

## 10.1 Intervals . Estimating Functions .

$\delta$ -neighborhood of  $a$  : the expressions below are all equivalent

$$x \in (a - \delta, a + \delta) \quad , \quad a - \delta < x < a + \delta \quad , \quad |x - a| < \delta \quad , \quad x \approx_{\delta} a$$

$$f(I) = \{f(x) : x \in I\}$$

$\Rightarrow f(I)$  : the range of  $f(x)$  over  $I$  , or the image of  $I$  under the mapping  $f$

Completeness Property for Functions :

- Suppose  $f(x)$  is defined on an interval  $I$ . If  $f(x)$  is bounded above on  $I$ , then  $\sup_I f(x)$  exists; if  $f(x)$  is bounded below, then  $\inf_I f(x)$  exists

Estimating Functions : Inequalities and Absolute Values

Error Function :  $\operatorname{erf} x = \int_0^x e^{-\frac{t^2}{2}} dt$

## 10.2 Approximating Functions

- Approximating functions over intervals which are small is often done using the first few terms of a power series

$$f(x) \approx_{\varepsilon} g(x) \Rightarrow \int_a^b f(x) dx \approx_{\varepsilon(b-a)} \int_a^b g(x) dx$$

## 10.3 Local Behavior

- Studying a function in a  $\delta$ -neighborhood of some point  $x_0$  is called studying its "local behavior at  $x_0$ "

Behavior at Infinity

Terminology ...

for  $x \gg 1$ , for large  $x$  = for  $x$  in some interval  $(a, \infty)$

for  $x \ll -1$ , for negatively large  $x$  = for  $x$  in some interval  $(-\infty, a)$

for  $|x| \gg 1$ , for large  $|x|$  = for  $|x| >$  some number  $a$

Local Properties at a point

$f(x)$  is **locally increasing** at  $x_0$  means  $f(x)$  is increasing for  $x \approx x_0$ ;

$f(x)$  is **locally bounded** at  $x_0$  means  $f(x)$  is bounded for  $x \approx x_0$ ;

$f(x)$  is **locally positive** at  $x_0$  means  $f(x)$  is positive for  $x \approx x_0$ ;

## 10.4 Local and Global Properties of Functions

- We say  $f(x)$  is **locally bounded** on the open interval  $I$  if it is locally bounded at every point of  $I$ :  
for all  $x_0 \in I$ ,  $f(x)$  is bounded for  $x \approx x_0$
- We say  $f(x)$  is **locally increasing** on the open interval  $I$  if it is locally increasing at every point  $x_0$  of  $I$

Local vs. Global

- Let  $f(x)$  be defined on an interval  $I = [a, b]$ . Then  **$f(x)$  locally bounded on  $I \Rightarrow f(x)$  bounded on  $I$**