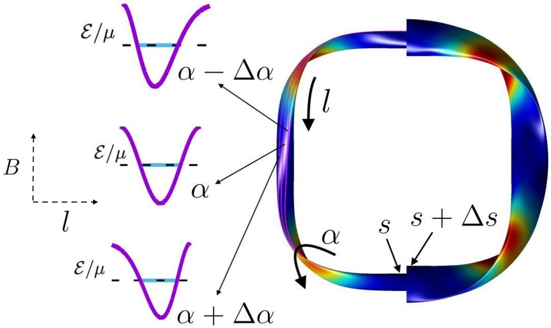
Robust fast ion confinement in quasi-symmetric stellarators

J. L. Velasco[1], I. Calvo[1] *et al.*

Laboratorio Nacional de Fusión, CIEMAT, Madrid, Spain

- Introduction
- Superbananas in QS fields
- Ripple wells
- Optimization strategies
- Discussion

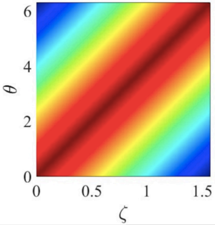
Neoclassical optimization, omnigeneity and quasi-symmetry



- Trapped particles *live* in regions of low field, moving back and forth along *l* at constant $\lambda = \mu/\mathcal{E}$.
- On a longer time scale, drift perpendicular to **B**
  - in the radial direction, $\overline{\mathbf{v}_M \cdot \nabla s}$, and/or
  - tangentially to the flux-surface, $\overline{\mathbf{v}_M \cdot \nabla\alpha}$.

Definition of omnigenous stellarator [Cary (1997) PRL]: $\overline{\mathbf{v}_M \cdot \nabla s} = 0$ for all particles.

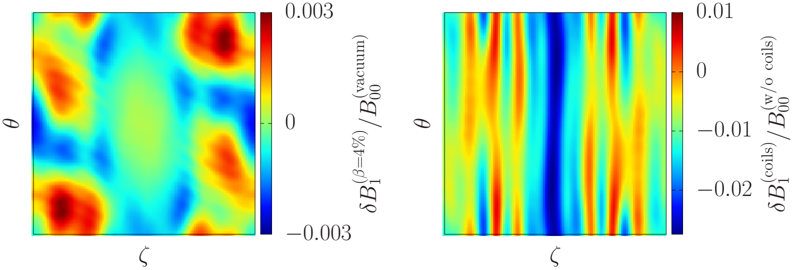
- Exact omnigeneity implies $\varepsilon_{eff} = 0$ and perfect fast ion confinement, as in tokamaks.
- Exactly quasi-isodynamic (QI) and quasi-symmetric (QS) stellarators are omnigenous.
- $\overline{\mathbf{v}_M \cdot \nabla s}$ is to a great extent independent of $\overline{\mathbf{v}_M \cdot \nabla\alpha}$.



Need for robust optimization

Very omnigenous stellarators are hard to find (and then build!).

- Approaching omnigeneity (and quasi-symmetry) **requires a very careful tailoring of** $B(\alpha, l)$, see e.g. [Landreman and Paul (2022) PRL].
- Plasma effects, error fields and additional optimization criteria may produce deviations of order 0.1 % - 10 %.



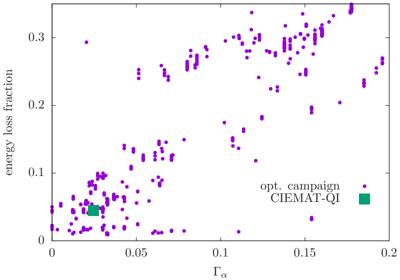
- Even small deviations can have important impact on energetic ion transport.

Robust optimization: good transport properties that resist uncontrolled deviations from omnigeneity.

Need for fast particle precession

Precession of trapped fast ions on the flux-surface has long been known to improve their confinement, see e.g. [Wobig (1993), PPCF].

