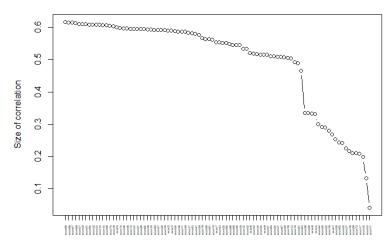
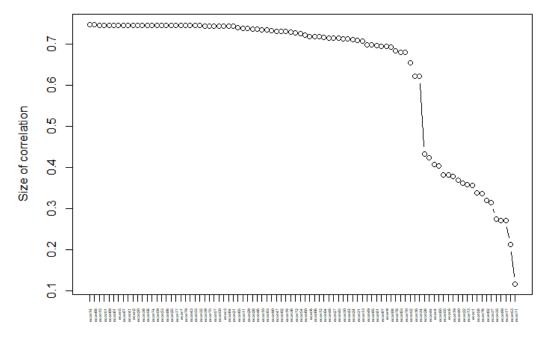
```
Statistical Modelling & Machine Learning #
              R Example4
 options(warn = -1) # Turn off warning message
 ####### Variable Importance: Regression problem #########
   Building data
 # 90 economic variables and sales variable (output) dat = read.table('building.csv', sep=',', header=T)
   Correlation coefficient -----
 VI = cor(dat)[,'price']
     = sort(abs(VI), decreasing = T)[-1]
 SVI
                                          econ57
   econ68
            econ86
                      econ87
                                econ69
                                                    econ50
                                                             econ51
econ32
                      econ33
                                econ14
                                          econ15
                                                    econ90
   econ64
                                                             econ75
econ39
                      econ74
                                econ46
                                          econ56
                                                    econ72
             econ5
                                                             econ23
0.60453730 0.60115498 0.59873700 0.59763375 0.59725241 0.59584429 0.59554659
                                                    econ82
   econ21
            econ28
                      econ65
                                econ38
                                          econ10
                                                             econ41
0.59517838 0.59486492 0.59482798 0.59426435 0.59390329 0.59294998 0.59285014
                                          econ54
                                                   econ36
   econ20
             econ18
                       econ2
                                 econ3
                                                             econ77
0.59162525
          0. 59154972 0. 59073885 0. 59021348 0. 58960077 0. 58789128 0. 58763764
   econ59
             econ11
                      econ83
                                econ47
                                          econ29
                                                    econ79
                                                             econ88
0. 58749289 0. 58404324 0. 58271133 0. 58018114 0. 57743482 0. 56642843 0. 56411320
             econ70
                       econ7
                                econ43
                                          econ25
                                                    econ84
                                                             econ66
   econ61
0.56404887 0.56210776 0.55472247 0.55350511 0.55298111 0.55210583 0.54885358
            econ48
                      econ30
                                econ12
                                          econ52
   econ60
                                                     econ6
                                                             econ42
0.54632685 0.54615987 0.54526351 0.53387115 0.53314601 0.52120916 0.51865266
   econ34
            econ45
                       econ9
                                econ27
                                          econ85
                                                   econ24
                                                             econ31
                    0.51504086 0.51490753 0.51001640 0.50990224 0.50903718
          0. 51511175
0. 51656070
                                          econ78
                                                    econ81
            econ49
                      econ16
                                econ13
   econ67
                                                             econ63
                               0. 50324426 0. 49319746 0. 48875751 0. 46591618
0.50847776
          0.50816509 0.50577267
                                                   econ40
             econ80
                                 econ4
                                          econ44
   econ26
                       econ8
                                                             econ22
0. 33372804 0. 33361651 0. 33291222 0. 33024927 0. 29966462 0. 29119779 0. 29012961
                                          econ55
   econ19
                      econ58
                                econ62
                                                    econ37
                                                             econ76
             econ1
0. 27901459 0. 26739827 0. 25257770 0. 24261738 0. 24170153 0. 22526512 0. 21588431
   econ89
             econ73
                      econ17
                                econ35
                                          econ53
                                                    econ71
0. 20988269 0. 20928031 0. 20824238 0. 19716845 0. 13182179 0. 04036955
                                  ylab='Size of correlation'
 plot(1:length(SVI), SVI, type='b',
     xlab = 'variables', main='Variable Importance', xaxt='n')
 axis(side=1, at=1:length(SVI), labels=names(SVI), cex.axis=0.3,las=2)
