BPhO Round 2 January 2015

In these questions you are asked to make reasoned estimates, assumptions and explanations. These assumptions and estimations must be clearly stated.

Qu 1.

(a)

(i) Venus is an inner planet. Therefore it will show a full range of phases (similar to our Moon), but its distance from earth will vary considerably as it is not in orbit about the Earth.
The Venus –Sun distance is 0.7 AU, although an estimate of about ½ AU will suffice to explain the observation. (This figure is deliberately not given in the data). The period of orbit about the sun will be shorter than that of the Earth and so it will change in appearance (size and shape) over a few days. This is an important observation by Galilea in developing support for the heliocentric model of the Solar System.
If we take 0.7 AU, then its angular size will vary by a factor of 1.7/0.3 = 6.
A sketch diagram is needed to explain the crescent shape clearly.

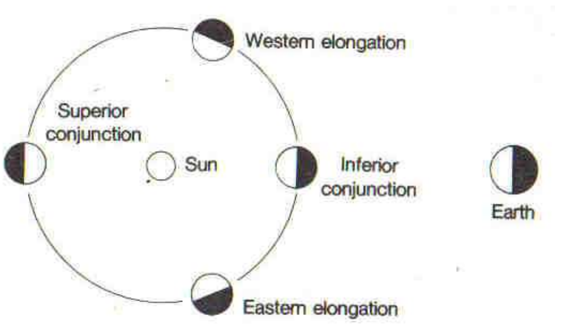


Figure 1. Illumination of surface of Venus, which will be seen as crescents from the Earth.

http://bdaugherty.tripod.com/gcseAstronomy/planetsInner.html

Figure 2. A composite sequence of images of Venus photographed with the 50 cm reflector at the Torbay Astronomical Society Observatory during its evening apparition between early May and late October 2002.

http://eaae-astronomy.org/WG3-SS/WorkShops/VenusOrbit.html

(ii) Calculate the angular diameter subtended at the eye to see if the change of apparent shape should be detectable.
Here the correct values are used, but any reasonable estimates would suffice $R_{\mathrm{V}} \approx R_{\mathrm{E}}$ and $r_{\mathrm{V}} \approx 0.7 r_{\mathrm{E}}$.
At its closest, the angular diameter of Venus is $\approx \frac{2r_{\mathrm{E}}}{(1.0-0.7)\ \mathrm{AU}} = 3 \times 10^{-4}\mathrm{rad}$
And at its furthest, the angular diameter subtended at the eye is $\approx \frac{2r_{\mathrm{E}}}{(1.0+0.7)\ \mathrm{AU}} = 5 \times 10^{-5}\mathrm{rad}$, a factor of 6 smaller.
Since the moon is about ½° in angular width, or $^{1}/_{120}{}^{\text{th}}$ radian, Venus is 1/170 th to 1/30 th of this, so that the crescent shape of Venus is just too small to be seen by the naked eye.

The brightness will vary, as the distance and area illuminated as seen from Earth will both vary during the course of the relative motion of Earth and Venus.

(iii) The ratio of the distances from the sun to the planets was known from Kepler's Laws, for example, but the actual size of the planetary orbits was unknown. Looking at Venus from two diametrically opposite points on the earth would have the two telescopes pointing in parallel directions less about 10^{-4} radians (see calculations above), or about 6 x 10^{-3} degrees. This is when Venus is observed at near its greatest angle from the Sun. But Venus is observed near sunrise of sunset, low in the horizon, so the observation is looking through a thick layer of atmosphere at that angle and this causes aberrations or distortion of the image and its position. It is a very mall angle to measure, beyond the technical level of telescope mounts of the 8th century for two separately mounted telescopes. The way to measure such a small angle would be to see the two different positions of Venus against a fixed background of stars. But low in the horizon it is difficult to observe nearby fainter stars, and Venus is moving relatively rapidly and not photography is available. The rate at which Venus traverses the background firmament is too rapid for such a small angle to be measured reliably. (By the 1840s, Bessel could measure the motion of a star through an angle of 10^{-7} radians (a second of arc), for a star at a distance of 1 parsec from the Earth, but this was done be timing when it crossed the meridian overhead, compared to other nearby stars, not measuring the angle directly).
In addition, Venus is not a point object to observe, but an extended object so hard to not the movement, and it has an atmosphere, so it does not have a sharp edge to measure.
By measuring it transit across the face of the sun, a timing method could be used for the two points on opposite sides of the earth, and timing of the entry and exact of the planet across the edge of the sun could be measured to a fraction of a second, giving a very fine measurement of the different angular line of sight for the two telescopes.

