Syntax and Semantics of V^-

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1 Syntax

NR: I recommend macros for optional and sequence, not literal brackets.
rab: Despite my best efforts, such a macro does not work in grammar mode.
We present a grammar of V^-:

⟨*program*⟩ ::= {⟨*def*⟩}

⟨*def*⟩ ::= `val` ⟨*name*⟩ ⟨*exp*⟩
| ⟨*exp*⟩

⟨*exp*⟩ ::= ⟨*integer-literal*⟩
| ⟨*name*⟩
| ⟨*guarded-if*⟩
| ⟨*lambda*⟩
| ⟨*value-constructor-name*⟩ {⟨*exp*⟩}
| ⟨*exp*⟩ ▌⟨*exp*⟩

⟨*lambda*⟩ ::= λ{⟨*name*⟩}`.` ⟨*exp*⟩

⟨*guarded-if*⟩ ::= `if` [⟨*guarded-exp*⟩ { [] ⟨*guarded-exp*⟩ }] `fi`

⟨*guarded-exp*⟩ ::= →⟨*exp*⟩
| `E` {⟨*name*⟩}`.` ⟨*guarded-exp*⟩
| ⟨*exp*⟩`;` ⟨*guarded-exp*⟩
| ⟨*name*⟩ = ⟨*exp*⟩`;` ⟨*guarded-exp*⟩
| ⟨*exp*⟩ = ⟨*exp*⟩`;` ⟨*guarded-exp*⟩
| ⟨*guarded-exp*⟩ ▌⟨*guarded-exp*⟩
| `one`({⟨*guarded-exp*⟩})
| `all`({⟨*guarded-exp*⟩})

⟨*name*⟩ ::= any token that is not an *int-lit*, does not contain whitespace, and is not a ⟨*value-constructor-name*⟩ or a reserved word.

⟨*value-constructor-name*⟩ ::= `cons` | `[]` | any token that begins with a capital letter or a colon

⟨*integer-literal*⟩ ::= token composed only of digits, possibly prefixed with a + or −.

2 Refinement ordering on environments

$$\rho \subseteq \rho' \text{ when } \operatorname{dom}\rho \subseteq \operatorname{dom}\rho'$$
$$\text{and } \forall x \in \operatorname{dom}\rho : \rho(x) \subseteq \rho'(x)$$

3 Forms of Judgement for V^-:

Metavariables	
$v,\ v'$	value
eq	equation
$\texttt{?t}\,.\,\{eq\}$	a temporarily-stuck equation
fail	failure
$\rho,\ \hat{\rho}$	environment: $name \to \mathcal{V}_\perp$
$\mathcal{T}$	Context of all temporarily stuck equations (a sequence)
e	An expression
ge	A guarded expression

Sequences	
ε	the empty sequence
$S_1 \cdot S_2$	Concatenate sequence S_1 and sequence S_2
$x \cdot S_2$	Cons x onto sequence S_2

Equations

An *equation* is added to the context of equations temporarily stuck equations or produces **reject**.

$$\langle \rho, eq \rangle \rightarrowtail \texttt{?t}\,.\,\{eq\} \quad \text{(EquationTempStuck)}$$

$$\langle \rho, eq \rangle \rightarrowtail \textbf{reject} \quad \text{(EquationReject)}$$

Expressions

An expression either evaluates to produce a sequence of values or fails. Failure is indicated by the empty sequence ε.

$$\langle \rho, e \rangle \Downarrow v \quad \text{(EvalSucc)}$$

$$\langle \rho, e \rangle \Downarrow \varepsilon \quad \text{(EvalFail)}$$

$$\langle \rho, \mathcal{T}, ge \rangle \Downarrow v \quad \text{(EvalGeSucc)}$$

$$\langle \rho, \mathcal{T}, ge \rangle \Downarrow \varepsilon \quad \text{(EvalGeFail)}$$

Temporarily stuck equations

$$\langle \rho, \texttt{?t}\,.\,\{eq\} \cdot \mathcal{T} \rangle \rightarrowtail \langle \hat{\rho}, \mathcal{T} \rangle \quad \text{(CtxToRho)}$$

