Mechanizing an elaboration algorithm for the Hindley-Damas-Milner system

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KU LEUVEN

Type checking, inference & elaboration

Type checking

```
bool p = let bool q = True in q \/ q
```

$\Rightarrow$

✓ (well-typed)

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Elaboration

```
p = let q = True in q \/ q
```

$\Rightarrow$

```
(p :: Bool) =
  let (q :: Bool) = True in q \/ q
```

Why not study elaboration

Plenty of research on type checking and type inference [GMM10; McB03; NN99; DM99; RC15]

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- Used to implement features like typeclasses [WB89]

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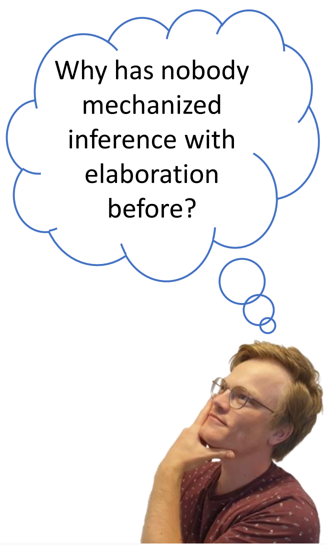

Limitation

Claim: type checking algorithms like algorithm $\mathcal{W}$ [DM82] cannot (fully) do elaboration

Example

```
let x =
      (λ y. unit)
      (λ z. z) in ...
```

Example

$$\texttt{let (x} :: \widehat{\alpha}\texttt{) =}$$
$$\texttt{(}\lambda\texttt{(y} :: \widehat{\beta}\texttt{). unit)}$$
$$\texttt{(}\lambda\texttt{(z} :: \widehat{\gamma}\texttt{). z) in ...}$$

To elaborate we must

- Assign existential type variables

Example

```
let (x :: Unit) =
      (λ(y :: γ̂ → γ̂). unit)
      (λ(z :: γ̂). z) in ...
```

To elaborate we must

- Assign existential type variables
- Solve constraints

Example

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let (x :: Unit) =
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Example

```
let (x :: ∀a. Unit) =
  Λa. (λ(y :: a → a). unit)
        (λ(z :: a). z) in ...
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To elaborate we must

- Assign existential type variables
- Solve constraints
- Generalize

Example

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let (x :: ∀a. Unit) =
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To elaborate we must

- Assign existential type variables
- Solve constraints
- Generalize

Since type inference algorithms like algorithm $\mathcal{W}$ are constraint based, they cannot perform this last step. The scope of existential type variables is lost.

Our algorithm

Solution? Maintain explicit scopes!

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$$\boxed{\Psi_{in} \vdash e : [A]\tau \leadsto t \dashv \Psi_{out}}$$

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Solution? Maintain explicit scopes!

$$\boxed{\overbrace{\Psi_{in} \vdash e}^{inputs} : \underbrace{[A]\tau \rightsquigarrow t \dashv \Psi_{out}}_{outputs}}$$

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Now with explicit scope information!

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A is a list of existential type variables $\overline{\widehat{\alpha}}$ in scope of inferred type τ

Our algorithm

Now with explicit scope information!

Solution? Maintain explicit scopes!

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A is a list of existential type variables $\overline{\widehat{\alpha}}$ in scope of inferred type τ

This makes generalization almost trivial:

$$\frac{\Psi_{in} \vdash e : [A]\tau \rightsquigarrow t \dashv \Psi_{out}}{\Psi_{in} \vdash e : \forall A.\tau \rightsquigarrow \Lambda A.t \dashv \Psi_{out}} \textsc{Gen}$$

State of affairs

Mechanization ongoing

- Coq proof assistant [BC13]
- Generalized rewriting [Soz]
- Locally nameless [Cha12]
- Ott/LNgen [Sew+10; AW10]

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Future work: extend with typeclasses

References

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