Under the multivariate regression setup, following the same routine of univariate case,

$$\text{the total sum of squares } \ \mathbf{T} = (\mathbf{Y} - \bar{\mathbf{Y}})'(\mathbf{Y} - \bar{\mathbf{Y}});$$
$$\text{the error sum of squares } \ \mathbf{E} = (\mathbf{Y} - \hat{\mathbf{Y}})'(\mathbf{Y} - \hat{\mathbf{Y}});$$
$$\text{the regression sum of squares } \ \mathbf{H} = (\hat{\mathbf{Y}} - \bar{\mathbf{Y}})'(\hat{\mathbf{Y}} - \bar{\mathbf{Y}}).$$

Denote $\mathbf{P} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ as the projection matrix of the space spanned by $\mathbf{X}$, $\mathbf{P}_1$ as the projection matrix of the space spanned by $\mathbf{1}$, the n-dimensional intercept column with all elements 1's. That is, $\mathbf{P}_1$ is indeed a special $\mathbf{P}$ where our $\mathbf{X}$ degenerates to $\mathbf{1}$. Furthermore, $\mathbf{I}$ is the $n \times n$ identity matrix, and $\mathbf{J}$ is the $n \times n$ matrix with all elements 1's.

1. Write out the explicit form with all elements of the matrix $\bar{\mathbf{Y}}$.
2. Show the following statements:
   - (a) $\mathbf{P}$ (and hence $\mathbf{P}_1$) and $\mathbf{I} - \mathbf{P}$ are idempotent, i.e., they are symmetric and their squares equal to themselves.
   - (b) $\hat{\mathbf{Y}} = \mathbf{P}\mathbf{Y}$; $\bar{\mathbf{Y}} = \mathbf{P}_1\mathbf{Y}$. (That means, as we stated before, $\bar{\mathbf{Y}}$ can be viewed as the fitted value of $\mathbf{Y}$ when regressing only the intercept.)
   - (c) $\mathbf{Y}'\hat{\mathbf{Y}} = \hat{\mathbf{Y}}'\hat{\mathbf{Y}}$; $\mathbf{Y}'\bar{\mathbf{Y}} = \bar{\mathbf{Y}}'\bar{\mathbf{Y}}$; $\hat{\mathbf{Y}}'\bar{\mathbf{Y}} = \bar{\mathbf{Y}}'\bar{\mathbf{Y}}$.
   - (d) $\mathbf{E} = \mathbf{Y}'\mathbf{Y} - \hat{\mathbf{B}}'\mathbf{X}'\mathbf{Y} = \mathbf{E} = \mathbf{Y}'\mathbf{Y} - \hat{\mathbf{Y}}'\hat{\mathbf{Y}} = \mathbf{T} = \mathbf{Y}'(\mathbf{I} - \mathbf{P})\mathbf{Y}$.
   - (e) $\mathbf{T} = \mathbf{Y}'\mathbf{Y} - n\bar{\mathbf{y}}\bar{\mathbf{y}}' = \mathbf{T} = \mathbf{Y}'\mathbf{Y} - \bar{\mathbf{Y}}'\bar{\mathbf{Y}} = \mathbf{T} = \mathbf{Y}'(\mathbf{I} - \mathbf{P}_1)\mathbf{Y} = \mathbf{Y}'(\mathbf{I} - \mathbf{J}/n)\mathbf{Y}$.
   - (f) $\mathbf{H} = \hat{\mathbf{B}}'\mathbf{X}'\mathbf{Y} - n\bar{\mathbf{y}}\bar{\mathbf{y}}' = \mathbf{H} = \hat{\mathbf{Y}}'\hat{\mathbf{Y}} - \bar{\mathbf{Y}}'\bar{\mathbf{Y}}$.