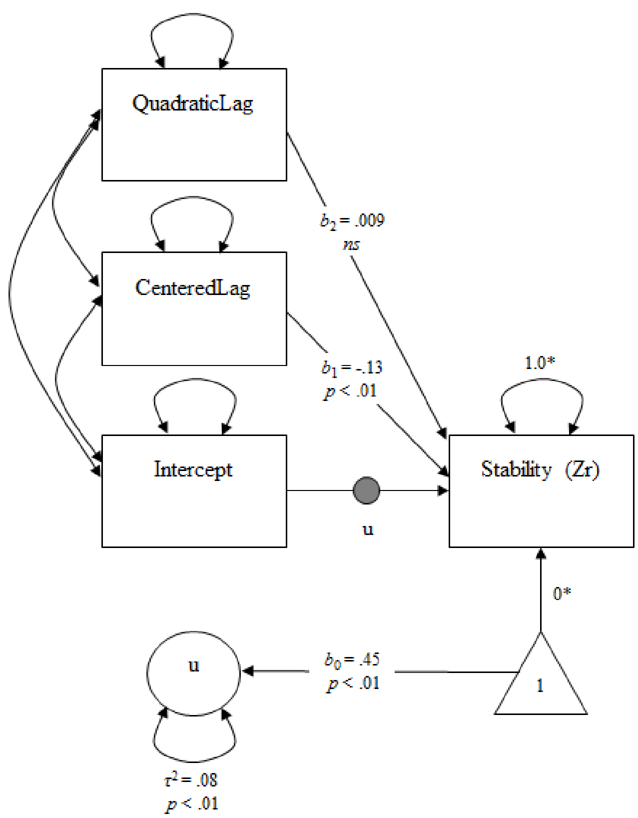
The SEM representation regresses the stability effect size (Z_r) onto predictors (Lag) and intercept (constant 1.0) just like the SPSS regression. However, instead of performing a weighted regression, Cheung (2008) showed that multiplying the dependent variable (Z_r) and all independent variables (Lag, Intercept) by $\sqrt{W}$ effectively performs the appropriate weighting. For example, the (fixed-effects) model of centered lag and quadratic lag predicting victimization stability could equivalently be estimated in Mplus using the following syntax:

```
TITLE: LAMMA fixed-effects quadratic model
DATA: File is Mplus.txt;  !Text file containing data
VARIABLE: NAMES Study W Zr Lag CLag QLag interc;
    USEVARIABLES ARE Zr interc CLag QLag;
DEFINE:
    Zr = SQRT(W) * Zr;
    interc = SQRT(W) * interc;
    CLag = SQRT(W) * CLag;
    QLag = SQRT(W) * QLag;
MODEL:
    [Zr@0.0];   !Fixes intercept at 0
    Zr@1.0;   !Fixes variance at 1
    Zr ON interc CLag QLag; !LAMMA quadratic model
```

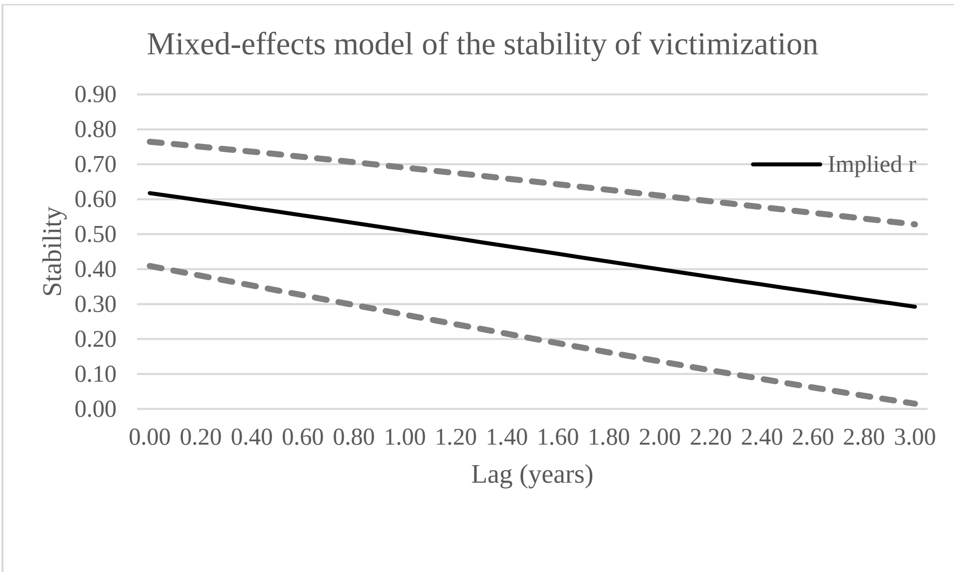
To extend this approach into a mixed-effects framework, one simply specifies a "random" model type and defines a latent variable (denoted U in the syntax below) that will capture both the average and variability in intercept (effect size when other predictors are zero).

The Mplus syntax for this mixed-effects analysis, and the results within a path-daigram figure, are shown next:

```
TITLE: LAMMA mixed-effects quadratic model
DATA: File is Mplus.txt;
VARIABLE: NAMES Study W Zr Lag CLag QLag interc;
    USEVARIABLES ARE Zr interc CLag QLag;
DEFINE:
    Zr = SQRT(W) * Zr;
    interc = SQRT(W) * interc;
    CLag = SQRT(W) * CLag;
    QLag = SQRT(W) * QLag;
ANALYSIS: TYPE=RANDOM;  !Specifies random slope analysis
MODEL:
    [Zr@0.0];   !Fixes intercept at 0
    Zr@1.0;   !Fixes variance at 1
    u | Zr ON interc;   !U as random effect
    Zr ON CLag QLag;
    [u*];  !Specifies estimation of random-effects mean
    u*;  !Specifies estimation of variance of random effect
```



In this model accounting for additional, residual heterogeneity, the linear effect of Lag remains significantly negative. However, Quadratic Lag is no longer statistically significant within this model (b = .009, p = .39). The following figure thus shows the model-implied stability of peer victimization across various lags:



Note that in contrast to previous figures of model-implied stability, which showed a single regression line for the stability r (as well as a second line for the transformed stability, Z_r), this model also shows dashed lines to represent the *distribution* of stability estimates at any given lag.