해석학 1 (타전공 학생용) 과제3

1.

① Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, evaluate $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$; **cite theorems used**

② Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be two convergent series with **non-negative terms**.

Prove that $\sum_{n=0}^{\infty} a_n b_n$ converges

2. Let $\sum_{n=0}^{\infty} a_n$ be a convergent series having the sum S. Let $\sum_{k=0}^{\infty} b_k$ be a new series, whose terms are formed by **grouping** the terms of $\sum_{n=0}^{\infty} a_n$ **in pairs**, and adding them. That is,

$$b_0 = a_0 + a_1, \quad b_1 = a_2 + a_3, \ \cdots, \ b_k = a_{2k} + a_{2k+1}, \ \cdots$$

Prove that $\sum_{k=0}^{\infty} b_k$ also converges, and to the same limit S

Hint: Use subsequence theorem

3. Test each of the following series for **convergence**

① $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ ② $\sum_{2}^{\infty} \frac{1}{n(\ln n)^p}$ $(p > 0)$

4. Test (each of the following series) for **conditional convergence**

① $\sum_{n=1}^{\infty} \frac{(-1)^n}{\tan^{-1} n}$ ② $\sum_{2}^{\infty} (-1)^n \frac{n^5}{2^n}$

5. Prove that if $| a_{n+1} / a_n | \leq | b_{n+1} / b_n |$, for $n \gg 1$, and $\sum_{n=0}^{\infty} b_n$ is absolutely convergent, then $\sum_{n=0}^{\infty} a_n$ is absolutely convergent.

6. Prove that if $| a_{n+1} / a_n | \leq r < 1$, for $n \gg 1$, then $\sum_{n=0}^{\infty} a_n$ converges