1. Let $Z = \sum_{j=1}^{c} Y_j$. Then the conditional dist. of $Y_1, \cdots, Y_c$ given $Z=n$ is

$$P(y_1, \cdots, y_c \mid Z=n) = \frac{P(y_1, \cdots, y_c, Z=n)}{P(Z=n)}$$

$$\left(\text{Note that } Z = \sum_{j=1}^{c} Y_j \sim \text{Poisson}\left(\sum_{j=1}^{c} \mu_j\right) \right)$$

$$= \frac{P(y_1)\, P(y_2) \cdots P(y_{c-1})\, P(n - y_1 - \cdots - y_{c-1})}{\frac{e^{-\sum_{j=1}^{c}\mu_j} \left(\sum_{j=1}^{c}\mu_j\right)^n}{n!}}$$

($\because$) $y_1, \cdots, y_c$ are indep.

$$= \frac{\frac{e^{-\mu_1}\mu_1^{y_1}}{y_1!} \frac{e^{-\mu_2}\mu_2^{y_2}}{y_2!} \cdots \frac{e^{-\mu_{c-1}}\mu_{c-1}^{y_{c-1}}}{y_{c-1}!} \frac{e^{-\mu_c}\mu_c^{n-y_1-\cdots-y_{c-1}}}{(n-y_1-\cdots-y_{c-1})!}}{\frac{e^{-\sum_{j=1}^{c}\mu_j} \left(\sum_{j=1}^{c}\mu_j\right)^n}{n!}}$$

$$= \frac{n!}{y_1! \cdots y_{c-1}!\, (n-y_1-\cdots-y_{c-1})!} \left(\frac{\mu_1}{\sum_{j=1}^{c}\mu_j}\right)^{y_1} \left(\frac{\mu_2}{\sum_{j=1}^{c}\mu_j}\right)^{y_2} \cdots \left(\frac{\mu_{c-1}}{\sum_{j=1}^{c}\mu_j}\right)^{y_{c-1}} \left(\frac{\mu_c}{\sum_{j=1}^{c}\mu_j}\right)^{n-y_1-\cdots-y_{c-1}}$$

it is multinomial dist $\left(n, \frac{\mu_1}{\sum_{j=1}^{c}\mu_j}, \cdots \frac{\mu_{c-1}}{\sum_{j=1}^{c}\mu_j}\right)$.

2. $\log\hat{\theta} = \log\frac{P_1}{1-P_1} - \log\frac{P_2}{1-P_2}$

$$= \log P_1 - \log(1-P_1) - \log P_2 + \log(1-P_2)$$

Using delta-method,

$$Var(\log\hat{\theta}) = \left(\frac{\partial \log\theta}{\partial \pi_1}, \frac{\partial \log\theta}{\partial \pi_2}\right)\begin{pmatrix} Var(P_1) & 0 \\ 0 & Var(P_2)\end{pmatrix}\begin{pmatrix}\frac{\partial\log\theta}{\partial\pi_1} \\ \frac{\partial\log\theta}{\partial\pi_2}\end{pmatrix}\Bigg|_{\underline{\pi}=\underline{\hat{\pi}}}$$

$$= \Bigg(\underbrace{\frac{1}{\hat{\pi}_1}+\frac{1}{1-\hat{\pi}_1}}_{=\frac{1}{\hat{\pi}_1(1-\hat{\pi}_1)}}, \underbrace{-\frac{1}{\hat{\pi}_2}-\frac{1}{1-\hat{\pi}_2}}_{=-\frac{1}{\hat{\pi}_2(1-\hat{\pi}_2)}}\Bigg)\begin{pmatrix}\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_{1+}} & 0 \\ 0 & \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_{2+}}\end{pmatrix}\begin{pmatrix}\frac{1}{\hat{\pi}_1(1-\hat{\pi}_1)} \\ -\frac{1}{\hat{\pi}_2(1-\hat{\pi}_2)}\end{pmatrix}$$

$$= \frac{1}{n_{1+}\hat{\pi}_1(1-\hat{\pi}_1)} + \frac{1}{n_{2+}\hat{\pi}_2(1-\hat{\pi}_2)}$$

$$= \frac{1}{\cancel{n_{1+}}\frac{n_{11}}{\cancel{n_{11}+n_{12}}}\frac{n_{12}}{n_{11}+n_{12}}} + \frac{1}{\cancel{n_{2+}}\frac{n_{21}}{\cancel{n_{21}+n_{22}}}\frac{n_{22}}{n_{21}+n_{22}}}$$

$$= \frac{n_{11}+n_{12}}{n_{11}n_{12}} + \frac{n_{21}+n_{22}}{n_{21}n_{22}}$$

$$= \frac{n_{11}n_{21}n_{22} + n_{12}n_{21}n_{22} + n_{21}n_{11}n_{12} + n_{22}n_{11}n_{12}}{n_{11}n_{12}n_{21}n_{22}}$$

$$= \frac{1}{n_{12}} + \frac{1}{n_{11}} + \frac{1}{n_{22}} + \frac{1}{n_{21}}$$

3.

		Y = 1	Y = 2	
X	1	π_1	$1-\pi_1$	1
	2	π_2	$1-\pi_2$	1

$$\pi_1 = P(Y=1 \mid X=1), \quad \pi_2 = P(Y=1 \mid X=2)$$

$$\gamma = P(X=1)$$

(a) $$P(X=1 \mid Y=1) = \frac{P(Y=1 \mid X=1)\,P(X=1)}{P(Y=1 \mid X=1)\,P(X=1) + P(Y=1 \mid X=2)\,P(X=2)}$$

$$= \frac{\pi_1 \gamma}{\pi_1 \gamma + \pi_2 (1-\gamma)}$$

(b) $$PPV = P(X=1 \mid Y=1) = \frac{(0.86)(0.01)}{(0.86)(0.01) + (0.12)(1-0.01)} = 0.0675$$

(c) $$P(X=1, Y=1) = P(Y=1 \mid X=1)\,P(X=1) = (0.86)(0.01) = 0.0086$$

$$P(X=1, Y=2) = P(Y=2 \mid X=1)\,P(X=1) = (1-0.86)(0.01) = 0.0014$$

$$P(X=2, Y=1) = P(Y=1 \mid X=2)\,P(X=2) = (0.12)(1-0.01) = 0.1188$$

$$P(X=2, Y=2) = P(Y=2 \mid X=2)\,P(X=2) = (1-0.12)(1-0.01) = 0.8712$$

4.

$$\hat{\theta} = \frac{(802)(494)}{(34)(53)} = 219.860 \Rightarrow \log \hat{\theta} = 5.393$$

Wald 95% C.I. for $\log \theta$

$$\log \hat{\theta} \pm 1.96 \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$$

$$= 5.393 \pm 1.96 (0.227)$$

$$= (4.948, 5.838)$$

$\Rightarrow$ 95% C.I. for θ

$$(e^{4.948}, e^{5.838})$$

$$= (140.893, 343.093)$$

$$> 1.$$

The odds of voting for Obama in 2012 voted ~~for~~ for Obama in 2008 are 219.86 times greater than the odds of voting for McCain in 2008.

The