```



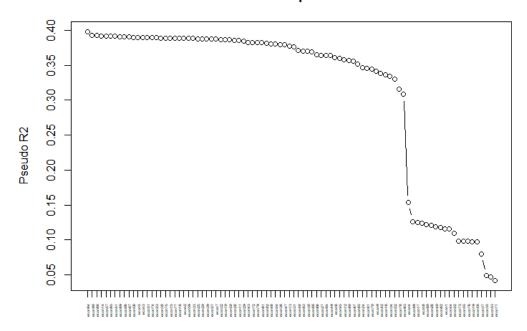
variables

```
# Spearman rank correlation coefficient -----
  SP = cor(dat, method='spearman')[,'price']
SPI = sort(abs(SP), decreasing = T)[-1]
  econ14
             econ68
                        econ15
                                  econ51
                                             econ69
                                                       econ61
                                                                   econ5
                                                                            econ87
0.7480006 0.7478559 0.7470943 0.7470746 0.7470564 0.7470406 0.7470166 0.7470160
   econ41
              econ2
                       econ20
                                  econ38
                                             econ56
                                                       econ74
                                                                  econ59
                                                                            econ23
0.7468722 0.7468707 0.7468707 0.7468707 0.7468707 0.7468707 0.7468269 0.7468192
  econ86
             econ25
                        econ77
                                             econ79
                                                       econ43
                                                                  econ33
                                   econ7
                                                                            econ32
0. 7466633 0. 7466291 0. 7465841 0. 7463867 0. 7463699 0. 7463527 0. 7461416 0. 7458426
                                  econ50
                                              econ3
  econ39
             econ75
                        econ57
                                                       econ64
                                                                  econ21
                                                                            econ65
0. 7453043 0. 7451003 0. 7449772 0. 7449110 0. 7446253 0. 7440360 0. 7440314 0. 7417415
                                  econ46
  econ11
             econ29
                        econ28
                                             econ10
                                                       econ83
                                                                  econ90
                                                                            econ47
0. 7385228 0. 7382788 0. 7371699 0. 7365226 0. 7360915 0. 7346095 0. 7331846 0. 7322964
                                  econ72
                                             econ54
  econ82
             econ18
                        econ36
                                                       econ60
                                                                   econ6
                                                                            econ66
0. 7322654 0. 7316920 0. 7304120 0. 7279187 0. 7262652 0. 7234224 0. 7194596 0. 7182935
                                                                  econ42
             econ84
                        econ48
                                  econ27
                                             econ45
                                                       econ30
   econ12
                                                                            econ24
0.7182107
           0.7169109
                      0. 7161513  0. 7159929  0. 7152081  0. 7142345  0. 7136541  0. 7114739
                        econ49
                                  econ85
                                                                   econ9
  econ31
             econ13
                                             econ81
                                                       econ67
                                                                            econ88
0.7093104 0.7088550 0.6993933 0.6990538 0.6977191 0.6955061 0.6951678 0.6939592
   econ78
             econ63
                        econ70
                                  econ52
                                             econ16
                                                       econ34
                                                                  econ26
                                                                             econ44
0. 6842120 0. 6815127 0. 6812766 0. 6544439 0. 6224187 0. 6220537 0. 4331914 0. 4234760
                                            econ19
                                                                            econ73
   econ4
             econ80
                       econ55
                                   econ8
                                                       econ40
                                                                  econ22
0. 4083398 0. 4047152 0. 3820735 0. 3817524 0. 3786940 0. 3695891 0. 3623909 0. 3580080
   econ1
             econ58
                       econ76
                                  econ62
                                            econ37
                                                       econ35
                                                                  econ89
