9.1 Modeling with Differential Equations

Models for Population Growth:

- Suppose an independent variable t = time and a dependent variable P = population. Then the rate of population growth is $\frac{dP}{dt} = kP$, where k is the proportionality constant, and it indicates that the growth rate, $\frac{dP}{dt}$, increases as the population increases.
- Many populations start by increasing in an exponential manner, but the population levels off when it approaches its carrying capacity M
- the equation below satisfies 2 important assumptions; $\frac{dP}{dt} \approx kP$ if P is small, $\frac{dP}{dt} < 0$ if $P > M$.
  => $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$, and it is called logistic differential equation
- We observe that the constant functions $P(t) = 0$ and $P(t) = M$ are solutions because one of the factors on the right side is 0, and those 2 constant solutions are called equilibrium solutions

A Model for the Motion of a Spring

- if the spring is stretched x units from its natural length, then it exerts a force that is proportional to x:
  Restoring Force $= -kx$, where k is a positive constant called spring constant

General Differential Equations

- in general, a differential equation is an equation that contains an unknown function and one or more of its derivative
- a function f is called a solution of a differential equation if the equation is satisfied when $y = f(x)$ and its derivatives are substituted into the equation
- When applying differential equations, we are usually not as interested in finding a family of solutions as we are finding a solution that satisfies some additional requirement
- In many physical problems, we need to find the particular solution that satisfies a condition of the form $y(t_0) = y_0$, and this is called initial condition, and the problem of finding a solution of the differential equation that satisfies the initial condition is called an initial-value problem

9.2 Direction Fields and Euler's Method

Direction Fields :

- short line segments at a number (x, y) with slope $x+y$

Euler's Method

- Euler's method does not produce the exact solution to an initial-value problem; it gives approximations. But by decreasing the step size, we obtain successively better approximations to the exact solution.
- Approximate values for the solution of the initial-value problem $y' = F(x, y)$, $y(x_0) = y_0$, with step size h, at $x_n = x_{n-1} + h$, are $y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$ $\quad n = 1, 2, 3, \ldots$

9.3 Separable Equations

- a first-order differential equation in which the expression for $\frac{dy}{dx}$ can be factored as a function of x times a function of y

$\Rightarrow \frac{dy}{dx} = g(x) f(y)$, or also $\frac{dy}{dx} = \frac{g(x)}{h(y)}$

Orthogonal Trajectories :

- Orthogonal Trajectories of a family of curves is a curve that intersects each curve of the family orthogonally