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1. (a) $P(X_1 < X_2 < X_3)$

$= P(X_1 = \min(X_1, X_2, X_3)) \cdot P(X_2 < X_3)$

$= \dfrac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \cdot \dfrac{\lambda_2}{\lambda_2 + \lambda_3}$

(b) $P(X_1 < X_2 \mid \max(X_1, X_2, X_3) = X_3)$

$= \dfrac{P(X_1 < X_2 < X_3)}{P(\max(X_1, X_2, X_3) = X_3)} = \dfrac{P(X_1 < X_2 < X_3)}{P(X_1 < X_2 < X_3) + P(X_2 < X_1 < X_3)} = \dfrac{\frac{1}{\lambda_2 + \lambda_3}}{\frac{1}{\lambda_2 + \lambda_3} + \frac{1}{\lambda_1 + \lambda_3}}$

(c) $E(\min(X_1, X_2, X_3)) = \dfrac{1}{\lambda_1 + \lambda_2 + \lambda_3}$

(d) $Var(\min(X_1, X_2, X_3)) = E(\min(X_1, X_2, X_3)^2) - E(\min(X_1, X_2, X_3))^2$

$E(\min(X_1, X_2, X_3)^2) = E(X_1^2) \cdot P(\min(X_1, X_2, X_3) = X_1) + E(X_2^2) \cdot P(\min(X_1, X_2, X_3) = X_2)$
$+ E(X_3^2) \cdot P(\min(X_1, X_2, X_3) = X_3)$

$= \dfrac{1}{\lambda_1 + \lambda_2 + \lambda_3} \left(\dfrac{\lambda_1}{\lambda_1^2} + \dfrac{\lambda_2}{\lambda_2^2} + \dfrac{\lambda_3}{\lambda_3^2} \right)$

$= \dfrac{1}{\lambda_1 + \lambda_2 + \lambda_3} \cdot \left(\dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2} + \dfrac{1}{\lambda_3} \right)$

2. T_1 : amount of time ~~spent by~~ 1:00 appointment last

T_2 : amount of time ~~spent by~~ 1:30 appointment last

$T_i \overset{iid}{\sim} Exp(\frac{1}{30})$

X : amount of time spent by 1:30 appointment

$E(X) = E(X \mid T_1 \le 30) \cdot P(T_1 \le 30) + E(X \mid T_1 > 30) \cdot P(T_1 > 30)$

$= E(T_2) \cdot P(T_1 \le 30) + E(T_1 + T_2) \cdot P(T_1 > 30)$

$= 30 \cdot (1 - e^{-1}) + 60 \cdot e^{-1}$

$= 30(1 + e^{-1})$

3. $N(t)$: # of parking surveillance occurance 2016711704 성예슨

$N(t) \sim PP(\lambda)$

$\sim Pois(\lambda t)$

$P(N(t)>0) = P(N(2)>0)$

$= 1 - P(N(2)=0)$

$= 1 - e^{-2\lambda}$

4. $N(t) \sim Pois(\lambda t)$

(a) Since $N(t)$ is given, just denote given $N(t)$ as n.

$X = (t-S_1) + (t-S_2) + \cdots + (t-S_n)$

$= \sum_{i=1}^{n} (t - S_n) = nt - \sum_{i=1}^{n} S_n$

$E(X|N(t)) = E(nt - \sum^{n} S_n | n)$

$= nt - E(\sum S_n | n)$

$= nt -$

5. $N(t) \sim Pois(\lambda t)$ 2016711704 성예슬

(a) $Cov(T, N(T))$

$= E(T \cdot N(T)) - E(T) \cdot E(N(T))$

$= E_T(E(T \cdot N(T) | T)) - E(T) \cdot E_T(E(N(T)|T))$

$= E_T(T \cdot E(N(T))) - \mu^2 \lambda$

$= E_T(\lambda T^2) - \mu^2 \lambda$

$= \lambda(\mu^2 + 6^2) - \mu^2 \lambda \quad = 6^2 \lambda$

(b) $var(N(T))$

$= E_T(Var(N(T)|T)) + Var_T(E(N(T)|T))$

$= E_T(\lambda T) + Var_T(\lambda T)$

$= \lambda \cdot \mu + \lambda^2 \cdot 6^2$

6. $0 \le s \le t$

(a) $E(B(t) | B(s) = y) = E(B(t) - B(s) + B(s) | B(s) = y)$

$= E(B(t) - B(s)) + y$

$= 0 + y = y$

(b) $B(t) \sim N(0, t)$ since $0 \le t \le 1$, $t^2 < t$

$tB(1) \sim N(0, t^2)$

$var(B(t) - tB(1)) = t - t^2 > 0.$

7. (a) $\binom{4}{2} p^2 (1-p)^2 = 6p^2(1-p)^2$

(b) $P(Z_2 = 1 | X_3 = 1) = \dfrac{P(Z_2 = 1, X_3 = 1)}{P(X_3 = 1)} = \dfrac{2 \times p^2 (1-p)}{\binom{3}{2} p^2 (1-p)} = \dfrac{2}{3}$

CamScanner로 스캔하기

8. $P(\max_{0 \le i \le t} |X_i| \ge a)$

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Let T_a be first time that X_i hits $a = \min_t \{ X_t \ge a \}$

$$P(|X_t| \ge a) = P(|X_t| \ge a \mid T_a \le t) \cdot P(T_a \le t) + P(|X_t| \ge a \mid T_a > t) \cdot P(T_a > t)$$