                                                                            econ17
0. 3566338 0. 3392029 0. 3360683 0. 3204578 0. 3156516 0. 2742122 0. 2716997 0. 2713140
  econ53
             econ71
0. 2130441 0. 1170807
  plot(1:length(SPI), SPI, type='b', ylab='Size of correlation',
    xlab ='variables', main='Variable Importance', xaxt='n')
  axis(side=1, at=1:length(SPI), labels=names(SPI), cex.axis=0.3,las=2)
```



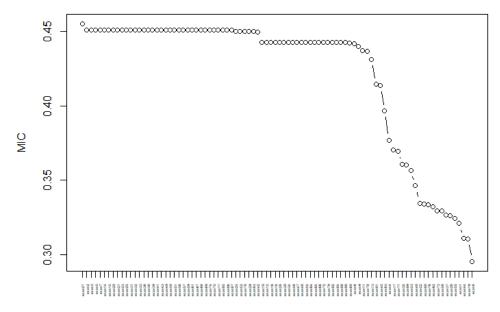
variables

```
fit = loess(price ~ dat[,j],
yhat = predict(fit, dat[,j])
                                 data=dat) # Local linear regression
   PR2[j] = 1-(sum((dat$price - yhat)^2)/sum((dat$price - mean(dat$price))^2))
  SPR2 = sort(PR2, decreasing = T)
  SPR<sub>2</sub>
                                              econ57
   econ64
              econ90
                        econ68
                                   econ14
                                                        econ65
                                                                   econ41
0.\ 39782881\ \ 0.\ 39311868\ \ 0.\ 39293505\ \ 0.\ 39187266\ \ 0.\ 39184300\ \ 0.\ 39143906\ \ 0.\ 39129056
                                                                  econ51
   econ69
              econ86
                        econ87
                                   econ56
                                              econ5
                                                         econ3
0. 39116206 0. 39103442 0. 39027272 0. 38992729 0. 38985100 0. 38983392 0. 38954506
   econ74
              econ23
                        econ38
                                   econ75
                                             econ15
                                                        econ77
                                                                   econ18
0. 38946368 0. 38928179 0. 38909528 0. 38900633 0. 38894807 0. 38892545 0. 38878446
                                             econ20
                                                        econ39
             econ59
                        econ33
                                   econ25
                                                                   econ21
    econ2
0. 38876378 0. 38865585
                      0. 38817614 0. 38810713 0. 38806374 0. 38804960 0. 38764551
                                   econ50
                                             econ28
    econ7
             econ32
                        econ10
                                                        econ11
                                                                   econ29
0.38751223
           0.38704344
                      0. 38689628 0. 38613038 0. 38563787 0. 38514273 0. 38406873
                                                        econ46
   econ43
              econ72
                        econ79
                                   econ61
                                              econ82
                                                                   econ54
0. 38303032 0. 38278444 0. 38257175 0. 38232618 0. 38100270 0. 38049542 0. 38002283
   econ36
              econ47
                        econ13
                                   econ31
                                              econ48
                                                        econ83
                                                                   econ30
0. 37981754 0. 37924352 0. 37712482 0. 37679239 0. 37171271 0. 37038657 0. 37004289
   econ88
              econ66
                        econ27
                                   econ84
                                              econ49
                                                                   econ45
                                                         econ6
0. 36955338 0. 36535822 0. 36434124 0. 36424892 0. 36378332 0. 36095139 0. 35953716
   econ12
              econ60
                        econ67
                                   econ85
                                              econ24
                                                        econ81
                                                                   econ70
0. 35780713
           0. 35636359
                      0. 35548371
                                  0. 35142339  0. 34675822  0. 34543402  0. 34496508
    econ9
             econ42
                        econ16
                                   econ34
                                             econ52
                                                        econ78
                                                                  econ63
0. 34106047
           0. 33880395 0. 33584800 0. 33398428 0. 33018205 0. 31556932 0. 30832680
                                             econ80
              econ89
                        econ17
                                    econ8
                                                        econ26
                                                                  econ40
    econ4
econ22
                                                        econ55
                        econ44
                                             econ73
