$index$, in, out, dec, alg	
var, x	expression variable
$tvar$, x	term variable
$skvar$, α, β	skollem variable

$Ex,\ e$	$::=$			Terms
	$\|$	x		Term variable
	$\|$	$\texttt{unit}$		Unit
	$\|$	$e_1\, e_2$		Term Application
	$\|$	$\lambda x.e$	bind x in e	Term Abstraction
	$\|$	$\mathbf{let}\, x = e_1\, \mathbf{in}\, e_2$	bind x in e_2	Let
$T,\ \tau$	$::=$			Types
	$\|$	α		Type variable
	$\|$	$\texttt{Unit}$		Unit
	$\|$	$\mathbf{Bool}$		Bool
	$\|$	$\tau_1 \to \tau_2$		Function
	$\|$	(τ)	S	Parenthesis
$Sc,\ \sigma$	$::=$			Type schemes
	$\|$	τ		Monotype
	$\|$	$\forall\, \alpha.\sigma$	bind α in σ	Forall
	$\|$	$[\tau/\alpha]\sigma$	M	Type substitution
	$\|$	$\{\tau/\alpha\}\sigma$	M	Non-opening Type substitution
	$\|$	$\theta\, \sigma$	M	Multiple Type substitution
	$\|$	$\forall A.\tau$	M	Closing w.r.t. A
$Tm,\ t,\ v$	$::=$			Elaborated terms
	$\|$	x		Term variable
	$\|$	$\texttt{unit}$		Unit
	$\|$	$\mathbf{true}$		True
	$\|$	$\mathbf{false}$		False
	$\|$	$t_1\, t_2$		Term Application
	$\|$	$t[\tau]$		Type Application
	$\|$	$\lambda(x : \tau).t$	bind x in t	Term Abstraction
	$\|$	$\Lambda\alpha.t$	bind α in t	Type Abstraction
	$\|$	(t)	S	Parenthesis
	$\|$	$[t_1/x]t_2$	M	Type substitution
	$\|$	$[t_1/x]t_2$	M	Type substitution
	$\|$	$[\tau/\alpha]t$	M	Type substitution
	$\|$	$\theta\, t$	M	Multiple Type substitution
	$\|$	$\Lambda A.t$	M	Closing w.r.t. A
	$\|$	t	M	Closing w.r.t. x
	$\|$	$[t_2/x]t_1$	M	Fake substitution
	$\|$	x	M	Term variable
	$\|$	$\lambda(x : \tau).t$	M	
$A,\ A$	$::=$			Type variable list
	$\|$	$\bullet$	M	Empty A
	$\|$	$A; \alpha$	M	Cons A
	$\|$	(A)	M	Parenthesis
	$\|$	$\langle \alpha \rangle$	M	Singleton
	$\|$	$\langle \alpha; \beta \rangle$	M	Twingleton
	$\|$	$A_1 +\!\!+\, A_2$	M	Append

