

2.7 Problems

$$2.1 \quad SE = \frac{s.d.}{\sqrt{n}} = \frac{3.12}{3} = 1.04$$

$$\text{Variance} = (s.d.)^2 = 9.7344$$

$$2.2 \quad \text{Mean} = \frac{\text{Sum}}{N} = \frac{399.851}{16} = 24.99$$

$$\text{Std. Dev.} = \sqrt{n} (SE \text{ Mean}) = 4 (0.159) = 0.636$$

2.3

Chapter 3

$$3.55 \quad a) \quad F_0 = 5.535 \Rightarrow P(F > F_0) = 0.0036$$

$$H_0: \mu_i = \mu_j \text{ for all}$$

$$H_a: \mu_i \neq \mu_j \text{ for at least one}$$

$$\alpha = 0.05 < P(F > F_0) = 0.0036, \text{ so we can reject the null hypothesis}$$

$$b) \quad E(MS_E) = \sigma^2$$

$$MS_E = \frac{SS_E}{N-a} = \frac{0.0876}{20} = 0.00438$$

$$E(MS_{\text{trt}}) = \sigma^2 + n \sigma_\tau^2$$

$$= 0.024244$$

$$\sigma_\tau^2 = \frac{MS_{\text{trt}} - MS_E}{n}$$

$$= 0.0139728$$

$$c) \quad L = \frac{1}{n} \left(\frac{MS_{\text{trt}}}{MS_E} \cdot \frac{1}{F_{\alpha/2, a-1, N-a}} - 1 \right)$$

$$= \frac{1}{5} \left(\frac{5.535}{3.51} - 1 \right)$$

$$= 0.1154$$

$$U = \frac{1}{n} \left(\frac{MS_{\text{trt}}}{MS_E} \cdot \frac{1}{F_{1-\alpha/2, a-1, N-a}} - 1 \right)$$

$$= \frac{1}{5} \left(\frac{5.535}{5.8} - 1 \right)$$

$$= -0.0091$$

$$95\% \text{ confidence interval for } \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma^2} : \frac{L}{1+L} \leq \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma^2} \leq \frac{U}{1+U}$$

$$0.1566 \leq$$

3.54

a)

This is a random effects experiment because we are randomly pooling looms, which is assumed to be the source of variation.

$$SS_{\tau\tau} = 0.0576, \quad df_{\tau\tau} = 4$$

$$\Rightarrow MS_{\tau\tau} = \frac{0.0576}{4} = 0.0144$$

$$SS_E = 0.58, \quad df_E = 20$$

$$\Rightarrow MS_E = \frac{0.58}{20} = 0.029$$

$$F_o = \frac{MS_{\tau\tau}}{MS_E} = 0.497$$

$$P(F > F_o) = 0.738$$

\therefore no rejection

$$b) \quad \sigma_\tau^2 = \frac{MS_{\tau\tau} - MS_E}{n}$$

Method of moment procedure give a negative σ_τ^2 , so the estimated σ_τ^2 is 0

$$c) \quad MS_E = \sigma^2 = 0.029$$

3.1

$$F_{0.025, 3, 5} = 7.76$$

$$F_{0.975, 3, 5} = \frac{1}{F_{0.025, 5, 3}} = \frac{1}{14.88} = 0.0672$$

$F_o = 3.26$ is within the 95% CI, so we cannot reject the null hypothesis

3.2

$$\alpha = 6$$

$$n = 3$$

3.3

$$MS_{\tau\tau} = \frac{SS_{\tau\tau}}{df_{\tau\tau}} = \frac{36.15}{3} = 12.05$$

$$SS_E = SS_r - SS_{\tau\tau} = 159.89$$

$$df_E = 19 - 3 = 16$$

$$MS_E = \frac{159.89}{16} = 9.993125$$

$$F_o = \frac{12.05}{9.99} = 1.21$$