TITLE: 중간고사! DATE:

1. 3 axioms of prob.

2. a WLLN
   b SLLN
   c CLT

3. 2-headed
   fair
   biased - H=95%
   P(2-headed | H)

4. $F(z) = 0$
   draw cdf of z
   & find var(z)

5. 25 people, equally like to have any of the 365 days
   A: no 2 of them share B-day
   Poisson approximation of P(A)

6. X & Y independently follows geometric dist. w p
   $U = \min(X, Y)$ & $V = X - Y$
   find joint pmf of U, V

7. X is from probability space (S, P)
   & $EX = \sum_i x_i P_X(x_i)$.
   show that $EX = \sum_{w \in S} X(w) \cdot P(w)$

8. E_n is a decreasing sequence of event
   show that $\lim_{n\to\infty} P(E_n) = P(\lim_{n\to\infty} E_n)$

TITLE: 중간 2 DATE:

1. $P(N=n \mid Y=y) = \frac{y^n e^{-y}}{n!}$, $n = 0, 1, 2 \cdots$

$f_Y(y) = \frac{\lambda e^{-\lambda y}}{(r-1)!}(\lambda y)^{r-1}$, $y > 0$

① $E(N)$

② $Var(N)$

③ $P(N=n)$

2. $S = \sum_{i=1}^{N} X_i$, X_i's are IID R.V and indep of N

MGF of X : $M_X(t)$ & MGF of N: $M_N(t)$

find MGF of S.

3. 3 urns : red, white, blue. initial urn is red, select 1 ball from an urn & replace the ball in that urn & select the color of the urn for next stage same as the color of the ball selected.

What will be the proportion of selecting each color of the ball in long-run?

in red urn, there are 1 red & 4 blue — total 5

white urn, " 3 white & 2 red & 2 blue — total 7

blue urn, 4 white & 3 red & 2 blue — total 9

4. Compute T_0 when $P_0 = \frac{1}{2}$, $P_1 = \frac{1}{4}$, $P_2 = \frac{1}{4}$.

5. (everlasting) tournament game among team 1, 2, 3.

initial game is team 1 & 2,

the probability that team i win against team j is p_{ij} (that means $p_{ij} + p_{ji} = 1$)

let X_n be the pair of the n^{th} game. show that it's a DTMC & compute probability transition matrix.

6. $P = \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0.5 \\ 0.1 & 0.2 & 0.3 & 0.4 \end{pmatrix}$ & $S = \{1, 2, 3, 4\}$

$P(X_0 = 1) = 1$

(a) $P(X_2 = 4)$

(b) $P(X_3 = 1, X_2 = 4, X_1 = 2)$

(c) $P(X_4 = 4 \mid X_5 = 2)$

(d) $E(X_3)$

7. $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$