Midterm

Name:

1. Suppose X_1, X_2 are independent discrete random variables with probability mass function for X_i , i = 1, 2, given by

$$\begin{array}{c|c|c|c|c} x & 0 & 1 & 2 \\ \hline p(x;\theta) & e^{-\theta} & \theta e^{-\theta} & 1 - e^{-\theta} - \theta e^{-\theta} \end{array}$$

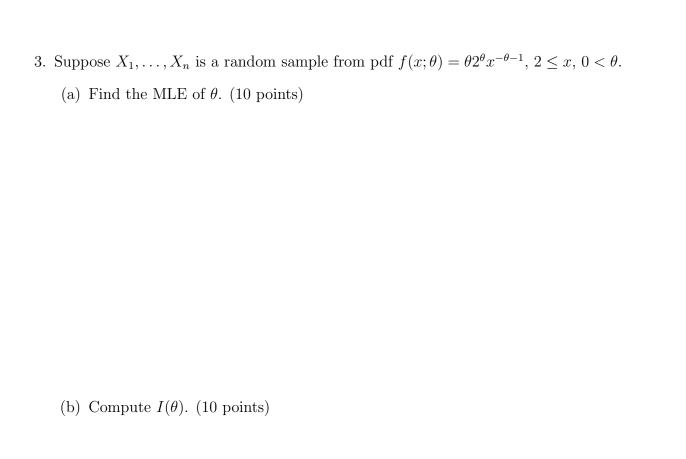
with $0 < \theta$. Show that $X_1 + X_2$ is not a sufficient statistic for θ . (10 points)

2. Suppose that X_1 and X_2 are iid random random variables with pdf

$$f(x; \lambda) = 3\lambda^3 x^2 e^{-\lambda^3 x^3}, \quad x > 0, \quad \lambda > 0.$$

(a) Find a sufficient statistic for λ . (10 points)

(b) If we observed $X_1=3$ and $X_2=1$, what is the MLE of λ ? (10 points)



(c) Find the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$, where $\hat{\theta}$ is the MLE of θ . (10 points)

4. Suppose X_1, \ldots, X_n is a random sample from $Beta(1, \theta), \theta > 0$. Define

$$\hat{\theta}_1 = -\frac{n}{\sum_{i=1}^n \log(1 - X_i)}$$
 and $\hat{\theta}_2 = \frac{1}{1 - \bar{X}}$.

Note that pdf of X_i is $f(x;\theta) = \theta(1-x)^{\theta-1}$, 0 < x < 1.

(a) Find asymptotic distributions of $\sqrt{n}(\hat{\theta}_1 - \theta)$ and $\sqrt{n}(\hat{\theta}_2 - \theta)$. (10 points)

(b) Compute asymptotic relative efficiency (ARE) of $\hat{\theta}_1$ to $\hat{\theta}_2$. (10 points)

5. Suppose X_1, \ldots, X_n is a random sample from the distribution having the pdf

$$f(x; \theta) = 2x/\theta^2, \quad 0 < x \le \theta, \quad -\infty < \theta < \infty.$$

(a) Find the MLE of η , where η is the median of the distribution. (10 points)

(b) Show that the MLE $\hat{\eta}$ is a consistent estimator for η . (10 points)