Design Principles for Precision Mechanisms

3. Flexure Mechanisms

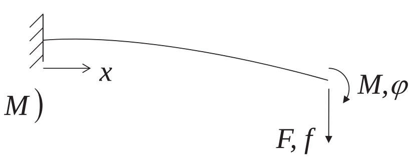
Flexure Mechanisms: Why?

- Miniaturization
- No friction
- Controlling DOFs

Flexure Mechanisms: Limitations

- Limited stroke
- Stress
- Stiffness/Energy

Deflection formula or Euler-Bernoulli equations



$$\frac{d^2 f}{dx^2} = \frac{1}{EI} M(x) = \frac{1}{EI}\left(F(L-x) + M\right)$$

$$\frac{df}{dx} = \frac{1}{EI}\left(FLx - F\tfrac{1}{2}x^2 + Mx + C_1\right) = \varphi$$

$$f = \frac{1}{EI}\left(F\tfrac{1}{2}Lx^2 - F\tfrac{1}{6}x^3 + \tfrac{1}{2}Mx^2 + C_1 x + C_2\right)$$

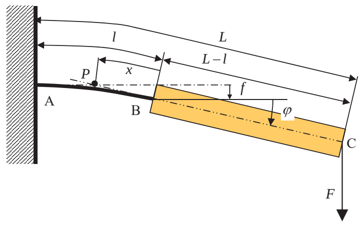
$$\frac{df}{dx}(x=0) = 0 \Rightarrow C_1 = 0$$

$$f(x=0) = 0 \Rightarrow C_2 = 0$$

$$f = \frac{ML^2}{2EI} + \frac{FL^3}{3EI}$$

$$\varphi = \frac{ML}{EI} + \frac{FL^2}{2EI}$$

Deflection of beam-end about instant centre of rotation P



$$\varphi = \frac{Fl^2}{2EI} + \frac{F(L-l)l}{EI}$$

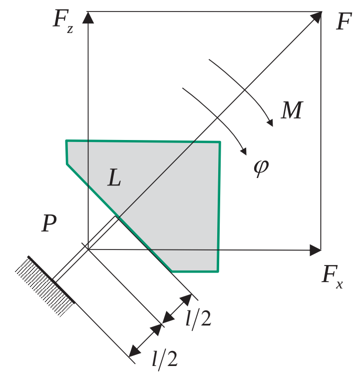
$$f = \frac{Fl^3}{3EI} + \frac{F(L-l)l^2}{2EI}$$

$$f = \varphi \cdot x$$

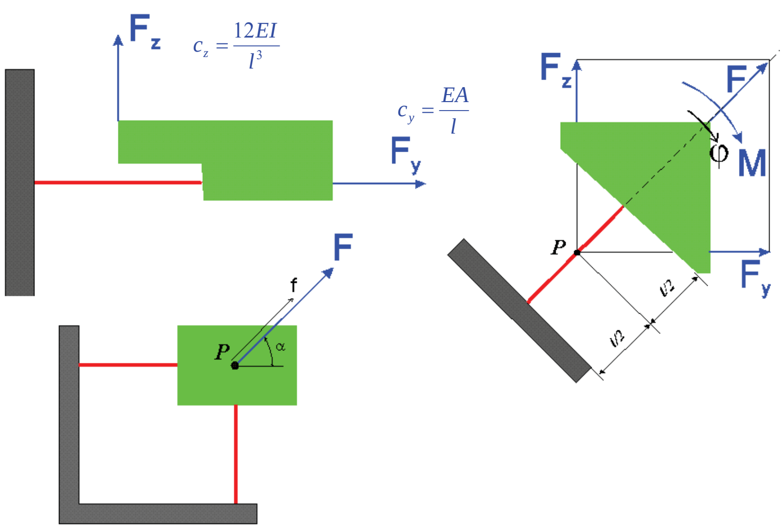
$$x = \frac{l(3L-l)}{3(2L-l)}.$$

1) Force F in B; $L = l$: $x = 2/3 \cdot l$
2) When $L \to \infty$ (load is moment), then $x \to l/2$

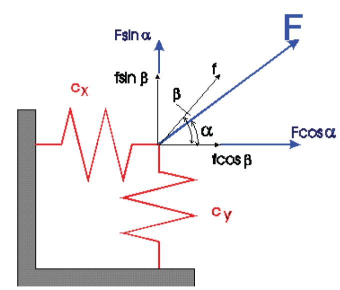
A single flexure may serve as a hinge when it is in line with the dominant load



Loadability and stiffness



Equal stiffness in every direction?



$$F\cos(\alpha) = c_x f \cos\beta \rightarrow \frac{F^2\cos^2(\alpha)}{c_x^{\ 2}} = f^2\cos^2(\beta)$$

$$F\sin(\alpha) = c_y f \sin\beta \rightarrow \frac{F^2\sin^2(\alpha)}{c_x^{\ 2}} = f^2\sin^2(\beta)$$

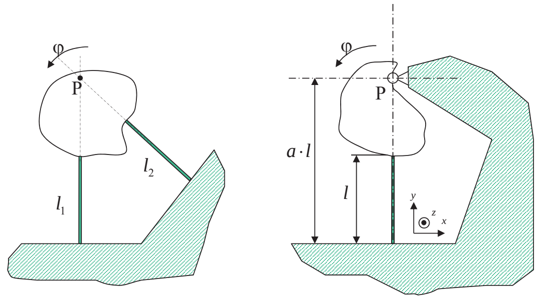
$$f(\alpha) = F\sqrt{\frac{\cos^2(\alpha)}{(c_x)^2} + \frac{\sin^2(\alpha)}{(c_y)^2}}$$

$f(0) = \frac{F}{c_x}$ principal direction

$f\left(\frac{\pi}{2}\right) = \frac{F}{c_y}$ principal direction

If $c_x = c_y = c$, then $f = \frac{F}{c}$, is independent of α, and $\beta = \alpha$

Generic model of a cross flexure

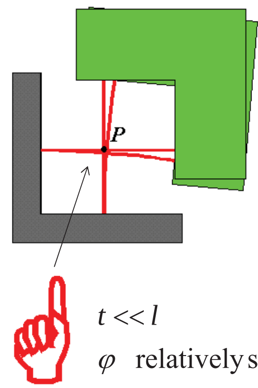


$$k_\varphi = \frac{M_\varphi}{\varphi} = 4\cdot\left\{\frac{1}{K_{z1}\cdot l_1}\cdot\left(1-3a_1+3a_1^2\right)+\frac{1}{K_{z2}\cdot l_2}\cdot\left(1-3a_2+3a_2^2\right)\right\}$$

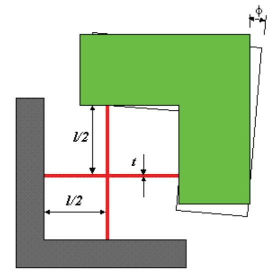
$K_z = \frac{1}{E \cdot I}$ for beams with cross sections with approximately the same thickness as width.

$K_z = \frac{(1-\nu^2)}{E \cdot I}$ for beams with cross sections with large width to thickness ratio.

Symmetric cross flexure



$$\left.\begin{array}{l} t << l \\ \varphi \ \text{ relatively small} \\ \text{Symmetry} \end{array}\right\}$$



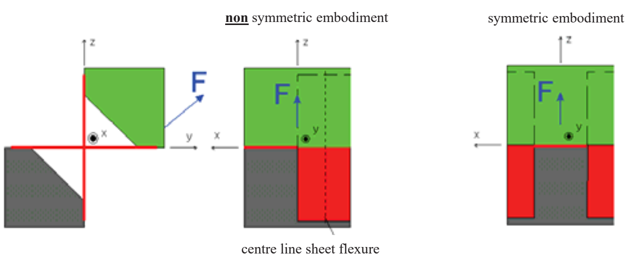
$$M_{in\,1\,LS} = \frac{EI}{L}\varphi$$

$$\sigma_\varphi = \frac{M_{in\,1\,LS}\left(\frac{1}{2}t\right)}{I} = \frac{\varphi E t}{2l}$$

$$k_\varphi = \frac{M_{tot}}{\varphi} = \frac{2EI}{l}$$

Asymmetric cross flexure

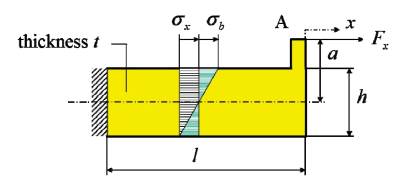
Cross spring hinge



Stiffness reduction due to eccentric tensile load

- The displacement x of point A:

[illegible] $\frac{\cdot l}{A} + \frac{F_x \cdot a}{F \cdot}$ [illegible]



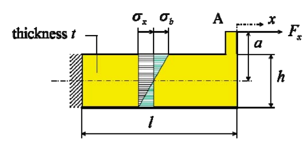
- The compliance in A is then:

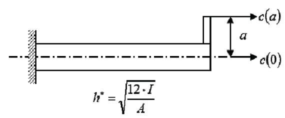
$$\frac{1}{c(a)} = \left\{\frac{l}{E\cdot A}\cdot\left(1+a^2\cdot\frac{A}{I}\right)\right\}^{-1}$$

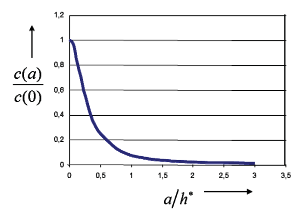
Stiffness reduction due to eccentric tensile load

- Stiffness ratio

$$\frac{c(a)}{c(0)} = \frac{1}{\left(1+a^2\cdot\right.}$$

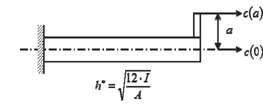




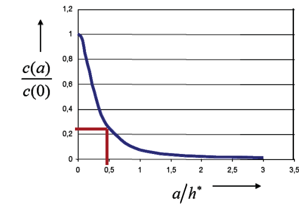


Stiffness reduction due to eccentric tensile load

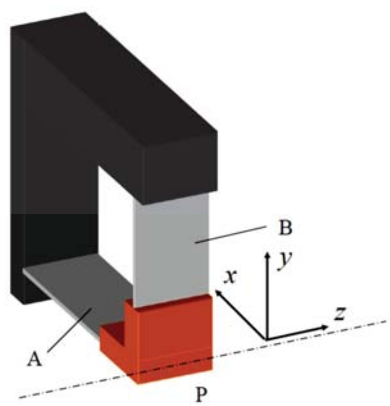
- Stiffness ratio



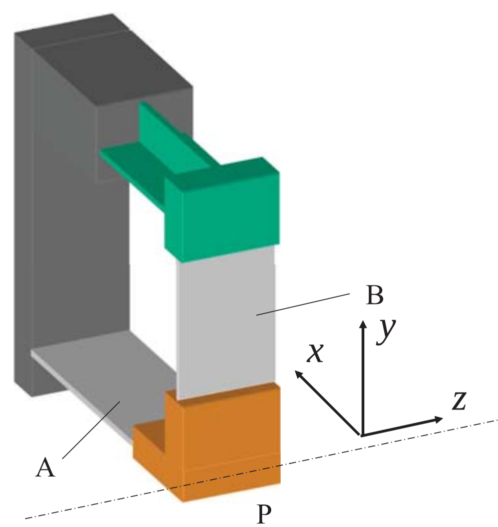
For a beam with a square cross-section and tensile force with an eccentricity a/h* = 0.5, the tensile stiffness drops by a factor of 4!



The over-constraint of a cross flexure

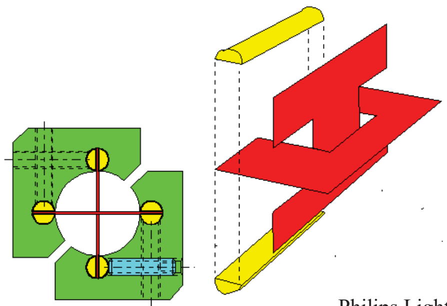


Resolving the overconstraint of a cross flexure



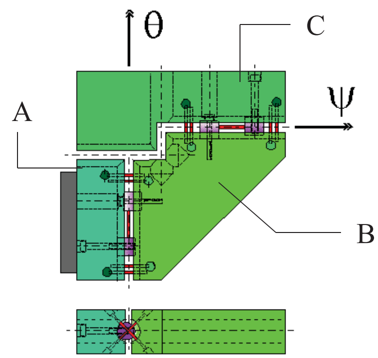
Examples of cross flexure

Embodiment of cross flexure



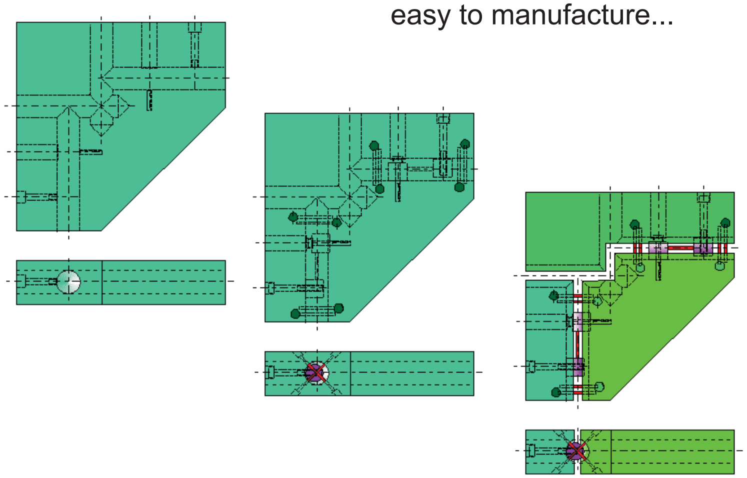
Philips Lighting version

Adjustment of two angles (1/2)



Two cascaded cross flexure mechanisms based on wire flexures

Adjustment two angles (2/2) easy to manufacture...



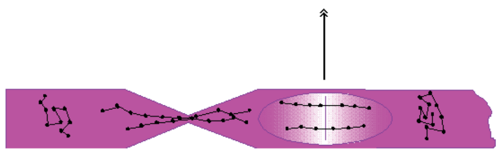
Commercial flexures

Riverhawk Company Headquarters is located in New Hartford, New York, USA
Mailing Address
215 Clinton Road
New Hartford, New York, 13413

http://www.flexpivots.com/ Riverhawk Company

Injection moulding of flexures in plastic (1/2)

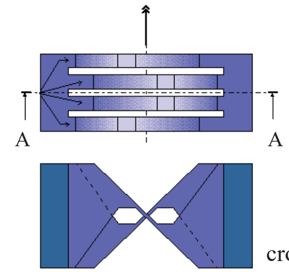
Injection molded plastic part with wire flexure functionality



All hinges are “through-injected”

Injection moulding of a cross flexure in plastic (2/2)

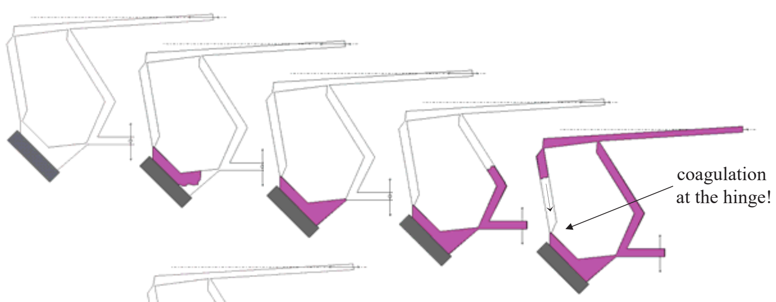
Symmetric injection molded cross flexure

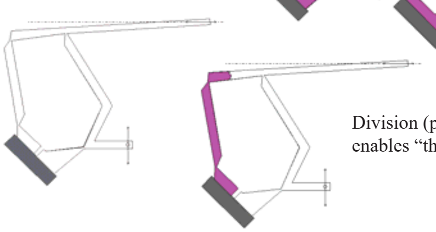


cross section A-A

Injection moulding of flexures in plastic

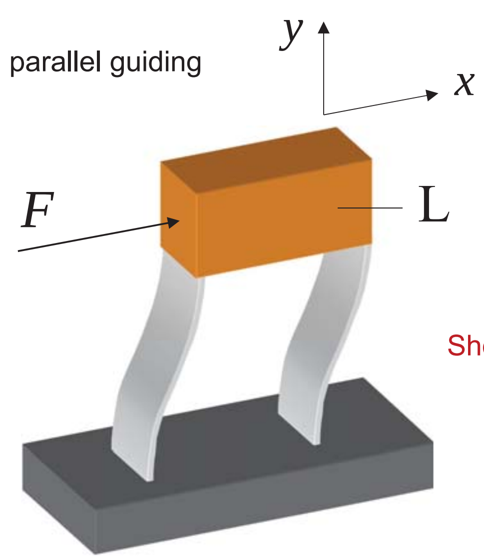
Polypropylene linkage as a station scale indicator





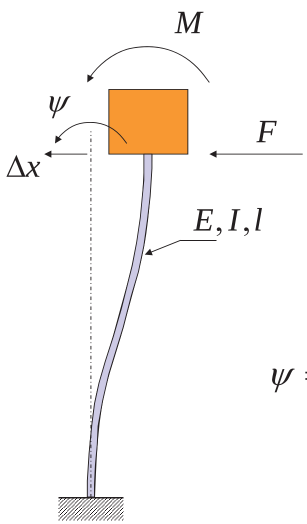
Division (pref. at the clamp section) enables “through-injection”of all hinges!

Flexure parallel guiding



Shortening effect!