$E,\ \Psi$	$::=$			Environment
	$\vert$	$\bullet$		Empty environment
	$\vert$	$\Psi; A$		Cons Existential environment
	$\vert$	$\Psi; x : \sigma$		Cons Variable
	$\vert$	$\Psi; x : \sigma$	M	
	$\vert$	$\Psi; \{t : [A]\sigma\}$		Cons Object
	$\vert$	$\Psi; \{t : \sigma\}$	S	Cons Object without a
	$\vert$	$\Psi; \{t\}$	M	Cons Object only t
	$\vert$	$\Psi; \{A\}$	M	Cons Object only a
	$\vert$	$\Psi_1 + \Psi_2$	M	Eironment append
	$\vert$	$\mathbf{append}\, \Psi\, \alpha$	M	Typevar append
	$\vert$	$\langle A \rangle$	M	OneA
	$\vert$	$[\tau/\alpha]\Psi$	M	Existential variable substitution
$Eqs,\ \Theta$	$::=$			
	$\vert$	$\bullet$		Empty equaltities
	$\vert$	$\tau_1\ \sim\ \tau_2; \Theta$		Cons Equality
	$\vert$	$\langle \tau_1\ \sim\ \tau_2 \rangle$	S	Singleton
	$\vert$	$[\tau/\alpha]\Theta$	M	Existential variable substitution
$(E \vdash \Theta)$	$::=$			E and Eqs pair (for unification)
	$\vert$	$\Psi \vdash Eqs$	M	Pair
$Sub,\ \theta$	$::=$			
	$\vert$	$\bullet$		Empty
	$\vert$	$[\tau/\alpha]\ \circ\ \theta$		Cons
	$\vert$	$\theta_1 + \theta_2$	M	Append
	$\vert$	(θ)	M	Parenthesis
$vars,\ L$	$::=$			
	$\vert$	$L_1 \cup L_2$	M	
	$\vert$	Ψ	M	
$terminals$	$::=$			
	$\vert$	λ		
	$\vert$	Λ		
	$\vert$	$\longrightarrow$		
	$\vert$	$\longrightarrow^*$		
	$\vert$	$\rightarrow$		
	$\vert$	$\rightsquigarrow$		
	$\vert$	$\bullet$		
	$\vert$	$\mathbf{let}$		
	$\vert$	$\mathbf{in}$		
	$\vert$	$\texttt{unit}$		
	$\vert$	$\texttt{Unit}$		
	$\vert$	$\forall$		
	$\vert$	$.$		
	$\vert$	$\vdash$		
	$\vert$	$\vdash_{\texttt{t}}$		
	$\vert$	$\vdash_{\texttt{T}}$		

| $\vdash_{\mathtt{Sc}}$
| $\Vdash$
| $\vdash_{\mathtt{tl}}$
| $\Vdash$
| $\dashv$
| $\sim$
| $\geq$
| $\geq_d$
| $\geq_{spec}$
| $\geq_a$
| $\leq$
| $\langle$
| $\rangle$
| $\circ$
| $+\!\!\!\!\!\!\!\!\;\Vert$
| $\coprod$
| $\mapsto$

formula ::=
| *judgement*
| $(\tau_1, \tau_2) \equiv (\tau_3, \tau_4)$
| $\alpha \in \Psi$
| $\alpha \# \Psi$
| $A \# L$
| $(x : \sigma) \in \Psi$
| $(x : \sigma) \in \Psi$
| $\Psi(x) = \sigma$
| $\Psi(x) = \sigma$
| $\alpha \notin \mathsf{fv}(\tau)$
| $x \notin \Psi$
| $(E \vdash \Theta) \longrightarrow^{*} (E \vdash \Theta)'$
| $\Psi \vdash_{\mathtt{t}} t$
| $\Psi \vdash_{\mathtt{T}} \tau$
| $\Psi \vdash_{\mathtt{Sc}} \sigma$
| $\Psi \vdash t_1 : \sigma \geq t_2 : \tau$
| $\Psi \Vdash t_1 : \sigma \geq_d t_2 : [A]\tau$
| $t_1 : \sigma \geq_a t_2 : [A]\tau$
| **val fv**
| **val** v
| **Alg** α
| **Dec** A

