Sample problmes

1. Suppose that $X_1, \ldots, X_n$ be a random sample from pdf $f(x;\lambda) = \frac{1}{\lambda}e^{-x/\lambda}$, $x > 0$, $\lambda > 0$.
   (a) Find the asymptotic distribution of $\sqrt{n}((\overline{X}_n)^2 - \lambda^2)$.
   (b) Find a function g such that the asymptotic distribution of $\sqrt{n}(g(\overline{X}_n) - g(\lambda))$ does not depend on λ.

2. Suppose that $X_1, \ldots, X_n$ is a random sample from $N(0, \theta)$, $0 < \theta < \infty$.
   (a) Find the MLE of θ.
   (b) Show that the MLE of θ is an unbiased estimator.
   (c) Compute the efficiency of the MLE of θ.

3. Suppose $X_1, \ldots, X_n$ is a random sample from pdf $f(x;\theta) = \theta x^{-\theta-1}$, $1 < x$, $0 < \theta$.
   (a) Find a sufficient statistic for θ.
   (b) Find the MLE of $\eta = P(X_1 > e)$.
   (c) Find the asymptotic distribution of $\hat{\eta}$, where $\hat{\eta}$ is the MLE of η.

4. Let $X_1, \cdots, X_n$ is a random sample from $N(\mu, 1)$ and $Y_1, \cdots, Y_m$ be a random sample from $N(\mu, 4)$. Assuming that X_i's and Y_j's are independnet for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$.
   (a) Show that $\hat{\mu}_1 = (\bar{X}_n + \bar{Y}_m)/2$ is unbiased for μ, where $\bar{X}_n = \sum_{i=1}^n X_i/n$ and $\bar{X}_m = \sum_{j=1}^m Y_j/m$.
   (b) Find the MLE of μ.
   (c) Show that $\hat{\mu}^{MLE}$ is more efficient estimator than $\hat{\mu}_1$.

5. Suppose that $X_1, \ldots, X_n$ is a random sample from $f(x;\theta) = e^{-(x-2\theta)}$, $x > 2\theta$, $-\infty < \theta < \infty$.
   (a) Find a sufficient statistic for θ.
   (b) Let T be the sufficient statistic obtained from (a). Find $\phi(T)$ that satisfies $E(\phi(T)) = \theta$.