Ch3. ARMA

Checking stationary condition

It only depends on AR(p) part and need to see whether roots are on unit-circle R function polyroot performs that. For example, consider AR(4) model with

$$X_t = 2.7607X_{t-1} - 3.8106X_{t-2} + 2.6535X_{t-3} - .9238X_{t-4} + Z_t$$

```
ch = polyroot(c(1, -2.7607, 3.8106, -2.6535, .9238))
Mod(ch)
```

```
## [1] 1.019877 1.020148 1.019877 1.020148
```

Concludion: Since all roots are outside unit-circle, they are stationary (even invertible).

Inverting ARMA(p, q) to MA(infty)

Use ARMAtoMA but be careful on their representation! You don't need to input 1 in characteristic polynomial and R follow our class notaton

$$X[t] = a[1]X[t-1] + \ldots + a[p]X[t-p] + Z[t] + b[1]Z[t-1] + . + b[q]Z[t-q],$$

so you need to multiply (-1) to the coefficients appear in the AR representation. For example

$$(1 - B + .25B^2)X_t = Z_t + Z_{t-1} \tag{1}$$

(example from TSTM Brockwell & Davis (1991, p.92)) gives $\psi_j = (1 + 3 * j) * 2^{-j}$. You need to input

```
ARMAtoMA(ar=c(1.0, -0.25),ma=c(1), lag.max=10)
```

```
##  [1] 2.00000000 1.75000000 1.25000000 0.81250000 0.50000000 0.29687500
##  [7] 0.17187500 0.09765625 0.05468750 0.03027344
```

```
j <- 1:10; (1 + 3*j)*2^(-j)
```

```
##  [1] 2.00000000 1.75000000 1.25000000 0.81250000 0.50000000 0.29687500
##  [7] 0.17187500 0.09765625 0.05468750 0.03027344
```

rather than write down coefficients directly as in the polyroot case (DO NOT write as ARMAtoMA(ar=c(-1, .25),ma=c(1), lag.max=10))

Calculating theoretical ACF/PACF for ARMA(p,q)

Again remark that the R uses the model representation

$$X[t] = a[1]X[t-1] + . + a[p]X[t-p] + Z[t] + b[1]Z[t-1] + . + b[q]Z[t-q]$$

Example (1) gives the theoretical annswer is $\gamma(k) = \sigma^2 2^{-k}(32/3 + 8k)$.

```
ARMAacf(ar=c(1.0, -0.25), ma=c(1.0), lag.max = 10)
```

```
##           0           1           2           3           4           5
## 1.000000000 0.875000000 0.625000000 0.406250000 0.250000000 0.148437500
##           6           7           8           9          10
## 0.085937500 0.048828125 0.027343750 0.015136719 0.008300781
```

```
n <- 1:10; 2^(-n) * (32/3 + 8 * n) /(32/3)
```

```
##  [1] 0.875000000 0.625000000 0.406250000 0.250000000 0.148437500
##  [6] 0.085937500 0.048828125 0.027343750 0.015136719 0.008300781
```

For PACF use

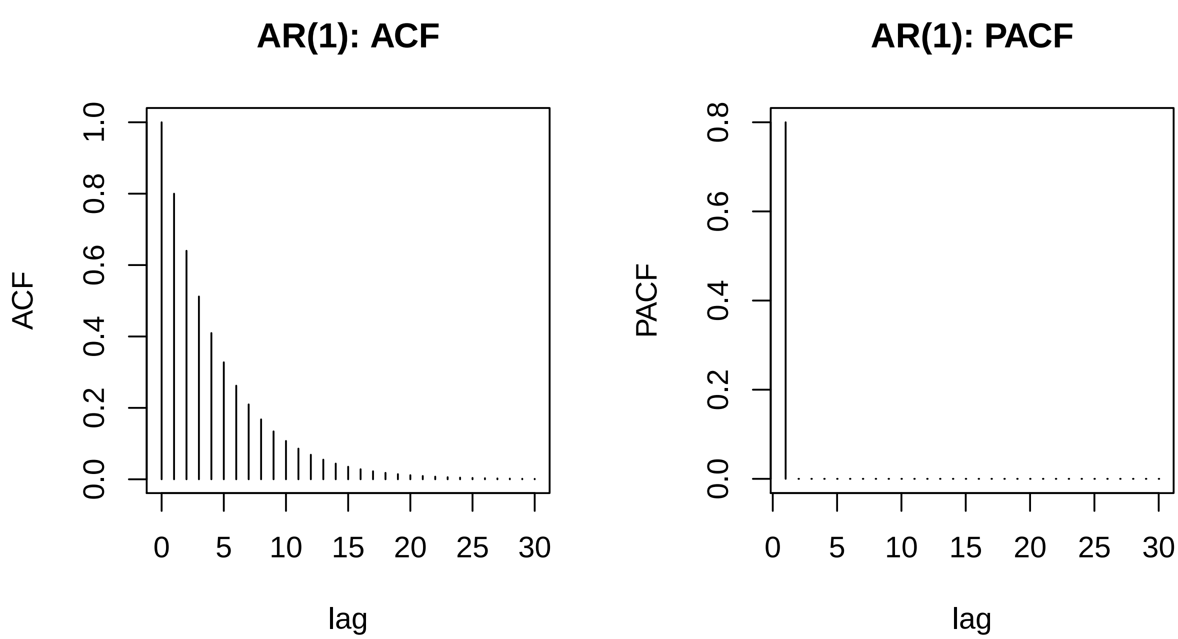
```
ARMAacf(ar=c(1.0, -0.25), ma=c(1.0), lag.max = 10, pacf = TRUE)
```

```
##  [1]  0.8750000 -0.6000000  0.3750000 -0.2727273  0.2142857 -0.1764706
##  [7]  0.1500000 -0.1304348  0.1153846 -0.1034483
```

Exercises:

AR(1) $X_t = .8X_{t-1} + Z_t$

```
ACF = ARMAacf(ar=c(.8), lag.max=30);
PACF = ARMAacf(ar=c(.8), lag.max=30, pacf=TRUE);
par(mfrow=c(1,2))
plot(0:30, ACF, type="h", xlab="lag", ylab="ACF"); title("AR(1): ACF");
plot(PACF, type="h", xlab="lag", ylab="PACF"); title("AR(1): PACF");
```



In-class practice

Find theoretical ACF/PACF for the follwing ARMA models:

- AR(2) $X_t = 1.5X_{t-1} - .75X_{t-2} + Z_t$
- MA(1) $X_t = .7X_{t-1} + Z_t$
- ARMA(1,1) $X_t = .7453X_{t-1} + Z_t + .32Z_{t-1}$