해석학 1 (타전공 학생용) 과제2

1.

① Prove that the sequence $a_n = \dfrac{1\cdot 3\cdot 5\cdots(2n+1)}{2\cdot 4\cdot 6\cdots 2n}$ is strictly increasing and *not* bounded above.

② Show that $\lim_{n\to\infty} \dfrac{1}{n}\left\{\dfrac{2\cdot 4\cdots 2n}{1\cdot 3\cdots(2n-1)}\right\}^2$ exits

2. Assume $\lim_{n\to\infty} a_{n+1}/a_n = L$, where $L<1$ and $a_n>0$. Prove that

(a) $\{a_n\}$ is decreasing for $n \gg 1$

(b) $\lim_{n\to\infty} a_n = 0$ [Give two proofs: an indirect one using (a), and a direct one]

3. Determine $\lim_{n\to\infty} \dfrac{1+\frac{1}{4}+\frac{1}{7}+\cdots+\frac{1}{3n-2}}{\ln n}$.

4. Prove that $\lim_{n\to\infty} \cos 3n$ does not exist.