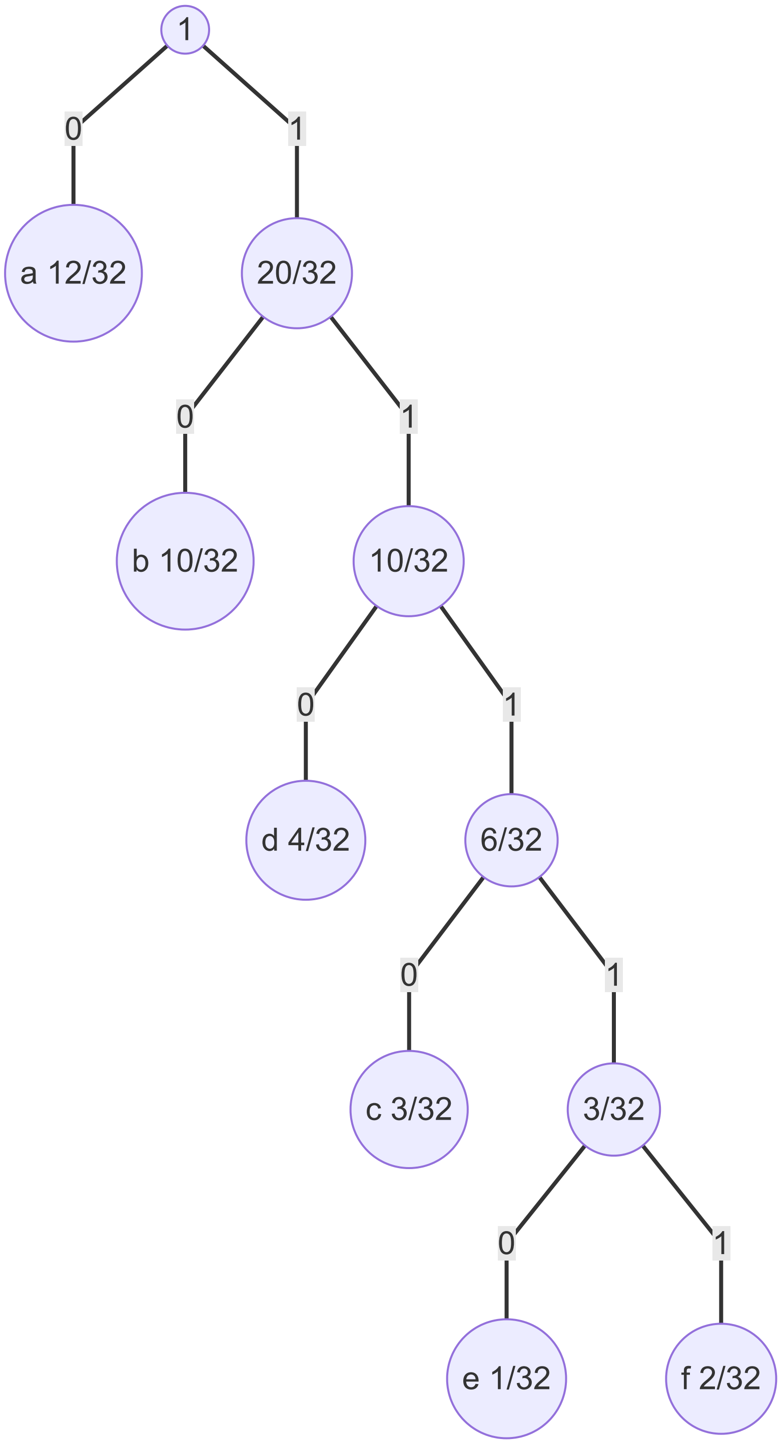
HW1

Problem 1

a

$$
\begin{aligned}
H(S) &= -\sum_{i=1}^{n} p_i \log p_i \\
&= -(\frac{12}{32}\log(\frac{12}{32}) + \frac{10}{32}\log(\frac{10}{32}) + \frac{3}{32}\log(\frac{3}{32}) + \frac{4}{32}\log(\frac{4}{32}) + \frac{1}{32}\log(\frac{1}{32}) + \frac{2}{32}\log(\frac{2}{32})) \\
&\approx 2.156
\end{aligned}
$$

b



a: 0

b: 10

c: 1110

d: 110

e: 11110

f: 11111

c

$$
\begin{aligned}
BR &= \sum_{i=1}^{n} p_i |codeword(a_i)| \\
&= 1\frac{12}{32} + 2\frac{10}{32} + 3\frac{4}{32} + 4\frac{3}{32} + 5\frac{1}{32} + 5\frac{2}{32} \\
&= \frac{71}{32} \\
&\approx 2.219
\end{aligned}
$$

d

```
aaaaabbbdddbdebaaaaccbaf
0000010101011011011010110111011000001110111010011111
```

