해석학 2 (타전공 학생용) 과제1

1. Find the second-order Taylor polynomial $T_2(x)$ for $f(x) = e^{-x}$ at the point $x = 0$

2. Find the third-order Taylor polynomial $T_3(x)$ for $f(x) = \dfrac{1}{\sqrt{1+x}}$ at the point $x = 0$

3. Prove the Taylor series at 0 of the following functions converges to the function for all x, or for the indicated values of x:

   (a) $\sin x$ (b) $\dfrac{1}{1-x}, \quad x \in (-1, 0]$ (c) $\ln(1+x), \quad x \in [0, 1)$

4. A polynomial $P(x)$ is said to have a k-fold zero at the point a if it has $(x-a)^k$ as a factor, but not $(x-a)^{k+1}$, i.e.,

   $P(x) = (x-a)^k Q(x)$, where $Q(x)$ is a polynomial, $Q(a) \neq 0$.

   (a) Prove: the point a is a k-fold zero for the polynomial $P(x) \Leftrightarrow$

   $$P(a) = P'(a) = \cdots = P^{(k-1)}(a) = 0, \quad P^{(k)}(a) \neq 0$$

   (b) For what value(s) of the constant b will $2x^3 - bx^2 + 1$ have a double zero at some point (i.e., a 2-fold zero)?