                                                                   econ76
   econ62
               econ1
0. 11775807 0. 11559373 0. 11535676 0. 10943468 0. 09876735 0. 09865478 0. 09847815
              econ58
                                   econ35
   econ19
                                              econ53
                        econ37
                                                        econ71
0. 09743884 0. 09718496 0. 07970306 0. 04856789 0. 04745673 0. 04211173
 axis(side=1, at=1:length(SPR2), labels=names(SPR2), cex.axis=0.3,las=2)
```

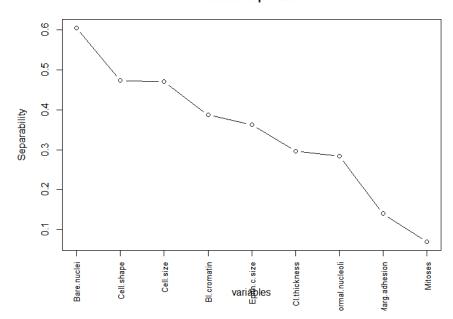


variables

```
# Maximal information coefficient (MIC) ------
  install.packages('minerva')
  library (minerva)
 MIC = mine(dat)
  MIC = MIC$MIC[, 'price']
  SMIC = sort(MIC, decreasing = T)[-1]
  SMI C
  econ27
                                            econ7
                                                      econ14
                                                                econ15
                                                                          econ20
              econ2
                        econ3
                                  econ5
0. 4550555 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285
  econ21
             econ23
                       econ25
                                  econ31
                                            econ32
                                                      econ33
                                                                 econ36
                                                                           econ38
0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285
                       econ43
                                 econ49
                                            econ50
  econ39
             econ41
                                                      econ51
                                                                 econ56
                                                                           econ57
0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285
  econ59
                                  econ68
                                            econ69
                                                      econ74
                                                                 econ75
             econ61
                       econ67
                                                                           econ77
0.4508285
          0.4508285
                     0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285 0. 4508285
                                                                 econ29
  econ85
             econ86
                       econ87
                                  econ24
                                            econ52
                                                      econ70
                                                                           econ83
0. 4508285 0. 4508285 0. 4508285 0. 4502464 0. 4502464 0. 4502464 0. 4499103 0. 4499103
   econ42
             econ10
                       econ12
                                  econ<sub>16</sub>
                                            econ18
                                                      econ28
                                                                 econ30
                                                                           econ34
0. 4497651 0. 4424887 0. 4424887 0. 4424887 0. 4424887 0. 4424887 0. 4424887 0. 4424887
  econ46
             econ47
                       econ48
                                  econ54
                                            econ64
                                                      econ65
                                                                 econ66
                                                                           econ72
0.4424887 0.4424887 0.4424887 0.4424887 0.4424887 0.4424887 0.4424887
  econ79
             econ82
                       econ84
                                  econ88
                                            econ90
                                                      econ60
                                                                  econ6
                                                                            econ9
0. 4424887 0. 4424887 0. 4424887 0. 4424887 0. 4424887 0. 4423206 0. 4416032 0. 4398774
                                 econ81
                                            econ45
                                                      econ63
  econ11
             econ78
                       econ13
                                                                  econ4
                                                                           econ17
0. 4371542 0. 4367981 0. 4311837
                                0. 4144450 0. 4136798 0. 3965958 0. 3769089 0. 3703032
                       econ89
                                  econ53
                                            econ40
                                                      econ22
  econ71
             econ35
                                                                 econ80
