BPO 2011 Paper 3

Some worked solutions

1(a)

Assumptions of Bernoulli's equation:

- incompressible liquid;
- non-turbulent flow;
- friction is negligible.

Distinguishing characteristics:

- Unlike a gas, the liquid cannot be compressed;
- The velocities of particles in the liquid do not fit a Maxwell-Boltzmann distribution;
- If the density of the gas is less than that of air, it will not go downwards at the "step" in the pipe.

$$(p_{in} - p_{out}) + \frac{1}{2}\rho(v_{in}^2 - v_{out}^2) + \rho g\,(h_{in} - h_{out}) = 0$$

1(d)

(i) The “temperature” given is actually an *energy*, as it has units of keV. At some temperature, the particles have sufficient thermal energy to overcome the repulsive interactions between them and initiate fusion when they collide. This is the “fusion temperature”.

(ii) Using the Boltzmann constant, k_b, we can convert temperatures into energies:

$$E = k_b T$$

We can also express temperatures as energies by rearranging the formula. 15 keV is equivalent to a temperature of 1.74×10^8 K, by using the formula above.

(iii) Paraffin is a hydrocarbon material derived from petroleum refining. As such, it contains a large number of hydrogen atoms per unit volume. Each H atom has a single proton as its nucleus, and the proton has very nearly the same rest mass as the neutron, so the energy transfer in the collision is efficient.

(iv) **The energy value in the question paper is wrong, the D-T reaction releases 17.59 MeV not 18.3 MeV.** With this in mind, the reason the neutron has a kinetic energy of 14.1 MeV is that the kinetic energies of the products are distributed in inverse proportion to the ratios of their masses:

$$\frac{m_{helium}}{m_{neutron}} = \frac{4}{1}$$

So the neutron will have energy of (0.8*17.59 MeV) = 14.1 MeV.

(v) Tritium is difficult to handle because (1) it is radioactive and (2) like hydrogen, it is difficult to contain, as most materials are permeable to it.

(vi) If substance A is said to have “a somewhat softer neutron spectrum” than substance B, this means that the average kinetic energy of the neutrons emitted from substance A is less than that for substance B.

(vii) Use a dummy mass, m_{dum}, for the photon:

$$E = hf = m_{dum}c^2$$
$$p = m_{dum}c$$
$$\text{So: } p = hf/c$$

Remember that this is not a strictly valid method, even though you get the right answer. The relativistic expression for the energy of a particle is:

$$E^2 = m^2c^4 + p^2c^2$$

…and this gives the right answer by a correct method, if you set the mass term to zero.

$$force = rate\ of\ change\ of\ momentum = \frac{power}{velocity}$$
$$pressure = \frac{force}{area}$$

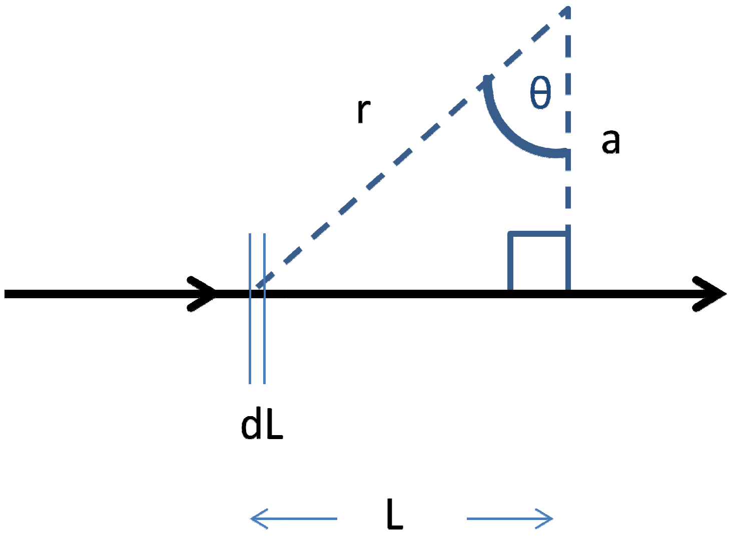
Surface area of sphere of radius R = 4πR

Putting in the values:

$$pressure = \frac{1\ MW}{3\ x\ 10^8\ m/s}\ x\ \frac{1}{4\pi\ x\ (10^{-3}\ m)^2}$$

$$pressure = 265\ N/m^2$$

2(a)



$\tan(\theta) = L/a$, so $dL/d\theta = a/\cos^2(\theta)$, and therefore $dL = d\theta.a/\cos^2(\theta)$

