실험계획법 HW 5.

1.

(a) # of replicates, $n=3$

total (1)=77 a=105 b=118 ab=149 c=126 ac=114 bc=165 abc=128

$A = \frac{1}{4n}[abc-bc+ac-c+ab-b+a-(1)] = 0.833$

$B = \frac{1}{4n}[abc+bc-ac-c+ab+b-a-(1)] = 11.5$

$C = \frac{1}{4n}[abc+bc+ac+c-ab-b-a-(1)] = 7$

$AB = \frac{1}{4n}[abc-bc-ac+c+ab-b-a+(1)] = -1.833$

$AC = \frac{1}{4n}[abc-bc+ac-c-ab+b-a+(1)] = -9$

$BC = \frac{1}{4n}[abc+bc-ac-c-ab-b+a+(1)] = -2.667$

$ABC = \frac{1}{4n}[abc-bc-ac+c-ab+b+a-(1)] = -2.333$

Effect	Estimate
A	0.833
B	11.5
C	7
AB	-1.833
AC	-9
BC	-2.667
ABC	-2.333

→ The effects B, C, and AC appear to be large (significant).

(b) ANOVA

Source	DF	Sum of Square	Mean Square	F value	P-value	
A	1	4.1667	4.1667	0.13	0.7241	
B	1	793.5	793.5	24.57	0.0001	< 0.05
AB	1	20.1667	20.1667	0.62	0.4409	
C	1	294	294	9.10	0.0082	< 0.05
AC	1	486	486	15.05	0.0013	< 0.05
BC	1	42.667	42.667	1.32	0.2673	
ABC	1	32.667	32.667	1.01	0.3295	
Error	16	516.667	32.292			
Total	23	2189.833				

→ The ANOVA table also shows that B, C and AC are significant effects.

(c) $y = 40.92 + 0.42X_A + 5.75X_B + 3.5X_C - 4.5X_AX_C - 0.92X_AX_B - 1.3X_BX_C - 1.12X_AX_BX_C$

(d) According to SAS output, the residuals seem okay. (no patterns) Normality does not work.

(e) According to the SAS output, the optimum condition is when A, B, C is (-, +, +).
It has the highest mean of 57 compare to the other conditions.

2. ABC and block are confounded with replicates

In this case, there is no df for SSE. Thus, we need a pooling hig-order interactions. Potentially significant effects (see SAS output) are B, C, AC, block. Even though A is not significant, it is a main effect. So, we include A, B, C, AC, block.

The model is

$$y_{ijkl} = \mu + \underset{\uparrow A}{\alpha_i} + \underset{\uparrow B}{\beta_j} + \underset{\uparrow C}{\gamma_k} + \underset{\uparrow AC}{(\alpha\gamma)_{ik}} + \underset{\uparrow block}{\eta_l} + \varepsilon_{ijkl}$$

ANOVA table

source	df	S.S	MS	F_0	P-value
A	1	15.125	15.125	0.55	0.5364
B	1	351.125	351.125	12.71	0.0705
C	1	210.125	210.125	7.61	0.1102
AC	1	406.125	406.125	14.70	0.0618
block	1	105.125	105.125	3.81	0.1904
Error	2	55.25	27.625		
Total	7	1142.875			

There are no significant effects.

We also check normality which is satisfied.

3.

Complete confounding with ABC

$$y_{ijklm} = \mu + \rho_i + \eta_j + (\rho\eta)_{ij} + \alpha_k + \beta_l + \gamma_m + (\alpha\beta)_{kl} + (\alpha\gamma)_{km} + (\beta\gamma)_{lm} + \varepsilon_{ijklm}$$

(ρ_i: Replicates, η_j: block)

$i = 1, 2, 3$; $j = 1, 2$; $k = 1, 2$; $l = 1, 2$; $m = 1, 2$.

Source	df	SS	MS	F_0	P-value
Rep.	2	1.083	0.542	0.01	0.985
block	1	32.667	32.667	0.89	0.365
Rep*block	2	74.083	37.042	1.01	0.394
A	1	4.167	4.167	0.11	0.742
B	1	793.500	793.500	21.57	0.001*
C	1	294.000	294.000	7.99	0.015*
AB	1	20.167	20.167	0.55	0.473
AC	1	486.000	486.000	13.21	0.003*
BC	1	42.667	42.667	1.16	0.303
Error	12	441.500	36.792		
Total	23	2189.833			

Factor B, C, and A*C are significant. Pairwise comparison can be conducted.

Residual are okay.

→ (−, +, +) is optimal.