Homework III (2021)

Do Problems 5.1 and 5.4 in Montgomery.

1. In the two-way ANOVA model, $y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijl}$ for $i = 1, \ldots, a$, $j = 1, \ldots, b$, and $k = 1, \ldots, n$ where $\sum_i \tau_i = 0$, $\sum_{j=1}^b \beta_j = 0$, $\sum_i (\tau\beta)_{ij} = \sum_j (\tau\beta)_{ij} = 0$ and $\epsilon_{ijk} \sim^{iid} N(0, \sigma^2)$. Then prove the following results:

   (a) $E(MS_A) = \sigma^2 + \frac{bn}{a-1} \sum_{i=1}^a \tau_i^2$.

   (b) $E(MS_B) = \sigma^2 + \frac{an \sum_{j=1}^b \beta_j^2}{b-1}$.

   (c) $E(MS_{AB}) = \sigma^2 + \frac{n \sum_{i=1}^a \sum_{j=1}^b (\tau\beta)_{ij}^2}{(a-1)(b-1)}$.

2. An experiment was conducted to determine the effects of various pesticides on the yield of fruit from three different varieties (B1, B2, B3) of lemon tree. The pesticide conditions utilized in the experiment were A1=Control (no pesticide), A2=Chemical X, A3=Chemical Y, A4=Mixture of chemicals X and Y. The lemon varieties used were B1=Standard variety (the variety that is currently most commonly grown), B2=New genetically engineered variety 1, and B3=New genetically engineered variety 2. Eight trees from each variety were randomly selected from an orchard. The four levels of pesticide were then randomly assigned to two trees of each variety and pesticide applications were made. Yields of fruit, in bushels per tree, were obtained after the test period. The data appear below:

		Pesticide		
Variety	A1	A2	A3	A4
B1	49, 39	50, 55	43, 38	53, 48
B2	55, 41	67, 58	53, 42	80, 73
B3	66, 68	85, 92	69, 62	89, 99

   (a) Write the appropriate two-way ANOVA model for analyzing these data. Perform the analysis of variance corresponding to this model and draw conclusions about the effects.

   (b) Construct and test a set of orthogonal contrasts to address the following questions about the marginal effects of the pesticide treatments:

      i. Does using chemical X increase the mean yield?

      ii. Does using chemical Y increase the mean yield?

      iii. Do chemicals X and Y interact, so that the effect of using chemical X differs when chemical Y is used from its effect when chemical Y is not used?

(c) Construct and test contrasts in the marginal variety means to address the following questions:

    i. Does genetically engineered variety 1 produce higher mean yields than the standard variety?

    ii. Does genetically engineered variety 2 produce higher mean yields than the standard variety?

(d) Use the contrasts that you formed in parts (b) and (c) to form interaction contrasts. (You should end up with $3 \times 2 = 6$ interaction contrasts.) Interpret these contrasts (i.e., what questions do they address?). Test these interaction contrasts as planned comparisons and state your conclusions.

(e) Were the contrasts in par (b) mutually orthogonal? How about the contrasts in part (c)?