$BR = \frac{52}{24} \approx 2.167$

problem 2

a

Firstly we need to know the probability of the binary sequence

$$
\begin{aligned}
P[000111] &= P[1^{st}bit = 0]P[0|0]P[0|0]P[0|1]P[1|1]P[1|1] \\
&\approx 0.019342
\end{aligned}
$$

L	R	Split	Δ	curr_symbol	action
0	1	$\frac{1}{2}$	1	0	choose left
0	$\frac{64}{128}$	$\frac{61}{128}$	$\frac{64}{128}$	0	choose left
0	$\frac{3904}{8192}$	$\frac{3721}{8192}$	$\frac{3904}{8192}$	0	choose left
0	$\frac{238144}{524288}$	$\frac{226981}{524288}$	$\frac{238144}{524288}$	1	choose right
$\frac{14526784}{33554432}$	$\frac{15241216}{33554432}$	$\frac{14560273}{33554432}$	$\frac{714432}{33554432}$	1	choose right
$\frac{931857472}{2147483648}$	$\frac{975437824}{2147483648}$	$\frac{933900301}{2147483648}$	$\frac{43580352}{2147483648}$	1	choose right
$\frac{933900301}{2147483648}$	$\frac{975437824}{2147483648}$		$\frac{41537523}{2147483648}$	end	final interval

So finally, L = $\frac{933900301}{2147483648}$,R= $\frac{975437824}{2147483648}$,$\Delta = \frac{41537523}{2147483648} \approx 0.019342$

$t = \lceil -log\Delta \rceil$ =6

$\frac{L+R}{2} = \frac{1909338125}{4294967296} = b0.01110001110011100011000000001101$

