1) $\cancel{E(S_4)}$

$$E(S_5) = E[T_1 + T_2 + T_3 + T_4 + T_5] \quad , \quad T_i \sim Exp(\mu)$$

$$= \frac{5}{\mu}$$

2) $$P(X > max(Y_1, Y_2)) = \int_0^\infty P(Y_1 < Y_2 < X \mid X) P(X) + P(Y_2 < Y_1 < X \mid x) P(x)\, dx$$

$$= \int_0^\infty e^{-\mu x} \cdot \left(\frac{\mu}{\lambda + 2\mu}\right) + e^{-\mu x} \cdot \left(\frac{\mu}{\lambda + 2\mu}\right) dx$$

$$= \int_0^\infty e^{-\mu x}\left(\frac{2\mu}{\lambda + 2\mu}\right) dx$$

$$= \frac{2\mu}{\lambda + 2\mu} \int_0^\infty e^{-\mu x}\, dx$$

$$= \frac{2\mu}{\lambda + 2\mu}\left(-\frac{1}{\mu} e^{-\mu x} \Big|_0^\infty\right)$$

$$= \frac{2}{\lambda + 2\mu}$$

3) a) i) $N(0) = 0$

ii) $\{N(t)\}$ ~~inde~~

has independent increments.

iii) $P(N(t+h) - N(t) = 1) = \lambda h + o(h)$

iv) $P(N(t+h) - N(t) \geq 2) = o(h)$

b) $N_s(0) = N(S+0) - N(S)$

$= N(0) = 0$, the first axiom satisfied

$N_s(t) = N(S+t) - N(s)$

$= N(t)$, has independent increments, (second axiom)

$P(N_s(t+h) - N_s(t) = 1) = \lambda h + o(h)$, satisfied

$P(N_s(t+h) - N_s(t) \geq 2) = o(h)$, satisfied

4) a) $E(N(8))$, $N(t) \sim PP(8t)$

$= 64$, $Var(N(8)) = 64$

b) $P(N(1) > 4) = 1 - P(N(1) \leq 3)$

$$= 1 - \left[\frac{e^{-8} 8^3}{3!} + \frac{e^{-8} 8^2}{2!} + \frac{e^{-8} 8}{1!} + e^{-8} \right]$$

$= 0.95762$

c) $P(N(\frac{1}{4}) = 0) = e^{-2} = 0.1353$

d) $cov(N(12) - N(10), N(11) - N(9))$

$= cov(N(12) - N(11) + N(11) - N(10), N(11) - N(10) + N(10) - N(9))$

$= cov(N(11) - N(10), N(11) - N(10))$

$= 8 = Var(N(12) - N(10))$

$= Var(N(11) - N(9))$

$\therefore Corr(N(12) - N(10), N(11) - N(9)) = \frac{1}{2}$

5) $X = \sum_{i=1}^{N(t)} Y_i$, X is the total amount of money

Y is money spent per ~~[illegible]~~ person , $Y \sim Exp(\frac{1}{2000})$

$E(X) = E(N(t))\,E(Y)$, the expected value of compound variable, $t=4, \lambda=5$

$= 20 \cdot (2000)$

$= \underline{40000}$, dollars

$Var(X) = Var(Y)\,E(N(t)) + [E(Y)]^2\,Var(N(t))$

$= (2000)^2 \cdot 20 + (2000)^2 \cdot 20$

$= \underline{160000000}$, dollars

6) $\text{cov}\left(N(t), \sum_{i=1}^{N(t)} X_i\right) = E\left[N(t) \cdot \sum_{i=1}^{N(t)} X_i\right] - E[N(t)] \cdot E\left[\sum_{i=1}^{N(t)} X_i\right]$

$E\left[E\left[N(t) \cdot \sum_{i=1}^{N(t)} X_i \mid N(t)\right]\right] = E\left[(N(t))^2\right] E(X) = (\lambda t + \lambda^2 t^2)\mu$

$E(N(t)) = \lambda t$

$E\left[\sum_{i=1}^{N(t)} X_i\right] = E(N(t)) E(X) = \lambda \mu t$

$\therefore \text{cov}\left(N(t), \sum_{i=1}^{N(t)} X_i\right) = \lambda \mu t + \lambda^2 t^2 \mu - \lambda^2 t^2 \mu$

$= \lambda \mu t$

7) a) $E[B(t) \mid B(s)=y]$

$$= E[B(t-s) + y]$$

$$= y$$

b) $\cancel{E[B(s) \mid B(t)=x]} =$,

$E[B(s) \mid B(t)=x]$, using Campell's Theorem

$$= \frac{s}{t} x$$

c) $E[B^2(t) \mid B(s)] = Var[B(t) \mid B(s)] + [E(B(t) \mid B(s))]$

$$= t-s$$

d) $cov(B^{\circ}(t), B^{\circ}(s)) = cov(B(t) - tB(1), B(s) - sB(1))$

$$= cov(B(t), B(s)) - s \cdot cov(B(t), B(1)) - t \cdot cov(B(1), B(s)) + ts \cdot cov(B(1), B(1))$$

$$= s - ts - ts + ts$$

$$= s(1-t)$$