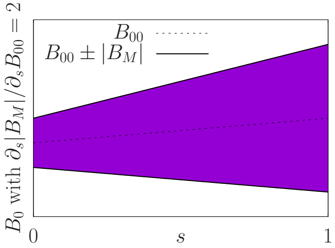
- $|\overline{\mathbf{v}_M \cdot \nabla s} / \overline{\mathbf{v}_M \cdot \nabla \alpha}|$ identified as key quantity for qualifying fast ion collisionless confinement [Nemov (2008) PoP].
- Optimization proxies Γ_c [Nemov (2008) PoP] and Γ_α [Velasco (2021) NF] try to achieve $|\overline{\mathbf{v}_M \cdot \nabla s} / \overline{\mathbf{v}_M \cdot \nabla \alpha}| \ll 1$.
    - Successfully employed in the optimization of configurations WISTELL-A [Bader (2020) JPP] and CIEMAT-QI [Sanchez (2023) NF].
    - Fair correlation between Γ_c and Γ_α with collisionless *prompt* losses of fast ions [Velasco (2021) NF, Leviness (2023) NF].
    - Fair correlation between Γ_c and Γ_α with *collisional* losses of fast ions [Leviness (2023) NF; Velasco (2023) NF].

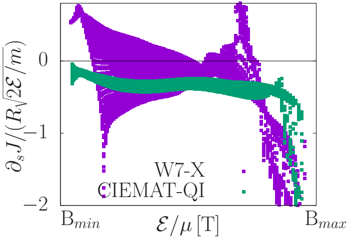


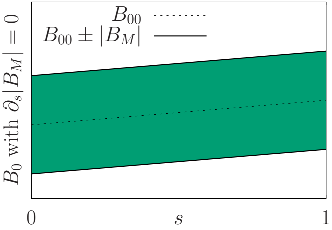
Maximizing $|\overline{\mathbf{v}_M \cdot \nabla \alpha}|$ allows for larger $|\overline{\mathbf{v}_M \cdot \nabla s}|$, i.e. contributes to robust optimization.

Superbananas in QI fields

In QI fields, it is crucial (see also next slides) to get rid of superbananas, i.e., orbits for which $\overline{\mathbf{v}_M \cdot \nabla\alpha} \approx 0$.







$$J(s,\alpha,\mathcal{E},\mu) \equiv 2\int_{l_{b_1}}^{l_{b_2}} \mathrm{d}l |v_\parallel| \,, \qquad \overline{\mathbf{v}_d \cdot \nabla s} = \frac{m}{Z_e \Psi_{LCMS}} \partial_\alpha J \,, \qquad \overline{\mathbf{v}_d \cdot \nabla\alpha} = -\frac{m}{Ze\Psi_{LCMS}} \partial_s J$$

Instead of very carefully designing the variation of $B - B_{00}$ on every flux-surface s, have a radial variation $\partial_s(B - B_{00})$ such that all trapped particles precess at low β [Velasco (2023) NF].

Control of $\partial_s(B - B_{00})$ in QI allows for robust optimization. However...

Superbananas in QS fields Figura?

In QS fields (either quasi-axis symmetric or quasi-helically symmetric), it is impossible to get rid of superbananas, i.e., orbits for which $\overline{\mathbf{v}_M \cdot \nabla\alpha} \approx 0$.

- Deeply trapped ions ($\mathcal{E}/\mu \approx B_{min}$) precess in the ion diamagnetic direction.
- Barely trapped ions ($\mathcal{E}/\mu \approx B_{max}$) precess in the direction opposite to the ion diamagnetic direction.
- Trapped ions with intermediate pitch-angle velocity (not far from barely passing) do not precess.
  - Result can be obtained in a first order expansion in the aspect ratio (see e.g. [Velasco (2023) NF]).
  - Barely modified by higher order terms [Rodriguez and Mackenbach (2023) JPP].

Yet, QS fields with excellent fast ion confinement have been obtained, see [Landreman (2022) PoP] and references therein.

Next slides:

- Understand mechanisms that mitigate superbananas and other loss mechanisms.
- Identify strategies to enhance these mechanisms.

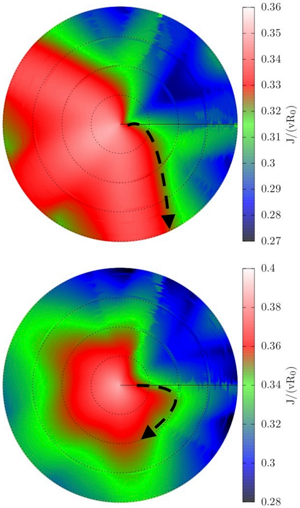
Radial globality of fast ion transport

Top polar $s-\alpha$ plot: a trapped ion at phase-space point with $\partial_s J = 0$ and $\partial_\alpha J > 0$ could be expected to drift at constant $\mathcal{E}/\mu$ in the radial direction towards the last-closed flux-surface ($s = 1$).

