$index$, in, out	
$termvar$, x, y	term variable
$dskvar$, a, b	Skolem variable
$skvar$, $\widehat{\mathrm{a}}$, $\widehat{\mathrm{b}}$	Skolem variable
$exvar$, $\widehat{\alpha}$, $\widehat{\beta}$	existential variable

e	$::=$			Terms
	$\|$	x		Term variable
	$\|$	$\texttt{unit}$		Term unit
	$\|$	$e_1\ e_2$		Application
	$\|$	$\lambda x.e$	bind x in e	Abstraction
	$\|$	$\textbf{let}\ x = e_1\ \textbf{in}\ e_2$	bind x in e_2	Let binding
	$\|$	(e)	S	Parenthesis
τ	$::=$			Dec Types
	$\|$	a		Skolem variable
	$\|$	$\texttt{Unit}$		Unit type
	$\|$	$\tau_1 \to \tau_2$		Function type
	$\|$	(τ)	S	Parenthesis
σ	$::=$			Dec Type schemes
	$\|$	τ		Monotype
	$\|$	$\forall\, a.\sigma$	bind a in σ	Forall
	$\|$	$[\tau/a]\sigma$	M	Skolem substitution
T	$::=$			Types
	$\|$	$\widehat{\mathrm{a}}$		Skolem variable
	$\|$	$\widehat{\alpha}$		Existential variable
	$\|$	$\texttt{Unit}$		Unit type
	$\|$	$T_1 \to T_2$		Function type
	$\|$	(T)	S	Parenthesis
S	$::=$			Type schemes
	$\|$	T		Monotype
	$\|$	$\forall\, \widehat{\mathrm{a}}.S$	bind $\widehat{\mathrm{a}}$ in S	Forall
	$\|$	$[T/\widehat{\alpha}]S$	M	Existential substitution
	$\|$	$[T/\widehat{\mathrm{a}}]S$	M	Skolem substitution
	$\|$	$\theta\, S$	M	Instatiation substitution
$\overline{a}$	$::=$			Decl Skolem variable list
	$\|$	$\bullet$	M	Empty environment
	$\|$	$\overline{a}; a$	M	Cons existential
	$\|$	$(\overline{a})$	M	Parenthesis
	$\|$	$\langle a \rangle$	M	Singleton
	$\|$	$\overline{a}_1 +\!\!+\ \overline{a}_2$	M	Append
o	$::=$			Objects
	$\|$	$[\overline{a}]\sigma$		Scheme
Γ	$::=$			Dec Environment
	$\|$	$\bullet$		Empty environment
	$\|$	$\Gamma; a$		Cons Sk
	$\|$	$\Gamma; x : \sigma$		Cons Variable
	$\|$	$\Gamma; \{o\}$		Cons Object
	$\|$	$\Gamma; \overline{a}$	M	Cons DA