One sheet flexure under parallel guiding



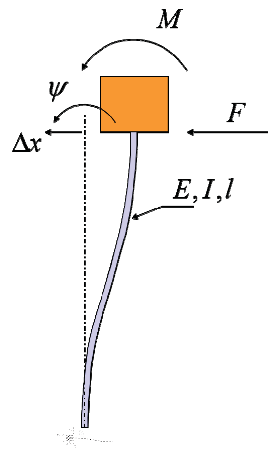
General equation:

$$\begin{bmatrix} F \\ M \end{bmatrix} = \frac{E \cdot I}{l^3} \cdot \begin{bmatrix} 12 & -6l \\ -6l & 4l^2 \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \psi \end{bmatrix}$$

Stiffness

$$\psi = 0 \quad \rightarrow \quad c_x = \frac{F}{\Delta x} = 12 \cdot \frac{E \cdot I}{l^3}$$

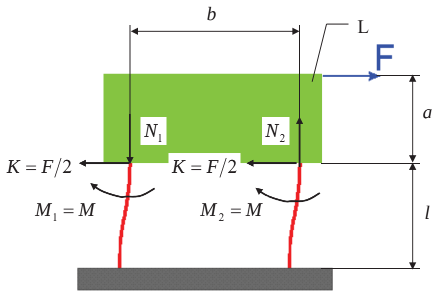
One sheet flexure under parallel guiding

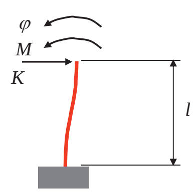


The maximum bending stress (parallel guiding) occurs at the clamp:

$$\sigma_{b,\max} = \frac{3 \cdot E \cdot h \cdot \Delta x}{l^2}$$

Force F evokes reactions N_1 and N_2



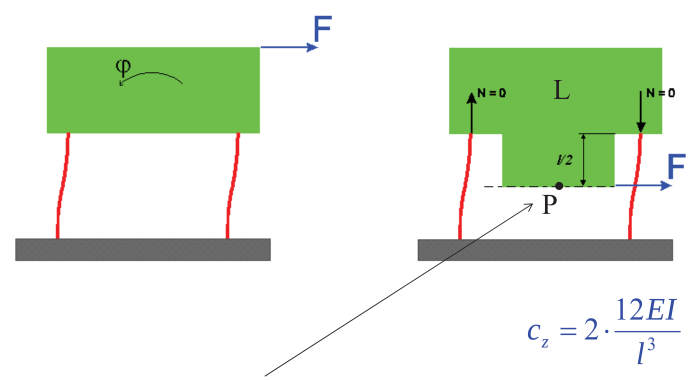


$$K_1 = K_2 = K$$
$$M_1 = M_2 = M$$
$$N_1 = N_2 = N$$
$$2M - N \cdot b + F \cdot a = 0$$

$$\varphi = \frac{K\ell^2}{2EI} - \frac{M\ell}{EI} = 0 \rightarrow M = \frac{K\ell}{2}$$

$$\boxed{N = F \cdot (l/2 + a)/b}$$

Sheet flexure <u>parallel</u> guiding

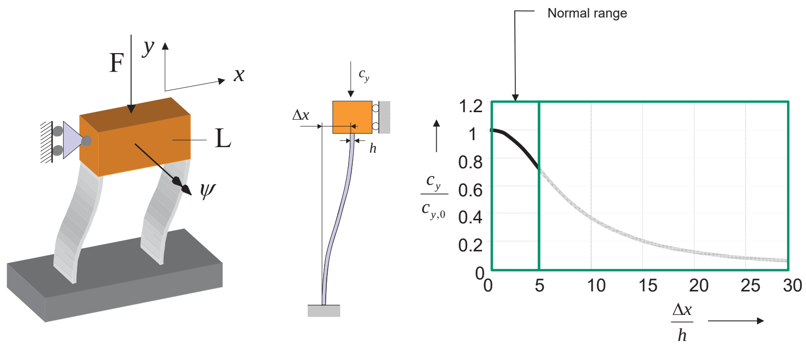


$$c_z = 2 \cdot \frac{12EI}{l^3}$$

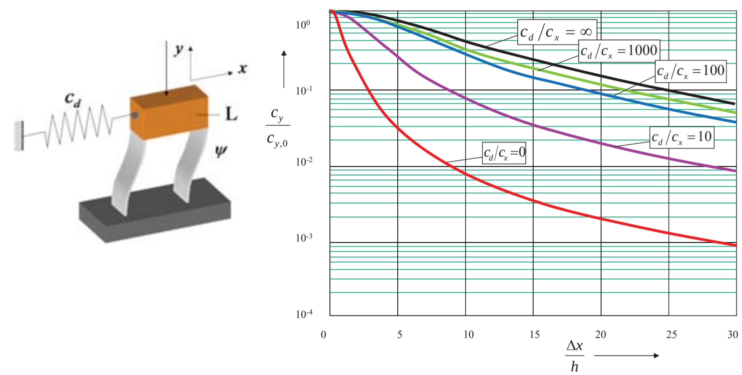
centre of compliance

Position dependent guiding stiffness

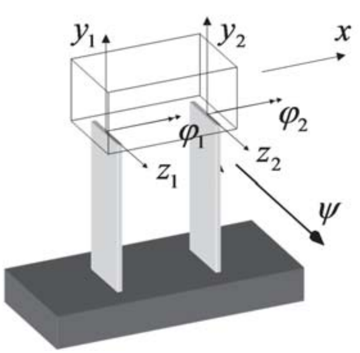
Supporting stiffness decreases with increasing deflection



Supporting stiffness also a function of drive stiffness

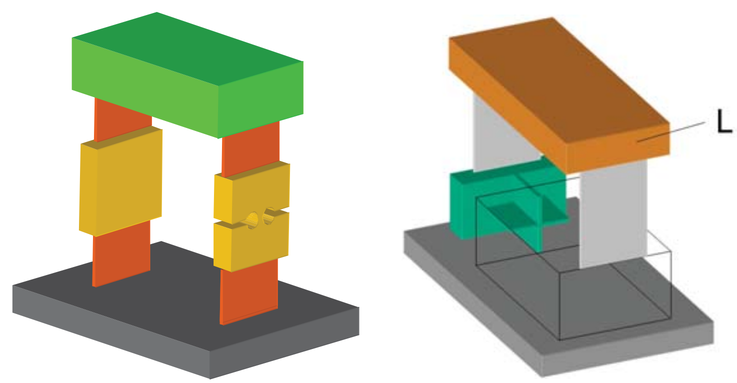


Overconstraint in φ direction

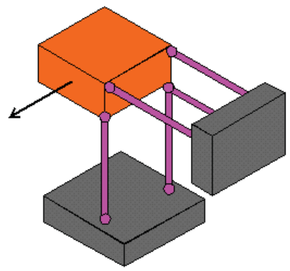


- One DOF (φ) is constrained twice
- One DOF remains: x

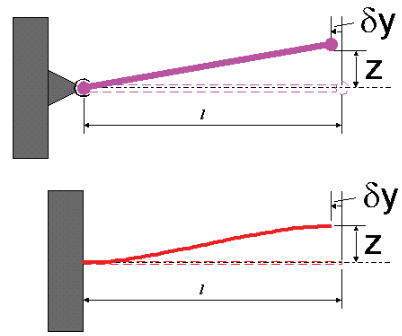
Relief of overconstraint in φ direction by additional rotational freedom in flexure



Principle of a wire flexure parallel guiding



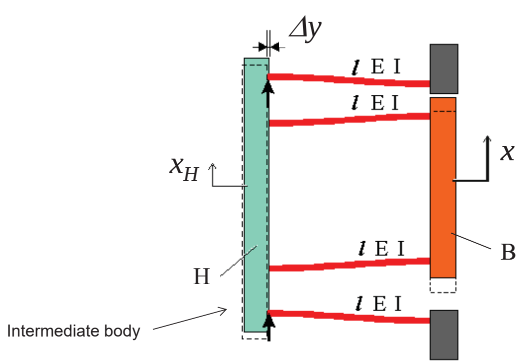
Second order shortening of sheet (wire) flexures



$$\delta y = 0.5\frac{z^2}{l}$$

$$\delta y = 0.6\frac{z^2}{l}$$

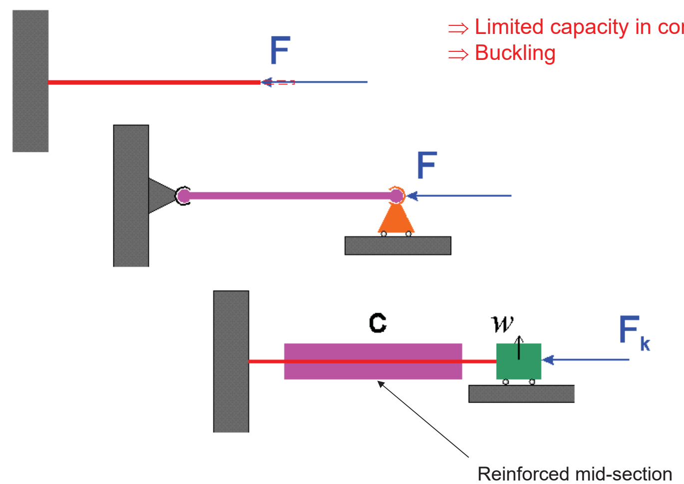
Compensating the "shortening effect"



