7 Small Physics

The rules, or laws, of classical mechanics break down typically in three cases. We have seen that when things start going quickly, we need to take special relativity into account. Another form of relativity – the general theory – is needed when things get very heavy, and the gravitational fields are strong. The third exception is very mysterious – and occurs often when we deal with very small objects like atoms and electrons. This is the realm of quantum physics, and many of its discoverers expressed horror or puzzlement at its conclusions and philosophy.

Having said that, there is no need to be frightened. While there is much we do not understand, a set of principles have been set up which allow us to perform accurate calculations. Furthermore, those calculations agree with experiment to a high degree of accuracy. The development of the transistor, hospital scanner, and many other useful devices testify to this. The situation is analogous to a lion-tamer who can get the lion to jump through a hoop, though she doesn't know what is going on inside the lion.

7.1 *Waves and Particles*

Quantum objects, like electrons and photons (packets of light) are difficult to describe. As physicists, we have two models, or descriptions, which we are comfortable using – the wave and the particle.

Waves can interfere, they have a wavelength, frequency and intensity, and they carry energy by means of fluctuations in a medium. The intensity is continuous – it can take any value.

Particles on the other hand, are lumps. They possess individual masses, energies and momenta. They most certainly do not interfere – if you add 1 apple to 1 apple, you always get 2 apples. Finally they only come in integer numbers. You can have one, or two, or 45 678 543; but you can't have half.

The electron fits neither description. Light fits neither description. The descriptions are too simplistic. However there are instances when the particle description fits well – but it doesn't always fit. There are also instances when the wave description fits well – but it doesn't always fit.

Given that a particular electron beam may behave like particles one minute, and waves the next, we need some kind of 'phrase book' to convert equivalent measurements from one description to the other. Quantum theory maintains that such a 'phrase book' exists.

The total number of particles (in the particle picture) is related to the intensity (in the wave picture). The exact conversion rate can be determined using the principle of conservation of energy.

The energy per particle (in the particle picture) is related to the frequency (in the wave picture) by the relationship

Energy of one particle (in J) = *h* × Frequency of wave (Hz), (1)

where h is the Planck constant, and has a value of 6.63×10^{-34} Js. You may also come across the constant 'h-bar' $\hbar \equiv h/2\pi$, which can be used in place of *h* if you wish to express your frequency as an angular frequency in radians per second.

The momentum per particle (in the particle picture) is related to the wavelength of the wave (in the wave picture) by the relationship

$$\text{Momentum of one particle (kg m/s)} = \frac{h}{\text{Wavelength (m)}} \quad . \qquad (2)$$

7.2 *Uncertainty*

The bridge between wave and particle causes interesting conclusions. We have seen in the chapter on Waves that a wave can have a well-defined frequency or duration (in time), but not both. This was expressed in the bandwidth theorem:

$$\Delta f \, \Delta t > 1 .$$

When combined with our wave-particle translation, we obtain a relationship between *energy* and time:

$$\Delta E \, \Delta t > h . \qquad (3)$$

In other words, only something that lasts a long time can have a very well known energy.

Let us have an example. Suppose a nucleus is unstable (radioactive), with a half-life *T*. Seeing as the emission of the radiation is a process that typically 'takes' a time *T*, the energy of the alpha particle (or whatever) has an inherent uncertainty of $\Delta E \approx h/T$. If we were watching a spectrometer, monitoring the radiation emitted, we would expect to see a spread of energies showing this level of uncertainty.

The bandwidth theorem also has something to say about wavelength:

$$\Delta\left(\frac{1}{\lambda}\right) \Delta x > 1 .$$

This has the quantum consequence:

$$\Delta p \, \Delta x > h \ . \qquad (4)$$

This is frequently stated as, “You can’t know both the momentum and the position of a particle accurately.” It might be better stated as, “Since it is a bit like a wave, it can not *have* both a well defined position and momentum.”

We can use this to make an estimate for the speed of an electron in an atom. Atoms have a size of about 10^{-10} m. Therefore, for an electron in an atom, $\Delta x \approx 10^{-10}\ \mathrm{m}$. So, using equation (4), $\Delta p \approx 10^{-23}\ \mathrm{kg\,m/s}$. Given the electron mass of about 10^{-30} kg, this gives us a speed of about 10^7 m/s – about a tenth the speed of light!

Caution: Please note that we haven’t defined precisely what we mean by uncertainty (Δ). That is why we have only been able to work with approximate quantities. In more advanced work, the definition can be tightened up (to mean, say, standard deviation). However it is better for us to leave things as they are. In any case, it is never wise to state uncertainties to more than one significant figure!

7.3 ***Atoms***

Putting things classically for a moment, the electron orbits the nucleus. While a quantum mechanic thinks this description very crude, we shall use it as a starting point.

Now, let’s imagine the electron as a wave. For the sake of visualization, think of it as a transverse wave on a string that goes round the nucleus, at a distance *R* from it. If the electron wave is to make sense, the string must join up to form a complete circle. Therefore the circumference must contain a whole number of wavelengths.

$$\begin{gathered} 2\pi R = n\lambda \\ 2\pi R = \frac{nh}{p} \\ pR = \frac{nh}{2\pi} = n\hbar \\ L = n\hbar \end{gathered} \qquad (5)$$

The conclusion of this argument is that the angular momentum of the electron, as it goes round the nucleus, must be in the $\hbar$-times table.

The argument is simplistic, in that the quantum picture does not involve a literal orbit. However, the quantum theory agrees with the reasoning above in its prediction of the angular momentum.