                                                                           econ76
0. 3696456 0. 3607008 0. 3601444
                                 0. 3566271  0. 3462738  0. 3345459  0. 3338895  0. 3334517
                       econ58
                                  econ37
                                            econ26
                                                      econ55
   econ62
             econ73
                                                                  econ1
                                                                           econ44
  3322830 0.3294336 0.3292661 0.3265118 0.3262451 0.3244009 0.3209021 0.3109958
  econ19
              econ8
0. 3106083 0. 2953973
                                         yl ab=' MI C'
  plot(1:length(SMIC), SMIC, type='b', ylab='MIC',
     xlab = 'variables', main='Variable Importance'
                                                         , xaxt='n')
  axis(side=1, at=1:length(SMIC), labels=names(SMIC), cex.axis=0.3,las=2)
```



```
####### Variable Importance: Classification problem #########
  # Data
  install.packages('mlbench')
  library(mlbench)
  data(BreastCancer)
  dat = BreastCancer[, -1]
  # Relief algorithm -----
  install.packages('CORELearn')
library(CORELearn)
  # Relief algorithm
  RE = attrEval (Class ~ ., data=dat, estimator='Relief',
                 Reliefl terations=30)
  SRE = sort(RE, decreasing = T)
  SRE
                                                              Marg. adhesi on 0. 3333333
                        BI.cromatin
                                           CI . thi ckness
                                                                                    Epi th. c. si ze
    Bare. nucl ei
                          0.5333333
                                                                                     0. 3333333
      0. 6330396
                                             0. 4666667
                                                                     Mi toses
      Cell. si ze
                         Cell.shape Normal.nucleoli
      0.3000000
                          0.2666667
                                             0.1666667
                                                                 0.1333333
  plot(1:length(SRE), SRE, type='b', ylab='Separability',
    xlab ='variables', main='Variable Importance', xaxt='n')
axis(side=1, at=1:length(SRE), labels=names(SRE), cex.axis=0.8, las=2)
>
                            Variable Importance
    9.0
   0.5
Separability
    0
    0.3
    0.2
                                                         nucleoli
                                                                Mitoses
                                           size
                       CI.thickness
                              arg.adhesion
                                           Cell.
                                  varia∰les
                                                         rmal.
  # ReliefF algorithm
  REF = attrEval (Class ~ ., data=dat, estimator='ReliefFequalK',
                  Reliefl terations=30)
  SREF = sort(REF, decreasing = T)
  SREF
                         Cel I . shape 0. 4733333
                                              Cell. si ze
                                                                Bl.cromatin
                                                                                   Epi th. c. si ze
    Bare. nucl ei
                                                                                     0.3633333
                                             0.4700000
                                                                 0.3866667
      0.6047962
                                           Marg. adhesi on
   CI. thickness Normal.nucleoli
                                                                      Mi toses
                                             0.1400000
                                                                 0.0700000
      0.2966667
                          0. 2833333
```



```
####### Variable Selection: Simulated Annealing ########
  install.packages('mvtnorm')
 library(mvtnorm)
  # Data generation
 set. seed(10)
 n = 500
 p = 20
S = \text{matri} x(0.3, \text{ nrow=p, ncol=p})
 X = rmvnorm(n, mean=rep(0, p), sigma=S)
 XN = NULL
 for (j in 1:p) XN = c(XN, paste('X', j, sep=''))
  colnames(X) = XN
 Y = 2 + 0.5*X[,1] - 0.3*X[,2] + 1.2*X[,3] + rnorm(n, sd=0.1)
 # Simulated Annealing ---
  install.packages('caret')
 library(caret)
 ctrl = safsControl(functions=caretSA, method='cv', number=5)
> obj = safs(x=X, y=Y, iters=20, safsControl=ctrl, method='lm')
 obj
```

Simulated Annealing Feature Selection

500 samples 20 predictors

Maximum search iterations: 20

Internal performance values: RMSE, Rsquared, MAE Subset selection driven to minimize internal RMSE

External performance values: RMSE, Rsquared, MAE Best iteration chose by minimizing external RMSE External resampling method: Cross-Validated (5 fold)

```
In the final search using the entire training set:
     14 features selected at iteration 20 including:
      X1 ...
