Experimental Design: Exam II (2021)

Name: Student id#: Department:

1. (20 points) For each of the following statements, answer right true (T) or false (F):
   (a) Rules for expected mean squares (EMS) work well in BIBD or Latin square design.
   (b) When we check the treatment effects in BIBD, we can use types I and II SS.
   (c) When the interaction of the two factors is significant in two-factor factorial design, the optimal level of the treatment combination is same to that of 'one-factor-a-time design'.
   (d) 2^k factorial design is often used to at the early stage of experimentation to detect potential candidate factors for more detailed investigation.
   (e) 2^k design with a block via confounding is an incomplete block design to sacrifice a specific treatment effect. Therefore, the specific treatment effect is partially confounded with the block.
   (f) In PROC MIXED using option METHOD=TYPE1, the estimation of variance components uses the maximum likelihood method.
   (g) Even though we use different kinds of constraints ($\sum_{i=1}^{a} \tau_i = 0$ or $\tau_a = 0$) in one-way ANOVA models, p-values of the test statistics for $H_0 : \tau_1 = \cdots = \tau_a = 0$ are same.
   (h) In a nested design, the levels of one factor, B, will not be identical across all levels of another factor, A. Factor A will contain different levels of factor B. In this case, the levels of A are said to be nested within the levels of B.
   (i) In an one-way ANOVA model with random effects, when the similarity of the observations within group is very high, the intraclass correlation coefficient is small.
   (j) In a split-plot designs, the experimental units are nested within the blocks, and a separate random assignment of units to treatments is made within each block.

2. (25 points) Consider the following SAS output from analysis of a balanced incomplete block design (BIBD). The statistical model is

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}.$$

We assume that all factors are fixed. τ_i and β_j are respectively treatment and block effects.

(a) Write out the remaining conditions for the above model.

(b) What are the hypotheses of interest? Should the hypothesis be rejected? Why, or why not?

(c) If the grand mean is 72.50, compute $\hat{\tau}_1, \cdots, \hat{\tau}_4$ (treatment effect).

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Source DF  Sum of Squares  Mean Square  F Value    Pr>F
Model   6      77.7500000   12.95833333   19.94  0.0024
Error   5       3.2500000    0.65000000
Total  11      81.0000000

Source DF  Type III SS    Mean Square  F Value   Pr>F
Block  3   66.08333333    22.02777778    33.89  .0010
Trt    3   22.75000000     7.58333333    11.67  .0107

Trt     y LSMEAN Standard Error LSMEAN Number
1   71.37500000      0.4868051             1
2   71.62500000      0.4868051             2
3   72.00000000      0.4868051             3
4   75.00000000      0.4868051             4
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3. (27 points) A horticulturist was interested in the phosphorus content in the leaves of a particular variety of apple tree. Five leaves from each of three randomly selected trees were measured for phosphorus content. The data from this study are as follows:

Tree	Phosphorus Content					Sum	Mean
1	.35	.40	.58	.50	.47	2.30	0.46
2	.65	.70	.90	.84	.79	3.88	0.78
3	.60	.80	.75	.73	.66	3.54	0.71
Total						9.72	0.65

Of central interest in this study was the tree-to-tree variability in phosphorous content. A partial ANOVA Table for these data is as follow:

Source of Variation	Sum of Squares	d.f.	Mean Squares	F
Treatment(Trees)	0.277	−−	−−	−−
Error	−−	−−	−−	
Total	0.374	−−		

(a) Fill in the ANOVA Table.

(b) State the hypothesis being tested by the F test in this table. Should the hypothesis be rejected? Why, or why not?

(c) Obtain point estimates of the within-tree and between-tree components of variance from these data.

(d) A 95% confidence interval for $\theta = \sigma_\tau^2/\sigma^2$ is (0.466, 133.57). Obtain a point estimate and a 95% confidence interval for the intraclass correlation coefficient.

4. (28 points) An experiment was conducted to compare 6 batches of auto body side panels in terms of deviations from nominal position (y). The engineer samples 2 "groups" of body panels from each batch (that is, the 2 "groups" for batch 1 differ from those from batch 2, etc..., implying "groups" are nested under batches). Each "group" has 3 individual body panels selected and measured (replicates) for y. Note that these are a random sample of batches (random effects), and the "groups" used are a sample from a larger population of "groups" (random effects).

$$\text{Model:} \quad y_{ijk} = \mu + \tau_i + \beta_{j(i)} + \epsilon_{ijk}, \text{ for } i = 1, \cdots, 6; \quad j = 1, 2; \quad k = 1, 2, 3,$$
$$\tau_i \sim N(0, \sigma_\alpha^2), \quad \beta_{j(i)} \sim N(0, \sigma_\beta^2), \quad \epsilon_{ijk} \sim N(0, \sigma^2).$$

(a) The 6 batch mean $y-$values are given below. Compute the overall mean, and obtain the sum of squares for batches:

$$\bar{y}_{1\cdot} = 4.00, \quad \bar{y}_{2\cdot} = 2.02, \quad \bar{y}_{3\cdot} = -4.57, \quad \bar{y}_{4\cdot} = -1.12, \quad \bar{y}_{5\cdot} = 4.05, \quad \bar{y}_{6\cdot} = -1.10.$$

(b) Complete the following partial ANOVA table:

Source	df	SS	MS	F	$F_{(0.05)}$
Batch					
Group(Batch)		62.5			
Error		438.57			
Total					

(c) Using 'rules for expected mean squares', find expectations of mean squares.

(d) Give unbiased estimates of each of the variance components:

$$\hat{\sigma}^2_{Batch} = \qquad \hat{\sigma}_{Group(Batch)})^2 = \qquad \hat{\sigma}^2 =$$

(e) Present null hypotheses, $F-$tests, and critical values.