	$\mid$	(Γ)	M	Parenthesis
	$\mid$	$\langle a \rangle$	M	Singleton Sk
	$\mid$	$\langle \overline{a} \rangle$	M	Singleton DA
	$\mid$	$\langle x : \sigma \rangle$	M	Singleton Var
	$\mid$	$\langle \{o\} \rangle$	M	Singleton Obj
	$\mid$	$\Gamma_1 + \Gamma_2$	M	Environment append
O	$::=$			Objects
	$\mid$	$[A]S$		Scheme
	$\mid$	S	M	Scheme without A
A	$::=$			Existential environment
	$\mid$	$\bullet$	M	Empty environment
	$\mid$	$A; \widehat{\alpha}$	M	Cons existential
	$\mid$	(A)	M	Parenthesis
	$\mid$	$\langle \widehat{\alpha} \rangle$	M	Singleton
	$\mid$	$\langle \widehat{\alpha}; \widehat{\beta} \rangle$	M	Singleton
	$\mid$	$A_1 \mathbin{+\!\!+} A_2$	M	Existential Environment append
Ψ	$::=$			Environment
	$\mid$	$\bullet$		Empty environment
	$\mid$	$\Psi; \widehat{\mathrm{a}}$		Cons Skolem variable
	$\mid$	$\Psi; A$		Cons Existential environment
	$\mid$	$\Psi; x : S$		Cons Variable
	$\mid$	$\Psi; \{O\}$		Cons Object
	$\mid$	$\langle A \rangle$	S	Singleton A
	$\mid$	$\langle x : S \rangle$	S	Singleton X
	$\mid$	$\langle \{O\} \rangle$	S	Singleton X
	$\mid$	(Ψ)	S	Parenthesis
	$\mid$	$\Psi_1 + \Psi_2$	M	Environment append
	$\mid$	$[T/\widehat{\alpha}]\Psi$	M	Substitution
	$\mid$	$\gamma\, \Psi$	M	Substitution
E	$::=$			
	$\mid$	$\bullet$		Empty equaltities
	$\mid$	$T_1 \sim T_2; E$		Cons Equality
	$\mid$	$\langle T_1 \sim T_2 \rangle$	S	Singleton
	$\mid$	$[T/\widehat{\alpha}]E$	M	Substitution
	$\mid$	$\gamma\, E$	M	Substitution'
	$\mid$	$(T_1 \sim T_2); E$	M	Parenthesis
γ	$::=$			
	$\mid$	$\bullet$		Empty substitution
	$\mid$	$\gamma \circ [T/\widehat{\alpha}]$		Cons substitution
	$\mid$	$[T/\widehat{\alpha}]$	M	Singleton
	$\mid$	(γ)	M	Parenthesis
	$\mid$	$\gamma_1 + \gamma_2$	M	Append
θ	$::=$			

	$\mid$	$\bullet$		Empty substitution
	$\mid$	$\theta \circ [\tau/\widehat{\alpha}]$		Cons substitution
	$\mid$	(θ)	M	Parenthesis
	$\mid$	$\theta_1 + \theta_2$	M	Append
terminals	$::=$			
	$\mid$	λ		
	$\mid$	Λ		
	$\mid$	$\longrightarrow$		
	$\mid$	$\rightarrow$		
	$\mid$	$\leadsto$		
	$\mid$	$\bullet$		
	$\mid$	**let**		
	$\mid$	**in**		
	$\mid$	`unit`		
	$\mid$	`Unit`		
	$\mid$	$\forall$		
	$\mid$	$\forall$		
	$\mid$	$.$		
	$\mid$	$\vdash$		
	$\mid$	$\models$		
	$\mid$	$\vdash_{\widehat{\mathtt{ty}}}$		
	$\mid$	$\vdash_{\mathtt{ty}}$		
	$\mid$	$\vdash$		
	$\mid$	$\Vdash$		
	$\mid$	$\Vdash_{\mathtt{mono}}$		
	$\mid$	$\Vdash_{\mathtt{poly}}$		
	$\mid$	$\dashv$		
	$\mid$	$\dashv\mid$		
	$\mid$	$\sim$		
	$\mid$	$\leq$		
	$\mid$	$\langle$		
	$\mid$	$\rangle$		
	$\mid$	$\hookrightarrow$		
	$\mid$	$\circ$		
	$\mid$	$\#$		
formula	$::=$			
	$\mid$	*judgement*		
	$\mid$	$\mathsf{gen}(\sigma_2, \overline{a}) = \sigma_1$		
	$\mid$	$\mathsf{gen}(\mathsf{S}_2, \mathsf{A}) = S_1$		
	$\mid$	$S_1 = S_2$		
	$\mid$	$\Psi_1 = \Psi_2$		
	$\mid$	$a \in \Gamma$		
	$\mid$	$a \notin \Gamma$		
	$\mid$	$x : \sigma \notin \Gamma$		
	$\mid$	$\widehat{\mathrm{a}} \in \Psi$		
	$\mid$	$\widehat{\alpha} \in \Psi$		
	$\mid$	$\widehat{\alpha} \# \Psi$		
	$\mid$	$x : S \in \Psi$		