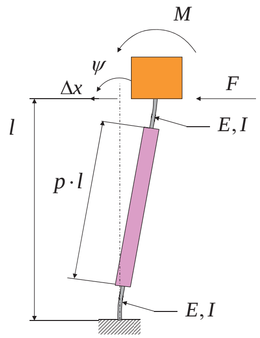
$$\Delta y = 0.6(\Delta x)^2/l$$

Sheet and wire flexures with reinforced mid-sections

⇒ Limited capacity in compression
⇒ Buckling

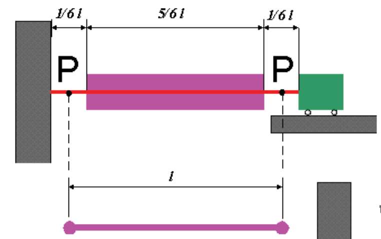


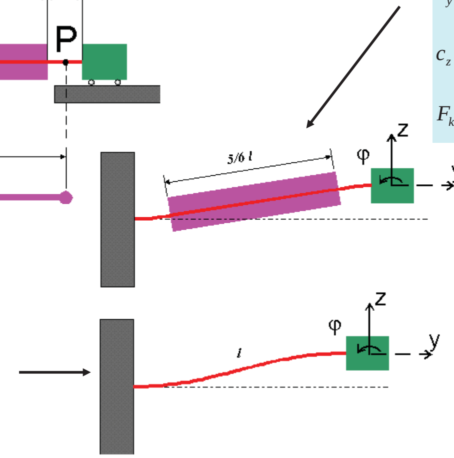
Stiffness matrix of a reinforced flexure



$$\begin{bmatrix} F \\ M \end{bmatrix} = E \cdot I \cdot \begin{bmatrix} \frac{1}{l^3} \cdot \frac{12}{1-p^3} & \frac{1}{l^2} \cdot \frac{-6}{(1-p^3)} \\ \frac{1}{l^2} \cdot \frac{-6}{(1-p^3)} & \frac{1}{l} \cdot \frac{(4+p+p^2)}{1-p^3} \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \psi \end{bmatrix}$$

Sheet and wire flexures with reinforced mid-sections

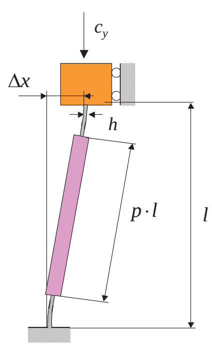


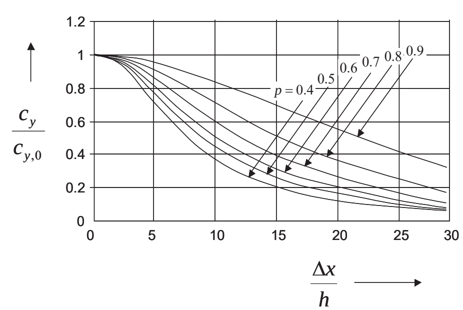


$$\sigma_{\max} = \frac{3Etz}{l^2}$$
$$c_y = \frac{3Etb}{l}$$
$$c_z \approx 1.2 \cdot \frac{Ebt^3}{l^3}$$
$$F_k = \frac{36\pi^2 EI}{l^2}$$

$$\sigma_{\max} = \frac{3Etz}{l^2}$$
$$c_y = \frac{Etb}{l}$$
$$c_z = \frac{Ebt^3}{l^3}$$
$$F_k = \frac{4\pi^2 EI}{l^2}$$

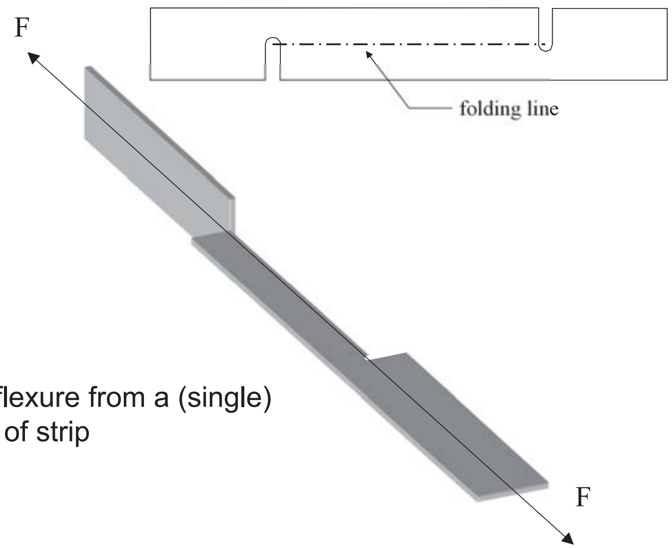
Normal stiffness c_y of a reinforced flexure





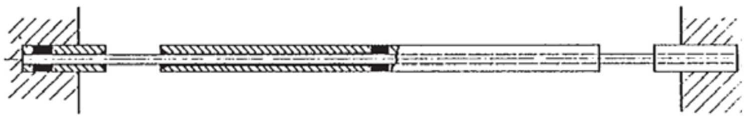
Basic formula for stiffness calculations on flexures

	leaf spring (width h)	stiffened leaf spring (width h)	wire spring (diameter d)	stiffened leaf spring (diameter d)
longitudinal stiffness c_{xx}	$\frac{EA}{\ell} = \frac{Eth}{\ell}$	$\frac{3Eth}{\ell}$	$\frac{EA}{\ell} = \frac{E\frac{\pi}{4}d^2}{\ell}$	$\frac{3E\frac{\pi}{4}d^2}{\ell}$
lateral stiffness c_{zz}	$\frac{12EI}{\ell^3} = \frac{Eht^3}{\ell^3}$	$\frac{72}{5}\frac{EI}{\ell^3} = 1{,}2\,\frac{Eht^3}{\ell^3}$	$12\,\frac{EI}{\ell^3} \simeq 0{,}6\,\frac{Ed^4}{\ell^3}$	$\frac{72}{5}\frac{EI}{\ell^3} \simeq 0{,}7\,\frac{Ed^4}{\ell^3}$
bending stress $\sigma_{\psi z}$	$\frac{3Etz}{\ell^2}$	$\frac{3Etz}{\ell^2}$	$\frac{3Edz}{\ell^2}$	$\frac{3Edz}{\ell^2}$
buckling load F_k	$\frac{4\pi^2 EI}{\ell^2}$	$\frac{\pi^2 EI}{(\ell/6)^2} = \frac{36\pi^2 EI}{\ell^2}$	$\frac{4\pi^2 EI}{\ell^2}$	$\frac{36\pi^2 EI}{\ell^2}$



Wire flexure from a (single) piece of strip

Wire flexure with reinforced mid-section

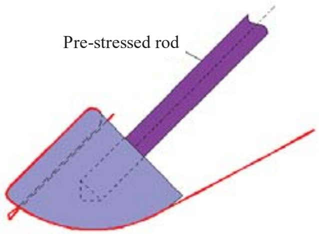


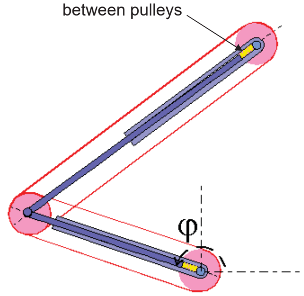
Examples of wire flexures

Polypropylene injection moulded product

Flattened steel wire

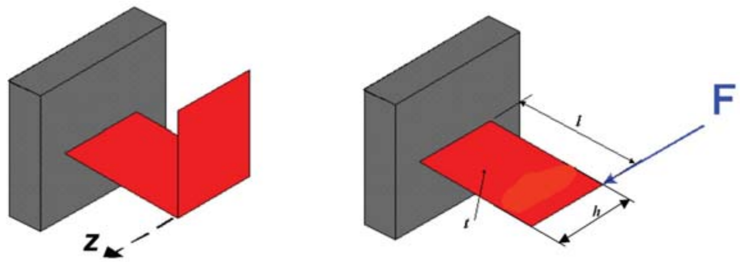
Circumventing the wire clamping issue





Folded sheet flexure

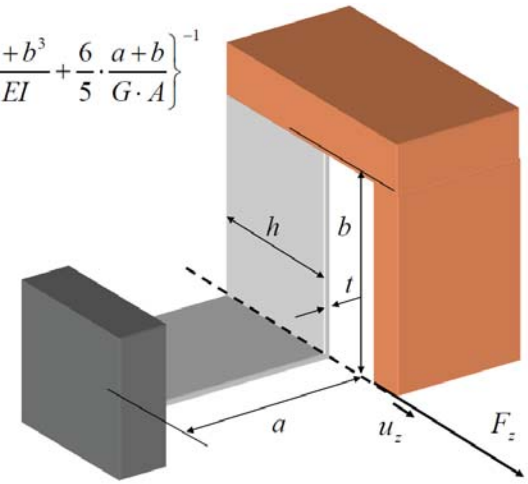
$\Rightarrow$ constrains only ONE DOF!



