Homework 2 (STA 3021)

Instructions: Recall Homework policy!

Reading: Chapter 2.

1. Fubini's theorem states that (if integral exists) double integral can be done by iterative integral, and even can change the order of integration. Namely,

$$\int\int_{\mathbb{R}^2} f(x,y)dA = \int_{\mathbb{R}}\left\{\int_{\mathbb{R}} f(x,y)dy\right\}dx = \int_{\mathbb{R}}\left\{\int_{\mathbb{R}} f(x,y)dx\right\}dy$$

   Identify Fubini's theorem by evaluating the following iterative integration

   (a) $\int_0^3\left(\int_1^2 x^2 y dy\right)dx$.

   (b) $\int_1^2\left(\int_0^3 x^2 y dx\right)dy$.

2. The headway X between two vehicles at a fixed instant is a random variable with

$$P(X \le t) = 1 - .6e^{-.02t} - .4e^{-.03t}, \quad t \ge 0.$$

   Find the expected value and the variance of the headway.

3. When the cumulative distribution has jumps, such random variable is called as mixed type. For example, consider the random variable has the cumulative distribution function as

$$F(x) = \begin{cases} 0, & \text{for } x < 1, \\ \frac{x^2-2x+2}{2}, & \text{for } 1 \le x < 2, \\ 1, & \text{for } x \ge 2. \end{cases}$$

   (a) Draw the distribution function and identify where jump occurs.

   (b) Expected value can be calculated as the following. Let S be the set of jump points then

$$E(g(X)) = \int_{\mathbb{R}} g(x)dF(x) = \sum_{i \in S} g(x_i)P(X = x_i) + \int_{\mathbb{R}} g(x)f_C(x)dx,$$

   where $f_C = F'$ at $\mathbb{R} \setminus S$. Find the variance of X.

4. Chapter 2 Exercise #34

5. Chapter 2 Exercise #68