TermTyping ::=
| $\Psi \vdash t : \sigma$ Term typing

WellFormedness	::=		
	|	$\mathtt{wf}(\Psi)$	Environment Well-formedness
Unification	::=		
	|	$\Psi_{in} \vdash \tau_1 \sim \tau_2 \dashv \Psi_{out}$	Unification Algorithm
	|	$(E \vdash \Theta)_{in} \longrightarrow (E \vdash \Theta)_{out}$	Unification Algorithm (Single-step)
DeclarativeSystem	::=		
	|	$\Psi \Vdash e : \tau \rightsquigarrow t$	Term Typing
	|	$\Psi \Vdash t_1 : \sigma \geq_{spec} t_2 : [A]\tau$	SubSump with A
	|	$\Psi \Vdash e : [A]\tau \rightsquigarrow t$	Term Typing with A
AlgorithmicSystem	::=		
	|	$\Psi_1 \vdash e : [A]\tau \rightsquigarrow t \dashv \Psi_2$	Type Inference
EironmentInstantiation	::=		
	|	$\Psi \vdash A \rightsquigarrow \theta_2$	A instantiation
	|	$\Psi, \theta_1 \vdash \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2$	Environment instantiation
	|	$\Psi, \theta_1 \| -' \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2$	Environment instantiation
judgement	::=		
	|	*TermTyping*	
	|	*WellFormedness*	
	|	*Unification*	
	|	*DeclarativeSystem*	
	|	*AlgorithmicSystem*	
	|	*EironmentInstantiation*	
user_syntax	::=		
	|	*index*	
	|	*var*	
	|	*tvar*	
	|	*skvar*	
	|	*Ex*	
	|	*T*	
	|	*Sc*	
	|	*Tm*	
	|	*A*	
	|	*E*	
	|	*Eqs*	
	|	$(E \vdash \Theta)$	
	|	*Sub*	
	|	*vars*	
	|	*terminals*	
	|	*formula*	

$\boxed{\Psi \vdash t : \sigma}$ Term typing

$$\frac{\Psi(x) = \sigma}{\Psi \vdash x : \sigma} \quad \textsc{TmTy\_\_Var}$$

$$\frac{}{\Psi \vdash \mathtt{unit} : \mathtt{Unit}} \quad \textsc{TmTy\_\_Unit}$$

$$\frac{}{\Psi \vdash \mathbf{true} : \mathbf{Bool}} \quad \textsc{TmTy\_\_True}$$

$$\frac{}{\Psi \vdash \mathbf{false} : \mathbf{Bool}} \quad \textsc{TmTy\_\_False}$$

$$\frac{\begin{array}{l}\Psi \vdash_{\mathtt{T}} \tau_1 \\ \Psi; x : \tau_1 \vdash t : \tau_2\end{array}}{\Psi \vdash \lambda(x : \tau_1).t : \tau_1 \to \tau_2} \quad \textsc{TmTy\_\_Abs}$$

$$\frac{\begin{array}{l}\Psi \vdash t_1 : \tau_1 \to \tau_2 \\ \Psi \vdash t_2 : \tau_1\end{array}}{\Psi \vdash t_1\, t_2 : \tau_2} \quad \textsc{TmTy\_\_App}$$

$$\frac{\Psi; \langle\alpha\rangle \vdash t : \sigma}{\Psi \vdash \Lambda\alpha.t : \forall\, \alpha.\sigma} \quad \textsc{TmTy\_\_TAbs}$$

$$\frac{\begin{array}{l}\Psi \vdash_{\mathtt{T}} \tau \\ \Psi \vdash t : \forall\, \alpha.\sigma\end{array}}{\Psi \vdash t[\tau] : \{\tau/\alpha\}\sigma} \quad \textsc{TmTy\_\_TApp}$$

$\boxed{\mathtt{wf}(\Psi)}$ Environment Well-formedness

$$\frac{}{\mathtt{wf}(\bullet)} \quad \textsc{WfE\_\_Nil}$$

$$\frac{\begin{array}{l}\mathtt{wf}(\Psi) \\ A \# \Psi\end{array}}{\mathtt{wf}(\Psi; A)} \quad \textsc{WfE\_\_A}$$

$$\frac{\begin{array}{l}\mathtt{wf}(\Psi) \\ x \notin \Psi \\ \Psi \vdash_{\mathtt{Sc}} \sigma\end{array}}{\mathtt{wf}(\Psi; x : \sigma)} \quad \textsc{WfE\_\_S}$$

$$\frac{\begin{array}{l}\mathtt{wf}(\Psi) \\ A \# \Psi \\ \Psi; A \vdash_{\mathtt{Sc}} \sigma \\ \Psi; A \vdash_{\mathtt{t}} t\end{array}}{\mathtt{wf}(\Psi; \{t : [A]\sigma\})} \quad \textsc{WfE\_\_O}$$