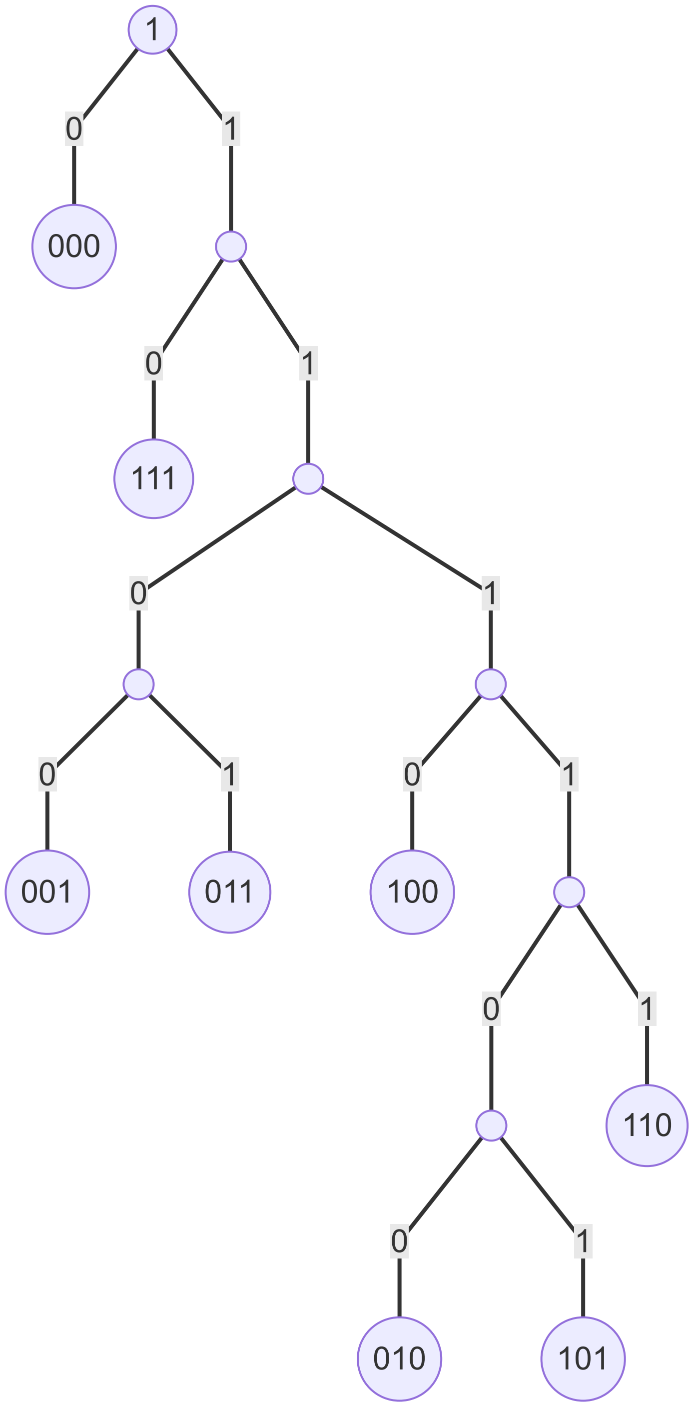
Take the first 6 bits after point

Then the coded bitstream is 011100

The length hasn't changed so the BR = 1

b

symbol	Cal	Probability
000	$P[1^{st}bit = 0]P[00]P[00]$	$\frac{3721}{8192}$
001	$P[1^{st}bit = 0]P[00]P[01]$	$\frac{183}{8192}$
010	$P[1^{st}bit = 0]P[01]P[10]$	$\frac{9}{8192}$
011	$P[1^{st}bit = 0]P[01]P[11]$	$\frac{183}{8192}$
100	$P[1^{st}bit = 1]P[10]P[00]$	$\frac{183}{8192}$
101	$P[1^{st}bit = 1]P[10]P[01]$	$\frac{9}{8192}$
110	$P[1^{st}bit = 1]P[11]P[10]$	$\frac{183}{8192}$
111	$P[1^{st}bit = 1]P[11]P[11]$	$\frac{3721}{8192}$



symbol	CodeWord
000	0
001	1100
010	111100
011	1101
100	1110
101	111101
110	11111

symbol	CodeWord
111	01

So 000111 -> 001

BR = 3/6 = 0.5

This bitrate is lower compared to BR = 1 in part a. So block-Huffman has achieved a better compression comparing to arithmetic coding in this scenario.

Problem 3

a

$x = 0^{15}1^{11}0^{10}1^{13}0^{14}1^{9}$

4 bit for each length so we can support at most 15 in length.

so x can be coded as x= (0,15)(1,11)(0,10)(1,13)(0,14)(1,9)

or in binary `x = (0,1111)(1,1011)(0,1010)(1,1101)(0,1110)(1,1001)`

word_length = 30

b

In x, $P(0) = \frac{11}{24}$,$P(1) = \frac{13}{24}$

MPB(x) is 1

$p * \frac{Ln2}{1-p} \approx 0.82$

The nearest power of 2 is $2^0 = 1$, so m=1

rewrite x as 0^x1

x=$0^{15}1(0^01)^{10}0^{10}1(0^01)^{12}0^{14}1(0^01)^{8}1$

code $0^{15}1$: q=15 so code ($0^{15}1$)=$1^{15}0$

code $0^{0}1$: q=0 so code ($0^{0}1$)=0

code $0^{10}1$: q=10 so code ($0^{10}1$)=$1^{10}0$

code $0^{14}1$: q=14 so code ($0^{14}1$)=$1^{14}0$

With head and tail, the coded word should be

$1, 0^{15}1, 0^{10}, 1^{10}0, 0^{12}, 1^{14}0, 0^{8}, 1$

word_length = 74

c

With differential GOLOMB, the original x can be transformed into $0^{15}1, 0^{10}1, 0^{9}1, 0^{12}1, 0^{13}1, 0^{8}(1)$

$P(1) = \frac{5}{72}, P(0) = \frac{67}{72}$

So MPB is 0

$p * \frac{Ln2}{1-p} \approx 9.24$

The nearest power of 2 is $2^3 = 8$ so m=8, log(m)=3

length of r is 3

code $0^{15}1$: 15=1*8+7, q=1,r=111 so code ($0^{15}1$)=10111

code $0^{10}1$: 10=1*8+2, q=1,r=010 so code ($0^{15}1$)=10010

code $0^{9}1$: 9=1*8+1, q=1,r=001 so code ($0^{15}1$)=10001

code $0^{12}1$: 12=1*8+4, q=1,r=100 so code ($0^{15}1$)=10100

code $0^{13}1$: 13=1*8+5, q=1,r=101 so code ($0^{15}1$)=10101

code $0^{8}1$: 15=1*8+0, q=1,r=000 so code ($0^{15}1$)=10000

So with head and tail, x is coded as

`0,10111,10010,10001,10100,10101,10000,0`

word_length=32

d

So we can see for this particular x

- run-length achieved word_length=30
- Golomb achieved word_length=74
- Differential Golomb achieved word_length=32

