Chapter 5

Markov's Inequality

Chebyshev's Inequality

Weak Law of Large Numbers

Continuous Mapping Theorem : 어떤 확률변수들이 확률수렴하면, 확률변수들의 선형결합도 확률수렴한다.

Delta Method

Chapter 6

Functional Invariance Property of MLE : 임의의 확률변수의 분포와 MLE를 알면, 해당 분포와 관련된 추정치는 반드시 MLE의 함수이다.

Jensen's Inequality : $\phi(E(X)) \le E(\phi(X))$

Rao-Cramer Lower Bound

Efficiency : $\frac{(nI(\theta))^{-1}}{Var(\hat{\theta})}$

Asymptotical Normality of MLE : $\sqrt{n}(\hat{\theta}-\theta) \xrightarrow{D} N(0, I^{-1}(\theta))$

Asymptotic Efficiency

Chapter 7

Sufficient Statistic : (i) Y is a statistic (ii) $P_\theta[(X_1,\dots,X_n) \in A \mid T(x)=y]$ does not depend on θ for each y and all A.

Factorization Theorem : exponential family가 아닐 경우 sufficient statistic을 알려주고, exponential family일 경우 Complete Sufficient Statistic

Rao-Blackwell Theorem : Rao-Blackwell에 의해, 좋은 함수는 CSS의 함수이어야 하는데, CSS는 유일하다.

Complete Sufficient Statistic

Minimum Variance Unbiased Estimator

Lehmann-Scheffe Theorem : $E(\phi(T))=\theta \Rightarrow \phi(T)$ is the MVUE

Chapter 8

Most Powerful Test

Uniformly Most Powerful Test

Neyman-Pearson Theorem : Δ가 1보다 작을수록 기각하고, 1에 가까울수록 기각하지 않겠다는 의미

Monotone Likelihood Ratio : MLR Property를 가지면 UMP가 존재하고, MP와 동일하다.

Likelihood Ratio Test : $-2\left(l(\theta_0)-l(\hat{\theta})\right) > k$

Wald Test : $\left(\sqrt{nI(\hat{\theta})}(\hat{\theta}-\theta_0)\right)^2 > k$

Score Test : $\left(\frac{l'(\theta_0)}{\sqrt{nI(\theta_0)}}\right)^2 > k$

Asymptotic Likelihood Ratio Test : $-2\log\Delta = -2\left(l(\theta_0)-l(\hat{\theta})\right) \xrightarrow{D} \chi^2(1)$