      external performance at this iteration is
         RMSE
                    Rsquared
                                            MAE
      0. 4314
                       0.8335
                                         0.3257
> install.packages('SIS')
Error in install packages: Updating loaded packages
> library(SIS)
> ?SIS
> # Data generation
> set. seed(0)
> n = 400; p = 50; rho = 0.5
> corrmat = diag(rep(1-rho, p)) + matrix(rho, p, p)
> corrmat[, 4] = sqrt(rho)
> corrmat[4, ] = Sq

> corrmat[4, 4] = 1

> corrmat[5, ] = 0

> corrmat[5, 5] = 1
                    ] = sqrt(rho)
> chol mat = chol (corrmat)
> x = matri x(rnorm(n*p, mean=0, sd=1), n, p)
> x = x\%*%chol mat
> # Linear regression
> set.seed(1)
> b = c(4, 4, 4, -6*sqrt(2), 4/3)
> y=x[, 1:5]\%*\%b + rnorm(n)
> # ISIS with regularization
> model 11=SIS(x, y, family='gaussian', tune='bic')
Iter 1 , screening: 1 2 3 4 5 6 7 8 9 10 11 18 21 22 24 25 26 29 30 31 32 33 34 35 36
41 42 43 44 46 47 48 50
Iter 1, selection: 1 2 3 4 5
Iter 1, serection: 1 2 3 4 5
Iter 1, conditional-screening: 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 2, screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 2, selection: 1 2 3 4 5
Model already selected
> model 11$i x
[1] 1 2 3 4 5
> model 12=SIS(x, y, family='gaussian', tune='bic', varISIS='aggr', seed=11)
Iter 1 , screening: 1 2 3 5 6 7 9 10 20 23 24 27 28 29 38 40 41 42 43 45 47 48
Iter 1 , selection: 1 2 3 5 6 7 9 10 20 23 24 27 28 29 38 40 41 42 43 45 47 48
Iter 1 , selection:
           , conditional-screening: 4 8 11 12 13 14 15 16 17 18 19 21 22 25 26 30 31 32 33
34 35 36 37 39 44 46 49 50
              screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 Iter 2, selection: 1 2 3 4 5 Iter 2, conditional-screening: 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 2
                                                 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50  
Iter 3 , screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 3 , selection: 1 2 3 4 5 Model already selected
> model 12$i x
[1] 1 2 3 4 5
```

During resampling, no variables were selected.

```
> # logistic regression
  set. seed(2)
> feta = x[, 1:5]%*%b; fprob = exp(feta)/(1+exp(feta))
> y = rbinom(n, 1, fprob)
> model 21=SIS(x, y, family='binomial', tune='bic', penalty='SCAD', perm=T, q=0.9)
Iter 1, screening: 1 2 3 5 29
Iter 1, selection: 1 2 3 5 29
Iter 1 , conditional-screening: 4 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 2 , screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 2 , selection: 1 2 3 4 5
Iter 2 , conditional -screening: 7 11 17 26 27 41 4
Iter 3 , screening: 1 2 3 4 5 7 11 17 26 27 41 46
Iter 3 , selection: 1 2 3 4 5
                                                                7 11 17 26 27 41 46
Model already selected
> model 21$i x [1] 1 2 3 4 5
> model 22=SIS(x, y, family='binomial', tune='bic', varISIS='aggr', seed=21)
Iter 1 , screening: 1 2 3 5 8 9 12 16 21 24 25 26 28 29 31 35 38 39 42 45 49 50
Iter 1 , selection: 1 2 3 5 8 9 12 21 24 25 26 28 31 35 38 39 50
Iter 1 , conditional-screening: 4 6 7 10 11 13 14 15 16 17 18 19 20 22 23 27 29 30 32
33 34 36 37 40 41 42 43 44 45 46 47 48 49
Iter 2 , screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 2 , selection: 1 2 3 4 5
Iter 2 , conditional -screening: 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 3 , screening: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Iter 3 , selection: 1 2 3 4 5
Model already selected
> model 22$i x
[1] 1 2 3 4 5
```