# Quiz 6

#1. Suppose that $X_1, \dots, X_n$ is a random sample from a distribution having pdf of form $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$,

zero elsewhere. For testing $H_0: \theta = 1$ against $H_1: \theta = 2$, find the best critical region.

$\rightarrow \quad L(\theta; x) = \prod_{i=1}^{n} \theta x_i^{\theta-1}$

$$\frac{L(\theta_0; x)}{L(\theta_1; x)} = \frac{\prod_{i=1}^{n} 1 \cdot x_i^{1-1}}{\prod_{i=1}^{n} 2 \cdot x_i^{2-1}} = \frac{1}{2^n \prod_{i=1}^{n} x_i} \le k$$

$$\Rightarrow \frac{1}{2^n \prod_{i=1}^{n} x_i} \le k$$

$$\Rightarrow \prod_{i=1}^{n} x_i \ge \frac{k}{2^n} = c \quad \text{where } c \text{ is some constant.}$$

$\therefore$ The best critical region is $\prod_{i=1}^{n} X_i \ge c$ (for some constant $c$).

#2. Let the random variable $X$ have the pdf $f(x;\theta) = \theta e^{-\theta x}$, $x > 0$.

For testing $H_0: \theta = 1$ against $H_1: \theta = 2$, find the critical region based on a random sample $X_1, X_2$ of size 2 from $f(x;\theta)$.

$\rightarrow \quad L(\theta; x) = \theta^2 e^{-\theta(x_1+x_2)}$

$$\frac{L(\theta_0; x)}{L(\theta_1; x)} = \frac{1^2 e^{-(x_1+x_2)}}{2^2 e^{-2(x_1+x_2)}} = \frac{1}{4} e^{x_1+x_2} \le k$$

$$\Rightarrow \frac{1}{4} e^{x_1+x_2} \le k$$

$$\Rightarrow e^{x_1+x_2} \le 4k$$

$$\Rightarrow x_1 + x_2 \le \log(4k) = c \quad \text{(for some constant } c\text{)}.$$

$\therefore$ The best critical region for random sample $X_1, X_2$ is $X_1 + X_2 \le c$ (for some constant $c$).