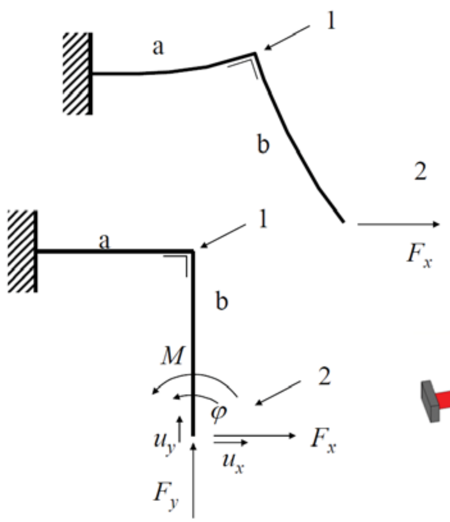
Folded sheet flexure

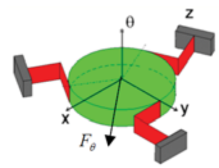
$$c_z = \frac{F_z}{u_z} = \left\{ \frac{a^3 + b^3}{3EI} + \frac{6}{5} \cdot \frac{a+b}{G \cdot A} \right\}^{-1}$$

$$A = t \cdot h$$

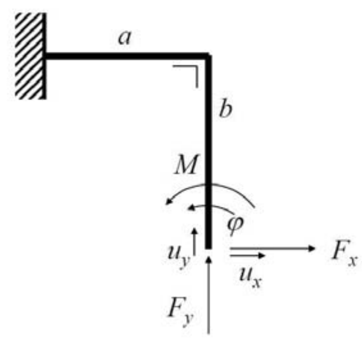


Folded sheet flexure

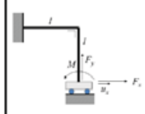
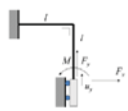
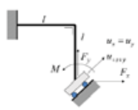
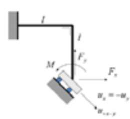




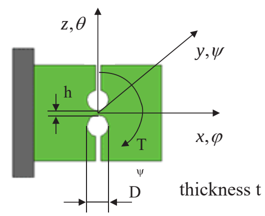
Folded sheet flexure

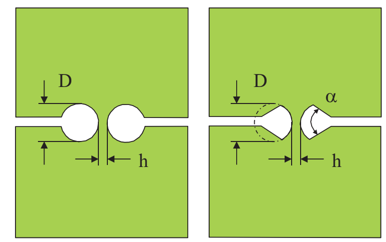


$$\begin{bmatrix} u_x \\ u_y \\ \varphi \end{bmatrix} = \frac{1}{E \cdot I} \cdot \begin{bmatrix} \frac{1}{3}b^3 + b^2 a & \frac{1}{2}a^2 b & \frac{1}{2}b^2 + ab \\ \frac{1}{2}ba^2 & \frac{1}{3}a^3 & \frac{1}{2}a^2 \\ \frac{1}{2}b^2 + ab & \frac{1}{2}a^2 & a+b \end{bmatrix} \cdot \begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix}$$

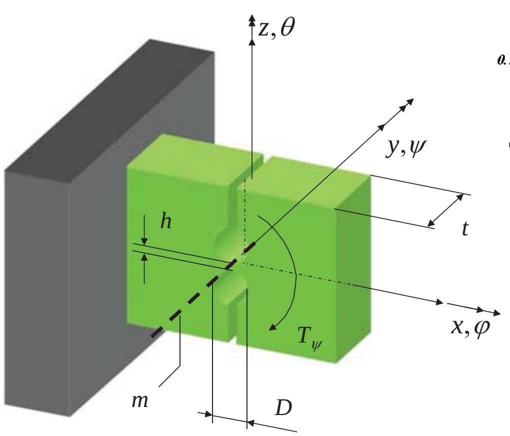
	$\begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix} = u_x \cdot \begin{bmatrix} \frac{15}{2l^3} \\ -\frac{9}{2l^3} \\ -\frac{9}{2l^2} \end{bmatrix} \cdot EI$ $I = \frac{1}{12} \cdot ht^3$ $c_x = \frac{F_x}{u_x} = \frac{15EI}{2l^3}$	$\sigma_{bend,F} = \frac{9}{4} \cdot \frac{E \cdot t \cdot u_x}{l^2}$ $\sigma_{bend,clamp} = \frac{3}{4} \cdot \frac{E \cdot t \cdot u_x}{l^2}$
	$\begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix} = u_y \cdot \begin{bmatrix} -\frac{9}{2l^3} \\ \frac{15}{2l^3} \\ \frac{3}{2l^2} \end{bmatrix} \cdot EI$ $c_y = \frac{F_y}{u_y} = \frac{15EI}{2l^3}$	$\sigma_{bend,F} = \frac{3}{4} \cdot \frac{E \cdot t \cdot u_y}{l^2}$ $\sigma_{bend,clamp} = \frac{9}{4} \cdot \frac{E \cdot t \cdot u_y}{l^2}$
	$\begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix} = u_{+x+y} \cdot \begin{bmatrix} \frac{6}{2l^3} \\ \frac{6}{2l^3} \\ -\frac{6}{2l^2} \end{bmatrix} \cdot EI$ $c_{+x+y} = \frac{F_{+x+y}}{u_{+x+y}} = \frac{3EI}{l^3} = \frac{2}{5} \cdot c_x$	$\sigma_{bend,F} = \frac{6}{4} \cdot \frac{E \cdot t \cdot u_{+x+y}}{l^2}$ $\sigma_{bend,clamp} = \frac{6}{4} \cdot \frac{E \cdot t \cdot u_{+x+y}}{l^2}$
	$\begin{bmatrix} F_x \\ F_y \\ M \end{bmatrix} = u_{+x-y} \cdot \begin{bmatrix} \frac{24}{2l^3} \\ -\frac{24}{2l^3} \\ -\frac{12}{2l^2} \end{bmatrix} \cdot EI$ $c_{+x-y} = \frac{F_{+x-y}}{u_{+x-y}} = \frac{12EI}{l^3} = \frac{8}{5} \cdot c_x$	$\sigma_{bend,F} = \frac{12}{4} \cdot \frac{E \cdot t \cdot u_{+x-y}}{l^2}$ $\sigma_{bend,clamp} = \frac{12}{4} \cdot \frac{E \cdot t \cdot u_{+x-y}}{l^2}$

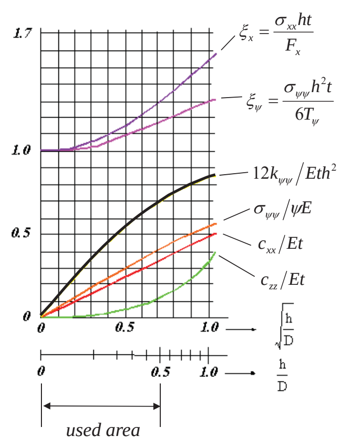
Hole flexures for enabling small rotations (< 1°)



