2. $\boxed{1}$ $P(X > \max(Y_1, Y_2))$

$$= P(X > \max(Y_1, Y_2) \cap Y_1 > Y_2) + P(X > \max(Y_1, Y_2) \cap Y_1 < Y_2)$$

$$= P(X > Y_1 > Y_2) + P(X > Y_2 > Y_1)$$

$$= \frac{\mu}{2\mu + \lambda} \cdot \frac{\mu}{\mu + \lambda} + \frac{\mu}{2\mu + \lambda} \frac{\mu}{\mu + \lambda} = \frac{2\mu^2}{(2\mu + \lambda)(\mu + \lambda)}$$

Using the fact

$$P(X_1 < X_2 < X_3) = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_2}{\lambda_2 + \lambda_3}$$

when $X_i \sim Exp(\lambda_i)$

$\boxed{2}$ Or, by conditioning on X

$$\int_0^\infty P(\max(Y_1, Y_2) < x \mid X = x)\, \lambda e^{-\lambda x} dx$$

$$= \int_0^\infty P(Y_1 < x, Y_2 < x)\, \lambda e^{-\lambda x} dx$$

$$= \int_0^\infty (1 - e^{-\mu x})^2 \lambda e^{-\lambda x} dx$$

$$= \frac{2\mu^2}{(\lambda + \mu)(\lambda + 2\mu)}$$

$\boxed{3}$ However, it is not correct to say

$$P(X > \max(Y_1, Y_2) \mid Y_1 > Y_2)\, P(Y_1 > Y_2)$$

$$= P(X > \max(Y_1, Y_2), Y_1 > Y_2 \mid Y_1 > Y_2)\, P(Y_1 > Y_2)$$

$$= P(X > Y_1 > Y_2 \mid Y_1 > Y_2)\, P(Y_1 > Y_2)$$

$$\neq P(X > Y_1)\, P(Y_1 > Y_2)$$