(a)
$$P(\forall) = P(0=1)P(\forall |0=1) + P(0=2)P(\forall |0=2)$$

= 0.5 $N(\forall |1, 2^2) + 0.5 N(\forall |2, 2^2)$
See R code

(b)
$$P(0=|y=1) = \frac{P(0=1,y=1)}{P(0=1,y=1) + P(0=2,y=1)}$$

$$= \frac{P(0=1)P(y=1|0=1)}{P(0=1)P(y=1|0=1) + P(0=2)P(y=1|0=2)}$$

$$= \frac{0.5N(1|1,2^{2})}{0.5N(1|1,2^{2}) + 0.5N(1|2,2^{2})}$$

$$= 0.53$$

(1) As
$$\sigma \rightarrow \infty$$
, the posterior density for approaches

the prior (the data contain no information):

$$P(0=|J=1) = \frac{0.5 N(1|1,\sigma^2)}{0.5 N(1|1,\sigma^2)} + 0.5 N(1|2,\sigma^2)$$

$$\longrightarrow \frac{1}{2} (G \rightarrow \infty)$$

As $\sigma \rightarrow 0$, the posterior density for 0 becames concentrated at 1:

$$P(\theta=1|Y=1) \longrightarrow 1$$

Use
$$P(GG) = P(M)P(GG|M) + P(D)P(GG|D)$$

$$= P(M)P + (1-P(M))P^{2}$$

$$\Rightarrow P(M) = \frac{P(GG) - P^{2}}{P(1-P)}$$

(a) $\times \sim \text{MegBin}(r, 0)$ Let $0 \sim \text{Beta}(d, \beta)$. Then $P(0|X) \propto O^{r}(1-0)^{X} \times O^{d-1}(1-0)^{\beta-1}$ $= O^{r+d-1}(1-0)^{X+\beta-1}$

which is Beta (rtd, xtb)

Thus, the conjugate prior for θ in NegBin(r, θ) is β eta(α , β).

(b) X~ Gamma (d,B), d is known Let B~ Gamma(a,b).

Then $P(\beta|X) < \beta^{X} e^{-\beta X} \times \beta^{A-1} e^{-b\beta}$ $= \beta^{A+A-1} - (x+b)\beta$ $= \beta^{A+A-1} e^{-(x+b)\beta}$ Which is Gammal d+a, >(t+b).

(() X ~ Gamma(<,B), B is known. In this case, we cannot write the distribution of a So none is available.

4. $O \sim \beta \operatorname{eta}(d, \beta)$ with E(0) = 0.6

$$Var(0) = 0.3^{2}$$

(a)
$$E(0) = \frac{d}{d+\beta} = 0.6$$

$$Var(0) = \frac{d\beta}{(d+\beta)^{2}(d+\beta+1)} = 0/3^{2} - 0$$

$$\frac{d\beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)} = \frac{\alpha}{\alpha+\beta} \frac{\beta}{(\alpha+\beta)} \frac{1}{\alpha+\beta+1}$$

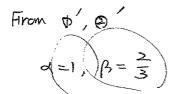
$$= \left(\frac{\alpha}{\alpha + \beta}\right) \left(1 - \frac{\alpha}{\alpha + \beta}\right) \left(\frac{1}{\alpha + \beta + 1}\right)$$

$$= (0.6)(1-0.6)(\frac{1}{\alpha+\beta+1}) = 0.09$$

$$\Rightarrow \alpha + \beta + 1 = \frac{24}{9}$$

$$\Rightarrow \alpha + \beta = \frac{5}{3} - \Theta'$$

$$\Rightarrow \alpha = \frac{3}{2}\beta - 0'$$



Thus,
$$0 \sim \beta eta(\alpha = 1, \beta = \frac{2}{3})$$
.

(b).
$$\forall 1, \dots, \forall 1000 \stackrel{\text{iid}}{\sim} \text{Bernoulli}(0).$$

$$\Rightarrow \overline{y} = 0.65.$$

$$\theta \sim \beta eta(1, \frac{2}{3})$$

$$P(0|y_{1},...,y_{1000}) \propto 0^{\frac{1000}{3}} (1-0)^{\frac{1000}{3}} \frac{1}{3} \frac{1}{3} \frac{1}{3}$$

$$= 0^{\frac{1000}{3}} \frac{1}{3} \frac{1}$$

which is Beta(651, 350+=) distribution

The posterior mean and variance are given by

$$E(0|Y_1, ---, Y_{1000}) = \frac{651}{651 + 350 + \frac{5}{3}} = 0.6499.$$

$$Var(0|31, \dots, 31000) = \frac{(651)(350+\frac{2}{3})}{(651+350+\frac{2}{3})^{2}(651+350+\frac{2}{3}+1)} = 0.00023$$