	|	$\widehat{\alpha} \notin fv_{\widehat{\alpha}}(T)$	
	|	$\sigma \hookrightarrow S$	
	|		
DecHelpers	$::=$		
	|	$\overline{a} \# \Gamma$	DA freshness
DecTyping	$::=$		
	|	$\Gamma \Vdash_{\mathtt{mono}} e : \tau$	Mono Term Typing
	|	$\Gamma \Vdash_{\mathtt{poly}} e : \sigma$	Poly Term Typing
	|	$\Gamma \vdash_{\mathtt{ty}} \sigma$	Dec Type Well-formedness
	|	$\mathtt{wf}\ (\Gamma)$	Scoping/Typing Environment Well-form
	|	$\Gamma \Vdash \sigma_1 \leq \sigma_2$	Type subsumption
AFreshNess	$::=$		
	|	$A \# \Psi$	A freshness
AlgorithmicSystem	$::=$		
	|	$\Psi_{in} \vdash e : [A]T \dashv \Psi_{out}$	Type Inference
	|	$\Psi_1 \vdash e : S \dashv \Psi_2$	Generalization
	|	$\Psi \vdash S \leq [A]T$	Polymorhpic Type Instantiation
	|	$\Psi_1 \vdash E \dashv \Psi_2$	Unification Algorithm
	|	$\Psi_{in} \vdash E \dashv \Psi_{out}, \gamma$	Unification Algorithm with substutution
	|	$\Psi_1 \vdash E_1 \rightarrow$ $\Psi_2 \vdash E_2, \gamma$	Unification Algorithm (Single-step)
WFAlgorithmic	$::=$		
	|	$\Psi \vdash_{\widehat{\mathtt{ty}}} S$	Type Well-formedness
	|	$\widehat{\mathtt{wf}}\ (\Psi)$	Scoping/Typing Environment Well-form
EnvironmentInstantiation	$::=$		
	|	$\Gamma_{in}, \theta_{in} \vdash A \rightsquigarrow \overline{a}, \theta$	A instantiation
	|	$\Gamma_{in}, \theta_{in} \vdash \Psi \rightsquigarrow \Gamma, \theta$	Environment instantiation
judgement	$::=$		
	|	*DecHelpers*	
	|	*DecTyping*	
	|	*AFreshNess*	
	|	*AlgorithmicSystem*	
	|	*WFAlgorithmic*	
	|	*EnvironmentInstantiation*	
user_syntax	$::=$		
	|	*index*	
	|	*termvar*	

$$
\begin{array}{ll}
| & dskvar \\
| & skvar \\
| & exvar \\
| & e \\
| & \tau \\
| & \sigma \\
| & T \\
| & S \\
| & \overline{a} \\
| & o \\
| & \Gamma \\
| & O \\
| & A \\
| & \Psi \\
| & E \\
| & \gamma \\
| & \theta \\
| & terminals \\
| & formula
\end{array}
$$

$\boxed{\overline{a}\#\Gamma}$ DA freshness

$$\frac{}{\bullet\#\Gamma}\quad\textsc{DFraNil}$$

$$\frac{\overline{a}\#\Gamma \qquad a \notin \Gamma;\overline{a}}{\overline{a};a\#\Gamma}\quad\textsc{DFraCons}$$

$\boxed{\Gamma \Vdash_{\texttt{mono}} e : \tau}$ Mono Term Typing

$$\frac{x : \sigma \notin \Gamma \qquad \Gamma \Vdash \sigma \leq \tau}{\Gamma \Vdash_{\texttt{mono}} x : \tau}\quad\textsc{TmVar}$$

$$\frac{}{\Gamma \Vdash_{\texttt{mono}} \texttt{unit} : \texttt{Unit}}\quad\textsc{TmUnit}$$

$$\frac{\Gamma \vdash_{\texttt{ty}} \tau_1 \qquad \Gamma; x : \tau_1 \Vdash_{\texttt{mono}} e : \tau_2}{\Gamma \Vdash_{\texttt{mono}} \lambda x.e : \tau_1 \to \tau_2}\quad\textsc{TmAbs}$$

$$\frac{\Gamma \Vdash_{\texttt{mono}} e_1 : \tau_1 \to \tau_2 \qquad \Gamma \Vdash_{\texttt{mono}} e_2 : \tau_1}{\Gamma \Vdash_{\texttt{mono}} e_1\ e_2 : \tau_2}\quad\textsc{TmApp}$$

$$\frac{\Gamma \Vdash_{\texttt{poly}} e_1 : \sigma \qquad \Gamma; x : \sigma \Vdash_{\texttt{mono}} e_2 : \tau}{\Gamma \Vdash_{\texttt{mono}} (\textbf{let}\ x = e_1\ \textbf{in}\ e_2) : \tau}\quad\textsc{TmLet}$$

