1

	Drugs	No Drugs
S	105	8
AD	12	2

$G_1^2 = .753$ (1df)
$\chi^2_{.05} = 3.84$ (1df)

(Not significant)

	Drugs	No Drugs
S+AD	117	10
N	18	19

$G_2^2 = 31.740$ (1df)
$\chi^2_{.05} = 3.84$ (1df)

(Significant)

	Drugs	No Drugs
S+AD+N	135	29
PD	47	52

$G_3^2 = 34.774$ (1df)
$\chi^2_{.05} = 3.84$ (1df)

(Significant)

	Drugs	No Drugs
S+AD+ N+PD	182	81
SS	0	13

$G_4^2 = 29.270$ (1df)
$\chi^2_{.05} = 3.84$ (1df)

(Significant)

G^2 for overall table $= 96.537$
$\chi^2_{.05} = 9.49$ (4 df)

Interpretation:

It appears that patients with schizophrenia or affective disorder are generally prescribed drugs, patients with neurosis or a personality disorder are prescribed drugs roughly half the time, and patients with special symptoms are rarely prescribed drugs.

The first 2×2 table produces a "small" G^2 since it considers patients with schizophrenia and patients with affective disorders; for both types of patients, the frequency with which drugs are prescribed is $\simeq 90\%$. In subsequent 2×2 tables, the pooled group of patients is always prescribed drugs at a considerably higher frequency than the group of patients serving as a reference. As a result, these tables produce "large" G^2 values.

2. Models

Let x = the score based on the smoking status of a student's parents, and let $y=1$ if the student smokes, 0 otherwise

The scores for x are $x_1=0$ (neither parent smokes), $x_2=2$ (one parent smokes), and $x_3=3$ (both parents smoke).

Let $\pi = P(Y=1)$.

For levels $i=1,2,3$, consider the models.

$$\log \frac{\pi_i}{1-\pi_i} = \alpha + \beta_i \quad \text{(model } M_S \text{ (saturated model))}$$

$$\log \frac{\pi_i}{1-\pi_i} = \alpha + \beta x \quad \text{(model } M_1\text{)}$$

In addition, we may consider the indep. model (M_0), given by $\log \frac{\pi_i}{1-\pi_i} = \alpha$.

Model M_S

I. Parameter Estimates

· The MLE's for parameters $(\alpha, \beta_1, \beta_2, \ )$ are

$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$
-1.827	0.349	0.588

Note the decreasing trend in the $\hat{\beta}_i$'s.

II Test

· The LR test for $H_0: \beta_1=\beta_2=\beta_3=0$ results in very low p-value. (LR stat. = 38.37, df = 2, p-value < .0001.)
see 'Type3' option

III Predicted Prob.'s

· The predicted probabilities $\hat{\pi}_1$, $\hat{\pi}_2$, and $\hat{\pi}_3$ based on both the observed data ($\hat{\pi}_i = n_{i1}/n_{i+}$) and the fitted model M_1 ($\hat{\pi}_i = e^{\hat{\alpha}+\hat{\beta}_i}/(1+e^{\hat{\alpha}+\hat{\beta}_i})$) are identical:

$\hat{\pi}_3 = 0.225$, $\hat{\pi}_2 = 0.186$, $\hat{\pi}_1 = 0.139$.

Note the decreasing trend in the $\hat{\pi}_i$'s, which is consistent with the decreasing trend in the $\hat{\beta}_i$'s.

IV Conclusion

The trends in the $\hat{\beta}_i$'s and the $\hat{\pi}_i$'s suggest that a linear logit model provide an adequate fit to the data. (Deviance (M1) $\doteq 0.367$)

Linear Logit Model (M1)

I. Parameter Estimates

The MLE's for α and β are $\hat{\alpha} = -1.844$, $\hat{\beta} = 0.196$. Note that the sign of $\hat{\beta}$ is positive, which suggests that as the smoking score for a student's parents increases, so does the probability that the student is a smoker.

II. Test for Slope

For testing $H_0: \beta = 0$, we have

LR stat. = 38.00, df = 1, p-value < .0001.

Wald Stat $= \left(\frac{0.196}{0.033}\right)^2 = 36.29$, p-value < .0001

There is strong evidence of a trend between the smoking score for a student's parents and the log odds of the student being a smoker.

III Tests for Goodness-of-fit.

H_0: Model M1 holds.

$G^2 = 0.367$ (1 df) P-value > 0.10

$X^2 = 0.366$ (1 df) "

There is no evidence that the linear logit model does not provide an adequate fit to the data.

IV <u>Conclusion</u>

The linear logit model appears more appropriate than the saturated logit model. The former model is more parsimonious than the latter and seems to provide an adequate fit to the data.

HW2-Problem3

```
data Problem1;
 input parent smoke nonsmoke @@;
   n=smoke+nonsmoke;
 cards;
 3 400 1380
 2 416 1823
 0 188 1168
 ;

proc genmod data=Problem1 descending;
  class parent(ref=first)/param=ref;
  model smoke/n=parent / dist=bin link=logit residuals type3;
run;

proc genmod data=Problem1 descending;
  model smoke/n=parent / dist=bin link=logit residuals type3;
run;
```

MS

M1

Ms

The GENMOD Procedure

Model Information

Data Set	WORK.PROBLEM1
Distribution	Binomial
Link Function	Logit
Response Variable (Events)	smoke
Response Variable (Trials)	n

Number of Observations Read	3
Number of Observations Used	3
Number of Events	1004
Number of Trials	5375

Class Level Information

Class	Value	Design Variables	
parent	0	0	0
	2	1	0
	3	0	1

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	0	0.0000	.
Scaled Deviance	0	0.0000	.
Pearson Chi-Square	0	0.0000	.
Scaled Pearson X2	0	0.0000	.
Log Likelihood		-2569.0722	

Algorithm converged.