(b) The rate of flow of water is to be considered. If this is constant, then for a constant mass (or volume) rate of flow, if the aperture is decreased then the speed at that point must be increased. The inlet pressure is determined by the head of water (the height of the tank) and that is constant. The outlet pressure is much lower, and small changes made by sticking a finger of the end of the hose do not affect the pressure difference very much. If the end of the hose is blocked, then the rate of flow will not be maintained constant and the pressure along the hose will tend to zero.

(c) As the model of the atom is that it is mainly "empty space", an alpha (helium nucleus) passing through will ionise the gas atoms by removing electrons. This is a few eV per atom and as an alpha come from a nuclear decay of several MeV, hundreds of thousands of atoms will be ionised on route. The small mass of the electrons will not cause the massive alpha particle (8000 m_e) to deviate much from its path, and will also not slow it down much at first, as the energy losses will be relatively small. However, as the alpha does finally slow, it will spend more time in one unit of length, and so will cause ionisation of more of the nearby atoms, hence producing more dark pixels, a measure of the degree of ionisation of the gas. After the rise at the end of the graph, the drop is because the alphas have lost their energy very gradually in many repeated collisions of an identical nature. So they will statistically travel a similar distance (unlike a one off loss of energy, which would cause a lot of variation in path length). So the alphas travel similar distance, and then stop, and pick a couple of electrons to cause no further ionisation.

(d)

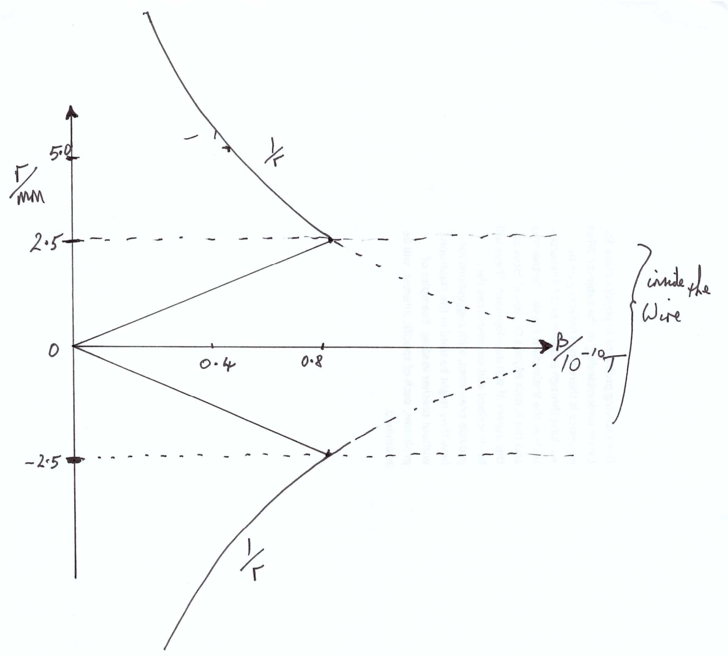
(i) $eV = \frac{1}{2}mv^2$ as the electrons start from rest in a uniform field. So $v_{\text{max}} = 1.9 \times 10^7 \text{m s}^{-1}$

(ii) Uniform field, so half the distance for half the potential, so speed is $v = \frac{v_{\text{max}}}{\sqrt{2}} = 1.3 \times 10^7 \text{ m s}^{-1}$

(iii) Equivalent to a long thin wire. $B = \frac{\mu_0 I}{2\pi r}$ to give $B = 4 \times 10^{-11}$ T.

(iv) Outside the beam, the field falls off as 1/*r*, whilst in the centre, the field is zero, by symmetry.