$\boxed{\Gamma \Vdash_{\texttt{poly}} e : \sigma}$ Poly Term Typing

$$\frac{\overline{a}\#\Gamma \qquad \Gamma;\overline{a} \Vdash_{\texttt{mono}} e : \tau \qquad \mathsf{gen}(\tau, \overline{a}) = \sigma}{\Gamma \Vdash_{\texttt{poly}} e : \sigma}\quad\textsc{TmGen}$$

$\boxed{\Gamma \vdash_{\texttt{ty}} \sigma}$ Dec Type Well-formedness

$$\frac{a \in \Gamma}{\Gamma \vdash_{\texttt{ty}} a}\quad\textsc{WfDTySk}$$

$$\frac{}{\Gamma \vdash_{\texttt{ty}} \texttt{Unit}}\quad\textsc{WfDTyUnit}$$

$$\frac{\Gamma \vdash_{\texttt{ty}} \tau_1 \qquad \Gamma \vdash_{\texttt{ty}} \tau_2}{\Gamma \vdash_{\texttt{ty}} \tau_1 \to \tau_2}\quad\textsc{WfDTyArr}$$

$$\frac{\Gamma; a \vdash_{\texttt{ty}} \sigma}{\Gamma \vdash_{\texttt{ty}} \forall a.\sigma}\quad\textsc{WfDTyAbs}$$

$\boxed{\texttt{wf}\ (\Gamma)}$ Scoping/Typing Environment Well-formedness

$$\frac{}{\texttt{wf}\ (\bullet)}\quad\textsc{WfDEnvNil}$$

$$\frac{\texttt{wf}\ (\Gamma) \qquad a \notin \Gamma}{\texttt{wf}\ (\Gamma; a)}\quad\textsc{WfDEnvSk}$$

$$\frac{\begin{array}{c}\texttt{wf}\ (\Gamma)\\ \Gamma \vdash_{\texttt{ty}} \sigma\end{array}}{\texttt{wf}\ (\Gamma; x : \sigma)}\ \textsc{WfDEnvSch}$$

$\boxed{\Gamma \Vdash \sigma_1 \leq \sigma_2}$ Type subsumption

$$\frac{}{\Gamma \Vdash \tau \leq \tau}\ \textsc{SubSumpMono}$$

$$\frac{\Gamma; a \Vdash \sigma_1 \leq \sigma_2}{\Gamma \Vdash \sigma_1 \leq \forall a.\sigma_2}\ \textsc{SubSumpSkol}$$

$$\frac{\begin{array}{l}\Gamma \vdash_{\texttt{ty}} \tau_1\\ \Gamma \Vdash [\tau_1/a]\sigma \leq \tau_2\end{array}}{\Gamma \Vdash \forall a.\sigma \leq \tau_2}\ \textsc{SubSumpInst}$$

$\boxed{A\#\Psi}$ A freshness

$$\frac{}{\bullet\#\Psi}\ \textsc{FrANil}$$

$$\frac{\begin{array}{l}A\#\Psi\\ \widehat{\alpha}\#\Psi; A\end{array}}{A; \widehat{\alpha}\#\Psi}\ \textsc{FrACons}$$

$\boxed{\Psi_{in} \vdash e : [A]T \dashv \Psi_{out}}$ Type Inference

$$\frac{\begin{array}{l}x : S \in \Psi\\ \Psi \vdash S \leq [A]T\end{array}}{\Psi \vdash x : [A]T \dashv \Psi}\ \textsc{InfVar}$$

$$\frac{}{\Psi \vdash \texttt{unit} : [\bullet]\texttt{Unit} \dashv \Psi}\ \textsc{InfUnit}$$

$$\frac{\begin{array}{l}\widehat{\alpha}\#\Psi_{in}\\ \Psi_{in}; \langle\widehat{\alpha}\rangle; x : \widehat{\alpha} \vdash e : [A_2]T_2 \dashv \Psi_{out}; A_1; x : T_1\end{array}}{\Psi_{in} \vdash \lambda x.e : [A_1 +\!\!+\ A_2]T_1 \to T_2 \dashv \Psi_{out}}\ \textsc{InfAbs}$$

