Goodness of Fit Test : statistical tests that determine whether a given probabilistic mechanism is appropriate

11.2 Goodness of Fit Tests When All Parameters Are Specified

- Let $H_0 : P(Y=i) = P_i \ , \quad i = 1, \dots, k$

  $H_a : P(Y=i) \neq P_i \ , \quad$ for some i , and let X_i denote the number of the Y_i's that equal i.

  $\Rightarrow E(X_i) = nP_i$, and we use those to calculate T to determine the approval or rejection.

  $\Rightarrow T = \sum_{i=1}^{k} \frac{(X_i - nP_i)^2}{nP_i} \sim \chi^2_{\alpha, k-1}$ as $n \to \infty$

An accepted rule of thumb as to how large n need be for the foregoing to be a good approximation is that it should be large enough so that $np_i \geq 1$ for each $i, i = 1, \dots, k$, and also at least 80 percent of the values np_i should exceed 5.

REMARKS

(a) A computationally simpler formula for T can be obtained by expanding the square in Equation 11.2.1 and using the results that $\sum_i p_i = 1$ and $\sum_i X_i = n$ (why is this true?):

$$T = \sum_{i=1}^{k} \frac{X_i^2 - 2np_iX_i + n^2p_i^2}{np_i} \tag{11.2.2}$$
$$= \sum_i X_i^2/np_i - 2\sum_i X_i + n\sum_i p_i$$
$$= \sum_i X_i^2/np_i - n$$

(b) The intuitive reason why T, which depends on the k values $X_1, \dots, X_k$, has only $k-1$ degrees of freedom is that 1 degree of freedom is lost because of the linear relationship $\sum_i X_i = n$.

(c) Whereas the proof that, asymptotically, T has a chi-square distribution is advanced, it can be easily shown when $k = 2$. In this case, since $X_1 + X_2 = n$, and $p_1 + p_2 = 1$, we see that

$$T = \frac{(X_1 - np_1)^2}{np_1} + \frac{(X_2 - np_2)^2}{np_2}$$
$$= \frac{(X_1 - np_1)^2}{np_1} + \frac{(n - X_1 - n[1-p_1])^2}{n(1-p_1)}$$
$$= \frac{(X_1 - np_1)^2}{np_1} + \frac{(X_1 - np_1)^2}{n(1-p_1)}$$
$$= \frac{(X_1 - np_1)^2}{np_1(1-p_1)} \quad \text{since} \quad \frac{1}{p} + \frac{1}{1-p} = \frac{1}{p(1-p)}$$

However, X_1 is a binomial random variable with mean np_1 and variance $np_1(1-p_1)$ and thus, by the normal approximation to the binomial, it follows that $(X_1 - np_1)/\sqrt{np_1(1-p_1)}$ has, for large n, approximately a standard normal distribution, and so its square has approximately a chi-square distribution with 1 degree of freedom.

* Determining hypotheses after observing data decreases the credibility of the study

11.2.1 Determing the Critical Region By Simulation

11.3 Goodness of Fit Tests When Some Parameters Are Unspecified

11.4 Tests of Independence in Contingency Tables