The current density is $\rho = \frac{I}{\pi r^2}$ so that the current inside a wire of diameter $r \le R$ is $I = \rho\pi r^2$.
The field is due to the current flowing within radius *r* only. So $B = \frac{\mu_0 \rho \pi r^2}{2\pi r} = \frac{1}{2}\mu_0 \rho r$



(v) The field will focus the beam (slightly), as can be seen by the greater filed density outside the beam (straight field lines repel each other) or from Flemings LHR, or $\vec{F} = I\vec{\ell} \times \vec{B}$.

Qu 2. Treating the key part of the question, of skiing down the straight slope.

A diagram showing the resolved forces is expected.
Forces:
The skier accelerates and so $ma = mg\sin\theta - \mu mg\, cos\theta$

$$a = 0.0732 \text{ m } s^{-1}$$

As the acceleration is constant, *s* = 171 m

and $h = s\sin 5° = 14.9$ m

Energy:

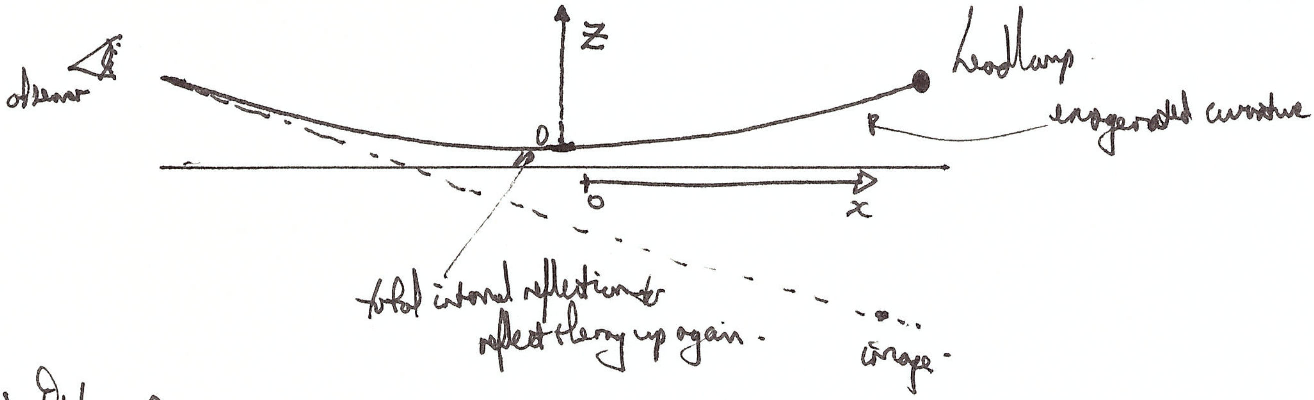
$$mgh - \frac{1}{2}mv^2 = F.s = \mu mg\cos\theta \,.\frac{h}{\sin\theta}$$

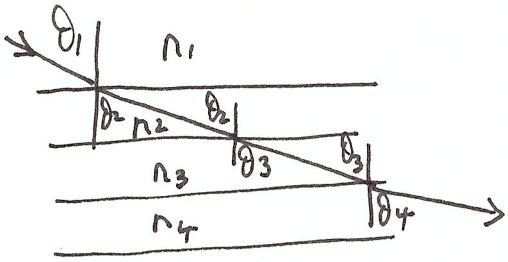
$$h\left(1 - \frac{\mu}{\tan\theta}\right) = \frac{v^2}{2g}$$

$$h = 14.9 \text{ m}$$

The effect of the curve at the bottom can also be considered. It is going to be small as the arc length s about 3 m, and so we can consider the speed to be approximately 5 m s^{-1} during this section. The centripetal acceleration due to the curve in the vertical plane is 5 m s^{-2}, which is about g/2. So the frictional force will be about 1½ times greater and the work done will be $1.5mg \times 0.01$. This produces an extra mgh amounting to an extra cm or so, a negligible amount. However turning to the left, or right, will possibly introduce a larger frictional force and may allow for a slightly higher starting height, although difficult to estimate.