$$\frac{\begin{array}{l}\widehat{\beta}\#\Psi_2; (A_1' +\!\!+\ A_2)\\ \Psi_{in} \vdash e_1 : [A_1]T \dashv \Psi_1\\ \Psi_1; \{[A_1]T\} \vdash e_2 : [A_2]T_1 \dashv \Psi_2; \{[A_1']T'\}\\ \Psi_2; A_1' +\!\!+\ (A_2; \widehat{\beta}); \{\widehat{\beta}\} \vdash \langle T' \sim T_1 \to \widehat{\beta}\rangle \dashv \Psi_{out}; A_{out}; \{T_2\}\end{array}}{\Psi_{in} \vdash e_1\, e_2 : [A_{out}]T_2 \dashv \Psi_{out}}\ \textsc{InfApp}$$

$$\frac{\begin{array}{l}\Psi_{in} \vdash e_1 : S \dashv \Psi\\ \Psi; x : S \vdash e_2 : [A_2]T_2 \dashv \Psi_{out}; x : S'\end{array}}{\Psi_{in} \vdash \textbf{let}\ x = e_1\ \textbf{in}\ e_2 : [A_2]T_2 \dashv \Psi_{out}}\ \textsc{InfLet}$$

$\boxed{\Psi_1 \vdash e : S \dashv \Psi_2}$ Generalization

$$\frac{\begin{array}{l}\Psi_{in} \vdash e : [A]T \dashv \Psi_{out}\\ \mathsf{gen}(\mathsf{T}, \mathsf{A}) = S\end{array}}{\Psi_{in} \vdash e : S \dashv \Psi_{out}}\ \textsc{InfGen}$$

$\boxed{\Psi \vdash S \leq [A]T}$ Polymorhpic Type Instantiation

$$\frac{}{\Psi \vdash T \leq [\bullet]T}\ \textsc{InstMono}$$

$$\frac{\widehat{\alpha}\#\Psi \quad \Psi; \langle\widehat{\alpha}\rangle \vdash [\widehat{\alpha}/\widehat{a}]S \leq [A]T}{\Psi \vdash \forall \widehat{a}.S \leq [\langle\widehat{\alpha}\rangle \mathbin{+\!\!+} A]T} \textsc{InstPoly}$$

$\boxed{\Psi_1 \vdash E \dashv \Psi_2}$ Unification Algorithm

$$\frac{\Psi_{in} \vdash E \dashv \Psi_{out}, \gamma}{\Psi_{in} \vdash E \dashv \Psi_{out}} \text{UU}$$

$\boxed{\Psi_{in} \vdash E \dashv \Psi_{out}, \gamma}$ Unification Algorithm with substutution

$$\frac{}{\Psi \vdash \bullet \dashv \Psi, \bullet} \textsc{UsNil}$$

$$\frac{\Psi_{in} \vdash E \to \Psi \vdash E', \gamma_1 \quad \gamma_1\, \Psi \vdash \gamma_1\, E' \dashv \Psi_{out}, \gamma_2}{\Psi_{in} \vdash E \dashv \Psi_{out}, \gamma_1 + \gamma_2} \textsc{UsCons}$$

$\boxed{\Psi_1 \vdash E_1 \to \Psi_2 \vdash E_2, \gamma}$ Unification Algorithm (Single-step)

$$\frac{}{\Psi \vdash \texttt{Unit} \sim \texttt{Unit}; E \to \Psi \vdash E, \bullet} \textsc{UssUnit}$$

$$\frac{}{\Psi \vdash \widehat{\alpha} \sim \widehat{\alpha}; E \to \Psi \vdash E, \bullet} \textsc{UssExA}$$

$$\frac{}{\Psi \vdash (T_1 \to T_2) \sim (T_3 \to T_4); E \to \Psi \vdash T_1 \sim T_3; T_2 \sim T_4; E, \bullet} \textsc{UssDistrArr}$$