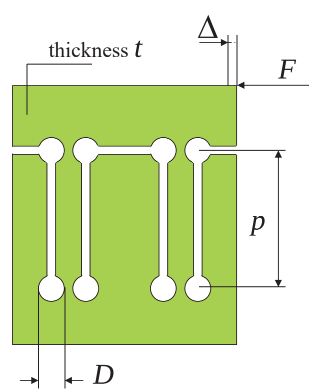


Dimensioning hole flexures

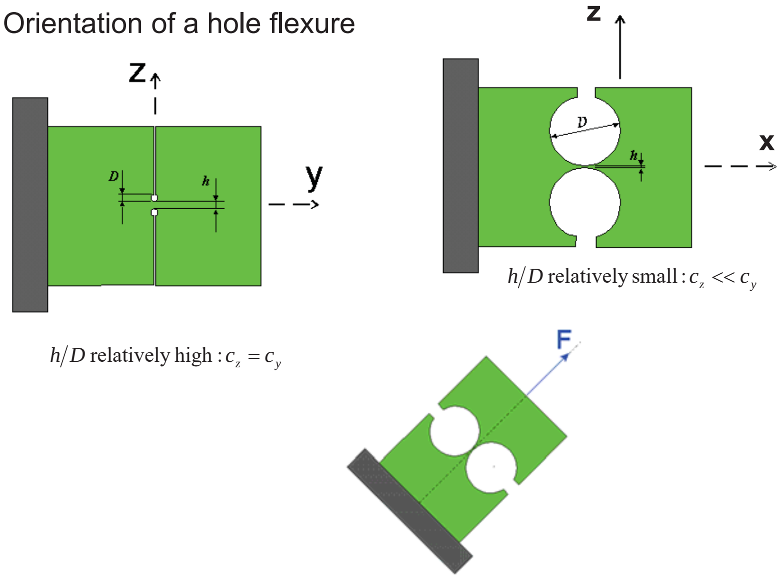




Example of dimensioning hole flexures



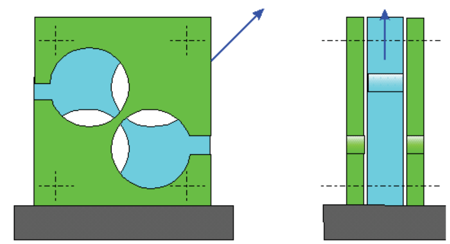
Orientation of a hole flexure



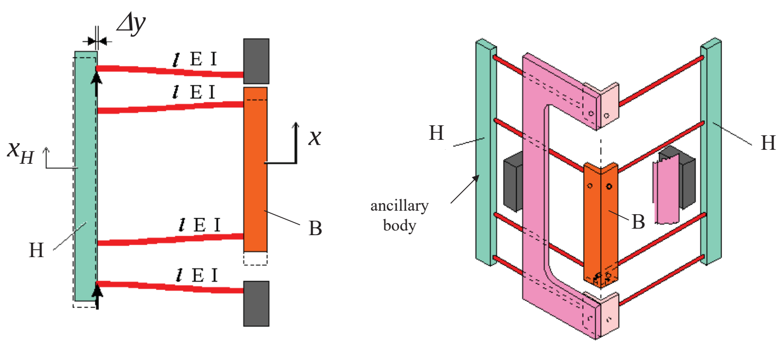
h/D relatively high : $c_z = c_y$

h/D relatively small : $c_z << c_y$

Cross pivot embodiment with hole flexures



Flexure guiding for a relatively large stroke (1/2)

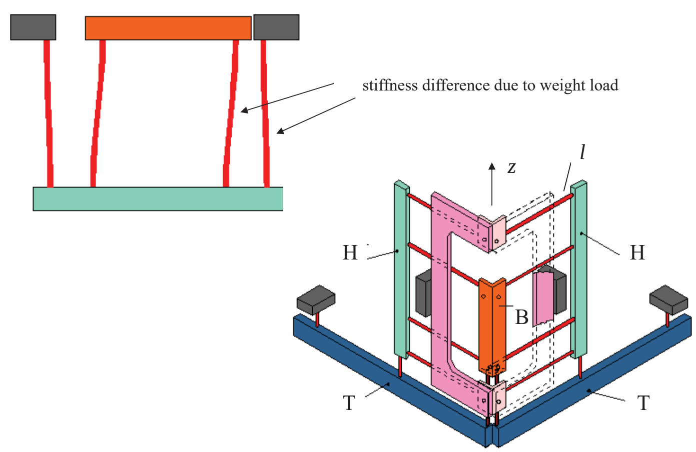


$$\Delta y = 0.6\frac{Z_H^2}{l}$$

When the lengths of the wire springs are equal, shortening compensation can be accomplished if :

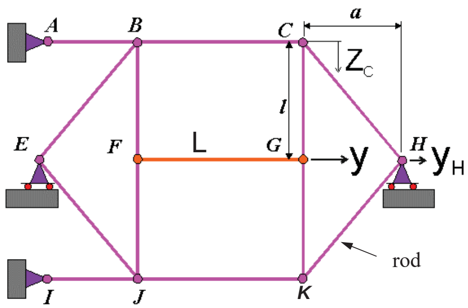
$$\frac{Z_H}{Z} = \frac{1}{2}$$

Elastic guiding for a relatively large stroke (2/2)



Linear guiding on basis of symmetry

“Aristoteles Calibrating Devices” (Arcade)

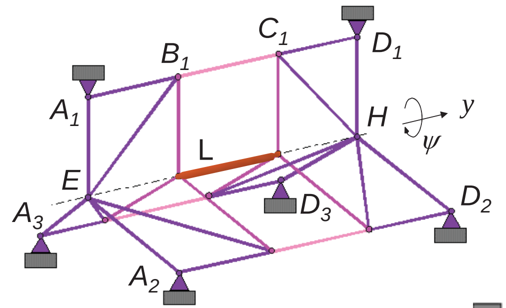


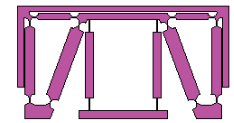
$$y << l \quad \Rightarrow \quad z_c = \frac{y^2}{2l}$$

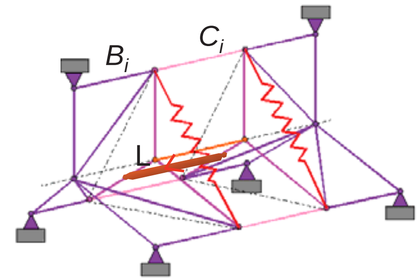
$$y_H = \frac{l}{a} z_C$$

$$y_H = \frac{y^2}{2a}$$

Spatial embodiment linear guiding (Arcade)

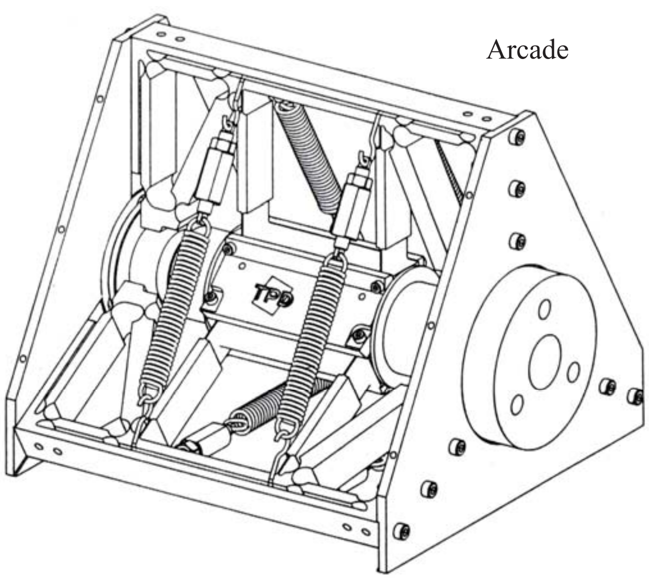






Additional springs for constant elastic energy: $f_c = 0.3$ Hz

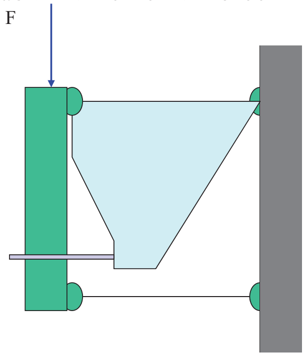
Arcade



Hole flexure applications

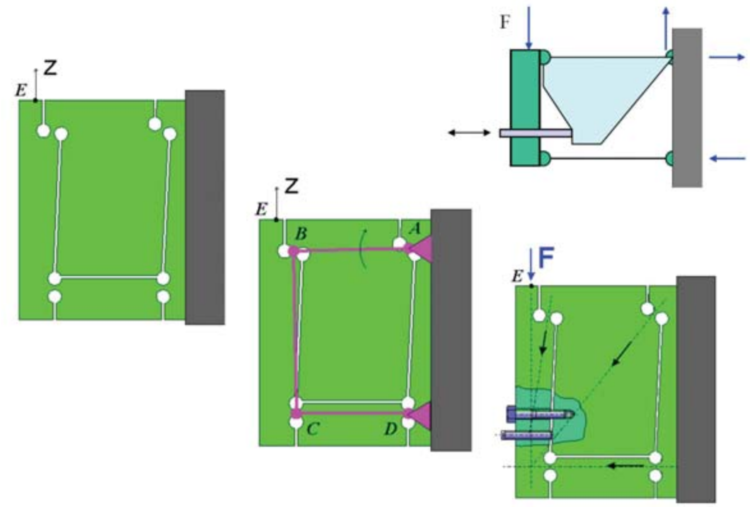
Convert this idea into a correct notch hinge mechanism.