Qu 3

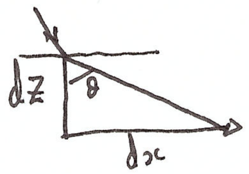




$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = \dots = C$$

(constant.)

so $C = n \sin\theta$



$$\tan\theta = \frac{dx}{dZ} \Rightarrow \left(\frac{dx}{dZ}\right)^2 = \tan^2\theta = \frac{\sin^2\theta}{1-\sin^2\theta} = \frac{(C/n)^2}{1-(C/n)^2}$$

$t: \tan\theta$
$c: \cos\theta$
$s = \sin\theta$

$c^2 + s^2 = 1$

$1 + t^2 = \frac{1}{c^2}$

$t^2 = \frac{1}{c^2} - 1$

$\frac{s^2}{c^2} = \frac{1-c^2}{c^2} = \frac{s^2}{1-s^2}$

$$\left(\frac{dx}{dZ}\right)^2 = \frac{C^2}{n^2\left(1-\frac{C^2}{n^2}\right)} = \frac{C^2}{n^2-C^2}$$

so $dx = \frac{C}{\sqrt{n^2-C^2}} \cdot dZ$ and $n^2 = n^2(Z)$

Given $n(Z) = n(0) + \alpha Z$, then $n(Z)^2 \simeq n(0)^2 + 2\alpha Z$

then $dx = \frac{C\, dZ}{\sqrt{n(0)^2 + 2\alpha Z - C^2}}$

Now when $Z \simeq 0$ then $\sin\theta_0 \simeq \frac{\pi}{2}$ and $n(0) = C$

so if we consider $Z = 0$ then $n(0)^2 \simeq C^2$

$\therefore$ $dx = \frac{C\, dZ}{\sqrt{2\alpha Z}}$ or $\int_0^x dx = \frac{C}{\sqrt{2\alpha}} \int_0^Z \frac{dZ}{Z^{\frac{1}{2}}}$

so $x = \frac{C\, 2 Z^{\frac{1}{2}}}{\sqrt{2\alpha}}$ $\left[x^2 = \frac{C^2 .2 .Z}{\alpha} \text{ - parabola in our approximations} \right]$.

for obtaining the x, Z relation this will be adequate.

Now $C \simeq 1$

and $n(Z) = n(0) + \alpha Z \simeq 1 + \alpha Z$; so $(1.0003) \simeq 1 + \alpha . 1$ when Z is 1 m say.

$1.0003 \simeq 1 + \alpha$

$\alpha \simeq 0.0003\ m^{-1}$

$\therefore$ $x = \frac{1}{0.025} . 2\sqrt{1}$ for headlamps 1 m above the ground, near where $Z = 0$

$x = 80$ m, so car to observer would be $\underline{160 \text{ metres}}$.

Some ideas and suggestions only

Qu 4 Rifle : 1 m long

4 cm diameter

Say, 2000 turns of wire as the solenoid.

$B = \mu_0 n I = 4\pi \times 10^{-7} \times 2000 \times I.$

$B = 2 \times 10^{-3} . I$

Attractive force on iron bullet.

Current has to turn off when bullet is midway through solenoid.

10g bullet, 200 m/s. $\Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} \times 10^{-2} \times 4 \times 10^4 = \underline{200\ J}$

capacitor as supply for current

trickle charging? Or perhaps minimal R, and low internal resistance of cells to charge quickly but without overheating.

half the solenoid to accelerate the bullet. Energy density of magnetic field is $\frac{B^2}{2\mu_0}$

So energy in half the solenoid : $\frac{B^2}{2\mu_0} \times \frac{l}{2} . \pi r^2 = 200\ J$

$B^2 = \frac{200 \times 2\mu_0 \times 2}{l \pi r^2} = \frac{800 \times 4\pi \times 10^{-7}}{0.5 \times \pi \times (2 \times 10^{-2})^2}$

$= \frac{1600 \times 10^{-7}}{10^{-4}} = 1.6$

$\therefore \underline{B = 1.3\ T}$

A very high field : $B = \mu_0 n I$

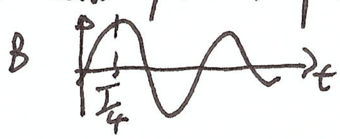
$\Rightarrow \underline{I = 520\ A}$ very large.