Bottom polar $s-\alpha$ plot: However,

- along the trajectory, $\partial_\alpha J$ and $\partial_s J$ will change;
- if, at some point, $\partial_\alpha J$ becomes 0 and $|\partial_s J|$ becomes large, the fast ion will precess on the flux-surface instead of drifting radially;
- the sign of $\partial_\alpha J$ could even reverse, and the trapped ion would drift inwards.

Local approach can be insufficient if not very close to omnigeneity.



Improving the local approach

We proceed as follows (we use for convenience $\lambda \equiv \mu/\mathcal{E}$):

- A trapped ion at phase-space point $(s,\lambda) = (s_0,\lambda_0)$, where $\partial_s J = 0$ and $\partial_\alpha J > 0$, will initially drift at constant λ towards $(s,\lambda) = (1,\lambda_0)$.
- In order to assess whether it actually reaches $s = 1$, we want to assess if, in $(s,\lambda) = (1,\lambda_0)$, the ion is still drifting mainly in the radial direction.
- This is determined by the size of $\partial_\alpha J/\partial_s J$ at $(s,\lambda) = (1,\lambda_0)$, which we estimate as follows:
  - Assuming a certain level of optimization, parametrized by δ (see next slide):

$$\partial_\alpha J|_{s=1,\lambda=\lambda_0} \sim \delta J|_{s=1,\lambda=\lambda_0} \sim \delta J|_{s=s_0,\lambda=\lambda_0}.$$

  - Taylor expanding from the initial point:

$$\partial_s J|_{s=1,\lambda=\lambda_0} \approx \partial_s^2 J|_{s=s_0,\lambda=\lambda_0}(1-s_0).$$

- Requiring that $\partial_\alpha J|_{s=1,\lambda=\lambda_0} \ll \partial_s J|_{s=1,\lambda=\lambda_0}$ leads to $\delta \ll \partial_s^2 J|_{s=s_0,\lambda=\lambda_0}(1-s_0)/J|_{s=s_0,\lambda=\lambda_0}$.

We can estimate of the level of optimization such that superbananas are not deleterious.

$\partial_s J$ and $\partial_\alpha J$ in optimized stellarators

The magnetic field strength B of a configuration sufficiently close to omnigeneity (either QI or QS) can be split into a omnigenous piece B_0 and a perturbation B_1, with

$$B = B_0 + B_1\,, \quad B_1 \sim \delta B_0\,, \quad 0 \leq \delta \ll 1\,, \quad |\nabla B_1| \ll |\nabla B_0|$$

The derivatives of J can then be expressed to first order in δ as

$$\begin{aligned} \partial_\alpha J(s,\alpha,\mathcal{E},\mu) &= \delta\partial_\alpha J_1(s,\alpha,\mathcal{E},\mu)\,,\\ \partial_s J(s,\alpha,\mathcal{E},\mu) &= \partial_s J_0(s,\mathcal{E},\mu) + \delta\partial_s J_1(s,\alpha,\mathcal{E},\mu)\,. \end{aligned}$$

- $J_0 \sim J_1$.
- $\partial_s J_0$ completely determined by B_0.
- $\partial_\alpha J_1$ set by both B_0 and B_1 (explicit expressions in [Calvo (2017) PPCF, (2018) JPP]).

Estimate of δ for a model field Revisar calculos

For QS field described by

$$B_0(s,\theta,\zeta) = B_{00} + B_{MN}(s)\cos(M\theta - N\zeta)\,,$$

following [Velasco (2023) NF], we can obtain:

$$\begin{aligned}
J_0 &= 2\sqrt{\mu}\frac{I_p+\iota I_t}{|N-M\iota|}\frac{|B_{MN}|^{1/2}}{B_{00}}\left[4E(\kappa^2)-4(\kappa^2-1)K(\kappa^2)\right]\,,\\
\partial_s J_0 &= 4\sqrt{\mu}\frac{I_p+\iota I_t}{|N-M\iota|}\frac{\partial_s B_{MN}}{B_{00}|B_{MN}|^{1/2}}\left[2E(\kappa^2)-K(\kappa^2)\right]\,,
\end{aligned}$$

and, at $\kappa_0^2 \equiv \kappa(\lambda_0) \approx 0.9$, where $\partial_s J_0 = 0$,

$$\partial_s^2 J_0 = 2\sqrt{\mu}\frac{I_p+\iota I_t}{|N-M\iota|}\frac{(\partial_s B_{MN})^2}{B_{00}|B_{MN}|^{3/2}}\frac{(0.5+\kappa_0^2)E(\kappa_0^2)}{\kappa_0^2(1-\kappa_0^4)}\,.$$

Here, $\kappa^2 = \frac{1-(\mu/\mathcal{E})(B_{00}-|B_{MN}|)}{2(\mu/\mathcal{E})|B_{MN}|}$ ($\kappa = 0/1$ for deeply/barely trapped particles).