Given that the angular momentum can only take certain values (we say that it is quantized – it comes in lumps), we conclude that the electron can only take certain energies. These are called the energy levels, and we can work out the energies as follows:[26]

$$\begin{aligned} \text{Kinetic Energy} &= \frac{L^2}{2I} = \frac{n^2\hbar^2}{2mR^2} \\ \text{Potential Energy} &= -\frac{Ze^2}{4\pi\varepsilon_0 R} \end{aligned}, \tag{6}$$

where Z is the number of protons in the nucleus. We are, of course, ignoring the other electrons in the atom – hence this model is only directly applicable to hydrogen.[27] Next, we use the relationships derived in section 1.2.2, where we showed that for a Coulomb attraction,

Potential energy = -2 × Kinetic Energy

$$\begin{aligned} \text{Potential energy} &= 2E \\ \text{Kinetic energy} &= -E \end{aligned} \tag{7}$$

where E is the total energy of the electron. We may use this information to eliminate the radius in equations (6), obtaining:

$$E = -\frac{Z^2 e^4 m}{2(4\pi\varepsilon_0\hbar)^2} \times \frac{1}{n^2}. \tag{8}$$

When dealing with atoms, the S.I. units can be frustrating. A more convenient unit for atomic energies is the electron-volt. This is the energy required to move an electron through a potential difference of one volt, and as such it is equal to about 1.60×10^{-19} J. In these units, equation (8) can be re-written:

$$E = -\frac{Z^2}{n^2} \times 13.6\,\mathrm{eV}. \tag{9}$$

This form should be remembered. It will help you to gain a 'feel' for the energies an electron can have in an atom, and as a result, it will help you spot errors more quickly.

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[26] The kinetic energy is calculated using the relationships derived in chapter 3. If you do not wish to go in there, a simpler derivation can be employed. L=mvR, where v is the speed. Therefore the kinetic energy $mv^2/2 = L^2/(2mR^2)$.

[27] Hydrogen, that is, and hydrogen-like ions: which are atoms that have had all the electrons removed apart from one.

When an electron moves from one level (*n* value) to another, energy is either required or given out. This is usually in the form of a photon of light that is absorbed or emitted. The energy of the photon is, as usual, given by the Planck constant, multiplied by the frequency (in Hz) of the light.

If an electron moves from orbit n_1 to n_2, where $n_2<n_1$, the frequency of photon emitted is therefore given by:

$$f = Z^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)3.29\times10^{15}\ \text{Hz}\,. \qquad (10)$$

Similarly, the formula gives the frequency of photon required to promote an electron from n_2 to n_1. The frequency of photon required to remove the electron completely from the atom (if it starts in level n_2) is also given by equation (10), if n_1 is taken as infinite.

7.4 Little Nuts

As far as Romans were concerned, the stones in the middle of olives were ‘little nuts’ or **nuclei**. We shall thus turn our attention to ‘nutty physics’.

The nuclear topics required for the International Olympiad are common to the A-level course. In this book we shall merely state what knowledge is needed. You will be able to find out more from your school textbook.

7.4.1 Types of radiation

Alpha decay: in which a helium nucleus (two protons and two neutrons) is ejected from the unstable nucleus.

Beta decay: in which some weird nucleonic processes go on. In all beta decays, the total number of nucleons (sometimes called the *mass number*) remains constant.

In β- decay (the most common), a neutron turns into a proton and an electron. The electron is ejected at speed from the nucleus.

There are two other forms of beta radiation. In β+ decay, a proton turns into a neutron and an anti-electron (or positron). The positron flies out of the nucleus, and annihilates the nearest electron it sees. The annihilation process produces two gamma rays.

The other permutation is electron capture (ε) in which an electron is captured from an inner (low *n*) orbit, and ‘reacts’ with a proton to make a neutron. This phenomenon is detected when another electron descends to fill the gap left by the captive – and gives out an X-ray photon as it does so.

Gamma decay: in which the nucleus re-organizes itself more efficiently, leading to a drop in its internal potential energy. This energy is released as a burst of electromagnetic radiation – a gamma ray photon. By convention high energy photons are called X-rays if they come from the electrons in an atom, and gamma rays if they come from a nucleus.

7.4.2 Radioactive decay

It is beyond the wit of a scientist to predict when a particular nucleus will decay. However we have so many radionuclides in a sample that the average behaviour can be modelled well.

The rate of decay (number of decays per second) is proportional to the number of nuclei remaining undecayed. This 'rate of decay' is called the activity, and is measured in Becquerels (Bq). We define a parameter λ to be the constant of proportionality:

$$\begin{aligned} I &= -\frac{dN}{dt} = \lambda N \\ N &= N_0 e^{-\lambda t} \\ I &= \lambda N_0 e^{-\lambda t} = I_0 e^{-\lambda t} \end{aligned} \tag{11}$$

where N_0 is the initial number of radionuclides, and I_0 is the initial activity.

The half-life (T) is the time taken for the activity (or the number of undecayed nuclei) to halve. This is inversely proportional to λ, as can be seen:

$$\begin{aligned} \tfrac{1}{2} &= \exp(-\lambda T) \\ \ln\tfrac{1}{2} &= -\lambda T \\ \ln 2 &= \lambda T \\ T &= \frac{\ln 2}{\lambda} \end{aligned} \quad . \tag{12}$$

If a half-life is too long to measure directly, the value of λ can be determined if I and N are known separately. I would be measured simply by counting the decays in one year (say), while N would be measured by putting a fraction of the sample through a mass spectrometer.

7.4.3 Nuclear Reactions

Now for the final technique: You will need to be able to calculate the energy released in a nuclear reaction. For this, add up the mass you started with, and add up the mass at the end. Some mass will have gone missing. Remembering that mass and energy are basically the same thing – the 'lost mass' is the energy released from the nuclei.

^{2}H	2.014102
^{3}H	3.016049
^{4}He	4.002604
n	1.008665