10.1 Intervals, Estimating Functions,

δ-neighborhood of a : the expressions below are all equivalent

$x \in (a-\delta, a+\delta)$, $a-\delta < x < a+\delta$, $|x-a| < \delta$, $x \underset{\delta}{\approx} a$

$f(I) = \{f(x) : x \in I\}$

$\Rightarrow$ $f(I)$; the range of $f(x)$ over I, or the image of I under the mapping f

Completeness Property for Functions :

- Suppose $f(x)$ is defined on an interval I. If $f(x)$ is bounded above on I, then $\sup_I f(x)$ exists; if $f(x)$ is bounded below, then $\inf_I f(x)$ exists

Estimating Functions : Inequalities and Absolute Values

Error Function : $\operatorname{erf} x = \int_0^x e^{-\frac{t^2}{2}} dt$

10.2 Approximating Functions

- Approximating functions over intervals which are small is often done using the first few terms of a power series

$$f(x) \underset{\varepsilon}{\approx} g(x) \Rightarrow \int_a^b f(x)\,dx \underset{\varepsilon(b-a)}{\approx} \int_a^b g(x)\,dx$$

10.3 Local Behavior

- Studying a function in a δ-neighborhood of some point x_0 is called studying its "local behavior at x_0"

Behavior at Infinity

Terminology ...

for $x \gg 1$, for large x = for x in some interval (a, ∞)

for $x \ll -1$, for negatively large x = for x in some interval $(-\infty, a)$

for $|x| \gg 1$, for large $|x|$ = for $|x| >$ some number a

Local Properties at a point

$f(x)$ is **locally increasing** at x_0 means $f(x)$ is increasing for $x \approx x_0$;
$f(x)$ is **locally bounded** at x_0 means $f(x)$ is bounded for $x \approx x_0$;
$f(x)$ is **locally positive** at x_0 means $f(x)$ is positive for $x \approx x_0$;

10.4 Local and Global Properties of Functions

- We say $f(x)$ is locally bounded on the open interval I if it is locally bounded at every point of I:

  for all $x_0 \in I$, $f(x)$ is bounded for $x \approx x_0$

- We say $f(x)$ is locally increasing on the open interval I if it is locally increasing at every point x_0 of I

Local vs. Global

- Let $f(x)$ be defined on an interval $I = [a, b]$. Then $f(x)$ locally bounded on $I \Rightarrow f(x)$ bounded on I