$\boxed{\Psi_{in} \vdash \tau_1 \sim \tau_2 \dashv \Psi_{out}}$ Unification Algorithm

$$\frac{\Psi_1 \vdash \langle \tau_1 \sim \tau_2 \rangle \longrightarrow^* \Psi_2 \vdash \bullet}{\Psi_1 \vdash \tau_1 \sim \tau_2 \dashv \Psi_2} \quad \textsc{U\_\_U}$$

$\boxed{\begin{array}{l}(E \vdash \Theta)_{in} \longrightarrow \\ (E \vdash \Theta)_{out}\end{array}}$ Unification Algorithm (Single-step)

$$\frac{}{\begin{array}{l}\Psi \vdash \mathtt{Unit} \sim \mathtt{Unit}; \Theta \longrightarrow \\ \Psi \vdash \Theta\end{array}} \quad \textsc{Uss\_\_Unit}$$

$$\frac{}{\begin{array}{l}\Psi \vdash \alpha \sim \alpha; \Theta \longrightarrow \\ \Psi \vdash \Theta\end{array}} \quad \textsc{Uss\_\_Ex}$$

$$\frac{}{\begin{array}{l}\Psi \vdash (\tau_1 \rightarrow \tau_2) \sim (\tau_3 \rightarrow \tau_4); \Theta \longrightarrow \\ \Psi \vdash \tau_1 \sim \tau_3; \tau_2 \sim \tau_4; \Theta\end{array}} \quad \textsc{Uss\_\_Arr}$$

$$\frac{\begin{array}{l}\alpha \notin \mathsf{fv}(\tau_3 \rightarrow \tau_4) \\ \langle \alpha_1; \alpha_2 \rangle \# \Psi_1; (A_1; \alpha +\!\!+ A_2) + \Psi_2 \\ \mathbf{Alg}\, \alpha_1 \\ \mathbf{Alg}\, \alpha_2 \\ (\alpha, (\tau_3 \rightarrow \tau_4)) \equiv (\tau_1, \tau_2)\end{array}}{\begin{array}{l}\Psi_1; (A_1; \alpha +\!\!+ A_2) + \Psi_2 \vdash \tau_1 \sim \tau_2; \Theta \longrightarrow \\ \Psi_1; (A_1; \alpha_1; \alpha_2 +\!\!+ A_2) + [\alpha_1 \rightarrow \alpha_2/\alpha]\Psi_2 \vdash (\alpha_1 \rightarrow \alpha_2) \sim (\tau_3 \rightarrow \tau_4); [\alpha_1 \rightarrow \alpha_2/\alpha]\Theta\end{array}} \quad \textsc{Uss\_\_Split}$$

$$\frac{\begin{array}{c}\alpha \in \Psi_1; A_1 \\ (\alpha, \beta) \equiv (\tau_1, \tau_2)\end{array}}{\begin{array}{l}\Psi_1; (A_1; \beta +\!\!+ A_2) + \Psi_2 \vdash \tau_1 \sim \tau_2; \Theta \longrightarrow \\ \Psi_1; (A_1 +\!\!+ A_2) + [\alpha/\beta]\Psi_2 \vdash [\alpha/\beta]\Theta\end{array}} \quad \textsc{Uss\_\_SubEx}$$

$$\frac{(\mathtt{Unit}, \alpha) \equiv (\tau_1, \tau_2)}{\begin{array}{l}\Psi_1; (A_1; \alpha +\!\!+ A_2) + \Psi_2 \vdash \tau_1 \sim \tau_2; \Theta \longrightarrow \\ \Psi_1; (A_1 +\!\!+ A_2) + [\mathtt{Unit}/\alpha]\Psi_2 \vdash [\mathtt{Unit}/\alpha]\Theta\end{array}} \quad \textsc{Uss\_\_SubUnit}$$

