Final

If you use some known property that we have studied, you should clearly mention which theorem or property you use. For example, if you need to use the property of Exponential family, you should first show that the density is in exponential familty, and explain what kind of property you used.

1. Suppose that $X_1, \ldots, X_n$ is a random sample from pdf $f(x;\theta) = e^{-(x-\theta)}$, $x \geq \theta$.

   (a) Show that the $Y_1 = \min(X_1, \ldots, X_n)$ is a complete and sufficient statistic for θ.

   (b) Find the MVUE of θ^2.

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from population with probability density function:

$$f(x;\theta) = \begin{cases} \frac{2x}{\theta} e^{-x^2/\theta} & x > 0 \\ 0 & elsewhere \end{cases}$$

   where $\theta > 0$. Find the MVUE of θ.

3. Suppose X_1 and X_2 are iid sample from

$$p(x;\beta) = \frac{\Gamma(2+x)}{\Gamma(2)x!\beta^2} \left(\frac{\beta}{\beta+1}\right)^{x+2}, \quad x = 0, 1, 2, 3, \cdots. \quad \beta > 0$$

   (a) Find the UMP test for $H_0 : \beta = 1$ versus $H_1 : \beta < 1$ with significant level $\alpha = 11/32$.

   (b) Compute the power of the UMP test for alternative $H_1 : \beta = 1/2$.

4. Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu_1, \theta_1)$ and that $Y_1, \ldots, Y_m \overset{iid}{\sim} N(\mu_2, 3\theta_2)$. Assume further that X_i and Y_j are independent for any i and j. Construct the LRT of size α for testing $H_0 : \theta_1 = \theta_2$ against $H_1 : \theta_1 \neq \theta_2$.

5. Suppose that $X_1, \ldots, X_n$ is a random sample from pdf $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$. Consider testing $H_0 : \theta = 2$ versus $H_1 : \theta \neq 2$.

   (a) Construct the LRT of size α.

   (b) Construct asymptotic versions of LRT, Score and Wald tests of size α.

6. Suppose that $p(x)$ and $q(x)$ are probability densities that are strictly positive and continuous on the interval $[0, 1]$. Let $X_1, \cdots, X_n$ be a random sample of size n from the pdf

$$f(x;\theta) = c(\theta)\{p(x)\}^{\theta}\{q(x)\}^{1-\theta}, \quad 0 \leq x \leq 1, \quad 0 \leq \theta \leq 1$$

   where

$$c(\theta) = \left[\int_0^1 \{p(x)\}^{\theta}\{q(x)\}^{1-\theta} dx\right]^{-1}.$$

   (a) Determine the family of uniformly most powerful critical region for testing the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : 0 < \theta < \theta_0$, for some given $\theta_0 \in (0, 1)$.

   (b) When $p(x) = 2x$ and $q(x) = 1$, $0 \leq x \leq 1$, $n = 1$, find the uniformly most powerful test of size α for $H_0 : \theta = 1/2$ versus $H_1 : 0 < \theta < 1/2$.