

# Syntax and Semantics of $V^-$

Roger Burtonpatel

October 22, 2023

## 1 Syntax

We present a grammar of  $V^-$ :

Programs	$P$	$::=$	$\{d\}$	definition
Definitions	$d$	$::=$	$\text{val } x = e$	bind name to expression
Expressions	$e$	$::=$	$x$	name
			$\text{if } g_\alpha \{ \{g_\alpha\} \} \text{ fi}$	if-fi
			$K\{e\}$	value constructor application
			$e_1 e_2$	function application
Guarded Expressions	$g_\alpha$	$::=$	$\rightarrow \alpha$	terminating $\alpha$
			$e; g_\alpha$	intermediate expression
			$E\{x\}.g_\alpha$	existential
			$e_1 = e_2; g_\alpha$	equation
Value Constructors	$K$	$::=$	$::$	cons
			$[]$	empty list
			$\#x$	name beginning with #
			$\mathbf{A-Z}x$	name beginning with capital letter
			$[- +](0-9)^+$	signed integer literal

A *name* is any token that is not an integer literal, does not contain whitespace, a bracket, or parenthesis, and is not a value constructor name or a reserved word.

*rab:* Would like help cleaning up the format on this, specifically with regards to the regex. The one downside of this nicer package is that descriptions will not wrap, so describing an integer literal in english isn't an option as far as I can tell.

## 2 Refinement ordering on environments

$$\begin{aligned} \rho \subseteq \rho' \text{ when } & \text{dom } \rho \subseteq \text{dom } \rho' \\ & \text{and } \forall x \in \text{dom } \rho : \rho(x) \subseteq \rho'(x) \end{aligned}$$

## 3 Forms of Judgement for $V^-$ :

---

<i>Metavariables</i>	
$\vartheta$	a value produced from evaluating $\alpha$ .
$eq$	equation
<b>reject</b>	equation rejection
$r$	$\vartheta \mid \mathbf{reject}$ : a result of $\vartheta$ or rejection
$\rho$	environment: $name \rightarrow \mathcal{V}_\perp$
$\rho\{x \mapsto y\}$	environment extended with name $x$ mapping to $y$
$\mathcal{T}$	Context of all temporarily stuck equations (a sequence)
$e$	An expression
$g$	A guarded expression

---

---

## Sequences

---

$\varepsilon$	the empty sequence
$S_1 \cdot S_2$	Concatenate sequence $S_1$ and sequence $S_2$
$x \cdot S_2$	Cons $x$ onto sequence $S_2$

---

## Expressions

An expression in core Verse evaluates to produce possibly-empty sequence of values. In  $V^-$ , values depend on  $\alpha$ . If  $\alpha$  is a Verse-like expression,  $\vartheta$  will be a value sequence. If it is an ML-like expression, it will be a single value.

A guarded expression evaluates to produce a **result**. A result is either a possibly-empty sequence of values or reject.

$$r ::= \vartheta \mid \mathbf{reject}$$

$$\rho; \mathcal{T} \vdash \alpha \Downarrow \vartheta \quad (\text{EVAL-EXPR})$$

$$\rho; \mathcal{T} \vdash g \Downarrow r \quad (\text{EVAL-GUARDED-EXPR})$$

$$\rho; \mathcal{T} \vdash g \rightsquigarrow \Leftarrow \quad (\text{NOTREE})$$

## 4 Sequences

The trivial sequence is  $\varepsilon$ . Sequences can be concatenated with infix  $\cdot$ . In an appropriate context, a value like  $x$  stands for the singleton sequence containing  $x$ .

$$\begin{aligned} \varepsilon \cdot ys &\equiv ys \\ ys \cdot \varepsilon &\equiv ys \\ (xs \cdot ys) \cdot zs &\equiv xs \cdot (ys \cdot zs) \end{aligned}$$

## 5 Rules (Big-step Operational Semantics) for $V^-$ :

### Evaluating Guarded Expressions

$$(\text{EVAL-ARROWEXPR}) \quad \frac{\rho; \varepsilon \vdash e \Downarrow \vartheta}{\rho; \varepsilon \vdash \rightarrow e \Downarrow \vartheta}$$

$$(\text{EVAL-EXISTS}) \quad \frac{\rho\{x \mapsto \perp\}; \mathcal{T} \vdash g \Downarrow r}{\rho; \mathcal{T} \vdash \exists x. g \Downarrow r}$$

$$(\text{G-MOVE-TO-CTX}) \quad \frac{\rho; eq \cdot \mathcal{T} \vdash g \Downarrow r}{\rho; \mathcal{T} \vdash eq; g \Downarrow r}$$

$$(\text{EVAL-MOVE-EQN}) \quad \frac{\rho; \mathcal{T} \vdash e \Downarrow \vartheta \quad \rho\{x \mapsto \vartheta\}; \mathcal{T} \cdot \mathcal{T}' \vdash g \Downarrow r'}{\rho; \mathcal{T} \cdot x = e \cdot \mathcal{T}' \vdash g \Downarrow r'}$$

$$(\text{G-VCON-SINGLE-FAIL}) \quad \frac{K \neq K'}{\rho; \mathcal{T} \cdot K = K' \cdot \mathcal{T}' \vdash g \Downarrow \mathbf{reject}}$$

$$(\text{G-VCON-SINGLE-SUCC}) \quad \frac{\rho; \mathcal{T} \cdot \mathcal{T}' \vdash g \Downarrow r}{\rho; \mathcal{T} \cdot K = K \cdot \mathcal{T}' \vdash g \Downarrow r}$$

$$(\text{G-VCON-MULTI-FAIL}) \quad \frac{K \neq K'}{\rho; \mathcal{T} \cdot K(e_1, \dots e_n) = K'(e'_1, \dots e'_n) \cdot \mathcal{T}' \vdash g \Downarrow \mathbf{reject}}$$

$$(\text{G-VCON-MULTI-SUCC}) \quad \frac{\rho; [e_i = e'_i \mid 1 \leq i \leq n] \cdot \mathcal{T} \cdot \mathcal{T}' \vdash g \Downarrow r}{\rho; \mathcal{T} \cdot K(e_1, \dots e_n) = K(e'_1, \dots e'_n) \cdot \mathcal{T}' \vdash g \Downarrow r}$$

## Evaluating General Expressions

$$(\text{IF-FI-SUCCESS}) \quad \frac{\rho; \mathcal{T} \vdash g \Downarrow \vartheta}{\rho; \mathcal{T} \vdash \text{IF } g \square \dots \text{FI} \Downarrow \vartheta}$$

$$(\text{IF-FI-REJECT}) \quad \frac{\rho; \mathcal{T} \vdash g \Downarrow \mathbf{reject} \quad \rho; \mathcal{T} \vdash \text{IF } \dots \text{FI} \Downarrow \vartheta}{\rho; \mathcal{T} \vdash \text{IF } g \square \dots \text{FI} \Downarrow \vartheta}$$

$$(\text{VCON-EMPTY}) \quad \frac{}{\rho; \mathcal{T} \vdash K \Downarrow K}$$

$$(\text{VCON-MULTI}) \quad \frac{\rho; \mathcal{T} \vdash e_i \Downarrow \vartheta_i \quad 1 \leq i \leq n}{\rho; \mathcal{T} \vdash K(e_1, \dots e_n) \Downarrow K(\vartheta_1, \dots \vartheta_i)}$$