Homework I (2021)

Please solve the following problems and then submit the pdf copy of them.

1. In class, we have learned the sample size calculation for two samples. That is, $y_{i1}, \ldots, y_{in} \overset{iid}{\sim} N(\mu_i, \sigma^2)$ for $i = 1, 2$. Show that under $H_0 : \Delta = \Delta_0$ versus $H_1 : \Delta \neq \Delta_0$,
$$n \approx \frac{(z_{\alpha/2} + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2},$$
where $\Delta = \mu_1 - \mu_2$ is the treu difference in means.

2. An experiment is conducted to compare four treatments. Five replicates are obtained per treatment.

Treatment			
1	2	3	4
41	36	103	53
46	7	75	30
56	53	84	70
56	31	82	80
69	76	80	42

   (a) Write out the cell means model,under the independent and normally distributed errors assumption.

   (b) Identify all parameters and clearly state ranges of subscripts.

   (c) Give an unbiased estimate of the common standard deviation σ.

   (d) Give the ANOVA table.

   (e) Test whether the underlying population means differ (show all parts of test, using a significance level of $\alpha = 0.05$).

3. In the single-factor ANOVA model, $y_{ij} = \mu + \tau_i + \epsilon_{ij}$ for $i = 1, \cdots, a$ and $j = 1, \cdots, n$ where $\sum_i \tau_i = 0$ and $\epsilon_{ij} \sim N(0, \sigma^2)$. Then prove the following results:

   (a) $E(MS_{Treatment}) = \sigma^2 + \frac{n}{a-1} \sum_{i=1}^{a} \tau_i^2$.

   (b) $E(MS_E) = \sigma^2$.

   (c) Under $H_0 : \tau_1 = \cdots = \tau_a = 0$, prove that
$$F = MS_{Treatment}/MS_E \sim F_{a-1,a(n-1)}.$$