8.1 Arc Length

- We are going to define the length of a general curve by first approximating it by a polygon and then taking a limit as the number of segments of the polygon is increased

$$\Rightarrow L = \lim_{n\to\infty} \sum_{i=1}^{n} |P_{i-1}P_i| \quad , \quad |P_{i-1}P_i| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$\approx \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} \quad , \quad \Delta y_i = f(x_i) - f(x_{i-1}) = f'(x_i^*)(x_i - x_{i-1}) = f'(x_i^*)\Delta x$$

$$\Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (f'(x_i^*)\Delta x)^2} = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2(1 + f'(x_i^*)^2)} = \lim_{n\to\infty} \sum_{i=1}^{n} [1 + f'(x_i^*)^2]\Delta x_i$$

$$\approx L = \int_a^b \sqrt{1 + (f(x_i^*))^2}\, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

- Because of the presence of the square root sign, the calculation of an arc length often leads to an integral that is very difficult or even impossible to evaluate explicitly. Thus we sometimes have to be content with finding an approximation to the length of a curve

  Endpoints, Midpoints, Simpson's ... etc

- Suppose $S(x)$ is arc length function $S(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$, then the differential of $S(x)$ is the rate of change of S with respect to x,

$$\Rightarrow \frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ , and it is always at least } 1$$

$$\Rightarrow ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\Rightarrow (ds)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right](dx)^2 = (dx)^2 + (dy)^2$$

8.2 Area of a Surface of Revolution

Surface Area of a circular Cylinder:

$$A = 2\pi r h$$

Surface Area of a cone:

$$A = \frac{1}{2} l^2 \theta = \frac{1}{2} l^2 \left(\frac{2\pi r}{l}\right) = \pi r l$$

Surface Area of Bands:

$$A = \pi r_2 (l_1 + l) - \pi r_1 l_1 = \pi[(r_2 - r_1) l_1 + r_2 l]$$

$$= 2\pi r l \quad , \text{ where } r = \frac{1}{2}(r_1 + r_2)$$

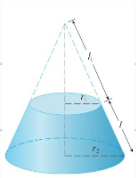

FIGURE 3

$\Rightarrow$ Surface Area

$$S = \int_a^b 2\pi \cdot f(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx$$

- remember $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$, then

$\therefore\ S = \int \underbrace{2\pi f(x)}\, ds$

this can be thought as the circumference of a circle traced out by the point (x, y) on the curve as it is rotated

8.3 Applications to Physics and Engineering

Hydrostatic | sure and Force :

$\Rightarrow \underset{\text{Force}}{\underline{F}} = \underset{\text{Mass}}{\underline{m}}\ \underset{\text{Acceleration}}{\underline{a}}$

$\approx F = m \underset{\text{acceleration due to gravity}}{\underline{g}}$

$= \rho g A d$, ρ = volume , A = area , d = depth of water

$\Rightarrow \underset{\text{Pressure}}{\underline{P}} = \frac{F}{A} = \rho g d$

$F =$