Final

Name: Student id:

1. Let $X_1, \ldots, X_n$ be a random sample from Poisson distribution with mean λ.

   (a) Find the MVUE of $\eta = (1+\lambda)e^{-\lambda} = P(X_1 \leq 1)$. (5 points)

   (b) Show that $\hat{\eta} \xrightarrow{p} \eta$, where $\hat{\eta}$ is the MVUE obtained in (a). (5 points)

2. Suppose that X is a random variable with probability mass function $p(x;\theta)$, where $\theta \in \Omega = \{0, 1\}$. When the probability distribution for X is given by

x	0	1	2	3	4	5
$p(x;\theta = 0)$	0.05	0.25	0.1	0.1	0.2	0.3
$p(x;\theta = 1)$	0.1	0.1	0.25	0.2	0.25	0.1

consider the hypothesis testing $H_0 : \theta = 0$ against $H_1 : \theta = 1$.

(a) When we observe one random variable $X \sim p(x;\theta)$, find the most powerful test of size $\alpha = 0.3$. (5 points)

(b) For two random variables $X_1, X_2 \overset{iid}{\sim} p(x;\theta)$, suppose that a test rejects H_0 if $X_1 X_2 \leq 1$. Determine the size and power of this test. (5 points)

3. Suppose that $X_1, \ldots, X_8$ are IID random variables from $N(\mu_1, 1)$ and that $Y_1, \ldots, Y_5$ are IID random variables from $N(\mu_2, 1)$. Assume that X_i's and Y_j's are independent for any i and j, $i = 1, \ldots, 8$, $j = 1, \ldots, 5$. For hypothesis test $H_0 : \mu_1 = 2\mu_2$ against $H_1 : \mu_1 \neq 2\mu_2$, answer the following questions.

   (a) Derive likelihood ratio test statistic that follows a normal distribution. (5 points)

   (b) Using (a), when the average of $X_1, \ldots, X_8$ is 6 and the average of $Y_1, \ldots, Y_5$ is 2, do hypothesis test under significant level $\alpha = 0.05$. (5 points)

4. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the following density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} x^{(1-\theta)/\theta}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta > 0$.

(a) For $H_0 : \theta = \theta_0$ versus $H_1 : \theta > \theta_0$, find a form of UMP test. (5 points)

(b) When we only have X_1, X_2, X_3 and their observed values are $X_1 = e^{-1/2}, X_2 = e^{-2}, X_3 = e^{-1}$, test hypothesis $H_0 : \theta = 1$ versus $H_1 : \theta > 1$ under significant level $\alpha = 0.05$ using (a). (5 points)