1) $\pi_3(x) = \dfrac{1}{1+\exp(\alpha_1+\beta_1 x)+\exp(\alpha_2+\beta_2 x)}$

a) If $\beta_1, \beta_2 > 0$, then $1+\exp(\alpha_1+\beta_1 x)+\exp(\alpha_2+\beta_2 x)$ is monotonically increasing in X, so $\pi_3(x)$ is decreasing in X.

b) If $\beta_1, \beta_2 < 0$, then $1+\exp(\alpha_1+\beta_1 x)+\exp(\alpha_2+\beta_2 x)$ is monotonically decreasing in X, so $\pi_3(x)$ is increasing in X.

c) If β_1 and β_2 have different signs, $|\beta_1|$ and $|\beta_2|$ affect $\pi_3(x)$ so it is not monotone.

2) a) Estimated coefficient of the treatment : 0.5806

Interpretation : Controlling for the gender, the estimated cumulative odds for an alternating therapy is in effective rather than non-effective direction are $e^{0.5806} = 1.79$ times the estimated odds for a sequential therapy.

b) Having the gender fixed as male, the cumulative odds of the response to chemotherapy below a given level for the patients with the alternating treatment is $\exp(0.49) = 1.63$ times the cumulative odds for those with the sequential treatment.

Having the gender fixed as female, the cumulative odds of the response to chemotherapy below a given level for the patients with the alternating treatment is $\exp(0.49+0.6) = 2.97$ times the cumulative odds for those with the sequential treatment.

Having the therapy fixed as the sequential, the cumulative odds of the response to chemotherapy below a given level for the female patients is $\exp(0.27) = 1.31$ times the cumulative odds for the male patients.

Having the therapy fixed as the alternating, the cumulative odds of the response to chemotherapy below a given level for the female patients is $\exp(0.27+0.6) = 2.4$ times the cumulative odds for the male patients.

c) no, because the difference in deviances is much smaller than the degree of freedom, 1.

In fact, the first cumulative logit model already had a fairly good fit.

3) a) Assumed Model : $\log(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$

$$L(\mu; n) = \prod_{i,j,k} \frac{e^{-\mu_{ijk}} \mu_{ijk}^{n_{ijk}}}{n_{ijk}!}$$

$l(\mu; n) = \sum_{i,j,k} -\mu_{ijk} + n_{ijk}\log\mu_{ijk} - \log(n_{ijk}!)$, using the assumed model

$$= \sum_{i,j,k} -\exp[\lambda+\lambda_i^X+\lambda_j^Y+\lambda_k^Z] + n_{ijk}(\lambda+\lambda_i^X+\lambda_j^Y+\lambda_k^Z) - \log(n_{ijk}!)$$

$$= n_{+++}\lambda + \sum_i n_{i++}\lambda_i^X + \sum_j n_{+j+}\lambda_j^Y + \sum_k n_{++k}\lambda_k^Z + \sum_{i,j,k} -\exp[\lambda+\lambda_i^X+\lambda_j^Y+\lambda_k^Z] - \log(n_{ijk}!)$$

$\therefore$ Using the factorization theorem, $(n_{i++}, n_{+j+}, n_{++k})$ are sufficient statistics for (X, Y, Z)

b) Assumed Model : $\log(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}$

$\Rightarrow \; l(\mu : n) = \sum_{i,j,k} -\mu_{ijk} + n_{ijk}\log\mu_{ijk} - \log(n_{ijk}!)$, using the assumed model

$= \sum_{i,j,k} -\exp[\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}] + n_{ijk}(\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}) - \log(n_{ijk}!)$

$= n_{+++}\lambda + \sum_i n_{i++}\lambda_i^X + \sum_j n_{+j+}\lambda_j^Y + \sum_k n_{++k}\lambda_k^Z + \sum_i\sum_j n_{ij+}\lambda_{ij}^{XY} + \sum_{i,j,k} -\exp[\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}] - \log(n_{ijk}!)$

$\therefore$ Using the factorization theorem, (n_{ij+}, n_{++k}) are sufficient statistics for (XY, Z)

c) Assumed Model : $\log(\mu_{ijk}) = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}$

$\Rightarrow \; l(\mu : n) = \sum_{i,j,k} -\mu_{ijk} + n_{ijk}\log\mu_{ijk} - \log(n_{ijk}!)$, using the assumed model

$= \sum_{i,j,k} -\exp[\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}] + n_{ijk}(\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}) - \log(n_{ijk}!)$

$= n_{+++}\lambda + \sum_i n_{i++}\lambda_i^X + \sum_j n_{+j+}\lambda_j^Y + \sum_k n_{++k}\lambda_k^Z + \sum_i\sum_j n_{ij+}\lambda_{ij}^{XY} + \sum_i\sum_k n_{i+k}\lambda_{ik}^{XZ} + \sum_j\sum_k n_{+jk}\lambda_{jk}^{YZ} + \sum_{i,j,k} -\exp[\lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY}] - \log(n_{ijk}!)$

$\therefore$ Using the factorization theorem, $(n_{ij+}, n_{i+k}, n_{+jk})$ are sufficient statistics for (XY, XZ, YZ)

4) a) Estimated Model : $\log\mu_{ijk} = 2.74 - 0.93 \cdot S + 4.51 \cdot E + 5.66 \cdot F$ $\left(S = \begin{cases} 1, & \text{None} \\ 0, & \text{seat belt} \end{cases}, \quad E = \begin{cases} 1, & \text{No} \\ 0, & \text{Yes} \end{cases}, \quad F = \begin{cases} 1, & \text{Nonfatal} \\ 0, & \text{Fatal} \end{cases} \right)$

Interpretation : The p-value of each coefficient indicates that they are all significant.

Not fastening a seat belt increases the count by $\exp(-0.93) = 0.395$ times.

Not ejecting increases the count by $\exp(4.51) = 90.92$ times.

Nonfatalness increases the count by $\exp(5.66) = 287.15$ times.

b) The p-values of both both fastening a seat belt and ejection greatly influence the severity of injury.

Not fastening seat belt increases the odds of fatalness by $\exp(1.72) = 5.58$ times, compared to fastening a seat belt.

Not ejecting increases the odds of fatalness by $\exp(-2.8) = 0.061$ times, compared to ejecting a seat belt.

c) The AIC of the logistic model is far smaller than that of log-linear model.

The logistic model fits better than the log-linear model.