

Exercise 1

Flip 0: 4913

Flip 1: 5087

Exercise 2

Flip 0: 3936

Flip 1: 6064

Exercise 3

$\Pr[\text{Exactly 2 h in 3 flips}] = 0.432851$ theory = 0.432

Exercise 4

- What is the probability that, in 3 flips, the third and only the third flip results in heads?
 - $\Pr[\text{only 3rd is heads}] = 0.095991$ theory = 0.096
- What is the probability that, with an unlimited number of flips, you need at least three flips to see heads for the first time?
 - $\Pr[\text{need at least 3 flips}] = 0.159825$ theory = 0.16
- Why are these two probabilities different?
 - The second condition doesn't require the third flip to be head.

Exercise 5

$\Pr[\text{odd+even}] = 0.249831$ theory = 0.25

Exercise 6:

- $\Pr[c1=\text{club given } c2=\text{club}] = 0.23604529756646053$ theory = 0.23529411764705882
- $\Pr[c1=\text{diamond given } c2=\text{club}] = 0.2500602361256124$ theory = 0.2549019607843137

Exercise 7:

- heads
- tails
- {heads,tails}
- {none}

Exercise 8:

2^n possible events

Exercise 9:

- 6 probabilities
- 36 probabilities for the sample space is 36

Exercise 10:

- Axiom 1: $\Pr[\Omega]=1$
- Axiom 2: $0 \leq \Pr[A] \leq 1$ for any event A.
- So $\Pr[A^c]=1-\Pr[A]$

Exercise 11:

- Using a couple of numbers, like (1,50), (2,40), The size should be $52*51 = 2652$
- The subset should be the $(\{1...13\}, \{1...13\})$ where the two number are not the same.
- The cardinality is $13*12 = 156$
- $\text{prob} = 156/2652 = 0.058$
- The subset should be $(\{1...13\}, \{1...52\})$, where two numbers are not the same.
 - the cardinality = $13*51 = 663$
 - The prob = $663/2652 = 0.25$

Exercise 12:

The sample space is unlimited, for the coin flip may never result in head.

Exercise 13:

$$P[\text{at least one head}] = 1 - P[\text{all tails}] = \frac{7}{8}$$

$$P[\text{allheads}] = \frac{1}{8}$$

$$P[\text{allhead and at least one head}] = \frac{1}{8}$$

$$P[\text{allheads} | \text{at least one head}] = \frac{1}{7}$$

Exercise 14:

Total sample space = $(\{1...52\}, \{1...52\} : (i,j) : i \neq j)$. = $52*51$ samples

event second is club = $(\{1...52\}, \{1...13\})$ where first and second is not the same. total $51*13$ samples.

$$P[\text{first is club}] = \frac{51*13}{51*52}$$

event *first is club and second is club* is $(\{1...13\}, \{1...13\})$ where first and second is not the same. total $13*12$ samples

$$P[\text{first is club and second is club}] = \frac{13*12}{51*52}$$

$$P[\text{first is club and second is club} | \text{first is club}] = \frac{12}{51}$$

Exercise 15:

$C1 = [1...13]$, cardinalities =13.

$C2 = [1...13]$, cardinalities =13.

Exercise 16:

$$\Pr[D1] = \frac{13}{52}$$

$$\Pr[D1 | C2] = \Pr[C2 | D1] \Pr[D1] / \Pr[C2] = (13/51) * (13/52) / (13/52) = \frac{13}{51}$$

Exercise 17:

$$\Pr[A|B] = 3/51$$

$$\Pr[B] = 4/52$$

$$\Pr[B'] = 48/52$$

$$\Pr[A|B'] = 4/51$$

$$\Pr[A] = \frac{3*4}{51*52} + \frac{4*48}{51*52} = \frac{4}{52}$$

Exercise 18:

$$P[S] = 0.05$$

$$P[S'] = 0.95$$

$$P[T|S] = 0.99$$

$$P[T|S'] = 0.03$$

$$P[T] = 0.99 * 0.05 + 0.95 * 0.03 = 0.078$$

$$P[S|T] = P[T|S]*P[S]/P[T] = 0.99 * 0.05/0.078 = 0.63$$

$$P[S'|T] = P[T|S']*P[S']/P[T] = 0.03 * 0.95/0.078 = 0.365$$

```
1 | False p:0.36590856466592914
2 | True  p:0.6340914353340709
```

Exercise 19:

- $\Pr[A > 1.0] = 0.367855$
 $\Pr[A > 0.5] = 0.606511$
- $\Pr[0.5 < A < 1]$
- No they are not, dot in continuous space doesn't make sense
- Yes

Exercise 20:

```
1 | Pr[A > 1.0] = 0.500319
2 | Pr[A > 0.5] = 0.750251
```

Exercise 21:

Uniform

```
1 | Pr[A > 1.0] = 0.500319
2 | Pr[A > 0.5] = 0.750251
3 | Pr[A > 1|A > 0.5] = 0.6668688212344934
```

Exp

```

1 | Pr[A > 1.0] = 0.367855
2 | Pr[A > 0.5] = 0.606511
3 | Pr[A > 1 | A > 0.5] = 0.6065100220770934

```

For Exponential, the $\Pr[A > 1 | A > 0.5]$ is very close to $\Pr[A > 0.5]$

Exercise 22:

Exp

```

1 | Avg[A > 1.0] = 1.9980611014979077
2 | Avg[A > 0.5] = 1.4989925090470337

```

Uniform

```

1 | Avg[A > 1.0] = 1.4993524154833202
2 | Avg[A > 0.5] = 1.249670823273795

```

Exercise 23:

The code below will always count one more bus than it actually should be.

For example, if the first bus is later than my arrival time, the numBuses given by the following code will be 1 instead of 0.

Exercise 24:

See `BusStopExample2.java`

prob: 0.9701

Exercise 25:

- Exponential
 - avg wait at 10 min: 1.0145516006133788
 - avg wait at 20 min: 1.0144012348073337
- Uniform
 - avg wait at 10 min: 0.6677589130029824
 - avg wait at 20 min: 0.6577483290960779
- So the average wait time doesn't depend on the time of arrival
- Both Uniform and Exponential arrival doesn't change for waiting 20 minute or 10 minute.
- The average wait time for exponential interval is the same at 10 minute and 20 minute. Which is not very intuitive.

Exercise 26:

`Pr[c2=1]=0.500163` `Pr[c2=1 | c1=1]=0.7494849748531853`

Exercise 27:

- My intuition tell me these events are not independent. For 0.5 is a small period. So If a bus arrives in period 0,0.5. Then the bus will not be very likely to arrive at 0.5,1.
- See `BusStopExample4.java`
- exp
 - $\Pr[B2]: 0.3075$
 $\Pr[B2 | B1] 0.0947$
- uniform
 - $\Pr[B2]: 0.2897$
 $\Pr[B2 | B1] 0.0569$
- We can see that in both exp and uniform interval, the B2 given B1 is both smaller than only B2.

Exercise 28:

Defect by A = $P[A] * P[AD] = 1.2\%$

Defect by B = $P[B] * P[BD] = 2.1\%$

Defect by C = $P[C] * P[CD] = 4\%$

Defected prob = $P[D] = 1.2+2.1+4 = 7.3\%$

So probability by A given defected = $P[A | D] = 1.2/7.3 = 16.4\%$