1) 1-1) $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \le \sum_{n=0}^{\infty} \frac{1}{(2n)^2} = \sum_{n=0}^{\infty} \frac{1}{4n^2}$

$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{4n^2} = \frac{1}{4} \sum_{n=0}^{\infty} \frac{1}{n^2}$ by Linearity Theorem

$\therefore$ Given $\sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, $\sum_{n=0}^{\infty} \frac{1}{4n^2} = \frac{\pi^2}{24} \ge \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ (Comparison Test)

$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ is convergent.

1-2) Because $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are convergent series with non-negative terms, they are both monotonically increasing and bounded above.

Let $\sum_{n=0}^{\infty} a_n \le L$ and $\sum_{n=0}^{\infty} b_n \le M$, then,

$\sum_{n=0}^{\infty} a_n b_n \le \sum_{n=0}^{\infty} L \cdot b_n \le \sum_{n=0}^{\infty} LM$, and $a_n b_n$ are non-negative terms.

$\therefore$ Because $\sum_{n=0}^{\infty} a_n b_n$ is monotonically increasing and bounded above by LM, $\sum_{n=0}^{\infty} a_n b_n$ converges by the Completeness Property.

2) $\sum_{k=0}^{\infty} b_k = b_1 + b_2 + \cdots + b_k$

$= (a_0 + a_1) + (a_2 + a_3) + \cdots + (a_{2k} + a_{2k+1})$

$= a_0 + a_1 + a_2 + a_3 + \cdots + a_{2k} + a_{2k+1}$, by the subsequence theorem,

$= \sum_{n=0}^{\infty} a_n = S$

3) 3-1) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{2\sqrt[n]{n!}}{n} = 0$, by n^{th} root test

3-2) $\sum_{2}^{\infty} \frac{1}{n(\ln n)^p}$

$\frac{a_{n+1}}{a_n} = \frac{n(\ln n)^p}{(n+1)(\ln (n+1))^p}$, $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, ratio test fails

$\frac{1}{n(\ln n)^p} \le \frac{1}{n \ln n}$, because $n \ge 2$ and $p > 0$,

$\frac{1}{n(\ln n)} \le \frac{1}{n}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n} = 0 < 1$

$\therefore$ By comparison test, $\sum_{2}^{\infty} \frac{1}{n(\ln n)^p}$ converges

4) 4-1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\tan^{-1} n} = \cdots + \frac{2}{\pi} - \frac{2}{\pi} + \frac{2}{\pi} - \frac{2}{\pi} + \cdots$ for $n \gg 1$, so because it is alternating in signs, it is not convergent.

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\tan^{-1} n}$ is not conditionally convergent

4-2) $\sum_{2}^{\infty} (-1)^n \frac{n^5}{2^n}$ is absolutely convergent since,

$\lim_{n\to\infty} |\sqrt[n]{a_n}| = \lim_{n\to\infty} \frac{n^{\frac{5}{n}}}{2} = \frac{1}{2} < 1$

$\therefore \sum_{2}^{\infty} (-1)^n \frac{n^5}{2^n}$ is not conditionally convergent

5) Let $\left|\frac{a_{n+1}}{a_n}\right| = x_n$ and $\left|\frac{b_{n+1}}{b_n}\right| = y_n$. By the Absolute Convergence Theorem, $\sum_{n=0}^{\infty} b_n$ is convergent. By the n^{th} term test of divergence, $\lim_{n\to\infty} b_n = 0$, which means $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = L < 1$.

Then it can be revised as, $\left|\frac{a_{n+1}}{a_n}\right| \le L$, and L becomes the upper bound for $\left|\frac{a_{n+1}}{a_n}\right|$.
Since $\left|\frac{a_{n+1}}{a_n}\right| \le L < 1$, by Limit Location Theorem, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \le L < 1$, and this satisfies the ratio test, so $\sum_{n=0}^{\infty} a_n$ is absolutely convergent

6) $\left|\frac{a_{n+1}}{a_n}\right| \le r < 1$ for $n \gg 1$

pf) $\left|\frac{a_{n+1}}{a_n}\right| = \frac{|a_{n+1}|}{|a_n|}$

$\Rightarrow |a_{n+1}| < |a_n| r$ for $n \gg 1$

$\Rightarrow |a_{n+2}| < |a_{n+1}| r < |a_n| r^2$

$\vdots$

$\Rightarrow |a_{n+k}| < |a_n| M^k$ for $k \ge 1$.

$\Rightarrow \sum_{k=1}^{\infty} |a_n| M^k$ is convergent by the Linearity Theorem, and by the Comparison test, $\sum_{k=1}^{\infty} |a_{n+k}|$ is also convergent. The Tail-convergence Theorem can be then applied to verify $\sum_{n=0}^{\infty} |a_n|$ is convergent.