Quiz 3

 # 1. Let X_1, X_2, X_3 be a random sample from a $N(\theta, 1)$. Suppose that the realized values of X_1, X_2, X_3 are $X_1 = 3$, $X_2 = 1$, $X_3 = 5$.

If θ is restricted by $\theta \le 2$. What is the MLE of θ?

$\rightarrow L(\theta) = \left(\frac{1}{\sqrt{2\pi}}\right)^3 \exp\left[-\frac{1}{2}\{(3-\theta)^2 + (1-\theta)^2 + (5-\theta)^2\}\right]$

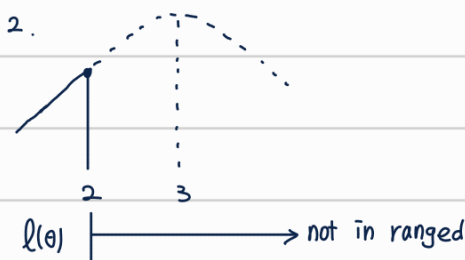$\ell(\theta) = \log L(\theta) = -\frac{3}{2}\log(2\pi) - \frac{1}{2}\{(3-\theta)^2 + (1-\theta)^2 + (5-\theta)^2\}$

We have to find the value that maximizes $\ell(\theta)$.

Since $\ell'(\theta) = \frac{d \log L(\theta)}{d\theta} = -\frac{1}{2}\{-2(3-\theta) - 2(1-\theta) - 2(5-\theta)\} = 9 - 3\theta$

and $\ell''(\theta) < 0$, $\ell(\theta)$ is maximized when $\hat{\theta} = 2$ if $\theta \le 2$.

Hence, the MLE is $\hat{\theta}^{MLE} = 2$.

$\therefore \hat{\theta}^{MLE} = 2$

(graph) $\ell(\theta)$ with marks at 2 and 3, $\rightarrow$ not in ranged

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 # 2. Let X and Y be a random sample from $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$.

If the observed value of X and Y are $X = e^{-3}$ and $Y = e^{-2}$.

What is the MLE of θ?

$\rightarrow L(\theta) = \theta^2 e^{-5(\theta-1)}$

$\ell(\theta) = \log L(\theta) = 2\log\theta - 5(\theta-1)$

$\ell'(\theta) = \frac{d \log L(\theta)}{d\theta} = \frac{2}{\theta} - 5 = 0$

$\Rightarrow \frac{2}{\theta} = 5$

$\Rightarrow \hat{\theta} = \frac{2}{5}$

$\therefore \hat{\theta}^{MLE} = \frac{2}{5}$