## 1.1 - A Nonparametric Test of Hypothesis and Confidence Interval for the mean

### 1.1.1 - Binomial Test

Tests of hypotheses for medians are typically used in the same situations that are apporopriate for tests of hypotheses for means. Except that, it is fixed,

$$H_0 : P = 0.5$$

$$H_a : P > 0.5$$

$$Z_B = \frac{B - 0.5n}{\sqrt{0.25n}}, \quad B = \text{\# of obs greater than or equal to } \theta_{0.5}$$

original median

### 1.1.2 - Confidence Interval (observations must be ordered)

$P(X_a < \theta_{0.5} < X_b) = 1 - \alpha$, at least "a" of the obs must fall less than $\theta_{0.5}$, and at most b-1 of the obs must fall less than or equal to $\theta_{0.5}$

$\sum_{k=a}^{b-1} \binom{n}{k} (0.5)^n$ = the probability that at least "a" and at most "b-1" of the obs fall less than $\theta_{0.5}$. We need to choose a and b summing to $1-\alpha$

### 1.2.1 Confidence Interval for the Population cdf

Let $\hat{F}(x)$ be an empirical cdf,

$$\sigma(\hat{F}(x)) = \sqrt{\frac{F(x)[1-F(x)]}{n}}$$

Confidence Interval

$$\hat{F}(x) \pm Z_{(1-\alpha/2)} \sqrt{\frac{\hat{F}(x)[1-\hat{F}(x)]}{n}}$$

## 1.2.2 Inferences for Percentiles

$\sum_{k=a}^{b-1} \binom{n}{k} p^k (1-p)^{n-k}$ , we must choose a & b satisfying $1-\alpha$

The equation results in the desired $100(1-\alpha)$ confidence interval.

the above process is not feasible with large $n$. In case of large $n$, we can assume normality and obtain $a$ / $b$ using,

$$\frac{a-np}{\sqrt{np(1-p)}} = -Z_{(1-\alpha/2)}, \quad \frac{b-1-np}{\sqrt{np(1-p)}} = Z_{(1-\alpha/2)}$$ , and from

the $a$/$b$ obtained, we need to round them to the nearest integers.

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## 1.3 A Comparison of Statistical Tests

### 1.3.1 Type I Errors (False Negative)

- The probability of Type I Error should be what we claim it to be ($\alpha = 0.05$)

* If two tests have the same probability of a Type I Error, then the one with the greater power is the preferred test.

- it is known that standard normal approximation is feasible for large or moderate sample sizes, but if observations come from a distribution that does not have a finite variance (Cauchy & Laplace), then standard normal approximation is inappropriate even for large sample sizes.

### 1.3.2 Power

- Generally, the binomial test will have higher power than the CLT test for heavier-tailed population distributions, but the opposite will be true for lighter-tailed distributions

**TABLE 1.3.1**
A Comparison of Power of the CLT and Binomial Tests $\mu_0 = 75$, $\mu = 75.8$, $\sigma = 2.5$, $\alpha = .05$

| *Population Distribution* | *Power of CLT Test* | *Power of Binomial Test* |
|---|---|---|
| Normal | .65 | .48 |
| Laplace | .65 | .76 |

| 1.3.3 | Derivations |
| --- | --- |

Derivation of Power for Samples from a Normal Population

power of CLT Test

$$= 1 - \Phi\left(Z_{(1-\alpha/2)} - \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right)$$, $\Phi$ = cdf of the standard normal distribution

Power of Binomial Test

$$= 1 - \Phi\left(Z_{(1-\alpha/2)}\sqrt{\frac{0.25}{p(1-p)}} - \frac{p - 0.5}{\sqrt{p(1-p)/n}}\right)$$, $p$ = the # of obs. less than or equal to $\bar{X}$.

Derivation of Power for Samples from a Laplace Population

$$\underline{P(X > x)} = 0.5 + 0.5\left(1 - e^{-\sqrt{2}|\mu - \bar{x}|/\sigma}\right)$$

and we apply this probability to the "Power of Binomial Test"