Chapter 9: Time Series Regression

Time Series Analysis
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- Intervention Analysis
- Outliers detection
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§9.1 Intervention Analysis

- Consider the monthly U.S. Airline miles, from January 1996 to May 2005.
- We find a sudden drop in the number of air passengers in September 2001 and several months thereafter.
- The terrorist attacks of September 2001 deeply depressed air traffic around that period, but air traffic gradually regained the losses as time went on. This is an example of an intervention that results in a change in the trend of a time series.

§9.1 Intervention Analysis

- Intervention Analysis aims at studying the effects of an intervention on a time series.
- We assume that the intervention affects the process by changing the mean function or trend of a time series. So the model is like

$$Y_t = m_t + N_t,$$

  where m_t is the change in the mean function and N_t is modeled as some ARIMA process, which represents the underlying time series were there no intervention.

- To model m_t, we first introduce the step function S_t and the pulse function P_t, where

$$S_t = I_{t \geq T}, \qquad P_t = (1 - B)S_t.$$

- Immediate and permanent shift in the mean function is modeled by $m_t = \omega S_t$. If a delay of d time unit happens, then $m_t = \omega S_{t-d}$.
- gradually impact can be modelled by an AR type model: $m_t = \delta m_{t-1} + \omega S_{t-1}$.
  It can be shown that $m_t = \omega \frac{1-\delta^{t-T}}{1-\delta}$ for $t \geq T$. The smaller $|\delta|$ is, the quicker the ultimate change is felt by the system.

- Short life intervention is modeled by $m_t = \omega P_t$.
  Intervention effect that gradually dies out can be modeled by $m_t = \delta m_{t-1} + \omega P_t$.
  To incorporate delay changes, we could consider models like $m_t = \delta m_{t-1} + \omega P_{t-1}$.
  In terms of backward operator, we have

$$m_t = \frac{\omega B}{1 - \delta B} P_t.$$

We could use combinations to model more sophisticated intervention effects.

$$m_t = \omega_0 P_t + \frac{\omega_1 B}{1 - \delta B} P_t + \frac{\omega_2 B}{1 - B} P_t$$

Or equivalently,

$$m_t = \frac{\omega(B)}{\delta(B)} P_t.$$

Example: $\omega_0 P_t + \frac{\omega_1 B}{1-\delta B} P_t = \frac{\omega_0 + \omega_1 - \omega_0 \delta B}{1-\delta B} P_t$

§9.2 Outliers detections

- Outliers refer to atypical observations that may arise because of measurement and/or copying errors or because of abrupt, short-term changes in the underlying process.
- Two kinds of outliers can be distinguished, namely additive outliers (AO) and innovative outliers (IO).
  Let Y_t be the unperturbed process and Y_t' be the perturbed one.
  AO: $Y_t' = Y_t + \omega P_t$.
  In other words, $Y_T' = Y_T + \omega$, but $Y_t' = Y_t$ for $t \neq T$.
  IO: $e_t' = e_t + \omega P_t$.
  Note the error terms affects the future, so it perturbs all observations on and after T, although with diminishing effect.

§9.3 Spurious Regression

- Consider the monthly milk production and the logarithms of monthly electricity production in the United States from January 1994 to December 2005.
- Are these two series really correlated to each other?
- Is the cross correlation function $\rho_k(X, Y) = corr(X_t, Y_{t-k})$ really meaningful?

Reference

Please read Chapter 11 of Cryer & Chan (2008).