# STA 3021: Stochastic Processes
## Midterm 1 (6:15 PM - 7:30 PM on Sep 27, 2021)

**Instructions:**

- This test is a closed book exam, but you are allowed to use calculator. Clarity of your answer will also be a part of credit. When needed, use the notation $\Phi(z) = P(Z < z)$ for a standard normal distribution $Z$. Show your ALL work neatly.
- Your answer sheets must be written in English.
- Remind that you can submit your answer sheets over icampus in a **pdf** file format ONLY.
- By submitting your report online, it is assumed that you agree with the following pledge;

  **Pledge**: *I have neither given nor received any unauthorized aid during this exam.*
- Don't forget to write down your name and student ID on your answer sheet.

1. (10 points) State the following theorems/definitions as precisely as you can.
   (a) Axioms of Probability.
   (b) Let $X_1, \ldots, X_n$ be a sequence of IID random variables with mean $\mu$ and variance $\sigma^2$. State the central limit theorem.

2. (10 points) A fair die is tossed until a 2 is obtained. If $X$ is the number of trials required to obtain the first 2, what is the smallest value of $x$ such that $P(X \leq x) \geq .5$?

3. (10 points) For a random variable $Z$ with cdf

$$F(z) = \begin{cases} 0, & z < 1, \\ \frac{z^2-2z+2}{2}, & 1 \leq z < 2, \\ 1, & z \geq 2 \end{cases}$$

   Sketch the cdf on a graph and find $\mathrm{E}(Z^2)$.

4. (10 points) In a class there are four freshman boys, six freshman girls, and six sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

5. (15 points) Consider $n$ people and suppose that each of them has a birthday that is equally likely to be any of the 365 days of the year. Furthermore, assume that their birthdays are independent, and let A be the event that no two of them share the same birthday. Employ the Poisson paradigm to approximate $P(A)$.

6. (15 points) Let $(X, Y)$ be a bivariate random variable with pdf

$$f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Find the marginal pdf of $X$ and $Y$.

7. (15 points) Let $X \sim \text{Gamma}(r, \lambda)$, $r > 0, \lambda > 0$ with pdf

$$\frac{1}{\Gamma(r)} \lambda^r x^{r-1} e^{-\lambda x} 1_{\{x>0\}}.$$

Find the MGF of $M_X(t)$.

8. (15 points) Show that

$$P\Big( \cup_{i=1}^n E_i \Big) \leq \sum_{i=1}^n P(E_i).$$