So more turns. Thicker wire but not for weight.

Smaller diameter needed. Some factors of 2 can be gained

Dynamics 0 to 200 m in 0.5 m $\Rightarrow t = \frac{1}{200}$ s.

$\therefore$ an LC oscillation to build up the B field. $\frac{T}{4} \approx \frac{1}{200}$ s.



$T = \frac{1}{50}$ s

For a solenoid, $N\phi = LI$.

$N . B \pi r^2 = LI$

$N . \mu_0 . \frac{N}{l} \cancel{I} \pi r^2 = L\cancel{I}$

$L = \frac{N^2 \mu_0 \pi r^2}{l} = 6 \times 10^{-3} H.$

$\left(= 2000^2 \times \mu_0 \pi \frac{(2 \times 10^{-2})^2}{1} \right).$

$f = 50\ Hz = \frac{1}{2\pi\sqrt{LC}}$

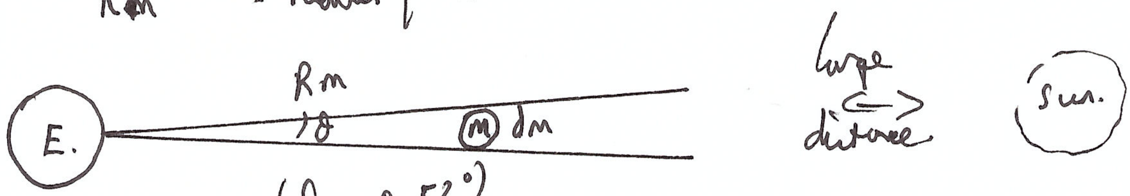
So $C \approx \frac{1}{600} F \approx 2\ mF.$

Then, $Q = I_{average} \times t = 250\ A \times \frac{1}{50} s = 5\ C.$

$\Rightarrow V = \frac{Q}{C} = \frac{5}{\frac{1}{600}} = \underline{3000\ V}.$

Qu. 5. d_e, d_m - diameters of earth, moon
R_m - radius of moon's orbit.

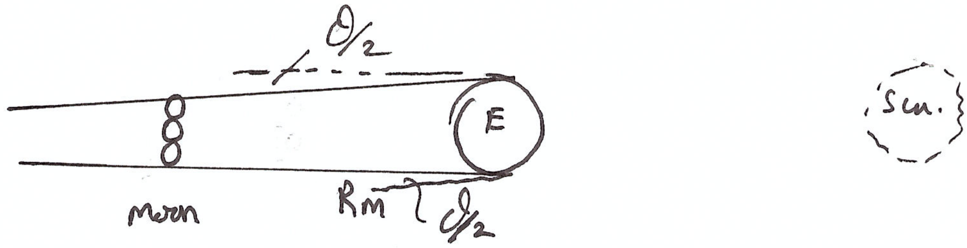
(1)



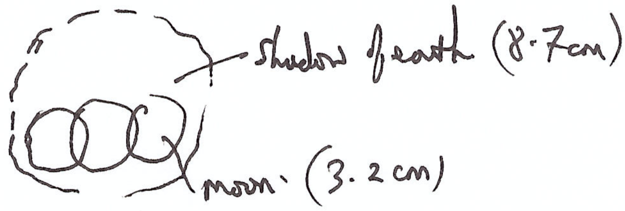
$(\theta \approx 0.52^\circ)$

$d_m = R_m \cdot \theta$

(2) From the image:



(3) measuring the picture:



$$\left(\frac{8.7}{3.2}\right) = \frac{d_e}{d_m} \simeq 2.7$$

$$\therefore \quad d_e = 2.7 d_m + 2 \times R_m \cdot \frac{\theta}{2}$$

$$d_e = 2.7 d_m + R_m \theta$$

and also $d_m = R_m \theta$

so $d_e = 3.7 d_m$

Hence $r_m = 1720$ km $(d_m = 3440$ km$)$

and $$R_m = \frac{d_m}{\theta} = \frac{3440}{\frac{0.52 \times 2\pi}{360}} = \underline{\underline{379{,}000 \text{ km}}}$$