Homework IV (2022)

1. When $J = 3$, suppose that

$$\pi_j(x) = \exp(\alpha_j + \beta_j x) \,/\, [1 + \exp(\alpha_1 + \beta_1 x) + \exp(\alpha_2 + \beta_2 x)]$$

for $j = 1, 2$. Show that $\pi_3(x)$ is (a) decreasing in x if $\beta_1 > 0$ and $\beta_2 > 0$, (b) increasing in x if $\beta_1 < 0$ and $\beta_2 < 0$, and (c) nonmonotone when β_1 and β_2 have different signs.

2. The following table results from a clinical trial for the treatment of small-cell lung cancer. Patients were randomly assigned to two treatment groups. The sequential therapy administered the same combination of chemotherapeutic agents in each treatment cycle. The alternating therapy used three different combinations, alternating from cycle to cycle.

		Response to Chemotherapy			
		Progressive	No	Partial	Complete
Therapy	Gender	Disease	Change	Remission	Remission
Sequential	Male	28	45	29	26
	Female	4	12	5	2
Alternating	Male	41	44	20	20
	Female	12	7	3	1

Source: Holtbrugge, W. and Schumacher, M. (1991). *Applied Statistics*, **40**: 249-259

   (a) Fit a cumulative logit model with main effects for treatment and gender. Interpret the estimated treatment effect.

   (b) Fit the model that also contains an interaction term between treatment and gender. Interpret the interaction term by showing how the estimated treatment effect varies by gender.

   (c) Does the interaction model give a significantly better fit?

3. Assume $n_{ijk} \sim^{indep.} \text{Poisson}(\mu_{ijk})$ for $i = 1, \ldots, I$; $j = 1, \ldots, J$; $k = 1, \ldots, K$. Then we have the following likelihood function:

$$L(\mu; n) = \prod_{i,j,k} \left(\frac{e^{-\mu_{ijk}} \mu_{ijk}^{n_{ijk}}}{n_{ijk}!} \right).$$

Using the likelihood, find the sufficient statistics for the parameters in the following loglinear models:

(a) (X, Y, Z).

(b) (XY, Z).

(c) (XY, XZ, YZ).

4. Consider the following record of automobile accident from the Florida in 1988.

Safety equipment in use	Whether ejected	Injury	
		Nonfatal	Fatal
Seat Belt	Yes	1,105	14
	No	411,111	483
None	Yes	4,624	497
	No	157,342	1,008

(a) Fit a loglinear model which includes the safety equipment, ejected and result of injury (nonfatal, fatal). Do not include interactions in your model. Interpret this model.

(b) Rather than using counts, fit a logistic model for the probability of fatality in terms of the safety equipment and ejected variables. Do not include interactions in this model. Interpret this model.

(c) How might you compare these two models?