

AR with adaptive Lasso

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Automatic variable selection using LASSO

Lasso type of estimation/variable selection can also be applied to time series. Recall Lasso is obtained (in the regression setting)

$$\min_{\beta_0, \beta} \left\{ \frac{1}{N} \sum_{i=1}^N (y_i - \beta_0 - x_i^T \beta)^2 \right\} \text{ subject to } \sum_{j=1}^p |\beta_j| \leq t.$$

or equivalently,

$$\min_{\beta \in \mathbb{R}^p} \left\{ \frac{1}{N} \|y - X\beta\|_2^2 + \lambda \|\beta\|_1 \right\}$$

Adaptive lasso is an improvement of lasso and conveniently implemented in parcor package. In the AR(p) setting, we can consider it as a regression model

$$X_t = \mu_s + \phi_1 X_{t-1} + \dots + \phi_p X_{t-p} + \epsilon_t$$

Then, applying lasso automatically select the order and zero coefficients all together.

```
rm(list=ls(all=TRUE))
library(parcor);

ar.adaplasso = function(y, p, nf){

  if(missing(nf)){ nf = 10 };
  if(missing(p)){ phat = ar(y, aic = TRUE, order.max=10)$order }

  # Check y is a vector
  y = as.vector(y);
  n = length(y);
  mu.s = mean(y);
  id = 1:n;
  X = NULL;
  for(j in 1:p){
    id1 = id-j;
    id2 = id1[id1 <= 0];
    id3 = id1[id1 > 0];
    X = cbind(X, c(rep(mu.s, length(id2)), y[id3]));
  }
  pp = adalasso(X, y, k=nf, intercept=TRUE);
  return(pp)
}
```

Small simulation result shows that

```
n=250;
#phi = c(.5, .3, .1);
phi = c(.7, .3, 0, -.2);
```

```

nrep=50;
order=5;
A = B = matrix(0, nrep, order);
for(r in 1:nrep){

data = arima.sim(n = n, list(ar = phi), sd = 1)
y = data/sd(data);
fit = ar.adaplasso(y, p=order)
fit = ar.adaplasso(y, p=order)
A[r,] = fit$coefficients.adalasso
B[r,] = fit$coefficients.lasso
print(r)
}

```

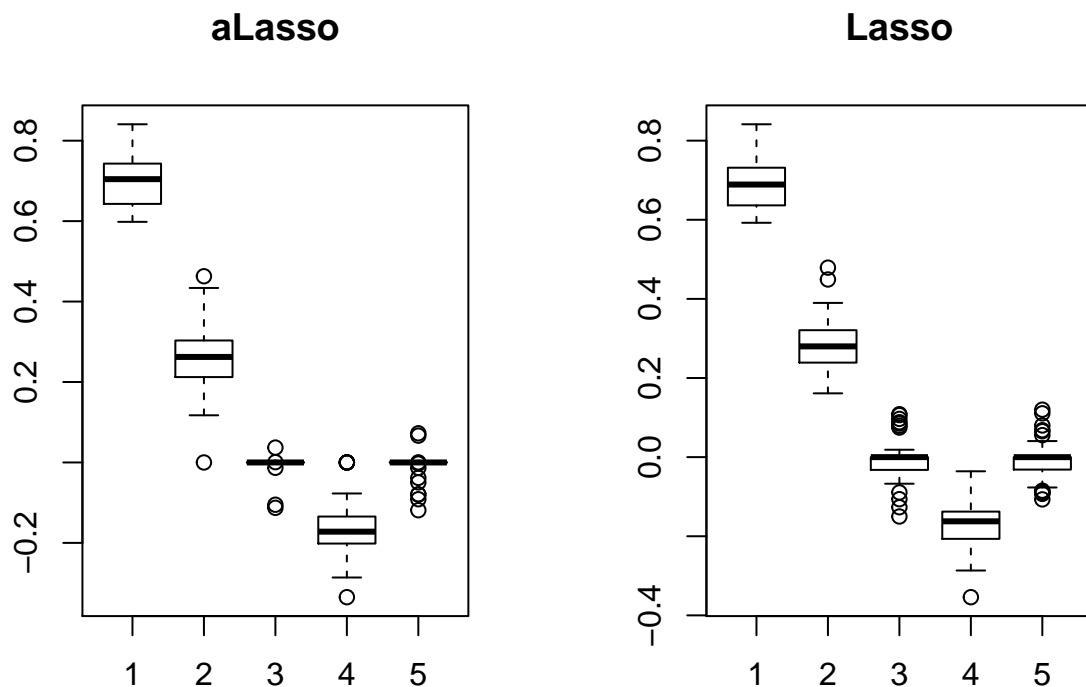
```

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```

```
par(mfrow=c(1,2))
boxplot(A, main="aLasso");
boxplot(B, main="Lasso");
```



In-class exercise

Use Australian wine example:

Apply adaptive lasso algorithm to find the best model. Also reestimate parameters using constraint optimization once you get the final model from adaptive lasso.

Cautions:

It looks like lasso seems work well in time series setting, however, tuning parameter λ in lasso is the key in finite sample performance. It is well documented that CV often fails in time series context.