Typesetting a math thesis with `ccool`

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Abstract

To test and showcase `ccool`[2], here's a portion of an applied math thesis[1, pp. 25-27] typeset using it. The source file, `thesis.tex`, is set up to accept `llncs`[3] as argument to `\documentclass`.

Contents

This document was generated using class other than llncs and 2020/04/12 v1.8 cool — Custom COntext Oriented for LaTeX.

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Listing 1. Preamble[a]

```
\usepackage{amsmath, amsthm, commath}
\usepackage{csquotes}
\usepackage[T1]{fontenc}
\newtheorem{lemma}{Lemma}
\AfterEndEnvironment{lemma}{\CcoolHook}
\newtheorem{definition}{Definition}
\AfterEndEnvironment{definition}{\CcoolHook}
```

Listing 2. Short

```
\Ccool{ SuchThat = { ;~ }, Time = { t }, Process = { \xi },
  StopT = T, EvalAt = \CcoolLambda{(#1)} }
[The CUSUM statistic process and the corresponding one-sided
  CUSUM stopping time are defined as follows:
\begin{definition}\label{the CUSUM statistic} Let~]
  { Scale = { \lambda }, Real = {\mathcal{R}}
  }+*s{{~\in~}}[~and~]
  { CUSUMthresh = { \nu } }+*o{$#1\in\Real^{+}$.}
  [~Define the following processes:]
  { LogWald = { u },  CUSUMst = { \StopT_{c} }, CUSUM = { y },
  LogWaldInf = { m } }+
  [\begin{enumerate}
  \item{$\LogWald_{\Time}\EvalAt{ \Scale } = \Scale
  \Process_{\Time} - \frac{1}{2} \Scale^2\Time$;
      $\LogWaldInf_{\Time}\EvalAt{ \Scale } = \inf_{ 0\le s \le
  \Time } \CUSUM_{s}\EvalAt{ \Scale }$.}
  \item{$\CUSUM_{\Time}\EvalAt{ \Scale } =
  \LogWaldInf_{\Time}\EvalAt{ \Scale } -
  \LogWald_{\Time}\EvalAt{ \Scale }\ge0$, which is the CUSUM
  statistic process.}
  \item{$\CUSUMst\EvalAt{ \Scale, \LogWaldInf } = \inf \left[
  \Time \ge 0 \SuchThat \CUSUM_{\Time}\EvalAt{\Scale} \ge
  \LogWaldInf \right]$, which is the CUSUM stopping time.}
  \end{enumerate}\end{definition}\par]{}
```

- -

The CUSUM statistic process and the corresponding one-sided CUSUM stopping time are defined as follows:

Definition 1. *Let* $\lambda \in \mathcal{R}$ *and* $\nu \in \mathcal{R}^+$. *Define the following processes:*

1. $u_t(\lambda) = \lambda\xi_t - \frac{1}{2}\lambda^2 t$; $m_t(\lambda) = \inf_{0 \le s \le t} y_s(\lambda)$.

2. $y_t(\lambda) = m_t(\lambda) - u_t(\lambda) \ge 0$, *which is the CUSUM statistic process.*

3. $T_c(\lambda, m) = \inf\left[t \ge 0; y_t(\lambda) \ge m\right]$, *which is the CUSUM stopping time.*