Also: $r = a/\cos(\theta)$, so $dL/r^2 = (d\theta.a/\cos^2(\theta)).(\cos^2(\theta)/a^2) = d\theta/a$

Integrate Biot-Savart law:

$$\int dB = \frac{\mu_0 \, i}{4\pi} \int_{-\infty}^{\infty} \frac{dL}{r^2} \sin(\theta)$$

$$B = \frac{\mu_0 \, i}{2\pi} \int_{0}^{\infty} \frac{dL}{r^2} \sin(\theta)$$

$$B = \frac{\mu_0 \, i}{2\pi} \int_{0}^{\frac{\pi}{2}} \frac{d\theta}{a} \sin(\theta)$$

$$B = \frac{\mu_0 \, i}{2\pi a} [-\cos(\theta)]_0^{\pi/2}$$

$$B = \frac{\mu_0 \, i}{2\pi a}$$

2(b)

Faraday's Law: $V \propto N\frac{d\emptyset}{dt}$ where Ø is the magnetic flux, V is the voltage, N is the number of turns and t is time.

$$\emptyset = \boldsymbol{B}.\boldsymbol{a}$$

$$\emptyset = \frac{\mu_0 I}{2\pi r}a$$

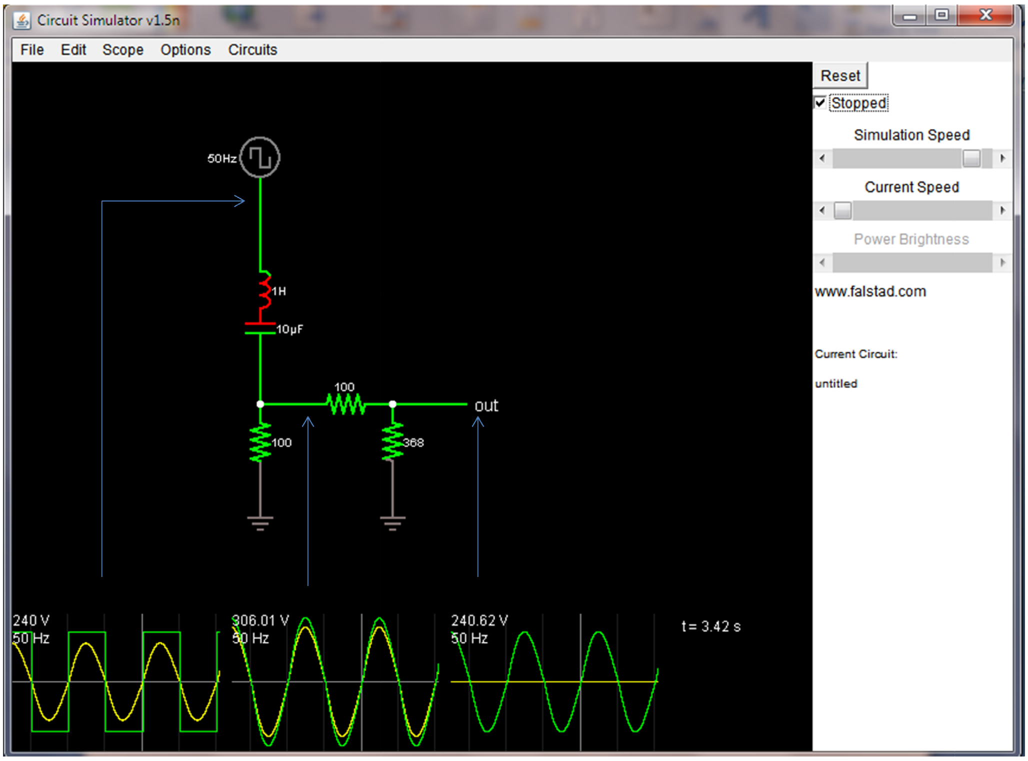
$$\frac{d\emptyset}{dt} = \frac{\mu_0 a}{2\pi r}\frac{dI}{dt}$$

Therefore: $V \propto N\frac{\mu_0 a}{2\pi r}\frac{dI}{dt}$

By Lenz's Law, the constant of proportionality is -1, so

$$V = -\frac{\mu_0 aN}{2\pi r}\frac{dI}{dt}$$

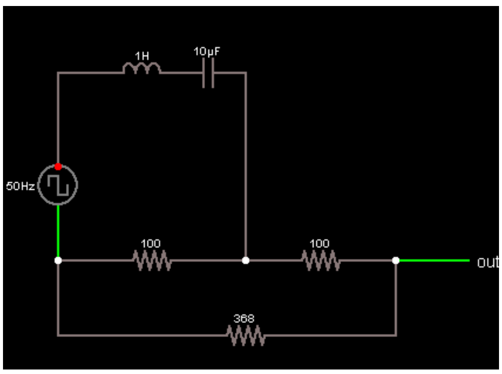
2(c)



This circuit will do it. It is a series RLC circuit followed by a voltage divider.