4 Sequences

The trivial sequence is ε. Sequences can be concatenated with infix $\cdot$. In an appropriate context, a value like x stands for the singleton sequence containing x.

$$\varepsilon \cdot ys \equiv ys$$

$$ys \cdot \varepsilon \equiv ys$$

$$(xs \cdot ys) \cdot zs \equiv xs \cdot (ys \cdot zs)$$

5 Rules (Big-step Operational Semantics) for V^{-}:

$$\text{(GE-CTX-STUCK)}\ \frac{\langle \rho, eq\rangle \rightarrowtail \texttt{?t}\,.\,\{eq\} \qquad \langle \rho, \texttt{?t}\,.\,\{eq\}\cdot\mathcal{T}\rangle \rightarrowtail \langle \hat{\rho}, \mathcal{T}\rangle \qquad \langle \hat{\rho}, \mathcal{T}, ge\rangle \Downarrow v}{\langle \rho, \mathcal{T}\cdot eq\cdot\mathcal{T}, ge\rangle \Downarrow v}$$

NR: The notation below has multiple horizontal lines. That makes it a derivation, not a rule. I'm having trouble figuring out what's being said here.

$$\text{(GE-EQ-SUCC)}\ \frac{\dfrac{\langle \rho, eq\rangle \rightarrowtail \texttt{?t}\,.\,\{eq\} \qquad \langle \rho, \texttt{?t}\,.\,\{eq\}\cdot\mathcal{T}\rangle \rightarrowtail \langle \hat{\rho}, \mathcal{T}\rangle \qquad \langle \hat{\rho}, \mathcal{T}, ge\rangle \Downarrow v}{\langle \rho, \mathcal{T}\cdot eq\cdot\mathcal{T}, ge\rangle \Downarrow v}\ \text{(GE-CTX-STUCK)}}{\langle \rho, \mathcal{T}, eq; ge\rangle \Downarrow v}$$

NR: If you're trying to pluck an equation out of a list of things, try "$\mathcal{T}\cdot eq\cdot\mathcal{T}'$."

$$\text{(GE-CTX-SUCC)}\ \frac{\langle \rho, eq\rangle \rightarrowtail \hat{\rho}REMOVETHIS \qquad \langle \hat{\rho}, \mathcal{T}, ge\rangle \Downarrow v}{\langle \rho, \mathcal{T}\cdot eq\cdot\mathcal{T}, ge\rangle \Downarrow v}$$

$$\text{(GE-EQ-SUCC)}\ \frac{\dfrac{\langle \rho, eq\rangle \rightarrowtail \hat{\rho}REMOVETHIS \qquad \langle \hat{\rho}, \mathcal{T}, ge\rangle \Downarrow v}{\langle \rho, \mathcal{T}\cdot eq\cdot\mathcal{T}, ge\rangle \Downarrow v}\ \text{(GE-CTX-SUCC)}}{\langle \rho, \mathcal{T}, eq; ge\rangle \Downarrow v}$$

$$\text{(GE-CTX-FAIL)}\ \frac{\langle \rho, eq\rangle \rightarrowtail \textbf{reject}}{\langle \rho, eq\cdot\mathcal{T}, ge\rangle \Downarrow \textbf{fail}}$$

$$\text{(GE-EQ-FAIL)}\ \frac{\dfrac{\langle \rho, eq\rangle \rightarrowtail \textbf{reject}}{\langle \rho, eq\cdot\mathcal{T}, ge\rangle \Downarrow \textbf{fail}}\ \text{(GE-CTX-FAIL)}}{\langle \rho, \mathcal{T}, eq; ge\rangle \Downarrow \textbf{fail}}$$

$$\text{(GE-EXP-FAIL)}\ \frac{\langle \rho, e\rangle \Downarrow \textbf{fail}}{\langle \rho, \mathcal{T}, e; ge\rangle \Downarrow \textbf{fail}}$$

$$\text{(GE-EQ-SUCC)}\ \frac{\langle \rho, e\rangle \Downarrow v' \qquad \langle \rho, \mathcal{T}, ge\rangle \Downarrow v}{\langle \rho, \mathcal{T}, e; ge\rangle \Downarrow v}$$