STA 3021: Stochastic Processes
Quiz 2 (Nov 28, 2017)

Student ID: ______________________ **Name:** ______________________

1. (5 points) For the transition probability matrix with state space $E = \{1, 2, 3, 4, 5\}$, do a complete classification of states, that is, identify communicating classes, periodic/aperiodic, postive/null recurrent or transient.

$$P = \begin{pmatrix} 0 & 0 & .4 & .6 & 0 \\ 0 & .2 & 0 & .5 & .3 \\ .5 & 0 & .5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ .3 & 0 & .5 & 0 & .2 \end{pmatrix}.$$

$\{1, 3, 4\}$ closed / aperiodic / positive recurrent

$\{2\}$ open / aperiodic / transient

$\{5\}$ open / aperiodic / transient.

2. (5 points) A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same the next day, and if it changes then it is equally likely to be either of the other two possibilities. In the long run, what proportion of days are sunny, cloudy (but dry), or rainy?

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\begin{cases} \pi_0 = \frac{1}{4}\pi_1 + \frac{1}{4}\pi_2 \\ \pi_1 = \frac{1}{2}\pi_0 + \frac{1}{2}\pi_1 + \frac{1}{4}\pi_2 \\ \pi_2 = \frac{1}{2}\pi_0 + \frac{1}{4}\pi_1 + \frac{1}{2}\pi_2 \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$\therefore \quad \pi_0 = \frac{1}{5}, \quad \pi_1 = \frac{2}{5}, \quad \pi_2 = \frac{2}{5}$

3. (5 points) The lifetimes of As dog and cat are independent exponential random variables with respective rates λ_d and λ_c. One of them has just died. Find the expected additional lifetime of the other pet.

T_d : lifetimes of a dog

T_c : lifetimes of a cat

A : additional lifetime of the other pet.

$$E(A) = E(A \mid T_d > T_c)\, P(T_d > T_c) + E(A \mid T_d < T_c)\, P(T_d < T_c)$$

$$= \frac{1}{\lambda_d} \cdot \frac{\lambda_c}{\lambda_d + \lambda_c} + \frac{1}{\lambda_c} \cdot \frac{\lambda_d}{\lambda_d + \lambda_c}$$

4. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ. Let S_n denote the time of the nth event. Find

(5)

(a) $E[S_4]$ $= E(T_1 + T_2 + T_3 + T_4)$, $T_i \overset{iid}{\sim} Exp(\lambda)$

$= 4E(T_1)$

$= \frac{4}{\lambda}$

(b) $Cov(S_3, S_4)$ $= Cov(T_1 + T_2 + T_3, T_1 + T_2 + T_3 + T_4)$, $T_i \overset{iid}{\sim} Exp(\lambda)$

$= Var(T_1) + Var(T_2) + Var(T_3)$

$= 3Var(T_1)$

$= \frac{3}{\lambda^2}$

End of quiz.