The right-hand wall is fixed. The blue part is adjustable by a screw (dark blue box) in the height direction. The blue part also has to sustain an external force F

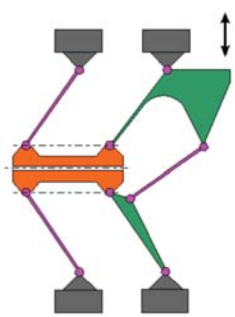


A screw constraining only the longitudinal direction

Adjusting mechanism with correct hinge orientation



Adjustable slit

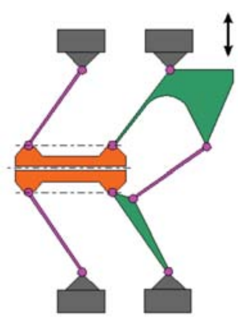


Mechanism’s principle

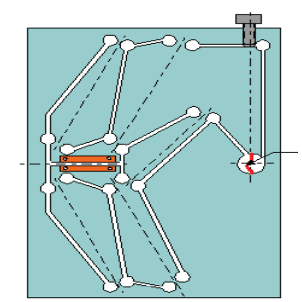
⇒ For an optical instrument

⇒ Adjustable in width from 0 to 1 mm, symmetrical with its bisector

Adjustable slit

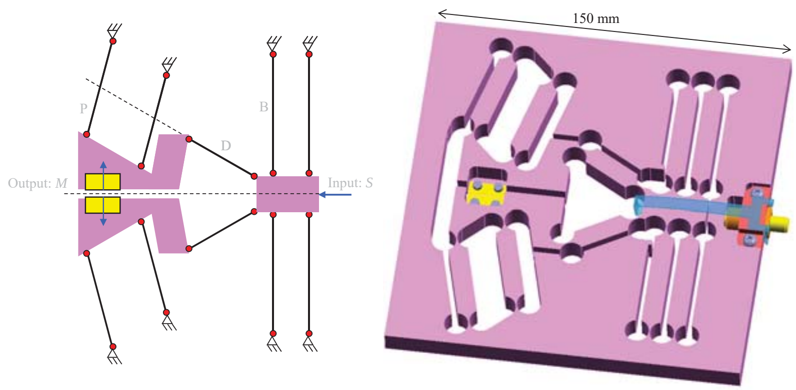


Mechanism’s principle

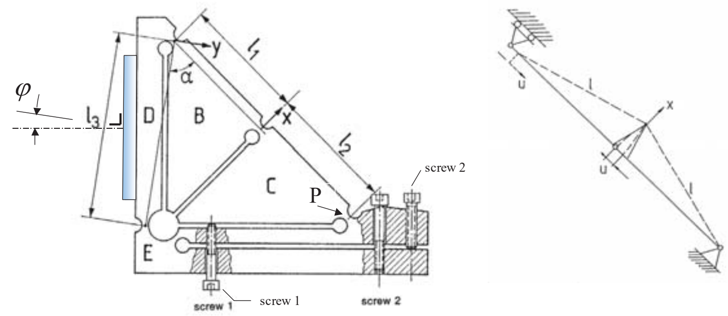


Embodiment design

Symmetric slit mechanism



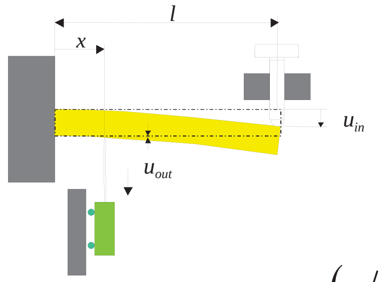
Mirror angle adjustment mechanism



⇒ Screw 2 for course adjustment
⇒ Screw 1 for fine adjustment

Elastic Adjustment

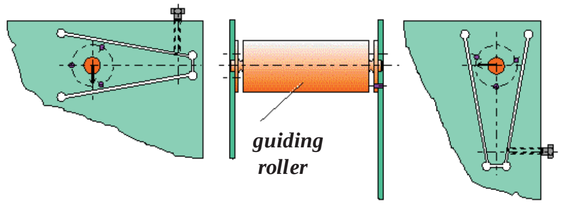
Reduction of input motion by deflection ratio



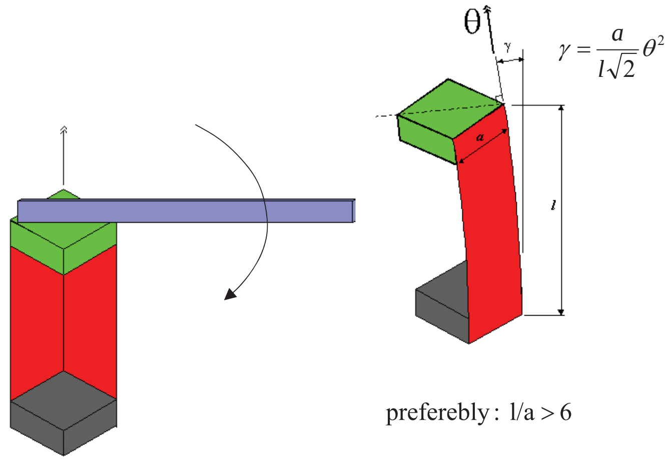
$$u_{out} = (x/l)^3 \cdot u_{in}$$

Elastic Adjustment

Tilt adjustment of roller by elastic/plastic deformation

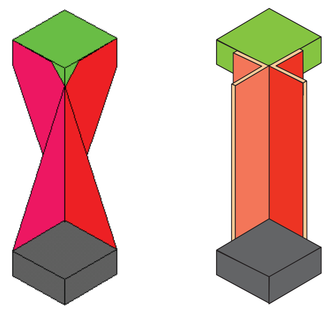


Torsion flexure hinge from folded strip

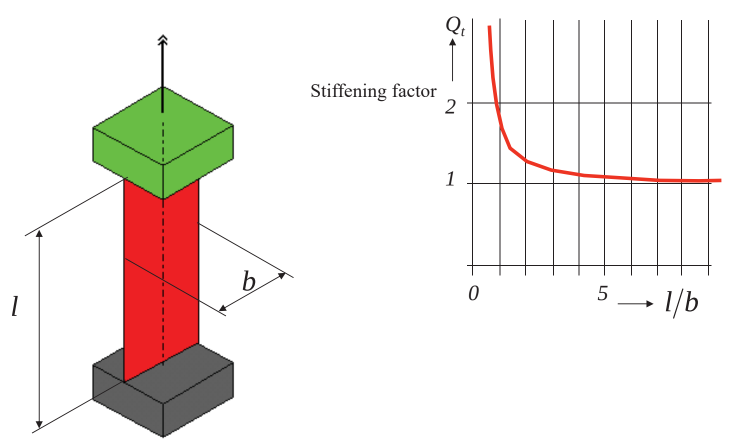


preferebly : l/a > 6

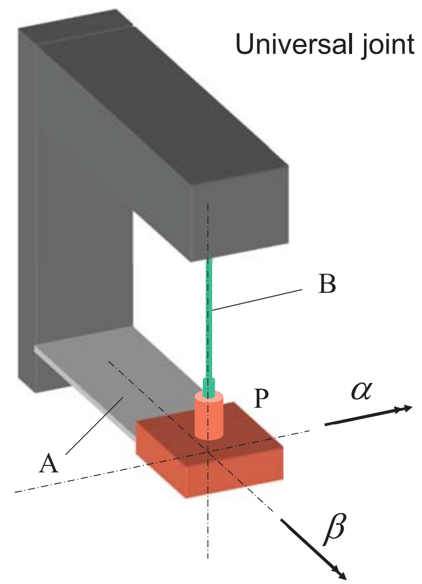
Torsion flexure from strip without the "bending-over-disadvantage"



Torsional stiffening due to "cross-section warping"

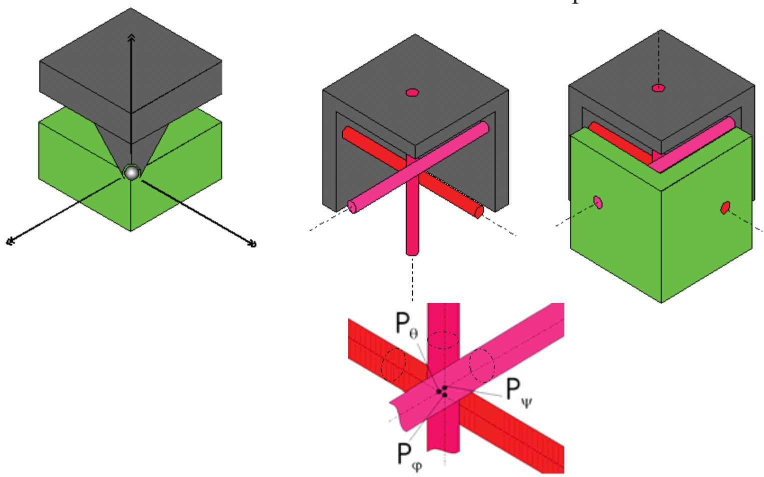


Universal joint function in flexures

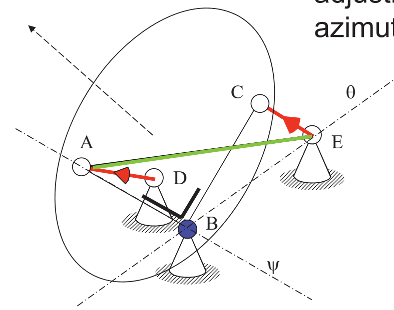


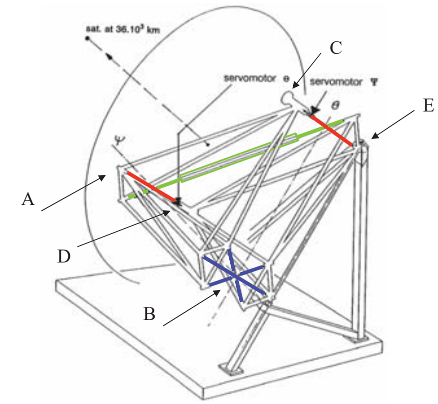
Elastic ball joint

No common pole…..