Listing 3. Long

```
  \Ccool
  {
    Cond = { \mid },
    Cont = { C },
    EssSup = { \textrm{ess\,sup} },
    Expect = { E },
    EvalAt = \CcoolLambda{(#1)},
    Event = \CcoolLambda{\left\{#1\right\}},
    Integrate = \CcoolLambda[4]{\int_{#1}^{#2} #3 \dif#4},
    Measure = { P },
    OpRHS = \CcoolLambda{\left[#1\right]},
    Sequence = \CcoolLambda{\left\{#1\right\}},
    SigmaAlg = { \mathcal{F} },
    SigmaAlgOp = { \operatorname{\sigma} },
    SuchThat = { ;~ },
    Time = { t },
    Union = { \cup }
  }
  [We begin by considering the observation process~]
  { Process = { \xi } }*o{$\Sequence{#1}_{\Time>0}$}
  [ with the following dynamics:]
  { Drift = { \mu_i }, Noise = { w }, ChangePoint = { \tau } }
  [\begin{displaymath}
    \Process_{\Time} = \left\{
      \begin{array}{ll}
        \dif\Noise_{\Time}  & \Time \le \ChangePoint \\
        \Drift\, \dif\Time + \dif\Noise_{\Time} &  t >
    \ChangePoint,~~i=1,2.
      \end{array}\right\}
  \end{displaymath}where $\ChangePoint$, the time of change, is
    assumed to be an unknown constant; $\Drift$, the possible
    drifts the process can change to, are assumed known, but the
    specific drift the process is changing to is assumed to be
    determined by nature and is thus unknown. Our goal is to
    detect the change and not to infer which of the change
    occurred.]{}
  \Ccool[The probability triple is~]
  { Space = { \Cont{[}0,\infty{)} }, IdxMeasure = {
    \Measure^i_{\ChangePoint} } }
  [\begin{equation*}
    \left(\Space, \SigmaAlg_{\infty},
    \Sequence{\SigmaAlg_{\Time}},
    \Measure\right)~\forall~i=1,2,~\rm{and}~\ChangePoint~\in~{[}0,\infty{)},
  \end{equation*}where $\Space$ is the space of continuous
    functions,
  $\SigmaAlg_{\Time} = \SigmaAlgOp\left\{\Process_{\Time}, \Time
```

```
  \ge 0\right\}$, $\SigmaAlg_{\infty} =
  \left\{\SigmaAlg_{\Time}\right\}_{\Time=\infty} =
  \Union_{\Time>0} \SigmaAlg_{\Time}$, and $\IdxMeasure$ is the
  family of probability measures generated by the observation
  process $\Sequence{\Process_{\Time}}$ when the change is
  $i=1,2$ and the change-point is $\ChangePoint$. Notice that
  $\Measure_{\Time=\infty}$ is the Wiener measure.\par]{}

\Ccool[The objective is to detect the change as soonas
  possible, which is achieved through the means of a stopping
  rule ]
{ StopT = T }*
[ adapted to the filtration $\SigmaAlg_{\Time}$. This means
  that at each instant $\Time$ the information that is
  available up to that instant. If~]
{ MeasureStop = { \Measure_{\ChangePoint} } }*
[~is the true distribution, then in the event that
  $\Event{\StopT \ge \ChangePoint}$ it is desired that the
  conditional expectation of $\StopT-\ChangePoint$ should be
  small. Notice that $\Event{\StopT \ge \ChangePoint} \in~$]
{ SigmaAlgStop = { \SigmaAlg_{\ChangePoint} } }*[.\par]{}

\Ccool[But  of course, $\forall~\Time > \ChangePoint$,
  $\left\{\StopT = \Time\right\} \in  \SigmaAlg_{t} \supset
  \SigmaAlgStop$. One of the possible performance measures of
  the  detection delay, suggested by Lorden in
  \textdel{\cite{Lorden:1971}},
considers the worst detection delay over all paths before the
  change and all possible change points $\ChangePoint$. It is]
{ Penalty = J }
[\begin{equation}
  {\Penalty}\EvalAt{\ChangePoint}=\sup_{\ChangePoint}\,
  \EssSup\, \Expect_{\ChangePoint}\OpRHS{
  (\StopT-\ChangePoint)^{+}\Cond\SigmaAlgStop },
  \label{oLorden}
\end{equation}  giving rise to the following constrained
  stochastic optimization problem:]{}

\Ccool{ OptimThresh = { \gamma } }
[\begin{equation}
  \begin{array}{c}
    \displaystyle \inf_{\StopT} {\Penalty}\EvalAt{\StopT}\\
    \Expect_{\infty}\OpRHS{\StopT} \geq \OptimThresh.
  \end{array}\label{eqnproblemL}
\end{equation}\par]{}

\Ccool[One can immediately notice that the small detection
