Confounding the 2^K factorial design in two blocks

$L = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_K x_K \pmod 2$: defining contrast (정의대비)

여기서 $\alpha_i = \begin{cases} 1 & \text{교락(confounding)될 효과에 } i\text{번째 요인이 나타날 때} \\ 0 & \text{o/w} \end{cases}$

$x_i = \begin{cases} 1 & \text{특정처리조합에 나타나는 } i\text{번째 요인의 수준으로서} \\ & \text{그 요인이 높은 수준에 있을 때.} \\ 0 & \text{낮은 수준에 있을 때.} \end{cases}$

ex1) 2^2 factorial design

AB와 block이 confounding 시킬 때,

$AB = A^{\alpha_1} B^{\alpha_2} = A^1 B^1 \Rightarrow \alpha_1 = \alpha_2 = 1$

$L = x_1 + x_2 \pmod 2$

(1) : $L = 0+0 = 0 \pmod 2 \Rightarrow 0$

a : $L = 1+0 = 1 \pmod 2 \Rightarrow 1$

b : $L = 0+1 = 1 \pmod 2 \Rightarrow 1$

ab : $L = 1+1 = 2 \pmod 2 \Rightarrow 0$

따라서

block 1	block 2
(1)	a
ab	b

ex2) 2^3 factorial design

- ABC가 block의 confounding 시킬 때,

$ABC = A^{\alpha_1} B^{\alpha_2} C^{\alpha_3} = A^1 B^1 C^1$

$L = x_1 + x_2 + x_3 \pmod 2$

(1) : $0+0+0 = 0 \pmod 2 \Rightarrow 0$

a : $1+0+0 = 1 \pmod 2 \Rightarrow 1$

b : $0+1+0 = 1$ " $\Rightarrow 1$

c : $0+0+1 = 1$ " $\Rightarrow 1$

ab : $1+1+0 = 2$ " $\Rightarrow 0$

ac : $1+0+1 = 2$ " $\Rightarrow 0$

bc : $0+1+1 = 2$ " $\Rightarrow 0$

abc : $1+1+1 = 3$ " $\Rightarrow 1$

block 1	block 2
(1)	a
ab	b
ac	c
bc	abc

In practice, we have three replicates,

Rep 1		Rep 2		Rep 3	
block 1	block 2	block 1	block 2	block 1	block 2
(1)	a	b	ab	ac	b
ab	b	c	(1)	ac	abc
ac	c	abc	bc	(1)	c
bc	abc	a	ac	bc	a

Replicates are considered as block effect.

In addition, there are two blocks in each replicate.

Then the model is

$$y_{ijklm} = \mu + \rho_i + \eta_j + (\rho\eta)_{ij} + \alpha_k + \beta_l + \gamma_m + (\alpha\beta)_{kl} + (\alpha\gamma)_{km} + (\beta\gamma)_{lm} + \varepsilon_{ijkl}$$

(ρ_i: replicates; η_j: block)

Note that there is no ABC effect.

Source	df	
Replicates	$r-1$	$r=3$
Blocks	1	
Rep * block	$r-1$	
A	1	
B	1	
C	1	
AB	1	
AC	1	
BC	1	
Error	$6(r-1)$	
Total	$8r-1$	

- AC가 block에 confounding 시킬 때,

$AC = A^1B^0C^1$

$L = x_1 + x_3 \pmod 2$

(1) : $0+0 = 0 \pmod 2 \Rightarrow 0$

a : $1+0 = 1$ " $\Rightarrow 1$

b : $0+0 = 0$ " $\Rightarrow 0$

c : $0+1 = 1$ " $\Rightarrow 1$

ab : $1+0 = 1$ " $\Rightarrow 1$

ac : $1+1 = 2$ " $\Rightarrow 0$

bc : $0+1 = 1$ " $\Rightarrow 1$

abc : $1+1 = 2$ " $\Rightarrow 0$

block 1	block 2
(1)	a
b	c
ac	ab
abc	bc

Confounding the 2^k factorial design in Four blocks

Select two effects ADE and BCE in 2^5 to be confounded.

Two defining contrasts:

$$L_1 = x_1 + x_4 + x_5 \pmod 2$$

$$L_2 = x_2 + x_3 + x_5 \pmod 2.$$

Then

$$(L_1, L_2) = \{(0,0), (0,1), (1,0), (1,1)\}$$

block 1	block 2	block 3	block 4
(1) abc	a be	b abce	e abcde
ad ace	d abde	abd ae	ade bd
bc cde	abc ce	c bcde	bce ac
abcd bce	bcd acde	acd de	ab cd
$L_1=0$	$L_1=1$	$L_1=0$	$L_1=1$
$L_2=0$	$L_2=0$	$L_2=1$	$L_2=1$.

ADE, BCE and ABCD are confounded

(∵) $ADE \times BCE = ABCDE^2 = ABCD$

$\uparrow$

$E^2=1$

Partial Confounding

Confound different interactions in different replicates.

For example,

2^3 in 4 replicates.

Each replicate has 2 blocks of size 4.

Confound AB in Rep. 1, AC in Rep 2, BC in Rep. 3 and ABC in Rep. 4.

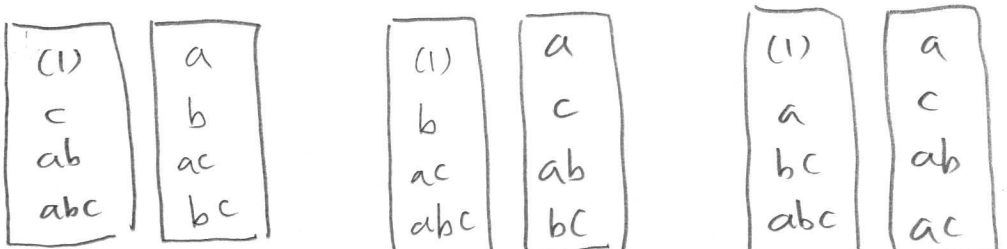
$AB = (1) + c + ab + abc) - (a + b + ac + bc)$

$AC = (1) + b + ac + abc) - (a + c + ab + bc)$

$BC = (1) + a + bc + abc) - (a + c + ab + ac)$

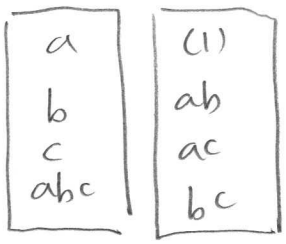
$ABC = (a + b + c + abc) - (1) + ab + ac + bc)$

Rep 1. (AB) Rep 2 (AC) Rep. 3 (BC)



block 1 block 2 block 3 block 4 block 5 block 6.

Rep 4 (ABC)



block 7 block 8

$$y_{ijkl} = \mu + \rho_i + \eta_{j(i)} + \alpha_k + \beta_l + \gamma_m + (\alpha\beta)_{kl} + (\alpha\gamma)_{km} + (\beta\gamma)_{lm} + (\alpha\beta\gamma)_{klm} + \varepsilon_{ijklm}$$

(replicates; block (nested within replicates); A; B; C)

source	df
Replicates	$r-1=4-1=3$
Blocks (Rep.)	$r(2-1)=4\times1=4$
A	1
B	1
C	1
$AB_{(II,III,IV)}$	1
AC (I, III, IV)	1
BC (I, II, IV)	1
ABC (I II III)	1
Error	17
Total	$8r-1=31$