

## Homework II (2022)

Please solve the following problems and then submit the pdf copy of them.

1. The following table classifies a sample of psychiatric patients by their diagnosis and by whether their treatment prescribed drugs.
  - (a) Obtain standardized Pearson residuals for independence, and interpret.
  - (b) Partition chi-squared into three components to describe differences and similarities among the diagnoses, by comparing (i) the first two rows, (ii) the third and fourth rows, and (iii) the last row to the first and second rows combined and the third and fourth rows combined.

Diagnosis	Drugs	No Drugs
Schizophrenia	105	8
Affective disorder	12	2
Neurosis	18	19
Personality disorder	47	52
Special symptoms	0	13

*Source:* Reprinted with permission from E. Helmes and G.C. Fekken, *J. Clin. Psychol.* **42**: 569-576 (1986).

2. Using models, analyze and interpret the data in the following table on smoking habits of students in Arizona high schools (Consider a proper model and conduct proper tests for corresponding parameters and goodness-of-fit).

	Student Smokes	Student Does Not Smoke
Both parents smoke	400	1380
One parent smokes	416	1823
Neither parent smokes	188	1168

3. Given  $\lambda$ , the distribution of  $Y$  follows  $Poisson(\lambda)$ ;  $\lambda$  follows  $Gamma(k, \mu/k)$ . The pdf of  $Gamma(\alpha, \beta)$  is

$$p(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right).$$

Then show the following results:

- (a) The marginal distribution of  $Y$  follows negative binomial distribution given by:

$$P(y; k, \mu) = \frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)} \left( \frac{k}{\mu+k} \right)^k \left( 1 - \frac{k}{\mu+k} \right)^y,$$

for  $y = 0, 1, 2, \dots$

- (b)  $E(Y) = \mu$ ;  $var(Y) = \mu + \frac{1}{k}\mu^2$ .