The input is a 50 Hz 240 V square-wave (left oscilloscope trace) where the high level is 240 volts above ground and the low level is zero volts. The signal after the capacitor is sinusoidal and in-phase, but the amplitude (306.01 V) is too high, hence the use of a voltage divider. The output is shown in the right-hand oscilloscope trace (voltage in green, current in yellow). The output voltage is in-phase with the input and of the same amplitude, and it has a sinusoidal form.

The circuit could also be drawn as shown below:

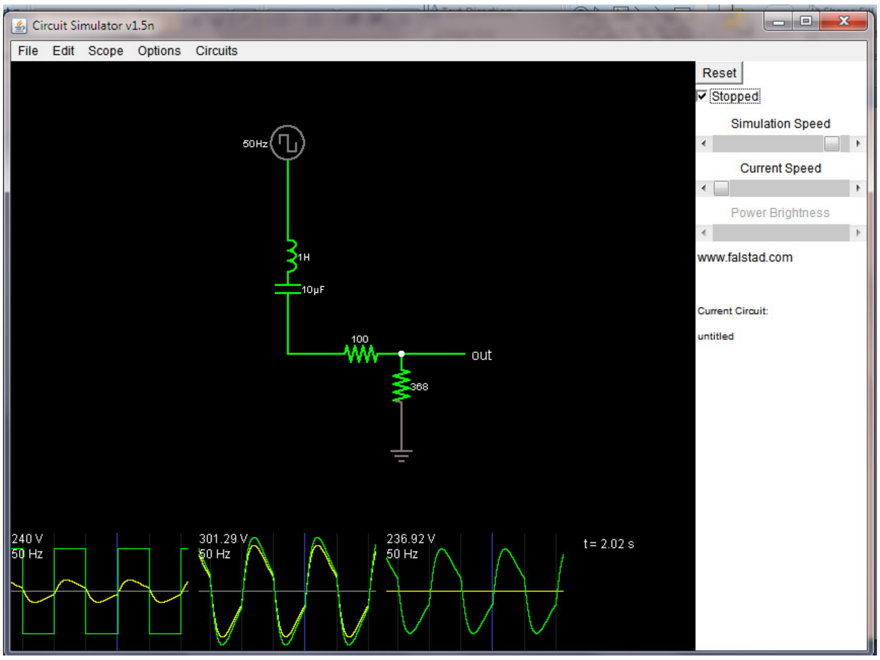


This makes it very apparent that the resonant RLC circuit is “tapped” to provide the input to the voltage divider.

I found the component values by trial-and-error using the circuit simulator.

All that is required of the LC combination is that its resonant frequency is 50 Hz. Since for resonance $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f$ = 50 Hz , C can be calculated if L is assumed to be equal to 1 H (or any other value).

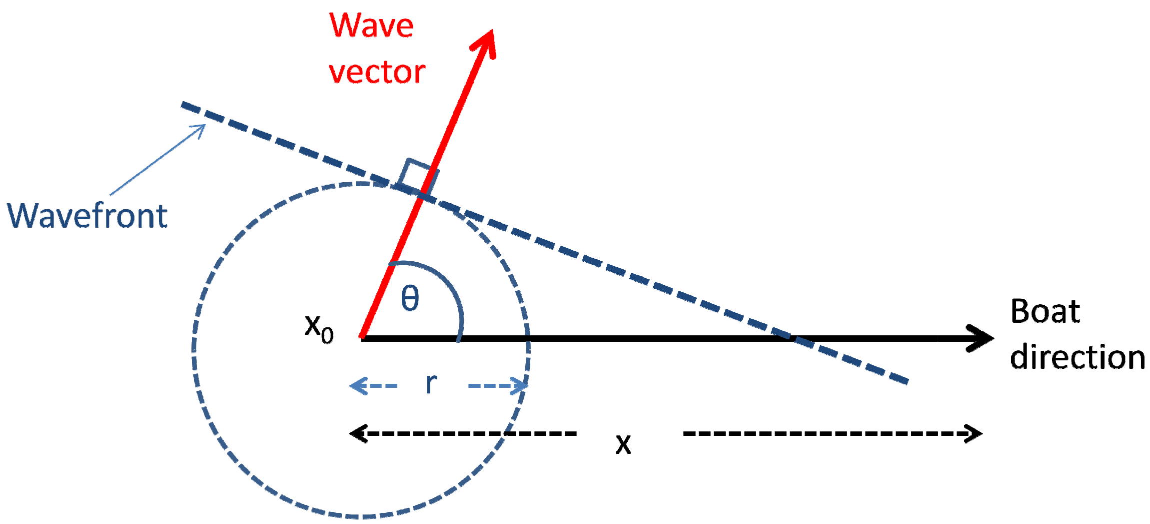
A student at A-level, under exam conditions, will do well to identify that an inductor and capacitor in series are required. It would surprise me if they also saw the need for the 100 Ω resistor between the capacitor and ground; without this resistor, the circuit output is not sinusoidal, as is shown in the figure below.



