9.1 Functions

9.2 Algebraic Operations on Functions

Definition 9.2 Given two functions $g : D_g \to \mathbb{R}$ and $f : D_f \to \mathbb{R}$, we define their **composition** $f \circ g : D_{f\circ g} \to \mathbb{R}$ by

(3) $$(f \circ g)(a) = f(g(a)), \quad a \in D_{f\circ g}\ ;$$

$$D_{f\circ g} = \{a \in \mathbb{R} : g \text{ is defined at } a \text{ and } f \text{ is defined at } g(a)\}\ .$$

Note that $f \circ g$ is read from right to left: first the mapping g is performed, then the mapping f; it is this convention that makes (3) true.

Two compositions interpreted geometrically by using the graph G_f of $f(x)$:

translation if $a > 0$,

the graph of $f(x+a)$ is the graph G_f moved to the *left* a units;

the graph of $f(x-a)$ is the graph G_f moved to the *right* a units;

change of scale if $a > 1$,

the graph of $f(x/a)$ is the graph G_f expanded horizontally by the factor a;

the graph of $f(ax)$ is the graph G_f compressed horizontally by $1/a$.

9.3 Some Properties of Functions

Definition 9.3A Let $f(x)$ be a function with domain D. We say f is

increasing	if	$f(a) \le f(b)$	for all pairs $a < b$ in D;
strictly increasing	if	$f(a) < f(b)$	for all pairs $a < b$ in D;
decreasing	if	$f(a) \ge f(b)$	for all pairs $a < b$ in D;
strictly decreasing	if	$f(a) > f(b)$	for all pairs $a < b$ in D;
monotone	if f is either increasing in D or decreasing in D;		
strictly monotone	if f is strictly increasing or strictly decreasing in D.		

Definition 9.3B

$f(x)$ is **even** if $f(-x) = f(x)$ for all $x \in D_f$;
$f(x)$ is **odd** if $f(-x) = -f(x)$ for all $x \in D_f$.

For both definitions the domain D_f of the function must be symmetric about the point 0 (i.e., $x \in D_f \Leftrightarrow -x \in D_f$), otherwise the equality makes no sense.

Geometrically, an even function is one whose graph is symmetric about the y-axis; an odd function is one whose graph is symmetric about the origin.

Definition 9.3C We say $f(x)$ is **periodic** if there is a $c > 0$ such that

$$f(x+c) = f(x) \qquad \text{for all } x \in D_f .$$

The number c is called a **period** of f; the smallest such c (if it exists) is called the *minimal period* of f, or simply the *period* of f.

9.4 Inverse Functions

9.5 The Elementary Functions

(a) the *rational* functions: those writable in the form $p(x)/q(x)$, where $p(x)$ and $q(x)$ are polynomials;

(b) the basic *trigonometric* functions: $\cos x$, $\sin x$, $\tan x$, their three reciprocals, and the six inverses $\cos^{-1}x, \sin^{-1}x, \ldots$;

(c) the *exponential* function e^x and its inverse, $\ln x$;

(d) the *algebraic* functions: those functions $y = y(x)$ which satisfy an equation of the form

(12) $$y^n + a_1(x)y^{n-1} + \ldots + a_n(x) \ = \ 0$$

- The elementary functions are then all functions that you can get from the four classes above by addition, multiplication, division, and composition of functions