$\boxed{\Psi \Vdash e : \tau \rightsquigarrow t}$ Term Typing

$$\frac{\begin{array}{l}\Psi(x) = \sigma \\ \Psi \vdash x : \sigma \geq t : \tau\end{array}}{\Psi \Vdash x : \tau \rightsquigarrow t} \quad \textsc{Dec\_\_Var}$$

$$\frac{}{\Psi \Vdash \mathtt{unit} : \mathtt{Unit} \rightsquigarrow \mathtt{unit}} \quad \textsc{Dec\_\_Unit}$$

$$\frac{\begin{array}{l}\Psi \vdash_{\mathtt{T}} \tau_1 \\ \Psi; x : \tau_1 \Vdash e : \tau_2 \rightsquigarrow t\end{array}}{\Psi \Vdash \lambda x.e : \tau_1 \rightarrow \tau_2 \rightsquigarrow \lambda(x : \tau_1).t} \quad \textsc{Dec\_\_Abs}$$

$$\frac{\begin{array}{l}\Psi \Vdash e_1 : \tau_1 \rightarrow \tau_2 \rightsquigarrow t_1 \\ \Psi \Vdash e_2 : \tau_1 \rightsquigarrow t_2\end{array}}{\Psi \Vdash e_1\, e_2 : \tau_2 \rightsquigarrow t_1\, t_2} \quad \textsc{Dec\_\_App}$$

$$\frac{\begin{array}{l}A \# \Psi \\ \Psi; A \Vdash e_1 : \tau_1 \rightsquigarrow t_1 \\ \Psi; x : \forall A.\tau_1 \Vdash e_2 : \tau \rightsquigarrow t_2\end{array}}{\Psi \Vdash \mathbf{let}\, x = e_1\, \mathbf{in}\, e_2 : \tau \rightsquigarrow [(\Lambda A.t_1)/x]t_2} \quad \textsc{Dec\_\_Let}$$

$\boxed{\Psi \Vdash t_1 : \sigma \geq_{spec} t_2 : [A]\tau}$ SubSump with A

$$\frac{}{\Psi \Vdash t : \tau \geq_{spec} t : [\bullet]\tau} \quad \textsc{SubSumpASpec\_\_M}$$

$$\frac{\begin{array}{l}\Psi; A_1 \vdash_{\mathtt{T}} \tau \\ \Psi; A_1 \Vdash t_1[\tau] : \{\tau/\alpha\}\sigma \geq_{spec} t_2 : [A_2]\tau\end{array}}{\Psi \Vdash t_1 : \forall\, \alpha.\sigma \geq_{spec} t_2 : [A_1 +\!\!+ A_2]\tau} \quad \textsc{SubSumpASpec\_\_S}$$

$\boxed{\Psi \Vdash e : [A]\tau \rightsquigarrow t}$ Term Typing with A

$$\frac{\begin{array}{l}\Psi(x) = \sigma \\ \Psi \Vdash x : \sigma \geq_d t : [A]\tau\end{array}}{\Psi \Vdash x : [A]\tau \rightsquigarrow t} \quad \textsc{DecA\_\_Var}$$

$$\frac{A \# \Psi}{\Psi \Vdash \texttt{unit} : [A]\texttt{Unit} \rightsquigarrow \texttt{unit}} \quad \textsc{DecA\_\_Unit}$$

$$\frac{\begin{array}{l}A_1 \# \Psi \\ \Psi; A_1 \vdash_{\mathrm{T}} \tau_1 \\ \Psi; A_1; x : \tau_1 \Vdash e : [A_2]\tau_2 \rightsquigarrow t\end{array}}{\Psi \Vdash \lambda x.e : [A_1 +\!\!+\ A_2]\tau_1 \to \tau_2 \rightsquigarrow \lambda(x : \tau_1).t} \quad \textsc{DecA\_\_Abs}$$

$$\frac{\begin{array}{l}\Psi \Vdash e_1 : [A_1]\tau_1 \to \tau_2 \rightsquigarrow t_1 \\ \Psi; A_1 \Vdash e_2 : [A_2]\tau_1 \rightsquigarrow t_2\end{array}}{\Psi \Vdash e_1\, e_2 : [A_1 +\!\!+\ A_2]\tau_2 \rightsquigarrow t_1\, t_2} \quad \textsc{DecA\_\_App}$$

