Final (STA 3001)
June 1, 2021 (6:15 PM to 7:30 PM)

Instruction

- This is a closed book, note exam proctored by WebEx. But, you are allowed to use calculator if necessary. By submitting your report online, it is assumed that you agree with the following pledge;

  Pledge: *I have neither given nor received any unauthorized aid during this exam.*

- Your answer sheets must be written in English, and keep in mind that you need to SHOW ALL YOUR WORK. Remind that you can submit your answer sheets over icampus in a pdf file format ONLY.

1. (20 points) State the followings as detail as you can
   (a) Write down all you know about stationarity of time series $\{X_t\}$.
   (b) State the Gauss-Markov Theorem in the ordinary linear regression.

2. (20 points) For the time series X_t given in the below, i) find the mean and autocovariance function of X_t and ii) determine whether it is a (weakly) stationary process.
   (a) $X_t = \cos(t) + Z_t Z_{t-1}$, $t = 0, \pm 1, \ldots$ where $Z_t \sim WN(0, \sigma^2)$.
   (b) $X_t = Z_0 \cos(ct)$, $t = 0, \pm 1, \pm 2, \ldots$, $Z_t \sim WN(0, \sigma^2)$.

3. (20 points) Find the region for $c \in \mathbb{R}$ such that the AR(2) process

   $$X_t = X_{t-1} + cX_{t-2} + Z_t$$

   is causal and stationary.

4. (20 points) Consider the regression with ARMA errors:

   $$Y = X\beta + \epsilon, \quad \epsilon \sim N(0, \Gamma).$$

   (a) Find the generalized least squares estimator minimizing

   $$(Y - X\beta)'\Gamma^{-1}(Y - X\beta)$$

   (You need to show all the intermediate steps to your final answer clearly.)
   (b) Calculate $E(\widehat{\beta}^{GLS})$
   (c) Calculate $\mathrm{Cov}(\widehat{\beta}^{GLS})$

5. (20 points) Consider the process

   $$X_t - .3X_{t-1} - .40X_{t-2} = Z_t + Z_{t-1} + .25Z_{t-2},$$

   where $Z_t \sim WN(0, \sigma^2)$ with $\sigma^2 = 1$.

(a) Answer the followings with your reasonings:

   i) stationary _____

   ii) causal _____

   iii) invertible _____

   iv) identifiable _____

(b) What are the optimal predictors for the two-step predictor, say $\widetilde{P}_t X_{t+2}$? Note that the predictor is based on the infinite past, that is the answer will be linear combination of X_s, $s \leq t$. Give the **explicit numerical coefficients only for X_t and X_{t-1}.**

Don't forget to write down your NAME, STUDENT ID in your answer sheets!