Chance Variation : inherent errors that we cannot control

Assignable Cause : errors caused that we can control

13.2 Control Charts for Average Values : The $\bar{X}$ Control Chart

- When the process is in control, the successive items produced have measurable characteristics that are independent, normal random variables with mean μ and variance σ^2
- if the process is in control throughout the production of subgroup i, then $\frac{(\bar{X}-\mu)}{\sigma/\sqrt{n}}$ has a standard normal distribution. * a standard normal variable Z will almost always be between -3 and $+3$. $\underbrace{\mu-\frac{3\sigma}{\sqrt{n}}}_{\text{lower control limits}} < \bar{X}_i < \underbrace{\mu+\frac{3\sigma}{\sqrt{n}}}_{\text{upper control limits}}$

- Suppose the process has gone out of contol by a chance that a shift has occurred in mean, $\mu \to \mu+\alpha$

$-3 < \sqrt{n}\frac{(\bar{X}-\mu)}{\sigma} < 3 \to -3-\frac{a\sqrt{n}}{\sigma} < \sqrt{n}\frac{(\bar{X}-\mu-\alpha)}{\sigma} < 3-\frac{a\sqrt{n}}{\sigma}$, hence, since $\bar{X}$ is normal with mean $\mu+\alpha$ and variance $\frac{\sigma^2}{n}$, $\sqrt{n}\frac{(\bar{X}-\mu-\alpha)}{\sigma}$ has a standard normal distribution.

$\Rightarrow P\left(-3-\frac{a\sqrt{n}}{\sigma} < Z < 3-\frac{a\sqrt{n}}{\sigma}\right) = \Phi\left(3-\frac{a\sqrt{n}}{\sigma}\right) - \Phi\left(-3-\frac{a\sqrt{n}}{\sigma}\right) \approx \Phi\left(3-\frac{a\sqrt{n}}{\sigma}\right)$, so the probability that it falls outside is approximately $1-\Phi\left(3-\frac{a\sqrt{n}}{\sigma}\right)$.

* the number of subgroups that will be needed to detect the shift has a geometric distribution with mean $\left[1-\Phi\left(3-\frac{a\sqrt{n}}{\sigma}\right)\right]^{-1}$

13.2.1 Case of Unknown μ and σ

- the unbiased estimator of μ is $\bar{\bar{X}}$, the average of $\bar{X}_i$'s.
- the unbiased estimator of σ is $\frac{E(S_1)}{c(n)}$, where $E(S_1) = \frac{\sqrt{2}\,\Gamma(\frac{n}{2})\sigma}{\sqrt{n-1}\,\Gamma(\frac{n-1}{2})}$, $c(n) = \frac{\sqrt{2}\,\Gamma(\frac{n}{2})}{\sqrt{n-1}\,\Gamma(\frac{n-1}{2})}$

$\Rightarrow$ Lower Control Limits $= \bar{\bar{X}} - \frac{3\bar{S}}{c(n)\sqrt{n}}$, Upper Control Limits $= \bar{\bar{X}} + \frac{3\bar{S}}{c(n)\sqrt{n}}$, and check if each of $\bar{X}_i$'s falls within these lower and upper limits.

13.3 S-Control Charts

- remember, $E(S_i) = c(n)\sigma$.

$$\Rightarrow Var(S_i) = E(S_i^2) - \{E(S_i)\}^2$$
$$= \sigma^2 - \{c(n)\}^2\sigma^2$$
$$= \sigma^2\{1-(c(n))^2\}$$

$$\Rightarrow P\{E(S_i) - 3\sqrt{Var(S_i)} < S_i < E(S_i) + 3\sqrt{Var(S_i)}\} = 0.99$$

$$\Rightarrow UCL = \sigma\left[c(n) + 3\sqrt{1-c^2(n)}\right]$$

$$LCL = \sigma\left[c(n) - 3\sqrt{1-c^2(n)}\right]$$, but if σ is unknown, it can be estimated from $\frac{\bar{S}}{c(n)}$

$$UCL = \bar{S}\left[1 + 3\sqrt{1/c^2(n) - 1}\right]$$
$$LCL = \bar{S}\left[1 - 3\sqrt{1/c^2(n) - 1}\right]$$

13.4 Control Charts For the Fraction Defective

- Let X denote the number of defective items in a subgroup of n items, then $X \sim binom(n, p)$. If $F = \frac{X}{n}$,

$\Rightarrow E(F) = \frac{E(X)}{n} = \frac{np}{n} = p$

$\Rightarrow \sqrt{Var(F)} = \sqrt{\frac{Var(X)}{n^2}} = \sqrt{\frac{np(1-p)}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$

$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$, $LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$

to start such a control chart, it is necessary to estimate p.

$\bar{F} = \frac{1}{K}\sum_{i=1}^{K} F_i$, $Var(\bar{F}) = \sqrt{\frac{\bar{F}(1-\bar{F})}{n}}$

$\Rightarrow UCL = \bar{F} + 3\sqrt{\frac{\bar{F}(1-\bar{F})}{n}}$, $LCL = p - 3\sqrt{\frac{\bar{F}(1-\bar{F})}{n}}$

13.5 Control Charts For Number of Defects

- Let X be a poisson random variable, then $E(X_i) = \lambda = Var(X_i) \Rightarrow UCL = \lambda + 3\sqrt{\lambda}$, $LCL = \lambda - 3\sqrt{\lambda}$

$\Rightarrow \bar{X} = \frac{1}{K}\sum_{i=1}^{K} X_i = Var(\bar{X})$, $\Rightarrow UCL = \bar{X} + 3\sqrt{\bar{X}}$, $LCL = \bar{X} - 3\sqrt{\bar{X}}$

...?

13.6 Other Control Charts for Detecting Changes in the Population Mean