8.2 Significance Levels

- the objective of a statistical test of H_0 is not to explicitly determine whether or not H_0 is true but rather to determine if its validity is consistent with the resultant data

Simple Hypothesis : A hypothesis that completely specifies the population distribution

Type I Error : False Negative

Type II Error : False Positive

8.3 Tests Concerning the Mean of a Normal Population

8.3.1 Case of Known Variance

$P[\text{Type II Error}]$ = Probability of accepting the null hypothesis when the true mean μ is unequal to μ_0

- It depends on the value of μ. Suppose $\underbrace{\beta(\mu)}_{\text{operating characteristic function}} = P_\mu\{\text{acceptance of } H_0\} = P_\mu\left\{-z_{\alpha/2} \le \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \le z_{\alpha/2}\right\}$

$$= \Phi\left\{\frac{\mu_0-\mu}{\sigma/\sqrt{n}} + z_{\alpha/2}\right\} - \Phi\left\{\frac{\mu_0-\mu}{\sigma/\sqrt{n}} - z_{\alpha/2}\right\}$$

☆ The function $1-\beta(\mu)$ is called the power-function of the test. The power of the test is equal to the probability of rejection when μ is the true value. The operating characteristic function is useful in determining how large the random sample need to be to meet certain specifications concerning Type II Errors.

$\Rightarrow \; = \Phi\left\{\frac{\mu_0-\mu}{\sigma/\sqrt{n}} + z_{\alpha/2}\right\} - \Phi\left\{\frac{\mu_0-\mu}{\sigma/\sqrt{n}} - z_{\alpha/2}\right\} \approx \beta$, from here, suppose $\mu_1 > \mu_0$. Then the equation implies

$$\frac{\sqrt{n}(\mu_0-\mu_1)}{\sigma} - z_{\alpha/2} \le -z_{\alpha/2}$$

$\Rightarrow \Phi\left\{\frac{\sqrt{n}(\mu_0-\mu_1)}{\sigma} - z_{\alpha/2}\right\} \le \Phi\{-z_{\alpha/2}\} = P(Z \le -z_{\alpha/2}) = P(Z \ge z_{\alpha/2}) = \alpha/2$, so we can assume

$\Rightarrow \Phi\left\{\frac{\sqrt{n}(\mu_0-\mu_1)}{\sigma} - z_{\alpha/2}\right\} \approx 0$, then it only remains,

$\Phi\left\{\frac{\mu_0-\mu}{\sigma/\sqrt{n}} + z_{\alpha/2}\right\} \approx \beta$, from here, since $\beta = P[Z > z_\beta] = P[Z < -z_\beta] = \Phi(-z_\beta)$, and $-z_\beta$ can be assumed

$\Rightarrow -z_\beta \approx (\mu_0-\mu_1)\frac{\sqrt{n}}{\sigma} + z_{\alpha/2}$, so

$\Rightarrow n \approx \frac{(z_{\alpha/2}+z_\beta)^2\sigma^2}{(\mu_1-\mu_0)^2}$

☆☆ the same approximation would result when $\mu_1 - \mu_0$

EXAMPLE 8.3d For the problem of Example 8.3a, how many signals need be sent so that the .05 level test of $H_0 : \mu = 8$ has at least a 75 percent probability of rejection when $\mu = 9.2$?

SOLUTION Since $z_{.025} = 1.96$, $z_{.25} = .67$, the approximation 8.3.7 yields

$$n \approx \frac{(1.96 + .67)^2}{(1.2)^2}4 = 19.21$$

Hence a sample of size 20 is needed. From Equation 8.3.4, we see that with $n = 20$

$$\beta(9.2) = \Phi\left(-\frac{1.2\sqrt{20}}{2} + 1.96\right) - \Phi\left(-\frac{1.2\sqrt{20}}{2} - 1.96\right)$$
$$= \Phi(-.723) - \Phi(-4.643)$$
$$\approx 1 - \Phi(.723)$$
$$\approx .235$$

Therefore, if the message is sent 20 times, then there is a 76.5 percent chance that the null hypothesis $\mu = 8$ will be rejected when the true mean is 9.2. ■

8.3.1.1 One-Sided Tests

8.3.2 Case of Unknown Variance: The t-Test

8.4 Testing the Equality of Means of Two Normal Populations

8.4.1 Case of Known Variances

8.4.2 Case of Unknown Variances

8.4.3 Case of Unknown and Unequal Variances

8.4.4 The Paired t-Test

8.5 Hypothesis Tests Concerning the Variance of a Normal Population

8.5.1 Testing for the Equality of Variances of Two Normal Populations

8.6 Hypothesis Tests in Bernoulli Populations

8.6.1 Testing the Equality of Parameters in Two Bernoulli Populations

Fisher-Irwin Test:

- let $P(X)$ be a pmf of hypergeometric distribution, then

  $p\text{-value} = 2 \cdot \min\left[P(X \leq x_1),\ P(X \geq x_1)\right]$

8.7 Tests Concerning the Mean of a Poisson Distribution

$p\text{-value} = 2 \cdot \min\left(P_{\lambda_0}\{X \geq x\},\ P_{\lambda_0}\{X \leq x\}\right)$