So the best technique for this x is ren-length

Problem 4

a

$x = 0^{15}1^{11}0^{10}1^{13}0^{14}1^{9}$

i	$\lceil \log i \rceil$	J	W	a	Dict[i]
1	0	empty	empty	0	0
2	1	1=1	0	0	00
3	2	10=2	00	0	000
4	2	11=3	000	0	0000
5	3	100=4	0000	0	00000
6	3	000=0	empty	1	1
7	3	110=6	1	1	11
8	3	111=7	11	1	111
9	4	1000=8	111	1	1111

i	$\lceil \log i \rceil$	J	W	a	Dict[i]
10	4	0110=6	1	0	10
11	4	0101=5	00000	0	000000
12	4	0011=3	000	1	0001
13	4	1001=9	1111	1	11111
14	4	1101=13	11111	1	111111
15	4	1010=10	10	0	100
16	5	01011=11	000000	0	0000000
17	5	00101=5	00000	1	000001
18	5	01110=14	111111	1	1111111
19	5	00110=6	1	empty	1

So we need to extract J and a from the dict

(-,0)(1,0)(10,0)(11,0)(100,0)(000,1)(110,1)(111,1)(1000,1)(0110,0)(0101,0)(0011,1)(1001,1)(1101,1)(1010,0)(01011,0)(00101,1)(01110,1)(00110,-)

word length = 84, $BR = 83/72 \approx 1.15$

This result is not as good as techniques in problem3

b

y = aaaabbbbabbbabaabaabbaaa

if we have more than 2 characters then we might need huffman code the string.

But for only two characters, we can assume a:0, b:1

so that y = 000011110111010|010|011000

i	$\lceil \log i \rceil$	J	W	a	Dict[i]
1	0	empty	empty	0	0
2	1	1=1	0	0	00
3	2	01=1	0	1	01
4	2	00=0	empty	1	1
5	3	100=4	1	1	11
6	3	011=3	01	1	011
7	3	100=4	1	0	10

i	$\lceil \log i \rceil$	J	W	a	Dict[i]
8	3	011=3	01	0	010
9	4	0110=6	011	0	0110
10	4	0010=2	00	empty	00

So the coded string is (-,0)(1,0)(01,1)(00,1)(100,1)(011,1)(100,0)(011,0)(0110,0)(0010,-)

WordLength =34

If we don't need to add mapping(a:0,b:1) to the code word then

$BR = 34/24 \approx 1.42$

If we need to add mapping to the coded string, then coded word length will need to increase and so will the BR

Problem 5

- The approach I'm to generalize the AC is:
  - Instead of splitting the interval into two, we can split the interval into three.
  - The characteristic of the AC doesn't change with this change.
    - The final interval's length is still the probability of the given string.
    - The Final interval is included in all the intervals in the path, so the coded word can be decoded with a similar decoding approach
- **The pseudo compress code goes as below**
- Let I = [L,R) where initially L=0, R=1;
- For i=1 to n do
  - Let $P_{ai} = Pr[a/x_1x_2...x_{(i-1)}]$,Let $P_{ci} = Pr[c/x_1x_2...x_{(i-1)}]$
  - Let $\Delta = R - L$;
  - Divide interval I into 3 subintervals: [L, L+$P_{ai}\Delta$),[L+$P_{ai}\Delta$,R -$P_{ci}\Delta$),[R -$P_{ci}\Delta$,R)
  - If x_i == a, reduce I to [L, L+$P_{ai}\Delta$)
  - If x_i == b, reduce I to [L+$P_{ai}\Delta$,R -$P_{ci}\Delta$)
  - if x_i == c, reduce I to [R -$P_{ci}\Delta$,R)
- Let t = $\lceil -log(R-L) \rceil$, and exprss $\frac{L+R}{2}$ in binary as $0.r_1r_2...r_t$
- Output := $r_1r_2...r_t$
- **For Decompress**
  - The decompress process remains mostly the same.
  - The only difference is we need to reproduce the 3 intervals our encoder generated. And pick the interval where our coded string value is in.
  - Following the decoding step, when T>=t, the original string will be completely decompressed.