  delay requirement is offset by the requirement that the
```

```
   frequency of "false reactions" be controlled. In other words,
   the meaning of the requirement that
   $\Expect_{\infty}\OpRHS{\StopT}\geq \OptimThresh$ is that,
   the mean time between alarms under the Wiener measure (i.e.
   the  measure corresponding to there not being any change) is
   at least  as big as $\OptimThresh$. One can also write
   $$\Expect_{\infty}\OpRHS{\StopT} = \Integrate{0}{\infty}{
   \Measure_{\infty}\OpRHS{\StopT > \Time} }{\Time}$$  and
   notice that the above requirement is equivalent to the
   requirement of a small
   $\Measure_{\infty}\OpRHS{\StopT{<}\Time}$, which is the
   probability of a false alarm (type I error).\par]{}

\Ccool[It is easily seen that, in seeking solutions to the
   above problem,  we can restrict our attention to stopping
   times that satisfy the false alarm constraint with equality.
   This is because, if $\Expect_{\infty}\OpRHS{\StopT} >
   \OptimThresh$, we can produce a stopping time that achieves
   the constraint with equality without increasing the
   detection delay, simply by randomizing between $\StopT$ and
   the stopping time that is identically $0$. This was first
   seen by Moustakides in the discrete case \char'[see
   \textdel{\cite{Moustakides:1986}}.\char']\par]{}

\Ccool[To this effect, we introduce the following definition:
\begin{definition}Define~]
  { StopTthresh = { \mathcal{K} } }+*
  [ to be the set of all stopping rules $\StopT$ that
  are adapted to $\SigmaAlg_{\Time}$ and that satisfy
  $\Expect_{\infty}\OpRHS{T}=\OptimThresh$.
\end{definition}\par]{}

\Ccool[The CUSUM statistic process and the corresponding
  one-sided CUSUM stopping time are defined as follows:
\begin{definition}\label{the CUSUM statistic}Let~]
  { Scale = { \lambda }, Real = {\mathcal{R}} }+*s{{~\in~}}
  [~Define the following processes:]
  { CUSUMthresh = { \nu } }+*o{$#1\in\Real^{+}$.}[]
  { LogWald = { u },  CUSUMst = { \StopT_{c} }, CUSUM = { y },
  LogWaldInf = { m } }+
  [\begin{enumerate}
  \item{$\LogWald _{\Time}\EvalAt{ \Scale } = \Scale
  \Process_{\Time} - \frac{1}{2} \Scale^2\Time$;
      $\LogWaldInf _{\Time}\EvalAt{ \Scale } = \inf_{ 0\le s
  \le \Time }  \CUSUM_{s}\EvalAt{ \Scale }$.}
  \item{$\CUSUM _{\Time}\EvalAt{ \Scale } =  \LogWaldInf
  _{\Time}\EvalAt{ \Scale }-   \LogWald _{\Time}\EvalAt{ \Scale
  }\ge0$, which is the CUSUM statistic process.}
```

```
    \item{$\CUSUMst\EvalAt{ \Scale, \LogWaldInf } = \inf \OpRHS{
    \Time \ge 0 \SuchThat \CUSUM_{\Time}\EvalAt{\Scale} \ge
    \LogWaldInf }$, which is the CUSUM stopping time.}
    \end{enumerate}\end{definition}\par]{}

  \Ccool[We are now in a position to examine two very important
    properties  of the one-sided CUSUM stopping time. The first
    is a characteristic specifically inherent in the CUSUM
    statistic, and is summarized in the following lemma:
    \begin{lemma}\label{l2} Fix $\ChangePoint \in
    {[}0,\infty{)}$. Let $\Time\ge\ChangePoint$, and consider the
    process $$\CUSUM_{\Time, \ChangePoint} =
    \LogWald_{\Time}-\LogWald_{\ChangePoint}-\inf_{\ChangePoint\le
    s\le \Time}(\LogWald_{\Time}-\LogWald_{\ChangePoint}).$$
    This is the CUSUM process when starting at time
    $\ChangePoint$. We have that $\CUSUM_{\Time}\ge
    \CUSUM_{\Time,\ChangePoint}$ with equality if
    $\CUSUM_{\ChangePoint}=0$. \end{lemma}\par
  \begin{proof} Note
    that\begin{equation}\CUSUM_{\Time}=\CUSUM_{\Time,\ChangePoint}
      +\left(\inf_{\ChangePoint\le s\le
    \Time}(\LogWald_{s}-\LogWald_{\ChangePoint})
        +\CUSUM_{\ChangePoint}\right)^{+}\ge
    \CUSUM_{t,\ChangePoint} \label{restart}
    \end{equation} and that $\inf_{\ChangePoint\le s\le
    \Time}(\LogWald_{s}-\LogWald_{\ChangePoint})\le0$.
  \end{proof}]{}
```