$$\frac{\begin{array}{l}\Psi \Vdash e_1 : [A_1]\tau_1 \rightsquigarrow t_1 \\ \Psi; x : \forall A_1.\tau_1; \{A_1\} \Vdash e_2 : [A_2]\tau \rightsquigarrow t_2\end{array}}{\Psi \Vdash \mathbf{let}\, x = e_1\, \mathbf{in}\, e_2 : [A_2]\tau \rightsquigarrow [(\Lambda A_1.t_1)/x]t_2} \quad \textsc{DecA\_\_Let}$$

$\boxed{\Psi_1 \vdash e : [A]\tau \rightsquigarrow t \dashv \Psi_2}$ Type Inference

$$\frac{\begin{array}{l}\Psi(x) = \sigma \\ x : \sigma \geq_a t : [A]\tau\end{array}}{\Psi \vdash x : [A]\tau \rightsquigarrow t \dashv \Psi} \quad \textsc{Inf\_\_Var}$$

$$\frac{}{\Psi \vdash \texttt{unit} : [\bullet]\texttt{Unit} \rightsquigarrow \texttt{unit} \dashv \Psi} \quad \textsc{Inf\_\_Unit}$$

$$\frac{\begin{array}{l}\alpha \# \Psi_{in} \\ \mathbf{Alg}\, \alpha \\ \Psi_{in}; \langle\alpha\rangle; x : \alpha \vdash e : [A_2]\tau_2 \rightsquigarrow t \dashv \Psi_{out}; A_1; x : \tau_1\end{array}}{\Psi_{in} \vdash \lambda x.e : [A_1 +\!\!+\ A_2]\tau_1 \to \tau_2 \rightsquigarrow \lambda(x : \tau_1).t \dashv \Psi_{out}} \quad \textsc{Inf\_\_Abs}$$

$$\frac{\begin{array}{l}\Psi_{in} \vdash e_1 : [A_1]\tau \rightsquigarrow t_1 \dashv \Psi_1 \\ \Psi_1; \{t_1 : [A_1]\tau\} \vdash e_2 : [A_2]\tau_1 \rightsquigarrow t_2 \dashv \Psi_2; \{t'_1 : [A'_1]\tau'\} \\ \alpha \# \Psi_2; (A'_1 +\!\!+\ A_2) \\ \mathbf{Alg}\, \alpha \\ \Psi_2; (A'_1 +\!\!+\ A_2; \alpha); \{t'_1\, t_2 : \alpha\} \vdash \tau' \sim \tau_1 \to \alpha \dashv \Psi_{out}; A_{out}; \{t_{out} : \tau_{out}\}\end{array}}{\Psi_{in} \vdash e_1\, e_2 : [A_{out}]\tau_{out} \rightsquigarrow t_{out} \dashv \Psi_{out}} \quad \textsc{Inf\_\_App}$$

$$\frac{\begin{array}{l}\Psi_{in} \vdash e_1 : [A_1]\tau \rightsquigarrow t_1 \dashv \Psi_1 \\ \Psi_1; x : \forall A_1.\tau; \{\Lambda A_1.t_1\}; \{A_1\} \vdash e_2 : [A_2]\tau_2 \rightsquigarrow t_2 \dashv \Psi_{out}; x : \sigma_{out}; \{t'_1\}; \{A'_1\}\end{array}}{\Psi_{in} \vdash \mathbf{let}\, x = e_1\, \mathbf{in}\, e_2 : [A_2]\tau_2 \rightsquigarrow [t'_1/x]t_2 \dashv \Psi_{out}} \quad \textsc{Inf\_\_Let}$$

$\boxed{\Psi \vdash A \rightsquigarrow \theta_2}$ A instantiation

$$\frac{}{\Psi \vdash \bullet \rightsquigarrow \bullet} \quad \textsc{AInst\_\_Nil}$$

$$\frac{\begin{array}{l}\Psi \vdash A \rightsquigarrow \theta_2 \\ \Psi \vdash_{\mathrm{T}} \tau\end{array}}{\Psi \vdash A; \alpha \rightsquigarrow [\tau/\alpha] \circ \theta_2} \quad \textsc{AInst\_\_C}$$