$$\frac{\widehat{\alpha} \notin fv_{\widehat{\alpha}}(T_1 \to T_2) \quad \langle\widehat{\alpha}_1; \widehat{\alpha}_2\rangle\#\Psi_1; (A_1; \widehat{\alpha} \mathbin{+\!\!+} A_2) + \Psi_2}{\Psi_1; (A_1; \widehat{\alpha} \mathbin{+\!\!+} A_2) + \Psi_2 \vdash \widehat{\alpha} \sim (T_1 \to T_2); E \to \Psi_1; (A_1; \widehat{\alpha}_1; \widehat{\alpha}_2 \mathbin{+\!\!+} A_2) + \Psi_2 \vdash (\widehat{\alpha}_1 \to \widehat{\alpha}_2 \sim T_1 \to T_2); E, [\widehat{\alpha}_1 \to \widehat{\alpha}_2/\widehat{\alpha}]} \textsc{UssSplitL}$$

$$\frac{\widehat{\alpha} \notin fv_{\widehat{\alpha}}(T_1 \to T_2) \quad \langle\widehat{\alpha}_1; \widehat{\alpha}_2\rangle\#\Psi_1; (A_1; \widehat{\alpha} \mathbin{+\!\!+} A_2) + \Psi_2}{\Psi_1; (A_1; \widehat{\alpha} \mathbin{+\!\!+} A_2) + \Psi_2 \vdash (T_1 \to T_2) \sim \widehat{\alpha}; E \to \Psi_1; (A_1; \widehat{\alpha}_1; \widehat{\alpha}_2 \mathbin{+\!\!+} A_2) + \Psi_2 \vdash (T_1 \to T_2 \sim \widehat{\alpha}_1 \to \widehat{\alpha}_2); E, [\widehat{\alpha}_1 \to \widehat{\alpha}_2/\widehat{\alpha}]} \textsc{UssSplitR}$$

$$\frac{\widehat{\alpha} \in \Psi_1; A_1}{\Psi_1; (A_1; \widehat{\beta} \mathbin{+\!\!+} A_2) + \Psi_2 \vdash \widehat{\alpha} \sim \widehat{\beta}; E \to \Psi_1; (A_1 \mathbin{+\!\!+} A_2) + \Psi_2 \vdash E, [\widehat{\alpha}/\widehat{\beta}]} \textsc{UssSubExL}$$

$$\frac{\widehat{\alpha} \in \Psi_1; A_1}{\Psi_1; (A_1; \widehat{\beta} \mathbin{+\!\!+} A_2) + \Psi_2 \vdash \widehat{\beta} \sim \widehat{\alpha}; E \to \Psi_1; (A_1 \mathbin{+\!\!+} A_2) + \Psi_2 \vdash E, [\widehat{\alpha}/\widehat{\beta}]} \textsc{UssSubExR}$$

$$\frac{}{\Psi_1; (A_1; \widehat{\alpha} \mathbin{+\!\!+} A_2) + \Psi_2 \vdash \texttt{Unit} \sim \widehat{\alpha}; E \to \Psi_1; (A_1 \mathbin{+\!\!+} A_2) + \Psi_2 \vdash E, [\texttt{Unit}/\widehat{\alpha}]} \textsc{UssSubUnitAL}$$

$$\frac{}{\begin{array}{l}\Psi_1; (A_1; \widehat{\alpha} +\!\!+\ A_2) + \Psi_2 \vdash \widehat{\alpha} \sim \texttt{Unit}; E \rightarrow \\ \Psi_1; (A_1 +\!\!+\ A_2) + \Psi_2 \vdash E, [\texttt{Unit}/\widehat{\alpha}]\end{array}} \textsc{UssSubUnitAR}$$

$\boxed{\Psi \vdash_{\widehat{\texttt{ty}}} S}$ Type Well-formedness

$$\frac{\widehat{\mathrm{a}} \in \Psi}{\Psi \vdash_{\widehat{\texttt{ty}}} \widehat{\mathrm{a}}} \textsc{WfTySk}$$

$$\frac{\widehat{\alpha} \in \Psi}{\Psi \vdash_{\widehat{\texttt{ty}}} \widehat{\alpha}} \textsc{WfTyEx}$$

$$\frac{}{\Psi \vdash_{\widehat{\texttt{ty}}} \texttt{Unit}} \textsc{WfTyUnit}$$

$$\frac{\Psi \vdash_{\widehat{\texttt{ty}}} T_1 \quad \Psi \vdash_{\widehat{\texttt{ty}}} T_2}{\Psi \vdash_{\widehat{\texttt{ty}}} T_1 \rightarrow T_2} \textsc{WfTyArr}$$

$$\frac{\Psi; \widehat{\mathrm{a}} \vdash_{\widehat{\texttt{ty}}} S}{\Psi \vdash_{\widehat{\texttt{ty}}} \forall \widehat{\mathrm{a}}.S} \textsc{WfTyAbs}$$

