Midterm

Name:

1. Suppose X_1, X_2 are independent discrete random variables with probability mass function for X_i, $i = 1, 2$, given by

x	0	1	2
$p(x;\theta)$	$e^{-\theta}$	$\theta e^{-\theta}$	$1 - e^{-\theta} - \theta e^{-\theta}$

with $0 < \theta$. Show that $X_1 + X_2$ is *not* a sufficient statistic for θ. (10 points)

2. Suppose that X_1 and X_2 are iid random random variables with pdf

$$f(x;\lambda) = 3\lambda^3 x^2 e^{-\lambda^3 x^3}, \quad x > 0, \quad \lambda > 0.$$

(a) Find a sufficient statistic for λ. (10 points)

(b) If we observed $X_1 = 3$ and $X_2 = 1$, what is the MLE of λ? (10 points)

3. Suppose $X_1, \ldots, X_n$ is a random sample from pdf $f(x;\theta) = \theta 2^{\theta} x^{-\theta-1}$, $2 \leq x$, $0 < \theta$.

   (a) Find the MLE of θ. (10 points)

   (b) Compute $I(\theta)$. (10 points)

   (c) Find the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$, where $\hat{\theta}$ is the MLE of θ. (10 points)

4. Suppose $X_1, \ldots, X_n$ is a random sample from $Beta(1, \theta)$, $\theta > 0$. Define

$$\hat{\theta}_1 = -\frac{n}{\sum_{i=1}^{n} \log(1 - X_i)} \quad \text{and} \quad \hat{\theta}_2 = \frac{1}{1 - \bar{X}}.$$

Note that pdf of X_i is $f(x; \theta) = \theta(1 - x)^{\theta - 1}$, $0 < x < 1$.

(a) Find asymptotic distributions of $\sqrt{n}(\hat{\theta}_1 - \theta)$ and $\sqrt{n}(\hat{\theta}_2 - \theta)$. (10 points)

(b) Compute asymptotic relative efficiency (ARE) of $\hat{\theta}_1$ to $\hat{\theta}_2$. (10 points)

5. Suppose $X_1, \ldots, X_n$ is a random sample from the distribution having the pdf

$$f(x;\theta) = 2x/\theta^2, \quad 0 < x \le \theta, \qquad -\infty < \theta < \infty.$$

(a) Find the MLE of η, where η is the median of the distribution. (10 points)

(b) Show that the MLE $\hat{\eta}$ is a consistent estimator for η. (10 points)