1) a) Given : $p(Y|r) = \sqrt{\frac{r}{2\pi}}\, e^{-\frac{ry^2}{2}}$

$p(r) = \left[\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} / \Gamma\left(\frac{\nu}{2}\right)\right] r^{\frac{\nu}{2}-1} e^{-\frac{\nu}{2}r}$

$\Rightarrow p(y) = \int_0^\infty p(y,r)\, dr$

$= \int_0^\infty p(y|r)\, p(r)\, dr$

$= \int_0^\infty \sqrt{\frac{r}{2\pi}}\, e^{-\frac{ry^2}{2}} \left[\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} / \Gamma\left(\frac{\nu}{2}\right)\right] r^{\frac{\nu}{2}-1} e^{-\frac{\nu}{2}r}\, dr$

$= \frac{1}{\sqrt{2\pi}} \left[\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} / \Gamma\left(\frac{\nu}{2}\right)\right] \int_0^\infty r^{\frac{\nu}{2}+\frac{1}{2}-1} e^{-\frac{(\nu+y^2)}{2}r}\, dr$

$= \frac{1}{\sqrt{2\pi}} \left[\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} / \Gamma\left(\frac{\nu}{2}\right)\right] \left[\Gamma\left(\frac{\nu+1}{2}\right) / \left(\frac{y^2+\nu}{2}\right)^{\frac{\nu+1}{2}}\right]$

$= \frac{1}{\sqrt{2\pi}} \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \left[\Gamma\left(\frac{\nu+1}{2}\right) / \Gamma\left(\frac{\nu}{2}\right)\right] \left[1 / \left(\frac{y^2+\nu}{2}\right)^{\frac{\nu+1}{2}}\right]$, $\frac{y^2+\nu}{2} = \frac{y^2+\nu}{\nu} \cdot \frac{\nu}{2}$

$= \frac{1}{\sqrt{2\pi}} \left(\frac{\nu}{2}\right)^{-\frac{1}{2}} \left[\Gamma\left(\frac{\nu+1}{2}\right) / \Gamma\left(\frac{\nu}{2}\right)\right] \left(\frac{y^2+\nu}{\nu}\right)^{-\frac{\nu+1}{2}}$

$= \left[\Gamma\left(\frac{\nu+1}{2}\right) / \Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi\nu}\right] \left(1 + \frac{y^2}{\nu}\right)^{-\frac{\nu+1}{2}}$

b) (코드)

c) (코드)

2) a) Let X_j be the number of vehicles found on the j-th street, i.e., $f(X_1, \dots, X_{10} | \theta_1, \dots, \theta_{10}) = \binom{N}{X_1, \dots, X_{10}} \theta_1^{X_1} \cdots \theta_{10}^{X_{10}}$

$\Rightarrow \theta_j \sim Beta(\alpha, \beta)$

$\Rightarrow p(\alpha, \beta) \propto (\alpha+\beta)^{-\frac{5}{2}}$, non-informative hyperprior distribution

$\Rightarrow$ Let $\underset{\sim}{X} = (X_1, X_2, \dots, X_{10})$ $\underset{\sim}{\theta} = (\theta_1, \theta_2, \dots, \theta_{10})$.

$\Rightarrow P(\underset{\sim}{\theta}, \alpha, \beta | \underset{\sim}{X}) \propto P(\underset{\sim}{X} | \underset{\sim}{\theta}) P(\underset{\sim}{\theta} | \alpha, \beta) P(\alpha, \beta)$

$\propto \theta_1^{X_1} \theta_2^{X_2} \cdots \theta_{10}^{X_{10}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^{10} \left(\prod_{i=1}^{10} \theta_i\right)^{\alpha-1} e^{-\beta \Sigma \theta_i} (\alpha+\beta)^{-\frac{5}{2}}$

$\Rightarrow P(\theta_j | \underset{\sim}{\theta}_{-j}, \alpha, \beta, \underset{\sim}{X}) \propto \theta_j^{X_j+\alpha-1} e^{-\beta\theta_j} \sim Gamma(X_j+\alpha, \beta)$

$\Rightarrow P(\alpha | \underset{\sim}{\theta}, \beta, \underset{\sim}{X}) \propto \left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^{10} \left(\prod_{i=1}^{10} \theta_i\right)^{\alpha-1} (\alpha+\beta)^{-\frac{5}{2}}$

$\Rightarrow P(\beta | \underset{\sim}{\theta}, \alpha, \underset{\sim}{X}) \propto \left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^{10} e^{-\beta \Sigma \theta_i} (\alpha+\beta)^{-\frac{5}{2}}$

$\Rightarrow$ ⋮

3) a) (코드) at least around 200000 times

b) (코드) at least around 200000 times

4) (코드)