Experimental Design

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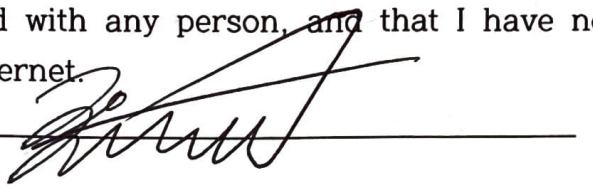
I certify that the work presented for this exam is my own, that I have not consulted with any person, and that I have not written the work using the Internet.

Signature: 

Problem 1.

(a)	False	(f)	True
(b)	False	(g)	True
(c)	False	(h)	False
(d)	True	(i)	False
(e)	True	(j)	True

Problem 2.

(a) i) There are k different treatments for each block, where $k < a$

ii) There are r different blocks for each treatment, where $r < b$

iii) There are λ pairs for each pair of treatment

$$\lambda = \frac{r(k-1)}{a-1}$$

(b) ~~Yes, because the Sum of Squares~~

$H_0: \tau_i = 0$ for all i

$H_a: \tau_i \neq 0$ for at least one i

Yes, since both of p-values of Type III indicate they are significant (p-value < 0.025)

We can reject the null hypothesis.

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(c)

$\tau_1 = 71.375 - 72.5 = -1.125$

$\tau_2 = 71.625 - 72.5 = -0.875$

$\tau_3 = 72 - 72.5 = -0.5$

$\tau_4 = 75 - 72.5 = 2.5$

Problem 3.

(a)

Sources of Variation	Sum of Squares	d.f.	Mean Squares	F
Treatment (Trees)	0.277	2	0.1385	17.1
Error	0.097	12	0.0081	
Total	0.374	14		

$a = 3$

$df_{trt} = 3 - 1 = 2$

$df_E = 15 - 3 = 12$

$df_{total} = 15 - 1 = 14$

$F_0 = 17.1 > F_{0.05,2,12} = 3.89$

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(b) $H_0: \tau_1 = \tau_2 = \tau_3 = 0$

$H_a: \tau_i \neq 0$ for at least one

$\therefore$ Since $F_0 = 17.1 > F_{0.05,2,12} = 3.89$, we can reject the null hypothesis.

(c) $E(MS_{trt}) = \sigma^2 + n\sigma_\tau^2$, $E(MS_E) = \sigma^2 = 0.0081$

$0.1385 = 0.0081 + 5\sigma_\tau^2$

$\sigma_\tau^2 = 0.02608$

(d) $E\left(\frac{\sigma_\tau^2}{\sigma^2}\right) = \frac{0.466 + 133.57}{2} = 67.018$

$\frac{L(\theta)}{1+L(\theta)}$ & $\frac{U(\theta)}{1+U(\theta)}$

$\Rightarrow [0.318, 0.993]$ = 95% CI

point estimate $= \frac{0.318 + 0.993}{2} = 0.655$

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Problem 4.

(a) 6 batches, 2 groups, 3 obs for each group

The overall mean = 0.5467

$SS_{batch} = 6 \sum_{i=1}^{6} (y_{i\cdot\cdot})^2 - n\bar{y}^2$

$= 6[15.7 + 3.8 + 20.6 + 0.96 + 16.1 + 0.91]$

$= 348.42$

(b)

Source	df	SS	MS	F	$F_{0.05}$
Batch	5	348.42	69.684	4.61	2.55
Group	1	62.5	62.5	4.13	4.18
Error	29	438.57	15.123		
Total	35	849.49			

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(c)

	R i	R j	R k	
T_i	1	2	3	$\sigma^2 + 6\sigma^2_{Batch}$
β_j	6	1	3	$\sigma^2 + 18\sigma^2_{Group}$
$Error_{ijk}$	1	1	1	σ^2

$E(MS_E) = \sigma^2 = 15.123$

$E(MS_{Batch}) = 15.123 + 6\sigma^2_{Batch} = 69.684$

$\sigma^2_{Batch} = 9.0935$

(d)

$E(MS_E) = \sigma^2 = 15.123$

$E(MS_{Batch}) = 15.123 + 6\sigma^2_{Batch} = 69.684$

$\therefore \sigma^2_{Batch} = 9.0935$

$E(MS_{Group}) = 15.123 + 18\sigma^2_{Group} = 62.5$

$\therefore \sigma^2_{Group} = 2.632$

(e)

H_0: $T_i = 0$ for all i

H_a: $T_i \neq 0$ for at least one

=> Since F_0 $4.61 > 2.55$ $F_{0.05,5,29}$, we can reject the null hypothesis.

Batch is significant

H_0': $\beta_j = 0$ for all j

H_a': $\beta_j \neq 0$ for at least one

=> Since F_0 $4.13 < 4.18$ $F_{0.05,1,29}$, we cannot reject the null hypothesis.

Group is not significant