We begin by considering the observation process $\{\xi\}_{t>0}$ with the following dynamics:

$$\xi_t = \left\{ \begin{array}{ll} \mathrm{d}w_t & t \le \tau \\ \mu_i \ \mathrm{d}t + \mathrm{d}w_t & t > \tau, \ \ i = 1, 2. \end{array} \right\}$$

where τ, the time of change, is assumed to be an unknown constant; μ_i, the possible drifts the process can change to, are assumed known, but the specific drift the process is changing to is assumed to be determined by nature and is thus unknown. Our goal is to detect the change and not to infer which of the change occurred. The probability triple is

$$\big(C[0,\infty), \mathcal{F}_\infty, \{\mathcal{F}_t\}, P \big) \ \forall \ i = 1, 2, \ \text{and} \ \tau \ \in \ [0, \infty),$$

where $C[0, \infty)$ is the space of continuous functions, $\mathcal{F}_t = \sigma\{\xi_t, t \ge 0\}$, $\mathcal{F}_\infty = \{\mathcal{F}_t\}_{t=\infty} = \cup_{t>0}\mathcal{F}_t$, and P_τ^i is the family of probability measures generated by the observation process $\{\xi_t\}$ when the change is $i = 1, 2$ and the change-point is τ. Notice that $P_{t=\infty}$ is the Wiener measure. The objective is to detect the change as soonas possible, which is achieved through the means of a stopping rule T adapted to the filtration $\mathcal{F}_t$. This

means that at each instant t the information that is available up to that instant. If P_τ is the true distribution, then in the event that $\{T \geq \tau\}$ it is desired that the conditional expectation of $T - \tau$ should be small. Notice that $\{T \geq \tau\} \in \mathcal{F}_\tau$.

But of course, $\forall\, t > \tau$, $\{T = t\} \in \mathcal{F}_t \supset \mathcal{F}_\tau$. One of the possible performance measures of the detection delay, suggested by Lorden in [], considers the worst detection delay over all paths before the change and all possible change points τ. It is

$$J(\tau) = \sup_\tau \operatorname{ess\,sup} E_\tau \left[(T - \tau)^+ | \mathcal{F}_\tau \right], \tag{1}$$

giving rise to the following constrained stochastic optimization problem:

$$\begin{aligned} \inf_T J(T) \\ E_\infty\left[T\right] \geq \gamma. \end{aligned} \tag{2}$$

One can immediately notice that the small detection delay requirement is offset by the requirement that the frequency of "false reactions" be controlled. In other words, the meaning of the requirement that $E_\infty\left[T\right] \geq \gamma$ is that, the mean time between alarms under the Wiener measure (i.e. the measure corresponding to there not being any change) is at least as big as γ. One can also write

$$E_\infty\left[T\right] = \int_0^\infty P_\infty\left[T > t\right] \mathrm{d}t$$

and notice that the above requirement is equivalent to the requirement of a small $P_\infty\left[T < t\right]$, which is the probability of a false alarm (type I error). It is easily seen that, in seeking solutions to the above problem, we can restrict our attention to stopping times that satisfy the false alarm constraint with equality. This is because, if $E_\infty\left[T\right] > \gamma$, we can produce a stopping time that achieves the constraint with equality without increasing the detection delay, simply by randomizing between T and the stopping time that is identically 0. This was first seen by Moustakides in the discrete case [see [].]

To this effect, we introduce the following definition:

Definition 2. *Define $\mathcal{K}$ to be the set of all stopping rules T that are adapted to $\mathcal{F}_t$ and that satisfy $E_\infty\left[T\right] = \gamma$.*

The CUSUM statistic process and the corresponding one-sided CUSUM stopping time are defined as follows:

Definition 3. *Let $\lambda \in \mathcal{R}$ Define the following processes:$\nu \in \mathcal{R}^+$.*

1. *$u_t(\lambda) = \lambda\xi_t - \frac{1}{2}\lambda^2 t$; $m_t(\lambda) = \inf_{0 \leq s \leq t} y_s(\lambda)$.*

2. *$y_t(\lambda) = m_t(\lambda) - u_t(\lambda) \geq 0$, which is the CUSUM statistic process.*

3. $T_c(\lambda, m) = \inf \big[t \geq 0; \ y_t(\lambda) \geq m\big]$, *which is the CUSUM stopping time.*

We are now in a position to examine two very important properties of the one-sided CUSUM stopping time. The first is a characteristic specifically inherent in the CUSUM statistic, and is summarized in the following lemma:

Lemma 1. *Fix $\tau \in [0, \infty)$. Let $t \geq \tau$, and consider the process*

$$y_{t,\tau} = u_t - u_\tau - \inf_{\tau \leq s \leq t}(u_t - u_\tau).$$

This is the CUSUM process when starting at time τ. We have that $y_t \geq y_{t,\tau}$ with equality if $y_\tau = 0$.

Proof. Note that

$$y_t = y_{t,\tau} + \left(\inf_{\tau \leq s \leq t}(u_s - u_\tau) + y_\tau\right)^+ \geq y_{t,\tau} \qquad (3)$$

and that $\inf_{\tau \leq s \leq t}(u_s - u_\tau) \leq 0$. $\qquad \square$