Final (STA 3001)
June 15, 2020

Instruction

- You are allowed to use any resources on your own, however, you are not permitted to discuss with anyone. The submitted answer sheets should be your own work. Otherwise, it may subject to serious violation of the student code of conduct. By submitting your report online, it is assumed that you agree with the following pledge;

  Pledge: *I have neither given nor received any unauthorized aid during this exam.*

- Your answer sheets must be written in English, and keep in mind that you need to SHOW ALL YOUR WORK. Remind that you can submit your answer sheets over icampus in a pdf file format ONLY.

1. Discuss the differences between strict- and weak- stationarity.

2. Let $X_t = \beta_0 + \beta_1 t + Y_t$, where $\{Y_t\}$, $t = \{0, \pm 1, \pm 2, \ldots\}$ is an IID random variables with mean 0 and variance σ^2. Find the mean and autocovariance function of $\{W_t\}$

$$W_t = \frac{1}{2q+1} \sum_{j=-q}^{q} X_{t+j}$$

3. Consider the stationary process generated by

$$X_t = \alpha + \phi X_{t-1} + Z_t + \theta Z_{t-1}, \quad Z_t \sim WN(0, \sigma^2)$$

   where $E(X_t) = \mu$ and $|\theta| < 1$, $|\phi| < 1$.
   (a) Show that the process is weakly stationary.
   (b) Find $\gamma(3)$.

4. Sketch the region on the plane that the MA(2) process given in the below is invertible

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}, \quad Z_t \sim WN(0, \sigma^2).$$

5. Consider the process

$$X_t - .1X_{t-1} - .12X_{t-2} = Z_t - .7Z_{t-1},$$

   where $Z_t \sim WN(0, \sigma^2)$ with $\sigma^2 = 1$.
   (a) Answer the followings with your reasonings:
   i) stationary _____
   ii) causal _____
   iii) invertible _____
   iv) identifiable _____

(b) What are the optimal predictors for the two-step predictor, say $\widetilde{P}_t X_{t+2}$? Note that the predictor is based on the infinite past, that is the answer will be linear combination of X_s, $s \leq t$. Give the **explicit numerical coefficients only for X_t and X_{t-1}**.

Don't forget to write down your NAME, STUDENT ID in your answer sheets!