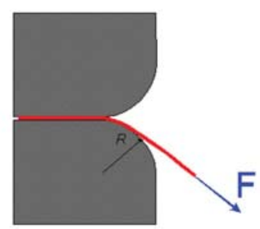


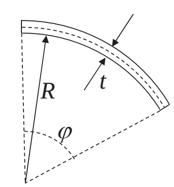
Flexure based dish antenna adjustment in meridional (θ) and azimuthal (ψ) directions





Clamping flexures with high tensile forces at varying angles



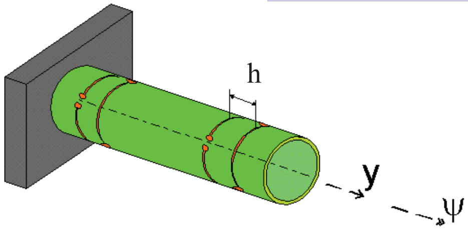


$$\varepsilon = \frac{\Delta l}{l} = \frac{\frac{1}{2}t\varphi}{\varphi R} = \frac{t}{2R}$$

$$\sigma = E\varepsilon = \frac{Et}{2R}$$

Flexures in tube walls: hollow tie rod

A high value for h gives a high *y*-stiffness



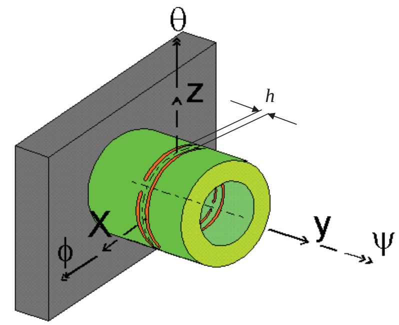
Equivalent to double cardan coupling

Concertina bush

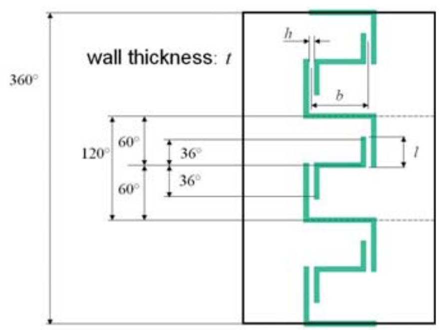
Compliance in z, θ and φ through slots in tube wall

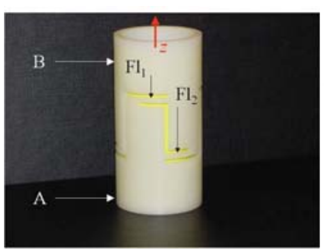
x, z, ψ : high stiffness

y, θ, φ : low stiffness

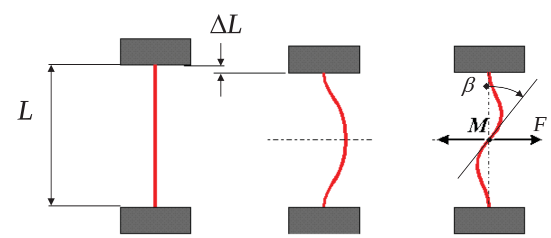


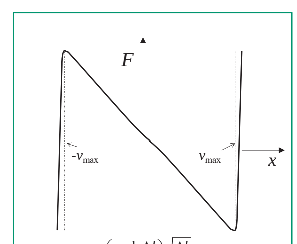
Straight guiding flexure mechanism in tube wall





Spring structures with negative spring stiffness (1/2)

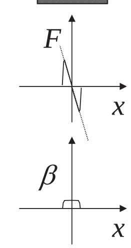




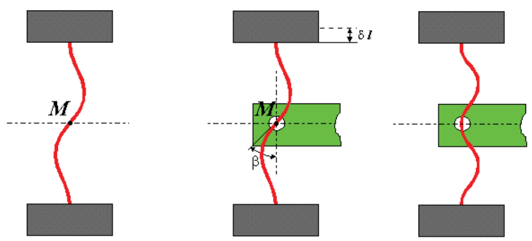
$v_{max} \approx \left(1-\frac{1}{4}\frac{\Delta l}{l}\right)\sqrt{\frac{\Delta l}{3l}}$

$$c = -\frac{210EI}{l^3}$$

$$\sigma_{b,max} = 8.9 \cdot Eh\sqrt{\frac{\Delta L}{L^3}}$$

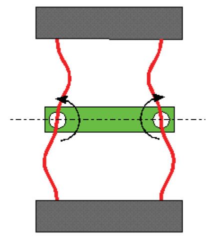


Spring structures with negative spring stiffness (2/2)

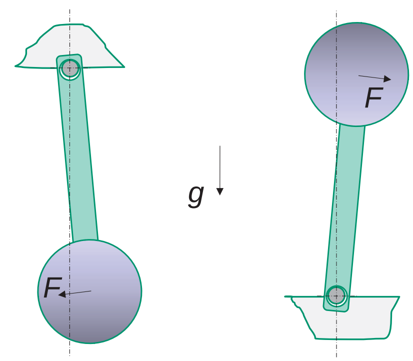


$$c = -\frac{576EI}{l^3}$$

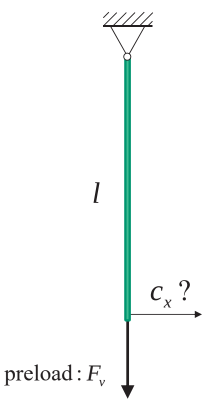
$$\sigma_{b,\max} = 12.7 \cdot Eh\sqrt{\frac{\Delta L}{L^3}}$$

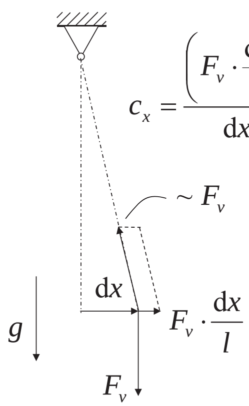


Pendulum and inverse-pendulum



Lateral stiffness <u>preloaded</u> tensile members

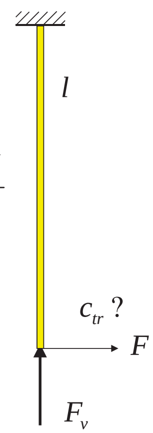




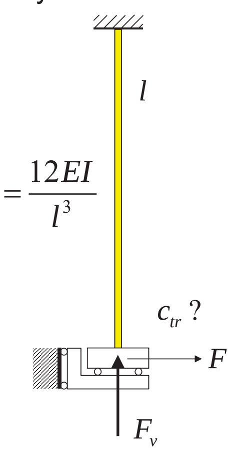
$$c_x = \frac{\left(F_v \cdot \frac{dx}{l}\right)}{dx} = \frac{F_v}{l}$$

Pendulum frequency $fe = \frac{1}{2\pi}\sqrt{\frac{c_x}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

Lateral stiffness of a compressively loaded flexure



$$c_b = \frac{3EI}{l^3}$$

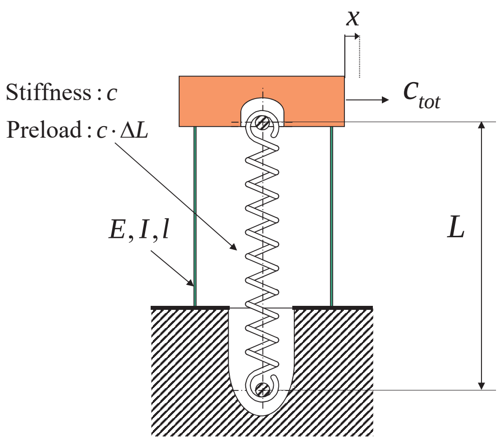


$$c_b = \frac{12EI}{l^3}$$

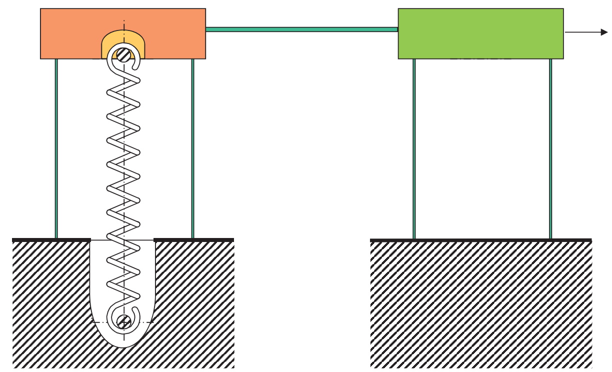
$$c_{tr} = c + c_b = -\frac{F_v}{l} + \frac{3EI}{l^3}$$

$$c_{tr} = c + c_b = -\frac{F_v}{l} + \frac{12EI}{l^3}$$

Flexure parallel guidance with preload for negative stiffness in *x*-direction



Flexure parallel guidance with preload for negative stiffness in *x*-direction and 1 DOF coupling to functional parallel guiding



End of chapter 3