$\boxed{\widehat{\texttt{wf}}\ (\Psi)}$ Scoping/Typing Environment Well-formedness

$$\frac{}{\widehat{\texttt{wf}}\ (\bullet)} \textsc{WfEnvNil}$$

$$\frac{\widehat{\texttt{wf}}\ (\Psi) \quad A \# \Psi}{\widehat{\texttt{wf}}\ (\Psi; A)} \textsc{WfEnvA}$$

$$\frac{\widehat{\texttt{wf}}\ (\Psi) \quad \Psi \vdash_{\widehat{\texttt{ty}}} S}{\widehat{\texttt{wf}}\ (\Psi; x : S)} \textsc{WfEnvSch}$$

$$\frac{\widehat{\texttt{wf}}\ (\Psi) \quad A \# \Psi \quad \Psi; A \vdash_{\widehat{\texttt{ty}}} S}{\widehat{\texttt{wf}}\ (\Psi; \{[A]S\})} \textsc{WfEnvObjSch}$$

$\boxed{\Gamma_{in}, \theta_{in} \vdash A \rightsquigarrow \overline{a}, \theta}$ A instantiation

$$\frac{}{\Gamma, \theta \vdash \bullet \rightsquigarrow \bullet, \bullet} \textsc{AInstNil}$$

$$\frac{\Gamma_{in}, \theta_{in} \vdash A \rightsquigarrow \overline{a}_1, \theta \quad \Gamma_{in}; \overline{a}_1 +\!\!+\ \overline{a}_2 \vdash_{\texttt{ty}} \tau}{\Gamma_{in}, \theta_{in} \vdash A; \widehat{\alpha} \rightsquigarrow (\overline{a}_1 +\!\!+\ \overline{a}_2), \theta \circ [\tau/\widehat{\alpha}]} \textsc{AInstCons}$$

$\boxed{\Gamma_{in}, \theta_{in} \vdash \Psi \rightsquigarrow \Gamma, \theta}$ Environment instantiation

$$\frac{}{\Gamma, \theta \vdash \bullet \rightsquigarrow \bullet, \bullet} \textsc{EInstNil}$$

$$\frac{\Gamma_{in}, \theta_{in} \vdash \Psi \rightsquigarrow \Gamma, \theta_1 \quad \Gamma_{in} + \Gamma, \theta_{in} + \theta_1 \vdash A \rightsquigarrow \overline{a}, \theta_2}{\Gamma_{in}, \theta_{in} \vdash \Psi; A \rightsquigarrow \Gamma; \overline{a}, \theta_1 + \theta_2} \textsc{EInstA}$$

$$\frac{\begin{array}{c}\Gamma_{in}, \theta_{in} \vdash \Psi \rightsquigarrow \Gamma, \theta \\ \sigma \hookrightarrow (\theta_{in} + \theta)\, S\end{array}}{\Gamma_{in}, \theta_{in} \vdash \Psi; x : S \rightsquigarrow \Gamma; x : \sigma, \theta} \quad \textsc{EInstSch}$$

$$\frac{\begin{array}{l}\Gamma_{in}, \theta_{in} \vdash \Psi \rightsquigarrow \Gamma, \theta \\ (\Gamma_{in} + \Gamma), (\theta_{in} + \theta) \vdash A \rightsquigarrow \overline{a}, \theta' \\ \sigma \hookrightarrow (\theta_{in} + (\theta + \theta'))\, S\end{array}}{\Gamma_{in}, \theta_{in} \vdash \Psi; \{[A]S\} \rightsquigarrow \Gamma; \{[\overline{a}]\sigma\}, \theta} \quad \textsc{EInstObjSch}$$