Analysis Of Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept		1	-1.8266	0.0786	-1.9806	-1.6726	540.29	<.0001
parent	2	1	0.3491	0.0955	0.1618	0.5363	13.35	0.0003
parent	3	1	0.5882	0.0970	0.3982	0.7783	36.81	<.0001
Scale		0	1.0000	0.0000	1.0000	1.0000		

NOTE: The scale parameter was held fixed.

The GENMOD Procedure

LR Statistics For Type 3 Analysis

Source	DF	Chi-Square	Pr > ChiSq

HW3-Problem1

parent	2	$\boxed{38.37}$	<.0001

Observation Statistics

Observation	Resraw	Reschi	Resdev	StResdev	StReschi	Reslik
1	1.11E-14	6.305E-16	0	.	.	.
2	-5.25E-15	-2.85E-16	0	.	.	.
3	2.143E-14	1.684E-15	0	.	.	.

M1

The GENMOD Procedure

Model Information

Data Set	WORK.PROBLEM1
Distribution	Binomial
Link Function	Logit
Response Variable (Events)	smoke
Response Variable (Trials)	n

Number of Observations Read	3
Number of Observations Used	3
Number of Events	1004
Number of Trials	5375

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	1	0.3669	0.3669
Scaled Deviance	1	0.3669	0.3669
Pearson Chi-Square	1	0.3663	0.3663
Scaled Pearson X2	1	0.3663	0.3663
Log Likelihood		-2569.2556	

Algorithm converged.

Analysis Of Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	-1.8443	0.0734	-1.9882	-1.7003	630.57	<.0001
parent	1	0.1958	0.0325	0.1321	0.2595	36.29	<.0001
Scale	0	1.0000	0.0000	1.0000	1.0000		

NOTE: The scale parameter was held fixed.

LR Statistics For Type 3 Analysis

Source	DF	Chi-Square	Pr > ChiSq
parent	1	$\boxed{38.00}$	<.0001

The GENMOD Procedure

Observation Statistics

Observation	Resraw	Reschi	Resdev	StResdev	StReschi	Reslik
1	5.6783188	0.3240957	0.3235431	0.6042227	0.6052546	0.6049589
2	-8.517483	-0.459213	-0.460401	-0.60682	-0.605255	-0.606156
3	2.8391588	0.2245414	0.2240615	0.6039611	0.6052545	0.6050767

3. $Y|\lambda \sim \text{Poisson}(\lambda)$ $\qquad p(y|\lambda) = \dfrac{e^{-\lambda}\lambda^{y}}{y!}$

$\lambda \sim \text{Gamma}(k, \mu/k)$ $\qquad p(\lambda) = \dfrac{(\frac{k}{\mu})^{k}}{\Gamma(k)} \lambda^{k-1} e^{-\frac{k}{\mu}\lambda}$

(a) The joint pdf of (y, λ) is

$$
\begin{aligned}
p(y,\lambda) &= p(y|\lambda)\,p(\lambda) \\
&= \frac{e^{-\lambda}\lambda^{y}}{y!}\,\frac{(\frac{k}{\mu})^{k}}{\Gamma(k)}\lambda^{k-1}e^{-\frac{k}{\mu}\lambda} \\
&= \frac{(\frac{k}{\mu})^{k}}{y!\,\Gamma(k)}\lambda^{y+k-1}e^{-(1+\frac{k}{\mu})\lambda}.
\end{aligned}
$$

The marginal dist. of y is

$$
p(y) = \frac{(\frac{k}{\mu})^{k}}{y!\,\Gamma(k)}\,\frac{\Gamma(y+k)}{(\frac{\mu+k}{\mu})^{y+k}}\underbrace{\int_{0}^{\infty}\frac{(\frac{\mu+k}{\mu})^{y+k}}{\Gamma(y+k)}\lambda^{y+k-1}e^{-\overset{\frac{\mu+k}{\mu}}{\overset{\parallel}{(1+\frac{k}{\mu})}}\lambda}\,d\lambda}_{=1}
$$

$$
\begin{aligned}
&= \frac{\Gamma(y+k)}{\Gamma(y+1)\Gamma(k)}\left(\frac{\frac{k}{\mu}}{\frac{\mu+k}{\mu}}\right)^{k}\left(\frac{1}{\frac{\mu+k}{\mu}}\right)^{y} \\
&= \frac{\Gamma(y+k)}{\Gamma(y+1)\Gamma(k)}\left(\frac{k}{\mu+k}\right)^{k}\left(\frac{\mu}{\mu+k}\right)^{y} \\
&= \frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)}\left(\frac{k}{\mu+k}\right)^{k}\left(1-\frac{k}{\mu+k}\right)^{y}.
\end{aligned}
$$

for $y = 0, 1, 2, \cdots$

(b) $E(Y) = E(E(Y|\lambda)) = E(\lambda) = k\frac{\mu}{k} = \mu$

$$
\begin{aligned}
Var(Y) &= E(Var(Y|\lambda)) + Var(E(Y|\lambda)) \\
&= E(\lambda) + Var(\lambda) \\
&= \mu + \frac{1}{k}\mu^{2}
\end{aligned}
$$