1) a) Stationarity is the conditions you need for time-series analysis. It is as similar as the assumption of normality in random errors in regression analysis. Failing to have stationarity... leads to ~~co~~ comprehensive analytical errors, ends up having too high variations. It has following 2 conditions.

i) constant mean : no seasonality or trend

ii) constant variance : the variance is constant, and the covariance only depends on time difference.

b) Gauss-Markov Theorem : 1. $\hat{\beta}^{OLS}$ is the best linear unbiased estimator of β.

2. $\hat{\beta}^{OLS}$ has the smaller estimation variance.

2) a) i) $E(X_t) = E(\cos(t) + Z_t Z_{t-1})$, since $E(Z_t, Z_{t-1}) = cov(Z_t, Z_{t-1}) = 0$

$= E(\cos(t))$

$$E(X_{t-k} X_t) = E[(\cos(t-k) + Z_{t-k} Z_{t-k-1})(\cos(t) + Z_t Z_{t-1})]$$

$$= \cos(t-k)\cos(t) + \cos(t) + Z_{t-k} Z_{t-k-1}$$

$$+ \cos(t-k) Z_t Z_{t-1} + Z_t Z_{t-1} Z_{t-k} Z_{t-k-1}$$

ii) It is not (weakly) stationary. Both the mean and covariance depends on t.

b) $E(X_t) = E(Z_0 \cos(ct)) = 0$

$$E(X_{t-k} X_t) = E[(Z_0 \cos(ct - ck))(Z_0 \cos(ct)]$$

$$= E(Z_0^2 \cos(ct)[\cos(ct)\cos(ck) + \sin(ct)\sin(ck)])$$

$$= \sigma^2 E[\cos(ct)[\cos(ct)\cos(ck) + \sin(ct)\sin(ck)]]$$

ii) It is not (weakly) stationary. The covariance is dependent on t.

3) $X_t - X_{t-1} - cX_{t-2} = Z_t$

$(1 - B - cB^2) X_t = Z_t$

$c = (-\infty, -1) \cup (1, \infty)$

4)

$\hat{\beta}^{GLS} \sim N(\beta, (X'\Gamma^{-1}X)^{-1})$ ...

a) 1 - Estimate the $\hat{\beta}^{OLS}$

2 - Take residuals from $\hat{\beta}^{OLS}$

3 - apply ARMA(p, q) process on residuals

4 - Estimate Γ again from the ARMA model, and calculate estimator again.

5 - 2~4 repetition

b) $E(\hat{\beta}^{GLS}) = \beta$

c) $cov(\hat{\beta}^{GLS}) = (X'\Gamma^{-1}X)^{-1}$

5) a) $X_t - 0{,}3X_{t-1} - 0{,}4X_{t-2} = Z_t + Z_{t-1} + 0{,}25Z_{t-2}$

$$(1 - 0{,}3B - 0{,}4B^2)X_t = (1 + B + 0{,}25B^2)Z_t$$

0,5
-0,8

$$(1 - 0{,}8B)(1 + 0{,}5B)X_t = (1 + 0{,}5B)(1 + 0{,}5B)Z_t$$

i) Stationarity : Satisfied, $\phi(z)$ does not roots on the unit circle. $z_1 = \frac{5}{4}$, $z_2 = -2$

ii) Causal : Satisfied, $\phi(z)$ has roots outside the unit circle. $z_1 = \frac{5}{4}$, $z_2 = -2$

iii) Invertible : Satisfied, $\theta(z)$ has a root outside the unit circle. $z = -2$

iv) Identifiable : failed, $\phi(z)$ and $\theta(z)$ have one common root. $z = -2$

b) $$\frac{(1 - 0{,}3B - 0{,}4B^2)}{(1 + 0{,}5B)(1 + 0{,}5B)}X_t = Z_t$$

$$(1 - 0{,}3B - 0{,}4B^2)(1 - 0.5B + 0{,}25B^2 - \dots)^2 X_t = Z_t$$

$$(1 - 0{,}3B - 0{,}4B^2)(1 - B + 0{,}75B^2 - \dots)X_t = Z_t$$

$$(1 - 1{,}3B + 0{,}32B^2 + \dots)X_t = Z_t$$

$\pi_1 = -1.3$, $\pi_2 = 0.32$