$\boxed{\Psi, \theta_1 \vdash \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2}$ Environment instantiation

$$\frac{}{\Psi, \theta \vdash \bullet \rightsquigarrow \bullet, \bullet} \quad \textsc{EInst\_\_Nil}$$

$$\frac{\begin{array}{l}\Psi, \theta_1 \vdash \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2 \\ A_{dec} \# \Psi_{dec} \\ \Psi + \Psi_{dec}; A_{dec} \vdash A_{alg} \rightsquigarrow \theta_3\end{array}}{\Psi, \theta_1 \vdash \Psi_{alg}; A_{alg} \rightsquigarrow \Psi_{dec}; A_{dec}, \theta_3 + \theta_2} \quad \textsc{EInst\_\_A}$$

$$\frac{\Psi, \theta_1 \vdash \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2}{\Psi, \theta_1 \vdash \Psi_{alg}; x : \sigma_{alg} \rightsquigarrow \Psi_{dec}; x : (\theta_2 + \theta_1)\, \sigma_{alg}, \theta_2} \quad \textsc{EInst\_\_S}$$

$$\frac{\begin{array}{l}\Psi, \theta_1 \vdash \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2 \\ A_{dec} \# \Psi_{dec} \\ \Psi + \Psi_{dec}; A_{dec} \vdash A_{alg} \rightsquigarrow \theta_3\end{array}}{\Psi, \theta_1 \vdash \Psi_{alg}; \{t_{alg} : [A_{alg}]\sigma_{alg}\} \rightsquigarrow \Psi_{dec}; \{(\theta_3 + \theta_2 + \theta_1)\, t_{alg} : [A_{dec}](\theta_3 + \theta_2 + \theta_1)\, \sigma_{alg}\}, \theta_2} \quad \textsc{EInst\_\_O}$$

$\boxed{\Psi, \theta_1 | -' \, \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2}$ Environment instantiation

$$\frac{}{\Psi, \theta | -' \bullet \rightsquigarrow \bullet, \bullet} \quad \textsc{EInstD\_\_Nil}$$

$$\frac{\begin{array}{l}\Psi, \theta_1 | -' \, \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2 \\ A_{dec} \# \Psi_{dec} \\ \mathbf{Dec}\, A_{dec} \\ \Psi + \Psi_{dec}; A_{dec} \vdash A_{alg} \rightsquigarrow \theta_3\end{array}}{\Psi, \theta_1 | -' \, \Psi_{alg}; A_{alg} \rightsquigarrow \Psi_{dec}; A_{dec}, \theta_3 + \theta_2} \quad \textsc{EInstD\_\_A}$$

$$\frac{\Psi, \theta_1 | -' \, \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2}{\Psi, \theta_1 | -' \, \Psi_{alg}; x : \sigma_{alg} \rightsquigarrow \Psi_{dec}; x : (\theta_2 + \theta_1)\, \sigma_{alg}, \theta_2} \quad \textsc{EInstD\_\_S}$$

$$\frac{\begin{array}{l}\Psi, \theta_1 | -' \, \Psi_{alg} \rightsquigarrow \Psi_{dec}, \theta_2 \\ A_{dec} \# \Psi_{dec} \\ \mathbf{Dec}\, A_{dec} \\ \Psi + \Psi_{dec}; A_{dec} \vdash A_{alg} \rightsquigarrow \theta_3\end{array}}{\Psi, \theta_1 | -' \, \Psi_{alg}; \{t_{alg} : [A_{alg}]\sigma_{alg}\} \rightsquigarrow \Psi_{dec}; \{(\theta_3 + \theta_2 + \theta_1)\, t_{alg} : [A_{dec}](\theta_3 + \theta_2 + \theta_1)\, \sigma_{alg}\}, \theta_2} \quad \textsc{EInstD\_\_O}$$