I would also be very surprised if the student saw the need for the voltage divider. If they *did* see this and attempted to calculate the resistor values, this should be done using the voltage divider formula $V_{out} = \left(\frac{R_2}{R_1 + R_2}\right) V_{in}$.

3(b)

The direction of the wave is (by definition) perpendicular to the wavefront.



If the boat is at x_0 at t = 0, then at t =1 second:

r = 2 m

x = 5 m

Therefore cos(θ) = r/x = 2/5 , giving θ = 1.16 radians (66.4°) to 3 significant figures.

3(c)

In time t, the particle travels a distance $x_p = v_p t$ from the origin. Light emitted from the particle's position at t = 0 travels a distance $x_{em} = v_{em}t$.

Analogously to part (b), a diagram can be drawn showing the angle between the EM wave and the particle's velocity.

$$\cos(\theta) = \frac{x_{em}}{x_p}$$

$$= \frac{v_{em}t}{v_p t}$$

The refractive index of a material is called "n" and given by $= \frac{c}{v_{em}}$, where c is the speed of light in vacuum. So we can get $\cos(\theta) = \frac{c}{nv_p}$.

Using $= \frac{v_p}{c}$, we get $\cos\theta = \frac{1}{n\beta}$.

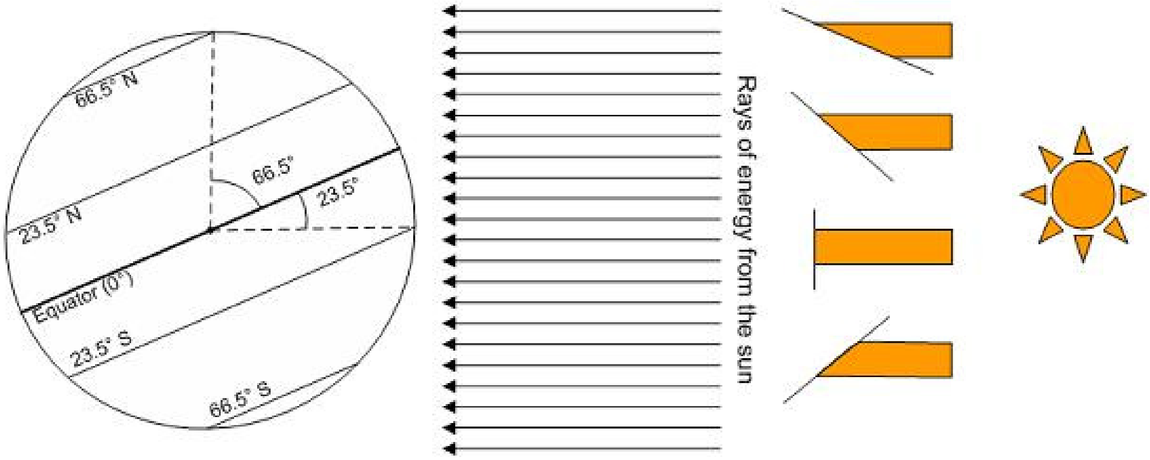
The Cerenkov radiation is *not* due to transitions between atomic or molecular energy levels. No credit will be given for attempts to explain the blue light using ideas based on energy level transitions. The radiation actually has a continuous spectrum, mainly distributed in the ultraviolet spectral region. Human eyes can't see UV radiation, but the Cerenkov radiation spectrum does have a low-energy tail which extends into the blue, which is what we see.

3(d)

$$temperature \propto power\ absorbed$$

$$power\ absorbed \propto flux$$

$$flux \propto effective\ area\ exposed\ to\ sunlight$$



Lamberts Cosine Law

$E_\theta = E_0 \cos(\theta)$

$\theta = |\beta - \delta|$

– Where
- θ = solar zenith (angle betw. Sun and perpendicular to surface)
- β = observer latitude
- δ = solar declination

The cosine relationship suggests that the temperature should be lowest at the poles and highest at the Equator. This is basically what is observed.