Superbananas are estimated to be not deleterious for $\delta \ll \left(\frac{\partial_s B_{MN}}{B_{MN}}|_{s=s_0}\right)^2(1-s_0)$.

Interpretation and relevance for QI and QS Figura?

Condition $\delta \ll \left(\frac{\partial_s B_{MN}}{B_{MN}} |_{s=s_0} \right)^2 (1 - s_0)$ is qualitatively different for QI or QS, due to distinct $B_{MN} \sim s^{M/2}$ behaviour close to the axis (and beyond, in practice):

- For a QS, $M > 0$, and the condition becomes $\delta \ll 1$. Sorprendentemente poco exigente.
- For a QI, $M = 0$, and $|\partial_s B_{MN} / B_{MN}|$ can in principle be very small.

Quasi-symmetric fields tend to be naturally robust against superbananas.

In quasi-isodynamic fields, superbananas need to be removed (e.g. as in [Velasco (2023) NF]).

Interpretation:

- Superbananas lie at constant $(\mu/\mathcal{E})B_{max}$, so the position in $\mu/\mathcal{E}$ varies radially, and rate grows with $\partial_s B_{MN}$.
- The *width* of the superbananas in $\mu/\mathcal{E}$ decreases with $\partial_s B_{MN}$.
- The larger $(\partial_s B_{MN})^2$, the harder for superbananas at $s = s_0$ and $s = 1$ to *overlap*.

Result should hold qualitatively for more general QS and QI fields, beyond the models.

Consequences for QS stellarator optimization

Good fast ion confinement not extremely close from quasi-symmetry (without resorting to omnigeneity)?.

If we can obtain a magnetic field strength B that can be split into a QS piece B_0 and a perturbation δB_1, with

$$B = B_0 + B_1\,, \quad B_1 \sim \delta B_0\,, \quad 0 \leq \delta \ll 1\,, \quad |\nabla B_1| \ll |\nabla B_0|$$

then

- $|\partial_\alpha J / \partial_s J| \sim \delta \ll 1$ for most particles.
  - Earlier estimates in QI [Velasco (2021) NF, Sanchez (2023) NF] tell us that $\delta \sim 0.1$ could be enough.
- $|\partial_\alpha J / \partial_s J|$ can become very large in a particular region of phase space close to B_{max} but, for QS, without very negative consequences!

However, we shouldn't use Taylor expasion in $(1 - s_0) \ll 1$ with $s_0 \approx 0$.

Practically speaking, we have learnt to devise strategies to mitigate superbananas.

- Larger $|\partial_s B_{MN}|$ (i.e., larger $\partial_s B_{max}$) helps.
- Finite β will separate (in $\mu/\mathcal{E}$) the superbananas at $s = s_0$ and $s = 1$, but it will also make them wider.
- Shaping of $B_0(M\theta - N\zeta)$ (equivalently, triangularity [Rodriguez and Mackenbach (2023) JPP]) could help.

Enhancing the precession velocity of trapped particles Comprobar calculos

Deeply trapped particles:

For a general QS whose main helicity is B_{MN}, following [Velasco (2023) NF], we can obtain for deeply trapped particles ($\mathcal{E}/\mu = B_{min}$)

$$\partial_s J_0 \quad = \quad -2\pi\sqrt{\mu}\frac{I_p + \iota I_t}{|N - M\iota|}\sqrt{\frac{2N_{fp}^2}{b_2}}\frac{\partial_s B_{min}}{B_{00}} ,$$

with b_2 the second derivative of B along the field line evaluated at B_{min}.

- Larger $\partial_s B_{min}$ (connected to larger $\partial_s B_{MN}$) makes trapped ions rotate faster.
- A broad mirror (small b_2) will make $\partial_s J_0$ larger (observed in QI [Drevlak (2018) NF; Sanchez (2023) NF]).

Barely passing particles:

The precession of particles with $\mathcal{E}/\mu = B_{max}$ is completely determined (and proportional to) $\partial_s B_{max}$, see e.g. [Calvo (2017) PPCF].

Increasing $\partial_s B_{MN}$ and a broad well increases the precession velocity of trapped particles.

If deeply and barely trapped particles precess faster with opposite sign, smaller $\mathcal{E}/\mu$-region of slow precession.