STA 3021: Stochastic Processes
Quiz 1 (April 4, 2018)

Student ID: ______________________ **Name:** ______________________

1. (3 points) Write down three axioms of probability on $(S, \mathcal{F}, P)$ as precisely as you can.

i) $P(A) \geq 0$ for all $A \in \mathcal{F}$

ii) $P(S) = 1$

iii) $A_1, A_2, \in \mathcal{F}$ and disjoint, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

2. (7 points) Let $(S, \mathcal{F}, P)$ is a given probability model, that is, $P(\cdot)$ satisfies three axioms to be a probability measure. Then, for the conditional probability $P(\cdot|B)$ with $P(B) > 0$, show that it also satisfies axioms of probability.

i) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} \geq 0$ since $P(\cdot)$ is a probability

ii) $P(S|B) = \dfrac{P(B)}{P(B)} = 1$

iii) For mutually disjoint events,

$$P\left(\bigcup A_i \mid B\right) = \frac{P\left(\bigcup_{i=1}^{\infty} A_i \cap B\right)}{P(B)}$$

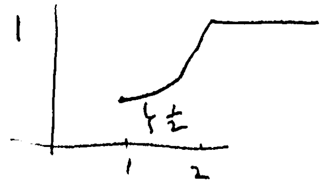
$$= \frac{P\left(\bigcup_{i=1}^{\infty} (A_i \cap B)\right)}{P(B)}$$

$$= \frac{\sum_{i=1}^{\infty} P(A_i \cap B)}{P(B)} = \sum_{i=1}^{\infty} P(A_i | B)$$

Since $(A_i \cap B)$ are also mutually disjoint

3. (5 points) For a random variable X with cdf

$$F(x) = \begin{cases} 0, & \text{for } x < 1, \\ \frac{x^2-2x+2}{2}, & \text{for } 1 \le x < 2, \\ 1, & \text{for } x \ge 2. \end{cases}$$



find EX^4.

$$EX^4 = 1^4 P(X=1) + \int_1^2 x^4 (x-1)\, dx$$

$$= 4.8$$

4. (5 points) Data indicate that the number of traffic accidents in Berkeley on a rainy day is a Poisson random variable with mean 9, whereas on a dry day it is a Poisson random variable with mean 3. Let X denote the number of traffic accidents tomorrow. If it will rain tomorrow with probability .6, find Var(X).

$X|\text{rain} \sim \text{Poisson}(9)$ $\quad X|\text{dry} \sim \text{Poisson}(3)$ $\quad P(\text{rain}) = .6$

$$EX = E(X|\text{rain})P(\text{rain}) + E(X|\text{dry})P(\text{dry}) = 6.6$$

$$EX^2 = 58.8$$

$$\text{Var}X = EX^2 - (EX)^2 = 15.24$$

End of quiz