16년, 15년

14. 확률변수 X의 누적분포함수가 다음과 같을 때,

$$F(x)=\begin{cases}0 & ,x<0\\ 0.5x+0.25 & ,0\le x<0.5\\ 1.2(x-0.5)^2+0.6 & ,0.5\le x<1\\ 1 & ,x\ge 1\end{cases} \quad \text{or} \quad F(x)=\begin{cases}0 & ,x<0\\ 0.5x+0.25 & ,0\le x<0.5\\ 1.2(x-0.5)^2+0.7 & ,0.5\le x<1\\ 1 & ,x\ge 1\end{cases}$$

(a) 이산형과 연속형 누적분포함수로 분해하고 밀도함수로 표현하여라.

14) a) $F(x)=\begin{cases}0 & , x<0\\ 0.5x+0.25 & , 0\le x<0.5\\ 1.2(x-0.5)^2+0.6 & , 0.5\le x<1\\ 1 & , 1\le x\end{cases}$

$p = 0.25 + (0.6-0.5) + (1-0.9) = 0.45$

$\Rightarrow F(x) = 0.45F^d(x) + 0.55F^c(x)$

$0.45F^d(x) = 0.25I(0\le x<\infty) + 0.1I(0.5\le x<\infty) + 0.1I(1\le x<\infty)$

$F^d(x) = \frac{5}{9}I(0\le x<\infty) + \frac{2}{9}I(0.5\le x<\infty) + \frac{2}{9}I(1\le x<\infty)$

$0.55F^c(x) = 0.5xI(0\le x<0.5) + (1.2(x-0.5)^2+0.25)I(0.5\le x<1) + I(1\le x<\infty)$

$F^c(x) = \frac{10}{11}xI(0\le x<0.5) + \left(\frac{24}{11}(x-0.5)^2+\frac{5}{11}\right) + I(1\le x<\infty)$

b) $E(X) = \int_0^1 1-F(x)\,dx$

$= \int_0^{0.5} 0.75-0.5x\,dx + \int_{0.5}^1 0.4-1.2(x-0.5)^2\,dx$

$= \left(0.75x-0.25x^2\Big|_0^{0.5}\right) + \left(0.4x-0.4x^3+0.6x^2-0.3x\Big|_{0.5}^1\right) = 0.3125 + (0.3-0.15) = 0.4625$

c) $E(X^2) = \int_0^1 2x(1-F(x))\,dx$

$= 2\left\{\int_0^{0.5} 0.75x-0.5x^2\,dx + \int_{0.5}^1 0.1x-1.2x^3+1.2x^2\,dx\right\}$

$= 2\left\{\left(\frac{3}{8}x^2-\frac{1}{6}x^3\Big|_0^{0.5}\right) + \left(\frac{1}{20}x^2-\frac{12}{40}x^4+\frac{12}{30}x^3\Big|_{0.5}^1\right)\right\}$

$= 2\left\{0.0729167 + \frac{1}{20} - \frac{12}{40} + \frac{12}{30} - \frac{1}{20}\left(\frac{1}{2}\right)^2 + \frac{12}{40}\left(\frac{1}{2}\right)^4 - \frac{12}{30}\left(\frac{1}{2}\right)^3\right\}$

$= 0.3583$

17년, 16년, 15년, 14년. 08년 (p.82 연습17)

12. 확률변수 X에 대하여 다음을 증명하여라.

$$\sum_{n=1}^{\infty} P(|X|\ge n) \le E|X| \le 1 + \sum_{n=1}^{\infty} P(|X|\ge n)$$

12) Let $A_n = \{n \le |X| \le n+1\}$, then $E(|X|) = \int_{-\infty}^{\infty} |x|f(x)\,dx$

$= \int_{\{\sum_{n=0}^{\infty} A_n\}} |x|f(x)\,dx$

$\Rightarrow \sum_{n=0}^{\infty} nP(A_n) \le E(|X|) \le \sum_{n=0}^{\infty}(n+1)P(A_n) = 1 + \sum_{n=1}^{\infty} nP(A_n)$

$$\sum_{n=0}^{\infty} n P(A_n) = 0 \cdot P(0 \le |X| < 1) + 1 \cdot P(1 \le |X| < 2) + 2 \cdot P(2 \le |X| < 3) + 3 \cdot P(3 \le |X| < 4) + \cdots$$

$$= P(1 \le |X| < 2) + P(2 \le |X| < 3) + P(3 \le |X| < 4) + \cdots$$

$$+ P(2 \le |X| < 3) + P(3 \le |X| < 4) + \cdots$$

$$+ P(3 \le |X| < 4) + \cdots$$

$$= P(|X| \ge 1) + P(|X| \ge 2) + P(|X| \ge 3) + \cdots$$

$$= \sum_{n=1}^{\infty} P(|X| \ge n)$$

$$\therefore \sum_{n=1}^{\infty} P(|X| \ge n) \le E(|X|) \le 1 + \sum_{n=1}^{\infty} P(|X| \ge n)$$